Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.3-Miscellaneous/52-1.3.2-Algebraic-functions

Nasser M. Abbasi

September 28, 2022 Compiled on September 28, 2022 at 2:40am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1025]. This is test number [52].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.41 (1019)	0.59 (6)
Mathematica	97.76 (1002)	2.24 (23)
Maple	82.15 (842)	17.85 (183)
Fricas	76.98 (789)	23.02 (236)
Giac	49.95 (512)	50.05 (513)
Mupad	44.39 (455)	55.61 (570)
Maxima	37.56 (385)	62.44 (640)
Sympy	26.05 (267)	73.95 (758)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

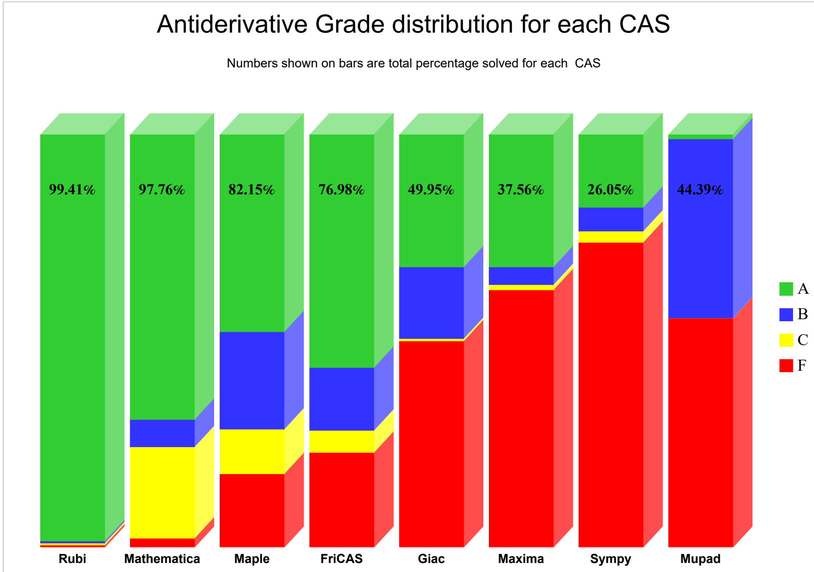
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

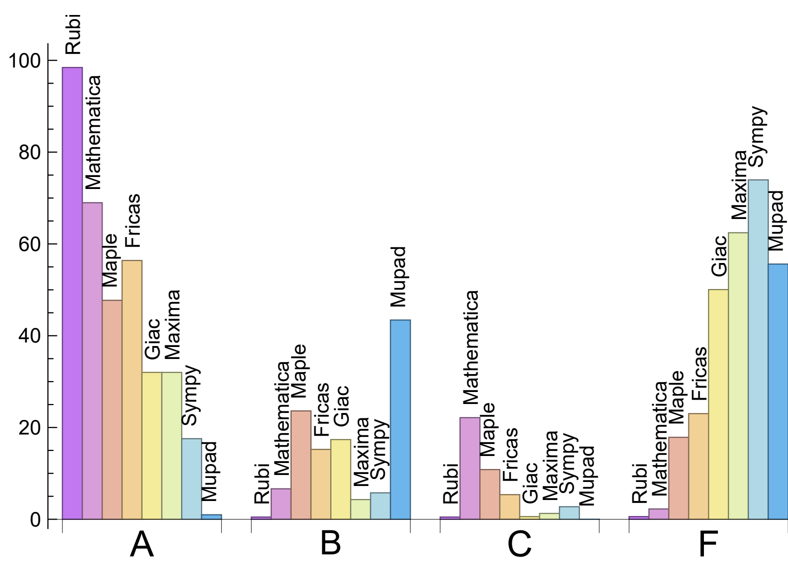
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.44	0.49	0.49	0.59
Mathematica	68.98	6.63	22.15	2.24
Fricas	56.39	15.22	5.37	23.02
Maple	47.71	23.61	10.83	17.85
Giac	32.00	17.37	0.59	50.05
Maxima	32.00	4.29	1.27	62.44
Sympy	17.56	5.76	2.73	73.95
Mupad	N/A	43.41	0.00	55.61

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	6	100.00 %	0.00 %	0.00 %
Mathematica	23	100.00 %	0.00 %	0.00 %
Maple	183	100.00 %	0.00 %	0.00 %
Fricas	236	42.80 %	29.66 %	27.54 %
Giac	513	75.44 %	9.55 %	15.01 %
Maxima	640	97.19 %	0.00 %	2.81 %
Sympy	758	87.86 %	11.21 %	0.92 %
Mupad	570	91.75 %	8.25 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

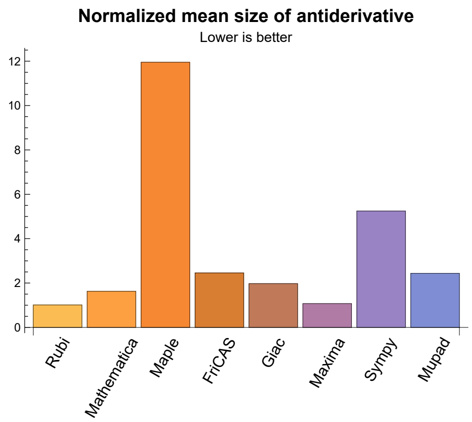
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

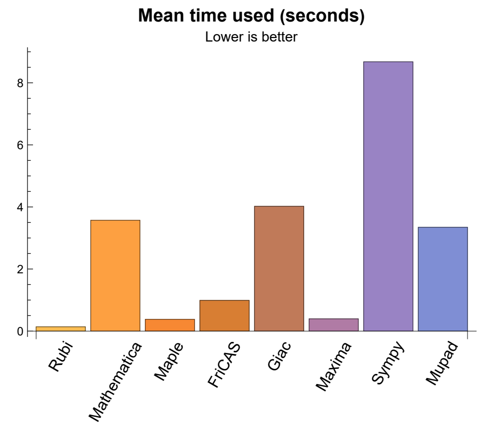
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	143.78	1.01	89.00	1.00
Mathematica	3.57	264.46	1.63	82.00	1.00
Maple	0.38	1472.94	11.95	94.00	1.25
Maxima	0.40	93.77	1.07	43.00	0.91
Fricas	0.99	252.12	2.46	75.00	1.34
Sympy	8.68	1124.17	5.24	60.00	1.00
Giac	4.02	227.53	1.97	58.50	1.18
Mupad	3.35	193.30	2.44	50.00	1.05

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
12
10
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
8
6
4
2
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{901, 902, 903, 904, 905, 906, 907, 908, 909, 910\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{166, 167, 196, 226\}$

Mathematica $\{1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 90, 96, 99, 107, 108, 111, 112, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 151, 164, 165, 167, 196, 561, 562, 764, 765, 766, 767, 768, 769, 770, 772, 773, 774, 775, 778, 780, 796, 797, 798, 799, 800, 801, 802, 803, 804, 885, 1010\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

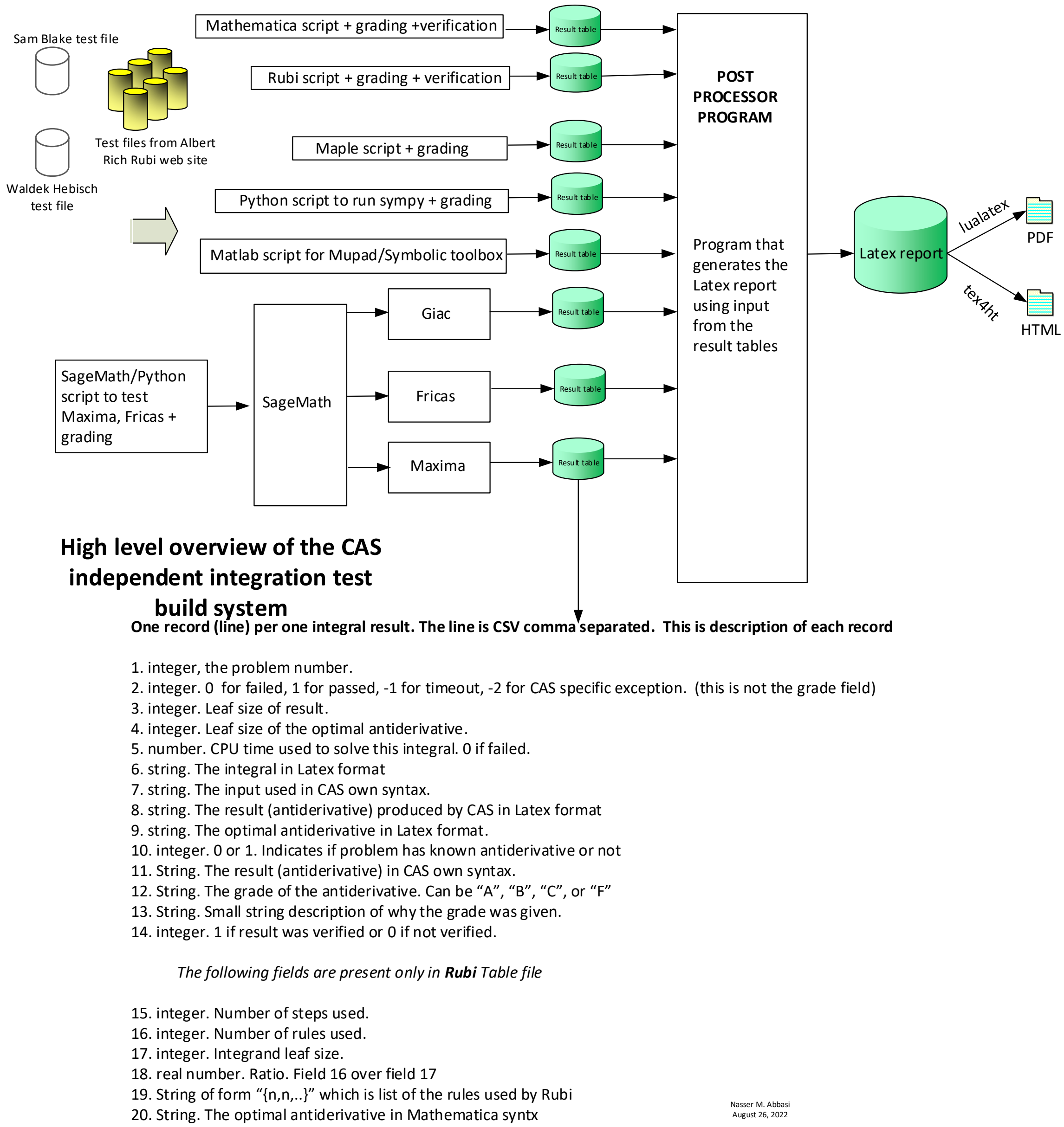
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1019, 1020, 1024 }

B grade: { 413, 726, 997, 1017, 1025 }

C grade: { 396, 941, 1018, 1021, 1022 }

F grade: { 197, 616, 617, 995, 996, 1023 }

2.1.2 Mathematica

A grade: { 23, 24, 25, 26, 29, 30, 31, 32, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 198, 199, 200, 201, 202, 203, 204, 205, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 261, 263, 264, 265, 266, 267, 268, 269, 272, 273, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 304, 305, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 327, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 347, 348, 349, 352, 353, 355, 356, 357, 358, 359, 360, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 388, 390, 393, 394, 395, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 424, 426, 427, 428, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 452, 453, 454, 455, 456, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 530, 531, 534, 535, 538, 539, 540, 541, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 602, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 733, 734, 735, 736, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 842, 848, 849, 850, 851, 854, 855, 856, 858, 859, 860, 862, 863, 865, 872, 874, 876, 877, 878, 879, 880, 881, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 920, 921, 922, 923, 924, 925, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 974, 975, 976, 977, 979, 980, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 1000, 1001, 1006, 1007, 1008, 1009, 1015, 1016, 1017, 1020, 1023, 1024, 1025 }

B grade: { 184, 423, 429, 448, 451, 457, 458, 459, 475, 476, 477, 479, 524, 562, 681, 682, 731, 732, 737, 738, 739, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 805, 843, 844, 845, 846, 847, 852, 853, 857, 861, 867, 868, 869, 870, 871, 873, 875, 918, 919, 926, 927, 961, 973, 978, 981, 1011, 1012, 1013, 1014 }

C grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 196, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 270, 271, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 306, 313, 314, 315, 316, 317, 325, 326, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 354, 361, 362, 363, 364, 365, 378, 380, 381, 383, 386, 387, 389, 391, 392, 397, 425, 450, 523, 525, 526, 527, 532, 533, 536, 537, 574, 580, 601, 603, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 796, 797, 798, 799, 800, 801, 802, 803, 804, 839, 840, 841, 864, 866, 882, 883, 884, 885, 913, 984, 998, 999, 1002, 1003, 1004, 1005, 1010, 1018, 1019, 1021, 1022 }

F grade: { 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 173, 195, 197, 528, 529, 542, 543, 558, 559, 587, 914 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 10, 11, 12, 13, 14, 15, 16, 17, 53, 54, 55, 57, 58, 59, 66, 68, 84, 85, 86, 93, 127, 128, 129, 130, 137, 138, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 176, 196, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 261, 270, 271, 272, 273, 274, 275, 285, 286, 287, 289, 290, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 340, 350, 351, 352, 353, 354, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 387, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 425, 426, 427, 428, 429, 430, 431, 438, 439, 440, 441, 444, 445, 446, 448, 450, 451, 455, 456, 474, 475, 530, 531, 538, 539, 565, 566, 567, 569, 570, 571, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 632, 633, 634, 636, 637, 638, 639, 640, 641, 643, 644, 645, 646, 647, 648, 649, 650, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 682, 685, 686, 687, 688, 689, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 727, 728, 733, 735, 736, 738, 741, 742, 743, 744, 748, 750, 752, 756, 757, 758, 805, 806, 807, 808, 809, 810, 813, 814, 815, 817, 818, 819, 820, 821, 822, 823, 824, 825, 828, 829, 830, 833, 834, 835, 836, 837, 839, 841, 842, 844, 845, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 858, 860, 864, 865, 868, 870, 872, 874, 877, 879, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 967, 969, 970, 971, 972, 975, 979, 980, 981, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1006, 1007, 1020, 1023 }

B grade: { 9, 52, 56, 64, 65, 67, 73, 83, 91, 92, 94, 95, 100, 125, 126, 135, 136, 139, 174, 175, 178, 179, 180, 182, 183, 184, 238, 260, 262, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 283, 284, 288, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 320, 321, 322, 323, 324, 331, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 355, 356, 357, 358, 359, 360, 361, 365, 408, 413, 422, 424, 432, 433, 442, 443, 447, 449, 452, 454, 457, 458, 459, 473, 476, 477, 478, 487, 523, 524, 525, 528, 529, 534, 535, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 568, 572, 601, 631, 635, 642, 651, 652, 681, 683, 684, 690, 697, 699, 700, 709, 726, 729, 730, 731, 732, 734, 737, 739, 740, 745, 746, 747, 749, 751, 753, 754, 755, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 811, 812, 816, 826, 827, 831, 832, 840, 843, 846, 857, 859, 861, 862, 863, 867, 869, 871, 873, 875, 878, 880, 881, 918, 919, 920, 921, 965, 966, 968, 973, 974, 976, 977, 978, 982, 983, 984, 997, 998, 1015, 1018 }

C grade: { 21, 22, 43, 44, 45, 46, 51, 74, 75, 76, 77, 82, 101, 102, 103, 104, 113, 114, 115, 116, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 226, 293, 378, 380, 381, 383, 386, 388, 389, 391, 406, 407, 409, 410, 434, 435, 436, 437, 468, 469, 470, 485, 486, 526, 527, 532, 533, 536, 537, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 796, 797, 798, 799, 800, 801, 802, 803, 804, 838, 866, 930, 954, 1002, 1003, 1004, 1005, 1009, 1010, 1016, 1017, 1019, 1021, 1022, 1024, 1025 }

F grade: { 5, 6, 7, 8, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 60, 61, 62, 63, 69, 70, 71, 72, 78, 79, 80, 81, 87, 88, 89, 90, 96, 97, 98, 99, 105, 106, 107, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 122, 123, 124, 131, 132, 133, 134, 140, 141, 142, 143, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 224, 225, 253, 254, 255, 256, 257, 258, 453, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 479, 480, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 562, 563, 564, 587, 588, 589, 590, 591, 592, 593, 594, 653, 654, 655, 656, 657, 876, 911, 912, 913, 914, 942, 995, 996, 999, 1000, 1001, 1008, 1011, 1012, 1013, 1014 }

2.1.4 Maxima

A grade: { 174, 175, 176, 179, 180, 227, 228, 229, 230, 231, 233, 234, 235, 236, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 290, 293, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 318, 319, 321, 322, 331, 332, 333, 334, 335, 336, 344, 345, 347, 348, 355, 356, 357, 358, 359, 370, 371, 372, 373, 374, 375, 376, 379, 384, 385, 396, 398, 399, 400, 419, 420, 421, 422, 423, 424, 425, 443, 444, 445, 446, 447, 448, 449, 450, 454, 455, 456, 530, 531, 538, 539, 544, 545, 546, 552, 553, 554, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 648, 649, 654, 655, 656, 670, 676, 677, 679, 680, 681, 683, 684, 685, 686, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 724, 725, 733, 734, 735, 737, 738, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 762, 763, 805, 808, 809, 810, 814, 815, 816, 817, 818, 819, 820, 822, 824, 825, 833, 835, 836, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 872, 874, 883, 884, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 935, 936, 937, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 952, 953, 954, 959, 960, 961, 963, 964, 971, 972, 975, 977, 979, 980, 981, 982, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1007, 1020, 1023 }

B grade: { 178, 182, 183, 184, 232, 238, 320, 323, 324, 337, 346, 349, 360, 369, 564, 566, 574, 580, 612, 613, 614, 615, 616, 617, 645, 653, 675, 678, 729, 730, 731, 732, 736, 746, 750, 829, 880, 948, 957, 958, 974, 976, 978, 1015 }

C grade: { 581, 582, 583, 584, 585, 586, 595, 596, 597, 598, 599, 600, 855 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 237, 239, 240, 241, 242, 243, 245, 255, 256, 257, 258, 259, 260, 261, 262, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 295, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 366, 367, 368, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 532, 533, 534, 535, 536, 537, 540, 541, 542, 543, 547, 548, 549, 550, 551, 555, 556, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 601, 629, 630, 631, 650, 651, 652, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 682, 687, 688, 689, 690, 691, 692, 701, 702, 703, 704, 705, 706, 710, 720, 721, 723, 726, 727, 728, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 806, 807, 811, 812, 813, 821, 823, 826, 827, 828, 830, 831, 832, 834, 837, 838, 850, 851, 853, 863, 869, 871, 873, 875, 876, 877, 878, 879, 881, 882, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 918, 919, 920, 921, 930, 931, 932, 933, 934, 938, 939, 940, 955, 956, 962, 965, 966, 967, 968, 969, 970, 973, 983, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.5 FriCAS

A grade: { 2, 11, 13, 53, 57, 66, 77, 84, 86, 93, 95, 103, 104, 105, 107, 109, 111, 113, 114, 117, 119, 121, 123, 128, 130, 136, 138, 153, 155, 157, 159, 169, 171, 198, 199, 200, 201, 202, 203, 204, 208, 209, 214, 215, 221, 222, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 276, 277, 278, 279, 280, 281, 282, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 307, 308, 309, 311, 312, 318, 319, 320, 323, 324, 331, 332, 333, 334, 335, 336, 337, 344, 345, 346, 349, 355, 356, 359, 366, 369, 370, 371, 372, 373, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 454, 455, 456, 457, 460, 461, 462, 463, 464, 466, 467, 468, 469, 470, 471, 473, 474, 475, 476, 479, 480, 481, 482, 485, 486, 487, 490, 491, 492, 495, 496, 497, 498, 501, 502, 503, 504, 507, 508, 509, 512, 514, 515, 516, 518, 519, 520, 522, 524, 525, 528, 529, 530, 531, 534, 535, 536, 538, 539, 540, 541, 542, 543, 544, 547, 548, 549, 551, 552, 553, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 646, 647, 648, 649, 656, 658, 659, 660, 661, 662, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 685, 686, 691, 692, 693, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 712, 713, 714, 715, 716, 717, 718, 719, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 751, 752, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 769, 770, 805, 806, 807, 808, 809, 810, 811, 812, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 835, 836, 837, 838, 848, 849, 850, 851, 852, 853, 854, 856, 858, 860, 862, 867, 872, 874, 877, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 915, 916, 917, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 960, 961, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 975, 977, 979, 980, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 1004, 1005, 1007, 1009, 1010, 1011, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025 }

B grade: { 21, 22, 43, 44, 45, 46, 51, 55, 68, 74, 75, 76, 78, 79, 80, 81, 82, 101, 102, 106, 108, 110, 112, 115, 116, 118, 120, 122, 124, 174, 175, 176, 178, 179, 180, 182, 183, 184, 205, 232, 238, 310, 321, 322, 347, 348, 357, 358, 360, 367, 368, 374, 375, 388, 408, 413, 422, 447, 458, 459, 465, 477, 478, 483, 484, 526, 527, 532, 533, 537, 545, 546, 550, 554, 566, 574, 580, 598, 629, 630, 631, 644, 645, 650, 651, 652, 653, 654, 655, 663, 681, 682, 683, 684, 687, 688, 689, 690, 694, 710, 720, 721, 726, 729, 730, 732, 750, 753, 759, 813, 832, 833, 834, 839, 840, 841, 842, 843, 844, 845, 846, 847, 859, 861, 863, 864, 865, 866, 868, 869, 870, 871, 873, 875, 878, 879, 918, 919, 935, 957, 958, 959, 968, 973, 974, 976, 978, 981, 996, 997, 998, 999, 1006, 1015, 1016, 1017 }

C grade: { 1, 9, 10, 12, 52, 56, 58, 64, 65, 67, 73, 83, 85, 91, 92, 94, 100, 125, 126, 127, 129, 131, 133, 134, 135, 137, 139, 140, 141, 142, 143, 152, 154, 156, 158, 168, 170, 378, 380, 381, 383, 387, 389, 391, 392, 397, 601, 766, 767, 768, 771, 772, 773, 855, 857 }

F grade: { 3, 4, 5, 6, 7, 8, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 47, 48, 49, 50, 54, 59, 60, 61, 62, 63, 69, 70, 71, 72, 87, 88, 89, 90, 96, 97, 98, 99, 132, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 210, 211, 212, 213, 216, 217, 218, 219, 220, 223, 224, 225, 226, 253, 254, 255, 256, 257, 258, 270, 271, 272, 273, 274, 275, 283, 284, 285, 286, 287, 288, 302, 303, 304, 305, 306, 313, 314, 315, 316, 317, 325, 326, 327, 328, 329, 330, 338, 339, 340, 341, 342, 343, 350, 351, 352, 353, 354, 361, 362, 363, 364, 365, 386, 393, 394, 395, 396, 453, 472, 488, 489, 493, 494, 499, 500, 505, 506, 510, 511, 513, 517, 521, 523, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 590, 591, 592, 593, 594, 613, 614, 615, 616, 617, 657, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 876, 913, 914, 930, 1000, 1001, 1002, 1003, 1008 }

2.1.6 Sympy

A grade: { 23, 24, 25, 29, 30, 31, 36, 37, 38, 152, 153, 154, 155, 156, 157, 158, 159, 168, 169, 170, 171, 206, 212, 249, 250, 251, 252, 379, 388, 420, 421, 422, 445, 446, 447, 452, 454, 455, 456, 524, 525, 527, 531, 537, 539, 545, 546, 547, 553, 554, 555, 564, 565, 567, 569, 570, 571, 575, 576, 602, 606, 607, 608, 609, 610, 612, 618, 619, 620, 621, 622, 634, 635, 636, 641, 643, 658, 659, 662, 663, 664, 665, 668, 669, 670, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 686, 687, 688, 689, 690, 693, 694, 696, 697, 698, 699, 700, 707, 708, 709, 711, 722, 724, 725, 726, 735, 806, 808, 809, 810, 811, 812, 814, 815, 817, 818, 819, 820, 833, 835, 836, 839, 841, 848, 849, 850, 852, 854, 856, 858, 860, 868, 870, 872, 874, 878, 879, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 916, 917, 924, 928, 936, 937, 939, 943, 944, 945, 946, 948, 949, 950, 952, 953, 954, 966, 977, 984, 1007, 1015, 1020 }

B grade: { 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 197, 244, 245, 403, 408, 412, 413, 416, 417, 418, 443, 469, 470, 486, 487, 498, 504, 526, 530, 532, 533, 536, 538, 566, 603, 604, 605, 611, 623, 624, 642, 685, 695, 714, 762, 763, 829, 837, 877, 933, 957, 958, 959, 963, 964, 967, 1023 }

C grade: { 26, 32, 39, 207, 208, 209, 213, 214, 215, 221, 222, 380, 468, 588, 589, 590, 729, 731, 840, 843, 846, 862, 941, 942, 951, 965, 971, 972 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 27, 28, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 160, 161, 162, 163, 164, 165, 166, 167, 172, 173, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 210, 211, 216, 217, 218, 219, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 246, 247, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 381, 382, 383, 384, 385, 386, 387, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 404, 405, 406, 407, 409, 410, 411, 414, 415, 419, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 444, 448, 449, 450, 451, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 528, 529, 534, 535, 540, 541, 542, 543, 544, 548, 549, 550, 551, 552, 556, 557, 558, 559, 560, 561, 562, 563, 568, 572, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 613, 614, 615, 616, 617, 625, 626, 627, 628, 629, 630, 631, 632, 633, 637, 638, 639, 640, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 660, 661, 666, 667, 671, 672, 673, 674, 691, 692, 701, 702, 703, 704, 705, 706, 710, 712, 713, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 730, 732, 733, 734, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 807, 813, 816, 821, 822, 823, 824, 825, 826, 827, 828, 830, 831, 832, 834, 838, 842, 844, 845, 847, 851, 853, 855, 857, 859, 861, 863, 864, 865, 866, 867, 869, 871, 873, 875, 876, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 911, 912, 913, 914, 915, 918, 919, 920, 921, 922, 923, 925, 926, 927, 929, 930, 931, 932, 934, 935, 938, 940, 947, 955, 956, 960, 961, 962, 968, 969, 970, 973, 974, 975, 976, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1021, 1022, 1024, 1025 }

2.1.7 Giac

A grade: { 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 263, 264, 265, 276, 277, 289, 290, 291, 293, 295, 296, 297, 298, 300, 318, 319, 344, 345, 367, 369, 371, 372, 373, 374, 375, 376, 377, 379, 382, 384, 385, 390, 398, 399, 400, 403, 409, 416, 417, 418, 452, 454, 455, 456, 473, 474, 475, 498, 504, 530, 531, 538, 539, 544, 545, 546, 547, 548, 550, 551, 552, 553, 554, 555, 556, 564, 565, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 602, 603, 604, 605, 607, 608, 609, 610, 618, 619, 620, 622, 623, 624, 629, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 670, 675, 676, 677, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 716, 718, 720, 721, 722, 723, 724, 725, 727, 728, 731, 733, 734, 735, 736, 737, 738, 739, 741, 742, 743, 744, 745, 748, 749, 755, 756, 762, 763, 806, 807, 808, 809, 810, 814, 816, 817, 818, 819, 820, 821, 822, 823, 830, 833, 834, 835, 836, 837, 838, 840, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 872, 874, 877, 882, 883, 891, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 915, 916, 917, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 959, 960, 961, 962, 963, 964, 965, 966, 967, 970, 971, 972, 973, 974, 975, 976, 977, 979, 980, 982, 984, 1007, 1020 }

B grade: { 174, 175, 176, 178, 179, 180, 183, 184, 227, 261, 267, 268, 269, 278, 301, 320, 322, 323, 324, 331, 332, 333, 346, 348, 349, 355, 356, 357, 368, 388, 401, 402, 404, 405, 406, 407, 408, 410, 411, 412, 413, 414, 415, 419, 420, 421, 422, 424, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 449, 457, 458, 459, 476, 477, 478, 526, 527, 532, 533, 536, 537, 566, 582, 583, 584, 585, 586, 606, 621, 625, 626, 627, 628, 630, 631, 651, 652, 653, 654, 655, 656, 678, 679, 680, 714, 715, 717, 719, 726, 729, 730, 732, 740, 746, 747, 750, 751, 753, 754, 757, 758, 759, 760, 761, 811, 812, 813, 815, 826, 827, 828, 831, 832, 839, 841, 842, 843, 844, 845, 846, 847, 867, 868, 870, 878, 879, 880, 881, 918, 919, 955, 956, 957, 958, 968, 969, 978, 981, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 998, 1006, 1015, 1021, 1022, 1023 }

C grade: { 294, 596, 600, 824, 825, 892 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 181, 182, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 255, 256, 257, 258, 260, 262, 266, 270, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 292, 299, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 321, 325, 326, 327, 328, 329, 330, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 347, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 365, 366, 370, 378, 380, 381, 383, 386, 387, 389, 391, 392, 393, 394, 395, 396, 397, 423, 425, 448, 450, 451, 453, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 528, 529, 534, 535, 540, 541, 542, 543, 549, 557, 558, 559, 560, 561, 562, 563, 581, 587, 588, 589, 590, 591, 592, 593, 594, 595, 597, 598, 599, 601, 611, 612, 613, 614, 615, 616, 617, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 752, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 829, 863, 864, 866, 869, 871, 873, 875, 876, 884, 885, 886, 887, 888, 889, 890, 911, 912, 913, 914, 920, 921, 941, 942, 995, 996, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

2.1.8 Mupad

A grade: { 901, 902, 903, 904, 905, 906, 907, 908, 909, 910 }

B grade: { 14, 15, 16, 17, 43, 44, 45, 46, 47, 48, 49, 50, 51, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 93, 94, 95, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 175, 176, 178, 179, 180, 182, 183, 184, 198, 199, 200, 201, 202, 203, 204, 205, 209, 215, 221, 222, 227, 230, 232, 238, 244, 245, 247, 249, 250, 251, 252, 289, 290, 291, 292, 293, 320, 333, 346, 357, 369, 370, 379, 384, 385, 396, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 455, 456, 498, 504, 516, 518, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 536, 537, 538, 539, 544, 545, 546, 547, 548, 552, 553, 554, 555, 556, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 590, 595, 596, 597, 599, 600, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 628, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 649, 656, 670, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 707, 708, 709, 711, 712, 722, 724, 725, 726, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 750, 751, 762, 763, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 824, 825, 826, 827, 828, 830, 831, 833, 835, 836, 837, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 854, 855, 856, 857, 858, 859, 860, 861, 862, 865, 867, 868, 870, 872, 874, 876, 877, 878, 879, 880, 881, 884, 885, 886, 887, 888, 889, 890, 900, 915, 916, 917, 918, 919, 922, 923, 924, 925, 928, 930, 931, 933, 935, 936, 937, 939, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 957, 958, 959, 960, 963, 964, 965, 966, 967, 968, 969, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 997, 1007, 1015, 1020, 1021, 1022, 1023 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 87, 88, 89, 90, 91, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 172, 173, 177, 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 206, 207, 208, 210, 211, 212, 213, 214, 216, 217, 218, 219, 220, 223, 224, 225, 226, 228, 229, 231, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 246, 248, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 371, 372, 373, 374, 375, 376, 377, 378, 380, 381, 382, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 453, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 520, 521, 522, 523, 534, 535, 540, 541, 542, 543, 549, 550, 551, 557, 558, 559, 560, 561, 562, 563, 587, 588, 589, 591, 592, 593, 594, 598, 601, 613, 614, 625, 626, 627, 629, 630, 631, 646, 647, 648, 650, 651, 652, 653, 654, 655, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 701, 702, 703, 704, 705, 706, 710, 713, 714, 715, 716, 717, 718, 719, 720, 721, 723, 727, 728, 749, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 817, 818, 819, 820, 821, 822, 823, 829, 832, 834, 838, 850, 851, 852, 853, 863, 864, 866, 869, 871, 873, 875, 882, 883, 891, 892, 893, 894, 895, 896, 897, 898, 899, 911, 912, 913, 914, 920, 921, 926, 927, 929, 932, 934, 938, 940, 955, 956, 961, 962, 970, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1024, 1025 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	148	139	0	74	0	0	-1
N.S.	1	1.00	1.02	0.96	0.00	0.51	0.00	0.00	-0.01
time (sec)	N/A	0.106	20.219	1.378	0.000	0.129	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	148	143	0	69	0	0	-1
N.S.	1	1.00	0.92	0.89	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.134	20.101	1.286	0.000	0.167	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	146	143	0	0	0	0	-1
N.S.	1	1.00	0.90	0.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	20.105	1.441	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	150	139	0	0	0	0	-1
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	20.097	1.259	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	164	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	10.154	0.044	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	166	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	10.156	0.049	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	167	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	10.095	0.046	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	167	0	0	0	0	0	-1
N.S.	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	10.104	0.043	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	169	495	0	349	0	0	-1
N.S.	1	1.00	0.68	1.99	0.00	1.40	0.00	0.00	-0.00
time (sec)	N/A	0.177	10.161	0.397	0.000	0.225	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	136	132	0	57	0	0	-1
N.S.	1	1.00	0.93	0.90	0.00	0.39	0.00	0.00	-0.01
time (sec)	N/A	0.138	20.153	0.892	0.000	0.175	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	136	143	0	56	0	0	-1
N.S.	1	1.00	0.83	0.87	0.00	0.34	0.00	0.00	-0.01
time (sec)	N/A	0.123	20.113	0.841	0.000	0.254	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	134	132	0	211	0	0	-1
N.S.	1	1.00	0.80	0.79	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.121	20.128	0.886	0.000	0.160	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	138	139	0	203	0	0	-1
N.S.	1	1.00	0.88	0.89	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.112	20.099	0.794	0.000	0.126	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	128	123	0	0	0	0	164
N.S.	1	1.00	0.39	0.37	0.00	0.00	0.00	0.00	0.50
time (sec)	N/A	0.458	20.076	0.406	0.000	0.000	0.000	0.000	0.216

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	128	133	0	0	0	0	180
N.S.	1	1.00	0.34	0.35	0.00	0.00	0.00	0.00	0.47
time (sec)	N/A	0.487	20.084	0.341	0.000	0.000	0.000	0.000	2.420

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	126	124	0	0	0	0	164
N.S.	1	1.00	0.34	0.33	0.00	0.00	0.00	0.00	0.44
time (sec)	N/A	0.390	20.064	0.370	0.000	0.000	0.000	0.000	0.050

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	130	133	0	0	0	0	179
N.S.	1	1.00	0.38	0.39	0.00	0.00	0.00	0.00	0.53
time (sec)	N/A	0.381	20.072	0.451	0.000	0.000	0.000	0.000	0.046

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	311	0	0	0	0	0	-1
N.S.	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	1.920	0.044	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	7.225	0.041	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	7.584	0.039	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	0	3064	0	712	0	0	-1
N.S.	1	1.00	0.00	20.84	0.00	4.84	0.00	0.00	-0.01
time (sec)	N/A	0.036	6.331	27.584	0.000	2.615	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	B	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	0	3250	0	720	0	0	-1
N.S.	1	1.00	0.00	20.44	0.00	4.53	0.00	0.00	-0.01
time (sec)	N/A	0.042	6.687	27.102	0.000	2.587	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	163	0	0	0	212	0	-1
N.S.	1	1.00	0.42	0.00	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.177	7.038	0.007	0.000	0.000	2.984	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	142	0	0	0	160	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.66	0.00	-0.00
time (sec)	N/A	0.170	6.911	0.006	0.000	0.000	1.960	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	111	0	0	0	114	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.59	0.00	-0.01
time (sec)	N/A	0.100	6.855	0.006	0.000	0.000	1.587	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	75	0	0	0	82	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.53	0.00	-0.01
time (sec)	N/A	0.065	6.204	0.004	0.000	0.000	1.322	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	17.019	0.005	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.529	17.820	0.043	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	392	0	0	0	206	0	-1
N.S.	1	1.00	1.26	0.00	0.00	0.00	0.66	0.00	-0.00
time (sec)	N/A	0.115	10.349	0.004	0.000	0.000	2.590	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	287	0	0	0	155	0	-1
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.61	0.00	-0.00
time (sec)	N/A	0.095	10.281	0.006	0.000	0.000	2.205	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	201	0	0	0	110	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.75	0.00	-0.01
time (sec)	N/A	0.070	10.129	0.006	0.000	0.000	1.583	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	163	0	0	0	78	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.63	0.00	-0.01
time (sec)	N/A	0.044	10.094	0.022	0.000	0.000	1.075	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	7.029	0.040	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	761	761	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	20.248	0.043	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1513	1513	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.324	20.350	0.041	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	166	0	0	0	204	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.67	0.00	-0.00
time (sec)	N/A	0.121	10.125	0.006	0.000	0.000	2.728	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	145	0	0	0	153	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.82	0.00	-0.01
time (sec)	N/A	0.125	10.083	0.007	0.000	0.000	2.123	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	95	0	0	0	109	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.77	0.00	-0.01
time (sec)	N/A	0.068	10.038	0.006	0.000	0.000	1.614	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	78	0	0	0	78	0	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.64	0.00	-0.01
time (sec)	N/A	0.043	10.039	0.023	0.000	0.000	1.126	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	7.330	0.040	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	760	760	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	20.223	0.040	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1357	1357	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.179	20.275	0.040	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	258	0	75	0	0	70
N.S.	1	1.00	1.05	6.97	0.00	2.03	0.00	0.00	1.89
time (sec)	N/A	0.069	1.249	1.329	0.000	0.439	0.000	0.000	3.593

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	41	253	0	76	0	0	74
N.S.	1	1.00	1.02	6.32	0.00	1.90	0.00	0.00	1.85
time (sec)	N/A	0.076	1.284	1.302	0.000	0.437	0.000	0.000	3.631

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	262	0	238	0	0	62
N.S.	1	1.00	1.03	6.89	0.00	6.26	0.00	0.00	1.63
time (sec)	N/A	0.071	1.262	1.362	0.000	0.431	0.000	0.000	2.861

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	249	0	241	0	0	63
N.S.	1	1.00	1.05	6.38	0.00	6.18	0.00	0.00	1.62
time (sec)	N/A	0.073	1.245	1.320	0.000	0.408	0.000	0.000	2.843

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	106
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.68
time (sec)	N/A	0.124	5.138	0.212	0.000	0.000	0.000	0.000	5.812

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	0	0	0	0	0	107
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.65
time (sec)	N/A	0.134	5.120	0.214	0.000	0.000	0.000	0.000	5.851

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	102
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.55
time (sec)	N/A	0.138	5.129	0.054	0.000	0.000	0.000	0.000	3.677

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	0	0	0	0	0	103
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	1.56
time (sec)	N/A	0.128	5.134	0.056	0.000	0.000	0.000	0.000	3.647

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	889	0	300	0	0	95
N.S.	1	1.00	1.04	18.14	0.00	6.12	0.00	0.00	1.94
time (sec)	N/A	0.079	1.021	0.248	0.000	0.445	0.000	0.000	4.731

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	336	262	0	157	0	0	-1
N.S.	1	1.00	2.13	1.66	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.141	20.305	1.181	0.000	0.193	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	335	257	0	146	0	0	-1
N.S.	1	1.00	1.94	1.49	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.164	20.307	2.011	0.000	0.169	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	333	266	0	0	0	0	-1
N.S.	1	1.00	1.89	1.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.154	20.256	1.976	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	338	253	0	415	0	0	-1
N.S.	1	1.00	2.00	1.50	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.152	20.198	1.938	0.000	0.182	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	340	264	0	949	0	0	-1
N.S.	1	1.00	2.14	1.66	0.00	5.97	0.00	0.00	-0.01
time (sec)	N/A	0.157	20.312	0.294	0.000	0.259	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	340	261	0	917	0	0	-1
N.S.	1	1.00	1.94	1.49	0.00	5.24	0.00	0.00	-0.01
time (sec)	N/A	0.183	20.307	0.264	0.000	0.297	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	338	270	0	947	0	0	-1
N.S.	1	1.00	1.90	1.52	0.00	5.32	0.00	0.00	-0.01
time (sec)	N/A	0.159	20.224	0.296	0.000	0.231	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	342	255	0	0	0	0	-1
N.S.	1	1.00	2.01	1.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.168	20.226	0.243	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	336	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	11.066	0.046	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	399	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	10.817	0.055	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	400	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.298	10.270	0.047	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	387	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	10.249	0.051	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	380	900	0	393	0	0	-1
N.S.	1	1.00	1.43	3.40	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.216	11.001	0.258	0.000	0.346	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	207	258	0	82	0	0	-1
N.S.	1	1.00	1.43	1.78	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.143	10.252	1.236	0.000	0.133	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	209	253	0	76	0	0	-1
N.S.	1	1.00	1.31	1.58	0.00	0.48	0.00	0.00	-0.01
time (sec)	N/A	0.159	10.245	1.187	0.000	0.187	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	207	262	0	245	0	0	-1
N.S.	1	1.00	1.27	1.61	0.00	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.155	10.180	1.204	0.000	0.132	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	249	0	241	0	0	-1
N.S.	1	1.00	1.34	1.60	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.154	10.230	1.091	0.000	0.136	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	324	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	10.851	0.049	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	388	0	0	0	0	0	-1
N.S.	1	1.00	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	10.595	0.046	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	389	0	0	0	0	0	-1
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	10.216	0.047	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	375	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	10.182	0.046	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	372	892	0	350	0	0	-1
N.S.	1	1.00	1.51	3.63	0.00	1.42	0.00	0.00	-0.00
time (sec)	N/A	0.200	10.789	0.260	0.000	0.174	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	31	240	0	44	0	0	205
N.S.	1	1.00	1.35	10.43	0.00	1.91	0.00	0.00	8.91
time (sec)	N/A	0.040	0.656	0.383	0.000	0.374	0.000	0.000	0.225

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	240	0	47	0	0	221
N.S.	1	1.00	1.22	8.89	0.00	1.74	0.00	0.00	8.19
time (sec)	N/A	0.043	0.687	0.359	0.000	0.383	0.000	0.000	0.184

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	21	240	0	40	0	0	205
N.S.	1	1.00	0.84	9.60	0.00	1.60	0.00	0.00	8.20
time (sec)	N/A	0.039	0.662	0.459	0.000	0.362	0.000	0.000	2.551

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	240	0	38	0	0	221
N.S.	1	1.00	0.92	9.60	0.00	1.52	0.00	0.00	8.84
time (sec)	N/A	0.043	0.668	0.420	0.000	0.356	0.000	0.000	2.529

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	48	0	0	587	0	0	65
N.S.	1	1.00	0.96	0.00	0.00	11.74	0.00	0.00	1.30
time (sec)	N/A	0.090	3.695	0.054	0.000	0.834	0.000	0.000	3.444

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	0	0	630	0	0	67
N.S.	1	1.00	0.96	0.00	0.00	12.12	0.00	0.00	1.29
time (sec)	N/A	0.091	3.608	0.048	0.000	0.846	0.000	0.000	3.591

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	592	0	0	74
N.S.	1	1.00	0.96	0.00	0.00	11.17	0.00	0.00	1.40
time (sec)	N/A	0.095	3.662	0.049	0.000	0.810	0.000	0.000	5.453

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	0	0	641	0	0	78
N.S.	1	1.00	0.96	0.00	0.00	12.09	0.00	0.00	1.47
time (sec)	N/A	0.097	3.655	0.048	0.000	0.833	0.000	0.000	5.373

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	503	0	294	0	0	67
N.S.	1	1.00	0.96	10.93	0.00	6.39	0.00	0.00	1.46
time (sec)	N/A	0.078	1.382	0.381	0.000	0.441	0.000	0.000	3.126

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	273	246	0	63	0	0	327
N.S.	1	1.00	1.96	1.77	0.00	0.45	0.00	0.00	2.35
time (sec)	N/A	0.104	20.215	0.253	0.000	0.110	0.000	0.000	2.711

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	271	246	0	55	0	0	359
N.S.	1	1.00	1.77	1.61	0.00	0.36	0.00	0.00	2.35
time (sec)	N/A	0.112	20.206	0.253	0.000	0.111	0.000	0.000	0.190

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	269	246	0	59	0	0	327
N.S.	1	1.00	1.72	1.58	0.00	0.38	0.00	0.00	2.10
time (sec)	N/A	0.106	20.129	0.280	0.000	0.109	0.000	0.000	0.120

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	275	246	0	44	0	0	359
N.S.	1	1.00	1.83	1.64	0.00	0.29	0.00	0.00	2.39
time (sec)	N/A	0.115	20.147	0.250	0.000	0.122	0.000	0.000	2.543

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	419	0	0	0	0	0	-1
N.S.	1	1.00	1.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.234	10.906	0.049	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	447	0	0	0	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	10.871	0.049	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	448	0	0	0	0	0	-1
N.S.	1	1.00	1.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	10.724	0.049	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	422	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	10.211	0.047	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	384	521	0	400	0	0	-1
N.S.	1	1.00	1.74	2.36	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.205	10.799	0.253	0.000	0.210	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	193	240	0	51	0	0	207
N.S.	1	1.00	1.50	1.86	0.00	0.40	0.00	0.00	1.60
time (sec)	N/A	0.098	10.204	0.264	0.000	0.095	0.000	0.000	0.057

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	195	240	0	47	0	0	224
N.S.	1	1.00	1.34	1.66	0.00	0.32	0.00	0.00	1.54
time (sec)	N/A	0.102	10.190	0.248	0.000	0.105	0.000	0.000	0.066

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	193	240	0	47	0	0	208
N.S.	1	1.00	1.30	1.62	0.00	0.32	0.00	0.00	1.41
time (sec)	N/A	0.097	10.136	0.296	0.000	0.109	0.000	0.000	2.534

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	195	240	0	38	0	0	223
N.S.	1	1.00	1.39	1.71	0.00	0.27	0.00	0.00	1.59
time (sec)	N/A	0.107	10.123	0.247	0.000	0.116	0.000	0.000	0.064

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	407	0	0	0	0	0	-1
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	11.019	0.047	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	371	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.202	10.532	0.047	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	372	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	10.173	0.051	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	410	0	0	0	0	0	-1
N.S.	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	10.186	0.048	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	295	509	0	356	0	0	-1
N.S.	1	1.00	1.46	2.52	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.186	10.469	0.250	0.000	0.142	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	245	0	205	0	0	-1
N.S.	1	1.00	1.17	5.83	0.00	4.88	0.00	0.00	-0.02
time (sec)	N/A	0.079	1.648	0.972	0.000	0.478	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	243	0	207	0	0	-1
N.S.	1	1.00	1.07	5.28	0.00	4.50	0.00	0.00	-0.02
time (sec)	N/A	0.077	1.627	0.886	0.000	0.436	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	245	0	50	0	0	-1
N.S.	1	1.00	1.07	5.57	0.00	1.14	0.00	0.00	-0.02
time (sec)	N/A	0.069	1.617	0.958	0.000	0.367	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	247	0	59	0	0	-1
N.S.	1	1.00	1.16	5.61	0.00	1.34	0.00	0.00	-0.02
time (sec)	N/A	0.065	1.631	0.896	0.000	0.379	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1240	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	17.97	0.00	0.00	-0.01
time (sec)	N/A	0.148	7.180	0.051	0.000	1.154	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1294	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	18.23	0.00	0.00	-0.01
time (sec)	N/A	0.133	7.236	0.048	0.000	1.065	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1245	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	17.29	0.00	0.00	-0.01
time (sec)	N/A	0.129	7.192	0.048	0.000	1.079	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1303	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	18.10	0.00	0.00	-0.01
time (sec)	N/A	0.117	7.143	0.052	0.000	1.092	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	82	0	0	1273	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	17.44	0.00	0.00	-0.01
time (sec)	N/A	0.141	7.421	0.044	0.000	0.779	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	84	0	0	1330	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	17.73	0.00	0.00	-0.01
time (sec)	N/A	0.140	7.349	0.052	0.000	0.752	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	85	0	0	1278	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	16.82	0.00	0.00	-0.01
time (sec)	N/A	0.136	7.215	0.044	0.000	0.770	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	85	0	0	1339	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	17.62	0.00	0.00	-0.01
time (sec)	N/A	0.124	7.042	0.045	0.000	0.783	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	49	245	0	50	0	0	-1
N.S.	1	1.00	1.17	5.83	0.00	1.19	0.00	0.00	-0.02
time (sec)	N/A	0.069	1.581	0.792	0.000	0.383	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	247	0	59	0	0	-1
N.S.	1	1.00	1.07	5.37	0.00	1.28	0.00	0.00	-0.02
time (sec)	N/A	0.072	1.537	0.753	0.000	0.372	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	245	0	204	0	0	-1
N.S.	1	1.00	1.07	5.57	0.00	4.64	0.00	0.00	-0.02
time (sec)	N/A	0.068	1.510	0.810	0.000	0.381	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	243	0	206	0	0	-1
N.S.	1	1.00	1.16	5.52	0.00	4.68	0.00	0.00	-0.02
time (sec)	N/A	0.063	1.556	0.717	0.000	0.396	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	84	0	0	1236	0	0	-1
N.S.	1	1.00	1.22	0.00	0.00	17.91	0.00	0.00	-0.01
time (sec)	N/A	0.129	7.010	0.048	0.000	1.155	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	84	0	0	1288	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	18.14	0.00	0.00	-0.01
time (sec)	N/A	0.124	7.052	0.052	0.000	1.089	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	85	0	0	1239	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	17.21	0.00	0.00	-0.01
time (sec)	N/A	0.127	7.180	0.049	0.000	1.114	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	87	0	0	1299	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	18.04	0.00	0.00	-0.01
time (sec)	N/A	0.117	7.213	0.047	0.000	1.109	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	86	0	0	1270	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	17.40	0.00	0.00	-0.01
time (sec)	N/A	0.124	7.382	0.045	0.000	0.782	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	0	0	1324	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	17.65	0.00	0.00	-0.01
time (sec)	N/A	0.132	7.284	0.046	0.000	0.756	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	89	0	0	1273	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	16.75	0.00	0.00	-0.01
time (sec)	N/A	0.125	7.295	0.046	0.000	0.782	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	89	0	0	1335	0	0	-1
N.S.	1	1.00	1.17	0.00	0.00	17.57	0.00	0.00	-0.01
time (sec)	N/A	0.120	7.140	0.045	0.000	0.784	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	269	245	0	55	0	0	-1
N.S.	1	1.00	1.86	1.69	0.00	0.38	0.00	0.00	-0.01
time (sec)	N/A	0.134	20.310	0.825	0.000	0.124	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	267	245	0	210	0	0	-1
N.S.	1	1.00	1.84	1.69	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.151	20.287	0.763	0.000	0.119	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	291	260	0	746	0	0	-1
N.S.	1	1.00	1.68	1.50	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.174	20.359	0.286	0.000	0.172	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	291	264	0	714	0	0	-1
N.S.	1	1.00	1.56	1.41	0.00	3.82	0.00	0.00	-0.01
time (sec)	N/A	0.196	20.387	0.275	0.000	0.202	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	289	262	0	748	0	0	-1
N.S.	1	1.00	1.52	1.38	0.00	3.94	0.00	0.00	-0.01
time (sec)	N/A	0.169	20.288	0.283	0.000	0.167	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	293	258	0	708	0	0	-1
N.S.	1	1.00	1.60	1.41	0.00	3.87	0.00	0.00	-0.01
time (sec)	N/A	0.168	20.275	0.275	0.000	0.223	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	438	0	0	6491	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	19.55	0.00	0.00	-0.00
time (sec)	N/A	0.395	11.259	0.053	0.000	43.684	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	466	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.379	11.100	0.049	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	467	0	0	6499	0	0	-1
N.S.	1	1.00	1.35	0.00	0.00	18.84	0.00	0.00	-0.00
time (sec)	N/A	0.376	10.415	0.050	0.000	38.134	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	441	0	0	6561	0	0	-1
N.S.	1	1.00	1.28	0.00	0.00	19.02	0.00	0.00	-0.00
time (sec)	N/A	0.363	11.018	0.050	0.000	44.147	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	209	255	0	58	0	0	-1
N.S.	1	1.00	1.54	1.88	0.00	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.153	10.327	0.791	0.000	0.109	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	232	257	0	55	0	0	-1
N.S.	1	1.00	1.53	1.69	0.00	0.36	0.00	0.00	-0.01
time (sec)	N/A	0.157	10.458	0.734	0.000	0.129	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	230	255	0	213	0	0	-1
N.S.	1	1.00	1.40	1.55	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.152	10.256	0.760	0.000	0.122	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	211	253	0	203	0	0	-1
N.S.	1	1.00	1.35	1.62	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.144	10.121	0.696	0.000	0.122	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	225	255	0	214	0	0	-1
N.S.	1	1.00	1.53	1.73	0.00	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.156	10.409	0.813	0.000	0.114	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	427	0	0	1288	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	4.63	0.00	0.00	-0.00
time (sec)	N/A	0.310	10.814	0.054	0.000	0.852	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	454	0	0	1354	0	0	-1
N.S.	1	1.00	1.59	0.00	0.00	4.73	0.00	0.00	-0.00
time (sec)	N/A	0.309	10.841	0.049	0.000	0.848	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	455	0	0	1295	0	0	-1
N.S.	1	1.00	1.61	0.00	0.00	4.59	0.00	0.00	-0.00
time (sec)	N/A	0.299	10.220	0.050	0.000	0.857	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	430	0	0	1348	0	0	-1
N.S.	1	1.00	1.55	0.00	0.00	4.85	0.00	0.00	-0.00
time (sec)	N/A	0.299	10.320	0.049	0.000	0.882	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	214	275	0	0	0	0	-1
N.S.	1	1.00	0.68	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.897	10.405	0.293	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	235	264	0	0	0	0	-1
N.S.	1	1.00	0.71	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.918	10.517	0.281	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	233	273	0	0	0	0	-1
N.S.	1	1.00	0.72	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.659	10.138	0.284	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	233	266	0	0	0	0	-1
N.S.	1	1.00	0.73	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.748	10.507	0.281	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	213	275	0	0	0	0	-1
N.S.	1	1.00	0.59	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.763	10.327	0.267	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	235	268	0	0	0	0	-1
N.S.	1	1.00	0.68	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	10.451	0.257	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	233	277	0	0	0	0	-1
N.S.	1	1.00	0.68	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.625	10.124	0.257	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	233	266	0	0	0	0	-1
N.S.	1	1.00	0.64	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.644	10.407	0.254	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	149	132	0	33	56	0	334
N.S.	1	1.00	1.19	1.06	0.00	0.26	0.45	0.00	2.67
time (sec)	N/A	0.034	10.291	0.414	0.000	0.094	2.515	0.000	0.139

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	157	125	0	29	99	0	373
N.S.	1	1.00	1.13	0.90	0.00	0.21	0.71	0.00	2.68
time (sec)	N/A	0.039	10.340	0.427	0.000	0.104	4.235	0.000	3.637

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	150	132	0	52	94	0	334
N.S.	1	1.00	1.06	0.93	0.00	0.37	0.66	0.00	2.35
time (sec)	N/A	0.035	10.383	0.691	0.000	0.103	4.219	0.000	2.716

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	155	125	0	54	61	0	376
N.S.	1	1.00	1.14	0.92	0.00	0.40	0.45	0.00	2.76
time (sec)	N/A	0.039	10.380	0.705	0.000	0.132	2.658	0.000	4.451

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	149	132	0	33	56	0	334
N.S.	1	1.00	1.17	1.04	0.00	0.26	0.44	0.00	2.63
time (sec)	N/A	0.031	10.188	0.265	0.000	0.095	2.532	0.000	2.647

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	158	125	0	29	99	0	373
N.S.	1	1.00	1.12	0.89	0.00	0.21	0.70	0.00	2.65
time (sec)	N/A	0.040	10.365	0.283	0.000	0.110	4.227	0.000	3.242

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	151	132	0	51	94	0	334
N.S.	1	1.00	1.05	0.92	0.00	0.35	0.65	0.00	2.32
time (sec)	N/A	0.032	10.309	0.547	0.000	0.122	4.233	0.000	2.702

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	156	125	0	53	61	0	376
N.S.	1	1.00	1.13	0.91	0.00	0.38	0.44	0.00	2.72
time (sec)	N/A	0.038	10.304	0.518	0.000	0.130	2.577	0.000	4.099

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	194	240	0	0	0	0	207
N.S.	1	1.00	0.58	0.72	0.00	0.00	0.00	0.00	0.62
time (sec)	N/A	0.437	10.166	0.257	0.000	0.000	0.000	0.000	0.225

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	195	240	0	0	0	0	224
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.59
time (sec)	N/A	0.489	10.179	0.255	0.000	0.000	0.000	0.000	2.742

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	193	240	0	0	0	0	208
N.S.	1	1.00	0.52	0.64	0.00	0.00	0.00	0.00	0.56
time (sec)	N/A	0.405	10.091	0.254	0.000	0.000	0.000	0.000	2.647

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	196	240	0	0	0	0	223
N.S.	1	1.00	0.57	0.70	0.00	0.00	0.00	0.00	0.65
time (sec)	N/A	0.400	10.142	0.248	0.000	0.000	0.000	0.000	2.611

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	211	274	0	0	0	0	356
N.S.	1	1.00	0.47	0.61	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.925	10.376	0.261	0.000	0.000	0.000	0.000	0.128

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	233	265	0	0	0	0	387
N.S.	1	1.00	0.49	0.56	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	1.033	10.469	0.256	0.000	0.000	0.000	0.000	0.093

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	B
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	231	274	0	0	0	0	355
N.S.	1	1.00	0.49	0.58	0.00	0.00	0.00	0.00	0.75
time (sec)	N/A	0.878	10.195	0.276	0.000	0.000	0.000	0.000	2.675

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	213	265	0	0	0	0	388
N.S.	1	1.00	0.46	0.57	0.00	0.00	0.00	0.00	0.84
time (sec)	N/A	0.924	10.294	0.254	0.000	0.000	0.000	0.000	0.097

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	134	129	0	31	42	0	207
N.S.	1	1.00	1.12	1.08	0.00	0.26	0.35	0.00	1.72
time (sec)	N/A	0.032	10.244	0.250	0.000	0.108	1.448	0.000	2.637

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	140	122	0	26	65	0	223
N.S.	1	1.00	1.04	0.91	0.00	0.19	0.49	0.00	1.66
time (sec)	N/A	0.040	10.250	0.247	0.000	0.105	1.530	0.000	0.065

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	136	129	0	27	60	0	207
N.S.	1	1.00	0.99	0.94	0.00	0.20	0.44	0.00	1.51
time (sec)	N/A	0.040	10.197	0.261	0.000	0.101	1.442	0.000	2.617

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	138	122	0	28	46	0	223
N.S.	1	1.00	1.05	0.93	0.00	0.21	0.35	0.00	1.70
time (sec)	N/A	0.042	10.176	0.242	0.000	0.102	1.611	0.000	2.656

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	159	0	0	0	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.240	0.045	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.159	10.116	0.043	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	415	253	490	6397	835	495
N.S.	1	1.00	0.83	2.59	1.58	3.06	39.98	5.22	3.09
time (sec)	N/A	0.075	0.126	0.214	0.287	0.519	2.091	3.969	3.204

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	283	184	348	3704	577	363
N.S.	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.88
time (sec)	N/A	0.050	0.090	0.240	0.295	0.441	1.298	4.879	2.950

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	95	167	122	222	1906	361	247
N.S.	1	1.00	1.01	1.78	1.30	2.36	20.28	3.84	2.63
time (sec)	N/A	0.031	0.085	0.204	0.278	0.353	0.779	3.989	2.946

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	94	0	0	0	741	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	7.48	0.00	-0.01
time (sec)	N/A	0.043	0.075	0.023	0.000	0.000	2.987	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	252	1565	601	1565	26746	2660	1410
N.S.	1	1.00	0.86	5.32	2.04	5.32	90.97	9.05	4.80
time (sec)	N/A	0.143	0.235	0.246	0.300	0.364	9.821	4.660	3.734

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	211	893	474	1216	18328	2034	1136
N.S.	1	1.00	0.85	3.60	1.91	4.90	73.90	8.20	4.58
time (sec)	N/A	0.111	0.185	0.250	0.300	0.357	6.595	3.755	3.392

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	297	700	359	893	11851	1477	878
N.S.	1	1.00	1.46	3.45	1.77	4.40	58.38	7.28	4.33
time (sec)	N/A	0.078	0.177	0.247	0.297	0.374	3.790	4.124	3.192

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	188	0	0	0	4760	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	22.78	0.00	-0.00
time (sec)	N/A	0.089	0.161	0.037	0.000	0.000	4.822	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	402	3780	1153	3564	75191	0	2500
N.S.	1	1.00	0.88	8.24	2.51	7.76	163.81	0.00	5.45
time (sec)	N/A	0.224	0.371	0.293	0.312	0.404	87.439	0.000	7.140

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	345	2972	953	2919	56151	4934	2500
N.S.	1	1.00	0.87	7.51	2.41	7.37	141.80	12.46	6.31
time (sec)	N/A	0.193	0.321	0.279	0.304	0.433	37.089	4.116	5.598

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	706	2280	770	2313	40536	3874	2001
N.S.	1	1.00	2.09	6.77	2.28	6.86	120.28	11.50	5.94
time (sec)	N/A	0.147	0.438	0.258	0.307	0.455	68.294	4.464	4.338

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	332	0	0	0	17258	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	48.21	0.00	-0.00
time (sec)	N/A	0.153	0.299	0.037	0.000	0.000	14.403	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	284	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.627	0.487	0.057	0.000	0.000	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	292	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	0.539	0.055	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	239	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.235	0.053	0.000	0.000	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	0.164	0.050	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	237	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.185	0.049	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	222	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.129	0.126	0.059	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	244	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.455	0.287	0.042	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	273	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.261	0.052	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.407	0.309	0.051	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.391	0.155	0.052	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1480	1480	820	1126	0	0	0	0	-1
N.S.	1	1.00	0.55	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.982	6.413	0.643	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	F	F	F	F	B	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	0	0	0	0	0	636	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	4.71	0.00	-0.01
time (sec)	N/A	0.055	0.619	0.044	0.000	0.000	27.233	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	23	1640	0	19	0	0	273
N.S.	1	1.00	1.44	102.50	0.00	1.19	0.00	0.00	17.06
time (sec)	N/A	0.042	0.882	0.410	0.000	0.478	0.000	0.000	0.201

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	23	732	0	28	0	0	292
N.S.	1	1.00	1.15	36.60	0.00	1.40	0.00	0.00	14.60
time (sec)	N/A	0.050	0.907	0.398	0.000	0.500	0.000	0.000	2.831

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	1656	0	25	0	0	276
N.S.	1	1.00	1.17	92.00	0.00	1.39	0.00	0.00	15.33
time (sec)	N/A	0.041	0.925	0.365	0.000	0.506	0.000	0.000	2.771

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	25	724	0	28	0	0	289
N.S.	1	1.00	1.39	40.22	0.00	1.56	0.00	0.00	16.06
time (sec)	N/A	0.045	0.923	0.326	0.000	0.368	0.000	0.000	0.106

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	37	4397	0	181	0	0	632
N.S.	1	1.00	1.23	146.57	0.00	6.03	0.00	0.00	21.07
time (sec)	N/A	0.057	1.619	0.378	0.000	0.376	0.000	0.000	2.822

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	1908	0	191	0	0	677
N.S.	1	1.00	0.97	50.21	0.00	5.03	0.00	0.00	17.82
time (sec)	N/A	0.070	1.707	0.370	0.000	0.394	0.000	0.000	0.141

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	4437	0	187	0	0	629
N.S.	1	1.00	0.97	123.25	0.00	5.19	0.00	0.00	17.47
time (sec)	N/A	0.065	1.681	0.369	0.000	0.426	0.000	0.000	2.801

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	39	1888	0	185	0	0	680
N.S.	1	1.00	1.22	59.00	0.00	5.78	0.00	0.00	21.25
time (sec)	N/A	0.064	1.675	0.332	0.000	0.378	0.000	0.000	0.120

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	186	269	0	0	175	0	-1
N.S.	1	1.00	0.52	0.76	0.00	0.00	0.49	0.00	-0.00
time (sec)	N/A	0.156	10.113	0.238	0.000	0.000	2.303	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	146	248	0	0	138	0	-1
N.S.	1	1.00	0.45	0.76	0.00	0.00	0.42	0.00	-0.00
time (sec)	N/A	0.120	9.009	0.237	0.000	0.000	2.015	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	109	129	0	95	88	0	-1
N.S.	1	1.00	0.69	0.82	0.00	0.60	0.56	0.00	-0.01
time (sec)	N/A	0.072	6.279	0.223	0.000	0.124	1.688	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	89	85	0	40	37	0	37
N.S.	1	1.00	0.85	0.81	0.00	0.38	0.35	0.00	0.35
time (sec)	N/A	0.013	3.733	0.211	0.000	0.098	0.387	0.000	2.644

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	421	404	0	0	0	0	-1
N.S.	1	1.00	0.58	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	9.835	0.252	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1221	1221	394	402	0	0	0	0	-1
N.S.	1	1.00	0.32	0.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.289	11.508	0.224	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	157	218	0	0	141	0	-1
N.S.	1	1.00	0.53	0.74	0.00	0.00	0.48	0.00	-0.00
time (sec)	N/A	0.109	10.105	0.234	0.000	0.000	1.987	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	133	197	0	0	105	0	-1
N.S.	1	1.00	0.51	0.75	0.00	0.00	0.40	0.00	-0.00
time (sec)	N/A	0.086	10.081	0.223	0.000	0.000	1.647	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	79	96	0	72	61	0	-1
N.S.	1	1.00	0.65	0.79	0.00	0.60	0.50	0.00	-0.01
time (sec)	N/A	0.043	10.046	0.207	0.000	0.119	1.079	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	28	36	0	37
N.S.	1	1.00	0.84	0.80	0.00	0.32	0.41	0.00	0.42
time (sec)	N/A	0.007	10.022	0.202	0.000	0.148	0.350	0.000	2.630

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	200	169	0	0	0	0	-1
N.S.	1	1.00	0.49	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	10.202	0.263	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	448	421	0	0	0	0	-1
N.S.	1	1.00	0.73	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.519	10.953	0.832	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	614	483	0	0	0	0	-1
N.S.	1	1.00	0.93	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.782	11.414	0.225	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	126	261	0	0	0	0	-1
N.S.	1	1.00	0.42	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.095	10.077	0.211	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	108	239	0	0	0	0	-1
N.S.	1	1.00	0.40	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	10.054	0.227	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	59	115	0	76	61	0	57
N.S.	1	1.00	0.52	1.01	0.00	0.67	0.54	0.00	0.50
time (sec)	N/A	0.030	10.042	0.196	0.000	0.101	3.025	0.000	2.877

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	94	0	63	36	0	37
N.S.	1	1.00	0.51	0.87	0.00	0.58	0.33	0.00	0.34
time (sec)	N/A	0.014	5.186	0.193	0.000	0.083	0.394	0.000	2.659

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	455	496	0	0	0	0	-1
N.S.	1	1.00	0.56	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	10.636	0.211	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.353	0.052	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	274	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	0.243	0.052	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1605	1605	455	1153	0	0	0	0	-1
N.S.	1	1.00	0.28	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.056	10.972	0.323	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	132	200	119	233	0	355	234
N.S.	1	1.00	0.82	1.24	0.74	1.45	0.00	2.20	1.45
time (sec)	N/A	0.052	0.086	0.013	0.275	0.350	0.000	3.371	2.973

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	136	74	0	72	-1
N.S.	1	1.00	0.44	0.42	0.95	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.047	0.015	0.062	0.288	0.333	0.000	6.007	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	47	74	0	72	-1
N.S.	1	1.00	0.44	0.42	0.33	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.043	0.015	0.060	0.305	0.332	0.000	5.314	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	60	98	74	0	48	50
N.S.	1	1.00	0.95	0.91	1.48	1.12	0.00	0.73	0.76
time (sec)	N/A	0.049	0.015	0.050	0.277	0.319	0.000	5.802	2.831

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	63	60	47	74	0	72	-1
N.S.	1	1.00	0.44	0.42	0.33	0.52	0.00	0.50	-0.01
time (sec)	N/A	0.042	0.012	0.063	0.280	0.351	0.000	5.080	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	26	60	73	0	25	40
N.S.	1	1.00	0.91	0.81	1.88	2.28	0.00	0.78	1.25
time (sec)	N/A	0.016	0.008	0.051	0.281	0.330	0.000	3.167	2.835

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	61	58	43	72	0	46	-1
N.S.	1	1.00	0.45	0.43	0.32	0.53	0.00	0.34	-0.01
time (sec)	N/A	0.018	0.009	0.025	0.273	0.323	0.000	3.549	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	62	59	171	73	0	73	-1
N.S.	1	1.00	0.45	0.42	1.23	0.53	0.00	0.53	-0.01
time (sec)	N/A	0.039	0.016	0.038	0.277	0.333	0.000	3.368	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	62	60	48	72	0	69	-1
N.S.	1	1.00	0.46	0.45	0.36	0.54	0.00	0.51	-0.01
time (sec)	N/A	0.034	0.018	0.036	0.296	0.332	0.000	3.565	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	65	61	176	76	0	91	-1
N.S.	1	1.00	0.46	0.44	1.26	0.54	0.00	0.65	-0.01
time (sec)	N/A	0.041	0.017	0.054	0.278	0.338	0.000	5.776	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	134	236	0	433	0	177	-1
N.S.	1	1.00	0.53	0.93	0.00	1.71	0.00	0.70	-0.00
time (sec)	N/A	0.095	0.151	0.060	0.000	0.411	0.000	4.758	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	29	138	70	87	0	28	62
N.S.	1	1.00	0.91	4.31	2.19	2.72	0.00	0.88	1.94
time (sec)	N/A	0.020	0.012	0.042	0.288	0.362	0.000	4.296	2.708

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	123	205	0	402	0	153	-1
N.S.	1	1.00	0.59	0.99	0.00	1.94	0.00	0.74	-0.00
time (sec)	N/A	0.044	0.143	0.041	0.000	0.390	0.000	3.277	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	111	221	0	391	0	185	-1
N.S.	1	1.00	0.58	1.15	0.00	2.04	0.00	0.96	-0.01
time (sec)	N/A	0.075	0.108	0.049	0.000	0.406	0.000	3.891	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	121	215	0	396	0	185	-1
N.S.	1	1.00	0.58	1.03	0.00	1.90	0.00	0.89	-0.00
time (sec)	N/A	0.064	0.143	0.052	0.000	0.377	0.000	4.637	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	118	238	0	411	0	204	-1
N.S.	1	1.00	0.58	1.18	0.00	2.03	0.00	1.01	-0.00
time (sec)	N/A	0.082	0.117	0.066	0.000	0.372	0.000	5.482	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	59	0	141	0	71	-1
N.S.	1	1.00	0.83	0.77	0.00	1.83	0.00	0.92	-0.01
time (sec)	N/A	0.032	0.057	0.030	0.000	0.372	0.000	5.720	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	19	19	42	28	19
N.S.	1	1.00	1.00	1.24	0.90	0.90	2.00	1.33	0.90
time (sec)	N/A	0.012	0.004	0.048	0.279	0.375	0.300	4.639	2.644

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	0	19	46	28	19
N.S.	1	1.00	1.00	1.24	0.00	0.90	2.19	1.33	0.90
time (sec)	N/A	0.005	0.028	0.020	0.000	0.367	0.328	4.787	2.739

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	59	64	80	138	0	59	-1
N.S.	1	1.00	0.83	0.90	1.13	1.94	0.00	0.83	-0.01
time (sec)	N/A	0.037	0.039	0.029	0.484	0.361	0.000	3.534	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	32	37	46	32	0	81	54
N.S.	1	1.00	0.67	0.77	0.96	0.67	0.00	1.69	1.12
time (sec)	N/A	0.029	0.044	0.039	0.288	0.352	0.000	3.941	2.830

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	78	81	121	175	0	103	-1
N.S.	1	1.00	0.75	0.78	1.16	1.68	0.00	0.99	-0.01
time (sec)	N/A	0.045	0.072	0.054	0.491	0.374	0.000	4.805	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	63	58	85	75	144	137	109
N.S.	1	1.00	0.46	0.42	0.62	0.54	1.04	0.99	0.79
time (sec)	N/A	0.070	0.048	0.020	0.269	0.380	12.485	4.799	2.965

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	52	47	64	63	116	109	88
N.S.	1	1.00	0.51	0.46	0.63	0.62	1.14	1.07	0.86
time (sec)	N/A	0.054	0.031	0.012	0.275	0.377	6.987	5.285	2.901

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	41	36	43	51	87	81	67
N.S.	1	1.00	0.62	0.55	0.65	0.77	1.32	1.23	1.02
time (sec)	N/A	0.043	0.029	0.013	0.278	0.376	3.500	4.547	2.885

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	31	26	25	37	58	17	28
N.S.	1	1.00	0.86	0.72	0.69	1.03	1.61	0.47	0.78
time (sec)	N/A	0.012	0.005	0.038	0.261	0.383	1.698	4.100	2.769

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	96	0	118	0	0	190	-1
N.S.	1	1.00	0.82	0.00	1.01	0.00	0.00	1.62	-0.01
time (sec)	N/A	0.048	0.066	0.013	0.506	0.000	0.000	3.195	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	107	0	138	0	0	212	-1
N.S.	1	1.00	0.80	0.00	1.04	0.00	0.00	1.59	-0.01
time (sec)	N/A	0.047	0.112	0.007	0.489	0.000	0.000	3.128	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	68	0	0	0	0	0	-1
N.S.	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	7.585	0.008	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	52	0	0	0	0	0	-1
N.S.	1	1.00	0.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	7.170	0.006	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	55	0	0	0	0	0	-1
N.S.	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	7.717	0.007	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	57	0	0	0	0	0	-1
N.S.	1	1.00	0.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.025	0.007	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	76	68	0	80	0	61	-1
N.S.	1	1.00	1.07	0.96	0.00	1.13	0.00	0.86	-0.01
time (sec)	N/A	0.017	0.114	0.263	0.000	0.365	0.000	5.446	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	24	0	0	-1
N.S.	1	1.00	1.23	2.38	0.00	0.50	0.00	0.00	-0.02
time (sec)	N/A	0.027	4.065	0.038	0.000	0.097	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	55	28	0	43	0	61	-1
N.S.	1	1.00	1.72	0.88	0.00	1.34	0.00	1.91	-0.03
time (sec)	N/A	0.008	0.045	0.243	0.000	0.402	0.000	4.792	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	8	0	0	-1
N.S.	1	1.00	1.50	3.58	0.00	0.67	0.00	0.00	-0.08
time (sec)	N/A	0.009	10.021	0.023	0.000	0.085	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	198	527	391	528	0	211	-1
N.S.	1	1.00	0.81	2.16	1.60	2.16	0.00	0.86	-0.00
time (sec)	N/A	0.296	1.641	0.099	0.510	0.372	0.000	5.265	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	161	342	253	394	0	155	-1
N.S.	1	1.00	1.00	2.12	1.57	2.45	0.00	0.96	-0.01
time (sec)	N/A	0.166	0.888	0.076	0.521	0.394	0.000	7.738	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	200	138	292	0	123	-1
N.S.	1	1.00	1.28	1.94	1.34	2.83	0.00	1.19	-0.01
time (sec)	N/A	0.083	0.374	0.060	0.493	0.362	0.000	6.355	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	147	179	140	815	0	0	-1
N.S.	1	1.00	1.31	1.60	1.25	7.28	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.465	0.065	0.516	0.455	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	137	326	138	314	0	213	-1
N.S.	1	1.00	1.08	2.57	1.09	2.47	0.00	1.68	-0.01
time (sec)	N/A	0.112	0.617	0.089	0.511	0.453	0.000	3.361	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	174	559	249	416	0	481	-1
N.S.	1	1.00	0.84	2.69	1.20	2.00	0.00	2.31	-0.00
time (sec)	N/A	0.191	1.179	0.089	0.497	0.664	0.000	5.839	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	222	849	387	554	0	871	-1
N.S.	1	1.00	0.70	2.67	1.22	1.74	0.00	2.74	-0.00
time (sec)	N/A	0.310	1.495	0.109	0.536	1.240	0.000	6.959	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	255	552	0	0	0	0	-1
N.S.	1	1.00	0.71	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	1.577	0.078	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	208	356	0	0	0	0	-1
N.S.	1	1.00	0.78	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	1.057	0.052	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	86	184	0	0	0	0	-1
N.S.	1	1.00	0.44	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.847	0.036	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	111	194	0	0	0	0	-1
N.S.	1	1.00	0.46	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	1.116	0.051	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	238	443	0	0	0	0	-1
N.S.	1	1.00	0.74	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	1.629	0.057	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	424	302	708	0	0	0	0	-1
N.S.	1	1.00	0.71	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	1.774	0.072	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	294	1027	426	526	0	420	-1
N.S.	1	1.00	1.04	3.64	1.51	1.87	0.00	1.49	-0.00
time (sec)	N/A	0.352	4.636	0.149	0.516	0.581	0.000	4.807	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	177	679	282	396	0	321	-1
N.S.	1	1.00	0.89	3.41	1.42	1.99	0.00	1.61	-0.01
time (sec)	N/A	0.216	2.251	0.116	0.524	0.532	0.000	5.636	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	136	432	177	303	0	247	-1
N.S.	1	1.00	0.96	3.06	1.26	2.15	0.00	1.75	-0.01
time (sec)	N/A	0.104	1.398	0.099	0.518	0.448	0.000	5.490	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	187	401	185	983	0	0	-1
N.S.	1	1.00	1.24	2.66	1.23	6.51	0.00	0.00	-0.01
time (sec)	N/A	0.188	1.894	0.076	0.503	0.733	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	641	177	330	0	0	-1
N.S.	1	1.00	0.88	3.88	1.07	2.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	4.023	0.134	0.535	0.662	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	186	1042	282	422	0	0	-1
N.S.	1	1.00	0.73	4.07	1.10	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.239	4.474	0.149	0.516	1.226	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	245	1498	425	552	0	0	-1
N.S.	1	1.00	0.67	4.09	1.16	1.51	0.00	0.00	-0.00
time (sec)	N/A	0.371	4.936	0.151	0.525	4.463	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	290	933	0	0	0	0	-1
N.S.	1	1.00	0.74	2.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	4.757	0.114	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	235	738	0	0	0	0	-1
N.S.	1	1.00	0.76	2.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	4.089	0.084	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	206	527	0	0	0	0	-1
N.S.	1	1.00	0.79	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	3.988	0.042	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	228	670	0	0	0	0	-1
N.S.	1	1.00	0.74	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	4.376	0.071	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	275	791	0	0	0	0	-1
N.S.	1	1.00	0.72	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	4.815	0.079	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	357	1197	0	0	0	0	-1
N.S.	1	1.00	0.74	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.460	5.341	0.094	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	85	52	0	55	0	18	55
N.S.	1	1.00	1.67	1.02	0.00	1.08	0.00	0.35	1.08
time (sec)	N/A	0.026	0.104	0.248	0.000	0.368	0.000	5.547	2.666

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	119	78	76	77	0	30	88
N.S.	1	1.00	1.65	1.08	1.06	1.07	0.00	0.42	1.22
time (sec)	N/A	0.039	0.211	0.209	0.489	0.398	0.000	5.376	0.210

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	85	68	0	55	0	22	56
N.S.	1	1.00	1.60	1.28	0.00	1.04	0.00	0.42	1.06
time (sec)	N/A	0.030	0.162	0.232	0.000	0.353	0.000	5.518	2.674

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	95	80	0	65	0	0	101
N.S.	1	1.00	0.84	0.71	0.00	0.58	0.00	0.00	0.89
time (sec)	N/A	0.070	0.390	0.205	0.000	0.339	0.000	0.000	2.658

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	114	121	82	0	47	134
N.S.	1	1.00	1.00	1.08	1.14	0.77	0.00	0.44	1.26
time (sec)	N/A	0.066	0.634	0.418	0.482	0.379	0.000	4.360	2.994

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	42	0	32	0	40	-1
N.S.	1	1.00	0.94	0.81	0.00	0.62	0.00	0.77	-0.02
time (sec)	N/A	0.031	0.026	0.199	0.000	0.344	0.000	3.970	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	60	0	42	0	61	-1
N.S.	1	1.00	0.96	0.88	0.00	0.62	0.00	0.90	-0.01
time (sec)	N/A	0.061	0.037	0.200	0.000	0.358	0.000	3.030	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	224	527	387	520	0	210	-1
N.S.	1	1.00	0.80	1.88	1.38	1.85	0.00	0.75	-0.00
time (sec)	N/A	0.295	1.541	0.091	0.522	0.392	0.000	3.760	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	161	341	249	388	0	153	-1
N.S.	1	1.00	0.95	2.02	1.47	2.30	0.00	0.91	-0.01
time (sec)	N/A	0.151	0.919	0.069	0.521	0.374	0.000	5.547	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	136	200	138	288	0	126	-1
N.S.	1	1.00	1.28	1.89	1.30	2.72	0.00	1.19	-0.01
time (sec)	N/A	0.074	0.399	0.062	0.513	0.373	0.000	6.344	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	147	179	140	821	0	0	-1
N.S.	1	1.00	1.31	1.60	1.25	7.33	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.458	0.049	0.521	0.458	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	143	326	138	310	0	216	-1
N.S.	1	1.00	1.10	2.51	1.06	2.38	0.00	1.66	-0.01
time (sec)	N/A	0.112	0.659	0.082	0.496	0.439	0.000	6.012	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	173	559	252	420	0	486	-1
N.S.	1	1.00	0.79	2.56	1.16	1.93	0.00	2.23	-0.00
time (sec)	N/A	0.173	1.231	0.089	0.554	0.664	0.000	3.527	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	258	554	0	0	0	0	-1
N.S.	1	1.00	0.64	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	1.586	0.060	0.000	0.000	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	212	358	0	0	0	0	-1
N.S.	1	1.00	0.68	1.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.155	1.050	0.046	0.000	0.000	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	86	127	0	0	0	0	-1
N.S.	1	1.00	0.34	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.077	0.852	0.036	0.000	0.000	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	111	297	0	0	0	0	-1
N.S.	1	1.00	0.38	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	1.105	0.049	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	372	238	444	0	0	0	0	-1
N.S.	1	0.99	0.63	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	1.460	0.049	0.000	0.000	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	353	247	1027	428	750	0	0	-1
N.S.	1	1.00	0.70	2.90	1.21	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.361	4.618	0.153	0.518	0.645	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	179	679	281	554	0	0	-1
N.S.	1	1.00	0.89	3.36	1.39	2.74	0.00	0.00	-0.00
time (sec)	N/A	0.234	2.215	0.109	0.511	0.593	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	138	432	180	403	0	0	-1
N.S.	1	1.00	0.95	2.96	1.23	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.120	1.414	0.100	0.497	0.515	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	189	401	185	1177	0	0	-1
N.S.	1	1.00	1.24	2.64	1.22	7.74	0.00	0.00	-0.01
time (sec)	N/A	0.197	1.700	0.070	0.516	0.871	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	148	641	178	433	0	0	-1
N.S.	1	1.00	0.87	3.77	1.05	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.143	2.120	0.117	0.504	1.026	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	189	1042	281	584	0	0	-1
N.S.	1	1.00	0.74	4.09	1.10	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.245	4.429	0.136	0.540	1.859	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	271	936	0	0	0	0	-1
N.S.	1	1.00	0.60	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.361	4.731	0.128	0.000	0.000	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	219	643	0	0	0	0	-1
N.S.	1	1.00	0.58	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	4.103	0.076	0.000	0.000	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	203	514	0	0	0	0	-1
N.S.	1	1.00	0.62	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.134	4.004	0.041	0.000	0.000	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	223	654	0	0	0	0	-1
N.S.	1	1.00	0.59	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	4.300	0.072	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	266	866	0	0	0	0	-1
N.S.	1	1.00	0.60	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.346	4.835	0.087	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	217	145	533	328	423	0	219	-1
N.S.	1	1.00	0.67	2.47	1.52	1.96	0.00	1.01	-0.00
time (sec)	N/A	0.290	0.240	0.094	0.513	0.389	0.000	4.228	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	107	353	218	325	0	159	-1
N.S.	1	1.00	0.76	2.50	1.55	2.30	0.00	1.13	-0.01
time (sec)	N/A	0.178	0.142	0.063	0.518	0.376	0.000	3.538	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	85	180	126	267	0	127	120
N.S.	1	1.00	1.23	2.61	1.83	3.87	0.00	1.84	1.74
time (sec)	N/A	0.036	0.102	0.094	0.501	0.376	0.000	4.430	3.007

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	102	235	159	927	0	0	-1
N.S.	1	1.00	1.06	2.45	1.66	9.66	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.140	0.049	0.535	0.416	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	110	454	156	433	0	281	-1
N.S.	1	1.00	1.06	4.37	1.50	4.16	0.00	2.70	-0.01
time (sec)	N/A	0.133	0.213	0.075	0.497	0.392	0.000	5.761	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	152	923	322	577	0	713	-1
N.S.	1	1.00	0.87	5.30	1.85	3.32	0.00	4.10	-0.01
time (sec)	N/A	0.231	0.268	0.086	0.507	0.489	0.000	4.447	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	216	1518	557	755	0	1414	-1
N.S.	1	1.00	0.82	5.73	2.10	2.85	0.00	5.34	-0.00
time (sec)	N/A	0.360	0.418	0.108	0.554	0.645	0.000	4.427	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	293	662	0	0	0	0	-1
N.S.	1	1.00	0.80	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	10.577	0.075	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	250	406	0	0	0	0	-1
N.S.	1	1.00	0.89	1.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.208	10.388	0.045	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	98	199	0	0	0	0	-1
N.S.	1	1.00	0.46	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.100	10.046	0.025	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	141	272	0	0	0	0	-1
N.S.	1	1.00	0.53	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	10.399	0.049	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	314	571	0	0	0	0	-1
N.S.	1	1.00	0.87	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	10.663	0.069	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	402	955	0	0	0	0	-1
N.S.	1	1.00	0.86	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.410	10.729	0.069	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	150	1018	368	427	0	527	-1
N.S.	1	1.00	0.60	4.09	1.48	1.71	0.00	2.12	-0.00
time (sec)	N/A	0.339	0.247	0.131	0.523	0.420	0.000	3.670	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	113	593	247	335	0	438	-1
N.S.	1	1.00	0.66	3.45	1.44	1.95	0.00	2.55	-0.01
time (sec)	N/A	0.223	0.181	0.098	0.522	0.419	0.000	4.034	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	87	336	156	269	0	387	61
N.S.	1	1.00	0.93	3.57	1.66	2.86	0.00	4.12	0.65
time (sec)	N/A	0.045	0.113	0.121	0.512	0.409	0.000	5.255	3.739

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	139	652	201	1073	0	0	-1
N.S.	1	1.00	1.10	5.17	1.60	8.52	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.192	0.071	0.529	0.466	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	820	202	404	0	0	-1
N.S.	1	1.00	0.90	5.94	1.46	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.213	0.108	0.502	0.443	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	153	1653	313	557	0	0	-1
N.S.	1	1.00	0.75	8.06	1.53	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.215	0.148	0.508	0.701	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	226	2605	534	733	0	0	-1
N.S.	1	1.00	0.77	8.92	1.83	2.51	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.446	0.153	0.536	0.951	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	308	1098	0	0	0	0	-1
N.S.	1	1.00	0.76	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.425	10.594	0.103	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	270	820	0	0	0	0	-1
N.S.	1	1.00	0.82	2.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	10.532	0.076	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	243	515	0	0	0	0	-1
N.S.	1	1.00	0.93	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	10.385	0.040	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	278	873	0	0	0	0	-1
N.S.	1	1.00	0.89	2.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	10.490	0.069	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	329	1039	0	0	0	0	-1
N.S.	1	1.00	0.85	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	10.573	0.072	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	430	1666	0	0	0	0	-1
N.S.	1	1.00	0.87	3.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.557	10.764	0.088	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	148	533	340	425	0	226	-1
N.S.	1	1.00	0.66	2.37	1.51	1.89	0.00	1.00	-0.00
time (sec)	N/A	0.294	0.242	0.079	0.511	0.374	0.000	3.407	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	354	223	333	0	167	-1
N.S.	1	1.00	0.73	2.39	1.51	2.25	0.00	1.13	-0.01
time (sec)	N/A	0.183	0.139	0.065	0.524	0.385	0.000	4.464	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	88	184	129	267	0	129	111
N.S.	1	1.00	1.22	2.56	1.79	3.71	0.00	1.79	1.54
time (sec)	N/A	0.039	0.106	0.235	0.511	0.359	0.000	4.816	3.342

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	312	155	972	0	0	-1
N.S.	1	1.00	1.05	3.25	1.61	10.12	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.105	0.043	0.534	0.461	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	114	452	173	451	0	292	-1
N.S.	1	1.00	1.06	4.19	1.60	4.18	0.00	2.70	-0.01
time (sec)	N/A	0.147	0.226	0.072	0.509	0.423	0.000	4.951	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	149	923	359	593	0	778	-1
N.S.	1	1.00	0.84	5.21	2.03	3.35	0.00	4.40	-0.01
time (sec)	N/A	0.224	0.257	0.086	0.526	0.542	0.000	4.860	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	297	664	0	0	0	0	-1
N.S.	1	1.00	0.67	1.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	10.174	0.058	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	253	409	0	0	0	0	-1
N.S.	1	1.00	0.71	1.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	9.038	0.046	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	107	164	0	0	0	0	-1
N.S.	1	1.00	0.37	0.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.107	8.826	0.025	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	151	345	0	0	0	0	-1
N.S.	1	1.00	0.44	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.192	9.157	0.046	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	431	314	593	0	0	0	0	-1
N.S.	1	0.99	0.72	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	10.631	0.052	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	183	1240	389	675	0	597	-1
N.S.	1	1.00	0.59	4.00	1.25	2.18	0.00	1.93	-0.00
time (sec)	N/A	0.344	0.346	0.120	0.516	0.476	0.000	4.070	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	144	783	262	541	0	510	-1
N.S.	1	1.00	0.77	4.19	1.40	2.89	0.00	2.73	-0.01
time (sec)	N/A	0.259	0.245	0.095	0.540	0.476	0.000	4.990	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	114	478	161	395	0	449	61
N.S.	1	1.00	1.14	4.78	1.61	3.95	0.00	4.49	0.61
time (sec)	N/A	0.049	0.169	0.272	0.526	0.396	0.000	4.839	3.930

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	140	1014	201	1477	0	0	-1
N.S.	1	1.00	1.04	7.57	1.50	11.02	0.00	0.00	-0.01
time (sec)	N/A	0.229	0.419	0.074	0.533	0.513	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	148	1088	247	599	0	0	-1
N.S.	1	1.00	1.01	7.45	1.69	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.245	0.109	0.520	0.484	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	192	1947	450	961	0	0	-1
N.S.	1	1.00	0.91	9.18	2.12	4.53	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.354	0.142	0.546	0.774	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	296	1159	0	0	0	0	-1
N.S.	1	1.00	0.61	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	10.580	0.113	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	255	667	0	0	0	0	-1
N.S.	1	1.00	0.62	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	10.391	0.076	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	241	466	0	0	0	0	-1
N.S.	1	1.00	0.68	1.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	10.342	0.044	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	268	686	0	0	0	0	-1
N.S.	1	1.00	0.65	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	10.452	0.074	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	319	1080	0	0	0	0	-1
N.S.	1	1.00	0.65	2.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.422	10.580	0.082	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	48	0	169	0	0	-1
N.S.	1	1.00	0.79	0.64	0.00	2.25	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.940	0.230	0.000	0.423	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	40	0	153	0	68	-1
N.S.	1	1.00	1.04	0.80	0.00	3.06	0.00	1.36	-0.02
time (sec)	N/A	0.009	0.154	0.222	0.000	0.426	0.000	4.532	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	34	17	0	98	0	58	-1
N.S.	1	1.00	1.42	0.71	0.00	4.08	0.00	2.42	-0.04
time (sec)	N/A	0.006	0.133	0.200	0.000	0.409	0.000	3.666	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	41	17	0	28	17
N.S.	1	1.00	1.00	0.78	1.78	0.74	0.00	1.22	0.74
time (sec)	N/A	0.003	0.146	0.200	0.642	0.345	0.000	3.859	2.693

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	25	50	25	0	0	29
N.S.	1	1.00	0.61	0.51	1.02	0.51	0.00	0.00	0.59
time (sec)	N/A	0.007	0.832	0.204	0.670	0.332	0.000	0.000	2.672

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	26	29	0	29	-1
N.S.	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03
time (sec)	N/A	0.006	0.011	0.205	0.483	0.358	0.000	6.821	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	26	29	0	29	-1
N.S.	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03
time (sec)	N/A	0.010	0.004	0.200	0.859	0.327	0.000	4.337	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	38	40	41	0	42	-1
N.S.	1	1.00	0.62	0.54	0.56	0.58	0.00	0.59	-0.01
time (sec)	N/A	0.010	0.012	0.212	0.597	0.354	0.000	4.090	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	-1
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	-0.02
time (sec)	N/A	0.010	0.050	0.231	0.656	0.390	0.000	3.349	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	42	37	42	256	0	48	-1
N.S.	1	1.00	0.86	0.76	0.86	5.22	0.00	0.98	-0.02
time (sec)	N/A	0.013	0.003	0.219	0.658	0.337	0.000	3.566	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	30	44	32	127	0	43	-1
N.S.	1	1.00	0.68	1.00	0.73	2.89	0.00	0.98	-0.02
time (sec)	N/A	0.010	0.021	0.199	0.629	0.336	0.000	3.743	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	27	0	42	0	27	-1
N.S.	1	1.00	0.95	0.61	0.00	0.95	0.00	0.61	-0.02
time (sec)	N/A	0.005	0.030	0.201	0.000	0.332	0.000	3.617	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	43	76	0	31	0	0	-1
N.S.	1	1.00	0.52	0.92	0.00	0.37	0.00	0.00	-0.01
time (sec)	N/A	0.019	10.015	0.211	0.000	0.073	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	19	18	17	19	19
N.S.	1	1.00	1.00	0.86	0.86	0.82	0.77	0.86	0.86
time (sec)	N/A	0.002	0.005	0.191	0.487	0.376	0.145	3.334	2.674

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	27	81	0	12	36	0	-1
N.S.	1	1.00	0.21	0.62	0.00	0.09	0.27	0.00	-0.01
time (sec)	N/A	0.058	10.030	0.202	0.000	0.076	0.491	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	27	62	0	9	0	0	-1
N.S.	1	1.00	0.50	1.15	0.00	0.17	0.00	0.00	-0.02
time (sec)	N/A	0.014	10.023	0.197	0.000	0.085	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	0	76	0	30	-1
N.S.	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	-0.05
time (sec)	N/A	0.007	0.015	0.200	0.000	0.334	0.000	3.667	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	27	116	0	30	0	0	-1
N.S.	1	1.00	0.17	0.73	0.00	0.19	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.014	0.205	0.000	0.077	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	30	0	22	18
N.S.	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.86
time (sec)	N/A	0.003	0.025	0.239	0.482	0.328	0.000	3.554	2.870

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	19	0	12	20
N.S.	1	1.00	1.00	0.80	1.12	0.76	0.00	0.48	0.80
time (sec)	N/A	0.003	0.010	0.197	0.516	0.341	0.000	3.269	2.912

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	29	1521	0	0	0	0	-1
N.S.	1	1.00	0.10	5.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.175	10.023	0.428	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	29	270	0	19	0	0	-1
N.S.	1	1.00	0.11	1.04	0.00	0.07	0.00	0.00	-0.00
time (sec)	N/A	0.049	10.018	0.283	0.000	0.076	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	32	321	0	85	14	35	-1
N.S.	1	1.00	1.39	13.96	0.00	3.70	0.61	1.52	-0.04
time (sec)	N/A	0.011	0.089	0.345	0.000	0.384	0.449	4.063	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	27	232	0	11	0	0	-1
N.S.	1	1.00	0.23	2.00	0.00	0.09	0.00	0.00	-0.01
time (sec)	N/A	0.054	10.017	0.369	0.000	0.075	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	0	68	0	31	-1
N.S.	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	-0.04
time (sec)	N/A	0.007	0.015	0.199	0.000	0.327	0.000	3.453	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	27	1784	0	14	0	0	-1
N.S.	1	1.00	0.09	5.72	0.00	0.04	0.00	0.00	-0.00
time (sec)	N/A	0.194	10.008	0.393	0.000	0.072	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	27	353	0	37	0	0	-1
N.S.	1	1.00	0.10	1.26	0.00	0.13	0.00	0.00	-0.00
time (sec)	N/A	0.058	10.025	0.325	0.000	0.077	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	36	0	0	0	0	-1
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.010	0.026	0.276	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	35	0	0	0	0	-1
N.S.	1	1.00	0.83	0.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.028	0.215	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	37	0	0	0	0	-1
N.S.	1	1.00	0.85	0.71	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.029	0.234	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	80	33	30	18	0	0	0	43
N.S.	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	1.26
time (sec)	N/A	0.022	0.053	0.247	0.329	0.000	0.000	0.000	2.888

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	191	0	250	0	0	-1
N.S.	1	1.00	0.93	1.68	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.039	1.913	0.226	0.000	0.131	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	20	0	20	16
N.S.	1	1.00	1.00	1.06	1.06	1.25	0.00	1.25	1.00
time (sec)	N/A	0.004	0.002	0.029	0.286	0.340	0.000	3.701	3.400

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	32	32	0	32	26
N.S.	1	1.00	1.00	1.04	1.23	1.23	0.00	1.23	1.00
time (sec)	N/A	0.007	0.005	0.314	0.285	0.329	0.000	4.371	2.918

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	44	44	0	44	36
N.S.	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00
time (sec)	N/A	0.011	0.007	24.714	0.303	0.365	0.000	4.475	3.103

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	90	0	94	0	390	179
N.S.	1	1.00	0.95	0.61	0.00	0.64	0.00	2.65	1.22
time (sec)	N/A	0.101	0.482	0.035	0.000	0.355	0.000	3.674	2.954

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	101	66	0	70	0	206	129
N.S.	1	1.00	1.06	0.69	0.00	0.74	0.00	2.17	1.36
time (sec)	N/A	0.059	0.361	0.031	0.000	0.317	0.000	4.342	2.703

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	35	40	0	29	136	75	79
N.S.	1	1.00	0.74	0.85	0.00	0.62	2.89	1.60	1.68
time (sec)	N/A	0.033	0.214	0.008	0.000	0.324	0.478	4.388	2.706

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	160	73	0	318	0	1016	2500
N.S.	1	1.00	1.65	0.75	0.00	3.28	0.00	10.47	25.77
time (sec)	N/A	0.072	0.504	0.010	0.000	0.366	0.000	4.701	18.080

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	187	88	0	399	0	1190	2642
N.S.	1	1.00	1.82	0.85	0.00	3.87	0.00	11.55	25.65
time (sec)	N/A	0.080	0.809	0.014	0.000	0.361	0.000	9.226	18.884

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	187	604	0	196	0	797	1358
N.S.	1	1.00	0.82	2.65	0.00	0.86	0.00	3.50	5.96
time (sec)	N/A	0.262	0.524	0.020	0.000	0.330	0.000	3.948	81.167

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	131	431	0	149	0	445	1012
N.S.	1	1.00	0.79	2.61	0.00	0.90	0.00	2.70	6.13
time (sec)	N/A	0.151	0.354	0.016	0.000	0.342	0.000	3.556	37.521

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	114	81	377	0	103	388	189	110
N.S.	1	1.81	1.29	5.98	0.00	1.63	6.16	3.00	1.75
time (sec)	N/A	0.073	0.214	0.015	0.000	0.346	0.511	3.759	0.239

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	137	258	0	290	0	194	524
N.S.	1	1.00	1.03	1.94	0.00	2.18	0.00	1.46	3.94
time (sec)	N/A	0.161	0.274	0.015	0.000	0.353	0.000	4.352	11.141

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	128	274	0	367	0	311	2500
N.S.	1	1.00	0.91	1.94	0.00	2.60	0.00	2.21	17.73
time (sec)	N/A	0.158	0.516	0.017	0.000	0.376	0.000	5.043	28.822

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	138	294	0	208	0	1447	529
N.S.	1	1.00	0.37	0.78	0.00	0.55	0.00	3.86	1.41
time (sec)	N/A	0.277	0.776	0.053	0.000	0.329	0.000	5.581	3.303

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	93	222	0	159	942	866	385
N.S.	1	1.00	0.36	0.85	0.00	0.61	3.61	3.32	1.48
time (sec)	N/A	0.172	0.611	0.054	0.000	0.337	0.904	4.559	3.211

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	151	55	146	0	106	384	427	252
N.S.	1	2.36	0.86	2.28	0.00	1.66	6.00	6.67	3.94
time (sec)	N/A	0.075	0.485	0.032	0.000	0.342	0.771	5.241	2.995

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	244	181	0	516	0	2652	2500
N.S.	1	1.00	1.55	1.15	0.00	3.29	0.00	16.89	15.92
time (sec)	N/A	0.162	0.957	0.010	0.000	0.351	0.000	5.519	27.722

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	223	112	252	0	675	0	2594	2500
N.S.	1	1.38	0.69	1.56	0.00	4.17	0.00	16.01	15.43
time (sec)	N/A	0.194	10.282	0.017	0.000	0.375	0.000	24.437	33.215

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.004	0.065	0.015	0.000	0.323	0.202	4.663	2.968

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	13	63	13	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00
time (sec)	N/A	0.005	0.067	0.015	0.000	0.321	0.183	4.355	2.937

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	16	0	15	51	15	15
N.S.	1	1.00	1.00	0.70	0.00	0.65	2.22	0.65	0.65
time (sec)	N/A	0.016	0.108	0.007	0.000	0.326	0.190	6.275	2.838

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	45
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.18
time (sec)	N/A	0.078	0.134	0.220	0.494	0.346	0.000	3.350	3.021

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	59	34	51	270	76	563
N.S.	1	1.00	1.23	1.23	0.71	1.06	5.62	1.58	11.73
time (sec)	N/A	0.069	0.122	0.219	0.492	0.341	159.229	4.090	14.086

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	25	24	15	23	144	51	33
N.S.	1	1.00	1.32	1.26	0.79	1.21	7.58	2.68	1.74
time (sec)	N/A	0.041	0.089	0.211	0.480	0.358	53.901	4.007	2.977

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	37	58	17	40	61	48	206
N.S.	1	1.00	1.95	3.05	0.89	2.11	3.21	2.53	10.84
time (sec)	N/A	0.016	0.085	0.207	0.492	0.404	16.248	4.816	7.725

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	81	51	41	41	0	0	122
N.S.	1	1.00	2.53	1.59	1.28	1.28	0.00	0.00	3.81
time (sec)	N/A	0.062	0.179	0.211	0.489	0.321	0.000	0.000	4.098

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	39	50	24	44	0	149	120
N.S.	1	1.00	1.50	1.92	0.92	1.69	0.00	5.73	4.62
time (sec)	N/A	0.057	0.099	0.217	0.489	0.352	0.000	4.472	3.795

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	58	54	44	0	0	189
N.S.	1	1.00	1.26	1.71	1.59	1.29	0.00	0.00	5.56
time (sec)	N/A	0.067	0.157	0.214	0.495	0.342	0.000	0.000	4.879

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	155	90	0	122	0	451	179
N.S.	1	1.00	1.05	0.61	0.00	0.83	0.00	3.07	1.22
time (sec)	N/A	0.086	1.092	0.037	0.000	0.339	0.000	3.962	2.920

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	113	66	0	92	0	255	129
N.S.	1	1.00	1.19	0.69	0.00	0.97	0.00	2.68	1.36
time (sec)	N/A	0.069	0.738	0.032	0.000	0.326	0.000	4.901	2.862

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	40	0	50	0	107	79
N.S.	1	1.00	1.51	0.85	0.00	1.06	0.00	2.28	1.68
time (sec)	N/A	0.039	0.492	0.008	0.000	0.329	0.000	5.185	2.906

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	263	73	0	158	0	1093	213
N.S.	1	1.00	2.71	0.75	0.00	1.63	0.00	11.27	2.20
time (sec)	N/A	0.063	1.399	0.009	0.000	0.351	0.000	4.708	4.330

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	81	88	0	182	0	1402	1637
N.S.	1	1.00	0.79	0.85	0.00	1.77	0.00	13.61	15.89
time (sec)	N/A	0.063	10.151	0.013	0.000	0.385	0.000	7.404	10.926

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	123	120	0	243	0	1895	1610
N.S.	1	1.00	0.72	0.70	0.00	1.42	0.00	11.08	9.42
time (sec)	N/A	0.079	10.138	0.013	0.000	0.350	0.000	15.752	11.847

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	213	517	0	479	0	511	1107
N.S.	1	1.00	1.09	2.65	0.00	2.46	0.00	2.62	5.68
time (sec)	N/A	0.252	1.177	0.020	0.000	0.355	0.000	4.668	18.151

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	161	385	0	372	0	272	129
N.S.	1	1.00	1.13	2.71	0.00	2.62	0.00	1.92	0.91
time (sec)	N/A	0.172	0.666	0.013	0.000	0.366	0.000	4.735	0.246

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	232	266	0	346	0	306	2500
N.S.	1	1.00	1.72	1.97	0.00	2.56	0.00	2.27	18.52
time (sec)	N/A	0.131	0.911	0.018	0.000	0.381	0.000	5.135	19.761

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	215	272	0	317	0	438	2500
N.S.	1	1.00	1.56	1.97	0.00	2.30	0.00	3.17	18.12
time (sec)	N/A	0.081	1.203	0.014	0.000	0.349	0.000	5.166	17.444

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	148	313	0	126	0	532	787
N.S.	1	1.00	1.20	2.54	0.00	1.02	0.00	4.33	6.40
time (sec)	N/A	0.140	1.847	0.016	0.000	0.362	0.000	7.315	12.320

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	164	457	0	182	0	802	1290
N.S.	1	1.00	0.94	2.63	0.00	1.05	0.00	4.61	7.41
time (sec)	N/A	0.156	10.274	0.017	0.000	0.332	0.000	6.365	18.735

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	114	246	0	225	0	932	429
N.S.	1	1.00	0.41	0.89	0.00	0.81	0.00	3.36	1.55
time (sec)	N/A	0.228	10.268	0.056	0.000	0.337	0.000	4.236	3.342

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	197	172	0	167	0	480	268
N.S.	1	1.00	1.21	1.06	0.00	1.02	0.00	2.94	1.64
time (sec)	N/A	0.157	1.063	0.032	0.000	0.342	0.000	6.789	3.360

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	127	148	0	321	0	2374	762
N.S.	1	1.00	0.82	0.95	0.00	2.07	0.00	15.32	4.92
time (sec)	N/A	0.140	10.216	0.010	0.000	0.362	0.000	5.704	7.023

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	223	143	237	0	260	0	2318	559
N.S.	1	1.42	0.91	1.51	0.00	1.66	0.00	14.76	3.56
time (sec)	N/A	0.150	10.152	0.014	0.000	0.362	0.000	23.327	7.488

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	275	146	300	0	297	0	2766	287
N.S.	1	1.68	0.89	1.83	0.00	1.81	0.00	16.87	1.75
time (sec)	N/A	0.126	10.158	0.016	0.000	0.358	0.000	55.114	5.743

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	63	23	44	65	54	209
N.S.	1	1.00	1.45	2.03	0.74	1.42	2.10	1.74	6.74
time (sec)	N/A	0.030	0.049	0.243	0.491	0.347	1.186	2.730	8.122

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	44	33	31	32	0	77	42
N.S.	1	1.00	1.16	0.87	0.82	0.84	0.00	2.03	1.11
time (sec)	N/A	0.238	0.013	0.264	0.481	0.332	0.000	3.176	3.065

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	59	34	51	270	76	381
N.S.	1	1.00	1.25	1.23	0.71	1.06	5.62	1.58	7.94
time (sec)	N/A	0.172	0.017	0.260	0.486	0.396	143.804	3.931	10.394

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	25	26	17	25	144	54	25
N.S.	1	1.00	1.19	1.24	0.81	1.19	6.86	2.57	1.19
time (sec)	N/A	0.078	0.005	0.254	0.492	0.348	54.637	3.038	3.061

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	38	59	20	41	63	49	205
N.S.	1	1.00	1.73	2.68	0.91	1.86	2.86	2.23	9.32
time (sec)	N/A	0.036	0.006	0.253	0.502	0.374	21.115	3.384	3.714

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	81	51	41	41	0	0	122
N.S.	1	1.00	2.53	1.59	1.28	1.28	0.00	0.00	3.81
time (sec)	N/A	0.137	0.017	0.258	0.494	0.351	0.000	0.000	4.127

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	39	50	24	44	0	149	118
N.S.	1	1.00	1.50	1.92	0.92	1.69	0.00	5.73	4.54
time (sec)	N/A	0.144	0.006	0.260	0.481	0.387	0.000	4.203	3.787

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	57	51	43	0	0	186
N.S.	1	1.00	1.27	1.73	1.55	1.30	0.00	0.00	5.64
time (sec)	N/A	0.151	0.010	0.255	0.484	0.345	0.000	0.000	4.777

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	81	48	0	36	0	0	93
N.S.	1	1.00	2.89	1.71	0.00	1.29	0.00	0.00	3.32
time (sec)	N/A	0.219	0.184	0.220	0.000	0.393	0.000	0.000	4.493

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	42	62	0	37	224	41	200
N.S.	1	1.00	1.27	1.88	0.00	1.12	6.79	1.24	6.06
time (sec)	N/A	0.100	0.089	0.227	0.000	0.354	16.551	3.725	10.853

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	86	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.454	0.009	0.000	0.000	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	159	341	252	150	279	163	-1
N.S.	1	1.00	0.91	1.95	1.44	0.86	1.59	0.93	-0.01
time (sec)	N/A	0.093	0.781	0.214	0.282	0.359	5.474	3.635	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	118	124	95	110	116	103	210
N.S.	1	1.00	0.87	0.91	0.70	0.81	0.85	0.76	1.54
time (sec)	N/A	0.080	0.600	0.209	0.272	0.342	2.155	3.373	4.657

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	83	75	46	74	54	65	136
N.S.	1	1.00	1.22	1.10	0.68	1.09	0.79	0.96	2.00
time (sec)	N/A	0.032	0.097	0.243	0.282	0.349	1.171	3.306	3.888

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	337	1325	0	187	0	219	-1
N.S.	1	1.00	2.88	11.32	0.00	1.60	0.00	1.87	-0.01
time (sec)	N/A	0.073	0.902	0.029	0.000	0.353	0.000	5.204	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	378	4161	0	290	0	398	-1
N.S.	1	1.00	2.50	27.56	0.00	1.92	0.00	2.64	-0.01
time (sec)	N/A	0.079	1.447	0.041	0.000	0.376	0.000	4.250	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	464	9871	0	521	0	679	-1
N.S.	1	1.00	2.40	51.15	0.00	2.70	0.00	3.52	-0.01
time (sec)	N/A	0.092	1.907	0.063	0.000	0.408	0.000	4.980	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	212	0	0	408	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.127	1.475	0.018	0.000	0.417	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	170	0	0	335	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	1.83	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.281	0.011	0.000	0.406	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	142	0	0	299	0	0	-1
N.S.	1	1.00	0.97	0.00	0.00	2.03	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.894	0.006	0.000	0.405	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	0	0	296	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.868	0.010	0.000	0.416	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	169	0	0	490	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.113	1.413	0.007	0.000	0.426	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	209	0	0	784	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	3.94	0.00	0.00	-0.01
time (sec)	N/A	0.130	1.434	0.008	0.000	0.452	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	26	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.63	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.041	0.037	0.000	0.381	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	59	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.527	0.012	0.000	0.379	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	35	60	0	34	415	0	-1
N.S.	1	1.00	0.78	1.33	0.00	0.76	9.22	0.00	-0.02
time (sec)	N/A	0.007	0.059	0.033	0.000	0.381	0.592	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	55	0	28	197	0	-1
N.S.	1	1.00	0.76	1.34	0.00	0.68	4.80	0.00	-0.02
time (sec)	N/A	0.005	0.060	0.025	0.000	0.376	0.578	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	31	64	0	30	197	0	-1
N.S.	1	1.00	0.76	1.56	0.00	0.73	4.80	0.00	-0.02
time (sec)	N/A	0.005	0.046	0.026	0.000	0.365	0.633	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	72	0	0	70	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	1.06	0.00	0.00	-0.02
time (sec)	N/A	0.023	3.958	0.010	0.000	0.414	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	134	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	1.121	0.009	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	490	809	0	326	0	373	-1
N.S.	1	1.00	1.62	2.67	0.00	1.08	0.00	1.23	-0.00
time (sec)	N/A	0.272	5.830	0.361	0.000	0.353	0.000	3.779	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	213	300	0	204	0	224	-1
N.S.	1	1.00	0.90	1.27	0.00	0.86	0.00	0.95	-0.00
time (sec)	N/A	0.172	10.297	0.301	0.000	0.346	0.000	3.647	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	478	173	0	117	0	111	-1
N.S.	1	1.00	4.05	1.47	0.00	0.99	0.00	0.94	-0.01
time (sec)	N/A	0.045	2.264	0.309	0.000	0.378	0.000	2.914	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	570	4918	0	362	0	523	-1
N.S.	1	1.00	2.65	22.87	0.00	1.68	0.00	2.43	-0.00
time (sec)	N/A	0.148	1.804	0.040	0.000	5.667	0.000	3.878	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	551	58069	0	803	0	1243	-1
N.S.	1	1.00	2.07	218.30	0.00	3.02	0.00	4.67	-0.00
time (sec)	N/A	0.163	3.771	0.099	0.000	3.037	0.000	4.272	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	300	295126	0	1942	0	3015	-1
N.S.	1	1.00	0.91	894.32	0.00	5.88	0.00	9.14	-0.00
time (sec)	N/A	0.205	10.554	0.145	0.000	26.982	0.000	11.771	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	985	0	0	858	0	0	-1
N.S.	1	1.00	2.66	0.00	0.00	2.32	0.00	0.00	-0.00
time (sec)	N/A	0.421	5.003	0.019	0.000	0.548	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	443	0	0	612	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	2.03	0.00	0.00	-0.00
time (sec)	N/A	0.301	2.841	0.013	0.000	0.496	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	327	0	0	600	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	2.58	0.00	0.00	-0.00
time (sec)	N/A	0.225	1.506	0.007	0.000	0.504	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	341	0	0	620	0	0	-1
N.S.	1	1.00	1.40	0.00	0.00	2.54	0.00	0.00	-0.00
time (sec)	N/A	0.218	1.580	0.010	0.000	0.519	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	395	0	0	1312	0	0	-1
N.S.	1	1.00	1.47	0.00	0.00	4.88	0.00	0.00	-0.00
time (sec)	N/A	0.281	2.574	0.009	0.000	0.650	0.000	0.000	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	557	0	0	2523	0	0	-1
N.S.	1	1.00	1.66	0.00	0.00	7.53	0.00	0.00	-0.00
time (sec)	N/A	0.372	3.640	0.010	0.000	1.492	0.000	0.000	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	138	216	0	158	0	0	-1
N.S.	1	1.00	0.84	1.32	0.00	0.96	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.416	0.242	0.000	0.376	0.000	0.000	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	167	0	78	14884	0	-1
N.S.	1	1.00	0.85	1.55	0.00	0.72	137.81	0.00	-0.01
time (sec)	N/A	0.045	0.237	0.016	0.000	0.352	15.841	0.000	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	120	0	32	2147	0	-1
N.S.	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02
time (sec)	N/A	0.016	0.130	0.013	0.000	0.394	1.653	0.000	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.155	0.026	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.171	0.031	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	150	0	0	159	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.450	0.037	0.000	0.376	0.000	0.000	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	100	0	0	79	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.238	0.023	0.000	0.353	0.000	0.000	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	0	0	33	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.145	0.024	0.000	0.357	0.000	0.000	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.162	0.044	0.000	0.000	0.000	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.167	0.043	0.000	0.000	0.000	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	157	0	0	201	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.376	0.022	0.000	0.390	0.000	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	111	0	0	110	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.222	0.023	0.000	0.358	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	48	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.184	0.022	0.000	0.362	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	0	15	311	15	15
N.S.	1	1.00	1.00	0.00	0.00	0.88	18.29	0.88	0.88
time (sec)	N/A	0.036	0.023	0.023	0.000	0.352	1.538	3.346	3.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.074	0.023	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.088	0.026	0.000	0.000	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	173	0	0	204	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.080	0.452	0.039	0.000	0.372	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	123	0	0	113	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.240	0.039	0.000	0.375	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	0	0	51	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.185	0.040	0.000	0.384	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	0	18	36	18	18
N.S.	1	1.00	1.00	0.00	0.00	0.90	1.80	0.90	0.90
time (sec)	N/A	0.038	0.022	0.041	0.000	0.355	0.959	3.546	3.193

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.077	0.039	0.000	0.000	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.094	0.041	0.000	0.000	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	280	0	0	570	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	1.56	0.00	0.00	-0.00
time (sec)	N/A	0.327	6.918	0.074	0.000	0.434	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	186	0	0	224	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.177	1.587	0.006	0.000	0.417	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.065	1.038	0.005	0.000	0.383	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	2.471	0.092	0.000	0.000	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	1.426	0.092	0.000	0.000	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	89	0	0	80	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.029	0.006	0.000	0.383	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.029	0.091	0.000	0.000	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	228	0	0	346	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.282	6.544	0.097	0.000	0.391	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	135	0	0	124	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.222	2.443	0.088	0.000	0.367	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	41
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.170	0.235	0.090	0.000	0.347	0.000	0.000	3.084

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	112	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	3.620	0.086	0.000	0.000	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	0	0	41	0	0	39
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	0.95
time (sec)	N/A	0.308	0.013	0.089	0.000	0.347	0.000	0.000	3.076

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	175	0	0	223	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.167	0.092	0.000	0.388	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.040	0.088	0.000	0.365	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	152	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.390	0.134	0.091	0.000	0.000	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	0	0	117	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.482	0.016	0.089	0.000	0.360	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	772	339	0	0	0	0	-1
N.S.	1	1.00	4.04	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	3.926	0.251	0.000	0.000	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	191	193	0	150	70	0	50
N.S.	1	1.00	2.36	2.38	0.00	1.85	0.86	0.00	0.62
time (sec)	N/A	0.042	0.079	0.059	0.000	0.406	0.330	0.000	3.113

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	233	193	0	152	63	0	28
N.S.	1	1.00	3.19	2.64	0.00	2.08	0.86	0.00	0.38
time (sec)	N/A	0.031	0.092	0.053	0.000	0.396	0.324	0.000	3.141

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	70	0	154	70	69	54
N.S.	1	1.00	2.29	1.84	0.00	4.05	1.84	1.82	1.42
time (sec)	N/A	0.044	0.042	0.036	0.000	0.434	0.374	4.315	3.315

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	87	72	0	156	63	65	67
N.S.	1	1.00	2.29	1.89	0.00	4.11	1.66	1.71	1.76
time (sec)	N/A	0.043	0.041	0.037	0.000	0.427	0.403	4.334	3.118

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	78	0	146	0	0	196
N.S.	1	1.00	0.00	2.05	0.00	3.84	0.00	0.00	5.16
time (sec)	N/A	0.066	0.248	0.044	0.000	0.420	0.000	0.000	3.419

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	0	72	0	146	0	0	139
N.S.	1	1.00	0.00	1.89	0.00	3.84	0.00	0.00	3.66
time (sec)	N/A	0.063	0.250	0.044	0.000	0.469	0.000	0.000	5.302

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	35	147	78	38	42
N.S.	1	1.00	1.00	1.00	0.83	3.50	1.86	0.90	1.00
time (sec)	N/A	0.048	0.017	0.031	0.515	0.355	0.382	8.134	3.100

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	70	160	75	41	199
N.S.	1	1.00	1.05	0.95	1.59	3.64	1.70	0.93	4.52
time (sec)	N/A	0.049	0.017	0.023	0.510	0.361	0.379	6.919	3.107

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	74	0	212	90	73	278
N.S.	1	1.00	2.12	1.85	0.00	5.30	2.25	1.82	6.95
time (sec)	N/A	0.085	0.034	0.100	0.000	0.363	0.689	5.749	3.209

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	85	77	0	215	80	69	30
N.S.	1	1.00	2.12	1.92	0.00	5.38	2.00	1.72	0.75
time (sec)	N/A	0.084	0.036	0.101	0.000	0.362	0.695	6.520	3.143

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	78	0	148	0	0	-1
N.S.	1	1.45	1.00	1.86	0.00	3.52	0.00	0.00	-0.02
time (sec)	N/A	0.148	0.943	0.056	0.000	0.360	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	61	42	74	0	148	0	0	-1
N.S.	1	1.45	1.00	1.76	0.00	3.52	0.00	0.00	-0.02
time (sec)	N/A	0.154	0.246	0.056	0.000	0.361	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	154	73	73	233
N.S.	1	1.00	2.15	1.85	0.00	3.85	1.82	1.82	5.82
time (sec)	N/A	0.060	0.034	0.040	0.000	0.372	0.651	5.180	3.488

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	86	74	0	156	66	69	67
N.S.	1	1.00	2.15	1.85	0.00	3.90	1.65	1.72	1.68
time (sec)	N/A	0.074	0.032	0.038	0.000	0.357	0.660	5.090	3.381

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	42	35	147	78	38	42
N.S.	1	1.00	1.00	1.00	0.83	3.50	1.86	0.90	1.00
time (sec)	N/A	0.046	0.015	0.023	0.518	0.358	0.479	4.547	3.433

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	42	70	160	75	41	923
N.S.	1	1.00	1.05	0.95	1.59	3.64	1.70	0.93	20.98
time (sec)	N/A	0.046	0.016	0.023	0.515	0.362	0.478	5.463	3.527

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	78	0	148	0	0	-1
N.S.	1	1.00	1.00	1.86	0.00	3.52	0.00	0.00	-0.02
time (sec)	N/A	0.154	2.283	0.034	0.000	0.370	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	74	0	148	0	0	-1
N.S.	1	1.00	1.00	1.76	0.00	3.52	0.00	0.00	-0.02
time (sec)	N/A	0.145	0.419	0.034	0.000	0.388	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	84	0	167	0	0	-1
N.S.	1	1.00	0.00	2.00	0.00	3.98	0.00	0.00	-0.02
time (sec)	N/A	0.172	0.431	0.055	0.000	0.386	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	B	F	A	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	0	78	0	167	0	0	-1
N.S.	1	1.00	0.00	1.86	0.00	3.98	0.00	0.00	-0.02
time (sec)	N/A	0.164	0.438	0.055	0.000	0.359	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	125	4947	125	233	0	155	167
N.S.	1	1.00	0.93	36.92	0.93	1.74	0.00	1.16	1.25
time (sec)	N/A	0.252	0.164	0.057	0.273	0.377	0.000	4.000	3.644

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	3410	62	161	88	72	123
N.S.	1	1.00	0.96	49.42	0.90	2.33	1.28	1.04	1.78
time (sec)	N/A	0.146	0.053	0.037	0.286	0.372	2.875	4.942	3.498

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	26	1931	21	105	29	22	45
N.S.	1	1.00	1.13	83.96	0.91	4.57	1.26	0.96	1.96
time (sec)	N/A	0.058	0.022	0.034	0.284	0.370	1.938	3.328	3.469

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	69	2175	0	316	88	94	1270
N.S.	1	1.00	0.78	24.72	0.00	3.59	1.00	1.07	14.43
time (sec)	N/A	0.170	0.075	0.040	0.000	0.421	3.845	3.411	3.948

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	139	2459	0	530	0	210	2500
N.S.	1	1.00	0.92	16.28	0.00	3.51	0.00	1.39	16.56
time (sec)	N/A	0.243	0.212	0.050	0.000	0.771	0.000	2.891	5.563

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	99	3485	0	1168	0	0	-1
N.S.	1	1.00	0.67	23.71	0.00	7.95	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.257	0.042	0.000	0.490	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	63	1995	0	510	0	107	-1
N.S.	1	1.00	0.61	19.37	0.00	4.95	0.00	1.04	-0.01
time (sec)	N/A	0.047	0.202	0.034	0.000	0.376	0.000	3.738	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	105	2289	0	581	0	211	-1
N.S.	1	1.00	0.66	14.31	0.00	3.63	0.00	1.32	-0.01
time (sec)	N/A	0.167	0.351	0.042	0.000	0.419	0.000	3.140	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	125	647	125	191	0	156	200
N.S.	1	1.00	0.89	4.62	0.89	1.36	0.00	1.11	1.43
time (sec)	N/A	0.214	0.161	0.177	0.294	0.386	0.000	3.189	3.740

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	555	62	118	95	72	119
N.S.	1	1.00	0.90	7.60	0.85	1.62	1.30	0.99	1.63
time (sec)	N/A	0.139	0.054	0.070	0.275	0.348	3.205	3.685	3.564

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	501	22	61	32	23	60
N.S.	1	1.00	1.12	19.27	0.85	2.35	1.23	0.88	2.31
time (sec)	N/A	0.078	0.027	0.066	0.279	0.382	1.999	3.235	3.508

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	69	621	0	232	97	94	156
N.S.	1	1.00	0.74	6.68	0.00	2.49	1.04	1.01	1.68
time (sec)	N/A	0.154	0.072	0.240	0.000	0.379	4.233	3.730	4.308

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	139	774	0	445	0	211	248
N.S.	1	1.00	0.90	5.03	0.00	2.89	0.00	1.37	1.61
time (sec)	N/A	0.211	0.199	0.206	0.000	0.447	0.000	4.203	5.463

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	296	1421	0	0	0	0	-1
N.S.	1	1.00	0.95	4.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.340	10.323	0.087	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	1505	0	0	0	0	-1
N.S.	1	1.00	0.00	4.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	10.105	0.086	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	0	1201	0	0	0	0	-1
N.S.	1	1.00	0.00	4.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.185	10.047	0.066	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	496	2227	0	0	0	0	-1
N.S.	1	1.00	1.55	6.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	10.445	0.135	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	604	1771	0	0	0	0	-1
N.S.	1	1.00	1.86	5.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	13.349	0.092	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	320	0	0	0	0	0	-1
N.S.	1	1.00	2.37	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.489	0.004	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	156	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.361	0.007	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	32	0	61	25	32	26	25
N.S.	1	1.00	1.19	0.00	2.26	0.93	1.19	0.96	0.93
time (sec)	N/A	0.072	0.047	0.007	0.278	0.328	15.586	3.309	3.346

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.010	0.204	0.477	0.348	0.071	4.210	0.047

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.006	0.020	0.205	0.490	0.356	0.116	4.141	3.080

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	20	19	19	22	20	19
N.S.	1	1.00	0.93	0.74	0.70	0.70	0.81	0.74	0.70
time (sec)	N/A	0.008	0.018	0.203	0.281	0.330	0.073	4.441	3.103

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	33	92	24	24	0	24	24
N.S.	1	1.00	1.03	2.88	0.75	0.75	0.00	0.75	0.75
time (sec)	N/A	0.010	0.023	0.197	0.271	0.342	0.000	3.903	0.030

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	22	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76
time (sec)	N/A	0.008	0.016	0.197	0.263	0.319	0.072	5.036	0.032

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	14	14	15	15	14
N.S.	1	1.00	0.90	0.75	0.70	0.70	0.75	0.75	0.70
time (sec)	N/A	0.007	0.011	0.202	0.279	0.343	0.050	4.526	0.077

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	46	45	47	68	45	73
N.S.	1	1.00	1.00	0.74	0.73	0.76	1.10	0.73	1.18
time (sec)	N/A	0.026	0.041	0.204	0.497	0.345	0.182	4.072	3.076

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	72	173	49	49	0	49	49
N.S.	1	1.00	0.99	2.37	0.67	0.67	0.00	0.67	0.67
time (sec)	N/A	0.020	0.033	0.208	0.298	0.343	0.000	4.360	0.036

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	117	83	82	76	0	82	82
N.S.	1	1.00	0.90	0.64	0.63	0.58	0.00	0.63	0.63
time (sec)	N/A	0.032	0.042	0.216	0.277	0.362	0.000	4.477	0.153

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	126	172	272	638	0	139	223
N.S.	1	1.00	0.63	0.86	1.36	3.19	0.00	0.70	1.12
time (sec)	N/A	0.278	0.049	0.211	0.522	1.147	0.000	4.248	0.122

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.010	0.204	0.511	0.344	0.173	3.613	0.139

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.012	0.012	0.229	0.274	0.394	0.057	3.304	0.042

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	62	66	83	120	0	83	42
N.S.	1	1.00	0.57	0.61	0.77	1.11	0.00	0.77	0.39
time (sec)	N/A	0.054	0.079	0.299	0.498	0.350	0.000	3.431	0.085

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	61	60	62	0	60	78
N.S.	1	1.00	1.09	0.80	0.79	0.82	0.00	0.79	1.03
time (sec)	N/A	0.024	0.047	0.313	0.508	0.350	0.000	4.304	3.080

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	110	76	75	71	0	75	75
N.S.	1	1.00	0.92	0.64	0.63	0.60	0.00	0.63	0.63
time (sec)	N/A	0.030	0.032	0.301	0.286	0.353	0.000	4.175	0.152

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	127	166	293	547	0	140	208
N.S.	1	1.00	0.63	0.83	1.46	2.72	0.00	0.70	1.03
time (sec)	N/A	0.157	0.047	0.318	0.524	1.165	0.000	4.465	0.061

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	36	15	44	0	0	32
N.S.	1	1.00	0.97	1.00	0.42	1.22	0.00	0.00	0.89
time (sec)	N/A	0.024	1.008	0.306	0.321	0.349	0.000	0.000	3.257

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	27	5	35	0	76	23
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	2.62	0.79
time (sec)	N/A	0.022	9.307	0.339	0.319	0.329	0.000	5.050	3.096

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	27	5	35	0	56	23
N.S.	1	1.00	0.93	0.93	0.17	1.21	0.00	1.93	0.79
time (sec)	N/A	0.015	5.941	0.295	0.304	0.339	0.000	4.681	3.064

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	25	5	33	0	51	21
N.S.	1	1.00	0.96	1.00	0.20	1.32	0.00	2.04	0.84
time (sec)	N/A	0.009	4.779	0.293	0.318	0.327	0.000	4.297	3.044

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	24	5	32	0	42	20
N.S.	1	1.00	1.00	1.00	0.21	1.33	0.00	1.75	0.83
time (sec)	N/A	0.021	1.684	0.296	0.305	0.320	0.000	4.076	3.079

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	27	5	35	0	60	23
N.S.	1	1.00	0.90	0.93	0.17	1.21	0.00	2.07	0.79
time (sec)	N/A	0.021	3.454	0.317	0.317	0.330	0.000	3.427	3.102

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.118	0.049	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	112	0	0	0	121	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.88	0.00	-0.01
time (sec)	N/A	0.083	0.155	0.026	0.000	0.000	12.597	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	75	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.028	0.099	0.009	0.000	0.000	2.671	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	50	0	0	0	34	0	51
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.85	0.00	1.28
time (sec)	N/A	0.008	0.041	0.008	0.000	0.000	0.647	0.000	3.135

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.136	0.036	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.114	0.034	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	99	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.156	0.047	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	155	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.222	0.058	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	8	44	0	0	29
N.S.	1	1.00	1.00	1.06	0.24	1.33	0.00	0.00	0.88
time (sec)	N/A	0.021	0.059	0.390	0.292	0.374	0.000	0.000	3.358

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	31	5	41	0	15	25
N.S.	1	1.00	1.03	1.00	0.16	1.32	0.00	0.48	0.81
time (sec)	N/A	0.023	0.033	0.342	0.302	0.355	0.000	4.731	3.137

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	7	41	0	0	27
N.S.	1	1.00	1.00	1.07	0.25	1.46	0.00	0.00	0.96
time (sec)	N/A	0.017	0.044	0.329	0.284	0.378	0.000	0.000	3.105

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	30	4	51	0	0	-1
N.S.	1	1.00	1.00	1.07	0.14	1.82	0.00	0.00	-0.04
time (sec)	N/A	0.012	0.053	0.329	0.289	0.351	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	7	41	0	0	22
N.S.	1	1.00	1.00	1.08	0.27	1.58	0.00	0.00	0.85
time (sec)	N/A	0.021	0.045	0.298	0.294	0.328	0.000	0.000	3.345

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	5	44	0	5	25
N.S.	1	1.00	1.00	1.00	0.16	1.42	0.00	0.16	0.81
time (sec)	N/A	0.022	0.053	0.339	0.281	0.416	0.000	3.701	3.465

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	540	1145	0	235	0	0	-1
N.S.	1	1.00	1.33	2.82	0.00	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.319	21.939	0.362	0.000	0.104	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.73
time (sec)	N/A	0.007	10.036	0.322	0.274	0.333	0.072	2.813	3.519

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	72	14	13	20	31	13	19
N.S.	1	1.00	4.24	0.82	0.76	1.18	1.82	0.76	1.12
time (sec)	N/A	0.006	10.039	0.325	0.281	0.348	0.093	3.328	3.163

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.006	10.027	0.349	0.277	0.345	0.083	3.695	3.098

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	31	11	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	2.07	0.73	0.73
time (sec)	N/A	0.006	10.042	0.385	0.268	0.332	0.079	3.627	3.108

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	21	11	11	10	23	10
N.S.	1	1.00	1.00	1.62	0.85	0.85	0.77	1.77	0.77
time (sec)	N/A	0.007	0.013	0.319	0.269	0.333	0.051	3.327	3.232

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	34	33	32	39	33	32
N.S.	1	1.00	1.02	0.77	0.75	0.73	0.89	0.75	0.73
time (sec)	N/A	0.017	0.022	0.320	0.270	0.329	0.063	3.363	0.111

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	16	15	15	17	16	15
N.S.	1	1.00	0.90	0.76	0.71	0.71	0.81	0.76	0.71
time (sec)	N/A	0.009	0.012	0.329	0.267	0.326	0.048	3.354	3.005

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	34	27	26	25	27	26	25
N.S.	1	1.00	1.03	0.82	0.79	0.76	0.82	0.79	0.76
time (sec)	N/A	0.015	0.018	0.273	0.265	0.337	0.058	3.114	3.095

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	36	35	30	29	42	38	29
N.S.	1	1.00	1.09	1.06	0.91	0.88	1.27	1.15	0.88
time (sec)	N/A	0.016	0.029	0.312	0.270	0.330	0.199	3.407	3.024

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	17	10	12	10	10	36	0	10
N.S.	1	1.70	1.00	1.20	1.00	1.00	3.60	0.00	1.00
time (sec)	N/A	0.040	0.024	0.319	0.270	0.338	1.669	0.000	3.273

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	10	12	86	10	8	0	39
N.S.	1	1.00	0.59	0.71	5.06	0.59	0.47	0.00	2.29
time (sec)	N/A	0.027	0.005	0.327	0.286	0.365	31.217	0.000	3.368

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	38	101	0	0	0	-1
N.S.	1	1.00	0.92	1.03	2.73	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.063	4.043	0.435	0.390	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	36	99	0	0	0	-1
N.S.	1	1.00	0.91	1.03	2.83	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.060	10.248	0.429	0.379	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	35	96	0	0	0	148
N.S.	1	1.00	0.91	1.03	2.82	0.00	0.00	0.00	4.35
time (sec)	N/A	0.077	7.168	0.419	0.385	0.000	0.000	0.000	9.778

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	99	0	0	0	37
N.S.	1	0.00	0.92	1.03	2.68	0.00	0.00	0.00	1.00
time (sec)	N/A	2.421	9.057	0.404	0.378	0.000	0.000	0.000	9.680

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	B	F(-1)	F(-1)	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	0	34	38	99	0	0	0	146
N.S.	1	0.00	0.92	1.03	2.68	0.00	0.00	0.00	3.95
time (sec)	N/A	2.271	9.513	0.390	0.387	0.000	0.000	0.000	9.216

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	105	78	151	94	139	151	167
N.S.	1	1.00	0.57	0.42	0.82	0.51	0.75	0.82	0.90
time (sec)	N/A	0.185	0.081	0.359	0.264	0.393	3.728	4.627	3.370

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	93	66	112	81	110	127	124
N.S.	1	1.00	0.67	0.48	0.81	0.59	0.80	0.92	0.90
time (sec)	N/A	0.118	0.062	0.298	0.269	0.377	3.198	5.570	0.063

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	68	54	72	67	94	131	79
N.S.	1	1.00	0.76	0.61	0.81	0.75	1.06	1.47	0.89
time (sec)	N/A	0.068	0.043	0.290	0.276	0.506	2.487	4.245	0.030

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	35	35	49	68	82	36
N.S.	1	1.00	1.00	0.85	0.85	1.20	1.66	2.00	0.88
time (sec)	N/A	0.021	0.020	0.262	0.278	0.369	0.075	4.088	0.046

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	63	51	70	118	65	59	130
N.S.	1	1.00	1.11	0.89	1.23	2.07	1.14	1.04	2.28
time (sec)	N/A	0.045	0.054	0.299	0.512	0.344	13.603	4.000	0.088

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	64	63	73	147	139	80	131
N.S.	1	1.00	1.19	1.17	1.35	2.72	2.57	1.48	2.43
time (sec)	N/A	0.048	0.104	0.304	0.496	0.361	35.463	3.846	0.124

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	82	113	181	292	105	80
N.S.	1	1.00	0.96	1.02	1.41	2.26	3.65	1.31	1.00
time (sec)	N/A	0.054	0.173	0.313	0.509	0.362	114.485	3.004	3.400

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	232	383	268	286	0	915	-1
N.S.	1	1.00	0.71	1.17	0.82	0.88	0.00	2.81	-0.00
time (sec)	N/A	0.177	0.219	0.104	0.477	0.387	0.000	3.238	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	147	183	167	184	0	549	-1
N.S.	1	1.00	0.66	0.82	0.75	0.82	0.00	2.45	-0.00
time (sec)	N/A	0.116	0.131	0.099	0.266	0.410	0.000	3.129	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	84	94	93	103	0	279	-1
N.S.	1	1.00	0.63	0.71	0.70	0.77	0.00	2.10	-0.01
time (sec)	N/A	0.067	0.068	0.092	0.267	0.445	0.000	3.623	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	41	43	50	0	99	44
N.S.	1	1.00	0.98	0.73	0.77	0.89	0.00	1.77	0.79
time (sec)	N/A	0.022	0.034	0.066	0.285	0.417	0.000	5.400	3.395

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	124	152	0	194	0	150	-1
N.S.	1	1.00	1.07	1.31	0.00	1.67	0.00	1.29	-0.01
time (sec)	N/A	0.105	0.230	0.123	0.000	0.400	0.000	3.938	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	144	166	0	1003	0	232	-1
N.S.	1	1.00	1.05	1.21	0.00	7.32	0.00	1.69	-0.01
time (sec)	N/A	0.117	0.392	0.135	0.000	0.401	0.000	3.651	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	217	373	0	2856	0	895	-1
N.S.	1	1.00	0.97	1.67	0.00	12.75	0.00	4.00	-0.00
time (sec)	N/A	0.314	1.807	0.142	0.000	0.622	0.000	3.333	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	235	288	243	228	0	341	317
N.S.	1	1.00	1.02	1.25	1.06	0.99	0.00	1.48	1.38
time (sec)	N/A	0.191	0.173	0.031	0.277	0.334	0.000	3.616	0.069

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	151	166	148	138	0	198	184
N.S.	1	1.00	1.00	1.10	0.98	0.91	0.00	1.31	1.22
time (sec)	N/A	0.116	0.111	0.026	0.277	0.361	0.000	3.896	3.213

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	85	81	71	109	105	89
N.S.	1	1.00	0.94	0.94	0.90	0.79	1.21	1.17	0.99
time (sec)	N/A	0.059	0.064	0.022	0.291	0.358	2.252	4.036	0.054

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	40	87	35	33	49	38	33
N.S.	1	1.00	0.98	2.12	0.85	0.80	1.20	0.93	0.80
time (sec)	N/A	0.016	0.025	0.025	0.314	0.363	0.260	3.886	0.053

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	62	69	95	125	85	88	181
N.S.	1	1.00	0.76	0.84	1.16	1.52	1.04	1.07	2.21
time (sec)	N/A	0.056	0.049	0.040	0.525	0.382	5.887	4.835	3.282

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	118	128	191	283	0	191	220
N.S.	1	1.00	0.91	0.98	1.47	2.18	0.00	1.47	1.69
time (sec)	N/A	0.126	0.181	0.043	0.513	0.403	0.000	3.670	3.591

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	198	227	367	534	0	375	1094
N.S.	1	1.00	0.97	1.11	1.80	2.62	0.00	1.84	5.36
time (sec)	N/A	0.200	0.376	0.053	0.521	0.743	0.000	4.131	5.009

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	283	277	251	392	0	324	461
N.S.	1	1.00	1.18	1.15	1.05	1.63	0.00	1.35	1.92
time (sec)	N/A	0.201	0.198	0.037	0.281	0.361	0.000	3.726	0.094

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	194	161	158	269	0	191	197
N.S.	1	1.00	1.17	0.97	0.95	1.62	0.00	1.15	1.19
time (sec)	N/A	0.132	0.125	0.030	0.275	0.348	0.000	3.140	3.197

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	108	87	90	163	131	102	98
N.S.	1	1.00	1.14	0.92	0.95	1.72	1.38	1.07	1.03
time (sec)	N/A	0.065	0.077	0.029	0.279	0.360	18.601	3.143	0.062

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	219	43	75	124	44	43
N.S.	1	1.00	0.91	4.66	0.91	1.60	2.64	0.94	0.91
time (sec)	N/A	0.023	0.041	0.034	0.285	0.350	0.620	3.604	0.047

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	107	118	176	444	153	174	125
N.S.	1	1.00	0.83	0.91	1.36	3.44	1.19	1.35	0.97
time (sec)	N/A	0.086	0.164	0.048	0.541	0.380	22.057	4.699	3.511

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	177	182	367	854	0	311	275
N.S.	1	1.00	0.88	0.90	1.82	4.23	0.00	1.54	1.36
time (sec)	N/A	0.175	0.308	0.059	0.662	0.562	0.000	4.922	0.731

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	301	303	659	1252	0	521	1441
N.S.	1	1.00	0.98	0.99	2.15	4.09	0.00	1.70	4.71
time (sec)	N/A	0.291	0.603	0.073	0.571	1.183	0.000	3.235	5.964

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	232	383	268	231	0	409	-1
N.S.	1	1.00	0.72	1.18	0.83	0.71	0.00	1.26	-0.00
time (sec)	N/A	0.168	0.204	0.101	0.339	0.425	0.000	3.601	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	147	183	167	140	0	238	-1
N.S.	1	1.00	0.66	0.82	0.75	0.63	0.00	1.07	-0.00
time (sec)	N/A	0.120	0.123	0.100	0.286	0.403	0.000	3.841	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	84	94	93	71	0	115	-1
N.S.	1	1.00	0.64	0.72	0.71	0.54	0.00	0.88	-0.01
time (sec)	N/A	0.069	0.065	0.069	0.271	0.402	0.000	3.841	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	41	42	34	0	38	44
N.S.	1	1.00	0.78	0.76	0.78	0.63	0.00	0.70	0.81
time (sec)	N/A	0.021	0.025	0.044	0.279	0.400	0.000	4.501	3.258

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	105	92	0	743	0	140	-1
N.S.	1	1.00	1.08	0.95	0.00	7.66	0.00	1.44	-0.01
time (sec)	N/A	0.062	0.153	0.122	0.000	0.417	0.000	4.262	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	170	259	0	2493	0	654	-1
N.S.	1	1.00	1.04	1.59	0.00	15.29	0.00	4.01	-0.01
time (sec)	N/A	0.154	0.516	0.128	0.000	0.484	0.000	3.804	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	249	427	0	4390	0	1303	-1
N.S.	1	1.00	0.95	1.64	0.00	16.82	0.00	4.99	-0.00
time (sec)	N/A	0.349	1.336	0.211	0.000	1.275	0.000	3.968	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	554	0	728	1416	0	5699	-1
N.S.	1	1.00	1.58	0.00	2.08	4.05	0.00	16.28	-0.00
time (sec)	N/A	0.204	0.785	0.005	0.291	0.500	0.000	4.133	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	285	0	402	712	0	2511	-1
N.S.	1	1.00	1.18	0.00	1.66	2.94	0.00	10.38	-0.00
time (sec)	N/A	0.132	0.455	0.005	0.300	0.417	0.000	3.245	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	128	0	187	294	0	806	-1
N.S.	1	1.00	0.88	0.00	1.29	2.03	0.00	5.56	-0.01
time (sec)	N/A	0.078	0.271	0.004	0.301	0.404	0.000	3.215	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	0	60	81	0	129	146
N.S.	1	1.00	0.85	0.00	0.97	1.31	0.00	2.08	2.35
time (sec)	N/A	0.030	0.092	0.004	0.284	0.405	0.000	2.679	3.583

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	136	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.169	0.005	0.000	0.000	0.000	0.000	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	70	0	164	122	0	-1
N.S.	1	1.00	0.83	0.75	0.00	1.76	1.31	0.00	-0.01
time (sec)	N/A	0.055	0.158	0.636	0.000	0.406	49.958	0.000	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	54	0	130	102	0	-1
N.S.	1	1.00	0.87	0.77	0.00	1.86	1.46	0.00	-0.01
time (sec)	N/A	0.042	0.127	0.622	0.000	0.404	36.796	0.000	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	40	0	103	0	0	-1
N.S.	1	1.00	0.94	0.82	0.00	2.10	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.102	0.574	0.000	0.364	0.000	0.000	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	-1
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.095	0.597	0.000	0.378	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	43	0	164	48	0	-1
N.S.	1	1.00	1.00	0.83	0.00	3.15	0.92	0.00	-0.02
time (sec)	N/A	0.033	0.133	0.454	0.000	0.359	5.549	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	59	0	262	70	0	-1
N.S.	1	1.00	0.89	0.79	0.00	3.49	0.93	0.00	-0.01
time (sec)	N/A	0.044	0.176	0.386	0.000	0.361	7.774	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	81	78	0	169	114	0	-1
N.S.	1	1.00	0.80	0.77	0.00	1.67	1.13	0.00	-0.01
time (sec)	N/A	0.054	0.158	0.611	0.000	0.376	45.403	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	60	0	135	95	0	-1
N.S.	1	1.00	0.87	0.79	0.00	1.78	1.25	0.00	-0.01
time (sec)	N/A	0.040	0.128	0.598	0.000	0.351	32.384	0.000	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	50	44	0	110	0	0	-1
N.S.	1	1.00	0.94	0.83	0.00	2.08	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.102	0.610	0.000	0.393	0.000	0.000	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	0	80	0	0	-1
N.S.	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.092	0.650	0.000	0.367	0.000	0.000	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	0	175	44	0	-1
N.S.	1	1.00	1.00	0.84	0.00	3.12	0.79	0.00	-0.02
time (sec)	N/A	0.031	0.137	0.389	0.000	0.380	7.428	0.000	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	70	65	0	277	63	0	-1
N.S.	1	1.00	0.86	0.80	0.00	3.42	0.78	0.00	-0.01
time (sec)	N/A	0.043	0.195	0.434	0.000	0.358	7.003	0.000	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	56	24	21	17
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74
time (sec)	N/A	0.005	0.018	0.359	0.494	0.364	0.585	3.811	3.108

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	0	78	0	0	-1
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03
time (sec)	N/A	0.022	0.100	0.613	0.000	0.387	0.000	0.000	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	32	0	116	0	0	-1
N.S.	1	1.00	1.00	0.86	0.00	3.14	0.00	0.00	-0.03
time (sec)	N/A	0.121	0.142	0.484	0.000	0.397	0.000	0.000	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	39	0	151	0	0	-1
N.S.	1	1.00	1.00	0.89	0.00	3.43	0.00	0.00	-0.02
time (sec)	N/A	0.262	0.182	0.434	0.000	0.386	0.000	0.000	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	46	0	174	0	0	-1
N.S.	1	1.00	1.00	0.90	0.00	3.41	0.00	0.00	-0.02
time (sec)	N/A	0.461	0.226	0.437	0.000	0.359	0.000	0.000	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	83	120	55	80	77	54
N.S.	1	1.00	0.92	1.09	1.58	0.72	1.05	1.01	0.71
time (sec)	N/A	0.019	0.104	0.382	0.502	0.357	168.851	2.511	3.502

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	65	69	67	50	60	67	44
N.S.	1	1.00	1.08	1.15	1.12	0.83	1.00	1.12	0.73
time (sec)	N/A	0.014	0.083	0.365	0.510	0.388	65.070	1.761	3.355

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	55	30	43	39	57	30
N.S.	1	1.00	1.05	1.25	0.68	0.98	0.89	1.30	0.68
time (sec)	N/A	0.010	0.064	0.113	0.530	0.345	22.771	2.263	3.575

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	13	16	12	8	37	14
N.S.	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.56
time (sec)	N/A	0.003	0.019	0.411	0.275	0.358	1.271	1.894	3.176

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	32	30	28	20	58	25
N.S.	1	1.00	1.14	1.52	1.43	1.33	0.95	2.76	1.19
time (sec)	N/A	0.008	0.028	0.383	0.281	0.350	2.057	2.343	3.107

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	32	37	38	38	34	68	28
N.S.	1	1.00	0.94	1.09	1.12	1.12	1.00	2.00	0.82
time (sec)	N/A	0.010	0.044	0.359	0.298	0.348	3.009	1.826	3.093

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	23	11	28	8	11	18
N.S.	1	1.00	2.22	2.56	1.22	3.11	0.89	1.22	2.00
time (sec)	N/A	0.003	0.021	0.344	0.488	0.368	1.562	2.124	3.108

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	12	0	28	10	13	18
N.S.	1	1.00	2.22	1.33	0.00	3.11	1.11	1.44	2.00
time (sec)	N/A	0.003	0.003	0.421	0.000	0.355	1.567	1.712	3.097

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	1059	16	67	14	16	26
N.S.	1	1.00	1.11	58.83	0.89	3.72	0.78	0.89	1.44
time (sec)	N/A	0.050	0.021	0.047	0.259	0.367	1.660	2.162	3.381

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	29	70	21	32	19	16	10
N.S.	1	1.00	1.81	4.38	1.31	2.00	1.19	1.00	0.62
time (sec)	N/A	0.042	0.017	0.410	0.274	0.390	0.077	1.880	3.345

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	50	49	36	94	29	26
N.S.	1	1.00	0.91	1.14	1.11	0.82	2.14	0.66	0.59
time (sec)	N/A	0.021	0.121	0.385	0.509	0.345	0.155	2.040	0.090

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	57	125	89	53	133	138	124
N.S.	1	1.00	0.47	1.03	0.74	0.44	1.10	1.14	1.02
time (sec)	N/A	0.077	0.030	0.495	0.503	0.359	1.181	2.303	3.273

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	42	0	268	49	41	42
N.S.	1	1.00	0.98	0.84	0.00	5.36	0.98	0.82	0.84
time (sec)	N/A	0.032	0.070	0.409	0.000	0.419	1.963	2.298	3.761

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	76	0	268	37	41	96
N.S.	1	1.00	0.98	1.52	0.00	5.36	0.74	0.82	1.92
time (sec)	N/A	0.046	0.005	0.400	0.000	0.415	35.720	1.558	4.516

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	0	382	97	55	56
N.S.	1	1.00	0.91	0.82	0.00	5.62	1.43	0.81	0.82
time (sec)	N/A	0.028	0.088	0.467	0.000	0.380	2.134	2.041	3.688

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	148	0	382	53	52	50
N.S.	1	1.00	0.91	2.18	0.00	5.62	0.78	0.76	0.74
time (sec)	N/A	0.357	0.048	0.389	0.000	0.416	150.729	1.486	3.899

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	45	0	39	0	27	39
N.S.	1	1.00	1.09	1.32	0.00	1.15	0.00	0.79	1.15
time (sec)	N/A	0.021	0.055	0.385	0.000	0.563	0.000	2.084	3.329

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	42	0	49	0	43	27
N.S.	1	1.00	0.68	0.57	0.00	0.66	0.00	0.58	0.36
time (sec)	N/A	0.030	0.080	0.375	0.000	0.620	0.000	2.392	3.162

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	13	16	17	16	13
N.S.	1	1.00	0.95	0.74	0.68	0.84	0.89	0.84	0.68
time (sec)	N/A	0.003	0.010	0.021	0.268	0.339	0.069	2.162	0.030

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	50	67	86	100	55	162
N.S.	1	1.00	0.96	0.93	1.24	1.59	1.85	1.02	3.00
time (sec)	N/A	0.059	0.077	5.076	0.510	0.403	0.639	2.322	0.095

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	18	12	12	32	12	12
N.S.	1	1.00	1.00	1.29	0.86	0.86	2.29	0.86	0.86
time (sec)	N/A	0.013	0.010	0.053	0.282	0.370	1.830	1.736	0.188

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	91	46	63	94	50	71
N.S.	1	1.00	0.92	1.49	0.75	1.03	1.54	0.82	1.16
time (sec)	N/A	0.029	0.056	0.140	0.476	0.345	0.998	2.260	3.238

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	30	32	39	30	32
N.S.	1	1.00	1.00	2.51	0.81	0.86	1.05	0.81	0.86
time (sec)	N/A	0.026	0.034	0.147	0.492	0.352	0.975	1.886	0.066

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	91	45	64	92	49	71
N.S.	1	1.00	0.92	1.49	0.74	1.05	1.51	0.80	1.16
time (sec)	N/A	0.022	0.040	0.136	0.506	0.372	0.962	1.924	0.119

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	29	54	21	21	36	22	25
N.S.	1	1.00	0.94	1.74	0.68	0.68	1.16	0.71	0.81
time (sec)	N/A	0.016	0.013	0.084	0.263	0.343	1.068	4.560	3.083

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	101	51	65	92	54	79
N.S.	1	1.00	0.92	1.55	0.78	1.00	1.42	0.83	1.22
time (sec)	N/A	0.026	0.039	0.135	0.483	0.379	0.931	2.438	3.158

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	42	0	51	0	45	-1
N.S.	1	1.00	0.92	0.68	0.00	0.82	0.00	0.73	-0.02
time (sec)	N/A	0.018	0.073	0.020	0.000	0.593	0.000	1.741	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	59	55	0	61	0	55	-1
N.S.	1	1.00	0.79	0.73	0.00	0.81	0.00	0.73	-0.01
time (sec)	N/A	0.030	0.086	0.020	0.000	0.557	0.000	2.097	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	65	48	0	59	0	53	-1
N.S.	1	1.00	0.96	0.71	0.00	0.87	0.00	0.78	-0.01
time (sec)	N/A	0.030	0.077	0.021	0.000	0.593	0.000	1.506	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	60	0	73	0	67	-1
N.S.	1	1.00	0.94	0.75	0.00	0.91	0.00	0.84	-0.01
time (sec)	N/A	0.028	0.080	0.024	0.000	0.590	0.000	5.285	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	83	67	0	101	0	88	-1
N.S.	1	1.00	0.76	0.61	0.00	0.93	0.00	0.81	-0.01
time (sec)	N/A	0.051	0.201	0.024	0.000	0.934	0.000	3.308	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	47	0	33	-1
N.S.	1	1.00	0.91	0.68	0.00	1.00	0.00	0.70	-0.02
time (sec)	N/A	0.021	0.058	0.020	0.000	0.529	0.000	1.755	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	52	49	48	58	60	102
N.S.	1	1.00	0.94	0.78	0.73	0.72	0.87	0.90	1.52
time (sec)	N/A	0.092	0.081	0.186	0.492	0.342	18.195	2.415	3.090

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	54	51	59	60	62	118
N.S.	1	1.00	0.94	0.76	0.72	0.83	0.85	0.87	1.66
time (sec)	N/A	0.078	0.066	0.183	0.501	0.374	33.979	2.128	0.033

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	32	117	46	23	44	46	43
N.S.	1	1.00	0.62	2.25	0.88	0.44	0.85	0.88	0.83
time (sec)	N/A	0.037	0.042	0.356	0.504	0.367	0.207	2.906	3.170

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	31	14	0	37	0	25	-1
N.S.	1	1.00	1.55	0.70	0.00	1.85	0.00	1.25	-0.05
time (sec)	N/A	0.073	0.063	0.349	0.000	0.549	0.000	2.538	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	31	39	34	29	29	36	30	53
N.S.	1	0.72	0.91	0.79	0.67	0.67	0.84	0.70	1.23
time (sec)	N/A	0.047	0.026	0.391	0.492	0.362	0.543	2.567	3.105

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	130	90	89	76	0	89	88
N.S.	1	1.00	1.12	0.78	0.77	0.66	0.00	0.77	0.76
time (sec)	N/A	0.094	0.053	0.013	0.283	0.337	0.000	2.397	0.127

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	0	82	-1
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01
time (sec)	N/A	0.039	0.054	0.161	0.275	0.352	0.000	2.838	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	41	40	39	216	268	-1
N.S.	1	1.00	0.97	0.64	0.62	0.61	3.38	4.19	-0.02
time (sec)	N/A	0.033	0.054	0.221	0.289	0.351	1.458	4.551	0.000

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	59	58	57	0	474	-1
N.S.	1	1.00	1.05	0.72	0.71	0.70	0.00	5.78	-0.01
time (sec)	N/A	0.055	0.061	0.177	0.285	0.340	0.000	3.222	0.000

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	58	54	53	35	0	82	-1
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01
time (sec)	N/A	0.031	0.002	0.115	0.281	0.385	0.000	2.525	0.000

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	168	121	120	76	0	7916	-1
N.S.	1	1.00	0.88	0.64	0.63	0.40	0.00	41.66	-0.01
time (sec)	N/A	0.253	0.158	0.355	0.269	0.384	0.000	35.370	0.000

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	186	154	153	85	0	271	-1
N.S.	1	1.00	0.80	0.66	0.66	0.36	0.00	1.16	-0.00
time (sec)	N/A	0.259	0.162	0.420	0.280	0.365	0.000	6.106	0.000

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	94	107	106	62	0	859	-1
N.S.	1	1.00	0.59	0.67	0.66	0.39	0.00	5.37	-0.01
time (sec)	N/A	0.200	0.112	0.284	0.273	0.397	0.000	5.690	0.000

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	31	16	0	35	0	25	-1
N.S.	1	1.00	1.55	0.80	0.00	1.75	0.00	1.25	-0.05
time (sec)	N/A	0.053	0.004	0.374	0.000	0.570	0.000	1.418	0.000

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	59	38	0	85	0	49	-1
N.S.	1	1.18	1.34	0.86	0.00	1.93	0.00	1.11	-0.02
time (sec)	N/A	0.023	0.074	0.022	0.000	0.878	0.000	3.252	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	51	50	58	45	99	61	50
N.S.	1	1.00	0.94	0.93	1.07	0.83	1.83	1.13	0.93
time (sec)	N/A	0.255	0.049	0.364	0.282	0.336	16.593	2.439	3.100

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	64	50	0	84	0	44	-1
N.S.	1	1.00	0.91	0.71	0.00	1.20	0.00	0.63	-0.01
time (sec)	N/A	0.025	0.084	0.021	0.000	0.827	0.000	2.070	0.000

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	15	17	15	15
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79
time (sec)	N/A	0.015	0.014	0.408	0.286	0.338	0.062	2.195	3.043

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	67	50	49	49	71	50	47
N.S.	1	1.00	0.87	0.65	0.64	0.64	0.92	0.65	0.61
time (sec)	N/A	0.049	0.034	0.398	0.304	0.366	0.391	2.177	0.048

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	224	76	289	0	302	56	171	255
N.S.	1	2.80	0.95	3.61	0.00	3.78	0.70	2.14	3.19
time (sec)	N/A	0.208	0.111	0.644	0.000	0.363	5.724	2.179	0.104

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	81	130	0	87	0	81	-1
N.S.	1	1.00	0.91	1.46	0.00	0.98	0.00	0.91	-0.01
time (sec)	N/A	0.185	0.102	0.415	0.000	2.044	0.000	2.586	0.000

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	55	68	0	62	0	65	-1
N.S.	1	1.00	0.90	1.11	0.00	1.02	0.00	1.07	-0.02
time (sec)	N/A	0.358	0.120	0.388	0.000	1.560	0.000	2.066	0.000

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	16	28	27	18	26	14	14
N.S.	1	1.00	2.00	3.50	3.38	2.25	3.25	1.75	1.75
time (sec)	N/A	0.002	0.021	0.377	0.276	0.334	0.461	1.571	0.158

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	32	27	27	0	22	12
N.S.	1	1.00	1.75	4.00	3.38	3.38	0.00	2.75	1.50
time (sec)	N/A	0.007	0.005	0.064	0.276	0.347	0.000	2.341	0.064

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	48	39	49	28	63	22	26
N.S.	1	1.00	2.18	1.77	2.23	1.27	2.86	1.00	1.18
time (sec)	N/A	0.003	0.043	0.382	0.277	0.343	0.834	1.493	3.719

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	48	45	51	42	0	35	35
N.S.	1	1.00	2.18	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.004	0.001	0.059	0.266	0.326	0.000	2.187	3.124

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	66	43	20	39	0	42	138
N.S.	1	1.00	1.83	1.19	0.56	1.08	0.00	1.17	3.83
time (sec)	N/A	0.004	0.114	0.401	0.504	0.364	0.000	1.854	5.090

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	66	59	41	36	0	51	41
N.S.	1	1.00	1.83	1.64	1.14	1.00	0.00	1.42	1.14
time (sec)	N/A	0.010	0.007	0.122	0.496	0.335	0.000	2.496	0.060

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	76	55	46	83	47	473
N.S.	1	1.00	1.07	1.10	0.80	0.67	1.20	0.68	6.86
time (sec)	N/A	0.009	0.105	0.411	0.299	0.345	5.163	1.819	12.863

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	74	79	138	64	0	62	119
N.S.	1	1.00	1.07	1.14	2.00	0.93	0.00	0.90	1.72
time (sec)	N/A	0.018	0.002	0.096	0.270	0.346	0.000	2.881	0.050

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	37	33	13	13	0	20	13
N.S.	1	1.00	2.47	2.20	0.87	0.87	0.00	1.33	0.87
time (sec)	N/A	0.008	0.012	0.113	0.480	0.351	0.000	3.938	0.179

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	51	30	15	15	0	16	15
N.S.	1	1.00	2.83	1.67	0.83	0.83	0.00	0.89	0.83
time (sec)	N/A	0.014	0.036	0.365	0.502	0.354	0.000	4.089	3.139

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	63	85	24	24	0	41	36
N.S.	1	1.00	2.62	3.54	1.00	1.00	0.00	1.71	1.50
time (sec)	N/A	0.040	0.067	0.365	0.496	0.348	0.000	5.860	0.180

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	77	80	59	105	0	74	31
N.S.	1	1.00	1.88	1.95	1.44	2.56	0.00	1.80	0.76
time (sec)	N/A	0.041	0.074	0.398	0.478	0.336	0.000	4.224	0.204

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	49	46	37	28	0	36	37
N.S.	1	1.00	1.53	1.44	1.16	0.88	0.00	1.12	1.16
time (sec)	N/A	0.008	0.014	0.107	0.494	0.371	0.000	4.595	3.127

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	66	39	43	32	0	29	43
N.S.	1	1.00	1.74	1.03	1.13	0.84	0.00	0.76	1.13
time (sec)	N/A	0.010	0.069	0.109	0.489	0.348	0.000	4.972	0.030

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	67	61	49	38	0	36	49
N.S.	1	1.00	1.60	1.45	1.17	0.90	0.00	0.86	1.17
time (sec)	N/A	0.011	0.091	0.064	0.502	0.332	0.000	6.286	3.171

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	67	60	70	58	0	40	51
N.S.	1	1.00	1.63	1.46	1.71	1.41	0.00	0.98	1.24
time (sec)	N/A	0.013	0.081	0.069	0.266	0.337	0.000	6.905	0.050

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	114	152	118	180	0	119	90
N.S.	1	1.00	1.50	2.00	1.55	2.37	0.00	1.57	1.18
time (sec)	N/A	0.028	0.236	0.049	0.493	0.333	0.000	5.282	0.273

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	76	80	54	0	74	57
N.S.	1	1.00	1.47	1.55	1.63	1.10	0.00	1.51	1.16
time (sec)	N/A	0.009	0.095	0.121	0.495	0.341	0.000	5.096	3.164

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	88	106	53	46	0	114	74
N.S.	1	1.00	1.91	2.30	1.15	1.00	0.00	2.48	1.61
time (sec)	N/A	0.015	0.171	0.133	0.471	0.340	0.000	5.304	3.239

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	36	27	17	0	17	17
N.S.	1	1.00	0.95	1.80	1.35	0.85	0.00	0.85	0.85
time (sec)	N/A	0.036	0.037	0.436	0.277	0.322	0.000	6.115	0.057

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	37	16	17	0	17	-1
N.S.	1	1.00	1.06	2.06	0.89	0.94	0.00	0.94	-0.06
time (sec)	N/A	0.021	0.029	0.411	0.285	0.350	0.000	4.407	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	66	81	103	83	0	129	62
N.S.	1	1.00	1.22	1.50	1.91	1.54	0.00	2.39	1.15
time (sec)	N/A	0.039	0.158	0.440	0.513	0.387	0.000	3.744	0.089

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	32	0	17	0	23	15
N.S.	1	1.00	1.73	2.91	0.00	1.55	0.00	2.09	1.36
time (sec)	N/A	0.002	0.024	0.392	0.000	0.359	0.000	4.818	3.125

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	34	0	0	-1
N.S.	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	-0.03
time (sec)	N/A	0.009	1.572	0.483	0.000	0.411	0.000	0.000	0.000

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	111	359	0	372	0	287	-1
N.S.	1	1.00	0.62	1.99	0.00	2.07	0.00	1.59	-0.01
time (sec)	N/A	0.141	0.276	0.583	0.000	0.387	0.000	6.022	0.000

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	94	1066	0	171	0	350	-1
N.S.	1	1.00	0.55	6.20	0.00	0.99	0.00	2.03	-0.01
time (sec)	N/A	0.111	0.383	0.167	0.000	0.383	0.000	2.385	0.000

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	115	5984	0	223	0	452	-1
N.S.	1	1.00	0.37	19.49	0.00	0.73	0.00	1.47	-0.00
time (sec)	N/A	0.189	0.456	0.184	0.000	0.352	0.000	2.944	0.000

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	71	0	77	0	81	-1
N.S.	1	1.00	1.20	1.09	0.00	1.18	0.00	1.25	-0.02
time (sec)	N/A	0.020	0.195	0.097	0.000	0.332	0.000	1.707	0.000

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	118	0	97	0	143	-1
N.S.	1	1.00	0.83	1.42	0.00	1.17	0.00	1.72	-0.01
time (sec)	N/A	0.023	0.312	0.081	0.000	0.334	0.000	1.905	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	83	146	0	129	0	184	-1
N.S.	1	1.00	0.82	1.45	0.00	1.28	0.00	1.82	-0.01
time (sec)	N/A	0.025	0.343	0.069	0.000	0.369	0.000	1.997	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	81	370	0	187	0	197	-1
N.S.	1	1.00	0.75	3.43	0.00	1.73	0.00	1.82	-0.01
time (sec)	N/A	0.081	0.230	0.540	0.000	0.399	0.000	2.168	0.000

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	2407	0	121	0	263	-1
N.S.	1	1.00	0.97	27.67	0.00	1.39	0.00	3.02	-0.01
time (sec)	N/A	0.047	0.341	0.200	0.000	0.352	0.000	2.221	0.000

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	109	14530	0	171	0	367	-1
N.S.	1	1.00	0.73	97.52	0.00	1.15	0.00	2.46	-0.01
time (sec)	N/A	0.067	0.398	0.356	0.000	0.347	0.000	2.019	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	42	191	59	58	182	58	38
N.S.	1	1.40	1.00	4.55	1.40	1.38	4.33	1.38	0.90
time (sec)	N/A	0.144	0.056	0.727	0.306	0.342	0.247	1.431	0.183

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	59	30	191	59	58	182	58	51
N.S.	1	1.40	0.71	4.55	1.40	1.38	4.33	1.38	1.21
time (sec)	N/A	0.157	0.027	0.518	0.313	0.336	1.002	2.622	3.421

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	58	0	0	-1
N.S.	1	1.00	2.73	10.29	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.051	20.451	0.638	0.000	0.123	0.000	0.000	0.000

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	36	0	0	-1
N.S.	1	1.00	4.13	15.26	0.00	0.58	0.00	0.00	-0.02
time (sec)	N/A	0.034	20.412	0.471	0.000	0.089	0.000	0.000	0.000

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	156	200	0	16	0	0	-1
N.S.	1	1.00	9.18	11.76	0.00	0.94	0.00	0.00	-0.06
time (sec)	N/A	0.010	30.168	0.516	0.000	0.091	0.000	0.000	0.000

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	261	963	0	119	0	0	-1
N.S.	1	1.00	3.58	13.19	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.035	20.620	0.493	0.000	0.075	0.000	0.000	0.000

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	298	1039	0	195	0	0	-1
N.S.	1	1.00	2.73	9.53	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.055	20.748	0.499	0.000	0.086	0.000	0.000	0.000

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	278	1050	0	58	0	0	-1
N.S.	1	1.00	2.73	10.29	0.00	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.049	22.683	0.559	0.000	0.102	0.000	0.000	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	256	946	0	36	0	0	-1
N.S.	1	1.00	4.13	15.26	0.00	0.58	0.00	0.00	-0.02
time (sec)	N/A	0.032	16.799	0.498	0.000	0.107	0.000	0.000	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	100	200	0	16	0	0	-1
N.S.	1	1.00	5.88	11.76	0.00	0.94	0.00	0.00	-0.06
time (sec)	N/A	0.009	31.921	0.513	0.000	0.081	0.000	0.000	0.000

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	298	963	0	119	0	0	-1
N.S.	1	1.00	4.08	13.19	0.00	1.63	0.00	0.00	-0.01
time (sec)	N/A	0.035	17.890	0.504	0.000	0.114	0.000	0.000	0.000

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	327	1039	0	195	0	0	-1
N.S.	1	1.00	3.00	9.53	0.00	1.79	0.00	0.00	-0.01
time (sec)	N/A	0.048	20.388	0.508	0.000	0.080	0.000	0.000	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	10468	5229	0	0	0	0	-1
N.S.	1	1.00	14.34	7.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.649	16.139	0.346	0.000	0.000	0.000	0.000	0.000

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	622	622	5218	4890	0	0	0	0	-1
N.S.	1	1.00	8.39	7.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.486	16.093	0.075	0.000	0.000	0.000	0.000	0.000

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	822	1056	0	0	0	0	-1
N.S.	1	1.00	3.62	4.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	11.409	0.058	0.000	0.000	0.000	0.000	0.000

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	5276	5024	0	0	0	0	-1
N.S.	1	1.00	7.83	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	16.110	0.076	0.000	0.000	0.000	0.000	0.000

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	7543	7887	0	0	0	0	-1
N.S.	1	1.00	11.38	11.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.556	14.866	0.345	0.000	0.000	0.000	0.000	0.000

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	1065	1704	0	0	0	0	-1
N.S.	1	1.00	4.53	7.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	11.546	0.053	0.000	0.000	0.000	0.000	0.000

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	748	748	7629	8103	0	0	0	0	-1
N.S.	1	1.00	10.20	10.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.552	16.114	0.087	0.000	0.000	0.000	0.000	0.000

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	6287	2655	0	0	0	0	-1
N.S.	1	1.00	13.91	5.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.421	16.110	0.148	0.000	0.000	0.000	0.000	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	3470	2519	0	0	0	0	-1
N.S.	1	1.00	8.74	6.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	16.061	0.047	0.000	0.000	0.000	0.000	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	540	530	0	0	0	0	-1
N.S.	1	1.00	3.75	3.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	11.007	0.036	0.000	0.000	0.000	0.000	0.000

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	3526	2601	0	0	0	0	-1
N.S.	1	1.00	8.07	5.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	16.069	0.041	0.000	0.000	0.000	0.000	0.000

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	6386	2757	0	0	0	0	-1
N.S.	1	1.00	12.35	5.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	16.125	0.058	0.000	0.000	0.000	0.000	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	7235	2694	0	0	0	0	-1
N.S.	1	1.00	12.97	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.412	16.123	0.053	0.000	0.000	0.000	0.000	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	4389	2551	0	0	0	0	-1
N.S.	1	1.00	9.42	5.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	13.081	0.049	0.000	0.000	0.000	0.000	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	813	788	0	0	0	0	-1
N.S.	1	1.00	4.54	4.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	11.788	0.036	0.000	0.000	0.000	0.000	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	3593	2616	0	0	0	0	-1
N.S.	1	1.00	7.58	5.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.326	13.757	0.042	0.000	0.000	0.000	0.000	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	6452	2777	0	0	0	0	-1
N.S.	1	1.00	10.92	4.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.438	16.107	0.058	0.000	0.000	0.000	0.000	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	8500	2733	0	0	0	0	-1
N.S.	1	1.00	14.53	4.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	16.149	0.056	0.000	0.000	0.000	0.000	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	5647	2582	0	0	0	0	-1
N.S.	1	1.00	11.64	5.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	13.201	0.054	0.000	0.000	0.000	0.000	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	1145	1147	0	0	0	0	-1
N.S.	1	1.00	2.95	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	13.981	0.036	0.000	0.000	0.000	0.000	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	2941	2607	0	0	0	0	-1
N.S.	1	1.00	9.46	8.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.241	13.780	0.044	0.000	0.000	0.000	0.000	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	5812	2780	0	0	0	0	-1
N.S.	1	1.00	9.99	4.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.496	16.124	0.071	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	927	965	0	0	0	0	-1
N.S.	1	1.00	7.19	7.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	10.580	0.891	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	4865	4426	0	0	0	0	-1
N.S.	1	1.00	11.29	10.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	16.041	0.647	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	249	961	0	0	0	0	-1
N.S.	1	1.00	2.31	8.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	10.392	0.949	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	602	2564	0	0	0	0	-1
N.S.	1	1.00	1.64	6.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	12.946	0.668	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	1148	1180	0	0	0	0	-1
N.S.	1	1.00	9.11	9.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	10.668	0.917	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	6019	5421	0	0	0	0	-1
N.S.	1	1.00	13.87	12.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.352	16.040	0.622	0.000	0.000	0.000	0.000	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	6084	5477	0	0	0	0	-1
N.S.	1	1.00	10.54	9.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	16.068	0.631	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	826	1182	0	0	0	0	-1
N.S.	1	1.00	6.35	9.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	10.096	0.413	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	4974	5427	0	0	0	0	-1
N.S.	1	1.00	11.20	12.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	15.786	0.133	0.000	0.000	0.000	0.000	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	257	76	57	81	0	0	158
N.S.	1	1.00	4.59	1.36	1.02	1.45	0.00	0.00	2.82
time (sec)	N/A	0.141	0.621	0.474	0.497	0.364	0.000	0.000	7.906

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	56	0	84	63	89	55
N.S.	1	1.00	0.75	0.86	0.00	1.29	0.97	1.37	0.85
time (sec)	N/A	0.106	0.176	0.110	0.000	0.338	2.893	3.265	0.043

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	101	53	58	0	78	0	80	52
N.S.	1	1.91	1.00	1.09	0.00	1.47	0.00	1.51	0.98
time (sec)	N/A	0.143	0.149	0.074	0.000	0.361	0.000	4.209	0.045

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	12	12	12	12
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86
time (sec)	N/A	0.079	0.007	0.506	0.515	0.381	0.053	2.790	3.387

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	46	45	43	60	43	43
N.S.	1	1.00	0.92	0.75	0.74	0.70	0.98	0.70	0.70
time (sec)	N/A	0.117	0.034	0.051	0.285	0.328	1.228	1.797	3.360

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	86	85	91	116	84	84
N.S.	1	1.00	0.90	0.75	0.75	0.80	1.02	0.74	0.74
time (sec)	N/A	0.135	0.051	0.085	0.288	0.349	1.852	1.696	0.060

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	103	102	0	180	70	92	46
N.S.	1	1.00	1.78	1.76	0.00	3.10	1.21	1.59	0.79
time (sec)	N/A	0.043	0.110	0.048	0.000	0.383	1.583	2.572	4.083

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	103	102	0	180	70	92	62
N.S.	1	1.00	1.78	1.76	0.00	3.10	1.21	1.59	1.07
time (sec)	N/A	0.097	0.004	0.049	0.000	0.406	3.652	2.556	4.045

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	39	0	182	0	123	67
N.S.	1	1.00	1.04	0.74	0.00	3.43	0.00	2.32	1.26
time (sec)	N/A	0.022	0.345	0.724	0.000	0.350	0.000	4.106	3.588

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	18	17	22	27	18	23
N.S.	1	1.00	1.04	0.78	0.74	0.96	1.17	0.78	1.00
time (sec)	N/A	0.005	0.016	0.473	0.299	0.341	0.066	3.774	0.030

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	24	26	17	25	139	54	35
N.S.	1	1.00	1.04	1.13	0.74	1.09	6.04	2.35	1.52
time (sec)	N/A	0.006	0.002	0.502	0.488	0.327	53.317	2.558	3.695

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	54	58	36	37	0	39	180
N.S.	1	1.00	1.64	1.76	1.09	1.12	0.00	1.18	5.45
time (sec)	N/A	0.011	0.050	0.498	0.283	0.361	0.000	2.611	7.557

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	2	31	-1
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.108	0.762	0.464	0.494	0.339	0.027	2.834	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	2	31	-1
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.037	0.002	0.488	0.507	0.376	0.026	2.404	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	35	2	2	2	31	-1
N.S.	1	1.00	1.27	0.78	0.04	0.04	0.04	0.69	-0.02
time (sec)	N/A	0.064	0.002	0.485	0.529	0.341	0.029	2.073	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	27	2	2	2	12	-1
N.S.	1	1.00	1.27	0.60	0.04	0.04	0.04	0.27	-0.02
time (sec)	N/A	0.091	3.888	0.559	0.505	0.326	0.027	2.504	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	39	41	0	6	0	6	-1
N.S.	1	1.00	0.75	0.79	0.00	0.12	0.00	0.12	-0.02
time (sec)	N/A	0.126	1.626	0.487	0.000	0.344	0.000	2.100	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	39	30	6	6	0	6	-1
N.S.	1	1.00	0.75	0.58	0.12	0.12	0.00	0.12	-0.02
time (sec)	N/A	0.082	1.608	0.500	0.522	0.342	0.000	2.297	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	40	0	8	0	8	-1
N.S.	1	1.00	0.99	0.59	0.00	0.12	0.00	0.12	-0.01
time (sec)	N/A	0.102	1.755	0.031	0.000	0.328	0.000	3.013	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	32	27	25	0	54	19
N.S.	1	1.00	1.26	1.03	0.87	0.81	0.00	1.74	0.61
time (sec)	N/A	0.013	0.273	0.570	0.538	0.343	0.000	2.342	3.327

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	25	25	0	54	19
N.S.	1	1.00	0.74	1.03	0.81	0.81	0.00	1.74	0.61
time (sec)	N/A	0.193	0.262	0.531	0.316	0.371	0.000	3.078	0.036

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	25	0	68	19
N.S.	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.096	0.285	0.044	0.000	0.324	0.000	2.594	0.065

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	81	0	25	0	68	19
N.S.	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.089	0.284	0.042	0.000	0.326	0.000	3.040	0.053

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	23	32	0	25	0	68	19
N.S.	1	1.00	0.74	1.03	0.00	0.81	0.00	2.19	0.61
time (sec)	N/A	0.447	0.280	0.500	0.000	0.334	0.000	5.717	3.324

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	38	58	85	44	216	0	-1
N.S.	1	1.00	0.88	1.35	1.98	1.02	5.02	0.00	-0.02
time (sec)	N/A	0.051	0.118	0.550	0.290	0.354	1.949	0.000	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	45	0	44	0	57	31
N.S.	1	1.00	1.26	1.07	0.00	1.05	0.00	1.36	0.74
time (sec)	N/A	0.088	0.106	0.487	0.000	0.339	0.000	2.350	3.388

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	50	262	0	80	0	133	60
N.S.	1	1.00	0.77	4.03	0.00	1.23	0.00	2.05	0.92
time (sec)	N/A	0.094	0.121	0.506	0.000	0.340	0.000	2.672	3.444

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	179	729	0	1122	0	536	-1
N.S.	1	1.00	0.92	3.74	0.00	5.75	0.00	2.75	-0.01
time (sec)	N/A	0.150	1.023	0.522	0.000	0.401	0.000	1.737	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	34	52	37	34	20
N.S.	1	1.00	1.00	0.96	1.21	1.86	1.32	1.21	0.71
time (sec)	N/A	0.010	0.027	0.531	0.505	0.339	10.747	2.804	3.850

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	27	0	52	0	34	-1
N.S.	1	1.00	1.00	0.96	0.00	1.86	0.00	1.21	-0.04
time (sec)	N/A	0.040	0.004	0.507	0.000	0.351	0.000	2.436	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.004	0.005	0.506	0.276	0.362	0.049	2.966	0.044

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	18	17	18	18
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82
time (sec)	N/A	0.055	0.001	0.556	0.281	0.349	0.252	2.317	0.029

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	0	11	36	11	11
N.S.	1	1.00	1.00	0.71	0.00	0.65	2.12	0.65	0.65
time (sec)	N/A	0.040	0.029	0.543	0.000	0.407	0.275	3.264	3.700

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	125	0	11	0	11	-1
N.S.	1	1.00	1.00	7.35	0.00	0.65	0.00	0.65	-0.06
time (sec)	N/A	0.032	0.029	0.562	0.000	0.404	0.000	1.651	0.000

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	24	7	6	19	7	19	6
N.S.	1	1.00	2.40	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.001	0.019	0.527	0.503	0.338	0.050	2.674	0.010

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	24	34	6	25	49	12	32
N.S.	1	1.00	2.40	3.40	0.60	2.50	4.90	1.20	3.20
time (sec)	N/A	0.001	0.002	0.523	0.551	0.347	0.546	3.250	0.148

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	24	7	6	19	7	19	6
N.S.	1	1.00	2.40	0.70	0.60	1.90	0.70	1.90	0.60
time (sec)	N/A	0.003	0.002	0.512	0.486	0.389	0.653	3.584	0.012

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	23	7	8	29	0	26	6
N.S.	1	1.00	1.92	0.58	0.67	2.42	0.00	2.17	0.50
time (sec)	N/A	0.005	0.053	0.595	0.489	0.352	0.000	3.123	3.125

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	44	31	8	29	39	13	30
N.S.	1	1.00	3.67	2.58	0.67	2.42	3.25	1.08	2.50
time (sec)	N/A	0.006	0.032	0.519	0.506	0.345	0.488	2.704	3.428

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	44	7	8	29	0	26	6
N.S.	1	1.00	3.67	0.58	0.67	2.42	0.00	2.17	0.50
time (sec)	N/A	0.006	0.002	0.572	0.514	0.374	0.000	3.168	3.370

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	23	5	8	29	0	24	4
N.S.	1	1.00	5.75	1.25	2.00	7.25	0.00	6.00	1.00
time (sec)	N/A	0.004	0.053	0.565	0.516	0.342	0.000	2.493	3.185

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	44	29	8	29	39	13	33
N.S.	1	1.00	11.00	7.25	2.00	7.25	9.75	3.25	8.25
time (sec)	N/A	0.004	0.032	0.618	0.493	0.345	0.488	2.856	0.082

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	44	5	8	29	0	24	4
N.S.	1	1.00	11.00	1.25	2.00	7.25	0.00	6.00	1.00
time (sec)	N/A	0.005	0.002	0.583	0.502	0.334	0.000	2.369	3.362

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.019	0.279	0.339	0.007	2.414	0.026

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.006	0.000	0.518	0.283	0.366	0.047	2.502	0.026

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	39	30	0	26	7	10	-1
N.S.	1	1.00	1.44	1.11	0.00	0.96	0.26	0.37	-0.04
time (sec)	N/A	0.004	0.020	0.523	0.000	0.432	0.864	2.167	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	39	30	0	26	0	10	-1
N.S.	1	1.00	1.44	1.11	0.00	0.96	0.00	0.37	-0.04
time (sec)	N/A	0.011	0.002	0.085	0.000	0.368	0.000	2.768	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	56	28	14	14	15	26	-1
N.S.	1	1.00	2.24	1.12	0.56	0.56	0.60	1.04	-0.04
time (sec)	N/A	0.003	0.014	0.583	0.285	0.329	0.756	3.443	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	56	28	0	14	0	21	-1
N.S.	1	1.00	2.24	1.12	0.00	0.56	0.00	0.84	-0.04
time (sec)	N/A	0.010	0.003	0.078	0.000	0.346	0.000	2.959	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.001	0.005	0.548	0.287	0.327	0.007	3.252	0.193

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	17	12	16	0	15	16
N.S.	1	1.00	1.82	1.55	1.09	1.45	0.00	1.36	1.45
time (sec)	N/A	0.001	0.028	0.559	0.300	0.331	0.000	2.596	3.557

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	7	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78
time (sec)	N/A	0.001	0.004	0.536	0.269	0.324	0.007	3.853	0.095

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	22	24	7	23	0	13	18
N.S.	1	1.00	2.44	2.67	0.78	2.56	0.00	1.44	2.00
time (sec)	N/A	0.001	0.030	0.559	0.280	0.354	0.000	2.642	3.635

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	12	10	9	9
N.S.	1	1.00	1.00	0.77	0.69	0.92	0.77	0.69	0.69
time (sec)	N/A	0.001	0.005	0.531	0.284	0.331	0.007	4.743	3.507

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	22	20	12	19	0	15	20
N.S.	1	1.00	1.69	1.54	0.92	1.46	0.00	1.15	1.54
time (sec)	N/A	0.001	0.030	0.605	0.274	0.335	0.000	2.298	3.523

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.004	0.554	0.311	0.352	0.007	3.188	3.419

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	24	27	7	26	0	13	22
N.S.	1	1.00	2.18	2.45	0.64	2.36	0.00	1.18	2.00
time (sec)	N/A	0.001	0.030	0.590	0.276	0.332	0.000	2.375	3.491

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	67	41	52	100	39	172
N.S.	1	1.00	1.23	1.91	1.17	1.49	2.86	1.11	4.91
time (sec)	N/A	0.005	0.078	0.557	0.525	0.354	0.920	3.398	6.136

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	49	86	0	96	0	0	-1
N.S.	1	1.00	1.40	2.46	0.00	2.74	0.00	0.00	-0.03
time (sec)	N/A	0.005	1.737	0.642	0.000	0.415	0.000	0.000	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	70	53	74	0	0	-1
N.S.	1	1.00	1.42	1.63	1.23	1.72	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.134	0.595	0.482	0.353	0.000	0.000	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	59	41	53	63	0	44	37
N.S.	1	1.00	1.69	1.17	1.51	1.80	0.00	1.26	1.06
time (sec)	N/A	0.041	0.122	0.617	0.477	0.351	0.000	3.413	3.204

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	134	65	93	0	0	-1
N.S.	1	1.00	1.47	2.63	1.27	1.82	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.161	0.698	0.480	0.370	0.000	0.000	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	94	99	65	82	0	87	82
N.S.	1	1.00	2.09	2.20	1.44	1.82	0.00	1.93	1.82
time (sec)	N/A	0.065	0.240	0.591	0.487	0.397	0.000	3.826	3.491

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	14	3	2	18	2	17	2
N.S.	1	1.00	7.00	1.50	1.00	9.00	1.00	8.50	1.00
time (sec)	N/A	0.001	0.002	0.612	0.488	0.344	0.046	3.735	0.006

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	32	29	0	27	0	0	-1
N.S.	1	1.00	16.00	14.50	0.00	13.50	0.00	0.00	-0.50
time (sec)	N/A	0.001	0.539	0.571	0.000	0.360	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	12	3	2	14	2	25	2
N.S.	1	1.00	6.00	1.50	1.00	7.00	1.00	12.50	1.00
time (sec)	N/A	0.001	0.011	0.539	0.481	0.346	0.046	3.678	0.028

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	42	29	0	81	0	0	-1
N.S.	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	-0.50
time (sec)	N/A	0.001	0.524	0.504	0.000	0.358	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	37	18	17	31	15	17	17
N.S.	1	1.00	1.61	0.78	0.74	1.35	0.65	0.74	0.74
time (sec)	N/A	0.002	0.003	0.502	0.492	0.380	0.062	3.330	0.031

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	50	42	0	60	0	0	-1
N.S.	1	1.00	2.17	1.83	0.00	2.61	0.00	0.00	-0.04
time (sec)	N/A	0.002	0.614	0.484	0.000	0.347	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	28	16	15	25	15	25	15
N.S.	1	1.00	1.33	0.76	0.71	1.19	0.71	1.19	0.71
time (sec)	N/A	0.002	0.022	0.510	0.504	0.379	0.061	2.738	0.028

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	70	47	0	120	0	0	-1
N.S.	1	1.00	3.33	2.24	0.00	5.71	0.00	0.00	-0.05
time (sec)	N/A	0.002	0.622	0.701	0.000	0.331	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	57	53	0	0	0	0	0	54
N.S.	1	1.16	1.08	0.00	0.00	0.00	0.00	0.00	1.10
time (sec)	N/A	0.013	0.022	0.031	0.000	0.000	0.000	0.000	4.673

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	33	21	0	30	58	30	20
N.S.	1	1.00	1.18	0.75	0.00	1.07	2.07	1.07	0.71
time (sec)	N/A	0.006	0.024	0.510	0.000	0.353	0.175	3.445	0.034

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	39	197	0	84	17	140	105
N.S.	1	1.00	1.05	5.32	0.00	2.27	0.46	3.78	2.84
time (sec)	N/A	0.028	0.042	0.148	0.000	0.386	0.079	3.415	0.143

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	68	33	0	90	27	88	57
N.S.	1	1.00	1.70	0.82	0.00	2.25	0.68	2.20	1.42
time (sec)	N/A	0.028	0.096	0.129	0.000	0.336	0.071	3.647	3.501

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	77	111	94	67	0	117	86
N.S.	1	1.00	1.43	2.06	1.74	1.24	0.00	2.17	1.59
time (sec)	N/A	0.088	0.106	0.273	0.496	0.354	0.000	3.988	3.328

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	111	0	67	0	117	86
N.S.	1	1.00	1.28	1.85	0.00	1.12	0.00	1.95	1.43
time (sec)	N/A	0.198	0.106	0.271	0.000	0.328	0.000	3.542	3.377

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	38	0	66	0	50	-1
N.S.	1	1.00	0.88	0.75	0.00	1.29	0.00	0.98	-0.02
time (sec)	N/A	0.073	0.111	0.050	0.000	0.327	0.000	3.584	0.000

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	38	54	66	0	50	-1
N.S.	1	1.00	0.88	0.75	1.06	1.29	0.00	0.98	-0.02
time (sec)	N/A	0.065	0.095	0.249	0.500	0.334	0.000	2.708	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	45	51	54	64	0	0	56
N.S.	1	1.00	0.88	1.00	1.06	1.25	0.00	0.00	1.10
time (sec)	N/A	0.099	0.004	0.247	0.498	0.331	0.000	0.000	4.782

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	51	0	64	0	0	56
N.S.	1	1.00	0.98	0.94	0.00	1.19	0.00	0.00	1.04
time (sec)	N/A	0.031	0.255	0.015	0.000	0.323	0.000	0.000	0.061

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	23	20	0	20	0	0	20
N.S.	1	1.00	0.85	0.74	0.00	0.74	0.00	0.00	0.74
time (sec)	N/A	0.020	0.023	0.026	0.000	0.363	0.000	0.000	3.419

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	13	0	16	0	0	138
N.S.	1	1.00	0.86	0.93	0.00	1.14	0.00	0.00	9.86
time (sec)	N/A	0.098	0.135	0.259	0.000	0.357	0.000	0.000	3.380

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	16	0	0	10
N.S.	1	1.00	1.00	1.08	0.00	1.33	0.00	0.00	0.83
time (sec)	N/A	0.045	0.002	0.241	0.000	0.360	0.000	0.000	0.051

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	29	31	0	30	0	0	48
N.S.	1	1.00	0.81	0.86	0.00	0.83	0.00	0.00	1.33
time (sec)	N/A	0.094	0.009	0.213	0.000	0.323	0.000	0.000	3.492

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	31	0	30	0	0	43
N.S.	1	1.00	0.88	0.94	0.00	0.91	0.00	0.00	1.30
time (sec)	N/A	0.125	0.004	0.215	0.000	0.329	0.000	0.000	3.494

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	103	0	208	0	92	-1
N.S.	1	1.00	1.00	1.47	0.00	2.97	0.00	1.31	-0.01
time (sec)	N/A	0.048	0.380	0.231	0.000	0.361	0.000	2.289	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	56	63	0	38	0	46	-1
N.S.	1	1.00	0.67	0.76	0.00	0.46	0.00	0.55	-0.01
time (sec)	N/A	0.098	0.036	0.322	0.000	0.353	0.000	3.107	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	73	44	63	0	36	0	18	-1
N.S.	1	1.55	0.94	1.34	0.00	0.77	0.00	0.38	-0.02
time (sec)	N/A	0.293	5.524	0.231	0.000	0.369	0.000	5.509	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	70	75	0	74	0	39	-1
N.S.	1	1.00	0.57	0.61	0.00	0.60	0.00	0.32	-0.01
time (sec)	N/A	0.178	0.030	0.233	0.000	0.356	0.000	3.272	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	74	62	0	117	0	86	-1
N.S.	1	1.00	0.56	0.47	0.00	0.88	0.00	0.65	-0.01
time (sec)	N/A	0.048	0.277	0.089	0.000	0.346	0.000	4.033	0.000

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	54	49	0	83	0	67	-1
N.S.	1	1.00	0.60	0.54	0.00	0.92	0.00	0.74	-0.01
time (sec)	N/A	0.033	0.147	0.084	0.000	0.332	0.000	4.599	0.000

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	42	0	75	0	49	-1
N.S.	1	1.00	0.82	0.69	0.00	1.23	0.00	0.80	-0.02
time (sec)	N/A	0.022	0.071	0.063	0.000	0.348	0.000	6.355	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	82	79	0	142	0	88	-1
N.S.	1	1.00	0.75	0.72	0.00	1.30	0.00	0.81	-0.01
time (sec)	N/A	0.044	0.114	0.161	0.000	0.348	0.000	5.721	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	106	217	0	205	0	170	-1
N.S.	1	1.00	0.74	1.51	0.00	1.42	0.00	1.18	-0.01
time (sec)	N/A	0.058	0.300	0.157	0.000	0.326	0.000	5.327	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	28	0	31	0	30	23
N.S.	1	1.00	0.89	1.00	0.00	1.11	0.00	1.07	0.82
time (sec)	N/A	0.079	0.090	0.215	0.000	0.380	0.000	5.222	3.515

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.023	0.072	0.029	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.024	0.122	0.028	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.070	0.020	0.029	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.077	0.017	0.027	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.011	0.005	0.007	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.009	0.006	0.008	0.000	0.000	0.000	0.000	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.006	0.006	0.005	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.007	0.008	0.006	0.000	0.000	0.000	0.000	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.006	0.010	0.007	0.000	0.000	0.000	0.000	0.000

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.007	0.010	0.008	0.000	0.000	0.000	0.000	0.000

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	0	0	135	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	2.87	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.259	180.000	0.000	1.152	0.000	0.000	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	0	0	146	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	3.04	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.253	180.000	0.000	1.252	0.000	0.000	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	531	0	0	0	0	0	-1
N.S.	1	1.00	3.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	4.001	0.055	0.000	0.000	0.000	0.000	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-1)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	10.096	0.050	0.000	0.000	0.000	0.000	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	28	27	25	0	27	27
N.S.	1	1.00	1.02	0.68	0.66	0.61	0.00	0.66	0.66
time (sec)	N/A	0.029	0.033	0.222	0.516	0.364	0.000	4.155	3.367

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	22
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.85
time (sec)	N/A	0.026	0.031	0.222	0.489	0.364	1.112	2.387	3.361

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	48	30	30	39	30	42
N.S.	1	1.00	1.00	1.14	0.71	0.71	0.93	0.71	1.00
time (sec)	N/A	0.103	0.031	0.218	0.515	0.374	39.361	4.537	0.026

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	48	50	0	75	0	44	17
N.S.	1	1.00	2.40	2.50	0.00	3.75	0.00	2.20	0.85
time (sec)	N/A	0.006	0.022	0.223	0.000	0.351	0.000	3.996	3.460

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	52	61	0	84	0	40	21
N.S.	1	1.00	2.60	3.05	0.00	4.20	0.00	2.00	1.05
time (sec)	N/A	0.006	0.023	0.224	0.000	0.394	0.000	4.228	3.470

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	160	247	0	1515	0	0	-1
N.S.	1	1.00	1.32	2.04	0.00	12.52	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.256	0.260	0.000	0.854	0.000	0.000	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	186	397	0	2407	0	0	-1
N.S.	1	1.00	1.03	2.19	0.00	13.30	0.00	0.00	-0.01
time (sec)	N/A	0.190	0.532	0.228	0.000	41.376	0.000	0.000	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	17	16	16	0	11	16
N.S.	1	1.00	1.00	0.65	0.62	0.62	0.00	0.42	0.62
time (sec)	N/A	0.005	7.084	0.010	0.303	0.350	0.000	5.057	3.541

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	43	23	22	32	0	22	22
N.S.	1	1.00	1.65	0.88	0.85	1.23	0.00	0.85	0.85
time (sec)	N/A	0.008	0.046	0.240	0.522	0.341	0.000	4.979	3.529

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	13	12	11	11	12	11	9
N.S.	1	1.00	0.87	0.80	0.73	0.73	0.80	0.73	0.60
time (sec)	N/A	0.002	10.010	0.225	0.294	0.322	0.067	5.169	3.565

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	19	31	28	27	0	27	18
N.S.	1	1.00	0.86	1.41	1.27	1.23	0.00	1.23	0.82
time (sec)	N/A	0.003	0.055	0.235	0.284	0.357	0.000	3.639	3.504

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	41	13	9	17	0	17	-1
N.S.	1	1.00	3.42	1.08	0.75	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.006	0.044	0.238	0.548	0.340	0.000	3.171	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	43	15	11	17	0	17	-1
N.S.	1	1.00	3.58	1.25	0.92	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.006	0.036	0.240	0.530	0.340	0.000	2.876	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	14	13	13	10	13	10
N.S.	1	1.00	0.80	0.93	0.87	0.87	0.67	0.87	0.67
time (sec)	N/A	0.003	0.029	0.225	0.298	0.414	0.046	4.337	3.640

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	82	54	42	49	0	53	-1
N.S.	1	1.00	1.52	1.00	0.78	0.91	0.00	0.98	-0.02
time (sec)	N/A	0.027	0.100	0.257	0.522	0.363	0.000	5.062	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	47	342	0	0	0	45	27
N.S.	1	1.00	0.80	5.80	0.00	0.00	0.00	0.76	0.46
time (sec)	N/A	0.045	0.343	0.529	0.000	0.000	0.000	5.972	3.532

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	39	28	0	30	0	33	27
N.S.	1	1.00	0.66	0.47	0.00	0.51	0.00	0.56	0.46
time (sec)	N/A	0.032	0.027	0.220	0.000	0.361	0.000	5.353	3.544

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	51	38	0	40	0	51	-1
N.S.	1	1.00	0.54	0.40	0.00	0.43	0.00	0.54	-0.01
time (sec)	N/A	0.059	0.031	0.221	0.000	0.407	0.000	5.998	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	0	20	29	18	19
N.S.	1	1.00	1.00	1.11	0.00	1.11	1.61	1.00	1.06
time (sec)	N/A	0.011	0.030	0.229	0.000	0.362	0.407	4.883	3.499

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	82	81	0	62	0	60	-1
N.S.	1	1.00	0.77	0.76	0.00	0.58	0.00	0.56	-0.01
time (sec)	N/A	0.020	0.072	0.117	0.000	0.394	0.000	4.077	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	14	64	0	14	25
N.S.	1	1.00	1.00	0.96	0.56	2.56	0.00	0.56	1.00
time (sec)	N/A	0.003	0.034	0.224	0.280	0.418	0.000	3.460	3.508

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	33	35	49	33	35
N.S.	1	1.00	1.00	0.81	0.79	0.83	1.17	0.79	0.83
time (sec)	N/A	0.020	0.029	0.159	0.505	0.370	0.086	2.679	3.448

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	25	27	37	25	27
N.S.	1	1.00	1.00	0.81	0.78	0.84	1.16	0.78	0.84
time (sec)	N/A	0.016	0.023	0.158	0.487	0.324	0.079	3.291	0.035

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	53	0	95	0	45	-1
N.S.	1	1.00	0.64	0.70	0.00	1.25	0.00	0.59	-0.01
time (sec)	N/A	0.015	0.228	0.011	0.000	0.365	0.000	3.280	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	53	0	64	48	79	46
N.S.	1	1.00	0.89	1.15	0.00	1.39	1.04	1.72	1.00
time (sec)	N/A	0.060	0.084	0.061	0.000	0.388	1.503	4.233	0.036

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	14	0	13	-1
N.S.	1	1.00	1.00	0.90	0.00	0.70	0.00	0.65	-0.05
time (sec)	N/A	0.004	0.236	0.246	0.000	0.376	0.000	2.847	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	A	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	18	20	38	19	68	0	14
N.S.	1	1.03	0.51	0.57	1.09	0.54	1.94	0.00	0.40
time (sec)	N/A	0.008	0.113	0.274	0.519	0.352	0.663	0.000	3.591

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	0	13	16	100	0	13
N.S.	1	1.00	1.00	0.00	0.87	1.07	6.67	0.00	0.87
time (sec)	N/A	0.012	0.174	0.047	0.347	0.331	65.654	0.000	3.684

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	28	38	37	24	46	37	37
N.S.	1	1.00	0.53	0.72	0.70	0.45	0.87	0.70	0.70
time (sec)	N/A	0.014	0.016	0.223	0.373	0.337	5.972	5.291	0.043

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	25	25	27	25	25
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.87	0.81	0.81
time (sec)	N/A	0.011	0.019	0.016	0.367	0.321	0.103	4.103	0.069

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	9
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82
time (sec)	N/A	0.002	0.027	0.220	0.283	0.372	0.064	5.639	3.516

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	4	4	5	4	4
N.S.	1	1.00	1.00	0.83	0.67	0.67	0.83	0.67	0.67
time (sec)	N/A	0.002	0.010	0.223	0.472	0.352	0.080	4.875	0.143

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	17	16	16	0	25	16
N.S.	1	1.00	0.85	0.85	0.80	0.80	0.00	1.25	0.80
time (sec)	N/A	0.003	0.014	0.226	0.486	0.355	0.000	4.956	3.508

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	26	11	12	11	11
N.S.	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65
time (sec)	N/A	0.002	0.002	0.018	0.275	0.339	0.063	4.583	0.024

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	11	11	15	11	12
N.S.	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63
time (sec)	N/A	0.003	0.002	0.033	0.277	0.384	0.830	3.702	0.025

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	42	28	27	25	37	27	27
N.S.	1	1.00	1.02	0.68	0.66	0.61	0.90	0.66	0.66
time (sec)	N/A	0.008	0.024	0.220	0.510	0.347	1.890	3.154	0.027

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	86	64	63	65	39	64	73
N.S.	1	1.00	1.28	0.96	0.94	0.97	0.58	0.96	1.09
time (sec)	N/A	0.021	0.045	0.227	0.505	0.367	0.602	2.742	3.832

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.016	0.267	0.333	0.009	3.152	0.002

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.001	0.024	0.264	0.335	0.008	3.871	0.023

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	46	7	7	8	7	7
N.S.	1	1.00	1.00	4.18	0.64	0.64	0.73	0.64	0.64
time (sec)	N/A	0.001	0.000	0.223	0.266	0.321	2.136	5.443	0.026

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	49	0	192	0	123	-1
N.S.	1	1.00	1.23	0.80	0.00	3.15	0.00	2.02	-0.02
time (sec)	N/A	0.016	0.017	0.306	0.000	0.367	0.000	3.908	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	78	55	0	202	0	131	-1
N.S.	1	1.00	1.20	0.85	0.00	3.11	0.00	2.02	-0.02
time (sec)	N/A	0.016	0.080	0.299	0.000	0.387	0.000	5.542	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.005	0.019	0.232	0.503	0.374	0.120	4.657	3.339

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	21	21	26	22	9
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69
time (sec)	N/A	0.008	0.019	0.234	0.490	0.354	0.175	5.454	0.027

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	58	77	131	202	80	233
N.S.	1	1.00	1.00	0.81	1.07	1.82	2.81	1.11	3.24
time (sec)	N/A	0.074	0.064	0.309	0.525	0.344	0.752	5.066	4.234

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	23	23	15	22	0	48	22
N.S.	1	1.00	0.62	0.62	0.41	0.59	0.00	1.30	0.59
time (sec)	N/A	0.017	0.012	0.226	0.268	0.335	0.000	4.878	3.513

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	41	13	9	17	0	17	-1
N.S.	1	1.00	3.42	1.08	0.75	1.42	0.00	1.42	-0.08
time (sec)	N/A	0.005	0.003	0.235	0.524	0.326	0.000	4.318	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	71	67	0	89	0	51	-1
N.S.	1	1.00	0.75	0.71	0.00	0.94	0.00	0.54	-0.01
time (sec)	N/A	0.035	0.119	0.013	0.000	0.885	0.000	4.121	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	24	23	21	184	23	16
N.S.	1	1.00	0.80	0.69	0.66	0.60	5.26	0.66	0.46
time (sec)	N/A	0.008	0.013	0.062	0.279	0.361	0.609	3.731	3.506

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	30	28	27	22	265	27	24
N.S.	1	1.00	0.81	0.76	0.73	0.59	7.16	0.73	0.65
time (sec)	N/A	0.010	0.014	0.044	0.313	0.328	0.607	3.666	3.578

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	40	48	0	33	32	29	39
N.S.	1	1.00	1.38	1.66	0.00	1.14	1.10	1.00	1.34
time (sec)	N/A	0.017	0.087	0.240	0.000	0.344	1.380	3.142	3.897

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	42	48	0	36	87	32	40
N.S.	1	1.00	1.68	1.92	0.00	1.44	3.48	1.28	1.60
time (sec)	N/A	0.017	0.089	0.249	0.000	0.359	2.176	2.837	3.648

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	0	15	56	15	22
N.S.	1	1.00	1.00	0.76	0.00	0.71	2.67	0.71	1.05
time (sec)	N/A	0.015	0.056	0.218	0.000	0.322	0.166	4.876	0.041

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	49	175	0	97	0	105	127
N.S.	1	1.00	0.75	2.69	0.00	1.49	0.00	1.62	1.95
time (sec)	N/A	0.036	0.116	0.139	0.000	0.332	0.000	5.392	3.499

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	28	0	41	0	63	64
N.S.	1	1.00	1.61	0.90	0.00	1.32	0.00	2.03	2.06
time (sec)	N/A	0.028	0.141	0.132	0.000	0.340	0.000	3.695	0.188

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	57	54	0	54	0	50	-1
N.S.	1	1.00	0.70	0.66	0.00	0.66	0.00	0.61	-0.01
time (sec)	N/A	0.029	0.098	0.225	0.000	0.572	0.000	5.436	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	56	55	57	155	55	95
N.S.	1	1.00	1.00	0.76	0.74	0.77	2.09	0.74	1.28
time (sec)	N/A	0.076	0.047	0.224	0.492	0.348	2.111	3.798	3.412

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	81	80	80	221	80	130
N.S.	1	1.00	1.00	0.70	0.70	0.70	1.92	0.70	1.13
time (sec)	N/A	0.103	0.057	0.225	0.494	0.354	2.431	3.182	0.087

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	22	51	0	20	0	4	4
N.S.	1	1.00	5.50	12.75	0.00	5.00	0.00	1.00	1.00
time (sec)	N/A	0.028	0.103	0.080	0.000	0.333	0.000	2.953	0.032

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	50	40	0	31	18
N.S.	1	1.00	1.00	1.86	2.27	1.82	0.00	1.41	0.82
time (sec)	N/A	0.008	0.025	0.060	0.277	0.331	0.000	3.309	3.387

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	37	26	0	28	20
N.S.	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83
time (sec)	N/A	0.007	0.020	0.098	0.486	0.347	0.000	4.678	0.037

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	28	34	45	51	40	0	35	24
N.S.	1	1.17	1.42	1.88	2.12	1.67	0.00	1.46	1.00
time (sec)	N/A	0.008	0.057	0.059	0.277	0.379	0.000	5.627	0.030

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	60	38	38	32	38	20
N.S.	1	1.00	1.00	2.50	1.58	1.58	1.33	1.58	0.83
time (sec)	N/A	0.012	0.019	0.056	0.277	0.330	1.768	4.706	0.037

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	48	45	51	42	0	35	35
N.S.	1	1.00	2.18	2.05	2.32	1.91	0.00	1.59	1.59
time (sec)	N/A	0.004	0.015	0.054	0.275	0.348	0.000	4.998	0.040

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	49	44	35	25	0	35	23
N.S.	1	1.00	1.69	1.52	1.21	0.86	0.00	1.21	0.79
time (sec)	N/A	0.009	0.009	0.105	0.484	0.330	0.000	3.694	3.367

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	49	28	36	35	0	25	26
N.S.	1	1.00	1.48	0.85	1.09	1.06	0.00	0.76	0.79
time (sec)	N/A	0.008	0.077	0.237	0.474	0.330	0.000	4.582	3.468

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	38	7	6	16	0	25	6
N.S.	1	1.00	4.75	0.88	0.75	2.00	0.00	3.12	0.75
time (sec)	N/A	0.003	0.026	0.228	0.496	0.344	0.000	4.265	0.014

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	29	11	18	0	18	13
N.S.	1	1.00	0.85	2.23	0.85	1.38	0.00	1.38	1.00
time (sec)	N/A	0.007	0.013	0.225	0.278	0.360	0.000	4.301	3.518

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	41	0	33	0	73	17
N.S.	1	1.00	1.09	1.86	0.00	1.50	0.00	3.32	0.77
time (sec)	N/A	0.021	0.039	0.232	0.000	0.327	0.000	4.634	3.583

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	39	32	11	13	15	13	45
N.S.	1	1.00	1.62	1.33	0.46	0.54	0.62	0.54	1.88
time (sec)	N/A	0.020	0.018	0.244	0.805	0.369	0.322	2.962	0.099

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.006	0.038	0.230	0.488	0.375	0.000	3.123	3.472

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.007	0.002	0.226	0.286	0.347	0.000	1.948	3.529

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.009	0.002	0.018	0.387	0.355	0.000	2.443	3.593

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.010	0.003	0.019	0.527	0.335	0.000	2.440	3.529

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.009	0.002	0.018	0.302	0.324	0.000	2.795	3.524

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	29	27	62	0	61	28
N.S.	1	1.00	1.96	1.04	0.96	2.21	0.00	2.18	1.00
time (sec)	N/A	0.009	0.003	0.018	0.314	0.346	0.000	4.491	3.515

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	0	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	0.00	1.90	0.82
time (sec)	N/A	0.011	0.009	0.230	0.286	0.342	0.000	3.147	3.903

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	0	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	0.00	1.90	0.82
time (sec)	N/A	0.012	0.003	0.226	0.262	0.328	0.000	2.606	3.557

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	0	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	0.00	1.90	0.82
time (sec)	N/A	0.011	0.002	0.228	0.331	0.334	0.000	2.211	3.475

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	37	36	87	0	76	33
N.S.	1	1.00	1.40	0.92	0.90	2.18	0.00	1.90	0.82
time (sec)	N/A	0.013	0.003	0.019	0.397	0.334	0.000	3.079	3.654

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	A	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	110	0	0	68	0	0	-1
N.S.	1	0.00	1.75	0.00	0.00	1.08	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.526	0.008	0.000	0.891	0.000	0.000	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	F	F	B	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	91	0	0	127	0	0	-1
N.S.	1	0.00	1.38	0.00	0.00	1.92	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.638	0.031	0.000	190.194	0.000	0.000	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	319	83	438	0	383	0	218	649
N.S.	1	4.09	1.06	5.62	0.00	4.91	0.00	2.79	8.32
time (sec)	N/A	0.407	0.348	1.132	0.000	0.404	0.000	2.450	4.380

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	97	753	0	770	0	444	-1
N.S.	1	1.00	0.77	5.98	0.00	6.11	0.00	3.52	-0.01
time (sec)	N/A	0.111	0.125	0.773	0.000	0.382	0.000	2.256	0.000

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	96	0	0	62	0	0	-1
N.S.	1	1.00	4.36	0.00	0.00	2.82	0.00	0.00	-0.05
time (sec)	N/A	0.041	0.210	0.055	0.000	0.767	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.094	0.837	0.034	0.000	0.000	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.101	0.836	0.034	0.000	0.000	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	330	1528	0	0	0	0	-1
N.S.	1	1.00	1.79	8.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	10.399	0.560	0.000	0.000	0.000	0.000	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	90	1036	0	0	0	0	-1
N.S.	1	1.00	0.69	7.91	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	20.068	0.054	0.000	0.000	0.000	0.000	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	419	514	0	318	0	0	-1
N.S.	1	1.00	7.76	9.52	0.00	5.89	0.00	0.00	-0.02
time (sec)	N/A	0.163	10.561	0.082	0.000	7.989	0.000	0.000	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	416	517	0	319	0	0	-1
N.S.	1	1.00	7.85	9.75	0.00	6.02	0.00	0.00	-0.02
time (sec)	N/A	0.166	10.529	0.085	0.000	8.148	0.000	0.000	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	90	69	0	154	0	164	-1
N.S.	1	1.00	1.07	0.82	0.00	1.83	0.00	1.95	-0.01
time (sec)	N/A	0.082	0.237	0.543	0.000	0.373	0.000	2.140	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	7	22	26	15	13
N.S.	1	1.00	0.85	1.15	0.35	1.10	1.30	0.75	0.65
time (sec)	N/A	0.004	0.004	0.025	0.485	0.376	0.164	1.915	3.413

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.012	0.010	0.000	0.000	0.000	0.000	0.000

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	242984	0	326	0	0	-1
N.S.	1	1.00	0.94	2761.18	0.00	3.70	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.724	0.374	0.000	2.234	0.000	0.000	0.000

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	15147	269221	0	333	0	0	-1
N.S.	1	1.00	172.12	3059.33	0.00	3.78	0.00	0.00	-0.01
time (sec)	N/A	0.217	16.386	0.333	0.000	2.282	0.000	0.000	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	-1
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.389	4.938	0.034	0.000	5.846	0.000	0.000	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	114	0	0	161	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.401	4.487	0.038	0.000	5.476	0.000	0.000	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	107	0	0	161	0	0	-1
N.S.	1	1.00	2.33	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.744	0.035	0.039	0.000	5.763	0.000	0.000	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	114	0	0	161	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	3.50	0.00	0.00	-0.02
time (sec)	N/A	0.762	0.010	0.038	0.000	5.274	0.000	0.000	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	27	147	94	96	17	58	132
N.S.	1	1.00	1.42	7.74	4.95	5.05	0.89	3.05	6.95
time (sec)	N/A	0.369	0.266	0.404	0.326	0.359	157.078	3.326	6.098

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	180	2499	0	458	0	0	-1
N.S.	1	1.00	2.00	27.77	0.00	5.09	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.361	9.401	0.000	5.249	0.000	0.000	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	425	181	690	0	289	0	0	-1
N.S.	1	4.13	1.76	6.70	0.00	2.81	0.00	0.00	-0.01
time (sec)	N/A	0.404	1.155	5.433	0.000	3.241	0.000	0.000	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	47	53	101	0	51	0	0	-1
N.S.	1	0.96	1.08	2.06	0.00	1.04	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.151	0.500	0.000	0.395	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	383	555	0	477	0	0	-1
N.S.	1	1.00	4.79	6.94	0.00	5.96	0.00	0.00	-0.01
time (sec)	N/A	0.281	10.561	0.278	0.000	29.132	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.001	0.000	0.221	0.271	0.342	0.006	1.984	0.005

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	122	56	234	0	73	0	193	549
N.S.	1	2.90	1.33	5.57	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.108	0.201	0.225	0.000	0.363	0.000	2.615	3.925

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	149	56	234	0	73	0	193	549
N.S.	1	3.55	1.33	5.57	0.00	1.74	0.00	4.60	13.07
time (sec)	N/A	0.258	0.173	0.244	0.000	0.361	0.000	1.427	3.895

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	A	A	A	A	B	B	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	31	32	16	31	73	81	31
N.S.	1	0.00	0.91	0.94	0.47	0.91	2.15	2.38	0.91
time (sec)	N/A	0.047	0.093	0.250	0.483	0.358	20.915	2.577	3.452

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	182	1597	0	164	0	0	-1
N.S.	1	1.00	1.03	9.02	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.060	6.520	0.679	0.000	0.445	0.000	0.000	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	243	100	2992	0	97	0	0	-1
N.S.	1	2.43	1.00	29.92	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.093	5.043	1.035	0.000	0.420	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [613] had the largest ratio of [176]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	4	1.00	19	0.210
2	A	4	4	1.00	23	0.174
3	A	4	4	1.00	21	0.190
4	A	4	4	1.00	21	0.190
5	A	4	4	1.00	33	0.121
6	A	4	4	1.00	35	0.114
7	A	4	4	1.00	36	0.111
8	A	4	4	1.00	36	0.111
9	A	4	4	1.00	24	0.167
10	A	4	4	1.00	20	0.200
11	A	4	4	1.00	24	0.167
12	A	4	4	1.00	22	0.182
13	A	4	4	1.00	22	0.182
14	A	8	8	1.00	15	0.533
15	A	8	8	1.00	17	0.471
16	A	8	8	1.00	15	0.533
17	A	8	8	1.00	17	0.471
18	A	1	1	1.00	25	0.040
19	A	3	3	1.00	25	0.120
20	A	1	1	1.00	25	0.040
21	A	1	1	1.00	21	0.048
22	A	1	1	1.00	24	0.042
23	A	11	9	1.00	19	0.474
24	A	9	8	1.00	19	0.421
25	A	8	7	1.00	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	5	1.00	17	0.294
27	A	13	12	1.00	19	0.632
28	A	20	17	1.00	19	0.895
29	A	10	6	1.00	19	0.316
30	A	8	6	1.00	19	0.316
31	A	7	6	1.00	19	0.316
32	A	5	4	1.00	17	0.235
33	A	10	9	1.00	19	0.474
34	A	17	13	1.00	19	0.684
35	A	32	17	1.00	19	0.895
36	A	10	8	1.00	19	0.421
37	A	8	7	1.00	19	0.368
38	A	7	6	1.00	19	0.316
39	A	5	4	1.00	17	0.235
40	A	10	9	1.00	19	0.474
41	A	18	15	1.00	19	0.790
42	A	30	18	1.00	19	0.947
43	A	2	2	1.00	28	0.071
44	A	2	2	1.00	32	0.062
45	A	2	2	1.00	30	0.067
46	A	2	2	1.00	30	0.067
47	A	2	2	1.00	53	0.038
48	A	2	2	1.00	55	0.036
49	A	2	2	1.00	56	0.036
50	A	2	2	1.00	56	0.036
51	A	2	2	1.00	30	0.067
52	A	4	4	1.00	24	0.167
53	A	4	4	1.00	28	0.143
54	A	4	4	1.00	26	0.154
55	A	4	4	1.00	26	0.154
56	A	4	4	1.00	24	0.167
57	A	4	4	1.00	28	0.143
58	A	4	4	1.00	26	0.154
59	A	4	4	1.00	26	0.154
60	A	4	4	1.00	38	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	40	0.100
62	A	4	4	1.00	41	0.098
63	A	4	4	1.00	41	0.098
64	A	4	4	1.00	29	0.138
65	A	4	4	1.00	20	0.200
66	A	4	4	1.00	24	0.167
67	A	4	4	1.00	22	0.182
68	A	4	4	1.00	22	0.182
69	A	4	4	1.00	34	0.118
70	A	4	4	1.00	36	0.111
71	A	4	4	1.00	37	0.108
72	A	4	4	1.00	37	0.108
73	A	4	4	1.00	25	0.160
74	A	2	2	1.00	20	0.100
75	A	2	2	1.00	22	0.091
76	A	2	2	1.00	20	0.100
77	A	2	2	1.00	22	0.091
78	A	2	2	1.00	43	0.047
79	A	2	2	1.00	44	0.045
80	A	2	2	1.00	45	0.044
81	A	2	2	1.00	46	0.043
82	A	2	2	1.00	30	0.067
83	A	4	4	1.00	22	0.182
84	A	4	4	1.00	22	0.182
85	A	4	4	1.00	20	0.200
86	A	4	4	1.00	24	0.167
87	A	4	4	1.00	35	0.114
88	A	4	4	1.00	35	0.114
89	A	4	4	1.00	36	0.111
90	A	4	4	1.00	38	0.105
91	A	4	4	1.00	29	0.138
92	A	4	4	1.00	18	0.222
93	A	4	4	1.00	18	0.222
94	A	4	4	1.00	16	0.250
95	A	4	4	1.00	20	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	4	1.00	31	0.129
97	A	4	4	1.00	31	0.129
98	A	4	4	1.00	32	0.125
99	A	4	4	1.00	34	0.118
100	A	4	4	1.00	25	0.160
101	A	2	2	1.00	30	0.067
102	A	2	2	1.00	36	0.056
103	A	2	2	1.00	34	0.059
104	A	2	2	1.00	32	0.062
105	A	2	2	1.00	58	0.034
106	A	2	2	1.00	61	0.033
107	A	2	2	1.00	62	0.032
108	A	2	2	1.00	61	0.033
109	A	2	2	1.00	52	0.038
110	A	2	2	1.00	55	0.036
111	A	2	2	1.00	56	0.036
112	A	2	2	1.00	55	0.036
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	36	0.056
115	A	2	2	1.00	34	0.059
116	A	2	2	1.00	32	0.062
117	A	2	2	1.00	58	0.034
118	A	2	2	1.00	61	0.033
119	A	2	2	1.00	62	0.032
120	A	2	2	1.00	61	0.033
121	A	2	2	1.00	52	0.038
122	A	2	2	1.00	55	0.036
123	A	2	2	1.00	56	0.036
124	A	2	2	1.00	55	0.036
125	A	4	4	1.00	23	0.174
126	A	4	4	1.00	25	0.160
127	A	4	4	1.00	25	0.160
128	A	4	4	1.00	29	0.138
129	A	4	4	1.00	27	0.148
130	A	4	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	42	0.095
132	A	4	4	1.00	44	0.091
133	A	4	4	1.00	45	0.089
134	A	4	4	1.00	45	0.089
135	A	4	4	1.00	21	0.190
136	A	4	4	1.00	25	0.160
137	A	4	4	1.00	23	0.174
138	A	4	4	1.00	23	0.174
139	A	4	4	1.00	23	0.174
140	A	4	4	1.00	38	0.105
141	A	4	4	1.00	40	0.100
142	A	4	4	1.00	41	0.098
143	A	4	4	1.00	41	0.098
144	A	6	6	1.00	25	0.240
145	A	6	6	1.00	29	0.207
146	A	6	6	1.00	27	0.222
147	A	6	6	1.00	27	0.222
148	A	6	6	1.00	27	0.222
149	A	6	6	1.00	31	0.194
150	A	6	6	1.00	29	0.207
151	A	6	6	1.00	29	0.207
152	A	5	5	1.00	21	0.238
153	A	5	5	1.00	25	0.200
154	A	5	5	1.00	23	0.217
155	A	5	5	1.00	23	0.217
156	A	5	5	1.00	23	0.217
157	A	5	5	1.00	27	0.185
158	A	5	5	1.00	25	0.200
159	A	5	5	1.00	25	0.200
160	A	8	8	1.00	16	0.500
161	A	8	8	1.00	18	0.444
162	A	8	8	1.00	16	0.500
163	A	8	8	1.00	18	0.444
164	A	8	8	1.00	22	0.364
165	A	8	8	1.00	24	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	8	1.00	22	0.364
167	A	8	8	1.00	24	0.333
168	A	6	6	1.00	18	0.333
169	A	6	6	1.00	20	0.300
170	A	6	6	1.00	18	0.333
171	A	6	6	1.00	20	0.300
172	A	1	1	1.00	31	0.032
173	A	3	3	1.00	30	0.100
174	A	2	1	1.00	18	0.056
175	A	2	1	1.00	16	0.062
176	A	2	1	1.00	15	0.067
177	A	3	2	1.00	18	0.111
178	A	2	1	1.00	20	0.050
179	A	2	1	1.00	18	0.056
180	A	2	1	1.00	17	0.059
181	A	3	2	1.00	20	0.100
182	A	2	1	1.00	20	0.050
183	A	2	1	1.00	18	0.056
184	A	2	1	1.00	17	0.059
185	A	3	2	1.00	20	0.100
186	A	7	2	1.00	20	0.100
187	A	7	2	1.00	20	0.100
188	A	7	2	1.00	20	0.100
189	A	5	2	1.00	20	0.100
190	A	5	2	1.00	18	0.111
191	A	5	2	1.00	17	0.118
192	A	8	3	1.00	20	0.150
193	A	8	3	1.00	20	0.150
194	A	5	2	1.00	22	0.091
195	A	8	3	1.00	20	0.150
196	A	13	12	1.00	19	0.632
197	F	0	0	N/A	0.	N/A
198	A	2	2	1.00	27	0.074
199	A	2	2	1.00	29	0.069
200	A	2	2	1.00	27	0.074

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	2	2	1.00	29	0.069
202	A	2	2	1.00	31	0.065
203	A	2	2	1.00	35	0.057
204	A	2	2	1.00	33	0.061
205	A	2	2	1.00	33	0.061
206	A	11	10	1.00	19	0.526
207	A	10	9	1.00	19	0.474
208	A	8	6	1.00	17	0.353
209	A	2	2	1.00	11	0.182
210	A	15	13	1.00	19	0.684
211	A	32	15	1.00	19	0.790
212	A	9	8	1.00	19	0.421
213	A	8	7	1.00	19	0.368
214	A	6	5	1.00	17	0.294
215	A	1	1	1.00	11	0.091
216	A	7	7	1.00	19	0.368
217	A	11	11	1.00	19	0.579
218	A	12	12	1.00	19	0.632
219	A	4	4	1.00	19	0.210
220	A	4	4	1.00	19	0.210
221	A	3	3	1.00	17	0.176
222	A	2	2	1.00	11	0.182
223	A	14	13	1.00	19	0.684
224	A	10	3	1.00	20	0.150
225	A	10	3	1.00	22	0.136
226	A	16	8	1.00	24	0.333
227	A	3	2	1.00	19	0.105
228	A	4	3	1.00	19	0.158
229	A	3	2	1.00	19	0.105
230	A	4	3	1.00	19	0.158
231	A	3	2	1.00	19	0.105
232	A	3	3	1.00	17	0.176
233	A	3	2	1.00	15	0.133
234	A	4	3	1.00	19	0.158
235	A	3	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	4	3	1.00	19	0.158
237	A	8	4	1.00	19	0.210
238	A	3	3	1.00	17	0.176
239	A	7	3	1.00	15	0.200
240	A	9	5	1.00	19	0.263
241	A	7	4	1.00	19	0.210
242	A	9	6	1.00	19	0.316
243	A	3	3	1.00	19	0.158
244	A	3	3	1.00	17	0.176
245	A	2	2	1.00	15	0.133
246	A	5	5	1.00	19	0.263
247	A	3	3	1.00	19	0.158
248	A	6	6	1.00	19	0.316
249	A	4	3	1.00	21	0.143
250	A	4	3	1.00	21	0.143
251	A	4	3	1.00	21	0.143
252	A	3	3	1.00	19	0.158
253	A	7	7	1.00	21	0.333
254	A	7	7	1.00	21	0.333
255	A	5	5	1.00	21	0.238
256	A	4	4	1.00	17	0.235
257	A	4	4	1.00	21	0.190
258	A	5	5	1.00	21	0.238
259	A	4	4	1.00	15	0.267
260	A	6	6	1.00	17	0.353
261	A	3	3	1.00	15	0.200
262	A	3	3	1.00	17	0.176
263	A	6	6	1.00	26	0.231
264	A	5	5	1.00	26	0.192
265	A	4	4	1.00	24	0.167
266	A	5	4	1.00	26	0.154
267	A	4	4	1.00	26	0.154
268	A	5	5	1.00	26	0.192
269	A	6	6	1.00	26	0.231
270	A	7	7	1.00	26	0.269

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	6	6	1.00	26	0.231
272	A	5	5	1.00	22	0.227
273	A	7	7	1.00	26	0.269
274	A	7	7	1.00	26	0.269
275	A	8	7	1.00	26	0.269
276	A	7	7	1.00	26	0.269
277	A	6	6	1.00	26	0.231
278	A	5	5	1.00	24	0.208
279	A	6	5	1.00	26	0.192
280	A	5	5	1.00	26	0.192
281	A	6	6	1.00	26	0.231
282	A	7	7	1.00	26	0.269
283	A	8	8	1.00	26	0.308
284	A	7	7	1.00	26	0.269
285	A	6	6	1.00	22	0.273
286	A	7	7	1.00	26	0.269
287	A	8	7	1.00	26	0.269
288	A	9	7	1.00	26	0.269
289	A	4	4	1.00	21	0.190
290	A	4	4	1.00	23	0.174
291	A	4	4	1.00	23	0.174
292	A	6	6	1.00	23	0.261
293	A	5	5	1.00	25	0.200
294	A	5	5	1.00	23	0.217
295	A	5	5	1.00	28	0.179
296	A	6	6	1.00	26	0.231
297	A	5	5	1.00	26	0.192
298	A	4	4	1.00	24	0.167
299	A	5	4	1.00	26	0.154
300	A	4	4	1.00	26	0.154
301	A	5	5	1.00	26	0.192
302	A	7	7	1.00	26	0.269
303	A	6	6	1.00	26	0.231
304	A	5	5	1.00	22	0.227
305	A	7	7	1.00	26	0.269

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	7	0.99	26	0.269
307	A	7	6	1.00	26	0.231
308	A	6	5	1.00	26	0.192
309	A	5	5	1.00	24	0.208
310	A	6	5	1.00	26	0.192
311	A	5	5	1.00	26	0.192
312	A	6	5	1.00	26	0.192
313	A	8	8	1.00	26	0.308
314	A	7	7	1.00	26	0.269
315	A	6	6	1.00	22	0.273
316	A	7	7	1.00	26	0.269
317	A	8	7	1.00	26	0.269
318	A	7	7	1.00	21	0.333
319	A	6	6	1.00	21	0.286
320	A	5	5	1.00	19	0.263
321	A	6	5	1.00	21	0.238
322	A	5	5	1.00	21	0.238
323	A	6	6	1.00	21	0.286
324	A	7	7	1.00	21	0.333
325	A	8	8	1.00	21	0.381
326	A	7	7	1.00	21	0.333
327	A	6	6	1.00	17	0.353
328	A	8	8	1.00	21	0.381
329	A	8	8	1.00	21	0.381
330	A	9	8	1.00	21	0.381
331	A	8	8	1.00	21	0.381
332	A	7	7	1.00	21	0.333
333	A	6	6	1.00	19	0.316
334	A	7	6	1.00	21	0.286
335	A	6	6	1.00	21	0.286
336	A	7	7	1.00	21	0.333
337	A	8	8	1.00	21	0.381
338	A	9	9	1.00	21	0.429
339	A	8	8	1.00	21	0.381
340	A	7	7	1.00	17	0.412

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	8	8	1.00	21	0.381
342	A	9	8	1.00	21	0.381
343	A	10	8	1.00	21	0.381
344	A	7	7	1.00	21	0.333
345	A	6	6	1.00	21	0.286
346	A	5	5	1.00	19	0.263
347	A	6	5	1.00	21	0.238
348	A	5	5	1.00	21	0.238
349	A	6	6	1.00	21	0.286
350	A	8	8	1.00	21	0.381
351	A	7	7	1.00	21	0.333
352	A	6	6	1.00	17	0.353
353	A	8	8	1.00	21	0.381
354	A	8	8	0.99	21	0.381
355	A	8	7	1.00	21	0.333
356	A	7	6	1.00	21	0.286
357	A	6	6	1.00	19	0.316
358	A	7	6	1.00	21	0.286
359	A	6	6	1.00	21	0.286
360	A	7	6	1.00	21	0.286
361	A	9	9	1.00	21	0.429
362	A	8	8	1.00	21	0.381
363	A	7	7	1.00	17	0.412
364	A	8	8	1.00	21	0.381
365	A	9	8	1.00	21	0.381
366	A	6	5	1.00	19	0.263
367	A	5	5	1.00	19	0.263
368	A	4	4	1.00	19	0.210
369	A	2	2	1.00	19	0.105
370	A	3	3	1.00	19	0.158
371	A	4	4	1.00	22	0.182
372	A	5	5	1.00	19	0.263
373	A	6	5	1.00	22	0.227
374	A	8	5	1.00	33	0.152
375	A	9	6	1.00	30	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	6	6	1.00	19	0.316
377	A	3	3	1.00	19	0.158
378	A	4	4	1.00	19	0.210
379	A	2	2	1.00	19	0.105
380	A	4	4	1.00	17	0.235
381	A	3	3	1.00	19	0.158
382	A	4	4	1.00	19	0.210
383	A	6	6	1.00	19	0.316
384	A	2	2	1.00	19	0.105
385	A	2	2	1.00	19	0.105
386	A	5	5	1.00	19	0.263
387	A	4	4	1.00	19	0.210
388	A	3	3	1.00	17	0.176
389	A	3	3	1.00	19	0.158
390	A	4	4	1.00	19	0.210
391	A	6	6	1.00	19	0.316
392	A	5	5	1.00	19	0.263
393	A	2	2	1.00	21	0.095
394	A	2	2	1.00	19	0.105
395	A	2	2	1.00	23	0.087
396	C	5	3	2.35	54	0.056
397	A	2	2	1.00	26	0.077
398	A	2	2	1.00	7	0.286
399	A	3	2	1.00	15	0.133
400	A	4	2	1.00	22	0.091
401	A	5	2	1.00	25	0.080
402	A	5	2	1.00	23	0.087
403	A	2	1	1.00	21	0.048
404	A	7	4	1.00	25	0.160
405	A	7	4	1.00	25	0.160
406	A	9	7	1.00	25	0.280
407	A	8	6	1.00	23	0.261
408	A	7	5	1.81	21	0.238
409	A	9	8	1.00	25	0.320
410	A	9	8	1.00	25	0.320

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	10	2	1.00	25	0.080
412	A	10	2	1.00	23	0.087
413	B	6	2	2.36	21	0.095
414	A	8	4	1.00	25	0.160
415	A	14	5	1.38	25	0.200
416	A	3	3	1.00	15	0.200
417	A	3	3	1.00	15	0.200
418	A	2	1	1.00	17	0.059
419	A	5	3	1.00	23	0.130
420	A	5	4	1.00	23	0.174
421	A	3	2	1.00	21	0.095
422	A	4	3	1.00	19	0.158
423	A	6	5	1.00	23	0.217
424	A	4	3	1.00	23	0.130
425	A	6	5	1.00	23	0.217
426	A	5	2	1.00	25	0.080
427	A	5	2	1.00	25	0.080
428	A	3	2	1.00	23	0.087
429	A	8	4	1.00	21	0.190
430	A	7	4	1.00	25	0.160
431	A	9	5	1.00	25	0.200
432	A	8	6	1.00	25	0.240
433	A	7	5	1.00	25	0.200
434	A	9	8	1.00	23	0.348
435	A	9	8	1.00	21	0.381
436	A	6	4	1.00	25	0.160
437	A	7	5	1.00	25	0.200
438	A	10	2	1.00	25	0.080
439	A	6	2	1.00	25	0.080
440	A	8	4	1.00	25	0.160
441	A	14	5	1.42	23	0.217
442	A	16	5	1.68	21	0.238
443	A	4	3	1.00	27	0.111
444	A	6	4	1.00	42	0.095
445	A	6	5	1.00	42	0.119

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	4	3	1.00	40	0.075
447	A	5	4	1.00	39	0.103
448	A	7	6	1.00	42	0.143
449	A	5	4	1.00	42	0.095
450	A	7	6	1.00	42	0.143
451	A	15	7	1.00	39	0.180
452	A	9	4	1.00	35	0.114
453	A	4	3	1.00	25	0.120
454	A	3	2	1.00	25	0.080
455	A	3	2	1.00	25	0.080
456	A	4	3	1.00	23	0.130
457	A	3	2	1.00	25	0.080
458	A	3	2	1.00	25	0.080
459	A	3	2	1.00	25	0.080
460	A	6	5	1.00	27	0.185
461	A	6	5	1.00	27	0.185
462	A	6	5	1.00	27	0.185
463	A	5	5	1.00	27	0.185
464	A	5	5	1.00	27	0.185
465	A	6	5	1.00	27	0.185
466	A	3	2	1.00	17	0.118
467	A	3	2	1.00	26	0.077
468	A	1	1	1.00	17	0.059
469	A	1	1	1.00	15	0.067
470	A	1	1	1.00	15	0.067
471	A	1	1	1.00	25	0.040
472	A	4	3	1.00	28	0.107
473	A	3	2	1.00	28	0.071
474	A	3	2	1.00	28	0.071
475	A	4	3	1.00	26	0.115
476	A	3	2	1.00	28	0.071
477	A	3	2	1.00	28	0.071
478	A	3	2	1.00	28	0.071
479	A	6	5	1.00	30	0.167
480	A	6	5	1.00	30	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	6	5	1.00	30	0.167
482	A	5	5	1.00	30	0.167
483	A	5	5	1.00	30	0.167
484	A	6	5	1.00	30	0.167
485	A	3	2	1.00	21	0.095
486	A	3	2	1.00	19	0.105
487	A	3	2	1.00	13	0.154
488	A	2	2	1.00	21	0.095
489	A	2	2	1.00	21	0.095
490	A	3	2	1.00	23	0.087
491	A	3	2	1.00	21	0.095
492	A	3	2	1.00	15	0.133
493	A	2	2	1.00	23	0.087
494	A	2	2	1.00	23	0.087
495	A	3	2	1.00	23	0.087
496	A	3	2	1.00	23	0.087
497	A	3	2	1.00	23	0.087
498	A	2	2	1.00	23	0.087
499	A	2	2	1.00	23	0.087
500	A	2	2	1.00	23	0.087
501	A	3	2	1.00	25	0.080
502	A	3	2	1.00	25	0.080
503	A	3	2	1.00	25	0.080
504	A	2	2	1.00	25	0.080
505	A	2	2	1.00	25	0.080
506	A	2	2	1.00	25	0.080
507	A	4	3	1.00	56	0.054
508	A	4	3	1.00	54	0.056
509	A	4	3	1.00	33	0.091
510	A	2	2	1.00	56	0.036
511	A	3	3	1.00	56	0.054
512	A	5	4	1.00	33	0.121
513	A	3	3	1.00	56	0.054
514	A	4	3	1.00	58	0.052
515	A	4	3	1.00	58	0.052

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	3	1.00	58	0.052
517	A	3	3	1.00	58	0.052
518	A	4	4	1.00	58	0.069
519	A	5	4	1.00	62	0.065
520	A	4	4	1.00	62	0.065
521	A	4	4	1.00	62	0.065
522	A	5	5	1.00	60	0.083
523	A	7	6	1.00	30	0.200
524	A	4	3	1.00	37	0.081
525	A	4	3	1.00	37	0.081
526	A	2	2	1.00	37	0.054
527	A	2	2	1.00	37	0.054
528	A	2	2	1.00	42	0.048
529	A	2	2	1.00	42	0.048
530	A	4	4	1.00	30	0.133
531	A	4	4	1.00	30	0.133
532	A	2	2	1.00	42	0.048
533	A	2	2	1.00	42	0.048
534	A	2	2	1.45	51	0.039
535	A	2	2	1.45	51	0.039
536	A	2	2	1.00	40	0.050
537	A	2	2	1.00	40	0.050
538	A	4	4	1.00	32	0.125
539	A	4	4	1.00	32	0.125
540	A	2	2	1.00	51	0.039
541	A	2	2	1.00	51	0.039
542	A	2	2	1.00	56	0.036
543	A	2	2	1.00	56	0.036
544	A	4	2	1.00	29	0.069
545	A	4	2	1.00	29	0.069
546	A	3	2	1.00	27	0.074
547	A	7	6	1.00	29	0.207
548	A	8	6	1.00	29	0.207
549	A	8	7	1.00	29	0.241
550	A	4	3	1.00	25	0.120

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	7	6	1.00	29	0.207
552	A	4	2	1.00	29	0.069
553	A	4	2	1.00	29	0.069
554	A	3	2	1.00	29	0.069
555	A	7	6	1.00	29	0.207
556	A	8	6	1.00	29	0.207
557	A	10	10	1.00	29	0.345
558	A	9	9	1.00	27	0.333
559	A	9	9	1.00	25	0.360
560	A	10	10	1.00	29	0.345
561	A	10	10	1.00	29	0.345
562	A	4	4	1.00	25	0.160
563	A	4	4	1.00	29	0.138
564	A	3	2	1.00	31	0.065
565	A	3	3	1.00	15	0.200
566	A	5	5	1.00	15	0.333
567	A	4	3	1.00	15	0.200
568	A	4	3	1.00	13	0.231
569	A	4	3	1.00	13	0.231
570	A	4	3	1.00	15	0.200
571	A	9	9	1.00	13	0.692
572	A	4	3	1.00	13	0.231
573	A	4	3	1.00	13	0.231
574	A	9	9	1.00	15	0.600
575	A	3	3	1.00	13	0.231
576	A	4	3	1.00	13	0.231
577	A	13	10	1.00	15	0.667
578	A	10	7	1.00	19	0.368
579	A	4	3	1.00	19	0.158
580	A	10	9	1.00	21	0.429
581	A	3	3	1.00	26	0.115
582	A	3	3	1.00	26	0.115
583	A	3	3	1.00	24	0.125
584	A	3	3	1.00	23	0.130
585	A	3	3	1.00	26	0.115

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
586	A	3	3	1.00	26	0.115
587	A	4	3	1.00	17	0.176
588	A	5	5	1.00	17	0.294
589	A	4	4	1.00	15	0.267
590	A	2	2	1.00	9	0.222
591	A	5	5	1.00	17	0.294
592	A	3	3	1.00	17	0.176
593	A	4	4	1.00	17	0.235
594	A	5	5	1.00	17	0.294
595	A	3	3	1.00	28	0.107
596	A	3	3	1.00	28	0.107
597	A	3	3	1.00	26	0.115
598	A	3	3	1.00	25	0.120
599	A	3	3	1.00	28	0.107
600	A	3	3	1.00	28	0.107
601	A	8	6	1.00	21	0.286
602	A	1	1	1.00	17	0.059
603	A	1	1	1.00	21	0.048
604	A	1	1	1.00	17	0.059
605	A	1	1	1.00	19	0.053
606	A	1	1	1.00	21	0.048
607	A	4	3	1.00	25	0.120
608	A	3	2	1.00	17	0.118
609	A	4	3	1.00	27	0.111
610	A	4	3	1.00	25	0.120
611	A	5	4	1.70	22	0.182
612	A	4	3	1.00	29	0.103
613	A	1	1	1.00	176	0.006
614	A	1	1	1.00	174	0.006
615	A	1	1	1.00	164	0.006
616	F	0	0	N/A	0.	N/A
617	F	0	0	N/A	0.	N/A
618	A	4	3	1.00	19	0.158
619	A	4	3	1.00	19	0.158
620	A	4	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	4	3	1.00	15	0.200
622	A	7	6	1.00	19	0.316
623	A	6	6	1.00	19	0.316
624	A	6	6	1.00	19	0.316
625	A	4	3	1.00	21	0.143
626	A	4	3	1.00	21	0.143
627	A	4	3	1.00	19	0.158
628	A	4	3	1.00	17	0.176
629	A	7	6	1.00	21	0.286
630	A	8	7	1.00	21	0.333
631	A	9	8	1.00	21	0.381
632	A	4	3	1.00	19	0.158
633	A	4	3	1.00	19	0.158
634	A	4	3	1.00	17	0.176
635	A	4	3	1.00	15	0.200
636	A	7	6	1.00	19	0.316
637	A	8	7	1.00	19	0.368
638	A	9	7	1.00	19	0.368
639	A	4	3	1.00	19	0.158
640	A	4	3	1.00	19	0.158
641	A	4	3	1.00	17	0.176
642	A	4	3	1.00	15	0.200
643	A	7	6	1.00	19	0.316
644	A	8	7	1.00	19	0.368
645	A	9	7	1.00	19	0.368
646	A	4	3	1.00	21	0.143
647	A	4	3	1.00	21	0.143
648	A	4	3	1.00	19	0.158
649	A	4	3	1.00	17	0.176
650	A	6	5	1.00	21	0.238
651	A	7	6	1.00	21	0.286
652	A	8	6	1.00	21	0.286
653	A	4	3	1.00	19	0.158
654	A	4	3	1.00	19	0.158
655	A	4	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	4	3	1.00	15	0.200
657	A	6	4	1.00	19	0.210
658	A	8	6	1.00	17	0.353
659	A	7	6	1.00	17	0.353
660	A	6	6	1.00	17	0.353
661	A	5	5	1.00	17	0.294
662	A	6	6	1.00	17	0.353
663	A	7	6	1.00	17	0.353
664	A	8	6	1.00	19	0.316
665	A	7	6	1.00	19	0.316
666	A	6	6	1.00	19	0.316
667	A	5	5	1.00	19	0.263
668	A	6	6	1.00	19	0.316
669	A	7	6	1.00	19	0.316
670	A	2	2	1.00	13	0.154
671	A	5	5	1.00	17	0.294
672	A	6	5	1.00	21	0.238
673	A	7	5	1.00	25	0.200
674	A	8	5	1.00	29	0.172
675	A	8	6	1.00	20	0.300
676	A	7	6	1.00	20	0.300
677	A	6	6	1.00	18	0.333
678	A	2	2	1.00	20	0.100
679	A	4	3	1.00	20	0.150
680	A	4	3	1.00	20	0.150
681	A	2	2	1.00	18	0.111
682	A	2	2	1.00	20	0.100
683	A	3	2	1.00	22	0.091
684	A	3	3	1.00	17	0.176
685	A	3	3	1.00	23	0.130
686	A	3	2	1.00	22	0.091
687	A	5	4	1.00	34	0.118
688	A	5	5	1.00	31	0.161
689	A	6	4	1.00	47	0.085
690	A	9	5	1.00	58	0.086

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
691	A	4	4	1.00	11	0.364
692	A	6	6	1.00	11	0.546
693	A	2	1	1.00	17	0.059
694	A	8	6	1.00	13	0.462
695	A	2	1	1.00	16	0.062
696	A	4	2	1.00	14	0.143
697	A	5	4	1.00	12	0.333
698	A	4	2	1.00	13	0.154
699	A	4	2	1.00	13	0.154
700	A	4	2	1.00	15	0.133
701	A	5	5	1.00	12	0.417
702	A	6	5	1.00	14	0.357
703	A	5	4	1.00	13	0.308
704	A	5	4	1.00	17	0.235
705	A	5	4	1.00	17	0.235
706	A	4	3	1.00	13	0.231
707	A	7	5	1.00	18	0.278
708	A	7	5	1.00	20	0.250
709	A	8	5	1.00	18	0.278
710	A	4	3	1.00	26	0.115
711	A	10	6	0.72	17	0.353
712	A	3	1	1.00	27	0.037
713	A	5	3	1.00	17	0.176
714	A	5	3	1.00	17	0.176
715	A	5	3	1.00	23	0.130
716	A	5	3	1.00	17	0.176
717	A	6	2	1.00	23	0.087
718	A	5	1	1.00	25	0.040
719	A	5	2	1.00	21	0.095
720	A	3	2	1.00	23	0.087
721	A	4	3	1.18	16	0.188
722	A	3	1	1.00	28	0.036
723	A	5	5	1.00	16	0.312
724	A	4	3	1.00	22	0.136
725	A	8	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	B	16	10	2.80	20	0.500
727	A	9	6	1.00	25	0.240
728	A	6	4	1.00	35	0.114
729	A	2	2	1.00	13	0.154
730	A	3	3	1.00	15	0.200
731	A	3	3	1.00	13	0.231
732	A	4	4	1.00	11	0.364
733	A	3	3	1.00	18	0.167
734	A	4	4	1.00	17	0.235
735	A	4	4	1.00	18	0.222
736	A	5	5	1.00	17	0.294
737	A	2	2	1.00	16	0.125
738	A	2	2	1.00	21	0.095
739	A	3	3	1.00	26	0.115
740	A	3	3	1.00	25	0.120
741	A	3	3	1.00	12	0.250
742	A	3	3	1.00	15	0.200
743	A	3	3	1.00	15	0.200
744	A	3	3	1.00	15	0.200
745	A	3	3	1.00	17	0.176
746	A	4	4	1.00	15	0.267
747	A	4	4	1.00	21	0.190
748	A	3	3	1.00	22	0.136
749	A	4	4	1.00	20	0.200
750	A	7	7	1.00	20	0.350
751	A	2	2	1.00	15	0.133
752	A	5	5	1.00	21	0.238
753	A	8	6	1.00	18	0.333
754	A	5	4	1.00	18	0.222
755	A	6	4	1.00	18	0.222
756	A	3	2	1.00	16	0.125
757	A	3	2	1.00	16	0.125
758	A	3	2	1.00	16	0.125
759	A	10	9	1.00	18	0.500
760	A	5	4	1.00	18	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	6	5	1.00	18	0.278
762	A	5	5	1.40	35	0.143
763	A	6	6	1.40	28	0.214
764	A	7	7	1.00	23	0.304
765	A	6	6	1.00	23	0.261
766	A	3	3	1.00	23	0.130
767	A	6	6	1.00	23	0.261
768	A	7	7	1.00	23	0.304
769	A	7	7	1.00	19	0.368
770	A	6	6	1.00	19	0.316
771	A	3	3	1.00	19	0.158
772	A	6	6	1.00	19	0.316
773	A	7	7	1.00	19	0.368
774	A	6	6	1.00	31	0.194
775	A	5	5	1.00	31	0.161
776	A	2	2	1.00	31	0.065
777	A	5	5	1.00	31	0.161
778	A	5	5	1.00	34	0.147
779	A	2	2	1.00	34	0.059
780	A	5	5	1.00	34	0.147
781	A	8	8	1.00	24	0.333
782	A	7	7	1.00	24	0.292
783	A	3	3	1.00	24	0.125
784	A	7	7	1.00	24	0.292
785	A	8	8	1.00	24	0.333
786	A	14	13	1.00	26	0.500
787	A	12	12	1.00	26	0.462
788	A	7	7	1.00	26	0.269
789	A	10	10	1.00	26	0.385
790	A	12	12	1.00	26	0.462
791	A	15	13	1.00	28	0.464
792	A	13	13	1.00	28	0.464
793	A	11	11	1.00	28	0.393
794	A	10	9	1.00	28	0.321
795	A	13	12	1.00	28	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
796	A	4	4	1.00	19	0.210
797	A	10	10	1.00	19	0.526
798	A	3	3	1.00	19	0.158
799	A	9	9	1.00	19	0.474
800	A	4	4	1.00	24	0.167
801	A	10	10	1.00	24	0.417
802	A	12	11	1.00	24	0.458
803	A	4	4	1.00	24	0.167
804	A	10	10	1.00	24	0.417
805	A	12	11	1.00	27	0.407
806	A	14	7	1.00	20	0.350
807	A	12	6	1.91	21	0.286
808	A	2	1	1.00	22	0.045
809	A	4	2	1.00	18	0.111
810	A	4	2	1.00	18	0.111
811	A	5	5	1.00	23	0.217
812	A	6	6	1.00	22	0.273
813	A	6	6	1.00	20	0.300
814	A	3	2	1.00	15	0.133
815	A	3	2	1.00	21	0.095
816	A	5	4	1.00	19	0.210
817	A	9	6	1.00	24	0.250
818	A	9	6	1.00	24	0.250
819	A	10	7	1.00	21	0.333
820	A	11	8	1.00	13	0.615
821	A	12	8	1.00	25	0.320
822	A	13	9	1.00	15	0.600
823	A	14	9	1.00	19	0.474
824	A	2	1	1.00	35	0.029
825	A	8	5	1.00	37	0.135
826	A	16	8	1.00	29	0.276
827	A	16	8	1.00	36	0.222
828	A	31	13	1.00	43	0.302
829	A	5	4	1.00	21	0.190
830	A	6	4	1.00	27	0.148

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	8	6	1.00	27	0.222
832	A	4	4	1.00	27	0.148
833	A	4	4	1.00	20	0.200
834	A	6	6	1.00	29	0.207
835	A	2	1	1.00	20	0.050
836	A	3	2	1.00	25	0.080
837	A	2	2	1.00	28	0.071
838	A	3	3	1.00	26	0.115
839	A	1	1	1.00	11	0.091
840	A	2	2	1.00	19	0.105
841	A	2	2	1.00	15	0.133
842	A	2	2	1.00	14	0.143
843	A	3	3	1.00	17	0.176
844	A	3	3	1.00	13	0.231
845	A	2	2	1.00	14	0.143
846	A	3	3	1.00	17	0.176
847	A	3	3	1.00	13	0.231
848	A	1	0	1.00	9	0.000
849	A	4	3	1.00	15	0.200
850	A	2	2	1.00	13	0.154
851	A	3	3	1.00	19	0.158
852	A	2	2	1.00	11	0.182
853	A	3	3	1.00	17	0.176
854	A	1	1	1.00	9	0.111
855	A	2	2	1.00	19	0.105
856	A	1	1	1.00	7	0.143
857	A	2	2	1.00	21	0.095
858	A	1	1	1.00	9	0.111
859	A	2	2	1.00	19	0.105
860	A	1	1	1.00	7	0.143
861	A	2	2	1.00	21	0.095
862	A	3	3	1.00	17	0.176
863	A	4	4	1.00	30	0.133
864	A	7	7	1.00	20	0.350
865	A	6	6	1.00	20	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
866	A	7	7	1.00	23	0.304
867	A	6	6	1.00	25	0.240
868	A	1	1	1.00	11	0.091
869	A	2	2	1.00	21	0.095
870	A	1	1	1.00	9	0.111
871	A	2	2	1.00	23	0.087
872	A	2	2	1.00	11	0.182
873	A	3	3	1.00	21	0.143
874	A	2	2	1.00	9	0.222
875	A	3	3	1.00	23	0.130
876	A	3	3	1.16	14	0.214
877	A	4	4	1.00	15	0.267
878	A	7	6	1.00	17	0.353
879	A	7	6	1.00	17	0.353
880	A	10	8	1.00	34	0.235
881	A	12	7	1.00	30	0.233
882	A	5	4	1.00	21	0.190
883	A	7	5	1.00	23	0.217
884	A	9	7	1.00	25	0.280
885	A	7	6	1.00	25	0.240
886	A	3	3	1.00	13	0.231
887	A	2	2	1.00	24	0.083
888	A	2	2	1.00	22	0.091
889	A	2	2	1.00	30	0.067
890	A	3	3	1.00	27	0.111
891	A	5	5	1.00	23	0.217
892	A	7	5	1.00	28	0.179
893	A	13	10	1.55	25	0.400
894	A	8	6	1.00	29	0.207
895	A	6	6	1.00	16	0.375
896	A	6	6	1.00	16	0.375
897	A	4	4	1.00	16	0.250
898	A	7	7	1.00	16	0.438
899	A	8	8	1.00	16	0.500
900	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
901	A	0	0	0.00	0	0.000
902	A	0	0	0.00	0	0.000
903	A	0	0	0.00	0	0.000
904	A	0	0	0.00	0	0.000
905	A	0	0	0.00	0	0.000
906	A	0	0	0.00	0	0.000
907	A	0	0	0.00	0	0.000
908	A	0	0	0.00	0	0.000
909	A	0	0	0.00	0	0.000
910	A	0	0	0.00	0	0.000
911	A	2	2	1.00	37	0.054
912	A	2	2	1.00	38	0.053
913	A	5	4	1.00	40	0.100
914	A	7	5	1.00	40	0.125
915	A	6	5	1.00	18	0.278
916	A	7	6	1.00	21	0.286
917	A	8	6	1.00	22	0.273
918	A	3	3	1.00	21	0.143
919	A	3	3	1.00	24	0.125
920	A	11	10	1.00	19	0.526
921	A	10	7	1.00	24	0.292
922	A	4	3	1.00	19	0.158
923	A	3	3	1.00	17	0.176
924	A	1	1	1.00	15	0.067
925	A	1	1	1.00	17	0.059
926	A	2	2	1.00	17	0.118
927	A	2	2	1.00	17	0.118
928	A	1	1	1.00	17	0.059
929	A	6	6	1.00	17	0.353
930	A	5	5	1.00	11	0.454
931	A	3	3	1.00	11	0.273
932	A	5	3	1.00	13	0.231
933	A	2	2	1.00	21	0.095
934	A	5	5	1.00	16	0.312
935	A	2	2	1.00	13	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
936	A	6	6	1.00	16	0.375
937	A	5	4	1.00	12	0.333
938	A	4	3	1.00	18	0.167
939	A	10	6	1.00	17	0.353
940	A	1	1	1.00	17	0.059
941	C	3	2	1.03	27	0.074
942	A	2	2	1.00	37	0.054
943	A	3	2	1.00	17	0.118
944	A	5	4	1.00	17	0.235
945	A	1	1	1.00	15	0.067
946	A	3	3	1.00	14	0.214
947	A	1	1	1.00	17	0.059
948	A	2	1	1.00	13	0.077
949	A	2	1	1.00	15	0.067
950	A	7	4	1.00	15	0.267
951	A	6	6	1.00	15	0.400
952	A	1	0	1.00	9	0.000
953	A	1	0	1.00	9	0.000
954	A	2	1	1.00	19	0.053
955	A	3	3	1.00	15	0.200
956	A	3	3	1.00	16	0.188
957	A	4	4	1.00	15	0.267
958	A	5	5	1.00	15	0.333
959	A	4	4	1.00	25	0.160
960	A	2	2	1.00	11	0.182
961	A	2	2	1.00	17	0.118
962	A	6	6	1.00	22	0.273
963	A	4	3	1.00	13	0.231
964	A	4	3	1.00	15	0.200
965	A	4	4	1.00	19	0.210
966	A	4	4	1.00	21	0.190
967	A	3	3	1.00	17	0.176
968	A	7	7	1.00	19	0.368
969	A	7	6	1.00	19	0.316
970	A	6	6	1.00	17	0.353

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
971	A	10	8	1.00	17	0.471
972	A	11	9	1.00	17	0.529
973	A	3	3	1.00	22	0.136
974	A	5	5	1.00	11	0.454
975	A	5	5	1.00	13	0.385
976	A	5	5	1.17	11	0.454
977	A	5	5	1.00	15	0.333
978	A	4	4	1.00	11	0.364
979	A	5	5	1.00	13	0.385
980	A	4	4	1.00	11	0.364
981	A	3	3	1.00	11	0.273
982	A	2	2	1.00	11	0.182
983	A	5	5	1.00	19	0.263
984	A	2	2	1.00	23	0.087
985	A	2	2	1.00	13	0.154
986	A	3	3	1.00	11	0.273
987	A	3	3	1.00	15	0.200
988	A	3	3	1.00	19	0.158
989	A	3	3	1.00	19	0.158
990	A	3	3	1.00	19	0.158
991	A	2	2	1.00	15	0.133
992	A	3	3	1.00	15	0.200
993	A	3	3	1.00	12	0.250
994	A	3	3	1.00	16	0.188
995	F	0	0	N/A	0.	N/A
996	F	0	0	N/A	0.	N/A
997	B	25	12	4.09	31	0.387
998	A	5	4	1.00	25	0.160
999	A	2	2	1.00	27	0.074
1000	A	2	2	1.00	33	0.061
1001	A	2	2	1.00	34	0.059
1002	A	7	6	1.00	51	0.118
1003	A	2	2	1.00	49	0.041
1004	A	2	2	1.00	43	0.047
1005	A	2	2	1.00	44	0.045

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1006	A	9	8	1.00	20	0.400
1007	A	3	3	1.00	15	0.200
1008	A	3	3	1.00	15	0.200
1009	A	1	1	1.00	52	0.019
1010	A	1	1	1.00	57	0.018
1011	A	2	2	1.00	59	0.034
1012	A	2	2	1.00	58	0.034
1013	A	3	3	1.00	58	0.052
1014	A	3	3	1.00	57	0.053
1015	A	3	3	1.00	66	0.045
1016	A	3	3	1.00	31	0.097
1017	B	42	15	4.13	29	0.517
1018	C	9	6	0.96	20	0.300
1019	A	2	2	1.00	46	0.043
1020	A	1	0	1.00	15	0.000
1021	C	12	9	2.90	17	0.529
1022	C	13	10	3.55	33	0.303
1023	F	0	0	N/A	0.	N/A
1024	A	1	1	1.00	38	0.026
1025	B	2	2	2.43	30	0.067

Chapter 3

Listing of integrals

Local contents

3.1 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] 2/9*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)*E
llipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2
))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1
/2))^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2159, 224, 2162, 209}

$$\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) + (2*2^(1/3
)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipti
cF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx}{3\;2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.22, size = 148, normalized size = 1.02

$$\frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A]
time = 1.38, size = 139, normalized size = 0.96

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\left(2^{\frac{2}{3}}-1\right)}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}}-1},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}\left(2^{\frac{2}{3}}-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1
/2),(-3/2+1/2*I*3^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 74, normalized size = 0.51

$$-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^3 - 2^{\frac{2}{3}}(x^5 + x^2) + 2^{\frac{1}{3}}(7x^4 + 4x) + 2\right)\sqrt{x^3 + 1}}{6(2x^6 + 3x^3 + 1)}\right) + \frac{2}{3}\cdot 2^{\frac{1}{3}}\text{weierstrassPInverse}(0, -4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 + x^2) + 2^(1/3)*(7*
x^4 + 4*x) + 2)*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2/3*2^(1/3)*weierstras
sPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x^3+1}\,(x+2^{2/3})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(1/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.2 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2159, 224, 2162, 209}

$$-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) - (2*2^(1/
3)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{\cdots}\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\cdots}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.10, size = 148, normalized size = 0.92

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A]
time = 1.29, size = 143, normalized size = 0.89

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}-2^{\frac{2}{3}}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/
2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [A]
time = 0.17, size = 69, normalized size = 0.43

$$-\frac{1}{9}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(5x^{3}-2^{\frac{2}{3}}(x^{5}-x^{2})-2^{\frac{1}{3}}(7x^{4}-4x)-2\right)\sqrt{-x^{3}+1}}{6(2x^{6}-3x^{3}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(-1/6*sqrt(3)*(5*x^3 - 2^(2/3)*(x^5 - x^2) - 2^(1/3)*(7*
x^4 - 4*x) - 2)*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
[Out] -int(1/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.3 $\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt[3]{2}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

```
[Out] -2/9*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9*2^(1/3)*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2159, 225, 2162, 212}

$$-\frac{2\sqrt[3]{2}\,\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3]) - (2*2^(
1/3)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx}{3\ 2^{2/3}} + \frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\cdots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\cdots}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\cdots\right.}}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.11, size = 146, normalized size = 0.90

$$-\frac{4i\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((-4*I)*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Ell
ipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3]
+ (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3
) - I*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A]
time = 1.44, size = 143, normalized size = 0.88

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1\right)}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right.}{\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)
))^(1/2),(3/2+1/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3
^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(1/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.4 $\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*2^(1/3)*(1+x)
*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-
3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2159, 225, 2162, 212}

$$\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3]) + (2*2^(1
/3)*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\ldots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F(\ldots}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.10, size = 150, normalized size = 0.96

$$\frac{4i\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(1+2\,2^{2/3}-i\sqrt{3}\right)\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((4*I)*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*
I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((1 + 2*2^(2/3) - I
*Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A]
time = 1.26, size = 139, normalized size = 0.89

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x \ldots\right.}}{\ldots}\right.}{3\sqrt{-x^3-1}\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x \ldots\right.}}{\ldots}\right.}{3\sqrt{-x^3-1}\left(2^{\frac{2}{3}}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2
/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[1]%
%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(1/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.5 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=280

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{s}\ldots\right.}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}}}$$

```
[Out] 2/9*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1/2))/b^(
1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2159, 224, 2162, 209}

$$\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])
/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx}{3\ 2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\Bigg(\mathrm{s}\cdots}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,\cdots\right.}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} + \frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{3\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.15, size = 164, normalized size = 0.59

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a + b*x^3])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^3}\cdot\left(2^{\frac{2}{3}}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(1/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.6 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

```
[Out] -2/9*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2159, 224, 2162, 209}

$$-\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]]
)/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(
1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^
(1/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)
*x)^2]*Sqrt[a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx=\frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{3\;2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$=-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\;F}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 166, normalized size = 0.58

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\;\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(1/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(1/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.7 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=297

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

```
[Out] -2/9*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)-2/9*2^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(b*x^3-a)^(1/2)/(-a^(
1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2159, 225, 2162, 212}

$$-\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3
]])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) - (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*
b^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{3\;2^{2/3}\sqrt[3]{a}} + \frac{\sqrt[3]{2}\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$= -\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\;\cdots}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}\cdots}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}} - \frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 167, normalized size = 0.56

$$\frac{2i\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3)
 + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))
*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(1/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(1/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(1/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.8 $\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=293

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\,\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{-\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

```
[Out] 2/9*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^(1/2))/b
^(1/3)*3^(1/2)/a^(1/2)+2/9*2^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(1/3)/(-b*x^3-a)^(1/2)/(-a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2159, 225, 2162, 212}

$$\frac{2\sqrt[3]{2}\,\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt{-\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]
])/(3*Sqrt[3]*Sqrt[a]*b^(1/3)) + (2*2^(1/3)*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b
^(1/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx}{3\;2^{2/3}\sqrt[3]{a}}+\frac{\sqrt[3]{2}\int\frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{a}}$$

$$=\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\;F\cdots}{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\sqrt[3]{b}}+\frac{2\sqrt[3]{2}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 167, normalized size = 0.57

$$-\frac{2i\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\;\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)|\sqrt[3]{-1}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)\sqrt[3]{b}\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] ((-2*I)*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^
(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3
) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3)
)*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[-a - b*x^3]
)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate(1/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a - bx^{3}} \cdot \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
[Out] int(1/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.9 $\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=249

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d}+\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c}{\left(1+\sqrt{3}\right)c}\right.\right.}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\,(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\sqrt{c^3+4d^3x^3}}$$

```
[Out] 2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3/2)/d*3^(1/
2)+2/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^(2/
3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(2/3
)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d
/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A]
time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2159, 224, 2162, 209}

$$\frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\,(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\sqrt{c^3+4d^3x^3}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]*
c^(3/2)*d) + (2*2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^(
2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
-7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*c
 + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2159

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[2/
(3*c), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(3*c), Int[(c - 2*d*x)/((c +
d*x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 - 4*a*
d^3, 0]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3c} + \frac{2\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3c}$$

$$= \frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{\ldots}\right.}{\left(1+\sqrt{\ldots}\right.}\right.\right.}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\sqrt{c^3+4d^3x^{3}}\ldots}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d} + \frac{2\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx\ldots}{\left(\left(1+\sqrt{3}\right)c\ldots\right.}}\ldots}{3\sqrt[4]{3}\,cd\sqrt{\frac{c\,(c\ldots}{\left(\left(1+\sqrt{3}\ldots\right.\right.}}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 169, normalized size = 0.68

$$-\frac{i2^{5/6}\sqrt{\frac{\sqrt[3]{2}\,c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}\,dx}{c}+\frac{4d^2x^2}{c^2}}\,\Pi\left(\frac{i\sqrt[3]{2}\,\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}\,c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)|\sqrt[3]{-1}\right)}{\left(2+\sqrt[3]{-2}\right)d\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] ((-I)*2^(5/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) -
 (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (
-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)
]/2^(1/6)], (-1)^(1/3)])/((2 + (-2)^(1/3))*d*Sqrt[c^3 + 4*d^3*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(200) = 400.
time = 0.40, size = 495, normalized size = 1.99

method	result
default	$2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\cdots}$
elliptic	$2\left(\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}\right)\sqrt{\frac{x-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}-\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}-\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}}}\sqrt{\frac{x+\frac{2^{\frac{1}{3}}c}{2d}}{\frac{\left(\frac{2^{\frac{1}{3}}}{4}+\frac{i\sqrt{3}\,2^{\frac{1}{3}}}{4}\right)c}{d}+\frac{2^{\frac{1}{3}}c}{2d}}}\sqrt{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(((x-(1
/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))
*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1
/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))
*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 349, normalized size = 1.40

$$\left[-\frac{\sqrt{3}\,\sqrt{-c}\,d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6+\sqrt{3}\,(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d^3x^3+c^3}\,\sqrt{-c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)-12\,c\sqrt{d^3}\,\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{18\,c^2d^3},\ -\frac{\sqrt{3}\,\sqrt{c}\,d^2\arctan\left(\frac{\sqrt{3}\,\sqrt{4\,d^3x^3+c^3}\,(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)\sqrt{c}}{3\,(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5)}\right)-6\,c\sqrt{d^3}\,\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,c^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4
 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 + sqrt(3)*(4*d^4*x^4 - 10*c*d^3*x^
3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d^6*
x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^
5*d*x + c^6)) - 12*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^
3), -1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d
^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^
3 + 2*c^4*d*x + c^5)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/
(c^2*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int(1/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.10 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=146

$$\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(9
+6*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2160, 224, 2165, 209}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[3])]
 + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellip
ticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)
*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{12\sqrt{3}}+\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}}$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.15, size = 136, normalized size = 0.93

$$-\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[1 + x^3])
```

Maple [A]
time = 0.89, size = 132, normalized size = 0.90

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\ldots}\right.}{3\sqrt{x^3+1}}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\ldots}\right.}{3\sqrt{x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2
),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 57, normalized size = 0.39

$$-\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)+\frac{1}{3}\sqrt{3}\,\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x - 6)*sqr
t(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + 1/3*sqrt(3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.11 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2
)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1
-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2160, 224, 2165, 209}

$$-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)]) - (Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3
/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\cdots\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.11, size = 136, normalized size = 0.83

$$\frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[1 - x^3])
```

Maple [A]
time = 0.84, size = 143, normalized size = 0.87

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/
2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*
3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A]
time = 0.25, size = 56, normalized size = 0.34

$$-\frac{1}{6}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}\left(x^2+4x-2\right)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\left(x^3-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4*x - 2)
 + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[2]%
%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argument Value
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.12 $\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=167

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2
)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/(x^3-1)^(1/2)/((-1
+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))
/(9+6*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2160, 225, 2165, 212}

$$-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]]/Sqrt[3*(3 + 2*Sqrt[
3])]) - (Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\int \frac{6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{2\sqrt{3}}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right.}}}{3^{3/4}\sqrt{-\frac{1-}{\left(1-\sqrt{3}\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.13, size = 134, normalized size = 0.80

$$\frac{4\sqrt{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*Sqrt[2]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*Elliptic
Pi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[
3])*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.89, size = 132, normalized size = 0.79

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\right.}{3\sqrt{x^3-1}}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{3}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3},\right.}{3\sqrt{x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(
1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 211, normalized size = 1.26

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\,\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2+4\left(2x^6+18x^5+42x^4+8x^3+\sqrt{3}\left(x^6+12x^5+18x^4+16x^3-12x^2-8\right)-24x+8\right)\sqrt{x^3-1}\,\sqrt{2\sqrt{3}-3}+16\sqrt{3}\left(x^7+2x^6+6x^5-5x^4+2x^3-6x^2+4x-4\right)-128x+112}{x^8-8x^7+16x^6+16x^5-56x^4-32x^3+64x^2+64x+16}\right)+\frac{1}{3}\sqrt{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 2
24*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6 + 12*x^
5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*sqrt(3)
- 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) -
 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 6
4*x + 16)) + 1/3*sqrt(3)*weierstrassPInverse(0, 4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{x\sqrt{x^3-1}-\sqrt{3}\,\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(1/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.13 $\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=157

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3-1)^(1/2)/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2)
)/(9+6*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2160, 225, 2165, 212}

$$\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}+\frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]]/Sqrt[3*(3 + 2*Sqrt[3]
)] + (Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/
4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2160

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-6
*a*(d^3/(c*(b*c^3 - 28*a*d^3))), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/(c*
(b*c^3 - 28*a*d^3)), Int[Simp[c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x, x]/((c + d*
x)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2*c^6 - 20*a
*b*c^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\int \frac{-6\left(1-\sqrt{3}\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{12\sqrt{3}} + \frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right.}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.10, size = 138, normalized size = 0.88

$$-\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((3*I + (1 + 2*I)*Sqrt[3])
*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.79, size = 139, normalized size = 0.89

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i(x\cdots}}{\cdots}\cdots\right.}{3\sqrt{-x^3-1}\left(\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i(x\cdots}}{\cdots}\cdots\right.}{3\sqrt{-x^3-1}\left(\frac{3}{2}+\sqrt{3}+\frac{i\sqrt{3}}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+
3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1
/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [A]
time = 0.13, size = 203, normalized size = 1.29

$$\frac{1}{12}\sqrt{2\sqrt{3}-3}\,\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3+\sqrt{3}\,(x^6-12x^5+18x^4-16x^3-12x^2-8)+24x+8\right)\sqrt{-x^3-1}\,\sqrt{2\sqrt{3}-3}-16\sqrt{3}\,(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 2
24*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6 - 12*x^
5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*sqrt(3)
 - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4)
+ 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 -
64*x + 16))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(x+1)\,(x^2-x+1)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
```

```
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.14 $\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=329

$$\frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right.}}}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}\right.}}}$$

```
[Out] 1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+
x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1/
2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/
(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2
)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3
*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x
+3^(1/2))^2)^(1/2)*(3/2*6^(1/2)+5/2*2^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(
1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2161, 224, 2167, 2138, 551, 585, 95, 210}

$$\frac{2\sqrt{26+15\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{4\sqrt[4]{3}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(97-56\sqrt{3};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{ArcTan}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[26 + 15*Sqr
t[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 -
 Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1
 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(
1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqr
t[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{26+15\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{2\sqrt{26+15\sqrt{\ldots}}}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.08, size = 128, normalized size = 0.39

$$-\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[1 + x^3])
```

Maple [A]
time = 0.41, size = 123, normalized size = 0.37

method	result
default	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$
elliptic	$\frac{\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},-\frac{3}{4}+\frac{i\sqrt{3}}{4},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^
(1/2)/(x^3+1)^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I
*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3+1)**(1/2),x)
[Out] Integral(1/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B]
time = 0.22, size = 164, normalized size = 0.50

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(1/2)*(x + 3)),x)
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.15 $\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=380

$$-\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{}{\left(1+\right.}}}$$

```
[Out] -1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1
-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1
/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*
(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1
/2))/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2161, 224, 2167, 2138, 551, 585, 95, 212}

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] -1/2*((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt
[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]
)/(Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2 + S
qrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1
 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])
*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (4*3^(1/4)*Sqrt[2 + Sqr
t[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304
*Sqrt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]]
)/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{1-x^3}}\,dx = \frac{\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} +$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} -$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} +$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} +$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}} - \frac{2\sqrt{2+\sqrt{3}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.08, size = 128, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[1 - x^3])
```

Maple [A]
time = 0.34, size = 133, normalized size = 0.35

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3+1}\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3+1}\left(\frac{5}{2}+\frac{i\sqrt{3}}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(5/
2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 + 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B]
time = 2.42, size = 180, normalized size = 0.47

$$-\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{2\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/2)*(x + 3)),x)
```

```
[Out] -(((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 +
 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/(2*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.16 $\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=374

$$-\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-\right.}}}{13\sqrt[4]{3}\sqrt{-\frac{1}{\left(1-\right.}}}$$

```
[Out] -2/39*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(5/2*6^(1/
2)-7/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-
1+x)/(1-x-3^(1/2))^2)^(1/2)-1/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1
/2))^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2
)^(1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)+4/13*3^(1/4)*(1
-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/
2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {2161, 225, 2167, 2138, 551, 585, 95, 212}

$$-\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -1/2*((1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt
[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]
)/(Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[62 -
 35*Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(13*3^(1/4)*Sqrt[
-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*S
qrt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/
(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1+x^3}}\,dx = \frac{\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\sqrt{62-35\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right.}{13\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt[2]{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{62-35}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.06, size = 126, normalized size = 0.34

$$-\frac{4\sqrt{2}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)}{\left(5i+\sqrt{3}\right)\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*Sqrt[1 + x + x^2]*EllipticP
i[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((5*I + Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.37, size = 124, normalized size = 0.33

method	result
default	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$
elliptic	$\frac{\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{3}{8}+\frac{i\sqrt{3}}{8},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{2\sqrt{x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8
+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 - 1)/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(x-1)\left(x^2+x+1\right)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x**3-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B]
time = 0.05, size = 164, normalized size = 0.44

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\;\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 - 1)^(1/2)*(x + 3)),x)
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.17 $\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=340

$$\frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\ldots\right)\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+\ldots\right.}}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/(
1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((-1
-x)/(1+x-3^(1/2))^2)^(1/2)+1/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/
2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)
^(1/2)*26^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1+x
)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^
2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+
x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2161, 225, 2167, 2138, 551, 585, 95, 210}

$$\frac{2(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{4\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(97-56\sqrt{3};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{ArcTan}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1
 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt
[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*(1 + x)*Sqrt[(1
 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sq
rt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((1 + x)/(1
- Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) - (4*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/
(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sq
rt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{\left(4\sqrt[4]{3}\,\sqrt{2-\cdots\right.}{\cdots}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} - \frac{\left(4\sqrt[4]{3}\,\sqrt{2-\cdots\right.}{\cdots}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\,\sqrt{2+\cdots}}{\cdots}$$

$$= \frac{2(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{4\sqrt[4]{3}\,\sqrt{2+\cdots}}{\cdots}$$

$$= \frac{(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{2(1+x)\sqrt{\cdots}}{\sqrt[4]{3}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.07, size = 130, normalized size = 0.38

$$-\frac{4\sqrt{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{7i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\left(7i+\sqrt{3}\right)\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-4*Sqrt[2]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2]*EllipticPi[
(2*Sqrt[3])/(7*I + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/((7*I + Sqrt[3])*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.45, size = 133, normalized size = 0.39

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x \cdots\right.}}{\cdots}\right.}{3\sqrt{-x^3-1}\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i\left(x \cdots\right.}}{\cdots}\right.}{3\sqrt{-x^3-1}\left(\frac{7}{2}+\frac{i\sqrt{3}}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(7/2+
1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(x+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B]
time = 0.05, size = 179, normalized size = 0.53

$$\frac{\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] (((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/
2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i
)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(- (3^(1/2)*1i)/4 -
3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2))^(1/2))
```

3.18 $\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd}-\frac{3\log\left(d(c-dx)+2^{2/3}d\sqrt[3]{-c^3+d^3x^3}\right)}{4\sqrt[3]{2}\,cd}$$

```
[Out] 1/8*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d-3/8*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3
-c^3)^(1/3))*2^(2/3)/c/d+1/4*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(
1/3))*3^(1/2))*3^(1/2)*2^(2/3)/c/d
```

Rubi [A]
time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2174}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd}-\frac{3\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd}+\frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/
(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c
- d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd} + \frac{\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd} - \frac{3\log\left(d(c-d\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.92, size = 311, normalized size = 2.24

$$\frac{\sqrt[3]{-\frac{1}{2}}\left(2i\sqrt{3}\tanh^{-1}\left(\frac{\sqrt[3]{2}\left(3+i\sqrt{3}\right)c+\sqrt[3]{2}\left(-3-i\sqrt{3}\right)dx+2i\sqrt{3}\sqrt[3]{-c^3+d^3x^3}}{6\sqrt[3]{-c^3+d^3x^3}}\right)+2\log\left(\sqrt{c}\sqrt{d}\left(-c+i\sqrt{3}c+dx-i\sqrt{3}dx+2\,2^{2/3}\sqrt[3]{-c^3+d^3x^3}\right)\right)-\log\left(-cd\left(\left(1+i\sqrt{3}\right)c^2+\left(1+i\sqrt{3}\right)d^2x^2-2(-2)^{2/3}dx\sqrt[3]{-c^3+d^3x^3}-4\sqrt[3]{2}\left(-c^3+d^3x^3\right)^{2/3}+2c\left(\left(-1-i\sqrt{3}\right)dx+(-2)^{2/3}\sqrt[3]{-c^3+d^3x^3}\right)\right)\right)\right)}{4cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ((-1/2)^(1/3)*((2*I)*Sqrt[3]*ArcTanh[(2^(1/3)*(3 + I*Sqrt[3])*c + 2^(1/3)*(
-3 - I*Sqrt[3])*d*x + (2*I)*Sqrt[3]*(-c^3 + d^3*x^3)^(1/3))/(6*(-c^3 + d^3*
x^3)^(1/3))] + 2*Log[Sqrt[c]*Sqrt[d]*(-c + I*Sqrt[3]*c + d*x - I*Sqrt[3]*d*
x + 2*2^(2/3)*(-c^3 + d^3*x^3)^(1/3))] - Log[-(c*d*((1 + I*Sqrt[3])*c^2 + (
1 + I*Sqrt[3])*d^2*x^2 - 2*(-2)^(2/3)*d*x*(-c^3 + d^3*x^3)^(1/3) - 4*2^(1/3
)*(-c^3 + d^3*x^3)^(2/3) + 2*c*((-1 - I*Sqrt[3])*d*x + (-2)^(2/3)*(-c^3 + d
^3*x^3)^(1/3))))]))/(4*c*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-c+dx)(c^2+cdx+d^2x^2)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d^3\,x^3-c^3\right)}^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.19 $\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=186

$$\frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd}-\frac{\log(c+dx)}{2cd}-\frac{\log\left(-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}+\frac{3}{}$$

```
[Out] -1/2*ln(d*x+c)/c/d-1/4*ln(-d*x+(d^3*x^3+2*c^3)^(1/3))/c/d+3/4*ln(d*(d*x+2*c
)-d*(d^3*x^3+2*c^3)^(1/3))/c/d+1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)^(1/3
))*3^(1/2))/c/d*3^(1/2)-1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3)
)*3^(1/2))*3^(1/2)/c/d
```

Rubi [A]
time = 0.13, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2175, 245, 2176}

$$\frac{\text{ArcTan}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,cd}-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}-\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd}-\frac{\log(c+dx)}{2cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sq
rt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d
) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) +
(3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2175

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[
1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c
+ d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3
- a*d^3, 0]
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx}{2c}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}\,cd} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd}$$

Mathematica [F]
time = 7.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(2\,c^3+d^3\,x^3\right)}^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.20 $\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2d}+\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2c^2d}-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\,\mathrm{lo}}{}$$

```
[Out] -1/2*ln(d*x+c)/c^2/d-1/4*ln(d*x-(d^3*x^3+2*c^3)^(1/3))/c^2/d+3/4*ln(d*(d*x+
2*c)-d*(d^3*x^3+2*c^3)^(1/3))/c^2/d-1/6*arctan(1/3*(1+2*d*x/(d^3*x^3+2*c^3)
^(1/3))*3^(1/2))/c^2/d*3^(1/2)+1/2*arctan(1/3*(1+2*(d*x+2*c)/(d^3*x^3+2*c^3
)^(1/3))*3^(1/2))*3^(1/2)/c^2/d
```

Rubi [A]

time = 0.05, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{2179\}$

$$-\frac{\text{ArcTan}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}\,c^2d}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2c^2d}-\frac{\log(c+dx)}{2c^2d}-\frac{\log\left(dx-\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] -1/2*ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*c^2*d)
+ (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(
2*c^2*d) - Log[c + d*x]/(2*c^2*d) - Log[d*x - (2*c^3 + d^3*x^3)^(1/3)]/(4*c
^2*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c^2*d)
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(2c^3+d^3x^3\right)^{2/3}}\,dx$$

Mathematica [F]
time = 7.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\,(2c^3+d^3x^3)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)),x]
```

```
[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(d^3x^3+2c^3)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

```
[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\,(2c^3+d^3x^3)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)
```

```
[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(2\,c^3+d^3\,x^3\right)^{2/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)
```

```
[Out] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)
```

3.21 $\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)\left(1+x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\left(2^{2/3}+x\right)}{\sqrt[3]{1+x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(1+\sqrt[3]{2}\,x\right)}{2^{2/3}}-\frac{\log\left(x-\sqrt[3]{1+x^3}\right)}{2\,2^{2/3}}+\frac{3\log\left(2+\right.}{}$$

```
[Out] -1/2*ln(1+2^(1/3)*x)*2^(1/3)-1/4*ln(x-(x^3+1)^(1/3))*2^(1/3)+3/4*ln(2+2^(1/
3)*x-2^(1/3)*(x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1+2*x/(x^3+1)^(1/3))*3^
(1/2))*2^(1/3)*3^(1/2)+1/2*arctan(1/3*(1+2*(2^(2/3)+x)/(x^3+1)^(1/3))*3^(1/
2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2179}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\left(x+2^{2/3}\right)}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(x-\sqrt[3]{x^3+1}\right)}{2\,2^{2/3}}+\frac{3\log\left(-\sqrt[3]{2}\sqrt[3]{x^3+1}+\sqrt[3]{2}\,x+2\right)}{2\,2^{2/3}}-\frac{\log\left(\sqrt[3]{2}\,x+1\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*x)/(1 + x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + (Sqrt[3]
*ArcTan[(1 + (2*(2^(2/3) + x))/(1 + x^3)^(1/3))/Sqrt[3]])/2^(2/3) - Log[1 +
 2^(1/3)*x]/2^(2/3) - Log[x - (1 + x^3)^(1/3)]/(2*2^(2/3)) + (3*Log[2 + 2^(
1/3)*x - 2^(1/3)*(1 + x^3)^(1/3)])/(2*2^(2/3))
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)(1+x^3)^{2/3}}\,dx = \int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)(1+x^3)^{2/3}}\,dx$$

Mathematica [F]
time = 6.33, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1+\sqrt[3]{2}\,x\right)(1+x^3)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)),x]
[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 27.58, size = 3064, normalized size = 20.84

method	result	size
trager	Expression too large to display	3064

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-1825528333966960
*x^6-3372637147591320*2^(1/3)*(x^3+1)^(1/3)+1382185738574984*x^3-3129477143
943360*2^(1/3)*x^4-3794491037031324*x^5*2^(2/3)-8449588288647072*2^(1/3)*x-
3129477143943360*2^(2/3)*x^2+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)*2^(1/3)*x^2+1432130219315922*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6
+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6-31321787721
21420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+6636187113750264*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+3532767618003008*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x^3+2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*
(x^3+1)^(2/3)*x^3+1221822944145530 4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)
^(2/3)*x^2-7245952797616344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x
^3+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+208180948
9180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-6471421936049328*2^(1/3
)*(x^3+1)^(2/3)*x^2+61051150340088*2^(2/3)*(x^3+1)^(2/3)*x^3+22188402286785
00*2^(2/3)*(x^3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)^2*(x^3+1)^(1/3)*x^4+3727651584179880*2^(1/3)*(x^3+1)^(1/3)*x^3-621275264
0299800*2^(2/3)*(x^3+1)^(1/3)*x+7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*2^(1/3)*(x^3+1)^(2/3)+216130034792674 8*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
```

)^2*(x^3+1)^(1/3)*x+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4+64
34683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+5504178119120758*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+18718371646419984*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-29915063
66664312*2^(2/3)*(x^3+1)^(2/3)+9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*(x^3+1)^(1/3)+1159971856461672*(x^3+1)^(2/3)*x^4+355014436588560*(x^3+
1)^(1/3)*x^5-11843923165977072*(x^3+1)^(2/3)*x-4082666020768440*(x^3+1)^(1/
3)*x^2-3429757412307240*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(x^3+1)^(1/
3)*x^4+12987025125355476*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3+1)^(2
/3)*x+5212685479353366*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3+1)^(1/3
)*x^2+2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(x^3+1)^(2
/3)*x+840505690860402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(x^3+1)^(1/
3)*x^2+16011331334883780*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(x^3+1)^(1
/3)*x-1244136642528564*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(x^3+1)^(2/3
)*x^3-1349025820696266*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3+1)^(2/3
)*x^4+96609493250466*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3+1)^(1/3)*
x^5-72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(x^3+1)^(2/3)*
x^4-1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(x^3+1)^(1/3
)*x^5+3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(x^3+1)^(2
/3)*x^2-3842311729647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(x^3+1)^
(1/3)*x^3)/(1+2^(1/3)*x)^6)-1/6*ln(-(-15559137585059152-936223178470608*x^6
-12498127505504256*2^(1/3)*(x^3+1)^(1/3)+14712078518823840*x^3-160495402023
5328*2^(1/3)*x^4-4279877387294208*x^5*2^(2/3)-23004340956706368*2^(1/3)*x-1
604954020235328*2^(2/3)*x^2+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2
)*2^(1/3)*x^2+2321435374812274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+
1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+101977140081
27436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+5665414413224496*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+3532767618003008*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x^3+2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*
(x^3+1)^(2/3)*x^3+2884227944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^
(2/3)*x^2-8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(x^3+1)^(1/3)*x^
3+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489
180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-11138422684341672*2^(1/3
)*(x^3+1)^(2/3)*x^2+3919074648194292*2^(2/3)*(x^3+1)^(2/3)*x^3-131559236900
0448*2^(2/3)*(x^3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*(x^3+1)^(1/3)*x^4+3288980922501120*2^(1/3)*(x^3+1)^(1/3)*x^3-2006278
3627256832*2^(2/3)*(x^3+1)^(1/3)*x-7115580883942020*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*2^(1/3)*(x^3+1)^(2/3)+2161300347926748*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*(x^3+1)^(1/3)*x+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4
+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5-10107
087250606332*2^(2/3)*(x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*
_Z+_Z^2)*(x^3+1)^(1/3)+3712807561447224*(x^3+1)^(2/3)*x^4-2960082830251008*
(x^3+1)^(1/3)*x^5-32590199706036744*(x^3+1)^(2/3)*x-12827025597754368*(x^3+

```
1)^(1/3)*x^2-10498622607665136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(x^3
+1)^(1/3)*x^4-7759251414704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3
+1)^(2/3)*x-3531674097632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(x^3+1
)^(1/3)*x^2+2613886855325640*RootOf(2^(2/3)+2^(...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(112) = 224.
time = 2.62, size = 712, normalized size = 4.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(-1/3*(13910019318573948542*sqrt(3)*(442971093109
30172741433829405399636654451725916403400759596345420183*x^16 + 46991175387
7577297266687493361266274298219751726156511748796788210304*x^13 - 168603219
036433260440647021325346295645242325246375460547582960409424*x^10 - 1978806
301182376573938292954227792627373330283397876582611558332893440*x^7 - 14400
90891687177581422918763089301968602581036872213084389912370301872*x^4 - 2^(
2/3)*(52271077453125107612995923977654758349394876922885552819209999866413*
x^15 + 59067454785454857729328582078834077849329928125521336059399799480517
2*x^12 + 306314261222931431619887382966630423064822217690279625339197857781
7900*x^9 + 7331049558697577809008352571597039403457968857066730277786114959
327080*x^6 + 77232448067562904437597705467808729717394447501735196355441861
14816064*x^3 + 291168089878390092195634857418355141558919044601510645260807
0501424800) + 6*2^(1/3)*(12601355996216322093314748679149120543302140685677
058235520929344665*x^14 - 5558690630019665139246271949192126784782079889001
9850227115938089718*x^11 - 450398920105320599307639536027883986131793624729
303407436233610788504*x^8 - 72188870588094826143251705267039410623833894384
4373553906510879866584*x^5 - 3386681580686843734363092730678494644056913607
51378507442472921774544*x^2) - 62367643045453979229021701235594440425380660
140976292433240780519680*x)*(x^3 + 1)^(2/3) - 13910019318573948542*sqrt(3)*
(20244151386762728582873176440916642276036913846721964342570319874272*x^17
+ 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 2179843197271775401147438396101666875537043663345199103065290718350660*
```

```
x^11 + 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 690583979302212649541846671752323578671762361564987198532372077617
072*x^5 + 42560446719395994043503690929493089250376947849898596094387069196
992*x^2 + 2^(2/3)*(58175953016441250552894129028785848895343146706912452780
410096144857*x^16 + 6033291234402259284595124428808463674980863404672105084
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 - 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
2325783379529417l024*x^4 - 774581653994506522185065060515457999562469670838
035710700279100960480*x) - 2*2^(1/3)*(4425033739586262364130843214610526559
1584981692216944246872622437586*x^15 + 937303319945530879145881930294041650
15738145719370012253256237142833*x^12 + 13218541316595455203956380934358348
61993288285254840631143087754453816*x^9 + 424770576770174688958921382572527
8162202431773760010908121531655858240*x^6 + 4593245463688643634993735851341
621838359838170188285500151733185855040*x^3 + 16158837376147892971429107707
86922880950970969890530541101538638738800))*(x^3 + 1)^(1/3) + sqrt(3)*(5808
458566248141380585366589250357524223410237450426571441100184341339711713923
78653765*x^18 + 85128502112016585963203224235079794367450370616046622522881
06173984889011398391939493844*x^15 + 46037674634299399876464933353333393651
798714498861959697684952859181279514449172348801132*x^12 + 1000163483533668
12357999723948540966952435611836580420294833827058766585456463611215562912*
x^9 + 913977586253668076790534210688867294404951076896021210254557365342556
42370122935700628112*x^6 + 276792064712221478189323489147072714065541212161
41734785863966451139338545569046396842944*x^3 - 13910019318573948542*2^(2/3
)*(3844366680114123938578119587438413410802428820066154040455085354797*x^17
 - 493131971154919078063173195983280278594703770406004388326552124793591*x^
14 - 2263656329733750526575239788393341804272268328404078377386979655411628
*x^11 - 3603296088959643040065882606156977332942778368970867958841266275405
688*x^8 - 23751439241454624747907892976430825810233524575836444336983180902
72160*x^5 - 538527827084536759298395164308728360347336217790784309877024260
129712*x^2) + 166920231822887382504*2^(1/3)*(135958920440428283662759820067
08049395032909698880004129949511339226*x^16 + 13513338488515825037717904859
5991346450771199327236207956421113461903*x^13 + 402245899028058436823068109
521885840258775610614711826343657868879359*x^10 + 5472587101498793346918329
99834525308297790387563356879645468036532966*x^7 + 363674199703640963884960
012124387263106254909521640663154302302116404*x^4 + 97123895740704644005292
055222464498011501842944639406026020532340120*x) - 180077408083879446119265
3903259802591850188394016866170707655609076236167687893936558400))/(4912705
745775473374655778624996785809194682896822406415994000025418186301732995555
53387*x^18 + 10277776658535231887928963830517649364075160462302952752368573
529738604577075128345830496*x^15 + 3805307460404116495559861338250665758871
8033800015428848687354515819408113275280820067228*x^12 + 104552977375786496
056156515228686393360634250389206134816652347595105200990156089430013680*x^
9 + 193789777878621710892383256210067417618988113173228069450235805883107523
1461508817660387440*x^6 + 176250773615214113270...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\left(x+1\right)\left(x^{2}-x+1\right)\right)^{\frac{2}{3}}\cdot\left(\sqrt[3]{2}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)
[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x^{3}+1\right)^{2/3}\,\left(2^{1/3}\,x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)
[Out] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)
```

3.22 $\int \frac{1}{\left(1-\sqrt[3]{2}\,x\right)(1-x^3)^{2/3}}\,dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}\,x\right)}{2^{2/3}}+\frac{\log\left(-x-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{3\log\left(-\right.}{}$$

```
[Out] 1/2*ln(1-2^(1/3)*x)*2^(1/3)+1/4*ln(-x-(-x^3+1)^(1/3))*2^(1/3)-3/4*ln(-2+2^(
1/3)*x+2^(1/3)*(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*x/(-x^3+1)^(1/3)
)*3^(1/2))*2^(1/3)*3^(1/2)-1/2*arctan(1/3*(1+(2*2^(2/3)-2*x)/(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)*2^(1/3)
```

Rubi [A]
time = 0.04, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {2179}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2\,2^{2/3}-2x}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}}+\frac{\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2\,2^{2/3}}-\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x^3}+\sqrt[3]{2}\,x-2\right)}{2\,2^{2/3}}+\frac{\log\left(1-\sqrt[3]{2}\,x\right)}{2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*2^(2/3) - 2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/2^(2/3)
) + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 -
 2^(1/3)*x]/2^(2/3) + Log[-x - (1 - x^3)^(1/3)]/(2*2^(2/3)) - (3*Log[-2 + 2
^(1/3)*x + 2^(1/3)*(1 - x^3)^(1/3)])/(2*2^(2/3))
```

Rule 2179

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(2/3)), x_Symbol] :> With[
{q = Rt[b, 3]}, Simp[(-d)*(ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/
(2*Sqrt[3]*q^2*c^2)), x] + (Simp[Sqrt[3]*d*(ArcTan[(1 + 2*q*((2*c + d*x)/(d
*(a + b*x^3)^(1/3))))/Sqrt[3]]/(2*q^2*c^2)), x] - Simp[d*(Log[c + d*x]/(2*q
^2*c^2)), x] - Simp[d*(Log[q*x - (a + b*x^3)^(1/3)]/(4*q^2*c^2)), x] + Simp
[3*d*(Log[q*(2*c + d*x) - d*(a + b*x^3)^(1/3)]/(4*q^2*c^2)), x])] /; FreeQ[
{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{\left(1-\sqrt[3]{2}\,x\right)\left(1-x^3\right)^{2/3}}\,dx=\int \frac{1}{\left(1-\sqrt[3]{2}\,x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Mathematica [F]
time = 6.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-\sqrt[3]{2}\,x\right)\left(1-x^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)),x]
[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 27.10, size = 3250, normalized size = 20.44

method	result	size
trager	Expression too large to display	3250

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(-(-15559137585059152-936223178470608*x^6-14712078518823840*x^3-12498
127505504256*2^(1/3)*(-x^3+1)^(1/3)-1604954020235328*2^(1/3)*x^4+4279877387
294208*x^5*2^(2/3)+23004340956706368*2^(1/3)*x-1604954020235328*2^(2/3)*x^2
+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+2321435374812
274*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^6+1876782797064098*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6-10197714008127436*2^(2/3)*RootOf(2^(2/3
)+2^(1/3)*_Z+_Z^2)*x^3-5665414413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(
1/3)*x^5-3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+321
7341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891803
4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x+2884227944870616*RootOf(2^(2/
3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(2/3)*x^2+8123294120973864*RootOf(2^(2/3)+2^(1
/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*x^3-2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)^2*(-x^3+1)^(2/3)*x^3-1113842268434167 2*2^(1/3)*(-x^3+1)^(2/3)*x^2-391
9074648194292*2^(2/3)*(-x^3+1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^
(1/3)*x^4-6964190009986188*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)
*x^4-3288980922501120*2^(1/3)*(-x^3+1)^(1/3)*x^3+20062783627256832*2^(2/3)*
(-x^3+1)^(1/3)*x-7115580883942020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(
-x^3+1)^(2/3)-216130034792 6748*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(
```

1/3)*x-10107087250606332*2^(2/3)*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+29
60082830251008*(-x^3+1)^(1/3)*x^5+32590199706036744*(-x^3+1)^(2/3)*x-128270
25597754368*(-x^3+1)^(1/3)*x^2+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)*x^4+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-55041781191
20758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+10391133689698608*RootOf(2^(2
/3)+2^(1/3)*_Z+_Z^2)*x-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x
^5+3775614346581480*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3
)*x^2+3842311729647552*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(
1/3)*x^3-6471910353179844*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^
(2/3)*x^3-10498622607665136*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1
)^(1/3)*x^4+7759251414704196*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+
1)^(2/3)*x-3531674097632562*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x^3+1
)^(1/3)*x^2-2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^
3+1)^(2/3)*x+840505690860402*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(-x^
3+1)^(1/3)*x^2+11688730639030284*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-
x^3+1)^(1/3)*x+1203809884289286*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-x
^3+1)^(2/3)*x^4+3218487773589102*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*(-
x^3+1)^(1/3)*x^5-72607968203490*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*(
-x^3+1)^(2/3)*x^4+1560939140169318*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3
)*(-x^3+1)^(1/3)*x^5)/(2^(1/3)*x-1)^6)*2^(1/3)+1/6*ln(-(-15559137585059152-
936223178470608*x^6-14712078518823840*x^3-12498127505504256*2^(1/3)*(-x^3+1
)^(1/3)-1604954020235328*2^(1/3)*x^4+4279877387294208*x^5*2^(2/3)+230043409
56706368*2^(1/3)*x-1604954020235328*2^(2/3)*x^2+7959206999356368*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+2321435374812274*2^(2/3)*RootOf(2^(2/3)+2
^(1/3)*_Z+_Z^2)*x^6+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x^6-10197714008127436*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3-56654
14413224496*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5-3532767618003008*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3+3217341937824168*RootOf(2^(2/3)
+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)^2*2^(2/3)*x+2884227944870616*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)
^(2/3)*x^2+8123294120973864*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)*
x^3-2613886855325640*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(2/3)*x^3-1
1138422684341672*2^(1/3)*(-x^3+1)^(2/3)*x^2-3919074648194292*2^(2/3)*(-x^3+
1)^(2/3)*x^3-1315592369000448*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*R
ootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3288980922501120*2^(1/3
)*(-x^3+1)^(1/3)*x^3+20062783627256832*2^(2/3)*(-x^3+1)^(1/3)*x-71155808839
42020*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^(2/3)-21613003479267
48*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x-10107087250606332*2^(
2/3)*(-x^3+1)^(2/3)-9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+
1)^(1/3)+3712807561447224*(-x^3+1)^(2/3)*x^4+2960082830251008*(-x^3+1)^(1/3
)*x^5+32590199706036744*(-x^3+1)^(2/3)*x-12827025597754368*(-x^3+1)^(1/3)*x
^2+7959206999356368*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^4+6434683875648336*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)*2^(2/3)+10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x-61508

```
00763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+3775614346581480*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(2/3)*x^2+3842311729647552*RootO
f(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*(-x^3+1)^(1/3)*x^3-6471910353179844*Ro
otOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*(-x^3+1)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")
[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(124) = 248$.
time = 2.59, size = 720, normalized size = 4.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/3*(13910019318573948542*sqrt(3)*(4429710931093
01727414338294053996366544517259164034007595963454201 83*x^16 - 469911753877
577297266687493361266274298219751726156511748796788210304*x^13 - 1686032190
364332604406470213253462956452423252463754605475829604 09424*x^10 + 19788063
011823765739382929542277926273733302833978765826115583328934 40*x^7 - 144009
089168717758142291876308930196860258103687221308438991237030 1872*x^4 + 2^(2
/3)*(52271077453125107612995923977654758349394876922885552819209999866413*x
^15 - 59067454785454857729328582078834077849329928125521336059399799480517 2
*x^12 + 30631426122293143161988738296663042306482221769027962533919785778 17
900*x^9 - 73310495586975778090083525715970394034579688570667302777861149593
27080*x^6 + 77232448067562904437597705467808729717394447501735196355441861 1
4816064*x^3 - 29116808987839009219563485741835514155891904460151064526080 70
501424800) + 6*2^(1/3)*(1260135599621632209331474867914912054330214068567 70
58235520929344665*x^14 + 55586906300196651392462719491921267847820798890019
850227115938089718*x^11 - 450398920105320599307639536027883986131793624729 3
034074362336107885 04*x^8 + 721888705880948261432517052670394106238338943844
37355390651087986658 4*x^5 - 3386681580686843734363092730678494644056913607 5
137850744247292177454 4*x^2) + 62367643045453979229021701235594440425380660 1
40976292433240780519680*x)*(-x^3 + 1)^(2/3) + 13910019318573948542*sqrt(3)*
(2024415138676272858287317644091664227603691384672196434257031987427 2*x^17
- 741146137078834990968958694956953525786968216162791369141561079231342*x^1
4 + 21798431972717754011474383961016668755370436633451991030652907183506 60*
```

```
x^11 - 21110249350284448030276350331723739969986388702750815288350190294268
08*x^8 + 690583979302212649541846671752323578671762361564987198532372077617
072*x^5 - 425604467193959940435036909294930892503769478498985960943870691 96
992*x^2 - 2^(2/3)*(581759530164412505528941290287858488953431467069124527 80
410096144857*x^16 - 603329123440225928459512442880846367498086340467210508 4
10170807919392*x^13 + 99321772442116051464080292497021614887213800679935641
7482692017634440*x^10 + 315373668616978600368729679828820826067145203897860
799345951918357208*x^7 - 15359897811758984549040097640804776981234391400095
2325783379529417 1024*x^4 + 774581653994506522185065060515457999562469670838
035710700279100960480*x) - 2*2^(1/3)*(442503373958626236413084321461052655 9
1584981692216944246872622437586*x^15 - 937303319945530879145881930294041650
1573814571937001225325623714283 3*x^12 + 1321854131659545520395638093435834 8
6199328828525484063114308775445381 6*x^9 - 424770576770174688958921382572527
8162202431773760010908121531655858240*x^6 + 459324546368864363499373585134 1
62183835983817018828550015173318585504 0*x^3 - 1615883737614789297142910770 7
86922880950970969890530541101538638738800))*(-x^3 + 1)^(1/3) + sqrt(3)*(580
8458566248141380585366589250357524223410237450426571441100184341339711713 92
378653765*x^18 - 85128502112016585963203224235079794367450370616046622522 88
1061739848890113983919394938 44*x^15 + 4603767463429939987646493335333339365
179871449886195969768495285918127951444917234880113 2*x^12 - 100016348353366
812357999723948540966952435611836580420294833827058766585456463611215562912
*x^9 + 9139775862536680767905342106888672944049510768960212102545573653425 5
642370122935700628112*x^6 - 2767920647122214781893234891470727140655412121 6
141734785863966451139338545569046396842944*x^3 + 13910019318573948542*2^(2/
3)*(384436668011412393857811958743841341080242882006615404045508535479 7*x^1
7 + 49313197115491907806317319598328027859470377040600438832655212479359 1*x
^14 - 22636563297337505265752397883933418042722683284040783773869796554116 2
8*x^11 + 360329608895964304006588260615697733294277836897086795884126627540
5688*x^8 - 237514392414546247479078929764308258102335245758364443369831809 0
272160*x^5 + 538527827084536759298395164308728360347336217790784309877024 26
0129712*x^2) + 1669202318228873825 04*2^(1/3)*(1359589204404282836627598200 6
70804939503290969888000412994951133922 6*x^16 - 135133384885158250377179048 5
95991346450771199327236207956421113461903*x^13 + 40224589902805843682306810
95218858402587756106147118263436578688793 59*x^10 - 547258710149879334691832
9998345253082977903875633568796454680365329 66*x^7 + 3636741997036409638849 6
00121243872631062549095216406631543023021164 04*x^4 - 971238957407046440052 9
20552224644980115018429446394060260205323401 20*x) - 18007740808387944611926
5390325980259185018839401686617070765560907623616768789393655840 0))/(491270
5745775473374655778624996785809194682896822406415994000025418186301732995 55
553387*x^18 - 10277776658535231887928963830517649364075160462302952752368 57
35297386045770751283458304 96*x^15 + 3805307460404116495559861338250665758 87
1803380001542884868735451581940811327528082006722 8*x^12 - 10455297737578649
605615651522868639336063425038920613481665234759510520099015608943001368 0*x
^9 + 193789778786217108923832562100674176189881131732280694502358058831075 2
314615088176603874 40*x^6 - 1762507736152141 1327...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt[3]{2}\,x\,(1-x^3)^{\frac{2}{3}}-(1-x^3)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(1-x^3)^{2/3}\,(2^{1/3}\,x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)
```

```
[Out] -int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)
```

3.23 $\int (c+dx)^4 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=387

$$\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)-\frac{4ac^3}{\cdots}$$

```
[Out] 3/2*a*c^2*d^2*(b*x^3+a)^(1/3)/b+1/18*a*d^4*x^2*(b*x^3+a)^(1/3)/b+1/30*(b*x^
3+a)^(1/3)*(5*d^4*x^5+24*c*d^3*x^4+45*c^2*d^2*x^3+40*c^3*d*x^2+15*c^4*x)+1/
2*a*c^4*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^
(2/3)+1/5*a*c*d^3*x^4*(1+b*x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a
)/(b*x^3+a)^(2/3)-2/3*a*c^3*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/18*a^
2*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)-4/9*a*c^3*d*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)+1/27*a^2*d^4*arctan(1/3*(1
+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1867, 1907, 252, 251, 337, 267, 372, 371, 327}

$$\frac{a^2d^4\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{5/3}}+\frac{a^2d^4\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}}-\frac{4ac^3d\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{2ac^3d\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}+\frac{ac^4x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}+\frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+\frac{acd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c^2*d^2*(a + b*x^3)^(1/3))/(2*b) + (a*d^4*x^2*(a + b*x^3)^(1/3))/(18*b
) + ((a + b*x^3)^(1/3)*(15*c^4*x + 40*c^3*d*x^2 + 45*c^2*d^2*x^3 + 24*c*d^3
*x^4 + 5*d^4*x^5))/30 - (4*a*c^3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a^2*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(5/3)) + (a*c^4*x*(1 + (b*x^3)/a)^(2
/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) +
 (a*c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x
^3)/a)])/(5*(a + b*x^3)^(2/3)) - (2*a*c^3*d*Log[b^(1/3)*x - (a + b*x^3)^(1/
3)])/(3*b^(2/3)) + (a^2*d^4*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(18*b^(5/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
```

```
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^4 \sqrt[3]{a+bx^3}\,dx = \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int \frac{\frac{c^4}{2}+\frac{4}{3}c^3dx \cdots}{\cdots}$$

$$= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+a\int \left(\frac{c^4}{2\left(a+bx \cdots\right.}\right.$$

$$= \frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x^3+24cd^3x^4+5d^4x^5\right)+\frac{1}{2}\left(ac^4\right)\int \frac{\cdots}{(a+\cdots}$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

$$= \frac{3ac^2d^2\sqrt[3]{a+bx^3}}{2b}+\frac{ad^4x^2\sqrt[3]{a+bx^3}}{18b}+\frac{1}{30}\sqrt[3]{a+bx^3}\left(15c^4x+40c^3dx^2+45c^2d^2x \cdots\right.$$

Mathematica [A]
time = 7.04, size = 163, normalized size = 0.42

$$\frac{\sqrt[3]{a+bx^3}\left(6bc^4x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d(12bc^3-ad^3)\,x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d^2\left((9c^2+d^2x^2)\,(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+6bcdx^4\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)}{6b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(6*b*c^4*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(12*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a
)] + d^2*((9*c^2 + d^2*x^2)*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 6*b*c*d*x^4
*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x^3)/a)])))/(6*b*(1 + (b*x^3)/a)^(1
/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \left(b\,x^3+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4*(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 2.98, size = 212, normalized size = 0.55

$$\frac{\sqrt[3]{a}\,c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{4\sqrt[3]{a}\,c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{4\sqrt[3]{a}\,cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}\,d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+6c^2d^2\left(\begin{cases}\frac{\sqrt[3]{a}\,x^3}{3} & \text{for}\: b=0\\ \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**4*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 4*a**(1/3)*c**3*d*x**2*gamma(2/3)*hyper((-1/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + 4*a**(1/3)*c*d**3*x**4*g
amma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3
)) + a**(1/3)*d**4*x**5*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*gamma(8/3)) + 6*c**2*d**2*Piecewise((a**(1/3)*x**3/3, Eq(b,
 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^3+a\right)}^{1/3}\,{\left(c+d\,x\right)}^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^4,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^4, x)
```

3.24 $\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx$

Optimal. Leaf size=242

$$\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)-\frac{ac^2d\tan^{-1}\left(\frac{1+\frac{\ldots}{\sqrt[3]{\ldots}}}{\ldots}\right)}{\sqrt{3}\,b^{2/3}\ldots}$$

```
[Out] 3/4*a*c*d^2*(b*x^3+a)^(1/3)/b+1/10*a*d^3*x*(b*x^3+a)^(1/3)/b+1/20*(b*x^3+a)
^(1/3)*(4*d^3*x^4+15*c*d^2*x^3+20*c^2*d*x^2+10*c^3*x)+1/10*a*(-a*d^3+5*b*c^
3)*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/
3)-1/2*a*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*a*c^2*d*arctan(1/3
*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1867, 1902, 1900, 267, 1907, 252, 251, 337}

$$-\frac{ac^2d\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}-\frac{ac^2d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{ax\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5bc^3-ad^3\right)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b\left(a+bx^3\right)^{2/3}}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] (3*a*c*d^2*(a + b*x^3)^(1/3))/(4*b) + (a*d^3*x*(a + b*x^3)^(1/3))/(10*b) +
((a + b*x^3)^(1/3)*(10*c^3*x + 20*c^2*d*x^2 + 15*c*d^2*x^3 + 4*d^3*x^4))/20
 - (a*c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]
*b^(2/3)) + (a*(5*b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*b*(a + b*x^3)^(2/3)) - (a*c^2*d*Log[b^(1/
3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^3 \sqrt[3]{a+bx^3}\, dx = \frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+a\int \frac{\frac{c^3}{2}+c^2dx+\frac{3}{4}cd^2x^2 \cdots}{(a+bx^3)^{2/3}}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int \frac{1}{5}(\cdots}{\cdots}$$

$$= \frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3+4d^3x^4\right)+\frac{a\int \frac{1}{5}(\cdots}{\cdots}$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

$$= \frac{3acd^2\sqrt[3]{a+bx^3}}{4b}+\frac{ad^3x\sqrt[3]{a+bx^3}}{10b}+\frac{1}{20}\sqrt[3]{a+bx^3}\left(10c^3x+20c^2dx^2+15cd^2x^3 \cdots\right.$$

Mathematica [A]
time = 6.91, size = 142, normalized size = 0.59

$$\frac{\sqrt[3]{a+bx^3}\left(4bc^3x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(6bc^2x^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+d\left(3c(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+bdx^4\,{}_2F_1\left(-\frac{1}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)\right)}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^3*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(6*b*c^2*x^2*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)] + d*(3*
c*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + b*d*x^4*Hypergeometric2F1[-1/3, 4/3,
7/3, -((b*x^3)/a)]))))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^3*(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [A]
time = 1.96, size = 160, normalized size = 0.66

$$\frac{\sqrt[3]{a}\,c^{3}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}\,c^{2}dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{\Gamma\left(\frac{5}{3}\right)}+\frac{\sqrt[3]{a}\,d^{3}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+3cd^{2}\left(\begin{cases}\frac{\sqrt[3]{a}\,x^{3}}{3} & \text{for}\ b=0\\\frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**3*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + a**(1/3)*c**2*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5
/3,), b*x**3*exp_polar(I*pi)/a)/gamma(5/3) + a**(1/3)*d**3*x**4*gamma(4/3)*
hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 3*c*d
**2*Piecewise((a**(1/3)*x**3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(b\,x^3+a\right)}^{1/3}\,{\left(c+d\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^3,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^3, x)
```

3.25 $\int (c+dx)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=192

$$\frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)-\frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\right.}{2\left(a+bx^3\right)}$$

```
[Out] 1/4*a*d^2*(b*x^3+a)^(1/3)/b+1/12*(b*x^3+a)^(1/3)*(3*d^2*x^3+8*c*d*x^2+6*c^2
*x)+1/2*a*c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x
^3+a)^(2/3)-1/3*a*c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/9*a*c*d*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1867, 1900, 267, 1907, 252, 251, 337}

$$-\frac{2acd\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{acd\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{2/3}}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}+\frac{ad^2\sqrt[3]{a+bx^3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (a*d^2*(a + b*x^3)^(1/3))/(4*b) + ((a + b*x^3)^(1/3)*(6*c^2*x + 8*c*d*x^2 +
 3*d^2*x^3))/12 - (2*a*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqr
t[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2
F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (a*c*d*Log[b^(1/3)
*x - (a + b*x^3)^(1/3)])/(3*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)^2\sqrt[3]{a+bx^3}\,dx = \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}+\frac{d^2x^2}{4}}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \frac{\frac{c^2}{2}+\frac{2cdx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx+\frac{1}{4}(ad^2)\int \frac{x^2}{(a+bx \ldots}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+a\int \left(\frac{c^2}{2\left(a+bx^3\right)^{2/3}}+ \ldots\right.$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{2}(ac^2)\int \frac{1}{\left(a+bx^3\right)^{2/3}} \ldots$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{1}{3}(2acd)\text{Subst}\left(\int \frac{x}{1-b \ldots}\right.$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\ldots\right.}{2\left(a+bx^3\right)^{2 \ldots}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\ldots\right.}{2\left(a+bx^3\right)^{2 \ldots}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)+\frac{ac^2x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\ldots\right.}{2\left(a+bx^3\right)^{2 \ldots}}$$

$$= \frac{ad^2\sqrt[3]{a+bx^3}}{4b}+\frac{1}{12}\sqrt[3]{a+bx^3}\left(6c^2x+8cdx^2+3d^2x^3\right)-\frac{2acd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{a+b\ldots}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}} \ldots$$

Mathematica [A]
time = 6.86, size = 111, normalized size = 0.58

$$\frac{\sqrt[3]{a+bx^3}\left(4bc^2x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(d(a+bx^3)\sqrt[3]{1+\frac{bx^3}{a}}+4bcx^2\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)\right)}{4b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] ((a + b*x^3)^(1/3)*(4*b*c^2*x*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3)*(1 + (b*x^3)/a)^(1/3) + 4*b*c*x^2*Hypergeometric2F1[-
1/3, 2/3, 5/3, -((b*x^3)/a)])))/(4*b*(1 + (b*x^3)/a)^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{2} \left(b\,x^{3}+a\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2*(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [A]
time = 1.59, size = 114, normalized size = 0.59

$$\frac{\sqrt[3]{a}\,c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2\sqrt[3]{a}\,cdx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+d^{2}\left(\begin{cases}\frac{\sqrt[3]{a}\,x^{3}}{3} & \text{for}\ b=0\\ \frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{4b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + d**2*Piecewise((a**(1/3)*x*
*3/3, Eq(b, 0)), ((a + b*x**3)**(4/3)/(4*b), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x)^2,x)
```

```
[Out] int((a + b*x^3)^(1/3)*(c + d*x)^2, x)
```

3.26 $\int (c+dx)\sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=155

$$\frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3}-\frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}+\frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}-\frac{ad\log\left(\sqrt[3]{b}\,x\right.}{6}$$

[Out] 1/6*(2*d*x^2+3*c*x)*(b*x^3+a)^(1/3)+1/2*a*c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-1/6*a*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/9*a*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1867, 1907, 252, 251, 337}

$$-\frac{ad\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}}-\frac{ad\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}}+\frac{1}{6}\sqrt[3]{a+bx^3}\left(3cx+2dx^2\right)+\frac{acx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)*(a + b*x^3)^(1/3),x]

[Out] ((3*c*x + 2*d*x^2)*(a + b*x^3)^(1/3))/6 - (a*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(2/3)) + (a*c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (a*d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(6*b^(2/3))

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1867

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Module[{q = Expon[Pq
, x], i}, Simp[(a + b*x^n)^p*Sum[Coeff[Pq, x, i]*(x^(i + 1)/(n*p + i + 1)),
 {i, 0, q}], x] + Dist[a*n*p, Int[(a + b*x^n)^(p - 1)*Sum[Coeff[Pq, x, i]*(
x^i/(n*p + i + 1)), {i, 0, q}], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x]
 && IGtQ[(n - 1)/2, 0] && GtQ[p, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int (c+dx)\sqrt[3]{a+bx^3}\,dx = \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + a\int \frac{\frac{c}{2}+\frac{dx}{3}}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + a\int \left(\frac{c}{2\left(a+bx^3\right)^{2/3}} + \frac{dx}{3\left(a+bx^3\right)^{2/3}}\right)dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{1}{2}(ac)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx + \frac{1}{3}(ad)\int \frac{x}{\left(a+bx^3\right)^{2/3}}\,dx$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{1}{3}(ad)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(ac\left(\right.\right.}{}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} + \frac{(ad)\text{Subst}\left(\right.}{}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(1-\right.}{9b}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\left(a+bx^3\right)^{2/3}} - \frac{ad\log\left(1-\right.}{9b}$$

$$= \frac{1}{6}\left(3cx+2dx^2\right)\sqrt[3]{a+bx^3} - \frac{ad\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}} + \frac{acx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\right.}{2\left(a+bx^3\right.}$$

Mathematica [A]
time = 6.20, size = 75, normalized size = 0.48

$$\frac{x\sqrt[3]{a+bx^3}\left(2c\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right) + dx\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(2*c*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)] +
 d*x*Hypergeometric2F1[-1/3, 2/3, 5/3, -((b*x^3)/a)]))/(2*(1 + (b*x^3)/a)^(
1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\left(b\,x^{3}+a\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)*(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.32, size = 82, normalized size = 0.53

$$\frac{\sqrt[3]{a}\,cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}\,dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**2*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)*(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)*(c + d*x),x)
[Out] int((a + b*x^3)^(1/3)*(c + d*x), x)
```

3.27 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$

Optimal. Leaf size=435

$$\frac{\sqrt[3]{a+bx^3}}{d}+\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}+\frac{\sqrt[3]{b}\,c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{\sqrt[3]{bc^3-ad^3}\,\tan^{-1}\left(\frac{1}{\ldots}\right.}{\sqrt{3}\,d\ldots}$$

```
[Out] (b*x^3+a)^(1/3)/d+x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d^3*x
^3/c^3)/c/(1+b*x^3/a)^(1/3)+1/3*(-a*d^3+b*c^3)^(1/3)*ln(d^3*x^3+c^3)/d^2+1/
2*b^(1/3)*c*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((
-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d^3+b*c^3)^(1/3)*ln((-
a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/d^2+1/3*b^(1/3)*c*arctan(1/3*(1+2*b^(
1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)-1/3*(-a*d^3+b*c^3)^(1/3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+1
/3*(-a*d^3+b*c^3)^(1/3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1
/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2181, 441, 440, 495, 337, 503, 455, 52, 60, 631, 210, 31}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{bc^3-ad^3}\,\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt[3]{bc^3-ad^3}\,\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt[3]{b}\,c\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt[3]{bc^3-ad^3}\,\log\left(c^3+d^3x^3\right)}{3d^2}-\frac{\sqrt[3]{bc^3-ad^3}\,\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2d^2}-\frac{\sqrt[3]{bc^3-ad^3}\,\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{b}\,c\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{\sqrt[3]{a+bx^3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b
*x^3)/a), -((d^3*x^3)/c^3)])/(c*(1 + (b*x^3)/a)^(1/3)) + (b^(1/3)*c*ArcTan[
(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c^3 - a
*d^3)^(1/3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/
Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)^(1/3)*ArcTan[(1 - (2*d*(a + b*x^
3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + ((b*c^3 - a*d^3)
^(1/3)*Log[c^3 + d^3*x^3])/(3*d^2) + (b^(1/3)*c*Log[b^(1/3)*x - (a + b*x^3)
^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c -
 (a + b*x^3)^(1/3)])/(2*d^2) - ((b*c^3 - a*d^3)^(1/3)*Log[(b*c^3 - a*d^3)^(
1/3) + d*(a + b*x^3)^(1/3)])/(2*d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 495

```
Int[((x_)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol
] :> Dist[b/d, Int[x*(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[
x*((a + b*x^n)^(p - 1)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, 1, 1
, n, p, -1, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx = \int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Mathematica [F]
time = 17.02, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x),x]
```

```
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x), x]
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{1/3}}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x),x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x), x)
```

3.28 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$

Optimal. Leaf size=818

$$-\frac{c^2\sqrt[3]{a+bx^3}}{d\left(c^3+d^3x^3\right)}-\frac{dx^2\sqrt[3]{a+bx^3}}{c^3+d^3x^3}+\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}-\frac{d^3x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\right.}{2c^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

```
[Out] -c^2*(b*x^3+a)^(1/3)/d/(d^3*x^3+c^3)-d*x^2*(b*x^3+a)^(1/3)/(d^3*x^3+c^3)+x*
(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^2/(1+b*x^3
/a)^(1/3)-1/2*d^3*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,2,7/3,-b*x^3/a,-d^3
*x^3/c^3)/c^5/(1+b*x^3/a)^(1/3)-1/6*b*c^2*ln(d^3*x^3+c^3)/d^2/(-a*d^3+b*c^3
)^(2/3)-1/9*a*d*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(2/3)-1/18*(-2*a*d^3+3*b*c
^3)*ln(d^3*x^3+c^3)/c/d^2/(-a*d^3+b*c^3)^(2/3)-1/2*b^(1/3)*ln(b^(1/3)*x-(b*
x^3+a)^(1/3))/d^2+1/3*a*d*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-
a*d^3+b*c^3)^(2/3)+1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^
3+a)^(1/3))/c/d^2/(-a*d^3+b*c^3)^(2/3)+1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*
(b*x^3+a)^(1/3))/d^2/(-a*d^3+b*c^3)^(2/3)-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)+2/9*a*d*arctan(1/3*(1+2*(-a*d^3+
b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(2/3)*3^(1/2)+1
/9*(-2*a*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1
/3))*3^(1/2))/c/d^2/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*
d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/d^2/(-a*d^3+b*c^3)^(2/3)*3
^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 17, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {2181, 441, 440, 480, 12, 503, 455, 43, 60, 631, 210, 31, 525, 524, 478, 598, 337}

$$-\frac{d^3\sqrt[3]{bx^3+a}\,F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x^4}{2c^5\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{d\sqrt[3]{bx^3+a}\,x^2}{c^3+d^3x^3}+\frac{\sqrt[3]{bx^3+a}\,F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)x}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{\sqrt[3]{b}\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}+\frac{2ad\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{bc^3-ad^3}x}{c\sqrt[3]{bx^3+a}}+1}{\sqrt{3}}\right)}{3\sqrt{3}cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\,\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{bx^3+a}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{ad\log\left(c^3+d^3x^3\right)}{9c\left(bc^3-ad^3\right)^{2/3}}-\frac{\left(3bc^3-2ad^3\right)\log\left(c^3+d^3x^3\right)}{18cd^2\left(bc^3-ad^3\right)^{2/3}}-\frac{bc^2\log\left(c^3+d^3x^3\right)}{6d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x-\sqrt[3]{bx^3+a}\right)}{2d^2}+\frac{ad\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{3c\left(bc^3-ad^3\right)^{2/3}}+\frac{\left(3bc^3-2ad^3\right)\log\left(\frac{\sqrt[3]{bc^3-ad^3}x}{c}-\sqrt[3]{bx^3+a}\right)}{6cd^2\left(bc^3-ad^3\right)^{2/3}}+\frac{bc^2\log\left(\sqrt[3]{bx^3+a}d+\sqrt[3]{bc^3-ad^3}\right)}{2d^2\left(bc^3-ad^3\right)^{2/3}}-\frac{c^2\sqrt[3]{bx^3+a}}{d\left(c^3+d^3x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
```

```
[Out] -((c^2*(a + b*x^3)^(1/3))/(d*(c^3 + d^3*x^3))) - (d*x^2*(a + b*x^3)^(1/3))/
(c^3 + d^3*x^3) + (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3
)/a), -((d^3*x^3)/c^3)])/(c^2*(1 + (b*x^3)/a)^(1/3)) - (d^3*x^4*(a + b*x^3)
^(1/3)*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*
(1 + (b*x^3)/a)^(1/3)) - (b^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/
```

```
3))/Sqrt[3]])/(Sqrt[3]*d^2) + (2*a*d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^3)^(2/3)) + ((
3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(
1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b*c^2*ArcTan[(1
 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2*(b
*c^3 - a*d^3)^(2/3)) - (b*c^2*Log[c^3 + d^3*x^3])/(6*d^2*(b*c^3 - a*d^3)^(2
/3)) - (a*d*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3)^(2/3)) - ((3*b*c^3 - 2
*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*d^2*(b*c^3 - a*d^3)^(2/3)) - (b^(1/3)*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*d^2) + (a*d*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(2/3)) + ((3*b*c^3 - 2*a*d^
3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(6*c*d^2*(b*c^3 -
a*d^3)^(2/3)) + (b*c^2*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2
*d^2*(b*c^3 - a*d^3)^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(e*x)^(m + 1))*(a + b*x^n)^(p + 1)*((c + d*x^n
)^q/(a*e*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p
 + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e,
m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 598

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a
+ b*x^n)^p*((e + f*x^n)/(c + d*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} \, dx = \int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} \, dx$$

Mathematica [F]
time = 17.82, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2,x]
[Out] Integrate[(a + b*x^3)^(1/3)/(c + d*x)^2, x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{1}{3}}}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
[Out] int((b*x^3+a)^(1/3)/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x+c)**2,x)
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\right)}^{1/3}}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x)^2,x)
[Out] int((a + b*x^3)^(1/3)/(c + d*x)^2, x)
```

3.29 $\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{3c^2d^2(a+bx^3)^{2/3}}{b}+\frac{4cd^3x(a+bx^3)^{2/3}}{3b}+\frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{4acd^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{2c^3dx^2}{}$$

```
[Out] 3*c^2*d^2*(b*x^3+a)^(2/3)/b+4/3*c*d^3*x*(b*x^3+a)^(2/3)/b+2*c^3*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)+1/5*d^4*x
^5*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 5/3],[8/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1
/2*c^4*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+2/3*a*c*d^3*ln(-b^(1/3)*x+(b*
x^3+a)^(1/3))/b^(4/3)+1/3*c^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^
(1/2))/b^(1/3)*3^(1/2)-4/9*a*c*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1907, 245, 372, 371, 267, 327}

$$-\frac{4acd^3\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{c^4\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{2acd^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{3b^{4/3}}-\frac{c^4\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{2c^3dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{3c^2d^2(a+bx^3)^{2/3}}{b}+\frac{4cd^3x(a+bx^3)^{2/3}}{3b}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)}{5\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c^2*d^2*(a + b*x^3)^(2/3))/b + (4*c*d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^
4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
- (4*a*c*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(4/3)) + (2*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/
3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*H
ypergeometric2F1[1/3, 5/3, 8/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(1/3)) - (c^4
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3)) + (2*a*c*d^3*Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^4}{\sqrt[3]{a+bx^3}}\,dx = \int \left(\frac{c^4}{\sqrt[3]{a+bx^3}} + \frac{4c^3dx}{\sqrt[3]{a+bx^3}} + \frac{6c^2d^2x^2}{\sqrt[3]{a+bx^3}} + \frac{4cd^3x^3}{\sqrt[3]{a+bx^3}} + \frac{d^4x^4}{\sqrt[3]{a+bx^3}}\right) dx$$

$$= c^4 \int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + \left(4c^3d\right) \int \frac{x}{\sqrt[3]{a+bx^3}}\,dx + \left(6c^2d^2\right) \int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + \left(4cd^3\right) \int$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b} + \frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{c^4\log\left(-\sqrt[3]{b}\,x +\right.}{2\sqrt[3]{b}}$$

$$= \frac{3c^2d^2\left(a+bx^3\right)^{2/3}}{b} + \frac{4cd^3x\left(a+bx^3\right)^{2/3}}{3b} + \frac{c^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{4acd^3\tan^{-1}\left(1+\right.}{3\sqrt{3}\,b}$$

Mathematica [**A**]
time = 10.35, size = 392, normalized size = 1.26

$$\frac{180b^{4/3}c^3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+18b^{4/3}d^4x^5\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{8}{3};-\frac{bx^3}{a}\right)+5c\left(54a\sqrt[3]{b}\,cd^2+24a\sqrt[3]{b}\,d^3x+54b^{4/3}cd^2x^3+24b^{4/3}d^3x^4+2\sqrt{3}\left(3bc^3-4ad^3\right)\sqrt[3]{a+bx^3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)+2\left(-3bc^3+4ad^3\right)\sqrt[3]{a+bx^3}\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+3bc^3\sqrt[3]{a+bx^3}\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)-4ad^3\sqrt[3]{a+bx^3}\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)\right)}{90b^{4/3}\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(1/3),x]
```

```
[Out] (180*b^(4/3)*c^3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/
3, -((b*x^3)/a)] + 18*b^(4/3)*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2
F1[1/3, 5/3, 8/3, -((b*x^3)/a)] + 5*c*(54*a*b^(1/3)*c*d^2 + 24*a*b^(1/3)*d^
3*x + 54*b^(4/3)*c*d^2*x^3 + 24*b^(4/3)*d^3*x^4 + 2*Sqrt[3]*(3*b*c^3 - 4*a*
d^3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]
] + 2*(-3*b*c^3 + 4*a*d^3)*(a + b*x^3)^(1/3)*Log[1 - (b^(1/3)*x)/(a + b*x^3
)^(1/3)] + 3*b*c^3*(a + b*x^3)^(1/3)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3
) + (b^(1/3)*x)/(a + b*x^3)^(1/3)] - 4*a*d^3*(a + b*x^3)^(1/3)*Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(90*b^(4/3)*
(a + b*x^3)^(1/3))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx+c\right)^{4}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^4 + integra
te((d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x)/(b*x^3 + a)^(1/3), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 2.59, size = 206, normalized size = 0.66

$$6c^2d^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(1/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(
2*b), True)) + c**4*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar
(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((1/3, 2/
3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3)) + 4*c*d**3*x*
*4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/
3)*gamma(7/3)) + d**4*x**5*gamma(5/3)*hyper((1/3, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(1/3)*gamma(8/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{(b\,x^3+a)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*x^3)^(1/3),x)
[Out] int((c + d*x)^4/(a + b*x^3)^(1/3), x)
```

3.30 $\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=255

$$\frac{3cd^2(a+bx^3)^{2/3}}{2b}+\frac{d^3x(a+bx^3)^{2/3}}{3b}+\frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{3c^2dx^2\sqrt[3]{1+\ldots}}{2\ldots}$$

```
[Out] 3/2*c*d^2*(b*x^3+a)^(2/3)/b+1/3*d^3*x*(b*x^3+a)^(2/3)/b+3/2*c^2*d*x^2*(1+b*
x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^3*l
n(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/6*a*d^3*ln(-b^(1/3)*x+(b*x^3+a)^(1/
3))/b^(4/3)+1/3*c^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
1/3)*3^(1/2)-1/9*a*d^3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1907, 245, 372, 371, 267, 327}

$$-\frac{ad^3\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}}+\frac{c^3\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{ad^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}}-\frac{c^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{3c^2dx^2\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}+\frac{3cd^2(a+bx^3)^{2/3}}{2b}+\frac{d^3x(a+bx^3)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(2/3))/(2*b) + (d^3*x*(a + b*x^3)^(2/3))/(3*b) + (c^3*
ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) -
(a*d^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^
(4/3)) + (3*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3
, -((b*x^3)/a)])/(2*(a + b*x^3)^(1/3)) - (c^3*Log[-(b^(1/3)*x) + (a + b*x^3
)^(1/3)])/(2*b^(1/3)) + (a*d^3*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(6*b^
(4/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^3}{\sqrt[3]{a+bx^3}}\,dx &= \int \left(\frac{c^3}{\sqrt[3]{a+bx^3}} + \frac{3c^2dx}{\sqrt[3]{a+bx^3}} + \frac{3cd^2x^2}{\sqrt[3]{a+bx^3}} + \frac{d^3x^3}{\sqrt[3]{a+bx^3}}\right)dx \\
&= c^3\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + \left(3c^2d\right)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx + \left(3cd^2\right)\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + d^3\int \frac{x^3}{\sqrt[3]{a+bx}} \\
&= \frac{3cd^2(a+bx^3)^{2/3}}{2b} + \frac{d^3x(a+bx^3)^{2/3}}{3b} + \frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{c^3\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a}\right.}{2\sqrt[3]{b}} \\
&= \frac{3cd^2(a+bx^3)^{2/3}}{2b} + \frac{d^3x(a+bx^3)^{2/3}}{3b} + \frac{c^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{ad^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}}{\sqrt[3]{a+}}}{\sqrt{3}}\right.}{3\sqrt{3}\,b^{4/3}}
\end{aligned}
$$

Mathematica [A]
time = 10.28, size = 287, normalized size = 1.13

$$
\frac{1}{18}\left(\frac{27c^2dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} + \frac{27\sqrt[3]{b}\,cd^2(a+bx^3)^{2/3} + 6\sqrt[3]{b}\,d^3x(a+bx^3)^{2/3} + 2\sqrt{3}\left(3bc^3-ad^3\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right) + \left(-6bc^3+2ad^3\right)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) + 3bc^3\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right) - ad^3\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{b^{4/3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(1/3),x]
```

```
[Out] ((27*c^2*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*
x^3)/a)])/(a + b*x^3)^(1/3) + (27*b^(1/3)*c*d^2*(a + b*x^3)^(2/3) + 6*b^(1/
3)*d^3*x*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] + (-6*b*c^3 + 2*a*d^3)*Log[1 - (b^(1/3)
*x)/(a + b*x^3)^(1/3)] + 3*b*c^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(b^(1/3)*x)/(a + b*x^3)^(1/3)] - a*d^3*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2
/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/b^(4/3))/18
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{\left(dx+c\right)^{3}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(1/3),x)
```

[Out] int((d*x+c)^3/(b*x^3+a)^(1/3),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")

[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^3 + integrate((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(b*x^3 + a)^(1/3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")

[Out] Timed out

Sympy [A]
time = 2.21, size = 155, normalized size = 0.61

$$3cd^2\left(\begin{cases}\frac{x^3}{3\sqrt[3]{a}} & \text{for } b=0\\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise}\end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**3/(b*x**3+a)**(1/3),x)

[Out] 3*c*d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), True)) + c**3*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(a**(1/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{{\left(b\,x^3+a\right)}^{1/3}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^3/(a + b*x^3)^(1/3), x)
```

3.31 $\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=147

$$\frac{d^2(a+bx^3)^{2/3}}{2b}+\frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}-\frac{c^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*d^2*(b*x^3+a)^(2/3)/b+c*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5
/3],-b*x^3/a)/(b*x^3+a)^(1/3)-1/2*c^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3
)+1/3*c^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/
2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1900, 267, 1907, 245, 372, 371}

$$\frac{c^2\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{c^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{d^2(a+bx^3)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(2*b^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{\sqrt[3]{a+bx^3}}\,dx &= d^2\int \frac{x^2}{\sqrt[3]{a+bx^3}}\,dx + \int \frac{c^2+2cdx}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \int \left(\frac{c^2}{\sqrt[3]{a+bx^3}} + \frac{2cdx}{\sqrt[3]{a+bx^3}}\right)dx \\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + c^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx + (2cd)\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx \\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} - \frac{c^2\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} + \frac{\left(2cd\sqrt[3]{1 \cdots\right.}{\cdots} \\
&= \frac{d^2\left(a+bx^3\right)^{2/3}}{2b} + \frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}} + \frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}} - \frac{c^2\,\mathrm{lo}\cdots}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 10.13, size = 201, normalized size = 1.37

$$\frac{d^2(a+bx^3)^{2/3}}{2b}+\frac{c^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{cdx^2\sqrt[3]{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}-\frac{c^2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{c^2\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(2/3))/(2*b) + (c^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^
(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) + (c*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((b*x^3)/a)])/(a + b*x^3)^(1/3) - (c^2*Log[1 - (
b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (c^2*Log[1 + (b^(2/3)*x^2)/(a
+ b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c^2 + integra
te((d^2*x^2 + 2*c*d*x)/(b*x^3 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(1/3), x)
```

Sympy [A]
time = 1.58, size = 110, normalized size = 0.75

$$d^2 \left(\begin{cases} \frac{x^3}{3\sqrt[3]{a}} & \text{for } b = 0 \\ \frac{(a+bx^3)^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases} \right) + \frac{c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^2 \Gamma\left(\frac{2}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(1/3)), Eq(b, 0)), ((a + b*x**3)**(2/3)/(2*b), T
rue)) + c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*a**(1/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{(b\,x^3+a)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)^2/(a + b*x^3)^(1/3), x)
```

3.32 $\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=124

$$\frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}$$

```
[Out] 1/2*d*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/(b*x^3+a)^
(1/3)-1/2*c*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)+1/3*c*arctan(1/3*(1+2*b^
(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1907, 245, 372, 371}

$$\frac{c\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{c\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] (c*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3))
 + (d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(2*(a + b*x^3)^(1/3)) - (c*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*b
^(1/3))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{\sqrt[3]{a+bx^3}}\,dx &= \int \left(\frac{c}{\sqrt[3]{a+bx^3}}+\frac{dx}{\sqrt[3]{a+bx^3}}\right)dx\\ &= c\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx+d\int \frac{x}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}-\frac{c\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}+\frac{\left(d\sqrt[3]{1+\frac{bx^3}{a}}\right)\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}}}{\sqrt[3]{a+bx^3}}\\ &= \frac{c\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{b}}+\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{a+bx^3}}-\frac{c\log\left(-\sqrt[3]{b}\,x+\sqrt[3]{a+b}\right.}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [A]
time = 10.09, size = 163, normalized size = 1.31

$$\frac{1}{6}\left(\frac{3dx^2\sqrt[3]{1+\frac{bx^3}{a}}\;{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{c\left(2\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)-2\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)+\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)\right)}{\sqrt[3]{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(1/3),x]
```

```
[Out] ((3*d*x^2*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x^3)/
a)])/(a + b*x^3)^(1/3) + (c*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3
```

```
)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(
2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3
))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/
x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3))*c + d*integra
te(x/(b*x^3 + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.07, size = 78, normalized size = 0.63

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((1/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{{\left(b\,x^3+a\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^(1/3),x)
```

```
[Out] int((c + d*x)/(a + b*x^3)^(1/3), x)
```

3.33 $\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{bc^3-ad^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{bc^3-ad^3}}+\frac{\log(\cdots}{3\sqrt[3]{\cdots}}$$

```
[Out] -1/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d^3*x^3/c^3)/
c^2/(b*x^3+a)^(1/3)+1/3*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3
+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)-1/2*ln((-a*d^3+b*c^
3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d^
3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(1/3)*3^(1/2)-1
/3*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3
+b*c^3)^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2181, 384, 525, 524, 455, 58, 631, 210, 31}

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^2\sqrt[3]{a+bx^3}}+\frac{\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{bc^3-ad^3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{bc^3-ad^3}}+\frac{\log\left(c^3+d^3x^3\right)}{3\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}-\frac{\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{bc^3-ad^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
```

```
[Out] -1/2*(d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
-((d^3*x^3)/c^3)])/(c^2*(a + b*x^3)^(1/3)) + ArcTan[(1 + (2*(b*c^3 - a*d^3)
^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*(b*c^3 - a*d^3)^(1/3)) -
 ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]]/(Sqrt[
3]*(b*c^3 - a*d^3)^(1/3)) + Log[c^3 + d^3*x^3]/(3*(b*c^3 - a*d^3)^(1/3)) -
Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(1/
3)) - Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)]/(2*(b*c^3 - a*d^3)^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx=\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F]
time = 7.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(1/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(1/3),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)), x)
```

3.34 $\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=761

$$\frac{c^2d^2(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)}-\frac{cd^3x(a+bx^3)^{2/3}}{(bc^3-ad^3)(c^3+d^3x^3)}-\frac{dx^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}+\frac{d^4x^5\sqrt[3]{1+\frac{bx}{a}}}{}$$

```
[Out] c^2*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-c*d^3*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)-d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,2,5
/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(1/3)+1/5*d^4*x^5*(1+b*x^3/a)^(1/3)
*AppellF1(5/3,1/3,2,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(1/3)+1/6*b*c^
2*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(4/3)+1/9*a*d^3*ln(d^3*x^3+c^3)/c/(-a*d^3+
b*c^3)^(4/3)+1/18*(-2*a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(4/3)
-1/3*a*d^3*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(4
/3)-1/6*(-2*a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(
-a*d^3+b*c^3)^(4/3)-1/2*b*c^2*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-
a*d^3+b*c^3)^(4/3)+2/9*a*d^3*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^
3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(4/3)*3^(1/2)+1/9*(-2*a*d^3+3*b*c^3)*
arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^
3+b*c^3)^(4/3)*3^(1/2)-1/3*b*c^2*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+
b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(4/3)*3^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {2181, 390, 384, 525, 524, 455, 44, 58, 631, 210, 31, 482, 12}

$$-\frac{dx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},2;\frac{5}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^3\sqrt[3]{a+bx^3}}+\frac{d^4x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},2;\frac{8}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{5c^6\sqrt[3]{a+bx^3}}+\frac{2ad^3\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c\,(bc^3-ad^3)^{4/3}}+\frac{(3bc^3-2ad^3)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,c\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc^3-ad^3)^{4/3}}-\frac{cd^3x(a+bx^3)^{2/3}}{(c^3+d^3x^3)(bc^3-ad^3)}+\frac{ad^3\log(c^3+d^3x^3)}{9c\,(bc^3-ad^3)^{4/3}}+\frac{(3bc^3-2ad^3)\log(c^3+d^3x^3)}{18c\,(bc^3-ad^3)^{4/3}}-\frac{ad^3\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c\,(bc^3-ad^3)^{4/3}}-\frac{(3bc^3-2ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{6c\,(bc^3-ad^3)^{4/3}}+\frac{bc^2\log(c^3+d^3x^3)}{6\,(bc^3-ad^3)^{4/3}}-\frac{bc^2\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\,(bc^3-ad^3)^{4/3}}+\frac{c^2d^2(a+bx^3)^{2/3}}{(c^3+d^3x^3)(bc^3-ad^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
```

```
[Out] (c^2*d^2*(a + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (c*d^3*x*(a
 + b*x^3)^(2/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) - (d*x^2*(1 + (b*x^3)/a)
^(1/3)*AppellF1[2/3, 1/3, 2, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a
+ b*x^3)^(1/3)) + (d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 2, 8/3,
 -((b*x^3)/a), -((d^3*x^3)/c^3)])/(5*c^6*(a + b*x^3)^(1/3)) + (2*a*d^3*ArcT
an[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqr
t[3]*c*(b*c^3 - a*d^3)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*ArcTan[(1 + (2*(b*c^3
- a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*
d^3)^(4/3)) - (b*c^2*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1
```

```
/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(4/3)) + (b*c^2*Log[c^3 + d^3*x^3])
/(6*(b*c^3 - a*d^3)^(4/3)) + (a*d^3*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(4/3)) + ((3*b*c^3 - 2*a*d^3)*Log[c^3 + d^3*x^3])/(18*c*(b*c^3 - a*d^3)^(
4/3)) - (a*d^3*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c*(
b*c^3 - a*d^3)^(4/3)) - ((3*b*c^3 - 2*a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*x)/
c - (a + b*x^3)^(1/3)])/(6*c*(b*c^3 - a*d^3)^(4/3)) - (b*c^2*Log[(b*c^3 - a
*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3)^(4/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F]
time = 20.25, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2\sqrt[3]{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(1/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(b\,x^3+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^2), x)
```

3.35 $\int \frac{1}{(c+dx)^3\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=1513

$$\frac{3c^4d^2(a+bx^3)^{2/3}}{2\,(bc^3-ad^3)\,(c^3+d^3x^3)^2}-\frac{3c^3d^3x(a+bx^3)^{2/3}}{2\,(bc^3-ad^3)\,(c^3+d^3x^3)^2}+\frac{4bc^4d^2(a+bx^3)^{2/3}}{3\,(bc^3-ad^3)^2\,(c^3+d^3x^3)}-\frac{cd^2(bc^3-3ad^3)\,(a+bx^3)^{2}}{3\,(bc^3-ad^3)^2\,(c^3+d^3x^3}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2-3/2*c^3*d^3*x*(b
*x^3+a)^(2/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+4/3*b*c^4*d^2*(b*x^3+a)^(2/3)/
(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/3*c*d^2*(-3*a*d^3+b*c^3)*(b*x^3+a)^(2/3)/(
-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/18*d^3*(-7*a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/18*d^3*(-5*a*d^3+9*b*c^3)*x*(b*x^3+a)^(2/
3)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-7/18*d^3*(a*d^3+3*b*c^3)*x*(b*x^3+a)^(2/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-3/2*d*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1
/3,3,5/3,-b*x^3/a,-d^3*x^3/c^3)/c^4/(b*x^3+a)^(1/3)+6/5*d^4*x^5*(1+b*x^3/a)
^(1/3)*AppellF1(5/3,1/3,3,8/3,-b*x^3/a,-d^3*x^3/c^3)/c^7/(b*x^3+a)^(1/3)+2/
9*b^2*c^4*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(7/3)+1/27*a^2*d^6*ln(d^3*x^3+c^3)
/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*b*c*(-3*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3
+b*c^3)^(7/3)+7/54*a*d^3*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3
)^(7/3)+1/54*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d
^3+b*c^3)^(7/3)-1/9*a^2*d^6*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^
2/(-a*d^3+b*c^3)^(7/3)-7/18*a*d^3*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*
x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(7/3)-1/18*(5*a^2*d^6-12*a*b*c^3*d^
3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3
)^(7/3)-2/3*b^2*c^4*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^
3)^(7/3)+1/6*b*c*(-3*a*d^3+b*c^3)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3)
)/(-a*d^3+b*c^3)^(7/3)+2/27*a^2*d^6*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/
c/(b*x^3+a)^(1/3))*3^(1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+7/27*a*d^3*(-a
*d^3+3*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(
1/2))/c^2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/27*(5*a^2*d^6-12*a*b*c^3*d^3+9*b^2
*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c^
2/(-a*d^3+b*c^3)^(7/3)*3^(1/2)-4/9*b^2*c^4*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(7/3)*3^(1/2)+1/9*b*c*(-3*a
*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2)
)/(-a*d^3+b*c^3)^(7/3)*3^(1/2)
```

Rubi [A]

time = 1.32, antiderivative size = 1513, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 17, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {2181, 425, 541, 12, 384, 525, 524, 455, 44, 58, 631, 210, 31, 482, 457, 79, 481}

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] (3*c^4*d^2*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) - (3*c^
3*d^3*x*(a + b*x^3)^(2/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (4*b*c^4
*d^2*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b*c
^3 - 3*a*d^3)*(a + b*x^3)^(2/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) + (d
^3*(3*b*c^3 - 7*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)^2*(c^3 + d^
3*x^3)) - (d^3*(9*b*c^3 - 5*a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*c^3 - a*d^3)
^2*(c^3 + d^3*x^3)) - (7*d^3*(3*b*c^3 + a*d^3)*x*(a + b*x^3)^(2/3))/(18*(b*
c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (3*d*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2
/3, 1/3, 3, 5/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^4*(a + b*x^3)^(1/3))
 + (6*d^4*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 3, 8/3, -((b*x^3)/a)
, -((d^3*x^3)/c^3)])/(5*c^7*(a + b*x^3)^(1/3)) + (2*a^2*d^6*ArcTan[(1 + (2*
(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b
*c^3 - a*d^3)^(7/3)) + (7*a*d^3*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a
*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3 - a*d
^3)^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*ArcTan[(1 + (2*(b*c^
3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^2*(b*c^3
- a*d^3)^(7/3)) - (4*b^2*c^4*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a
*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (b*c*(b*c^3 - 3*
a*d^3)*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])
/(3*Sqrt[3]*(b*c^3 - a*d^3)^(7/3)) + (2*b^2*c^4*Log[c^3 + d^3*x^3])/(9*(b*c
^3 - a*d^3)^(7/3)) + (a^2*d^6*Log[c^3 + d^3*x^3])/(27*c^2*(b*c^3 - a*d^3)^(
7/3)) - (b*c*(b*c^3 - 3*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^3)^(7/3
)) + (7*a*d^3*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(54*c^2*(b*c^3 - a*d^3)
^(7/3)) + ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[c^3 + d^3*x^3])/(54
*c^2*(b*c^3 - a*d^3)^(7/3)) - (a^2*d^6*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(9*c^2*(b*c^3 - a*d^3)^(7/3)) - (7*a*d^3*(3*b*c^3 - a*d^3
)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*
d^3)^(7/3)) - ((9*b^2*c^6 - 12*a*b*c^3*d^3 + 5*a^2*d^6)*Log[((b*c^3 - a*d^3
)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(18*c^2*(b*c^3 - a*d^3)^(7/3)) - (2*b^2*
c^4*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(7
/3)) + (b*c*(b*c^3 - 3*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/
3)])/(6*(b*c^3 - a*d^3)^(7/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-a)*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)
^(p + 1)*((c + d*x^n)^(q + 1)/(b*n*(b*c - a*d)*(p + 1))), x] + Dist[e^(2*n)
/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*
x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n
, p, q, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx = \int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx$$

Mathematica [F]
time = 20.35, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3 \sqrt[3]{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)),x]
```

```
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(1/3)), x]
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

```
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^{3}}\,\left(c+dx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3),x)
```

```
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x)^3), x)
```

3.36 $\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=306

$$\frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}-\frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{2ad^4\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}}+\frac{c^4x\left(1+\frac{b}{}\right.}{}$$

```
[Out] 6*c^2*d^2*(b*x^3+a)^(1/3)/b+1/3*d^4*x^2*(b*x^3+a)^(1/3)/b+c^4*x*(1+b*x^3/a)
^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)+c*d^3*x^4*(1+b*
x^3/a)^(2/3)*hypergeom([2/3, 4/3],[7/3],-b*x^3/a)/(b*x^3+a)^(2/3)-2*c^3*d*l
n(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)+1/3*a*d^4*ln(b^(1/3)*x-(b*x^3+a)^(1/3)
)/b^(5/3)-4/3*c^3*d*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)*3^(1/2)+2/9*a*d^4*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1907, 252, 251, 337, 267, 372, 371, 327}

$$-\frac{4c^3d\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{2ad^4\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}}-\frac{2c^3d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{ad^4\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3}}+\frac{c^4x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{6c^2d^2\sqrt[3]{a+bx^3}}{b}+\frac{cd^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (6*c^2*d^2*(a + b*x^3)^(1/3))/b + (d^4*x^2*(a + b*x^3)^(1/3))/(3*b) - (4*c^
3*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
) + (2*a*d^4*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt
[3]*b^(5/3)) + (c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3
, -((b*x^3)/a)])/(a + b*x^3)^(2/3) + (c*d^3*x^4*(1 + (b*x^3)/a)^(2/3)*Hyper
geometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (2*c^3*d*Log
[b^(1/3)*x - (a + b*x^3)^(1/3)])/b^(2/3) + (a*d^4*Log[b^(1/3)*x - (a + b*x^
3)^(1/3)])/(3*b^(5/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^4}{(a+bx^3)^{2/3}}\,dx = \int \left(\frac{c^4}{(a+bx^3)^{2/3}} + \frac{4c^3dx}{(a+bx^3)^{2/3}} + \frac{6c^2d^2x^2}{(a+bx^3)^{2/3}} + \frac{4cd^3x^3}{(a+bx^3)^{2/3}} + \frac{d^4x^4}{(a+bx^3)^{2/3}}\right)dx$$

$$= c^4\int \frac{1}{(a+bx^3)^{2/3}}\,dx + \left(4c^3d\right)\int \frac{x}{(a+bx^3)^{2/3}}\,dx + \left(6c^2d^2\right)\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx + \left(4cd^3\right.$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \left(4c^3d\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) - $$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{cd^3x^4\left(1+\right.}{}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{cd^3x^4\left(1+\right.}{}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{cd^3x^4\left(1+\right.}{}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{c^4x\left(1+\frac{bx^3}{a}\right)^{2/3}}{(a+}$$

$$= \frac{6c^2d^2\sqrt[3]{a+bx^3}}{b} + \frac{d^4x^2\sqrt[3]{a+bx^3}}{3b} - \frac{4c^3d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{2ad^4\tan^{-1}\left(\frac{1+}{}\right.}{3\sqrt{3}\,b}$$

Mathematica [A]
time = 10.13, size = 166, normalized size = 0.54

$$\frac{3bc^4x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right) + d\left(\left(6bc^3-ad^3\right)x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{a+bx^3}\right) + d\left(\left(18c^2+d^2x^2\right)\left(a+bx^3\right) + 3bcdx^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)}{3b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*b*c^4*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*((6*b*c^3 - a*d^3)*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
```

```
 b*x^3)] + d*((18*c^2 + d^2*x^2)*(a + b*x^3) + 3*b*c*d*x^4*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])))/(3*b*(a + b*x^3)^(2
/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^4}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^4/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A]
time = 2.73, size = 204, normalized size = 0.67

$$6c^2d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for } b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^4x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{4c^3dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{4cd^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{d^4x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4/(b*x**3+a)**(2/3),x)
```

```
[Out] 6*c**2*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b
, True)) + c**4*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*p
```

```
i)/a)/(3*a**(2/3)*gamma(4/3)) + 4*c**3*d*x**2*gamma(2/3)*hyper((2/3, 2/3),
(5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3)) + 4*c*d**3*x**4*g
amma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*g
amma(7/3)) + d**4*x**5*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(2/3)*gamma(8/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^4/(b*x^3 + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{(b\,x^3+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^4/(a + b*x^3)^(2/3), x)
```

3.37 $\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}} \, dx$

Optimal. Leaf size=187

$$\frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}-\frac{\sqrt{3}\,c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\right.}{2b\,(a+bx^3)^{2/3}}$$

```
[Out] 3*c*d^2*(b*x^3+a)^(1/3)/b+1/2*d^3*x*(b*x^3+a)^(1/3)/b+1/2*(-a*d^3+2*b*c^3)*
x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b/(b*x^3+a)^(2/3)-
3/2*c^2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-c^2*d*arctan(1/3*(1+2*b^(1/
3)*x/(b*x^3+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)
```

Rubi [A]
time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1902, 1900, 267, 1907, 252, 251, 337}

$$-\frac{\sqrt{3}\,c^2d\,\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{3c^2d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc^3-ad^3)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}}+\frac{3cd^2\sqrt[3]{a+bx^3}}{b}+\frac{d^3x\sqrt[3]{a+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (3*c*d^2*(a + b*x^3)^(1/3))/b + (d^3*x*(a + b*x^3)^(1/3))/(2*b) - (Sqrt[3]*
c^2*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/b^(2/3) + ((2*
b*c^3 - a*d^3)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((
b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3)) - (3*c^2*d*Log[b^(1/3)*x - (a + b*x^3)^
(1/3)])/(2*b^(2/3))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1902

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Expon[Pq, x
]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(q + n*p + 1)), Int[ExpandToSum
[b*(q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(q - n + 1)*x^(q - n), x]*(a + b*x^
n)^p, x], x] + Simp[Pqq*x^(q - n + 1)*((a + b*x^n)^(p + 1)/(b*(q + n*p + 1)
)), x]] /; NeQ[q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[
p + (q + 1)/(2*n)])] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx^3)^{2/3}}\,dx = \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx+6bcd^2x^2}{(a+bx^3)^{2/3}}\,dx}{2b}$$

$$= \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \frac{2bc^3-ad^3+6bc^2dx}{(a+bx^3)^{2/3}}\,dx}{2b} + \left(3cd^2\right)\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{\int \left(\frac{2bc^3\left(1-\frac{ad^3}{2bc^3}\right)}{(a+bx^3)^{2/3}} + \frac{6bc^2dx}{(a+bx^3)^{2/3}}\right)dx}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\int \frac{x}{(a+bx^3)^{2/3}}\,dx + \frac{(2bc^3-ad^3)\int \frac{1}{(a+bx^3)^{2/3}}}{2b}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \left(3c^2d\right)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{((2b}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} + \frac{(c^2}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} - \frac{c^2d}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc^3-ad^3)\,x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\,(a+bx^3)^{2/3}} - \frac{c^2d}{}$$

$$= \frac{3cd^2\sqrt[3]{a+bx^3}}{b} + \frac{d^3x\sqrt[3]{a+bx^3}}{2b} - \frac{\sqrt{3}\,c^2d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{(2bc^3-ad^3)\,x\left(1\right.}{2}$$

Mathematica [A]
time = 10.08, size = 145, normalized size = 0.78

$$\frac{4bc^3x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right) + d\left(6bc^2x^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{a+bx^3}\right) + d\left(12c(a+bx^3) + bdx^4\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)\right)\right)}{4b\,(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x^3)^(2/3),x]
```

```
[Out] (4*b*c^3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)] + d*(6*b*c^2*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)] +
```

```
d*(12*c*(a + b*x^3) + b*d*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3,
4/3, 7/3, -((b*x^3)/a)])))/(4*b*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
[Out] int((d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/(b*x^3 + a)^(2/3), x)
```

Sympy [A]
time = 2.12, size = 153, normalized size = 0.82

$$3cd^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for}\ b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^3x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{c^2dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}+\frac{d^3x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x**3+a)**(2/3),x)
[Out] 3*c*d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, T
rue)) + c**3*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/
```

```
a)/(3*a**(2/3)*gamma(4/3)) + c**2*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,
), b*x**3*exp_polar(I*pi)/a)/(a**(2/3)*gamma(5/3)) + d**3*x**4*gamma(4/3)*h
yper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*x^3 + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{(b\,x^3+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^3/(a + b*x^3)^(2/3), x)
```

3.38 $\int \frac{(c+dx)^2}{\left(a+bx^3\right)^{2/3}} \, dx$

Optimal. Leaf size=141

$$\frac{d^2 \sqrt[3]{a+bx^3}}{b}-\frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}-\frac{cd\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}$$

```
[Out] d^2*(b*x^3+a)^(1/3)/b+c^2*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b
*x^3/a)/(b*x^3+a)^(2/3)-c*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-2/3*c*d*a
rctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1900, 267, 1907, 252, 251, 337}

$$-\frac{2cd\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}-\frac{cd\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{b^{2/3}}+\frac{c^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}+\frac{d^2\sqrt[3]{a+bx^3}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (d^2*(a + b*x^3)^(1/3))/b - (2*c*d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1
/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3)) + (c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeomet
ric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(a + b*x^3)^(2/3) - (c*d*Log[b^(1/3)*x
 - (a + b*x^3)^(1/3)])/b^(2/3)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1900

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[Coeff[Pq, x, n -
 1], Int[x^(n - 1)*(a + b*x^n)^p, x], x] + Int[ExpandToSum[Pq - Coeff[Pq, x
, n - 1]*x^(n - 1), x]*(a + b*x^n)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq
, x] && IGtQ[n, 0] && Expon[Pq, x] == n - 1
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^2}{(a+bx^3)^{2/3}}\,dx &= d^2\int \frac{x^2}{(a+bx^3)^{2/3}}\,dx + \int \frac{c^2+2cdx}{(a+bx^3)^{2/3}}\,dx \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \int \left(\frac{c^2}{(a+bx^3)^{2/3}} + \frac{2cdx}{(a+bx^3)^{2/3}}\right)dx \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + c^2\int \frac{1}{(a+bx^3)^{2/3}}\,dx + (2cd)\int \frac{x}{(a+bx^3)^{2/3}}\,dx \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + (2cd)\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \cdots}{(a+bx^3)^{2/3}} \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{(2cd)\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \cdots\right.}{3\sqrt[3]{b}} \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \cdots \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{2cd\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{cd \cdots}{\cdots} \\
&= \frac{d^2\sqrt[3]{a+bx^3}}{b} - \frac{2cd\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{c^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \cdots
\end{aligned}
$$

Mathematica [A]
time = 10.04, size = 95, normalized size = 0.67

$$
\frac{bc^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right) + d\left(d(a+bx^3) + bcx^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{a+bx^3}\right)\right)}{b\,(a+bx^3)^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (b*c^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a
)] + d*(d*(a + b*x^3) + b*c*x^2*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a +
 b*x^3)]))/(b*(a + b*x^3)^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(b\,x^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [A]
time = 1.61, size = 109, normalized size = 0.77

$$d^2\left(\begin{cases}\frac{x^3}{3a^{\frac{2}{3}}} & \text{for}\ b=0\\ \frac{\sqrt[3]{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)+\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] d**2*Piecewise((x**3/(3*a**(2/3)), Eq(b, 0)), ((a + b*x**3)**(1/3)/b, True)
) + c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
```

```
3*a**(2/3)*gamma(4/3)) + 2*c*d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*x^3 + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{(b\,x^3+a)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x^3)^(2/3),x)
[Out] int((c + d*x)^2/(a + b*x^3)^(2/3), x)
```

3.39 $\int \frac{c+dx}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=121

$$-\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}+\frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}-\frac{d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}$$

```
[Out] c*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/(b*x^3+a)^(2/3)-
1/2*d*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)-1/3*d*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1907, 252, 251, 337}

$$-\frac{d\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}}-\frac{d\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}}+\frac{cx\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] -((d*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(2/3
))) + (c*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)
/a)])/(a + b*x^3)^(2/3) - (d*Log[b^(1/3)*x - (a + b*x^3)^(1/3)])/(2*b^(2/3)
)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 337

```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$\int \frac{c+dx}{(a+bx^3)^{2/3}}\,dx = \int \left(\frac{c}{(a+bx^3)^{2/3}} + \frac{dx}{(a+bx^3)^{2/3}}\right)dx$$

$$= c\int \frac{1}{(a+bx^3)^{2/3}}\,dx + d\int \frac{x}{(a+bx^3)^{2/3}}\,dx$$

$$= d\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right) + \frac{\left(c\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} + \frac{d\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} - \frac{d\text{Subst}(}{}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}}{1+\sqrt[3]{b}\,x+b^{2/}}\right.}{6b}$$

$$= \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}} + \frac{d\log\left(1+\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\right.}{6b^{2/3}}$$

$$= -\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}} + \frac{cx\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{(a+bx^3)^{2/3}} - \frac{d\log\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a+}}\right.}{3b^{2/3}}$$

Mathematica [A]

time = 10.04, size = 78, normalized size = 0.64

$$\frac{x\left(2c\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{bx^3}{a+bx^3}\right)\right)}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(2*c*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)
] + d*x*Hypergeometric2F1[2/3, 1, 5/3, (b*x^3)/(a + b*x^3)]))/(2*(a + b*x^3
)^(2/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{dx+c}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.13, size = 78, normalized size = 0.64

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**2*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(5/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*x^3 + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{{\left(b\,x^3+a\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x^3)^(2/3),x)
```

```
[Out] int((c + d*x)/(a + b*x^3)^(2/3), x)
```

3.40 $\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=332

$$\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}}+\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc^3-ad^3}\,x}{c\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\left(bc^3-ad^3\right)^{2/3}}-\frac{d\tan^{-1}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\left(bc^3-ad^3\right)^{2/3}}-\frac{d\log}{3\left(bc\right.}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d^3*x^3/c^3)/c/(b*x^3+
a)^(2/3)-1/3*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)
^(1/3)*x/c-(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/2*d*ln((-a*d^3+b*c^3)^(1
/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(2/3)+1/3*d*arctan(1/3*(1+2*(-a*d^3+b
*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(2/3)*3^(1/2)-1/3*
d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+
b*c^3)^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2181, 441, 440, 503, 455, 60, 631, 210, 31}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c\left(a+bx^3\right)^{2/3}}+\frac{d\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,\left(bc^3-ad^3\right)^{2/3}}-\frac{d\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,\left(bc^3-ad^3\right)^{2/3}}-\frac{d\log\left(c^3+d^3x^3\right)}{3\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}+\frac{d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{2\left(bc^3-ad^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d^3*x^
3)/c^3)])/(c*(a + b*x^3)^(2/3)) + (d*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x
)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(2/3)) - (d*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*
(b*c^3 - a*d^3)^(2/3)) - (d*Log[c^3 + d^3*x^3])/(3*(b*c^3 - a*d^3)^(2/3)) +
 (d*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(2*(b*c^3 - a*d^3
)^(2/3)) + (d*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(2*(b*c^3 -
 a*d^3)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
```

```
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx=\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F]
time = 7.33, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\left(a+bx^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)*(a + b*x^3)^(2/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(b\,x^{3}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^3)^{\frac{2}{3}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{(b\,x^3+a)}^{2/3}\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)), x)
```

3.41 $\int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}} \, dx$

Optimal. Leaf size=760

$$\frac{c^2 d^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)}+\frac{d^4 x^2 \sqrt[3]{a+bx^3}}{(bc^3-ad^3)(c^3+d^3x^3)}+\frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}}-\frac{d^3x^4\left(1+\frac{bx^3}{a}\right)^{2}}{2}$$

```
[Out] c^2*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+d^4*x^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,2,4/3,-
b*x^3/a,-d^3*x^3/c^3)/c^2/(b*x^3+a)^(2/3)-1/2*d^3*x^4*(1+b*x^3/a)^(2/3)*App
ellF1(4/3,2/3,2,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^5/(b*x^3+a)^(2/3)-1/3*b*c^2*d*
ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(5/3)-1/9*a*d^4*ln(d^3*x^3+c^3)/c/(-a*d^3+b*
c^3)^(5/3)-1/9*d*(-a*d^3+3*b*c^3)*ln(d^3*x^3+c^3)/c/(-a*d^3+b*c^3)^(5/3)+1/
3*a*d^4*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*d^3+b*c^3)^(5/3)
+1/3*d*(-a*d^3+3*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c/(-a*
d^3+b*c^3)^(5/3)+b*c^2*d*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3
+b*c^3)^(5/3)+2/9*a*d^4*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^
(1/3))*3^(1/2))/c/(-a*d^3+b*c^3)^(5/3)*3^(1/2)+2/9*d*(-a*d^3+3*b*c^3)*arcta
n(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2))/c/(-a*d^3+b*c
^3)^(5/3)*3^(1/2)-2/3*b*c^2*d*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c
^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(5/3)*3^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 760, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.790$, Rules used = {2181, 441, 440, 483, 12, 503, 455, 44, 60, 631, 210, 31, 525, 524, 482}

$$-\frac{d^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{2c^5\left(a+bx^3\right)^{2/3}}+\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{d^3x^3}{c^3}\right)}{c^2\left(a+bx^3\right)^{2/3}}+\frac{2d(3bc^3-ad^3)\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}}+\frac{2ad^4\text{ArcTan}\left(\frac{\frac{2x\sqrt[3]{bc^3-ad^3}}{c\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c(bc^3-ad^3)^{5/3}}-\frac{2bc^2d\text{ArcTan}\left(\frac{1-\frac{2d\sqrt[3]{a+bx^3}}{\sqrt[3]{bc^3-ad^3}}}{\sqrt{3}}\right)}{\sqrt{3}(bc^3-ad^3)^{5/3}}-\frac{d(3bc^3-ad^3)\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}}+\frac{d(3bc^3-ad^3)\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}}-\frac{ad^4\log(c^3+d^3x^3)}{9c(bc^3-ad^3)^{5/3}}+\frac{ad^4\log\left(\frac{x\sqrt[3]{bc^3-ad^3}}{c}-\sqrt[3]{a+bx^3}\right)}{3c(bc^3-ad^3)^{5/3}}+\frac{d^4x^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)}-\frac{bc^2d\log(c^3+d^3x^3)}{3(bc^3-ad^3)^{5/3}}+\frac{bc^2d\log\left(\sqrt[3]{bc^3-ad^3}+d\sqrt[3]{a+bx^3}\right)}{(bc^3-ad^3)^{5/3}}+\frac{c^2d^2\sqrt[3]{a+bx^3}}{(c^3+d^3x^3)(bc^3-ad^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]
```

```
[Out] (c^2*d^2*(a + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (d^4*x^2*(a
 + b*x^3)^(1/3))/((b*c^3 - a*d^3)*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/
3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^2*(a + b*
x^3)^(2/3)) - (d^3*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 2, 7/3, -((
b*x^3)/a), -((d^3*x^3)/c^3)])/(2*c^5*(a + b*x^3)^(2/3)) + (2*a*d^4*ArcTan[(
1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]
*c*(b*c^3 - a*d^3)^(5/3)) + (2*d*(3*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 -
a*d^3)^(1/3)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c*(b*c^3 - a*d^
3)^(5/3)) - (2*b*c^2*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^
```

```
(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c^3 - a*d^3)^(5/3)) - (b*c^2*d*Log[c^3 + d^3*x
^3])/(3*(b*c^3 - a*d^3)^(5/3)) - (a*d^4*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a
*d^3)^(5/3)) - (d*(3*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c*(b*c^3 - a*d^3
)^(5/3)) + (a*d^4*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*
c*(b*c^3 - a*d^3)^(5/3)) + (d*(3*b*c^3 - a*d^3)*Log[((b*c^3 - a*d^3)^(1/3)*
x)/c - (a + b*x^3)^(1/3)])/(3*c*(b*c^3 - a*d^3)^(5/3)) + (b*c^2*d*Log[(b*c^
3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(b*c^3 - a*d^3)^(5/3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
```

```
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}}\,dx = \int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}}\,dx$$

Mathematica [F]
time = 20.22, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2 \left(a+bx^3\right)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)),x]

[Out] Integrate[1/((c + d*x)^2*(a + b*x^3)^(2/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{2} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}} \left(c+dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{2/3}\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^2), x)
```

3.42 $\int \frac{1}{(c+dx)^3\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=1357

$$\frac{3c^4d^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+\frac{3c^2d^4x^2\sqrt[3]{a+bx^3}}{2\left(bc^3-ad^3\right)\left(c^3+d^3x^3\right)^2}+\frac{5bc^4d^2\sqrt[3]{a+bx^3}}{3\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}-\frac{cd^2\left(bc^3-6ad^3\right)\sqrt[3]{a+bx^3}}{6\left(bc^3-ad^3\right)^2\left(c^3+d^3x^3\right)}$$

```
[Out] 3/2*c^4*d^2*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+3/2*c^2*d^4*x^2*
(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)/(d^3*x^3+c^3)^2+5/3*b*c^4*d^2*(b*x^3+a)^(1/3
)/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)-1/6*c*d^2*(-6*a*d^3+b*c^3)*(b*x^3+a)^(1/3)
/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/6*d^4*(-4*a*d^3+9*b*c^3)*x^2*(b*x^3+a)^(1
/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+1/3*d^4*(2*a*d^3+3*b*c^3)*x^2*(b*x^3+a
)^(1/3)/c/(-a*d^3+b*c^3)^2/(d^3*x^3+c^3)+x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2
/3,3,4/3,-b*x^3/a,-d^3*x^3/c^3)/c^3/(b*x^3+a)^(2/3)-7/4*d^3*x^4*(1+b*x^3/a)
^(2/3)*AppellF1(4/3,2/3,3,7/3,-b*x^3/a,-d^3*x^3/c^3)/c^6/(b*x^3+a)^(2/3)+1/
7*d^6*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,3,10/3,-b*x^3/a,-d^3*x^3/c^3)/
c^9/(b*x^3+a)^(2/3)-5/9*b^2*c^4*d*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)+1/18
*b*c*d*(-6*a*d^3+b*c^3)*ln(d^3*x^3+c^3)/(-a*d^3+b*c^3)^(8/3)-1/9*a*d^4*(-a*
d^3+6*b*c^3)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)-1/18*d*(2*a^2*d^6-6*a
*b*c^3*d^3+9*b^2*c^6)*ln(d^3*x^3+c^3)/c^2/(-a*d^3+b*c^3)^(8/3)+1/3*a*d^4*(-
a*d^3+6*b*c^3)*ln((-a*d^3+b*c^3)^(1/3)*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c
^3)^(8/3)+1/6*d*(2*a^2*d^6-6*a*b*c^3*d^3+9*b^2*c^6)*ln((-a*d^3+b*c^3)^(1/3)
*x/c-(b*x^3+a)^(1/3))/c^2/(-a*d^3+b*c^3)^(8/3)+5/3*b^2*c^4*d*ln((-a*d^3+b*c
^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)-1/6*b*c*d*(-6*a*d^3+b*c^3
)*ln((-a*d^3+b*c^3)^(1/3)+d*(b*x^3+a)^(1/3))/(-a*d^3+b*c^3)^(8/3)+2/9*a*d^4
*(-a*d^3+6*b*c^3)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))
*3^(1/2))/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*d*(2*a^2*d^6-6*a*b*c^3*d^3+9
*b^2*c^6)*arctan(1/3*(1+2*(-a*d^3+b*c^3)^(1/3)*x/c/(b*x^3+a)^(1/3))*3^(1/2)
)/c^2/(-a*d^3+b*c^3)^(8/3)*3^(1/2)-10/9*b^2*c^4*d*arctan(1/3*(1-2*d*(b*x^3+
a)^(1/3)/(-a*d^3+b*c^3)^(1/3))*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)+1/9*b*
c*d*(-6*a*d^3+b*c^3)*arctan(1/3*(1-2*d*(b*x^3+a)^(1/3)/(-a*d^3+b*c^3)^(1/3)
)*3^(1/2))/(-a*d^3+b*c^3)^(8/3)*3^(1/2)
```

Rubi [A]

time = 1.18, antiderivative size = 1357, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 18, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.947$, Rules used = {2181, 441, 440, 483, 593, 12, 503, 455, 44, 60, 631, 210, 31, 525, 524, 482, 457, 79}

Antiderivative was successfully verified.

```
[In] Int[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
```

```
[Out] (3*c^4*d^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (3*c^
2*d^4*x^2*(a + b*x^3)^(1/3))/(2*(b*c^3 - a*d^3)*(c^3 + d^3*x^3)^2) + (5*b*c
^4*d^2*(a + b*x^3)^(1/3))/(3*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) - (c*d^2*(b
*c^3 - 6*a*d^3)*(a + b*x^3)^(1/3))/(6*(b*c^3 - a*d^3)^2*(c^3 + d^3*x^3)) +
(d^4*(9*b*c^3 - 4*a*d^3)*x^2*(a + b*x^3)^(1/3))/(6*c*(b*c^3 - a*d^3)^2*(c^3
 + d^3*x^3)) + (d^4*(3*b*c^3 + 2*a*d^3)*x^2*(a + b*x^3)^(1/3))/(3*c*(b*c^3
- a*d^3)^2*(c^3 + d^3*x^3)) + (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3
, 4/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(c^3*(a + b*x^3)^(2/3)) - (7*d^3*x^
4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 3, 7/3, -((b*x^3)/a), -((d^3*x^3
)/c^3)])/(4*c^6*(a + b*x^3)^(2/3)) + (d^6*x^7*(1 + (b*x^3)/a)^(2/3)*AppellF
1[7/3, 2/3, 3, 10/3, -((b*x^3)/a), -((d^3*x^3)/c^3)])/(7*c^9*(a + b*x^3)^(2
/3)) + (2*a*d^4*(6*b*c^3 - a*d^3)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3)*x)/(
c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*
(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*ArcTan[(1 + (2*(b*c^3 - a*d^3)^(1/3
)*x)/(c*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^2*(b*c^3 - a*d^3)^(8/3))
 - (10*b^2*c^4*d*ArcTan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))
/Sqrt[3]])/(3*Sqrt[3]*(b*c^3 - a*d^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Arc
Tan[(1 - (2*d*(a + b*x^3)^(1/3))/(b*c^3 - a*d^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3
]*(b*c^3 - a*d^3)^(8/3)) - (5*b^2*c^4*d*Log[c^3 + d^3*x^3])/(9*(b*c^3 - a*d
^3)^(8/3)) + (b*c*d*(b*c^3 - 6*a*d^3)*Log[c^3 + d^3*x^3])/(18*(b*c^3 - a*d^
3)^(8/3)) - (a*d^4*(6*b*c^3 - a*d^3)*Log[c^3 + d^3*x^3])/(9*c^2*(b*c^3 - a*
d^3)^(8/3)) - (d*(9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[c^3 + d^3*x^3]
)/(18*c^2*(b*c^3 - a*d^3)^(8/3)) + (a*d^4*(6*b*c^3 - a*d^3)*Log[((b*c^3 - a
*d^3)^(1/3)*x)/c - (a + b*x^3)^(1/3)])/(3*c^2*(b*c^3 - a*d^3)^(8/3)) + (d*(
9*b^2*c^6 - 6*a*b*c^3*d^3 + 2*a^2*d^6)*Log[((b*c^3 - a*d^3)^(1/3)*x)/c - (a
 + b*x^3)^(1/3)])/(6*c^2*(b*c^3 - a*d^3)^(8/3)) + (5*b^2*c^4*d*Log[(b*c^3 -
 a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(3*(b*c^3 - a*d^3)^(8/3)) - (b*c*d*(b
*c^3 - 6*a*d^3)*Log[(b*c^3 - a*d^3)^(1/3) + d*(a + b*x^3)^(1/3)])/(6*(b*c^3
 - a*d^3)^(8/3))
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 44</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
```

```
1, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 483

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x
^n)^(q + 1)/(a*e*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(p +
 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b
*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] &&
 IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 593

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*g*n*(b*c - a*d)*(p + 1)))
, x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c
 + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e -
 a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2181

```
Int[(Px_.)*((c_) + (d_.)*(x_))^(q_)*((a_) + (b_.)*(x_)^3)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(c^3 + d^3*x^3)^q*(a + b*x^3)^p, Px/(c^2 - c*d*x + d
^2*x^2)^q, x], x] /; FreeQ[{a, b, c, d, p}, x] && PolyQ[Px, x] && ILtQ[q, 0
] && RationalQ[p] && EqQ[Denominator[p], 3]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3\,(a+bx^3)^{2/3}}\,dx = \int \frac{1}{(c+dx)^3\,(a+bx^3)^{2/3}}\,dx$$

Mathematica [F]
time = 20.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3\,(a+bx^3)^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)),x]
[Out] Integrate[1/((c + d*x)^3*(a + b*x^3)^(2/3)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx+c\right)^{3} \left(b\,x^{3}+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
[Out] int(1/(d*x+c)^3/(b*x^3+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^{3}\right)^{\frac{2}{3}} \left(c+dx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3/(b*x**3+a)**(2/3),x)
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3/(b*x^3+a)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x + c)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,{\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3),x)
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x)^3), x)
```

3.43 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=37

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2162, 209}

$$\frac{2\ 2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{1+x^3}}\right)$$

$$= \frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 1.25, size = 39, normalized size = 1.05

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{1+x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.33, size = 258, normalized size = 6.97

method	result
trager	$-\frac{\text{RootOf}(\_Z^2+6\,2^{\frac{1}{3}}) \ln (\frac{12x\sqrt{x^3+1}+3\,\text{RootOf}(\_Z^2+6\,2^{\frac{1}{3}})\,2^{\frac{2}{3}}x^2-\text{RootOf}(\_Z^2+6\,2^{\frac{1}{3}})\,x^3+6\sqrt{x^3+1}\,2^{\frac{2}{3}}+6\,\text{RootOf}(\_Z}{(2^{\frac{1}{3}}x+2)^3}}{3}$
default	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}(\frac{3}{2}-}{}$
elliptic	$-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{6\,2^{\frac{2}{3}}(\frac{3}{2}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((
1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2))
)^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2
^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))
/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(27) = 54$.
time = 0.44, size = 75, normalized size = 2.03

$$\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}(7\,x^4+4\,x)-2^{\frac{1}{3}}(5\,x^3+2)\right)\sqrt{x^3+1}}{12\,(2\,x^6+3\,x^3+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(
7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\right)dx-\int\frac{2x}{x\sqrt{x^3+1}+2^{\frac{2}{3}}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Inte
gral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [B]
time = 3.59, size = 70, normalized size = 1.89

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,1\mathrm{i}+\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)\,{\left(\sqrt{3}\,1\mathrm{i}-\sqrt{x^3+1}+2^{1/3}\,\sqrt{3}\,x\,1\mathrm{i}\right)}^3}{{\left(x+2^{2/3}\right)}^6}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*
1i)/3
```

3.44 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2162, 209}

$$-\frac{2\ 2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\left(2\ 2^{2/3}\right)\operatorname{Subst}\left(\int \frac{1}{1+3x^2}\,dx, x, \frac{1-\sqrt[3]{2}\,x}{\sqrt{1-x^3}}\right)\right)$$

$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 1.28, size = 41, normalized size = 1.02

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.30, size = 253, normalized size = 6.32

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3+1}\,x-3\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)2^{\frac{2}{3}}x^2-\operatorname{RootOf}\left(\_Z^2+6\,2^{\frac{1}{3}}\right)x^3-6\sqrt{-x^3+1}\,2^{\frac{2}{3}}+6\operatorname{RootOf}\left(\_\right.}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$

elliptic	$\frac{4i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/
2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(
2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.
time = 0.44, size = 76, normalized size = 1.90

$$-\frac{1}{3}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{\sqrt{6}\,2^{\frac{1}{6}}\left(2\,x^{5}-2\,x^{2}+2^{\frac{2}{3}}(7\,x^{4}-4\,x)-2^{\frac{1}{3}}(5\,x^{3}-2)\right)\sqrt{-x^{3}+1}}{12\,(2\,x^{6}-3\,x^{3}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3} - 2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx - \int \frac{2x}{x\sqrt{1-x^3} - 2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integ
ral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [B]
time = 3.63, size = 74, normalized size = 1.85

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-\sqrt{3}\,1\mathrm{i}+2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)\left(\sqrt{3}\,1\mathrm{i}+\sqrt{1-x^3}-2^{1/3}\sqrt{3}\,x\,1\mathrm{i}\right)^3}{\left(x-2^{2/3}\right)^6}\right)\,1\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i)
*(3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6)*
1i)/3
```

3.45 $\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=38

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

```
[Out] -2/3*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2162, 212}

$$-\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\left((2\ 2^{2/3})\,\mathrm{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1-\sqrt[3]{2}\,x}{\sqrt{-1+x^3}}\right)\right)$$

$$= -\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 1.26, size = 39, normalized size = 1.03

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-1+x^3}}{\sqrt{3}\left(-1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.36, size = 262, normalized size = 6.89

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(-\frac{3\,2^{\frac{2}{3}}x^2\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)-12\sqrt{x^3-1}\,x+\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3-6\,\mathrm{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)2^{\frac{1}{3}}x+6}{\left(2^{\frac{1}{3}}x-2\right)^3}\right.}{3}$
default	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{6\,2^{\frac{2}{3}}}{}$
elliptic	$-\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{6\,2^{\frac{2}{3}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-6*2^(2/3)*(-3/2-1/2*I*3^(1/2))*
((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/
(-2^(2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1
/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(28) = 56$.
time = 0.43, size = 238, normalized size = 6.26

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^{9}-10368\,x^{6}+15360\,x^{3}+2\sqrt{6}\,2^{\frac{1}{6}}\left(126\,x^{14}+2664\,x^{11}-4608\,x^{5}+2304\,x^{2}+2^{\frac{2}{3}}(x^{16}+310\,x^{13}+2332\,x^{10}-2656\,x^{7}-256\,x^{4}+512\,x)+2^{\frac{1}{3}}(17\,x^{15}+1058\,x^{12}+2528\,x^{9}-5408\,x^{6}+2560\,x^{3}-512)\right)\sqrt{x^{3}-1}+24\cdot 2^{\frac{2}{3}}(x^{17}+121\,x^{14}+478\,x^{11}-1144\,x^{8}+608\,x^{5}-64\,x^{2})+48\cdot 2^{\frac{1}{3}}(5\,x^{16}+176\,x^{13}+83\,x^{10}-680\,x^{7}+544\,x^{4}-128\,x)-2048}{x^{18}-24\,x^{15}+240\,x^{12}-1280\,x^{9}+3840\,x^{6}-6144\,x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*
x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) +
 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt
(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 6
4*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*
x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2^{\frac{2}{3}}}{x\sqrt{x^{3}-1}-2^{\frac{2}{3}}\sqrt{x^{3}-1}}\,dx-\int\frac{2x}{x\sqrt{x^{3}-1}-2^{\frac{2}{3}}\sqrt{x^{3}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integ
ral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [B]
time = 2.86, size = 62, normalized size = 1.63

$$\frac{2^{2/3}\,\sqrt{3}\,\ln\left(\frac{\left(\sqrt{x^3-1}-\sqrt{3}+2^{1/3}\,\sqrt{3}\,x\right)^3\,\left(\sqrt{3}+\sqrt{x^3-1}-2^{1/3}\,\sqrt{3}\,x\right)}{{\left(x-2^{2/3}\right)}^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + 2^(2/3))/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^
(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3
```

3.46 $\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=39

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

```
[Out] 2/3*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2162, 212}

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx = \left(2\ 2^{2/3}\right)\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{1+\sqrt[3]{2}\,x}{\sqrt{-1-x^3}}\right)$$

$$= \frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 1.25, size = 41, normalized size = 1.05

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-1-x^3}}{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 1.32, size = 249, normalized size = 6.38

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)\ln\left(\frac{12\sqrt{-x^3-1}\,x-3\,2^{\frac{2}{3}}x^2\,\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)+\text{RootOf}\left(\_Z^2-6\,2^{\frac{1}{3}}\right)x^3+6\sqrt{-x^3-1}\,2^{\frac{2}{3}}-6\,\text{RootOf}}{\left(2^{\frac{1}{3}}x+2\right)^3}\right.}{3}$
default	$\frac{4i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\frac{1}{2}\right.}}{3}\right.}{3\sqrt{-x^3-1}}$

elliptic	$\frac{4i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^{3}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)),(
I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(29) = 58$.
time = 0.41, size = 241, normalized size = 6.18

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^{9}-10368\,x^{6}-15360\,x^{3}-2\sqrt{6}\,2^{\frac{1}{6}}\left(126\,x^{14}-2664\,x^{11}+4608\,x^{5}+2304\,x^{2}+2^{\frac{2}{3}}(x^{16}-310\,x^{13}+2332\,x^{10}+2656\,x^{7}-256\,x^{4}-512\,x)-2^{\frac{1}{3}}(17\,x^{15}-1058\,x^{12}+2528\,x^{9}+5408\,x^{6}+2560\,x^{3}+512)\right)\sqrt{-x^{3}-1}-24\cdot 2^{\frac{2}{3}}(x^{17}-121\,x^{14}+478\,x^{11}+1144\,x^{8}+608\,x^{5}+64\,x^{2})+48\cdot 2^{\frac{1}{3}}(5\,x^{16}-176\,x^{13}+83\,x^{10}+680\,x^{7}+544\,x^{4}+128\,x)-2048}{x^{18}+24\,x^{15}+240\,x^{12}+1280\,x^{9}+3840\,x^{6}+6144\,x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^5 + 2304
*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
```

```
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\right)dx-\int\frac{2x}{x\sqrt{-x^3-1}+2^{\frac{2}{3}}\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - In
tegral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [B]
time = 2.84, size = 63, normalized size = 1.62

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}+\sqrt{-x^3-1}+2^{1/3}\sqrt{3}\,x\right)^3\left(\sqrt{3}-\sqrt{-x^3-1}+2^{1/3}\sqrt{3}\,x\right)}{\left(x+2^{2/3}\right)^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(
3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3
```

3.47 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=63

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1
/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2162, 209}

$$\frac{2\ 2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a +
 b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx=\frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A]
time = 5.14, size = 65, normalized size = 1.03

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(1
/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{2^{\frac{2}{3}}a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/
2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a+bx^3}+\sqrt[3]{b}\,x\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)

[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

Mupad [B]
time = 5.81, size = 106, normalized size = 1.68

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}-\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)^3\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{b\,x^3+a}+2^{1/3}\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*a^(1/2)*1i - (a + b*x^3)^(1/2) + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)^3*(3^(1/2)*a^(1/2)*1i + (a + b*x^3)^(1/2) + 2^(1
/3)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

3.48 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\ \sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\ \sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\ \sqrt[6]{a}\ \sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2162, 209}

$$-\frac{2\ 2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\ \sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\ \sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\ \sqrt[6]{a}\ \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a -
 b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a -
 b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A]
time = 5.12, size = 67, normalized size = 1.03

$$\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{a-bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}\,x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1/
3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{2^{\frac{2}{3}}a^{\frac{1}{3}}+2b^{\frac{1}{3}}x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1
/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2
^(2/3)*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx-\int \frac{2\sqrt[3]{b}\,x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)
*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a
 - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3
+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.85, size = 107, normalized size = 1.65

$$\frac{2^{2/3}\sqrt{3}\,\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)\,{\left(\sqrt{3}\,\sqrt{a}\,1\mathrm{i}+\sqrt{a-b\,x^3}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\,1\mathrm{i}\right)}^3}{{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}^6}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (2^(2/3)*3^(1/2)*log((((a - b*x^3)^(1/2) - 3^(1/2)*a^(1/2)*1i + 2^(1/3)*3^(
1/2)*a^(1/6)*b^(1/3)*x*1i)*(3^(1/2)*a^(1/2)*1i + (a - b*x^3)^(1/2) - 2^(1/3
)*3^(1/2)*a^(1/6)*b^(1/3)*x*1i)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6)*1i)/(3*
a^(1/6)*b^(1/3))
```

3.49 $\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2162, 212}

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a
+ b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\left(2\,2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A]
time = 5.13, size = 68, normalized size = 1.03

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-a+bx^3}}{\sqrt{3}\left(\sqrt{a}-\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}\,x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sq
rt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^
(1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{2^{\frac{2}{3}} a^{\frac{1}{3}}+2 b^{\frac{1}{3}} x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/
2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^
(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}\sqrt[3]{a}}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx-\int \frac{2\sqrt[3]{b}\,x}{-2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(
b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3
)*x*sqrt(-a + b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt
(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-
a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 3.68, size = 102, normalized size = 1.55

$$\frac{\sqrt{3}\,4^{1/3}\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{3}\,\sqrt{a}-2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)\,{\left(\sqrt{b\,x^3-a}-\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)}^3}{{\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b
^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((b*x^3 - a)^(1/2) + 3^(1/2)*a^(1/2) - 2^(1/3)*3^(1/2
)*a^(1/6)*b^(1/3)*x)*((b*x^3 - a)^(1/2) - 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/2)
*a^(1/6)*b^(1/3)*x)^3)/(2^(2/3)*a^(1/3) - b^(1/3)*x)^6))/(3*a^(1/6)*b^(1/3)
)
```

3.50 $\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\ \sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\ \sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\ \sqrt[6]{a}\ \sqrt[3]{b}}$$

```
[Out] 2/3*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)^
(1/2))/a^(1/6)/b^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2162, 212}

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\ \sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\ \sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\ \sqrt[6]{a}\ \sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a
- b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
- b*x^3]])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx=\frac{\left(2\ 2^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-3ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A]
time = 5.13, size = 68, normalized size = 1.03

$$\frac{2\ 2^{2/3}\tanh^{-1}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt{a}+\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sq
rt[-a - b*x^3]),x]
```

```
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(
1/3)*x))])/(Sqrt[3]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{2^{\frac{2}{3}}a^{\frac{1}{3}}-2b^{\frac{1}{3}}x}{\left(2^{\frac{2}{3}}a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

```
[Out] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1
/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2
^(2/3)*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{a}}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\right)dx-\int\frac{2\sqrt[3]{b}\,x}{2^{\frac{2}{3}}\sqrt[3]{a}\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(
-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3
)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(
-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3
-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.65, size = 103, normalized size = 1.56

$$\frac{\sqrt{3}\,4^{1/3}\ln\left(\frac{\left(\sqrt{-b\,x^3-a}+\sqrt{3}\,\sqrt{a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)^3\left(\sqrt{3}\,\sqrt{a}-\sqrt{-b\,x^3-a}+2^{1/3}\,\sqrt{3}\,a^{1/6}\,b^{1/3}\,x\right)}{\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)^6}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) +
 b^(1/3)*x)),x)
```

```
[Out] (3^(1/2)*4^(1/3)*log((((- a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1
/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (- a - b*x^3)^(1/2) + 2^(1/3)*3
^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6))/(3*a^(1/6)*b^(
1/3))
```

3.51 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=49

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\,\sqrt{c}\,d}$$

```
[Out] 2/3*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d*3^(1/2)/c^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2162, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\,\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sq
rt[c]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{1+3c^3x^2}\,dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}}\right)}{d}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{\sqrt{3}\sqrt{c}\,d}$$

Mathematica [A]
time = 1.02, size = 51, normalized size = 1.04

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{c^3+4d^3x^3}}{\sqrt{3}\sqrt{c}\,(c+2dx)}\right)}{\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*Sqrt[c]*(c + 2*d*x))])/(Sqrt[3]*S
qrt[c]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.25, size = 889, normalized size = 18.14

method	result	size
default	Expression too large to display	889
elliptic	Expression too large to display	889

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1
/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*
3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1/
2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(
1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/
2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3+
c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(
1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2
^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/
2))+6*c/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/
```

```
2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)
-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)
*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*
c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*
I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d
^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*EllipticPi(
((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^
(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(
1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*
c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^
(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(39) = 78.
time = 0.44, size = 300, normalized size = 6.12

$$\left[\frac{\sqrt{3}\sqrt{-\frac{1}{c}}\log\left(\frac{2d^6x^6-36cd^5x^5-18c^2d^4x^4+28c^3d^3x^3+18c^4d^2x^2-c^6-\sqrt{3}\,(4cd^4x^4-10c^2d^3x^3-18c^3d^2x^2-8c^4dx-c^5)\sqrt{4d^3x^3+c^3}\sqrt{-\frac{1}{c}}}{d^6x^6+6cd^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}\right)}{6d}, -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{4d^3x^3+c^3}\,(2d^3x^3-6cd^2x^2-6c^2dx-c^3)}{3\,(8d^4x^4+4cd^3x^3+2c^3dx+c^4)\sqrt{c}}\right)}{3\sqrt{c}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28
*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3
 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5)*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6
*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c
^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*
d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*
d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{c}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3+4d^3x^3}+dx\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**
3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B]
time = 4.73, size = 95, normalized size = 1.94

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(-\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)^3\left(\sqrt{c^3+4\,d^3\,x^3}+\sqrt{3}\,c^{3/2}\,1\mathrm{i}+\sqrt{3}\,\sqrt{c}\,d\,x\,2\mathrm{i}\right)}{(c+d\,x)^6}\right)\,1\mathrm{i}}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1/
2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1/2
)*d*x*2i))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)
```

3.52 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=158

$$\frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
3+2*2^(1/3))*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\left(2-3\ 2^{2/3}\right)\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx + \frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\left(2-3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\ldots}}{3\sqrt[4]{3}\sqrt{\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.30, size = 336, normalized size = 2.13

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-4\sqrt{3}\left(-3+\sqrt[3]{2}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(125) = 250$.
time = 1.18, size = 262, normalized size = 1.66

method	result
default	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\cdots}\right.}{\cdots}$
elliptic	$\frac{6\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(2-3\,2^{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 6*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(2-3*2^(2/3))*(3/2-1/2*I*3^(1/2
))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.19, size = 157, normalized size = 0.99

$$\frac{1}{9}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18x^5-42x^4-10x^3+18x^2+2^{\frac{2}{3}}(2x^5-63x^4-15x^3+2x^2-36x-6)+2^{\frac{1}{3}}(6x^5-14x^4-45x^3+6x^2-8x-18)-24x-4\right)\sqrt{x^3+1}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}}{300\,(2x^6+3x^3+1)}\right)+\frac{2}{3}\left(2\cdot 2^{\frac{1}{3}}+3\right)\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/300*sqrt(3)*(18*x^5
 - 42*x^4 - 10*x^3 + 18*x^2 + 2^(2/3)*(2*x^5 - 63*x^4 - 15*x^3 + 2*x^2 - 36
*x - 6) + 2^(1/3)*(6*x^5 - 14*x^4 - 45*x^3 + 6*x^2 - 8*x - 18) - 24*x - 4)*
sqrt(x^3 + 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 + 3*x^3 + 1)) + 2/3
*(2*2^(1/3) + 3)*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x+2}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
```

```
[Out] int((3*x + 2)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.53 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=173

$$-\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\left(2+3\ 2^{2/3}\right)\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\left(2+3\ 2^{2/3}\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\cdots}}{3\sqrt[4]{3}\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.31, size = 335, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A]
time = 2.01, size = 257, normalized size = 1.49

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{\sqrt{-x^{3}+1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*(-2-3*2^(2/3))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1
/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [A]
time = 0.17, size = 146, normalized size = 0.84

$$-\frac{1}{9}\sqrt{3}\sqrt{12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\arctan\left(\frac{\sqrt{3}\left(18x^5-42x^4-10x^3-18x^2+2^{\frac{2}{3}}(2x^5+63x^4+15x^3-2x^2-36x-6)-2^{\frac{1}{3}}(6x^5-14x^4+45x^3-6x^2+8x-18)+24x+4\right)\sqrt{-x^3+1}\sqrt{12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}}{348\,(2x^6-3x^3+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)*arctan(1/348*sqrt(3)*(18*x^5
 - 42*x^4 - 10*x^3 - 18*x^2 + 2^(2/3)*(2*x^5 + 63*x^4 + 15*x^3 - 2*x^2 - 36
*x - 6) - 2^(1/3)*(6*x^5 - 14*x^4 + 45*x^3 - 6*x^2 + 8*x - 18) + 24*x + 4)*
sqrt(-x^3 + 1)*sqrt(12*2^(2/3) + 18*2^(1/3) + 4)/(2*x^6 - 3*x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{3x}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int\frac{2}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(2
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,x+2}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
[Out] int(-(3*x + 2)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.54 $\int \frac{2+3x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=176

$$-\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*(2+3*2^(2/3))*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9
*(3-2*2^(1/3))*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(2 + 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) + (2*(3 - 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(3-2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\left(3+\sqrt[3]{2}\right)\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x}}$$

$$= \frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\right|}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\left(2+3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3-2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1-x}{3\sqrt[4]{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.26, size = 333, normalized size = 1.89

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3i\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+4\sqrt{3}\left(3+\sqrt[3]{2}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[3]*(3 +
 2^(1/3))*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[
3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) +
 Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A]
time = 1.98, size = 266, normalized size = 1.51

method	result
default	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(-2}{}$
elliptic	$-\frac{6\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(-2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-2-3*2^(2/3))*(-3/2-1/2*I*3^
(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)
^(1/2)/(-2^(2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2
*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((3*x + 2)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{3x}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\, dx - \int \frac{2}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(3*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(2
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{3\,x+2}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(3*x + 2)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.55 $\int \frac{2+3x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=169

$$\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\cdots}{1-\cdots}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*(2-3*2^(2/3))*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(3+2*2^(1/3))*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(2 - 3*2^(2/3))*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*(3 + 2*2^(1/3))*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{2+3x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\left(-3+\sqrt[3]{2}\right)\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx + \frac{1}{3}\left(3+2\sqrt[3]{2}\right)\int \frac{1}{\sqrt{-1-x^3}}\,d$$

$$= \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right|}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\left(2-3\ 2^{2/3}\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\left(3+2\sqrt[3]{2}\right)\sqrt{2-\sqrt{3}}\,(1+x)}{3\sqrt[4]{3}\sqrt{}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 20.20, size = 338, normalized size = 2.00

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-4\sqrt{3}\left(-3+\sqrt[3]{2}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 4*Sqrt[3]*(-3 + 2^(1/3
))*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I
+ (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(
1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqrt[3
])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A]
time = 1.94, size = 253, normalized size = 1.50

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{\sqrt{-x^{3}-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{\sqrt{-x^{3}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1
/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*
I*3^(1/2)))^(1/2))-2/3*I*(2-3*2^(2/3))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(
1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/
2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(
1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(130) = 260$.
time = 0.18, size = 415, normalized size = 2.46

$$\frac{1}{18}\sqrt{3}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}\,\log\left(\frac{25x^{18}-36000x^{15}+435000x^{12}+526400x^{9}-259200x^{6}-384000x^{3}+2\sqrt{3}\left(6x^{16}-34x^{15}+1134x^{14}-1860x^{13}+2116x^{12}-23976x^{11}+13992x^{10}-5056x^{9}+15936x^{7}-10816x^{6}+41472x^{5}-1536x^{4}-5120x^{3}+20736x^{2}+3\cdot 2^{\frac{2}{3}}\left(3x^{16}-17x^{15}+42x^{14}-930x^{13}+1058x^{12}-888x^{11}+6996x^{10}-2528x^{9}+7968x^{7}-5408x^{6}+1536x^{5}-768x^{4}-2560x^{3}+768x^{2}-1536x-512\right)+2^{\frac{1}{3}}\left(2x^{16}-153x^{15}+378x^{14}-620x^{13}+9522x^{12}-7992x^{11}+4664x^{10}-22752x^{9}+5312x^{7}-48672x^{6}+13824x^{5}-512x^{4}-23040x^{3}+6912x^{2}-1024x-4608\right)-3072x-1024\right)\sqrt{-x^{3}-1}\sqrt{-12\cdot 2^{\frac{2}{3}}+18\cdot 2^{\frac{1}{3}}+4}-600\cdot 2^{\frac{2}{3}}\left(x^{17}-121x^{14}+478x^{11}+1144x^{8}+608x^{5}+64x^{2}\right)+1200\cdot 2^{\frac{1}{3}}\left(5x^{16}-176x^{13}+83x^{10}+680x^{7}+544x^{4}+128x\right)-51200}{x^{18}+24x^{15}+240x^{12}+1280x^{9}+3840x^{6}+6144x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4)*log((25*x^18 - 36000*x^15 +
 435000*x^12 + 526400*x^9 - 259200*x^6 - 384000*x^3 + 2*sqrt(3)*(6*x^16 - 3
4*x^15 + 1134*x^14 - 1860*x^13 + 2116*x^12 - 23976*x^11 + 13992*x^10 - 5056
*x^9 + 15936*x^7 - 10816*x^6 + 41472*x^5 - 1536*x^4 - 5120*x^3 + 20736*x^2
+ 3*2^(2/3)*(3*x^16 - 17*x^15 + 42*x^14 - 930*x^13 + 1058*x^12 - 888*x^11 +
 6996*x^10 - 2528*x^9 + 7968*x^7 - 5408*x^6 + 1536*x^5 - 768*x^4 - 2560*x^3
 + 768*x^2 - 1536*x - 512) + 2^(1/3)*(2*x^16 - 153*x^15 + 378*x^14 - 620*x^
13 + 9522*x^12 - 7992*x^11 + 4664*x^10 - 22752*x^9 + 5312*x^7 - 48672*x^6 +
 13824*x^5 - 512*x^4 - 23040*x^3 + 6912*x^2 - 1024*x - 4608) - 3072*x - 102
4)*sqrt(-x^3 - 1)*sqrt(-12*2^(2/3) + 18*2^(1/3) + 4) - 600*2^(2/3)*(x^17 -
121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 + 64*x^2) + 1200*2^(1/3)*(5*x^16 -
 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128*x) - 51200)/(x^18 + 24*x^15 +
 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4096))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x+2}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((3*x + 2)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((3*x + 2)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x+2}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((3*x + 2)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.56 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=159

$$\frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(
2^(1/3)*e+f)*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/
2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1
/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}e+f\right)F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}x+1\right)}{\sqrt{x^3+1}}\right)\left(e-2^{2/3}f\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt
[3]) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 +
 Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7
- 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}\,e-2f\right)\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx+\frac{1}{3}\left(\sqrt[3]{2}\,e+f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\left(e-2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,e+f\right)(1+x)\sqrt{\frac{}{(1}}}{3\sqrt[4]{3}\sqrt{\frac{}{(1}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.31, size = 340, normalized size = 2.14

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}\left(\sqrt[3]{2}\,e-2f\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(126) = 252$.
time = 0.29, size = 264, normalized size = 1.66

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}\right.}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-2^{\frac{2}{3}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(e-2^(2/3)*f)*(3/2-1/2*I*3^(1
/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3
^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(
1/2)/(2^(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.26, size = 949, normalized size = 5.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(2*2^(2/3)*f*e - 2*2^(1/3)*f^2 - e^2)*log(-(4*f^3*x^18 -
5760*f^3*x^15 + 69600*f^3*x^12 + 84224*f^3*x^9 - 41472*f^3*x^6 - 61440*f^3*
x^3 - 8192*f^3 + 4*sqrt(3)*(252*f^2*x^14 - 5328*f^2*x^11 + 9216*f^2*x^5 + 4
608*f^2*x^2 - (17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512)*
e^2 + 2*(f*x^16 - 310*f*x^13 + 2332*f*x^10 + 2656*f*x^7 - 256*f*x^4 - 512*f
*x)*e + 2^(2/3)*(2*f^2*x^16 - 620*f^2*x^13 + 4664*f^2*x^10 + 5312*f^2*x^7 -
 512*f^2*x^4 - 1024*f^2*x + 9*(7*x^14 - 148*x^11 + 256*x^5 + 128*x^2)*e^2 -
 (17*f*x^15 - 1058*f*x^12 + 2528*f*x^9 + 5408*f*x^6 + 2560*f*x^3 + 512*f)*e
) - 2^(1/3)*(34*f^2*x^15 - 2116*f^2*x^12 + 5056*f^2*x^9 + 10816*f^2*x^6 + 5
120*f^2*x^3 + 1024*f^2 - (x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4
- 512*x)*e^2 - 18*(7*f*x^14 - 148*f*x^11 + 256*f*x^5 + 128*f*x^2)*e))*sqrt(
x^3 + 1)*sqrt(2*2^(2/3)*f*e - 2*2^(1/3)*f^2 - e^2) - (x^18 - 1440*x^15 + 17
400*x^12 + 21056*x^9 - 10368*x^6 - 15360*x^3 - 2048)*e^3 - 24*2^(2/3)*(4*f^
3*x^17 - 484*f^3*x^14 + 1912*f^3*x^11 + 4576*f^3*x^8 + 2432*f^3*x^5 + 256*f
^3*x^2 - (x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 + 64*x^2)*e^3) +
48*2^(1/3)*(20*f^3*x^16 - 704*f^3*x^13 + 332*f^3*x^10 + 2720*f^3*x^7 + 2176
*f^3*x^4 + 512*f^3*x - (5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 1
28*x)*e^3))/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096)) + 2/3*(2^(1/3)*e + f)*weierstrassPInverse(0, -4, x), -1/9*sqrt(3)*sqr
t(-2*2^(2/3)*f*e + 2*2^(1/3)*f^2 + e^2)*arctan(-1/6*sqrt(3)*(4*f^2*x^5 + 4*
f^2*x^2 - (5*x^3 + 2)*e^2 - 2*(7*f*x^4 + 4*f*x)*e - 2^(2/3)*(14*f^2*x^4 + 8
*f^2*x - (x^5 + x^2)*e^2 + (5*f*x^3 + 2*f)*e) - 2^(1/3)*(10*f^2*x^3 + 4*f^2
 + (7*x^4 + 4*x)*e^2 - 2*(f*x^5 + f*x^2)*e))*sqrt(x^3 + 1)*sqrt(-2*2^(2/3)*
f*e + 2*2^(1/3)*f^2 + e^2)/(8*f^3*x^6 + 12*f^3*x^3 + 4*f^3 - (2*x^6 + 3*x^3
 + 1)*e^3)) + 2/3*(2^(1/3)*e + f)*weierstrassPInverse(0, -4, x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{(x + 1)(x^2 - x + 1)}\left(x + 2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e+f\,x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.57 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\cdots}{1+\cdots}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(
1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)
^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2164, 224, 2162, 209}

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}\,e-f\right)F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)\left(e+2^{2/3}f\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqr
t[3]) - (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7
 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}\,e+f\right)\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}\,e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\right|}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\sqrt{}}{3\sqrt[4]{3}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.31, size = 340, normalized size = 1.94

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt[3]{2}e+2f\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A]
time = 0.26, size = 261, normalized size = 1.49

method	result
default	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\cdots\right.}}{\cdots}\right.\cdots}{3\sqrt{-x^{3}+1}}$
elliptic	$\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\cdots\right.}}{\cdots}\right.\cdots}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-2^(2/3)*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(
1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*
3^(1/2)-2^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [A]
time = 0.30, size = 917, normalized size = 5.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(-2*2^(2/3)*f*e - 2*2^(1/3)*f^2 - e^2)*log((4*f^3*x^18 +
5760*f^3*x^15 + 69600*f^3*x^12 - 84224*f^3*x^9 - 41472*f^3*x^6 + 61440*f^3*
x^3 - 8192*f^3 + 4*sqrt(3)*(252*f^2*x^14 + 5328*f^2*x^11 - 9216*f^2*x^5 + 4
608*f^2*x^2 + (17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512)*
e^2 - 2*(f*x^16 + 310*f*x^13 + 2332*f*x^10 - 2656*f*x^7 - 256*f*x^4 + 512*f
*x)*e + 2^(2/3)*(2*f^2*x^16 + 620*f^2*x^13 + 4664*f^2*x^10 - 5312*f^2*x^7 -
 512*f^2*x^4 + 1024*f^2*x + 9*(7*x^14 + 148*x^11 - 256*x^5 + 128*x^2)*e^2 -
 (17*f*x^15 + 1058*f*x^12 + 2528*f*x^9 - 5408*f*x^6 + 2560*f*x^3 - 512*f)*e
) + 2^(1/3)*(34*f^2*x^15 + 2116*f^2*x^12 + 5056*f^2*x^9 - 10816*f^2*x^6 + 5
120*f^2*x^3 - 1024*f^2 + (x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4
+ 512*x)*e^2 - 18*(7*f*x^14 + 148*f*x^11 - 256*f*x^5 + 128*f*x^2)*e))*sqrt(
-x^3 + 1)*sqrt(-2*2^(2/3)*f*e - 2*2^(1/3)*f^2 - e^2) + (x^18 + 1440*x^15 +
17400*x^12 - 21056*x^9 - 10368*x^6 + 15360*x^3 - 2048)*e^3 + 24*2^(2/3)*(4*
f^3*x^17 + 484*f^3*x^14 + 1912*f^3*x^11 - 4576*f^3*x^8 + 2432*f^3*x^5 - 256
*f^3*x^2 + (x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 64*x^2)*e^3)
+ 48*2^(1/3)*(20*f^3*x^16 + 704*f^3*x^13 + 332*f^3*x^10 - 2720*f^3*x^7 + 21
76*f^3*x^4 - 512*f^3*x + (5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 -
 128*x)*e^3))/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 +
 4096)), -1/9*sqrt(3)*sqrt(2*2^(2/3)*f*e + 2*2^(1/3)*f^2 + e^2)*arctan(1/6*
sqrt(3)*(4*f^2*x^5 - 4*f^2*x^2 - (5*x^3 - 2)*e^2 - 2*(7*f*x^4 - 4*f*x)*e +
2^(2/3)*(14*f^2*x^4 - 8*f^2*x + (x^5 - x^2)*e^2 + (5*f*x^3 - 2*f)*e) - 2^(1
/3)*(10*f^2*x^3 - 4*f^2 - (7*x^4 - 4*x)*e^2 + 2*(f*x^5 - f*x^2)*e))*sqrt(-x
^3 + 1)*sqrt(2*2^(2/3)*f*e + 2*2^(1/3)*f^2 + e^2)/(8*f^3*x^6 - 12*f^3*x^3 +
 4*f^3 + (2*x^6 - 3*x^3 + 1)*e^3))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx-\int \frac{fx}{x\sqrt{1-x^3}-2^{\frac{2}{3}}\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(f*x
/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e+f\,x}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.58 $\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=178

$$-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*(e+2^(2/3)*f)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)-2/9
*(2^(1/3)*e-f)*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^
(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2164, 225, 2162, 212}

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}\,e-f\right)F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e + 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*S
qrt[3]) - (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e - f)*(1 - x)*Sqrt[(1 + x + x^2)/(
1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)],
-7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\left(-\sqrt[3]{2}\,e+f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{6}\left(\sqrt[3]{2}\,e+2f\right)\int \frac{2^{2/3}+2x}{\left(2^{2/3}-x\right)\sqrt{-1\cdots}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\cdots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\left(e+2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e-f\right)(1-x)\cdots}{3\sqrt[4]{3}\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.22, size = 338, normalized size = 1.90

$$\frac{2\sqrt[6]{2}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-if\sqrt{-i+\sqrt{3}-2ix}\left(-6i-3i\sqrt[3]{2}+2\sqrt{3}-\sqrt[3]{2}\sqrt{3}+\left(-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\sqrt{3}\left(\sqrt[3]{2}\,e+2f\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-I)*f*Sqrt[-I + Sqrt[3]
- (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^
(1/3) + 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] +
 (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqrt[3]*(2^(
1/3)*e + 2*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*S
qrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/
3) + Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.30, size = 270, normalized size = 1.52

method	result
default	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+-$ 2
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+-$ 2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-e-2^(2/3)*f)*(-3/2-1/2*I*
3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/
2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-
1)^(1/2)/(-2^(2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1
/2*I*3^(1/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.23, size = 947, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*sqrt(3)*sqrt(2*2^(2/3)*f*e + 2*2^(1/3)*f^2 + e^2)*log((4*f^3*x^18 + 5
760*f^3*x^15 + 69600*f^3*x^12 - 84224*f^3*x^9 - 41472*f^3*x^6 + 61440*f^3*x
^3 - 8192*f^3 + 4*sqrt(3)*(252*f^2*x^14 + 5328*f^2*x^11 - 9216*f^2*x^5 + 46
08*f^2*x^2 + (17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512)*e
^2 - 2*(f*x^16 + 310*f*x^13 + 2332*f*x^10 - 2656*f*x^7 - 256*f*x^4 + 512*f*
x)*e + 2^(2/3)*(2*f^2*x^16 + 620*f^2*x^13 + 4664*f^2*x^10 - 5312*f^2*x^7 -
512*f^2*x^4 + 1024*f^2*x + 9*(7*x^14 + 148*x^11 - 256*x^5 + 128*x^2)*e^2 -
(17*f*x^15 + 1058*f*x^12 + 2528*f*x^9 - 5408*f*x^6 + 2560*f*x^3 - 512*f)*e)
 + 2^(1/3)*(34*f^2*x^15 + 2116*f^2*x^12 + 5056*f^2*x^9 - 10816*f^2*x^6 + 51
20*f^2*x^3 - 1024*f^2 + (x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 +
 512*x)*e^2 - 18*(7*f*x^14 + 148*f*x^11 - 256*f*x^5 + 128*f*x^2)*e))*sqrt(x
^3 - 1)*sqrt(2*2^(2/3)*f*e + 2*2^(1/3)*f^2 + e^2) + (x^18 + 1440*x^15 + 174
00*x^12 - 21056*x^9 - 10368*x^6 + 15360*x^3 - 2048)*e^3 + 24*2^(2/3)*(4*f^3
*x^17 + 484*f^3*x^14 + 1912*f^3*x^11 - 4576*f^3*x^8 + 2432*f^3*x^5 - 256*f^
3*x^2 + (x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 64*x^2)*e^3) + 4
8*2^(1/3)*(20*f^3*x^16 + 704*f^3*x^13 + 332*f^3*x^10 - 2720*f^3*x^7 + 2176*
f^3*x^4 - 512*f^3*x + (5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 12
8*x)*e^3))/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 40
96)) + 2/3*(2^(1/3)*e - f)*weierstrassPInverse(0, 4, x), -1/9*sqrt(3)*sqrt(
-2*2^(2/3)*f*e - 2*2^(1/3)*f^2 - e^2)*arctan(1/6*sqrt(3)*(4*f^2*x^5 - 4*f^2
*x^2 - (5*x^3 - 2)*e^2 - 2*(7*f*x^4 - 4*f*x)*e + 2^(2/3)*(14*f^2*x^4 - 8*f^
2*x + (x^5 - x^2)*e^2 + (5*f*x^3 - 2*f)*e) - 2^(1/3)*(10*f^2*x^3 - 4*f^2 -
(7*x^4 - 4*x)*e^2 + 2*(f*x^5 - f*x^2)*e))*sqrt(x^3 - 1)*sqrt(-2*2^(2/3)*f*e
 - 2*2^(1/3)*f^2 - e^2)/(8*f^3*x^6 - 12*f^3*x^3 + 4*f^3 + (2*x^6 - 3*x^3 +
1)*e^3)) + 2/3*(2^(1/3)*e - f)*weierstrassPInverse(0, 4, x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\, dx - \int \frac{fx}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(f*x
/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e+f\,x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] int(-(e + f*x)/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.59 $\int \frac{e+fx}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=170

$$\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\ldots\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/9*(e-2^(2/3)*f)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9
*(2^(1/3)*e+f)*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(
1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)
^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(\sqrt[3]{2}\,e+f\right)F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e - 2^(2/3)*f)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sq
rt[3]) + (2*Sqrt[2 - Sqrt[3]]*(2^(1/3)*e + f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -
7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x
^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{6}\left(\sqrt[3]{2}\,e-2f\right)\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx + \frac{1}{3}\left(\sqrt[3]{2}\,e+f\right)\int \frac{1}{\sqrt{-1-x^3}}\,d$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e+f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right|}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\left(e-2^{2/3}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\,e+f\right)(1+x)\,\sqrt{}}{3\sqrt[4]{3}\sqrt{-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.23, size = 342, normalized size = 2.01

$$\frac{2\sqrt[6]{2}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(f\sqrt{-i+\sqrt{3}+2ix}\left(-6-3\sqrt[3]{2}-2i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}+\left(3\sqrt[3]{2}+4i\sqrt{3}+i\sqrt[3]{2}\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-2\sqrt{3}\left(\sqrt[3]{2}\,e-2f\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}\left(i+2i2^{2/3}+\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(f*Sqrt[-I + Sqrt[3] + (2*I)*x
]*(-6 - 3*2^(1/3) - (2*I)*Sqrt[3] + I*2^(1/3)*Sqrt[3] + (3*2^(1/3) + (4*I)*
Sqrt[3] + I*2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x
]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - 2*Sqrt[3]*(2^(1/3)*e -
 2*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/
(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/(Sqrt[3]*(I + (2*I)*2^(2/3) + Sqr
t[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.24, size = 255, normalized size = 1.50

method	result
default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\right.}}{}\right.}{3 \sqrt{-x^{3}-1}}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\right.}}{}\right.}{3 \sqrt{-x^{3}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2^(2/3)*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))
*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3
^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I
*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-(x+1)(x^2-x+1)}\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac {e+f\,x}{\sqrt {-x^3-1}\,\left (x+2^{2/3}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.60 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=316

$$\frac{2\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\cdots\right.}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b\cdots}$$

[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.29, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{a}\,f+\sqrt[3]{2}\,\sqrt[3]{b}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = \frac{1}{6}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx + \frac{1}{3}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

$$= \frac{2\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}} + \frac{2\sqrt{2+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.07, size = 336, normalized size = 1.06

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\sqrt[4]{3}\,f\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(-\sqrt[3]{b}\,e+2^{2/3}\sqrt[3]{a}\,f\right)\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{3}\,b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*f*((-
1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Elli
pticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*
a^(1/3))]) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f
)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[
3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((
1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*
b^(2/3)*Sqrt[a + b*x^3])
```

Maple [F]

time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{a+bx^{3}} \cdot \left(2^{\frac{2}{3}} \sqrt[3]{a}+\sqrt[3]{b}\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.61 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=324

$$-\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\ldots\right.}{3\sqrt[4]{3}\sqrt[3]{a}\ldots}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2164, 224, 2162, 209}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{3\sqrt{3}\sqrt{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1
/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 +
 Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/
3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)
*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x
^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int \frac{1}{\sqrt{a-bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\int \frac{}{(}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

$$= -\frac{2\left(\sqrt[3]{b}\,e+2^{2/3}\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}} - \frac{2\sqrt{2-}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.82, size = 399, normalized size = 1.23

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{b}\,e+2^{2/3}\sqrt[3]{a}\,f\right)\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="maxima")
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b}\, x\sqrt{a-bx^{3}}}\,dx-\int \frac{fx}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b}\, x\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*s
qrt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.62 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=333

$$-\frac{2\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{3\sqrt[4]{3}\sqrt[3]{a}}$$

```
[Out] -2/9*(b^(1/3)*e+2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)
*x)*3^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)-2/9*(2^(1/3)*b^(1/3)*e
-a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/
(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^
(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x
)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2164, 225, 2162, 212}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{2}\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\left(2^{2/3}\sqrt[3]{a}f+\sqrt[3]{b}e\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(
1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) - (2*Sqrt[2
 - Sqrt[3]]*(2^(1/3)*b^(1/3)*e - a^(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(
1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
 + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = -\left(\frac{1}{3}\left(-\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\int\frac{1}{\sqrt{-a+bx^3}}\,dx\right)+\frac{1}{6}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{2f}{\sqrt[3]{b}}\right)\cdots$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}\cdots$$

$$= -\frac{2\left(\sqrt[3]{b}\,e+2^{2/3}\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt{a}\,b^{2/3}}-\cdots 2\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.27, size = 400, normalized size = 1.20

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\left(\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)\right)+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt[3]{b}e+2^{2/3}\sqrt[3]{a}f\right)\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((-1)^(1/3) +
2^(2/3))*f*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1
)^(1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1
/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]) + ((
-1)^(1/3)*(1 + (-1)^(1/3))*(b^(1/3)*e + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) -
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1
/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]],
 (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx + e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"maxima")
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a + bx^{3}} + \sqrt[3]{b}\, x \sqrt{-a + bx^{3}}}\,dx - \int \frac{fx}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a + bx^{3}} + \sqrt[3]{b}\, x \sqrt{-a + bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] -Integral(e/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(f*x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*
x*sqrt(-a + b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm=
"giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.63 $\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=329

$$\frac{2\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{2}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\cdots\right.}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\cdots}$$

```
[Out] 2/9*(b^(1/3)*e-2^(2/3)*a^(1/3)*f)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*
x)*3^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*3^(1/2)/a^(1/2)+2/9*(2^(1/3)*b^(1/3)*e
+a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b
^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/
2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{a}\,f+\sqrt[3]{2}\,\sqrt[3]{b}\,e\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}+\frac{2\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e - 2^(2/3)*a^(1/3)*f)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1
/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]])/(3*Sqrt[3]*Sqrt[a]*b^(2/3)) + (2*Sqrt[2
- Sqrt[3]]*(2^(1/3)*b^(1/3)*e + a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=\frac{1}{6}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx+\frac{1}{3}\left(\frac{\sqrt[3]{2}}{\sqrt[3]{a}}\right.$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\frac{\sqrt[3]{2}\,e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{-\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

$$=\frac{2\left(\sqrt[3]{b}\,e-2^{2/3}\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2}}{}$$

<u>Mathematica</u> [**C**] Result contains complex when optimal does not.
time = 10.25, size = 387, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)f\left(\sqrt[3]{-1}\,\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[4]{3}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(-\sqrt[3]{b}\,e+2^{2/3}\sqrt[3]{a}\,f\right)\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*f*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x
)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*(1 + (-1)^(1/3))*(
-(b^(1/3)*e) + 2^(2/3)*a^(1/3)*f)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1
 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/
3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (
-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(
((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
[Out] int((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="maxima")
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-a-bx^{3}} \cdot \left(2^{\frac{2}{3}} \sqrt[3]{a}+\sqrt[3]{b}\, x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm
="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int((e + f*x)/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.64 $\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=265

$$\frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}c^{3/2}d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}}$$

```
[Out] 2/9*(-c*f+d*e)*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/c^(3
/2)/d^2*3^(1/2)+1/9*2^(1/3)*(c*f+2*d*e)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*
d*x+c*(1-3^(1/2)))/(2^(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((c^2-2^(2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/c/d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/
3)*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2164, 224, 2162, 209}

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}(cf+2de)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}+\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{c}(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)(de-cf)}{3\sqrt{3}c^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2*(d*e - c*f)*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])
/(3*Sqrt[3]*c^(3/2)*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c + 2^
(2/3)*d*x)*Sqrt[(c^2 - 2^(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c
+ 2^(2/3)*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sq
rt[3])*c + 2^(2/3)*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*d^2*Sqrt[(c*(c + 2^
(2/3)*d*x))/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3cd}$$

$$= \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(2de+cf)\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\Big(\sin^{-}}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\sqrt{c^3+4}$$

$$= \frac{2(de-cf)\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,c^{3/2}d^2} + \frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(2de+cf)\left(c+2^{2/3}dx\right)}{3\sqrt[4]{3}\,cd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.00, size = 380, normalized size = 1.43

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-f\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}dx}{c}+\frac{4d^2x^2}{c^2}}\,\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(f*Sqrt[((-2)^(1/
3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))
*c - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1
)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(
2/3)*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((
1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*Ell
ipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-
1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 +
(-2)^(1/3))*d^2*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*S
qrt[c^3 + 4*d^3*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 899 vs. 2(216) = 432.
time = 0.26, size = 900, normalized size = 3.40

method	result	size
default	Expression too large to display	900
elliptic	Expression too large to display	900

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2
^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4
*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x
+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d)
)^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^
(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x
^3+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
)^(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2
)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2))+2*(-c*f+d*e)/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/
```

```
3)+1/4*I*3^(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)
/((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d))^(1/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/
d+1/2*2^(1/3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/
4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/
d))^(1/2)/(4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/
d)*EllipticPi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/
4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((
1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*
c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(
1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/
4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.35, size = 393, normalized size = 1.48

$$\left[\frac{\sqrt{3}\,(cd^2f-d^3e)\sqrt{-c}\,\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6+\sqrt{3}\,(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d^3x^3+c^3}\,\sqrt{-c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)+6\,\sqrt{d^3}\,(c^2f+2\,cde)\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{18\,c^2d^4},\ \frac{\sqrt{3}\,(cd^2f-d^3e)\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{4\,d^3x^3+c^3}\,(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)\sqrt{c}}{3\,(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5)}\right)+3\,\sqrt{d^3}\,(c^2f+2\,cde)\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,c^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(sqrt(3)*(c*d^2*f - d^3*e)*sqrt(-c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 1
8*c^2*d^4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 + sqrt(3)*(4*d^4*x^4
- 10*c*d^3*x^3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sq
rt(-c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d
^2*x^2 + 6*c^5*d*x + c^6)) + 6*sqrt(d^3)*(c^2*f + 2*c*d*e)*weierstrassPInve
rse(0, -c^3/d^3, x))/(c^2*d^4), 1/9*(sqrt(3)*(c*d^2*f - d^3*e)*sqrt(c)*arct
an(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x -
 c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 + 2*c^4*d*x + c^5)) + 3*sqrt(d^3
)*(c^2*f + 2*c*d*e)*weierstrassPInverse(0, -c^3/d^3, x))/(c^2*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.65 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1+2^(1/3)*x)*3^(1/2)/(x^3+1)^(1/2))*3^(1/2)+2/9*(1+x)*
EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(
1/2))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2\,2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{x^3+1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} -$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{1+x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.25, size = 207, normalized size = 1.43

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 + x^3]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(114) = 228.
time = 1.24, size = 258, normalized size = 1.78

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - 2\,2^{\frac{2}{3}}\left(\frac{3}{2}-i\right.$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - 2\,2^{\frac{2}{3}}\left(\frac{3}{2}-i\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(3/2-1/2*I*3^(1/2))*((1
+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))
^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2^
(2/3)-1)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/
(2^(2/3)-1),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 82, normalized size = 0.57

$$-\frac{1}{9}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2\,x^5+2\,x^2-2^{\frac{2}{3}}(7\,x^4+4\,x)-2^{\frac{1}{3}}(5\,x^3+2)\right)\sqrt{x^3+1}}{12\,(2\,x^6+3\,x^3+1)}\right)+\frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(2/3)*arctan(-1/12*sqrt(3)*2^(2/3)*(2*x^5 + 2*x^2 - 2^(2/3)*
(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1)) + 2
/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
```

```
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{x^3+1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(x/((x^3 + 1)^(1/2)*(x + 2^(2/3))), x)
```

3.66 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=160

$$-\frac{2\ 2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/9*2^(2/3)*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3
^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2\ 2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/(3*Sqrt[3]) +
(2*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4
)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} -$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\Big(}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.25, size = 209, normalized size = 1.31

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[1 - x^3]
```

Maple [A]
time = 1.19, size = 253, normalized size = 1.58

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/
2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(
```

```
1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2
^(2/3)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)
```

Fricas [**A**]
time = 0.19, size = 76, normalized size = 0.48

$$-\frac{1}{9}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2\,x^{5}-2\,x^{2}+2^{\frac{2}{3}}(7\,x^{4}-4\,x)-2^{\frac{1}{3}}(5\,x^{3}-2)\right)\sqrt{-x^{3}+1}}{12\,(2\,x^{6}-3\,x^{3}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*2^(2/3)*arctan(1/12*sqrt(3)*2^(2/3)*(2*x^5 - 2*x^2 + 2^(2/3)*(
7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{1-x^{3}}-2^{\frac{2}{3}}\sqrt{1-x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,0]:[1,0,0,-2]%%},[2]%%%} Error: Bad Arg
ument
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{1-x^3}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(x/((1 - x^3)^(1/2)*(x - 2^(2/3))), x)
```

3.67 $\int \frac{x}{\left(2^{2/3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=163

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1-2^(1/3)*x)*3^(1/2)/(x^3-1)^(1/2))*3^(1/2)+2/9*(1-x)
*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1
/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3
^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\,x\right)}{\sqrt{x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{-1+x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}} \cdots}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3} \cdots\right)^2}} \cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.18, size = 207, normalized size = 1.27

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 + x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 + x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(127) = 254.
time = 1.20, size = 262, normalized size = 1.61

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{2\,2^{\frac{2}{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)-x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*2^(2/3)*(-3/2-1/2*I*3^(1/2))*
((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)/
(-2^(2/3)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1
/2))/(-2^(2/3)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 245, normalized size = 1.50

$$\frac{1}{18}\sqrt{3}\,2^{\frac{2}{3}}\log\left(\frac{x^{18}+1440\,x^{15}+17400\,x^{12}-21056\,x^{9}-10368\,x^{6}+15360\,x^{3}+2\sqrt{3}\,2^{\frac{2}{3}}\left(126\,x^{14}+2664\,x^{11}-4608\,x^{5}+2304\,x^{2}+2^{\frac{2}{3}}\left(x^{16}+310\,x^{13}+2332\,x^{10}-2656\,x^{7}-256\,x^{4}+512\,x\right)+2^{\frac{1}{3}}\left(17\,x^{15}+1058\,x^{12}+2528\,x^{9}-5408\,x^{6}+2560\,x^{3}-512\right)\right)\sqrt{x^{3}-1}+24\cdot 2^{\frac{2}{3}}\left(x^{17}+121\,x^{14}+478\,x^{11}-1144\,x^{8}+608\,x^{5}-64\,x^{2}\right)+48\cdot 2^{\frac{1}{3}}\left(5\,x^{16}+176\,x^{13}+83\,x^{10}-680\,x^{7}+544\,x^{4}-128\,x\right)-2048}{x^{18}-24\,x^{15}+240\,x^{12}-1280\,x^{9}+3840\,x^{6}-6144\,x^{3}+4096}\right)-\frac{2}{3}\,\mathrm{weierstrassPInverse}\left(0,4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(2/3)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368
*x^6 + 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 + 2664*x^11 - 4608*x^5 + 230
4*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x)
+ 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqr
t(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 -
64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128
*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4
096)) - 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1} - 2^{\frac{2}{3}}\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)-x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[1]%%%} / %%%{%%{[1,0,0]:[1,0,0,-2]%%},[1]%%%} Error: Bad A
rgumen
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x}{\sqrt{x^3-1}\,\left(x-2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 - 1)^(1/2)*(x - 2^(2/3))),x)
```

```
[Out] -int(x/((x^3 - 1)^(1/2)*(x - 2^(2/3))), x)
```

3.68 $\int \frac{x}{\left(2^{2/3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}}+\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh((1+2^(1/3)*x)*3^(1/2)/(-x^3-1)^(1/2))*3^(1/2)+2/9*(1+x
)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(
1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x
-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(\sqrt[3]{2}\,x+1\right)}{\sqrt{-x^3-1}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/(3*Sqrt[3])
+ (2*Sqrt[2 - Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Elli
pticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1
/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx - \frac{1}{3}\int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\left(1+\sqrt[3]{2}\,x\right)}{\sqrt{-1-x^3}}\right)}{3\sqrt{3}} + \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.23, size = 209, normalized size = 1.34

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i2^{2/3}\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*2^(2/3)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)),
 ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/((-1)^(1/3
) + 2^(2/3))))/Sqrt[-1 - x^3]
```

Maple [A]
time = 1.09, size = 249, normalized size = 1.60

method	result
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*I*2^(2/3)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2)
```

```
)^(1/2)/(-x^3-1)^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(2^(2/3)+1/2+1/2*I*3^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(119) = 238.
time = 0.14, size = 241, normalized size = 1.54

$$\frac{1}{18}\,\sqrt{3}\,2^{\frac{2}{3}}\log\left(\frac{x^{18}-1440\,x^{15}+17400\,x^{12}+21056\,x^{9}-10368\,x^{6}-15360\,x^{3}+2\,\sqrt{3}\,2^{\frac{2}{3}}\left(126\,x^{14}-2664\,x^{11}+4608\,x^{5}+2304\,x^{2}+2^{\frac{2}{3}}(x^{16}-310\,x^{13}+2332\,x^{10}+2656\,x^{7}-256\,x^{4}-512\,x)-2^{\frac{1}{3}}(17\,x^{15}-1058\,x^{12}+2528\,x^{9}+5408\,x^{6}+2560\,x^{3}+512)\right)\sqrt{-x^{3}-1}-24\cdot 2^{\frac{2}{3}}(x^{17}-121\,x^{14}+478\,x^{11}+1144\,x^{8}+608\,x^{5}+64\,x^{2})+48\cdot 2^{\frac{1}{3}}(5\,x^{16}-176\,x^{13}+83\,x^{10}+680\,x^{7}+544\,x^{4}+128\,x)-2048}{x^{18}+24\,x^{15}+240\,x^{12}+1280\,x^{9}+3840\,x^{6}+6144\,x^{3}+4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*2^(2/3)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368
*x^6 - 15360*x^3 + 2*sqrt(3)*2^(2/3)*(126*x^14 - 2664*x^11 + 4608*x^5 + 230
4*x^2 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x)
- 2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqr
t(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
 64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 12
8*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 +
4096))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(x+2^{\frac{2}{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)+x)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 2**(2/3))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^3-1}\,\left(x+2^{2/3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)
[Out] int(x/((- x^3 - 1)^(1/2)*(x + 2^(2/3))), x)
```

3.69 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=275

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}$$

```
[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3+a)^(
1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\,2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a +
 b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(
1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(
1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx=\frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

$$=-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.85, size = 324, normalized size = 1.18

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\sqrt[4]{3}\left(\sqrt[3]{-1}\,\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}}+\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[3]{-1}+2^{2/3}}\right)}{\sqrt{3}\,b^{2/3}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((3^(1/4)*((-1)
^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*Ellipt
icF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))
]], (-1)^(1/3)])/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^
(1/3))]) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x
)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(
2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3) + 2^(2/3))))/(Sqrt[3]*b^(2/3)*Sqrt[a + b*x^
3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3}+a}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b x^{3}} \cdot \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.70 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=283

$$-\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

[Out] -2/9*2^(2/3)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3+a)^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]

time = 0.27, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2164, 224, 2162, 209}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{2\,2^{2/3}\text{ArcTan}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}}$$

$$= -\frac{2\,2^{2/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 388, normalized size = 1.37

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}-b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{a-bx^{3}}+\sqrt[3]{b}\, x \sqrt{a-bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a-b\,x^3}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(x/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.71 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=292

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\ \cdots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)-2^(1/3)*b^(1/3)*x)*3^(1/2)/(b*x^3-a)^
(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^
(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{bx^3-a}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 + b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b
^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2^{2/3}\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\Big(\sin}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}}}$$

$$= -\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 389, normalized size = 1.33

$$-\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)-\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1)^(1/3) + 2
^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(
1/3)*b^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3)
 - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - ((-1)^
(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x
)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/
a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3
) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3
]))/(((-1)^(1/3) + 2^(2/3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(
(1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}} - b^{\frac{1}{3}} x\right) \sqrt{b\,x^{3} - a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
[Out] int(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2^{\frac{2}{3}} \sqrt[3]{a} \sqrt{-a + bx^{3}} + \sqrt[3]{b}\, x \sqrt{-a + bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-2**(2/3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}-b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)),x)
```

```
[Out] int(x/((b*x^3 - a)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)
```

3.72 $\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=288

$$-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}$$

```
[Out] -2/9*2^(2/3)*arctanh(a^(1/6)*(a^(1/3)+2^(1/3)*b^(1/3)*x)*3^(1/2)/(-b*x^3-a)
^(1/2))/a^(1/6)/b^(2/3)*3^(1/2)+2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2164, 225, 2162, 212}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\,\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\,\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a
 - b*x^3]])/(3*Sqrt[3]*a^(1/6)*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}-\frac{\int \frac{2^{2/3}\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\Bigg(\mathrm{si}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}}}$$

$$=-\frac{2\,2^{2/3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3\sqrt{3}\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 375, normalized size = 1.30

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\left(\sqrt[3]{-1}+2^{2/3}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\sqrt[6]{-1}-\frac{i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt[4]{3}}+\frac{\sqrt[3]{-1}\,2^{2/3}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{i\sqrt{3}}{\sqrt[3]{-1}+2^{2/3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(\sqrt[3]{-1}+2^{2/3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3) +
 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/
a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^
(1/3))*a^(1/3))]], (-1)^(1/3)])/3^(1/4)) + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1
/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqr
t[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/
((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/(((-1)^(1/3) + 2^(2/3)
)*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]
*Sqrt[-a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2^{\frac{2}{3}} a^{\frac{1}{3}}+b^{\frac{1}{3}} x\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
[Out] int(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a - bx^{3}} \cdot \left(2^{\frac{2}{3}} \sqrt[3]{a} + \sqrt[3]{b} x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] Integral(x/(sqrt(-a - b*x**3)*(2**(2/3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(2^{2/3}\,a^{1/3}+b^{1/3}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)), x)
```

3.73 $\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$

Optimal. Leaf size=246

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}\,d^2}+\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)}{\left(1+\sqrt{3}\right)}\right.\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\,(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}$$

```
[Out] -2/9*arctan((2*d*x+c)*3^(1/2)*c^(1/2)/(4*d^3*x^3+c^3)^(1/2))/d^2*3^(1/2)/c^
(1/2)+1/9*2^(1/3)*(c+2^(2/3)*d*x)*EllipticF((2^(2/3)*d*x+c*(1-3^(1/2)))/(2^
(2/3)*d*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^2-2^(
2/3)*c*d*x+2*2^(1/3)*d^2*x^2)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
d^2/(4*d^3*x^3+c^3)^(1/2)/(c*(c+2^(2/3)*d*x)/(2^(2/3)*d*x+c*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2164, 224, 2162, 209}

$$\frac{\sqrt[3]{2}\sqrt{2+\sqrt{3}}\,(c+2^{2/3}dx)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)c+2^{2/3}dx}{\left(1+\sqrt{3}\right)c+2^{2/3}dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\,(c+2^{2/3}dx)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\sqrt{c^3+4d^3x^3}}-\frac{2\text{ArcTan}\left(\frac{\sqrt{3}\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(3*Sqrt[3]
*Sqrt[c]*d^2) + (2^(1/3)*Sqrt[2 + Sqrt[3]]*(c + 2^(2/3)*d*x)*Sqrt[(c^2 - 2^
(2/3)*c*d*x + 2*2^(1/3)*d^2*x^2)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c + 2^(2/3)*d*x)/((1 + Sqrt[3])*c + 2^(2/3)*d*x)],
 -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c + 2^(2/3)*d*x))/((1 + Sqrt[3])*
c + 2^(2/3)*d*x)^2]*Sqrt[c^3 + 4*d^3*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2162

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[2*(e/d), Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + 2*d*(x/c))/
Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &
& EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3+4d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx}{3d}$$

$$= \frac{\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx+2\sqrt[3]{2}\,d^2x^2}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right.}{\left(1+\sqrt{3}\right.}\right.\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\left(c+2^{2/3}dx\right)}{\left(\left(1+\sqrt{3}\right)c+2^{2/3}dx\right)^2}}\,\sqrt{c^3+4d^3x^3}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt{c}\,(c+2dx)}{\sqrt{c^3+4d^3x^3}}\right)}{3\sqrt{3}\,\sqrt{c}\,d^2} + \frac{\sqrt[3]{2}\,\sqrt{2+\sqrt{3}}\,\left(c+2^{2/3}dx\right)\sqrt{\frac{c^2-2^{2/3}cdx}{\left(\left(1+\sqrt{3}\right)\right.}}}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c\,(}{\left(\left(1+\sqrt{\right.\right.}}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.79, size = 372, normalized size = 1.51

$$\frac{\sqrt[6]{2}\sqrt{\frac{\sqrt[3]{2}c+2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(-\sqrt{\frac{\sqrt[3]{-2}c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\sqrt[3]{-1}\left(2+\sqrt[3]{-2}\right)c-2\left(\sqrt[3]{-1}+2^{2/3}\right)dx\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)+\frac{\sqrt[3]{-1}2^{2/3}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{2^{2/3}-\frac{2\sqrt[3]{2}dx}{c}+\frac{4d^2x^2}{c^2}}\Pi\left(\frac{i\sqrt[3]{2}\sqrt{3}}{2+\sqrt[3]{-2}};\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}}{\sqrt[6]{2}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(2+\sqrt[3]{-2}\right)d^2\sqrt{\frac{\sqrt[3]{2}c+2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]),x]
```

```
[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)]*(-(Sqrt[((-2)^(1/3)
*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*((-1)^(1/3)*(2 + (-2)^(1/3))*c
 - 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^
(2/3)*d*x)/((1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)]) + ((-1)^(1/3)*2^(2/
3)*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))
*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1
/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/(
(1 + (-1)^(1/3))*c)]/2^(1/6)], (-1)^(1/3)])/Sqrt[3]))/((2 + (-2)^(1/3))*d^2
*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3
*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(197) = 394.
time = 0.26, size = 892, normalized size = 3.63

method	result	size
default	Expression too large to display	892
elliptic	Expression too large to display	892

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(
1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I
*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)*((x+1
/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^
(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1
/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(4*d^3*x^3
+c^3)^(1/2)*EllipticF(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^
(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^
(1/2),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*
2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1
/2))-2*c/d^2*((1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^
```

```
(1/2)*2^(1/3))*c/d)*((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/(((1/4*2^(1
/3)-1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1
/2)*((x+1/2*2^(1/3)*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+1/2*2^(1/
3)*c/d))^(1/2)*((x-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1
/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2)/(
4*d^3*x^3+c^3)^(1/2)/((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d)*Elliptic
Pi(((x-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2)
*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d))^(1/2),((1/4*2^(1/3)
+1/4*I*3^(1/2)*2^(1/3))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*
2^(1/3)+1/4*I*3^(1/2)*2^(1/3))*c/d+c/d),(((1/4*2^(1/3)+1/4*I*3^(1/2)*2^(1/3
))*c/d-(1/4*2^(1/3)-1/4*I*3^(1/2)*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2)
*2^(1/3))*c/d+1/2*2^(1/3)*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 350, normalized size = 1.42

$$\left[-\frac{\sqrt{3}\,\sqrt{-c}\,d^2\log\left(\frac{2\,d^6x^6-36\,cd^5x^5-18\,c^2d^4x^4+28\,c^3d^3x^3+18\,c^4d^2x^2-c^6-\sqrt{3}\,(4\,d^4x^4-10\,cd^3x^3-18\,c^2d^2x^2-8\,c^3dx-c^4)\sqrt{4\,d^3x^3+c^3}\,\sqrt{-c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)-6\,c\sqrt{d^3}\,\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{18\,cd^4},\ \frac{\sqrt{3}\,\sqrt{c}\,d^2\arctan\left(\frac{\sqrt{3}\,\sqrt{4\,d^3x^3+c^3}\,(2\,d^3x^3-6\,cd^2x^2-6\,c^2dx-c^3)\sqrt{c}}{3\,(8\,cd^4x^4+4\,c^2d^3x^3+2\,c^4dx+c^5)}\right)+3\,c\sqrt{d^3}\,\mathrm{weierstrassPInverse}\left(0,-\frac{c^3}{d^3},x\right)}{9\,cd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(sqrt(3)*sqrt(-c)*d^2*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4
 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*d^4*x^4 - 10*c*d^3*x^
3 - 18*c^2*d^2*x^2 - 8*c^3*d*x - c^4)*sqrt(4*d^3*x^3 + c^3)*sqrt(-c))/(d^6*
x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^
5*d*x + c^6)) - 6*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*d^4),
 1/9*(sqrt(3)*sqrt(c)*d^2*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x
^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)*sqrt(c)/(8*c*d^4*x^4 + 4*c^2*d^3*x^3 +
2*c^4*d*x + c^5)) + 3*c*sqrt(d^3)*weierstrassPInverse(0, -c^3/d^3, x))/(c*d
^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(c+dx)\sqrt{c^3+4d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/((c + d*x)*sqrt(c**3 + 4*d**3*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c^3+4\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int(x/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.74 $\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)$$

```
[Out] 2/3*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))
```

Rubi [A]

time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2163, 212}

$$\frac{2}{3}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{1+x^3}}\,dx &= 2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}}\right)\\ &= \frac{2}{3}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.66, size = 31, normalized size = 1.35

$$\frac{2}{3}\tanh^{-1}\left(\frac{\frac{1}{3}+\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 240, normalized size = 10.43

method	result
trager	$\frac{\ln\left(\frac{x^3+6x\sqrt{x^3+1}+12x^2+6\sqrt{x^3+1}-6x+10}{(x-2)^3}\right)}{3}$
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\cdots$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*
((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi
(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(17) = 34$.
time = 0.37, size = 44, normalized size = 1.91

$$\frac{1}{3}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int\frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(1/(x*sqrt(
x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B]
time = 0.23, size = 205, normalized size = 8.91

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x
 + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)
/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1
/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.75 $\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=27

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

```
[Out] -2/3*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2163, 212}

$$-\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{1-x^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}}\right)\right)\\ &= -\frac{2}{3}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.69, size = 33, normalized size = 1.22

$$-\frac{2}{3}\tanh^{-1}\left(\frac{\frac{1}{3}-\frac{2x}{3}+\frac{x^2}{3}}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]
[Out] (-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 240, normalized size = 8.89

method	result
trager	$\frac{\ln\left(-\frac{-x^3+6\sqrt{-x^3+1}\,x+12x^2-6\sqrt{-x^3+1}+6x+10}{(x+2)^3}\right)}{3}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2
)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2
*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.
time = 0.38, size = 47, normalized size = 1.74

$$\frac{1}{3}\log\left(-\frac{x^3-12\,x^2-6\,\sqrt{-x^3+1}\,(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int\left(-\frac{1}{x\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt
(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B]
time = 0.18, size = 221, normalized size = 8.19

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
*(ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i
)/2 - 3/2)))*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((1 - x^3)^(1/2)*(((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.76 $\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)$$

```
[Out] -2/3*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2163, 209}

$$-\frac{2}{3}\text{ArcTan}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{(2+x)\sqrt{-1+x^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}}\right)\right)\\ &= -\frac{2}{3}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.66, size = 21, normalized size = 0.84

$$\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{-1+x^3}}{(-1+x)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.46, size = 240, normalized size = 9.60

method	result
trager	$\frac{\mathrm{RootOf}(\_Z^2+1)\ln\left(-\frac{\mathrm{RootOf}(\_Z^2+1)x^3-12\,\mathrm{RootOf}(\_Z^2+1)x^2+6\sqrt{x^3-1}\,x-6x\,\mathrm{RootOf}(\_Z^2+1)-6\sqrt{x^3-1}-10\,\mathrm{RootO}}{(x+2)^3}\right.}{3}$
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(-}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2(-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((-1+x)/
(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/
2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Elliptic
Pi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.
time = 0.36, size = 40, normalized size = 1.60

$$-\frac{1}{3}\arctan\left(\frac{(x^3-12x^2-6x-10)\sqrt{x^3-1}}{6(x^4-x^3-x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1)
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{x\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt
(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B]
time = 2.55, size = 205, normalized size = 8.20

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticF(asin
((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2
)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1
/2)
```

3.77 $\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=25

$$\frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)$$

```
[Out] 2/3*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2163, 209}

$$\frac{2}{3}\text{ArcTan}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(2-x)\sqrt{-1-x^3}}\,dx &= 2\text{Subst}\left(\int \frac{1}{9+x^2}\,dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}}\right)\\ &= \frac{2}{3}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.67, size = 23, normalized size = 0.92

$$-\frac{2}{3}\tan^{-1}\left(\frac{3\sqrt{-1-x^3}}{(1+x)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.42, size = 240, normalized size = 9.60

method	result
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^3+12\,\mathrm{RootOf}\left(\_Z^2+1\right)x^2-6x\,\mathrm{RootOf}\left(\_Z^2+1\right)-6\sqrt{-x^3-1}\,x+10\,\mathrm{RootOf}\left(\_Z^2+1\right)-}{(x-2)^3}\right.}{3}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{3}\right.}{3\sqrt{-x^3-1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}\right.}}{3}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1
+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x
^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3
^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [A]
time = 0.36, size = 38, normalized size = 1.52

$$-\frac{1}{3}\arctan\left(\frac{(x^3+12x^2-6x+10)\sqrt{-x^3-1}}{6(x^4+x^3+x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int\frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(1/(x*sqr
t(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B]
time = 2.53, size = 221, normalized size = 8.84

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*(ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
-((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticPi((3^(1/2)*1i)/6
 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/
2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/
2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x^3
- x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.78 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\, x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\, x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=50

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2/3*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A]
time = 0.09, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2163, 212}

$$\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A]
time = 3.70, size = 48, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{3\sqrt[6]{a}\sqrt{a+bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (2*ArcTanh[(3*a^(1/6)*Sqrt[a + b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6)
*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))),
 x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(34) = 68$.
time = 0.83, size = 587, normalized size = 11.74

$$\left[\frac{1}{6}a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^{6}x^{18}+7800\,ab^{5}x^{15}+535272\,a^{2}b^{4}x^{12}+5147264\,a^{3}b^{3}x^{9}+10516992\,a^{4}b^{2}x^{6}+5922816\,a^{5}bx^{3}+557056\,a^{6}+144\,(7\,b^{5}x^{16}+1169\,ab^{4}x^{13}+20266\,a^{2}b^{3}x^{10}+66976\,a^{3}b^{2}x^{7}+58112\,a^{4}bx^{4}+10240\,a^{5}x)a^{\frac{2}{3}}b^{\frac{1}{3}}+72\,(b^{5}x^{17}+581\,ab^{4}x^{14}+19108\,a^{2}b^{3}x^{11}+106336\,a^{3}b^{2}x^{8}+137984\,a^{4}bx^{5}+50176\,a^{5}x^{2})a^{\frac{1}{3}}b^{\frac{2}{3}}+12\,\sqrt{bx^{3}+a}\left((b^{5}x^{16}+1568\,ab^{4}x^{13}+72520\,a^{2}b^{3}x^{10}+498304\,a^{3}b^{2}x^{7}+625664\,a^{4}bx^{4}+139264\,a^{5}x)a^{\frac{2}{3}}b^{\frac{2}{3}}+6\,(41\,ab^{5}x^{14}+4268\,a^{2}b^{4}x^{11}+52896\,a^{3}b^{3}x^{8}+116480\,a^{4}b^{2}x^{5}+48128\,a^{5}bx^{2})a^{\frac{1}{3}}+(25\,ab^{5}x^{15}+7202\,a^{2}b^{4}x^{12}+167392\,a^{3}b^{3}x^{9}+647296\,a^{4}b^{2}x^{6}+468992\,a^{5}bx^{3}+40960\,a^{6})b^{\frac{1}{3}}\right)\sqrt{\frac{1}{ab^{\frac{2}{3}}}}}{b^{6}x^{18}-48\,ab^{5}x^{15}+960\,a^{2}b^{4}x^{12}-10240\,a^{3}b^{3}x^{9}+61440\,a^{4}b^{2}x^{6}-196608\,a^{5}bx^{3}+262144\,a^{6}}\right),\ -\frac{1}{3}a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(\frac{\sqrt{bx^{3}+a}\left((11\,bx^{4}-16\,ax)a^{\frac{2}{3}}b^{\frac{2}{3}}+(b^{2}x^{5}+28\,abx^{2})a^{\frac{1}{3}}-(17\,abx^{3}-10\,a^{2})b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}{6\,(b^{2}x^{6}+2\,abx^{3}+a^{2})}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] [1/6*a^(1/3)*sqrt(1/(a*b^(2/3)))*log((b^6*x^18 + 7800*a*b^5*x^15 + 535272*a
^2*b^4*x^12 + 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 + 5922816*a^5*b*x^
3 + 557056*a^6 + 144*(7*b^5*x^16 + 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 + 6
6976*a^3*b^2*x^7 + 58112*a^4*b*x^4 + 10240*a^5*x)*a^(2/3)*b^(1/3) + 72*(b^5
*x^17 + 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 + 106336*a^3*b^2*x^8 + 137984*a
^4*b*x^5 + 50176*a^5*x^2)*a^(1/3)*b^(2/3) + 12*sqrt(b*x^3 + a)*((b^5*x^16 +
 1568*a*b^4*x^13 + 72520*a^2*b^3*x^10 + 498304*a^3*b^2*x^7 + 625664*a^4*b*x
^4 + 139264*a^5*x)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 + 4268*a^2*b^4*x^11 +
 52896*a^3*b^3*x^8 + 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*a^(1/3) + (25*a*
b^5*x^15 + 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 + 647296*a^4*b^2*x^6 + 46
8992*a^5*b*x^3 + 40960*a^6)*b^(1/3))*sqrt(1/(a*b^(2/3))))/(b^6*x^18 - 48*a*
b^5*x^15 + 960*a^2*b^4*x^12 - 10240*a^3*b^3*x^9 + 61440*a^4*b^2*x^6 - 19660
8*a^5*b*x^3 + 262144*a^6)), -1/3*a^(1/3)*sqrt(-1/(a*b^(2/3)))*arctan(1/6*sq
rt(b*x^3 + a)*((11*b*x^4 - 16*a*x)*a^(2/3)*b^(2/3) + (b^2*x^5 + 28*a*b*x^2)
*a^(1/3) - (17*a*b*x^3 - 10*a^2)*b^(1/3))*sqrt(-1/(a*b^(2/3)))/(b^2*x^6 + 2
*a*b*x^3 + a^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx-\int\frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x
)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x*
*3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*s
qrt(a + b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.44, size = 65, normalized size = 1.30

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)\,\left(\sqrt{b\,x^3+a}-\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.79 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=52

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

[Out] -2/3*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^(1/3)

Rubi [A]
time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2163, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(3*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9-ax^2}\,dx, x, \frac{\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 3.61, size = 50, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{3\sqrt[6]{a}\,\sqrt{a-bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-2*ArcTanh[(3*a^(1/6)*Sqrt[a - b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3)))
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(37) = 74$.
time = 0.85, size = 630, normalized size = 12.12

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{\frac{1}{ab^{\frac{2}{3}}}} \log\left(\frac{b^6x^{18} - 7800\,ab^5x^{15} + 535272\,a^2b^4x^{12} - 5147264\,a^3b^3x^9 + 10516992\,a^4b^2x^6 - 5922816\,a^5bx^3 + 557056\,a^6 + 144\,(7\,b^5x^{16} - 1169\,ab^4x^{13} + 20266\,a^2b^3x^{10} - 66976\,a^3b^2x^7 + 58112\,a^4bx^4 - 10240\,a^5x)a^{\frac{2}{3}}b^{\frac{1}{3}} - 72\,(b^5x^{17} - 581\,ab^4x^{14} + 19108\,a^2b^3x^{11} - 106336\,a^3b^2x^8 + 137984\,a^4bx^5 - 50176\,a^5x^2)a^{\frac{1}{3}}b^{\frac{2}{3}} - 12\,\left((b^5x^{16} - 1568\,ab^4x^{13} + 72520\,a^2b^3x^{10} - 498304\,a^3b^2x^7 + 625664\,a^4bx^4 - 139264\,a^5x)\sqrt{-bx^3+a}\,a^{\frac{2}{3}}b^{\frac{2}{3}} + 6\,(41\,ab^5x^{14} - 4268\,a^2b^4x^{11} + 52896\,a^3b^3x^8 - 116480\,a^4b^2x^5 + 48128\,a^5bx^2)\sqrt{-bx^3+a}\,a^{\frac{1}{3}} - (25\,ab^5x^{15} - 7202\,a^2b^4x^{12} + 167392\,a^3b^3x^9 - 647296\,a^4b^2x^6 + 468992\,a^5bx^3 - 40960\,a^6)\sqrt{-bx^3+a}\,b^{\frac{1}{3}}\right)\sqrt{\frac{1}{ab^{\frac{2}{3}}}}}{b^6x^{18} + 48\,ab^5x^{15} + 960\,a^2b^4x^{12} + 10240\,a^3b^3x^9 + 61440\,a^4b^2x^6 + 196608\,a^5bx^3 + 262144\,a^6}\right), \frac{1}{3} a^{\frac{1}{3}} \sqrt{-\frac{1}{ab^{\frac{2}{3}}}} \arctan\left(\frac{\left((11\,bx^4 + 16\,ax)\sqrt{-bx^3+a}\,a^{\frac{2}{3}}b^{\frac{2}{3}} - (b^2x^5 - 28\,abx^2)\sqrt{-bx^3+a}\,a^{\frac{1}{3}} + (17\,abx^3 + 10\,a^2)\sqrt{-bx^3+a}\,b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}{6\,(b^2x^6 - 2\,abx^3 + a^2)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*a^(1/3)*sqrt(1/(a*b^(2/3)))*log((b^6*x^18 - 7800*a*b^5*x^15 + 535272*a
^2*b^4*x^12 - 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 - 5922816*a^5*b*x^
3 + 557056*a^6 + 144*(7*b^5*x^16 - 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 - 6
6976*a^3*b^2*x^7 + 58112*a^4*b*x^4 - 10240*a^5*x)*a^(2/3)*b^(1/3) - 72*(b^5
*x^17 - 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 - 106336*a^3*b^2*x^8 + 137984*a
^4*b*x^5 - 50176*a^5*x^2)*a^(1/3)*b^(2/3) - 12*((b^5*x^16 - 1568*a*b^4*x^13
 + 72520*a^2*b^3*x^10 - 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 - 139264*a^5*
x)*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 - 4268*a^2*b^4*x^11
+ 52896*a^3*b^3*x^8 - 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*sqrt(-b*x^3 + a
)*a^(1/3) - (25*a*b^5*x^15 - 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 - 64729
6*a^4*b^2*x^6 + 468992*a^5*b*x^3 - 40960*a^6)*sqrt(-b*x^3 + a)*b^(1/3))*sqr
t(1/(a*b^(2/3))))/(b^6*x^18 + 48*a*b^5*x^15 + 960*a^2*b^4*x^12 + 10240*a^3*
b^3*x^9 + 61440*a^4*b^2*x^6 + 196608*a^5*b*x^3 + 262144*a^6)), 1/3*a^(1/3)*
sqrt(-1/(a*b^(2/3)))*arctan(1/6*((11*b*x^4 + 16*a*x)*sqrt(-b*x^3 + a)*a^(2/
3)*b^(2/3) - (b^2*x^5 - 28*a*b*x^2)*sqrt(-b*x^3 + a)*a^(1/3) + (17*a*b*x^3
+ 10*a^2)*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-1/(a*b^(2/3)))/(b^2*x^6 - 2*a*b*x
^3 + a^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\,\sqrt{a - bx^3} + \sqrt[3]{b}\,x\sqrt{a - bx^3}}\right) dx - \int \frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\,\sqrt{a - bx^3} + \sqrt[3]{b}\,x\sqrt{a - bx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),
x)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x*
*3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sq
rt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 3.59, size = 67, normalized size = 1.29

$$\frac{\ln\left(\frac{\left(\sqrt{a-b\,x^3}-\sqrt{a}\right)\,{\left(\sqrt{a-b\,x^3}+\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x\right)}^3}{x^3\,{\left(b^{1/3}\,x+2\,a^{1/3}\right)}^3}\right)}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

```
[Out] log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6)
*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))
```

3.80 $\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=53

$$-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

[Out] -2/3*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(1/3)

Rubi [A]
time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2163, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{bx^3-a}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(3*a^(1/6)*b^(1/3))

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{9+ax^2}\,dx,x,\frac{\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 3.66, size = 51, normalized size = 0.96

$$\frac{2\tan^{-1}\left(\frac{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
[Out] (2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(1/3) - b^(1/3)*x)^2])/(3*a^(1/6)
*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{a^{\frac{1}{3}}-b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))),
 x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(38) = 76$.
time = 0.81, size = 592, normalized size = 11.17

$$\left[\frac{1}{6}a^{\frac{1}{3}}\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}\log\left(\frac{b^{6}x^{18}-7800\,ab^{5}x^{15}+535272\,a^{2}b^{4}x^{12}-5147264\,a^{3}b^{3}x^{9}+10516992\,a^{4}b^{2}x^{6}-5922816\,a^{5}bx^{3}+557056\,a^{6}+144\,(7\,b^{5}x^{16}-1169\,ab^{4}x^{13}+20266\,a^{2}b^{3}x^{10}-66976\,a^{3}b^{2}x^{7}+58112\,a^{4}bx^{4}-10240\,a^{5}x)a^{\frac{2}{3}}b^{\frac{1}{3}}-72\,(b^{5}x^{17}-581\,ab^{4}x^{14}+19108\,a^{2}b^{3}x^{11}-106336\,a^{3}b^{2}x^{8}+137984\,a^{4}bx^{5}-50176\,a^{5}x^{2})a^{\frac{1}{3}}b^{\frac{2}{3}}-12\,\sqrt{bx^{3}-a}\left((b^{5}x^{16}-1568\,ab^{4}x^{13}+72520\,a^{2}b^{3}x^{10}-498304\,a^{3}b^{2}x^{7}+625664\,a^{4}bx^{4}-139264\,a^{5}x)a^{\frac{2}{3}}b^{\frac{2}{3}}+6\,(41\,ab^{5}x^{14}-4268\,a^{2}b^{4}x^{11}+52896\,a^{3}b^{3}x^{8}-116480\,a^{4}b^{2}x^{5}+48128\,a^{5}bx^{2})a^{\frac{1}{3}}-(25\,ab^{5}x^{15}-7202\,a^{2}b^{4}x^{12}+167392\,a^{3}b^{3}x^{9}-647296\,a^{4}b^{2}x^{6}+468992\,a^{5}bx^{3}-40960\,a^{6})b^{\frac{1}{3}}\right)\sqrt{-\frac{1}{ab^{\frac{2}{3}}}}}{b^{6}x^{18}+48\,ab^{5}x^{15}+960\,a^{2}b^{4}x^{12}+10240\,a^{3}b^{3}x^{9}+61440\,a^{4}b^{2}x^{6}+196608\,a^{5}bx^{3}+262144\,a^{6}}\right),\ \frac{1}{3}a^{\frac{1}{3}}\sqrt{\frac{1}{ab^{\frac{2}{3}}}}\arctan\left(\frac{\sqrt{bx^{3}-a}\left((11\,bx^{4}+16\,ax)a^{\frac{2}{3}}b^{\frac{2}{3}}-(b^{2}x^{5}-28\,abx^{2})a^{\frac{1}{3}}+(17\,abx^{3}+10\,a^{2})b^{\frac{1}{3}}\right)\sqrt{\frac{1}{ab^{\frac{2}{3}}}}}{6\,(b^{2}x^{6}-2\,abx^{3}+a^{2})}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="fricas")
```

```
[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 - 7800*a*b^5*x^15 + 535272*
a^2*b^4*x^12 - 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 - 5922816*a^5*b*x
^3 + 557056*a^6 + 144*(7*b^5*x^16 - 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 -
66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 - 10240*a^5*x)*a^(2/3)*b^(1/3) - 72*(b^
5*x^17 - 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 - 106336*a^3*b^2*x^8 + 137984*
a^4*b*x^5 - 50176*a^5*x^2)*a^(1/3)*b^(2/3) - 12*sqrt(b*x^3 - a)*((b^5*x^16
- 1568*a*b^4*x^13 + 72520*a^2*b^3*x^10 - 498304*a^3*b^2*x^7 + 625664*a^4*b*
x^4 - 139264*a^5*x)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 - 4268*a^2*b^4*x^11
+ 52896*a^3*b^3*x^8 - 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*a^(1/3) - (25*a
*b^5*x^15 - 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 - 647296*a^4*b^2*x^6 + 4
68992*a^5*b*x^3 - 40960*a^6)*b^(1/3))*sqrt(-1/(a*b^(2/3))))/(b^6*x^18 + 48*
a*b^5*x^15 + 960*a^2*b^4*x^12 + 10240*a^3*b^3*x^9 + 61440*a^4*b^2*x^6 + 196
608*a^5*b*x^3 + 262144*a^6)), 1/3*a^(1/3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*sq
rt(b*x^3 - a)*((11*b*x^4 + 16*a*x)*a^(2/3)*b^(2/3) - (b^2*x^5 - 28*a*b*x^2)
*a^(1/3) + (17*a*b*x^3 + 10*a^2)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(b^2*x^6 - 2*
a*b*x^3 + a^2))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\,\sqrt{-a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a+bx^{3}}}\right)dx-\int\frac{\sqrt[3]{b}\,x}{2\sqrt[3]{a}\,\sqrt{-a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x
)
```

```
[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*
x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x
*sqrt(-a + b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 5.45, size = 74, normalized size = 1.40

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3-a}+\sqrt{a}\,1\mathrm{i}\right)\,\left(\sqrt{a}+2\,a^{1/6}\,b^{1/3}\,x+\sqrt{b\,x^3-a}\,1\mathrm{i}\right)^3}{x^3\,\left(b^{1/3}\,x+2\,a^{1/3}\right)^3}\right)\,1\mathrm{i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] (log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2*
a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(1/
3))
```

3.81 $\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=53

$$\frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2/3*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(1
/3)
```

Rubi [A]
time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2163, 209}

$$\frac{2\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(3*a^(1/6)
*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{9+ax^2}\,dx, x, \frac{\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 3.66, size = 51, normalized size = 0.96

$$-\frac{2\tan^{-1}\left(\frac{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}\right)}{3\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(1/3) + b^(1/3)*x)^2])/(3*a^(1/6
)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{a^{\frac{1}{3}}+b^{\frac{1}{3}}x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3)))
, x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(37) = 74$.
time = 0.83, size = 641, normalized size = 12.09

$$\left[\frac{1}{6} a^{\frac{1}{3}} \sqrt{-\frac{1}{a b^{\frac{2}{3}}}} \log\left(\frac{b^{6} x^{18} + 7800 a b^{5} x^{15} + 535272 a^{2} b^{4} x^{12} + 5147264 a^{3} b^{3} x^{9} + 10516992 a^{4} b^{2} x^{6} + 5922816 a^{5} b x^{3} + 557056 a^{6} + 144 (7 b^{5} x^{16} + 1169 a b^{4} x^{13} + 20266 a^{2} b^{3} x^{10} + 66976 a^{3} b^{2} x^{7} + 58112 a^{4} b x^{4} + 10240 a^{5} x) a^{\frac{2}{3}} b^{\frac{1}{3}} + 72 (b^{5} x^{17} + 581 a b^{4} x^{14} + 19108 a^{2} b^{3} x^{11} + 106336 a^{3} b^{2} x^{8} + 137984 a^{4} b x^{5} + 50176 a^{5} x^{2}) a^{\frac{1}{3}} b^{\frac{2}{3}} + 12 \left((b^{5} x^{16} + 1568 a b^{4} x^{13} + 72520 a^{2} b^{3} x^{10} + 498304 a^{3} b^{2} x^{7} + 625664 a^{4} b x^{4} + 139264 a^{5} x) \sqrt{-b x^{3} - a} a^{\frac{2}{3}} b^{\frac{2}{3}} + 6 (41 a b^{5} x^{14} + 4268 a^{2} b^{4} x^{11} + 52896 a^{3} b^{3} x^{8} + 116480 a^{4} b^{2} x^{5} + 48128 a^{5} b x^{2}) \sqrt{-b x^{3} - a} a^{\frac{1}{3}} + (25 a b^{5} x^{15} + 7202 a^{2} b^{4} x^{12} + 167392 a^{3} b^{3} x^{9} + 647296 a^{4} b^{2} x^{6} + 468992 a^{5} b x^{3} + 40960 a^{6}) \sqrt{-b x^{3} - a} b^{\frac{1}{3}}\right) \sqrt{-\frac{1}{a b^{\frac{2}{3}}}}}{b^{6} x^{18} - 48 a b^{5} x^{15} + 960 a^{2} b^{4} x^{12} - 10240 a^{3} b^{3} x^{9} + 61440 a^{4} b^{2} x^{6} - 196608 a^{5} b x^{3} + 262144 a^{6}}\right), -\frac{1}{3} a^{\frac{1}{3}} \sqrt{\frac{1}{a b^{\frac{2}{3}}}} \arctan\left(\frac{\left((11 b x^{4} - 16 a x) \sqrt{-b x^{3} - a} a^{\frac{2}{3}} b^{\frac{2}{3}} + (b^{2} x^{5} + 28 a b x^{2}) \sqrt{-b x^{3} - a} a^{\frac{1}{3}} - (17 a b x^{3} - 10 a^{2}) \sqrt{-b x^{3} - a} b^{\frac{1}{3}}\right) \sqrt{\frac{1}{a b^{\frac{2}{3}}}}}{6 (b^{2} x^{6} + 2 a b x^{3} + a^{2})}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/6*a^(1/3)*sqrt(-1/(a*b^(2/3)))*log((b^6*x^18 + 7800*a*b^5*x^15 + 535272*
a^2*b^4*x^12 + 5147264*a^3*b^3*x^9 + 10516992*a^4*b^2*x^6 + 5922816*a^5*b*x
^3 + 557056*a^6 + 144*(7*b^5*x^16 + 1169*a*b^4*x^13 + 20266*a^2*b^3*x^10 +
66976*a^3*b^2*x^7 + 58112*a^4*b*x^4 + 10240*a^5*x)*a^(2/3)*b^(1/3) + 72*(b^
5*x^17 + 581*a*b^4*x^14 + 19108*a^2*b^3*x^11 + 106336*a^3*b^2*x^8 + 137984*
a^4*b*x^5 + 50176*a^5*x^2)*a^(1/3)*b^(2/3) + 12*((b^5*x^16 + 1568*a*b^4*x^1
3 + 72520*a^2*b^3*x^10 + 498304*a^3*b^2*x^7 + 625664*a^4*b*x^4 + 139264*a^5
*x)*sqrt(-b*x^3 - a)*a^(2/3)*b^(2/3) + 6*(41*a*b^5*x^14 + 4268*a^2*b^4*x^11
 + 52896*a^3*b^3*x^8 + 116480*a^4*b^2*x^5 + 48128*a^5*b*x^2)*sqrt(-b*x^3 -
a)*a^(1/3) + (25*a*b^5*x^15 + 7202*a^2*b^4*x^12 + 167392*a^3*b^3*x^9 + 6472
96*a^4*b^2*x^6 + 468992*a^5*b*x^3 + 40960*a^6)*sqrt(-b*x^3 - a)*b^(1/3))*sq
rt(-1/(a*b^(2/3))))/(b^6*x^18 - 48*a*b^5*x^15 + 960*a^2*b^4*x^12 - 10240*a^
3*b^3*x^9 + 61440*a^4*b^2*x^6 - 196608*a^5*b*x^3 + 262144*a^6)), -1/3*a^(1/
3)*sqrt(1/(a*b^(2/3)))*arctan(1/6*((11*b*x^4 - 16*a*x)*sqrt(-b*x^3 - a)*a^(
2/3)*b^(2/3) + (b^2*x^5 + 28*a*b*x^2)*sqrt(-b*x^3 - a)*a^(1/3) - (17*a*b*x^
3 - 10*a^2)*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(1/(a*b^(2/3)))/(b^2*x^6 + 2*a*b*
x^3 + a^2))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\,\sqrt{-a - bx^3} + \sqrt[3]{b}\,x\sqrt{-a - bx^3}}\,dx - \int \frac{\sqrt[3]{b}\,x}{-2\sqrt[3]{a}\,\sqrt{-a - bx^3} + \sqrt[3]{b}\,x\sqrt{-a - bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),
x)
```

```
[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*
x*sqrt(-a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, alg
orithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 5.37, size = 78, normalized size = 1.47

$$\frac{\ln\left(\frac{\left(\sqrt{-b\,x^3-a}-\sqrt{a}\;1{\rm i}\right)\,\left(2\,a^{1/6}\,b^{1/3}\,x-\sqrt{a}+\sqrt{-b\,x^3-a}\;1{\rm i}\right)^3}{x^3\,\left(b^{1/3}\,x-2\,a^{1/3}\right)^3}\right)\,1{\rm i}}{3\,a^{1/6}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] (log((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2)
+ 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b
^(1/3))
```

3.82 $\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

```
[Out] -2/3*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d/c^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2163, 212}

$$-\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx &= -\frac{(2c)\text{Subst}\left(\int \frac{1}{9-c^3x^2}\,dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}\,d}\end{aligned}$$

Mathematica [A]
time = 1.38, size = 44, normalized size = 0.96

$$-\frac{2\tanh^{-1}\left(\frac{3\sqrt{c}\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 503, normalized size = 10.93

method	result
default	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\cdots}\right.}{\sqrt{-8d^3x^3+c^3}}$
elliptic	$-\frac{4\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\cdots}\right.}{\sqrt{-8d^3x^3+c^3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(
1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-
1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1
/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/
2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2-1/
2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4*(1/2*(-1
/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1
/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3
```

```
^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2-1/
2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/2*c/d)/(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/2-1/2*I*3^(
1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1
/2*I*3^(1/2))*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(38) = 76$.
time = 0.44, size = 294, normalized size = 6.39

$$\left[\frac{\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x^3+c^3}\,\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{6\,\sqrt{c}\,d}, -\frac{\sqrt{-c}\,\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3)\sqrt{-8\,d^3x^3+c^3}\,\sqrt{-c}}{3\,(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5)}\right)}{3\,cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 15
6*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*
d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c
*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c
^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^
2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^
3 - 2*c^4*d*x + c^5))/(c*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right)dx-\int\frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x
) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**
3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [B]
time = 3.13, size = 67, normalized size = 1.46

$$\frac{\ln\left(\frac{\left(\sqrt{c^3-8\,d^3\,x^3}-c^{3/2}\right)\,{\left(\sqrt{c^3-8\,d^3\,x^3}+c^{3/2}+4\,\sqrt{c}\,d\,x\right)}^3}{x^3\,{\left(c+d\,x\right)}^3}\right)}{3\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2)
 + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)
```

3.83 $\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=139

$$\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7$$

$$3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}$$

```
[Out] 2/9*(e+2*f)*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/
2)
```

Rubi [A]
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2164, 224, 2163, 212}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,(e-f)F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 + (2*Sqrt[2 + Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 +
x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{1+x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{1+x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2}{9}(e+2f)\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\ldots}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)\ldots}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.21, size = 273, normalized size = 1.96

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+2\sqrt{3}(e+2f)\sqrt{-i+\sqrt{3}+2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}+2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(114) = 228$.
time = 0.25, size = 246, normalized size = 1.77

method	result
default	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2(e+}{}$
elliptic	$-\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2(-e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^
(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2
+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e+2*f)*(3/2-1/2*I*3^(1/2)
)*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1
/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2
)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2
*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 63, normalized size = 0.45

$$\frac{1}{9}\,(2\,f+e)\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)-\frac{2}{3}\,(f-e)\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*f + e)*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3
- 6*x^2 + 12*x - 8)) - 2/3*(f - e)*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{e}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx-\int\frac{fx}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x) - Integral(f*x/(x*sqr
t(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B]
time = 2.71, size = 327, normalized size = 2.35

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2
```

```
- x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, a
sin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/
2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)
```

3.84 $\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=153

$$-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

```
[Out] -2/9*(e-2*f)*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF(
(1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+
x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^
(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2164, 224, 2163, 212}

$$-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(e+f)F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 -
 x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{1-x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2}{9}(e-2f)\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right.}}}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.21, size = 271, normalized size = 1.77

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}\left(-1+i\sqrt{3}+x+i\sqrt{3}\,x\right)F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}\,(e-2f)\sqrt{-i+\sqrt{3}-2ix}\,\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}-2ix}\,\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[1 - x^3])
```

Maple [A]
time = 0.25, size = 246, normalized size = 1.61

method	result
default	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$
elliptic	$-\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-2*f)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*
3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1
/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [A]
time = 0.11, size = 55, normalized size = 0.36

$$-\frac{1}{9}\,(2\,f-e)\log\left(-\frac{x^{3}-12\,x^{2}-6\,\sqrt{-x^{3}+1}\,(x-1)-6\,x-10}{x^{3}+6\,x^{2}+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*f - e)*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x
^3 + 6*x^2 + 12*x + 8))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e+fx}{\sqrt{-\left(x-1\right)\left(x^{2}+x+1\right)}\,\left(x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B]
time = 0.19, size = 359, normalized size = 2.35

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(e-2\,f\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(x + 2)),x)

[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/
((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*((
3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)
*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1
i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2
 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
+ 1) + x^3)^(1/2))
```

3.85 $\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

```
[Out] -2/9*(e-2*f)*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(e+f)*(1-x)*EllipticF((1
-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+
1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1
/2)
```

Rubi [A]

time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2164, 225, 2163, 209}

$$-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,(e+f)F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}-\frac{2}{9}\text{ArcTan}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)(e-2f)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*(e - 2*f)*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]
]*(e + f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[
(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{6}(e-2f)\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx + \frac{1}{3}(e+f)\int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{2}{9}(e-2f)\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}\,(e+f)(1-x)\sqrt{\frac{1+x+x}{\left(1-\sqrt{3}-\right.}}}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.13, size = 269, normalized size = 1.72

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(-1+x)}{-3i+\sqrt{3}}}\left(3f\sqrt{i+\sqrt{3}+2ix}\left(-1+i\sqrt{3}+x+i\sqrt{3}\,x\right)F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)-2\sqrt{3}(e-2f)\sqrt{-i+\sqrt{3}-2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}-2ix}\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(3*f*Sqrt[I + Sqrt[3] + (2
*I)*x]*(-1 + I*Sqrt[3] + x + I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3
] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - 2*Sqrt[3]*
(e - 2*f)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt
[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*
I)*x]*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.28, size = 246, normalized size = 1.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+$ 2(e−
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+$ 2(e−

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-2*f)*(-3/2-1/2*I*3^(1/2
))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/
2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1
/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 59, normalized size = 0.38

$$\frac{1}{9}\,(2\,f-e)\arctan\left(\frac{(x^3-12\,x^2-6\,x-10)\sqrt{x^3-1}}{6\,(x^4-x^3-x+1)}\right)+\frac{2}{3}\,(f+e)\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*f - e)*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3
 - x + 1)) + 2/3*(f + e)*weierstrassPInverse(0, 4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{e+fx}{\sqrt{(x-1)\,(x^2+x+1)}\,\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B]
time = 0.12, size = 327, normalized size = 2.10

$$-\frac{2\,f\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e-2\,f)\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
- 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(
```

```
x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/
2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)*(e - 2*f)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/
2 + 1/2) + 1) + x^3)^(1/2))
```

3.86 $\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=150

$$\frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

```
[Out] 2/9*(e+2*f)*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))+2/9*(e-f)*(1+x)*EllipticF((1
+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2164, 225, 2163, 209}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,(e-f)F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}+\frac{2}{9}\text{ArcTan}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)(e+2f)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(e + 2*f)*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 + (2*Sqrt[2 - Sqrt[3]]
*(e - f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1
+ x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(2-x)\sqrt{-1-x^3}}\,dx = \frac{1}{3}(e-f)\int \frac{1}{\sqrt{-1-x^3}}\,dx + \frac{1}{6}(e+2f)\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{}\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2}{9}(e+2f)\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(e-f)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+\cdots\right.}}}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+\cdots\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.15, size = 275, normalized size = 1.83

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(1+x)}{-3i+\sqrt{3}}}\left(-3if\sqrt{i+\sqrt{3}-2ix}\left(-i-\sqrt{3}+\left(-i+\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)+2\sqrt{3}(e+2f)\sqrt{-i+\sqrt{3}+2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-3i+\sqrt{3}}\right)\right)}{\left(3i+\sqrt{3}\right)\sqrt{-i+\sqrt{3}+2ix}\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(1 + x))/(-3*I + Sqrt[3])]*((-3*I)*f*Sqrt[I + Sqrt[
3] - (2*I)*x]*(-I - Sqrt[3] + (-I + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 2*Sq
rt[3]*(e + 2*f)*Sqrt[-I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(
2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^
(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3*I + Sqrt[3])*Sqrt[-I + Sqrt[3]
 + (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.25, size = 246, normalized size = 1.64

method	result
default	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}\right.}}{}\right.}{3 \sqrt{-x^{3}-1}}$
elliptic	$\frac{2 i f \sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{1+x}{\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x-\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x-\frac{1}{2}\right.}}{}\right.}{3 \sqrt{-x^{3}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1
/2*I*3^(1/2)))^(1/2))+2/3*I*(e+2*f)*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2
))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [A]
time = 0.12, size = 44, normalized size = 0.29

$$-\frac{1}{9}\,(2\,f+e)\arctan\left(\frac{(x^3+12\,x^2-6\,x+10)\sqrt{-x^3-1}}{6\,(x^4+x^3+x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*f + e)*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x
^3 + x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{e}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx-\int\frac{fx}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x) - Integral(f*x/(x*s
qrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(f*x + e)/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B]
time = 2.54, size = 359, normalized size = 2.39

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(e+2\,f)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{3\,\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((- x^3 - 1)^(1/2)*(x - 2)),x)

[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(
1/2)*1i)/2 - 3/2))^(1/2)*(e + 2*f)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(
((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2
)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)
/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2
)*1i)/2 + 1/2))^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x +
(3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*e
llipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3
/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2))^(1/2))
```

3.87 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=297

$$\frac{2\left(\sqrt[3]{b}\,e+2\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{\ }}{\left(\left(1+\sqrt{3}\right.\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\right.\right.}}}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^
3+a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/(b*x^
3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2164, 224, 2163, 212}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{2\left(2\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx\right)-\frac{1}{3}\left(-\frac{e}{\sqrt[3]{a}}\right.$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}$$

$$= \frac{2\left(\sqrt[3]{b}\,e+2\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.91, size = 419, normalized size = 1.41

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\sqrt[4]{3}f\left(\left(i+\sqrt{3}\right)\sqrt[3]{a}-\left(-i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(\sqrt[3]{b}\,e+2\sqrt[3]{a}\,f\right)\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-1/2*(3^(1/4)*f*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
) - Integral(f*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3
)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.88 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{2\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{b}\,e+\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right.\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}}{\left(\left(1+\right.\right.}}}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*
x^3+a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)
*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^(2/3)/
(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2164, 224, 2163, 212}

$$-\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a-bx^3}}-\frac{2\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sq
rt[a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx + \frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

$$= -\frac{2\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.87, size = 447, normalized size = 1.47

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-i\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[
((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin
[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1
/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1
/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt
[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.89 $\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=313

$$-\frac{2\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e+\sqrt[3]{a}f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\cdots}{\left(\left(1-\cdots\right.\right.}}}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{\cdots}}{\left(\left(1\cdots\right.\right.}}}$$

```
[Out] -2/9*(b^(1/3)*e-2*a^(1/3)*f)*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)-2/9*(b^(1/3)*e+a^(1/3)*f)*(a^(1/3)-b^(1/3)*x)*E
llipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(b
*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2164, 225, 2163, 209}

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{2\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}}\right)\left(\sqrt[3]{b}e-2\sqrt[3]{a}f\right)}{9\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*(b^(1/3)*e - 2*a^(1/3)*f)*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqr
t[-a + b*x^3])])/(9*Sqrt[a]*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + a^
(1/3)*f)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]]
)/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - S
qrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=\frac{1}{6}\left(\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx+\frac{1}{3}\left(\frac{e}{\sqrt[3]{a}}+\right.$$

$$=-\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}+\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}{3\sqrt[4]{3}\,\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

$$=-\frac{2\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.72, size = 448, normalized size = 1.43

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{1}{2}if\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\left(\left(-3i+\sqrt{3}\right)\sqrt[3]{a}-\left(3i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-i\left(\sqrt[3]{b}\,e-2\sqrt[3]{a}\,f\right)\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-1/2*I)*f*Sqrt[
((-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*((-3*I + Sqrt[3])*a^(1/3) - (3*I + Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin
[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1
/3))]], (1 + I*Sqrt[3])/2] - I*(b^(1/3)*e - 2*a^(1/3)*f)*Sqrt[((-I)*(2*a^(1
/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(
1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt
[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + S
qrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a
^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3]
)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
[Out] int((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxim
a")
[Out] integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] Integral((e + f*x)/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] int((e + f*x)/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.90 $\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=310

$$\frac{2\left(\sqrt[3]{b}e+2\sqrt[3]{a}f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\cdots\right.\right.}}}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\cdots}{\left(\left(1-\cdots\right.\right.}}}$$

```
[Out] 2/9*(b^(1/3)*e+2*a^(1/3)*f)*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^
3-a)^(1/2))/b^(2/3)/a^(1/2)+2/9*(b^(1/3)*e-a^(1/3)*f)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*
I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(3/4)/a^(1/3)/b^(2/3)/(-b*x
^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2164, 225, 2163, 209}

$$\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{2\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)\left(2\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{9\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*(b^(1/3)*e + 2*a^(1/3)*f)*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt
[-a - b*x^3])])/(9*Sqrt[a]*b^(2/3)) + (2*Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - a^(
1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])
/(3*3^(1/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = -\left(\frac{1}{6}\left(-\frac{e}{\sqrt[3]{a}}-\frac{2f}{\sqrt[3]{b}}\right)\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx\right)-\frac{1}{3}\Big(-$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}-\frac{f}{\sqrt[3]{b}}\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}{3\sqrt[4]{3}\sqrt[3]{b}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}}}$$

$$= \frac{2\left(\sqrt[3]{b}\,e+2\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt{a}\,b^{2/3}}+\frac{2\sqrt{2-\sqrt{3}}\left(\frac{e}{\sqrt[3]{a}}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.21, size = 422, normalized size = 1.36

$$\frac{2\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{\sqrt[4]{3}f\left(\left(i+\sqrt{3}\right)\sqrt[3]{a}-\left(-i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(\sqrt[3]{b}\,e+2\sqrt[3]{a}\,f\right)\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-1/2*(3^(1/4)*f*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2])/Sqrt[2] + I*(
b^(1/3)*e + 2*a^(1/3)*f)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((
-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2
/3)]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) +
(I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))
/((-2 + (-1)^(1/3))*b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1
)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-2\sqrt[3]{a}\,\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\,dx-\int \frac{fx}{-2\sqrt[3]{a}\,\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x) - Integral(f*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*
x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac
")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] int(-(e + f*x)/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.91 $\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=221

$$-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}-\frac{\sqrt{2+\sqrt{3}}\,(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\Big(}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{}}$$

```
[Out] -2/9*(-c*f+d*e)*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/c^
(3/2)/d^2-1/9*(c*f+2*d*e)*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d
*x+c*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*
d*x+c^2)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^2/(-8*d^3*x^3+c^3)^(1/
2)/(c*(-2*d*x+c)/(-2*d*x+c*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2164, 224, 2163, 212}

$$-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,(cf+2de)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}-\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (-2*(d*e - c*f)*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(
9*c^(3/2)*d^2) - (Sqrt[2 + Sqrt[3]]*(2*d*e + c*f)*(c - 2*d*x)*Sqrt[(c^2 + 2
*c*d*x + 4*d^2*x^2)/((1 + Sqrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqr
t[3])*c - 2*d*x)/((1 + Sqrt[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*c*
d^2*Sqrt[(c*(c - 2*d*x))/((1 + Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{(de-cf)\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3cd} + \frac{(2de+cf)\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3cd}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(2de+cf)(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{}}{(1+\sqrt{}}\right.\right.}{3\sqrt[4]{3}\,cd^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}$$

$$= -\frac{2(de-cf)\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{9c^{3/2}d^2} - \frac{\sqrt{2+\sqrt{3}}\,(2de+cf)(c-2dx}{3\sqrt[4]{3}\,}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.80, size = 384, normalized size = 1.74

$$-\frac{i\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(f\sqrt{\frac{\left(-i+\sqrt{3}\right)c+2\left(i+\sqrt{3}\right)dx}{\left(-3i+\sqrt{3}\right)c}}\left(\left(-3i+\sqrt{3}\right)c-2\left(3i+\sqrt{3}\right)dx\right)F\left(\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ic+idx+\sqrt{3}\,dx}{3ic-\sqrt{3}\,c}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+4\sqrt{2}\,(de-cf)\sqrt{\frac{ic+idx+\sqrt{3}\,dx}{3ic-\sqrt{3}\,c}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{2}\sqrt{\frac{ic+idx+\sqrt{3}\,dx}{3ic-\sqrt{3}\,c}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{2\left(-2+\sqrt[3]{-1}\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] ((-1/2*I)*Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*(f*Sqrt[((-I + Sqrt[3])*c
+ 2*(I + Sqrt[3])*d*x)/((-3*I + Sqrt[3])*c)]*((-3*I + Sqrt[3])*c - 2*(3*I +
 Sqrt[3])*d*x)*EllipticF[ArcSin[Sqrt[2]*Sqrt[(I*c + I*d*x + Sqrt[3]*d*x)/((
3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2] + 4*Sqrt[2]*(d*e - c*f)*Sqrt[(I*c
 + I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*
x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[2]*Sqrt[(I*c
+ I*d*x + Sqrt[3]*d*x)/((3*I)*c - Sqrt[3]*c)]], (1 + I*Sqrt[3])/2]))/((-2 +
 (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3
 - 8*d^3*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(192) = 384.
time = 0.25, size = 521, normalized size = 2.36

method	result
default	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\cdots}{\left(-\frac{1}{2}-\cdots\right.}}\right.\cdots}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2f\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\cdots}{\left(-\frac{1}{2}-\cdots\right.}}\right.\cdots}{d\sqrt{-8d^3x^3+c^3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*
3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2
```

```
*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/
d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x
-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2
-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))+4/3*(-c
*f+d*e)/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/
2*I*3^(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d
-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/
2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)/c*Ellipti
cPi(((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d
-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/
(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.21, size = 400, normalized size = 1.81

$$\left[-\frac{3\sqrt{2}\sqrt{-d^3}\,(c^2f+2\,cde)\mathrm{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)+(cd^2f-d^3e)\sqrt{c}\,\log\left(\frac{8\,d^6x^6-240\,cd^5x^5+408\,c^2d^4x^4+88\,c^3d^3x^3+156\,c^4d^2x^2+12\,c^5dx+17\,c^6-3\,(8\,d^4x^4-52\,cd^3x^3+12\,c^2d^2x^2-4\,c^3dx+5\,c^4)\sqrt{-8\,d^3x^3+c^3}\sqrt{c}}{d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx+c^6}\right)}{18\,c^2d^4},-\frac{3\sqrt{2}\sqrt{-d^3}\,(c^2f+2\,cde)\mathrm{weierstrassPInverse}\left(0,\frac{c^3}{2\,d^3},x\right)-2\,(cd^2f-d^3e)\sqrt{-c}\,\arctan\left(\frac{(4\,d^3x^3-24\,cd^2x^2-6\,c^2dx-5\,c^3)\sqrt{-8\,d^3x^3+c^3}\sqrt{-c}}{3\,(16\,cd^4x^4-8\,c^2d^3x^3-2\,c^4dx+c^5)}\right)}{18\,c^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
[Out] [-1/18*(3*sqrt(2)*sqrt(-d^3)*(c^2*f + 2*c*d*e)*weierstrassPInverse(0, 1/2*c
^3/d^3, x) + (c*d^2*f - d^3*e)*sqrt(c)*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408
*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(
8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*
x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^
3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6)))/(c^2*d^4), -1/18*(3*sqrt(2)*sqrt(-d
^3)*(c^2*f + 2*c*d*e)*weierstrassPInverse(0, 1/2*c^3/d^3, x) - 2*(c*d^2*f -
 d^3*e)*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x - 5*c^3)*
sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^3 - 2*c^4*d*x +
 c^5)))/(c^2*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-\left(-c+2dx\right)\left(c^2+2cdx+4d^2x^2\right)}\,\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c +
 d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((e + f*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.92 $\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=129

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 4/9*arctanh(1/3*(1+x)^2/(x^3+1)^(1/2))-2/9*(1+x)*EllipticF((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2
))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2164, 224, 2163, 212}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(x+1)^2}{3\sqrt{x^3+1}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\mathrm{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (4*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] +
x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{1+x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{1+x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+$$

$$= \frac{4}{9}\tanh^{-1}\left(\frac{(1+x)^2}{3\sqrt{1+x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 193, normalized size = 1.50

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-2+\sqrt[3]{-1}}\right)}{\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 + x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(104) = 208$.
time = 0.26, size = 240, normalized size = 1.86

method	result
default	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{}}{}\right.}{}$
elliptic	$-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{4\left(\frac{3}{2}-\frac{i\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+4/3*(3/2-1/2*I*3^(1/2))*((1+x)/(
3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2
)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*Elliptic
Pi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/2-1/6*I*3^(1/2),((-3/2+1/2*I*3^(1/2)
)/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 51, normalized size = 0.40

$$\frac{2}{9}\log\left(\frac{x^3+12\,x^2+6\,\sqrt{x^3+1}\,(x+1)-6\,x+10}{x^3-6\,x^2+12\,x-8}\right)-\frac{2}{3}\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 +
12*x - 8)) - 2/3*weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 + 1) - 2*sqrt(x**3 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(x^3 + 1)*(x - 2)), x)
```

Mupad [B]
time = 0.06, size = 207, normalized size = 1.60

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{3\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^3 + 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((3^(
1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*
1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
)^(1/2))
```

3.93 $\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=145

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{1-x^3}}$$

```
[Out] 4/9*arctanh(1/3*(1-x)^2/(-x^3+1)^(1/2))-2/9*(1-x)*EllipticF((1-x-3^(1/2))/(
1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/
2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2164, 224, 2163, 212}

$$\frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right)-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (4*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/9 - (2*Sqrt[2 + Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(
1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
x)^2]*Sqrt[1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(2+x)\sqrt{1-x^3}}\,dx &= \frac{1}{3}\int \frac{1}{\sqrt{1-x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{1-x^3}}\,dx\\ &= -\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{4}{3}\cdots\\ &= \frac{4}{9}\tanh^{-1}\left(\frac{(1-x)^2}{3\sqrt{1-x^3}}\right) - \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\cdots}{1+\cdots}\right.\right.}{3\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}\cdots}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 195, normalized size = 1.34

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-2+\sqrt[3]{-1}}\right)}{\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A]
time = 0.25, size = 240, normalized size = 1.66

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x \cdots\right.}}{\cdots}\right.}{3\sqrt{-x^3+1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x \cdots\right.}}{\cdots}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/
2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Fricas [A]
time = 0.10, size = 47, normalized size = 0.32

$$\frac{2}{9}\,\log\left(-\frac{x^3-12\,x^2+6\,\sqrt{-x^3+1}\,(x-1)-6\,x-10}{x^3+6\,x^2+12\,x+8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/9*log(-(x^3 - 12*x^2 + 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2
+ 12*x + 8))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{-\left(x-1\right)\left(x^2+x+1\right)}\,\left(x+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 2)), x)
```

Mupad [B]
time = 0.07, size = 224, normalized size = 1.54

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{3\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 2*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.94 $\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=148

$$\frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 4/9*arctan(1/3*(1-x)^2/(x^3-1)^(1/2))-2/9*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2164, 225, 2163, 209}

$$\frac{4}{9}\text{ArcTan}\left(\frac{(1-x)^2}{3\sqrt{x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (4*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3]
- x)^2)]*Sqrt[-1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2+x)\sqrt{-1+x^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{-1+x^3}}\,dx - \frac{1}{3}\int \frac{2-2x}{(2+x)\sqrt{-1+x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1-x)^2}{3\sqrt{-1+x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 193, normalized size = 1.30

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{-2+\sqrt[3]{-1}}\right)}{\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(118) = 236$.
time = 0.30, size = 240, normalized size = 1.62

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4/3*(-3/2-1/2*I*3^(1/2))*((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Ellipti
cPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 47, normalized size = 0.32

$$\frac{2}{9}\arctan\left(\frac{\left(x^{3}-12\,x^{2}-6\,x-10\right)\sqrt{x^{3}-1}}{6\left(x^{4}-x^{3}-x+1\right)}\right)+\frac{2}{3}\,\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))
 + 2/3*weierstrassPInverse(0, 4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{(x-1)\left(x^{2}+x+1\right)}\,(x+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 2)), x)
```

Mupad [B]
time = 2.53, size = 208, normalized size = 1.41

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{3\sqrt{x^{3}+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x + 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2*ellipticPi((
3^(1/2)*1i)/6 + 1/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.95 $\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=140

$$\frac{4}{9}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

```
[Out] 4/9*arctan(1/3*(1+x)^2/(-x^3-1)^(1/2))-2/9*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2164, 225, 2163, 209}

$$\frac{4}{9}\text{ArcTan}\left(\frac{(x+1)^2}{3\sqrt{-x^3-1}}\right)-\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\,\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (4*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/9 - (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3]
+ x)^2)]*Sqrt[-1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(2-x)\sqrt{-1-x^3}}\,dx = -\left(\frac{1}{3}\int \frac{1}{\sqrt{-1-x^3}}\,dx\right)+\frac{1}{3}\int \frac{2+2x}{(2-x)\sqrt{-1-x^3}}\,dx$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + $$

$$= \frac{4}{9}\tan^{-1}\left(\frac{(1+x)^2}{3\sqrt{-1-x^3}}\right) - \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{3\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 195, normalized size = 1.39

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-2+\sqrt[3]{-1}}\right)}{\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2 - x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((2
*I)*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(
1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-2 + (-1)^(1/3))))/Sqrt
[-1 - x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(114) = 228$.
time = 0.25, size = 240, normalized size = 1.71

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^{3}-1}}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^{3}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(
(1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(
-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3
^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I
*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Fricas [A]
time = 0.12, size = 38, normalized size = 0.27

$$-\frac{2}{9}\arctan\left(\frac{(x^3+12x^2-6x+10)\sqrt{-x^3-1}}{6(x^4+x^3+x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] -2/9*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x**3-1)**(1/2),x)
[Out] -Integral(x/(x*sqrt(-x**3 - 1) - 2*sqrt(-x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x/(sqrt(-x^3 - 1)*(x - 2)), x)
```

Mupad [B]
time = 0.06, size = 223, normalized size = 1.59

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(3\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-2\,\Pi\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{6};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{3\sqrt{-x^3-1}\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((- x^3 - 1)^(1/2)*(x - 2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i
)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
 x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3*ellipticF(asin(((x + 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 2
*ellipticPi((3^(1/2)*1i)/6 + 1/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(3*(- x^3 - 1)^(1/2)
*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.96 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=260

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1-\sqrt{\ }}{(1+\sqrt{\ }}\right.\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(b*x^3+a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2164, 224, 2163, 212}

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\Bigg(\sin^{-1}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}$$

$$= \frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-}{\left(\left(1+\right.\right.}}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.02, size = 407, normalized size = 1.57

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\sqrt{2}\sqrt[4]{3}\left(\left(i+\sqrt{3}\right)\sqrt[3]{a}-\left(-i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+8i\sqrt[3]{a}\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{2\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[a + b*x^3])
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2\sqrt[3]{a}\,\sqrt{a+bx^{3}}+\sqrt[3]{b}\,x\sqrt{a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x
)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)
```

```
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)), x)
```

3.97 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=268

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\cdots\right.}{\left(1+\cdots\right.}\right.\right.}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-\cdots}}$$

```
[Out] 4/9*arctanh(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(-b*x^3+a)^(1/2))/a^(1/6)/b^(
2/3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^
(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2164, 224, 2163, 212}

$$\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (4*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 + Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx=\frac{\int \frac{1}{\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-}\cdots\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\cdots}$$

$$=\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}\cdots}{\left(\left(1+\cdots\right.\right.}}\cdots}{3\sqrt[4]{3}\,b^{2/3}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.53, size = 371, normalized size = 1.38

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(a - b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{a-b\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)
```

```
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)), x)
```

3.98 $\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=277

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{(1+\cdots}{(1-\cdots}\right.\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a+\cdots}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)-b^(1/3)*x)^2/a^(1/6)/(b*x^3-a)^(1/2))/a^(1/6)/b^(2/
3)-2/9*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(
1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2)
)*3^(3/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2164, 225, 2163, 209}

$$\frac{4\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{bx^3-a}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\middle|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=\frac{\int \frac{1}{\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}-\frac{\int \frac{2\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$=-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{\cdots}\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\cdots}$$

$$=\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}\cdots}{\left(\left(1-\cdots\right.\right.}}}{3\sqrt[4]{3}\,b^{2/3}\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 372, normalized size = 1.34

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\,x\right)}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2 + (-1)^(1/3)
)*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b
^(1/3)*x))/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1
)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] + (2*(-1)^(1/3
)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)
^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*Ell
ipticPi[(2*Sqrt[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1
/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/
3))*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)
)]*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2a^{\frac{1}{3}}+b^{\frac{1}{3}}x\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
[Out] int(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a+bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)
[Out] Integral(x/((2*a**(1/3) + b**(1/3)*x)*sqrt(-a + b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b^{1/3}\,x+2\,a^{1/3}\right)\,\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)
```

```
[Out] int(x/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)), x)
```

3.99 $\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=273

$$\frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\left(1+\right.\right.\right.}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a}}$$

```
[Out] 4/9*arctan(1/3*(a^(1/3)+b^(1/3)*x)^2/a^(1/6)/(-b*x^3-a)^(1/2))/a^(1/6)/b^(2
/3)-2/9*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1
/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*
3^(3/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2164, 225, 2163, 209}

$$\frac{4\text{ArcTan}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}}-\frac{2\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (4*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])])/(9*a^(1/6)
*b^(2/3)) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 + 4*Sqrt[3]])/(3*3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{1}{\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}} + \frac{\int \frac{2\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\left(2\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx}{3\sqrt[3]{b}}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\;F\Big(\sin \ldots}{3\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right) \ldots}}}$$

$$= \frac{4\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}}\right)}{9\sqrt[6]{a}\,b^{2/3}} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/ \ldots}{\left(\left(1 \ldots\right.\right.}}}{3\sqrt[4]{3}\,b^{2/3} \ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 410, normalized size = 1.50

$$\frac{\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\sqrt{2}\sqrt[4]{3}\left(\left(i+\sqrt{3}\right)\sqrt[3]{a}-\left(-i+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+8i\sqrt[3]{a}\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{2\left(-2+\sqrt[3]{-1}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(Sqrt[2]*3^(1/4)*
((I + Sqrt[3])*a^(1/3) - (-I + Sqrt[3])*b^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I
)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])
*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]) + (8*I)*a^(1/
3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3
))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqr
t[3])/(3*I + Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*
x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/(2*(-2 + (-1)^(1/3))*
b^(2/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*S
qrt[-a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(2a^{\frac{1}{3}}-b^{\frac{1}{3}}x\right)\sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
[Out] int(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")
[Out] -integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-2\sqrt[3]{a}\,\sqrt{-a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{-a-bx^{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2),x)
[Out] -Integral(x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)),
 x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\left(b^{1/3}\,x-2\,a^{1/3}\right)\,\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)
```

```
[Out] -int(x/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)), x)
```

3.100 $\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx$

Optimal. Leaf size=202

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}$$

```
[Out] 2/9*arctanh(1/3*(-2*d*x+c)^2/c^(1/2)/(-8*d^3*x^3+c^3)^(1/2))/d^2/c^(1/2)-1/
9*(-2*d*x+c)*EllipticF((-2*d*x+c*(1-3^(1/2)))/(-2*d*x+c*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((4*d^2*x^2+2*c*d*x+c^2)/(-2*d*x+c*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/d^2/(-8*d^3*x^3+c^3)^(1/2)/(c*(-2*d*x+c)/(-2*d*x+c
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2164, 224, 2163, 212}

$$\frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2}-\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(9*Sqrt[c]*d^2
) - (Sqrt[2 + Sqrt[3]]*(c - 2*d*x)*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/((1 + S
qrt[3])*c - 2*d*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c - 2*d*x)/((1 + Sqrt
[3])*c - 2*d*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^2*Sqrt[(c*(c - 2*d*x))/((1
+ Sqrt[3])*c - 2*d*x)^2]*Sqrt[c^3 - 8*d^3*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2163

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[-2*(e/d), Subst[Int[1/(9 - a*x^2), x], x, (1 + f*(x/e))^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2164

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[(2*d*e + c*f)/(3*c*d), Int[1/Sqrt[a + b*x^3], x], x] + Dis
t[(d*e - c*f)/(3*c*d), Int[(c - 2*d*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && (EqQ[b*c^3 - 4*a*d^
3, 0] || EqQ[b*c^3 + 8*a*d^3, 0]) && NeQ[2*d*e + c*f, 0]
```

Rubi steps

$$\int \frac{x}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx = \frac{\int \frac{1}{\sqrt{c^3-8d^3x^3}}\,dx}{3d} - \frac{\int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}}\,dx}{3d}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d^2x^2}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)c-2dx}{\left(1+\sqrt{3}\right)c-2dx}\right)\right.}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c-2dx)}{\left(\left(1+\sqrt{3}\right)c-2dx\right)^2}}\,\sqrt{c^3-8d^3x^3}}$$

$$= \frac{2\tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\,\sqrt{c^3-8d^3x^3}}\right)}{9\sqrt{c}\,d^2} - \frac{\sqrt{2+\sqrt{3}}\,(c-2dx)\sqrt{\frac{c^2+2cdx+4d}{\left(\left(1+\sqrt{3}\right)c-\right.}}}{3\sqrt[4]{3}\,d^2\sqrt{\frac{c(c}{\left(\left(1+\sqrt{3}\right.\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.47, size = 295, normalized size = 1.46

$$\frac{\sqrt{\frac{c-2dx}{\left(1+\sqrt[3]{-1}\right)c}}\left(\left(-2+\sqrt[3]{-1}\right)\left(\sqrt[3]{-1}\,c+2dx\right)\sqrt{\frac{\sqrt[3]{-1}\,\left(c+2\sqrt[3]{-1}\,dx\right)}{\left(1+\sqrt[3]{-1}\right)c}}\,F\left(\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)\middle|\sqrt[3]{-1}\right)+\frac{2\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)c\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{\frac{c^2+2cdx+4d^2x^2}{c^2}}\,\Pi\left(\frac{2\sqrt{3}}{3i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{3}}\right)}{\left(-2+\sqrt[3]{-1}\right)d^2\sqrt{\frac{c-2(-1)^{2/3}dx}{\left(1+\sqrt[3]{-1}\right)c}}\sqrt{c^3-8d^3x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]),x]
```

```
[Out] (Sqrt[(c - 2*d*x)/((1 + (-1)^(1/3))*c)]*((-2 + (-1)^(1/3))*((-1)^(1/3)*c +
2*d*x)*Sqrt[((-1)^(1/3)*(c + 2*(-1)^(1/3)*d*x))/((1 + (-1)^(1/3))*c)]*Ellip
ticF[ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-1)^(1/3)]
 + (2*(-1)^(1/3)*(1 + (-1)^(1/3))*c*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^
(1/3))*c)]*Sqrt[(c^2 + 2*c*d*x + 4*d^2*x^2)/c^2]*EllipticPi[(2*Sqrt[3])/(3*
I + Sqrt[3]), ArcSin[Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]], (-
1)^(1/3)])/Sqrt[3]))/((-2 + (-1)^(1/3))*d^2*Sqrt[(c - 2*(-1)^(2/3)*d*x)/((1
 + (-1)^(1/3))*c)]*Sqrt[c^3 - 8*d^3*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(173) = 346$.
time = 0.25, size = 509, normalized size = 2.52

method	result
default	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-\cdots}{\left(-\frac{1}{2}-\frac{i\sqrt{\cdots}}{}\right)\cdots}}\cdots\right.}{d\sqrt{-8d^3x^3+c^3}}$
elliptic	$\frac{2\left(\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}\right)\sqrt{\frac{x-\frac{c}{2d}}{\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}-\frac{c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)c}{2d}}}\sqrt{\frac{x-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}{\frac{c}{2d}-\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)c}{2d}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-\cdots}{\left(-\frac{1}{2}-\frac{i\sqrt{\cdots}}{}\right)\cdots}}\cdots\right.}{d\sqrt{-8d^3x^3+c^3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^
(1/2))*c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(
```

```
-1/2+1/2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-
1/2*(-1/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1
/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),((1/2*c/d-1/2*(-1/2-1
/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^(1/2))-4/3/d*(1/
2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d)*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*
c/d-1/2*c/d))^(1/2)*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1/2+1/
2*I*3^(1/2))*c/d))^(1/2)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-1
/2-1/2*I*3^(1/2))*c/d))^(1/2)/(-8*d^3*x^3+c^3)^(1/2)*EllipticPi(((x-1/2*c/d
)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^(1/2),2/3*(1/2*c/d-1/2*(-1/2-1/2*
I*3^(1/2))*c/d)/c*d,((1/2*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*c/d-1/2*(-
1/2+1/2*I*3^(1/2))*c/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 356, normalized size = 1.76

$$\left[\frac{\sqrt{c}\, d^2 \log\left(\frac{8\, d^6x^6 - 240\, cd^5x^5 + 408\, c^2d^4x^4 + 88\, c^3d^3x^3 + 156\, c^4d^2x^2 + 12\, c^5dx + 17\, c^6 + 3\, (8\, d^4x^4 - 52\, cd^3x^3 + 12\, c^2d^2x^2 - 4\, c^3dx + 5\, c^4)\sqrt{-8\, d^3x^3 + c^3}\sqrt{c}}{d^6x^6 + 6\, cd^5x^5 + 15\, c^2d^4x^4 + 20\, c^3d^3x^3 + 15\, c^4d^2x^2 + 6\, c^5dx + c^6}\right) - 3\sqrt{2}\sqrt{-d^3}\, c\,\mathrm{weierstrassPInverse}\left(0, \frac{c^3}{2\, d^3}, x\right)}{18\, cd^4}, \frac{2\sqrt{-c}\, d^2 \arctan\left(\frac{(4\, d^3x^3 - 24\, cd^2x^2 - 6\, c^2dx - 5\, c^3)\sqrt{-8\, d^3x^3 + c^3}\sqrt{-c}}{3\, (16\, cd^4x^4 - 8\, c^2d^3x^3 - 2\, c^4dx + c^5)}\right) - 3\sqrt{2}\sqrt{-d^3}\, c\,\mathrm{weierstrassPInverse}\left(0, \frac{c^3}{2\, d^3}, x\right)}{18\, cd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(sqrt(c)*d^2*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^
3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 + 3*(8*d^4*x^4 - 52*c*d^3
*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/
(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 +
 6*c^5*d*x + c^6)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0, 1/2*c^3/
d^3, x))/(c*d^4), 1/18*(2*sqrt(-c)*d^2*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2
 - 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2
*d^3*x^3 - 2*c^4*d*x + c^5)) - 3*sqrt(2)*sqrt(-d^3)*c*weierstrassPInverse(0
, 1/2*c^3/d^3, x))/(c*d^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(-c + 2dx\right)\left(c^2 + 2cdx + 4d^2x^2\right)}\,\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(-c + 2*d*x)*(c**2 + 2*c*d*x + 4*d**2*x**2))*(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{c^3-8\,d^3\,x^3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] int(x/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)), x)
```

3.101 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] -2*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [**A**]
time = 1.65, size = 49, normalized size = 1.17

$$-2\sqrt{1+\frac{2}{\sqrt{3}}}\,\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{1+x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
[Out] -2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x
+ x^2)]
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.97, size = 245, normalized size = 5.83

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(-\frac{6\,\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\,\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\text{RootOf}\left(\_Z^2\right.}{}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(32) = 64$.
time = 0.48, size = 205, normalized size = 4.88

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3-\sqrt{3}\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)+24x+8\right)\sqrt{x^3+1}\sqrt{2\sqrt{3}+3}+16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*s
qrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x
 + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*
x^2 - 64*x + 16))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1))
, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2
]%%%}
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.102 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] 2*arctanh((1-x)*(-3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[
3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.63, size = 49, normalized size = 1.07

$$2\sqrt{1+\frac{2}{\sqrt{3}}}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] 2*Sqrt[1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x +
 x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.89, size = 243, normalized size = 5.28

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\ln\left(-\frac{6\,\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)x^2+4\,\text{RootOf}\left(\_Z^2-24\sqrt{3}-36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\text{RootOf}\left(\_Z^2-2\right.}{}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$

elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)
/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3
+1)^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(35) = 70$.
time = 0.44, size = 207, normalized size = 4.50

$$\frac{1}{6}\sqrt{3}\,\sqrt{2\sqrt{3}+3}\,\log\left(\frac{x^{8}+16x^{7}+112x^{6}+16x^{5}+112x^{4}-224x^{3}+64x^{2}+4\left(2x^{6}+18x^{5}+42x^{4}+8x^{3}-\sqrt{3}\,(x^{6}+12x^{5}+18x^{4}+16x^{3}-12x^{2}-8)-24x+8\right)\sqrt{-x^{3}+1}\,\sqrt{2\sqrt{3}+3}-16\sqrt{3}\,(x^{7}+2x^{6}+6x^{5}-5x^{4}+2x^{3}-6x^{2}+4x-4)-128x+112}{x^{8}-8x^{7}+16x^{6}+16x^{5}-56x^{4}-32x^{3}+64x^{2}+64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{-(x-1)(x^2+x+1)}\left(x - 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

3.103 $\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

[Out] 2*arctan((1-x)*(-3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[3]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{\left(1-\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [**A**]
time = 1.62, size = 47, normalized size = 1.07

$$-2\sqrt{1+\frac{2}{\sqrt{3}}}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{-1+x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
+ x^2)]
```

Maple [**C**] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.96, size = 245, normalized size = 5.57

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\,\text{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\,\text{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\text{RootOf}\left(\_Z^2 \right.}{\ldots}\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)
```

Fricas [A]
time = 0.37, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2+4x-2\right)-6x+6\right)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x +
 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}-1}{\sqrt{(x-1)(x^2+x+1)}\left(x-1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3)))
, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

3.104 $\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

```
[Out] -2*arctan((1+x)*(-3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt
[3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{\left(1-\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.63, size = 51, normalized size = 1.16

$$2\sqrt{1+\frac{2}{\sqrt{3}}}\,\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{-1-x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
[Out] 2*Sqrt[1 + 2/Sqrt[3]]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x +
 x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.90, size = 247, normalized size = 5.61

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\ln\left(-\frac{6\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)x^2+4\operatorname{RootOf}\left(\_Z^2+24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\operatorname{RootOf}\left(\_Z^2+24\right.}{}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)
```

Fricas [A]
time = 0.38, size = 59, normalized size = 1.34

$$\frac{1}{3}\sqrt{3}\,\sqrt{2\sqrt{3}+3}\,\arctan\left(\frac{\sqrt{-x^3-1}\left(\sqrt{3}\,(x^2-4x-2)+6x+6\right)\sqrt{2\sqrt{3}+3}}{6\,(x^3+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4
*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{\sqrt{-\left(x + 1\right)\left(x^{2} - x + 1\right)}\left(x - \sqrt{3} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)
), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[-2,4]:[1,0,-3]%%},[2
]%%%}
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.105 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/((((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 7.18, size = 84, normalized size = 1.22

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
))/(a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [A]
time = 1.15, size = 1240, normalized size = 17.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^
```

```
16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b
^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b
^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt(3)
*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a + bx^3}\left(-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqr
t(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.106 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 7.24, size = 84, normalized size = 1.18

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.
time = 1.06, size = 1294, normalized size = 18.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
- sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x
))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 +
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
```

```
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3
+ a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a
))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a - bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.107 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 7.19, size = 85, normalized size = 1.18

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2))
/(a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [A]
time = 1.08, size = 1245, normalized size = 17.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*
```

```
x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a + bx^3}\left(-\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a
**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.108 $\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [A]
time = 7.14, size = 87, normalized size = 1.21

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
[Out] (-2*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
)/(a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(52) = 104.
time = 1.09, size = 1303, normalized size = 18.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
 - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*
x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
- 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7
*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
```

```
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
 a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a))
]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a-bx^3}\,\left(-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sq
rt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a
^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.109 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.42, size = 82, normalized size = 1.12

$$-\frac{2\sqrt[6]{\frac{b}{a}}\,\tanh^{-1}\left(\frac{\sqrt[6]{\frac{b}{a}}\,\sqrt{b+\frac{2b}{\sqrt{3}}}\,\sqrt{a+bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (-2*(b/a)^(1/6)*ArcTanh[((b/a)^(1/6)*Sqrt[b + (2*b)/Sqrt[3]]*Sqrt[a + b*x^3
])/(a*(b/a)^(2/3) + b*x)])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - s
qrt(3) + 1)), x)
```

Fricas [A]
time = 0.78, size = 1273, normalized size = 17.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a
^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
+ 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^
```

```
5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3
) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3+a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/
2) + 1)), x)
```

3.110 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{1+\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.35, size = 84, normalized size = 1.12

$$-\frac{2\sqrt[6]{\frac{b}{a}}\,\tanh^{-1}\left(\frac{\sqrt[6]{\frac{b}{a}}\,\sqrt{b+\frac{2b}{\sqrt{3}}}\,\sqrt{a-bx^3}}{-a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]

[Out] (-2*(b/a)^(1/6)*ArcTanh[((b/a)^(1/6)*Sqrt[b + (2*b)/Sqrt[3]]*Sqrt[a - b*x^3
])/(-(a*(b/a)^(2/3)) + b*x)])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) +
sqrt(3) - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(57) = 114$.
time = 0.75, size = 1330, normalized size = 17.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
+ 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x
^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a
```

```
^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
+ a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}-1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

3.111 $\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.21, size = 85, normalized size = 1.12

$$\frac{2\sqrt[6]{\frac{b}{a}}\,\tan^{-1}\left(\frac{\sqrt[6]{\frac{b}{a}}\,\sqrt{b+\frac{2b}{\sqrt{3}}}\,\sqrt{-a+bx^3}}{-a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]
```

```
[Out] (2*(b/a)^(1/6)*ArcTan[((b/a)^(1/6)*Sqrt[b + (2*b)/Sqrt[3]]*Sqrt[-a + b*x^3]
)/(-(a*(b/a)^(2/3)) + b*x)])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + s
qrt(3) - 1)), x)
```

Fricas [A]
time = 0.77, size = 1278, normalized size = 16.82

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*
a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 - 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b
```

```
^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{-a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- 1 + sqrt(3))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}-x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)),x)
```

```
[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1
/3) - 1)), x)
```

3.112 $\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{1-\left(3-2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.04, size = 85, normalized size = 1.12

$$\frac{2\sqrt[6]{\frac{b}{a}}\,\tan^{-1}\left(\frac{\sqrt[6]{\frac{b}{a}}\,\sqrt{b+\frac{2b}{\sqrt{3}}}\,\sqrt{-a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{-3+2\sqrt{3}}\,\sqrt{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (2*(b/a)^(1/6)*ArcTan[((b/a)^(1/6)*Sqrt[b + (2*b)/Sqrt[3]]*Sqrt[-a - b*x^3]
)/(a*(b/a)^(2/3) + b*x)])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) -
sqrt(3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(58) = 116.
time = 0.78, size = 1339, normalized size = 17.62

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 -
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*
x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*
```

```
a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
 - a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sqr
t(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}}{\sqrt{-a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1}{\sqrt{-b\,x^3-a}\,\left(x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)),x)
```

```
[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(
1/2) + 1)), x)
```

3.113 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] -2*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]
]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = -\left(2\mathrm{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.58, size = 49, normalized size = 1.17

$$-2\sqrt{-1+\frac{2}{\sqrt{3}}}\,\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{1+x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
[Out] -2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x +
 x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.79, size = 245, normalized size = 5.83

method	result
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(-\frac{6\,\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\,\mathrm{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2+4\sqrt{3}\,\mathrm{RootOf}\left(\_Z^2 \cdots\right.}{\cdots}\right.}{6}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\cdots\right.}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{4\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [**A**]
time = 0.38, size = 50, normalized size = 1.19

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-\sqrt{3}+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3)))
, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.114 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

[Out] 2*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))/(3+2*3^(1/2))^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]

[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.54, size = 49, normalized size = 1.07

$$2\sqrt{-1+\frac{2}{\sqrt{3}}}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1-x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] 2*Sqrt[-1 + 2/Sqrt[3]]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 + x + 
x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.75, size = 247, normalized size = 5.37

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\ln\left(-\frac{6\,\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)x^2-4\,\text{RootOf}\left(\_Z^2-36+24\sqrt{3}\right)\sqrt{3}\,x^2-4\sqrt{3}\,\text{RootOf}\left(\_Z^2-3\right.}{}\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$

elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)
/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3
+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2
*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2)),(I*3^(1/2
)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A]
time = 0.37, size = 59, normalized size = 1.28

$$\frac{1}{3}\sqrt{3}\,\sqrt{2\sqrt{3}-3}\ \arctan\left(\frac{\sqrt{-x^3+1}\left(\sqrt{3}\,(x^2+4x-2)+6x-6\right)\sqrt{2\sqrt{3}-3}}{6\,(x^3-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1+\sqrt{3}}{\sqrt{-(x-1)(x^2+x+1)}\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)
), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

3.115 $\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=44

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

```
[Out] 2*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3
]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = 2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.51, size = 47, normalized size = 1.07

$$-2\sqrt{-1+\frac{2}{\sqrt{3}}}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{-1+x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
[Out] -2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x
 + x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.81, size = 245, normalized size = 5.57

method	result
trager	$\frac{\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2-4\sqrt{3}\,\text{RootOf}\left(\_Z^2-24\right.}{}\right.}{6}$
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - \frac{4\left(-\frac{3}{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((-1+x)/(
-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2
)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticP
i(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(33) = 66$.
time = 0.38, size = 204, normalized size = 4.64

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4\left(2x^6+18x^5+42x^4+8x^3+\sqrt{3}\left(x^6+12x^5+18x^4+16x^3-12x^2-8\right)-24x+8\right)\sqrt{x^3-1}\sqrt{2\sqrt{3}-3}+16\sqrt{3}\left(x^7+2x^6+6x^5-5x^4+2x^3-6x^2+4x-4\right)-128x+112}{x^8-8x^7+16x^6+16x^5-56x^4-32x^3+64x^2+64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*s
qrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x
 - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*
x^2 + 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-1+\sqrt{3}}{\sqrt{(x-1)\left(x^2+x+1\right)}\,\left(x-\sqrt{3}-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1))
, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

3.116 $\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

[Out] -2*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))/(3+2*3^(1/2))^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\left(2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [A]
time = 1.56, size = 51, normalized size = 1.16

$$2\sqrt{-1+\frac{2}{\sqrt{3}}}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{-1-x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
[Out] 2*Sqrt[-1 + 2/Sqrt[3]]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x
+ x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.72, size = 243, normalized size = 5.52

method	result
trager	$-\frac{\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\ln\left(\frac{6\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)x^2-4\,\text{RootOf}\left(\_Z^2-24\sqrt{3}+36\right)\sqrt{3}\,x^2+4\sqrt{3}\,\text{RootOf}\left(\_Z^2-\right.\right.}{6}$
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/
2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(
1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(34) = 68$.
time = 0.40, size = 206, normalized size = 4.68

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4\left(2x^6-18x^5+42x^4-8x^3+\sqrt{3}\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)+24x+8\right)\sqrt{-x^3-1}\sqrt{2\sqrt{3}-3}-16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*
x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6
- 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*
sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-\left(x+1\right)\left(x^{2}-x+1\right)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))
), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[1,-1]:[1,0,-3]%%},[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%
%%} Er
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.117 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=69

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [**A**]
time = 7.01, size = 84, normalized size = 1.22

$$\frac{2\tan^{-1}\left(\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*ArcTan[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]*
a^(1/6)*Sqrt[a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+
a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
 a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [A]
time = 1.16, size = 1236, normalized size = 17.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672
*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a
^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7
- 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 1
6860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^
2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 -
 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^
3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724
*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 +
 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2
*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 2
33856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2
920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
```

```
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}\,\left(\sqrt[3]{a}+\sqrt{3}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**
(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.118 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

```
[Out] 2*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[a - b*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int\frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [**A**]
time = 7.05, size = 84, normalized size = 1.18

$$-\frac{2\tan^{-1}\left(\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-2*ArcTan[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x
^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.
time = 1.09, size = 1288, normalized size = 18.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1
840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x
^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672
*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x
^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x
 + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*
x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*
x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17
+ 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^
6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b
^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 931
84*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 +
 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*
x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^
3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 446
```

```
40*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^
6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2
*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 +
 a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{a - bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sq
rt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.119 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] 2*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/
a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*
x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2\sqrt[3]{a}\right)\mathrm{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$=\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [**A**]
time = 7.18, size = 85, normalized size = 1.18

$$-\frac{2\tanh^{-1}\left(\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]
]*a^(1/6)*Sqrt[-a + b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^
3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x -
 a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [A]
time = 1.11, size = 1239, normalized size = 17.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*
a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^
2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 -
 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16
860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2
*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 +
4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3
*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*
a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 -
37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*
b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 23
3856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 29
20*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x
^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt((2*sqr
t(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^1
6 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*
b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^
16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6
*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 1
3320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b
```

```
^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6
*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024
*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 32*sqrt(
3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^
12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b
^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a
^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4
096*a^8)), sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*
sqrt(1/3)*(a^(1/3)*b*x^2 + 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt(3)
*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3}\sqrt[3]{a} + \sqrt[3]{b}\,x}{\sqrt{-a + bx^3}\left(-\sqrt{3}\sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(
1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-s
qrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^(
1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.120 $\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=72

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

```
[Out] -2*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2)
)/a^(1/6)/b^(1/3)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b
*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
```

```
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx=-\frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$=-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Mathematica [**A**]
time = 7.21, size = 87, normalized size = 1.21

$$\frac{2\tanh^{-1}\left(\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt[6]{a}\,\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (2*ArcTanh[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(Sqrt[3 + 2*Sqrt[3]]
*a^(1/6)*Sqrt[-a - b*x^3])])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1+\sqrt{3}\right)\right)\sqrt{-b\,x^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

```
[Out] int((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3
-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(52) = 104$.
time = 1.11, size = 1299, normalized size = 18.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 18
40*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^
12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*
a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^
13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x
+ sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x
^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x
))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 -
 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6
*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^
5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 9318
4*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 -
2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x
^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*
x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
```

```
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{-a-bx^3}\left(\sqrt[3]{a}+\sqrt{3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/
2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a*
*(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x + a
^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

3.121 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctan((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2165, 209}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b
*x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3
]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a+bx^3}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [**A**]
time = 7.38, size = 86, normalized size = 1.18

$$\frac{2\sqrt[6]{\frac{b}{a}}\,\tan^{-1}\left(\frac{\sqrt{-1+\frac{2}{\sqrt{3}}}\,\sqrt{b}\,\sqrt[6]{\frac{b}{a}}\,\sqrt{a+bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[a + b*x^3]),x]
```

```
[Out] (2*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[a + b*
x^3])/(a*(b/a)^(2/3) + b*x)])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + s
qrt(3) + 1)), x)
```

Fricas [A]
time = 0.78, size = 1270, normalized size = 17.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 +
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 14
5152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x
^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4
*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*
a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 197
92*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17
+ 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2
*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b
```

```
^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^
(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
 a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}}-\sqrt{3}+1}{\sqrt{a+bx^3}\left(x\sqrt[3]{\frac{b}{a}}+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) +
 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{b\,x^3+a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/
3) + 1)), x)
```

3.122 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctan((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 209}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b
*x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]
])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{a-bx^3}}\,dx=\frac{2\text{Subst}\left(\int \frac{1}{1+\left(3+2\sqrt{3}\right)ax^2}\,dx,x,\frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$=\frac{2\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.28, size = 88, normalized size = 1.17

$$\frac{2\sqrt[6]{\frac{b}{a}}\,\tan^{-1}\left(\frac{\sqrt{-1+\frac{2}{\sqrt{3}}}\,\sqrt{b}\,\sqrt[6]{\frac{b}{a}}\,\sqrt{a-bx^3}}{-a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[a - b*x^3]),x]
```

```
[Out] (2*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[a - b*
x^3])/(-(a*(b/a)^(2/3)) + b*x)])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{-b\,x^{3}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) -
sqrt(3) - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(57) = 114$.
time = 0.76, size = 1324, normalized size = 17.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*
b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 -
2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 9350
4*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x
^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*
a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 +
5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3
*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*
x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*
```

```
a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3
 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
 1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 + 2*sqr
t(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) -
 sqrt(3) - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}-1}{\sqrt{a-b\,x^3}\,\left(\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

3.123 $\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=76

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

```
[Out] 2*arctanh((1-(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))/(b
/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{a}\left(1-x\sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}}\sqrt{a}\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a +
b*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^
3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1-\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1-\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [**A**]
time = 7.29, size = 89, normalized size = 1.17

$$-\frac{2\sqrt[6]{\frac{b}{a}}\tanh^{-1}\left(\frac{\sqrt{-1+\frac{2}{\sqrt{3}}}\,\sqrt{b}\,\sqrt[6]{\frac{b}{a}}\,\sqrt{-a+bx^3}}{-a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt
[-a + b*x^3]),x]

[Out] (-2*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[-a +
 b*x^3])/(-(a*(b/a)^(2/3)) + b*x)])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1-\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{b x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
[Out] int((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="maxima")
[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - s
qrt(3) - 1)), x)
```

Fricas [A]
time = 0.78, size = 1273, normalized size = 16.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="fricas")
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 -
4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145
152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^
2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*
x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a
^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 1979
2*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4
- 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 +
 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*
x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^
```

```
5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{-a + bx^3}\left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a
)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3)
- sqrt(3) - 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{3}+x\,{\left(\frac{b}{a}\right)}^{1/3}-1}{\sqrt{b\,x^3-a}\,\left(\sqrt{3}-x\,{\left(\frac{b}{a}\right)}^{1/3}+1\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)),x)
```

```
[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1
/3) + 1)), x)
```

3.124 $\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=76

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

```
[Out] -2*arctanh((1+(b/a)^(1/3)*x)*a^(1/2)*(3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))/
(b/a)^(1/3)/a^(1/2)/(3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2165, 212}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(x\sqrt[3]{\frac{b}{a}}+1\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a -
b*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x
^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{2\text{Subst}\left(\int \frac{1}{1-\left(3+2\sqrt{3}\right)ax^2}\,dx, x, \frac{1+\sqrt[3]{\frac{b}{a}}\,x}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{\frac{b}{a}}}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\left(1+\sqrt[3]{\frac{b}{a}}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{a}\,\sqrt[3]{\frac{b}{a}}}$$

Mathematica [A]
time = 7.14, size = 89, normalized size = 1.17

$$-\frac{2\sqrt[6]{\frac{b}{a}}\,\tanh^{-1}\left(\frac{\sqrt{-1+\frac{2}{\sqrt{3}}}\,\sqrt{b}\,\sqrt[6]{\frac{b}{a}}\,\sqrt{-a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{3+2\sqrt{3}}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt
[-a - b*x^3]),x]
```

```
[Out] (-2*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[-a -
 b*x^3])/(a*(b/a)^(2/3) + b*x)])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[b])
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x-\sqrt{3}}{\left(1+\left(\frac{b}{a}\right)^{\frac{1}{3}} x+\sqrt{3}\right) \sqrt{-b\,x^{3}-a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) +
sqrt(3) + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(58) = 116.
time = 0.78, size = 1335, normalized size = 17.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b
^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2
140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 +
4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504
*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4
- 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^
16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a
^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5
010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*
x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x
^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a
```

```
^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^1
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{-a - bx^3}\left(x\sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-
a)**(1/2),x)
```

```
[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3)
+ 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(\frac{b}{a}\right)^{1/3}-\sqrt{3}+1}{\sqrt{-b\,x^3-a}\,\left(\sqrt{3}+x\left(\frac{b}{a}\right)^{1/3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)),x)
```

```
[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(
1/3) + 1)), x)
```

3.125 $\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/(3
+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2166, 224, 2165, 209}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[3 + 2*Sqrt[3]])
+ (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{1+x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3+2\sqrt{3}}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.31, size = 269, normalized size = 1.86

$$\frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*
((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 2*Sqr
t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1
 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2
*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] -
(2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(119) = 238.
time = 0.82, size = 245, normalized size = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{}}{}\cdots\right.}{\cdots}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{}}{}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 55, normalized size = 0.38

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\arctan\left(\frac{\left(\sqrt{3}\left(x^2-4x-2\right)-6x-6\right)\sqrt{2\sqrt{3}-3}}{6\sqrt{x^3+1}}\right)+\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x -
 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+1}{\sqrt{(x+1)\left(x^2-x+1\right)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.126 $\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=145

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)
+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x
)/(1+x+3^(1/2))^2)^(1/2)-arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))/
(-3+2*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2166, 224, 2165, 212}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]]/Sqrt[-3 + 2*Sqrt[3]
]) + (Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ell
ipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/
4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{1+x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{1}{12}\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\sqrt{3}\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}+\frac{\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right.}}}{\sqrt[4]{3}\sqrt{\frac{1+}{\left(1+\sqrt{3}\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.29, size = 267, normalized size = 1.84

$$-\frac{2\sqrt{6}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(\sqrt{-i+\sqrt{3}+2ix}\left((1+2i)-i\sqrt{3}+\left((-2-i)+\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2i\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(-3+(2+i)\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I +
 Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (2*I
)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/
(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3]
 - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(119) = 238.
time = 0.76, size = 245, normalized size = 1.69

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 210, normalized size = 1.45

$$\frac{1}{12}\sqrt{3}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4\left(2x^6-18x^5+42x^4-8x^3-\sqrt{3}\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)+24x+8\right)\sqrt{x^3+1}\sqrt{2\sqrt{3}+3}+16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)+\text{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112
*x^4 + 224*x^3 + 64*x^2 - 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 - sqrt(3)*(x^6
 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(x^3 + 1)*sqrt(2*
sqrt(3) + 3) + 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16)) + weierstrassPInverse(0, -4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral((x + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.127 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=173

$$\frac{\left(e-f-\sqrt{3}\,f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2+\sqrt{3}}\,\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(
x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+arctan((1+x)*(3+2*3^(1/2))^(1/2)
/(x^3+1)^(1/2))*(e-f-f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2166, 224, 2165, 209}

$$\frac{\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)\left(e-\sqrt{3}\,f-f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] ((e - f - Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/S
qrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 + Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 + x)*S
qrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(
1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)
^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-\right.}{\left(1+\sqrt{3}\right.}}{\left(1+\sqrt{3}\right)\left(-28+\left(1\right.\right.}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{\left(e-f-\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2+\sqrt{3}}\left(e-\left(1-\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.36, size = 291, normalized size = 1.68

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\left(-\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])
```

Maple [A]
time = 0.29, size = 260, normalized size = 1.50

method	result

default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\right.}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(e-f-f\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(e-f-f*3^(1/2))*(3/2-1/2*I*
3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2
*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+
1)^(1/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2
*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 746, normalized size = 4.31

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(sqrt(3)*(f - e) - 3*f)*weierstrassPInverse(0, -4, x) + 1/12*sqrt(6*f
*e - 2*sqrt(3)*(f^2 + f*e + e^2) + 3*e^2)*log(-(2*f^2*x^8 - 32*f^2*x^7 + 22
4*f^2*x^6 - 32*f^2*x^5 + 224*f^2*x^4 + 448*f^2*x^3 + 128*f^2*x^2 + 256*f^2*
```

```
x + 4*(f*x^6 - 18*f*x^5 + 12*f*x^4 - 40*f*x^3 - 36*f*x^2 - 24*f*x + 2*(x^6
- 9*x^5 + 21*x^4 - 4*x^3 + 12*x + 4)*e + sqrt(3)*(f*x^6 - 6*f*x^5 + 24*f*x^
4 + 8*f*x^3 + 12*f*x^2 + 24*f*x + (x^6 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2
- 8)*e + 16*f) - 32*f)*sqrt(x^3 + 1)*sqrt(6*f*e - 2*sqrt(3)*(f^2 + f*e + e^
2) + 3*e^2) + 224*f^2 - (x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^
3 + 64*x^2 + 128*x + 112)*e^2 + 2*(f*x^8 - 16*f*x^7 + 112*f*x^6 - 16*f*x^5
+ 112*f*x^4 + 224*f*x^3 + 64*f*x^2 + 128*f*x + 112*f)*e - 16*sqrt(3)*(2*f^2
*x^7 - 4*f^2*x^6 + 12*f^2*x^5 + 10*f^2*x^4 + 4*f^2*x^3 + 12*f^2*x^2 + 8*f^2
*x + 8*f^2 - (x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4)*e^2 +
2*(f*x^7 - 2*f*x^6 + 6*f*x^5 + 5*f*x^4 + 2*f*x^3 + 6*f*x^2 + 4*f*x + 4*f)*e
))/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16)),
 -1/3*(sqrt(3)*(f - e) - 3*f)*weierstrassPInverse(0, -4, x) - 1/6*sqrt(-6*f
*e + 2*sqrt(3)*(f^2 + f*e + e^2) - 3*e^2)*arctan(-1/6*(3*f*x^2 - 6*f*x - 6*
(x + 1)*e - sqrt(3)*(f*x^2 + 2*f*x - (x^2 - 4*x - 2)*e + 4*f))*sqrt(x^3 + 1
)*sqrt(-6*f*e + 2*sqrt(3)*(f^2 + f*e + e^2) - 3*e^2)/(2*f^2*x^3 + 2*f^2 - (
x^3 + 1)*e^2 + 2*(f*x^3 + f)*e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{(x+1)(x^2-x+1)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.128 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=187

$$-\frac{\left(e+f+\sqrt{3}\,f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)}}}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/
(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-arctan((1-x)*(3+2*3^(1/2))^(1/
2)/(-x^3+1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {2166, 224, 2165, 209}

$$-\frac{\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\,\sqrt{1-x^3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)\left(e+\sqrt{3}\,f+f\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])
/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 + Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 - x
)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x
)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)/(1 + Sqrt[3] -
 x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(28-\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(6e-\left(1+\right.\right.}{}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2+\sqrt{3}}\left(e+\left(1-\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.39, size = 291, normalized size = 1.56

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])
```

Maple [A]
time = 0.28, size = 264, normalized size = 1.41

method	result

default	$\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$
elliptic	$\frac{2if\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-e-f-f*3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*
I*3^(1/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A]
time = 0.20, size = 714, normalized size = 3.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*sqrt(-6*f*e - 2*sqrt(3)*(f^2 - f*e + e^2) + 3*e^2)*log(-(2*f^2*x^8 +
32*f^2*x^7 + 224*f^2*x^6 + 32*f^2*x^5 + 224*f^2*x^4 - 448*f^2*x^3 + 128*f^2
*x^2 - 256*f^2*x + 4*(f*x^6 + 18*f*x^5 + 12*f*x^4 + 40*f*x^3 - 36*f*x^2 + 2
4*f*x - 2*(x^6 + 9*x^5 + 21*x^4 + 4*x^3 - 12*x + 4)*e + sqrt(3)*(f*x^6 + 6*
f*x^5 + 24*f*x^4 - 8*f*x^3 + 12*f*x^2 - 24*f*x - (x^6 + 12*x^5 + 18*x^4 + 1
6*x^3 - 12*x^2 - 8)*e + 16*f) - 32*f)*sqrt(-x^3 + 1)*sqrt(-6*f*e - 2*sqrt(3
)*(f^2 - f*e + e^2) + 3*e^2) + 224*f^2 - (x^8 + 16*x^7 + 112*x^6 + 16*x^5 +
 112*x^4 - 224*x^3 + 64*x^2 - 128*x + 112)*e^2 - 2*(f*x^8 + 16*f*x^7 + 112*
f*x^6 + 16*f*x^5 + 112*f*x^4 - 224*f*x^3 + 64*f*x^2 - 128*f*x + 112*f)*e +
16*sqrt(3)*(2*f^2*x^7 + 4*f^2*x^6 + 12*f^2*x^5 - 10*f^2*x^4 + 4*f^2*x^3 - 1
2*f^2*x^2 + 8*f^2*x - 8*f^2 - (x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2
+ 4*x - 4)*e^2 - 2*(f*x^7 + 2*f*x^6 + 6*f*x^5 - 5*f*x^4 + 2*f*x^3 - 6*f*x^2
 + 4*f*x - 4*f)*e))/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x
^2 + 64*x + 16)), -1/6*sqrt(6*f*e + 2*sqrt(3)*(f^2 - f*e + e^2) - 3*e^2)*ar
ctan(1/6*(3*f*x^2 + 6*f*x - 6*(x - 1)*e - sqrt(3)*(f*x^2 - 2*f*x + (x^2 + 4
*x - 2)*e + 4*f))*sqrt(-x^3 + 1)*sqrt(6*f*e + 2*sqrt(3)*(f^2 - f*e + e^2) -
 3*e^2)/(2*f^2*x^3 - 2*f^2 - (x^3 - 1)*e^2 - 2*(f*x^3 - f)*e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{1-x^3}-\sqrt{3}\,\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx-\int \frac{fx}{x\sqrt{1-x^3}-\sqrt{3}\,\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
) - Integral(f*x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3
)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.129 $\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=190

$$-\frac{\left(e+f+\sqrt{3}\,f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}-\frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right.}}}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}$$

```
[Out] -1/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f*(1-3^(
1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(1/4)/
(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-arctanh((1-x)*(3+2*3^(1/2))^(1
/2)/(x^3-1)^(1/2))*(e+f+f*3^(1/2))/(9+6*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2166, 225, 2165, 212}

$$-\frac{\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\left(1-\sqrt{3}\right)f\right)F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\left(e+\sqrt{3}\,f+f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((e + f + Sqrt[3]*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])]) - (Sqrt[2 - Sqrt[3]]*(e + (1 - Sqrt[3])*f)*(1 -
 x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 - x)/(1 - Sqrt[
3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-6e-\right.}{}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(e+\left(1-\sqrt{3}\right)f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\right.}{3^{3/4}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{\left(e+f+\sqrt{3}f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} - \frac{\sqrt{2-\sqrt{3}}\left(e+\left(\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.29, size = 289, normalized size = 1.52

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{-\frac{i(-1+x)}{3i+\sqrt{3}}}\left(-3if\sqrt{-i+\sqrt{3}-2ix}\left(-i\left((2+i)+\sqrt{3}\right)+\left((2-i)+\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\left(\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}+2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}+2ix}\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*((-3*I)*f*Sqrt[-I + Sqrt
[3] - (2*I)*x]*((-I)*((2 + I) + Sqrt[3]) + ((2 - I) + Sqrt[3])*x)*EllipticF
[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I +
Sqrt[3])] + 2*(Sqrt[3]*e + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqr
t[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqr
t[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))
/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.28, size = 262, normalized size = 1.38

method	result

default	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - 2\Big($
elliptic	$-\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}} - 2\Big($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-e-f-f*3^(1/2))*(-3/2-1/
2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/
2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(
x^3-1)^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3
/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((f*x + e)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.17, size = 748, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*(f + e) - 3*f)*weierstrassPInverse(0, 4, x) + 1/12*sqrt(6*f*e
 + 2*sqrt(3)*(f^2 - f*e + e^2) - 3*e^2)*log(-(2*f^2*x^8 + 32*f^2*x^7 + 224*
f^2*x^6 + 32*f^2*x^5 + 224*f^2*x^4 - 448*f^2*x^3 + 128*f^2*x^2 - 256*f^2*x
```

```
+ 4*(f*x^6 + 18*f*x^5 + 12*f*x^4 + 40*f*x^3 - 36*f*x^2 + 24*f*x - 2*(x^6 +
9*x^5 + 21*x^4 + 4*x^3 - 12*x + 4)*e + sqrt(3)*(f*x^6 + 6*f*x^5 + 24*f*x^4
- 8*f*x^3 + 12*f*x^2 - 24*f*x - (x^6 + 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 -
8)*e + 16*f) - 32*f)*sqrt(x^3 - 1)*sqrt(6*f*e + 2*sqrt(3)*(f^2 - f*e + e^2)
 - 3*e^2) + 224*f^2 - (x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3
+ 64*x^2 - 128*x + 112)*e^2 - 2*(f*x^8 + 16*f*x^7 + 112*f*x^6 + 16*f*x^5 +
112*f*x^4 - 224*f*x^3 + 64*f*x^2 - 128*f*x + 112*f)*e + 16*sqrt(3)*(2*f^2*x
^7 + 4*f^2*x^6 + 12*f^2*x^5 - 10*f^2*x^4 + 4*f^2*x^3 - 12*f^2*x^2 + 8*f^2*x
 - 8*f^2 - (x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4)*e^2 - 2*
(f*x^7 + 2*f*x^6 + 6*f*x^5 - 5*f*x^4 + 2*f*x^3 - 6*f*x^2 + 4*f*x - 4*f)*e))
/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16)), 1
/3*(sqrt(3)*(f + e) - 3*f)*weierstrassPInverse(0, 4, x) - 1/6*sqrt(-6*f*e -
 2*sqrt(3)*(f^2 - f*e + e^2) + 3*e^2)*arctan(1/6*(3*f*x^2 + 6*f*x - 6*(x -
1)*e - sqrt(3)*(f*x^2 - 2*f*x + (x^2 + 4*x - 2)*e + 4*f))*sqrt(x^3 - 1)*sqr
t(-6*f*e - 2*sqrt(3)*(f^2 - f*e + e^2) + 3*e^2)/(2*f^2*x^3 - 2*f^2 - (x^3 -
 1)*e^2 - 2*(f*x^3 - f)*e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx-\int \frac{fx}{x\sqrt{x^3-1}-\sqrt{3}\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(e/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
) - Integral(f*x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1
)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
[Out] \text{Hanged}
```

3.130 $\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=183

$$\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}+\frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}$$

```
[Out] 1/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f*(1-3^(1
/2)))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(1/4)/(
-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+arctanh((1+x)*(3+2*3^(1/2))^(1
/2)/(-x^3-1)^(1/2))*(e-f*(1+3^(1/2)))/(9+6*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2166, 225, 2165, 212}

$$\frac{\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\left(1-\sqrt{3}\right)f\right)F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((e - (1 + Sqrt[3])*f)*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]
])/Sqrt[3*(3 + 2*Sqrt[3])] + (Sqrt[2 - Sqrt[3]]*(e - (1 - Sqrt[3])*f)*(1 +
x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] +
x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(3/4)*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1-\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{2\sqrt{3}} + \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{\left(1+\sqrt{3}\right)}{\left(1+\sqrt{3}\right.}}{12\sqrt{3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(e-\left(1-\sqrt{3}\right)f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1}{1}\right.\right.}{3^{3/4}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3\left(3+2\sqrt{3}\right)}} + \frac{\sqrt{2-\sqrt{3}}\left(e-\left(\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.28, size = 293, normalized size = 1.60

$$\frac{2\sqrt{\frac{2}{3}}\sqrt{\frac{i(1+x)}{3i+\sqrt{3}}}\left(3f\sqrt{-i+\sqrt{3}+2ix}\left((-2-i)-\sqrt{3}+\left((1+2i)+i\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)+2\left(-\sqrt{3}e+\left(3+\sqrt{3}\right)f\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-2ix}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\left(3i+(1+2i)\sqrt{3}\right)\sqrt{i+\sqrt{3}-2ix}\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[2/3]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(3*f*Sqrt[-I + Sqrt[3] + (2*
I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqr
t[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] +
 2*(-(Sqrt[3]*e) + (3 + Sqrt[3])*f)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x
+ x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sq
rt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I +
 (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.28, size = 258, normalized size = 1.41

method	result
default	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(e-f-f*3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2
```

```
))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2
*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [A]
time = 0.22, size = 708, normalized size = 3.87

$$\left[\frac{1}{12}\sqrt{-6fe+2\sqrt{3}\left(f^2+fe+e^2\right)-3e^2}\log\left(-\frac{2f^2x^8-32f^2x^7+224f^2x^6-32f^2x^5+224f^2x^4+448f^2x^3+128f^2x^2+256f^2x+4\left(fx^6-18fx^5+12fx^4-40fx^3-36fx^2-24fx+2\left(x^6-9x^5+21x^4-4x^3+12x+4\right)e+\sqrt{3}\left(fx^6-6fx^5+24fx^4+8fx^3+12fx^2+24fx+\left(x^6-12x^5+18x^4-16x^3-12x^2-8\right)e+16f\right)-32f\right)\sqrt{-x^3-1}\sqrt{-6fe+2\sqrt{3}\left(f^2+fe+e^2\right)-3e^2}+224f^2-\left(x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+128x+112\right)e^2+2\left(fx^8-16fx^7+112fx^6-16fx^5+112fx^4+224fx^3+64fx^2+128fx+112f\right)e-16\sqrt{3}\left(2f^2x^7-4f^2x^6+12f^2x^5+10f^2x^4+4f^2x^3+12f^2x^2+8f^2x+8f^2-\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)e^2+2\left(fx^7-2fx^6+6fx^5+5fx^4+2fx^3+6fx^2+4fx+4f\right)e\right)}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right),\ -\frac{1}{6}\sqrt{6fe-2\sqrt{3}\left(f^2+fe+e^2\right)+3e^2}\arctan\left(-\frac{\left(3fx^2-6fx-6(x+1)e-\sqrt{3}\left(fx^2+2fx-\left(x^2-4x-2\right)e+4f\right)\right)\sqrt{-x^3-1}\sqrt{6fe-2\sqrt{3}\left(f^2+fe+e^2\right)+3e^2}}{6\left(2f^2x^3+2f^2-\left(x^3+1\right)e^2+2\left(fx^3+f\right)e\right)}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*sqrt(-6*f*e + 2*sqrt(3)*(f^2 + f*e + e^2) - 3*e^2)*log(-(2*f^2*x^8 -
32*f^2*x^7 + 224*f^2*x^6 - 32*f^2*x^5 + 224*f^2*x^4 + 448*f^2*x^3 + 128*f^2
*x^2 + 256*f^2*x + 4*(f*x^6 - 18*f*x^5 + 12*f*x^4 - 40*f*x^3 - 36*f*x^2 - 2
4*f*x + 2*(x^6 - 9*x^5 + 21*x^4 - 4*x^3 + 12*x + 4)*e + sqrt(3)*(f*x^6 - 6*
f*x^5 + 24*f*x^4 + 8*f*x^3 + 12*f*x^2 + 24*f*x + (x^6 - 12*x^5 + 18*x^4 - 1
6*x^3 - 12*x^2 - 8)*e + 16*f) - 32*f)*sqrt(-x^3 - 1)*sqrt(-6*f*e + 2*sqrt(3
)*(f^2 + f*e + e^2) - 3*e^2) + 224*f^2 - (x^8 - 16*x^7 + 112*x^6 - 16*x^5 +
 112*x^4 + 224*x^3 + 64*x^2 + 128*x + 112)*e^2 + 2*(f*x^8 - 16*f*x^7 + 112*
f*x^6 - 16*f*x^5 + 112*f*x^4 + 224*f*x^3 + 64*f*x^2 + 128*f*x + 112*f)*e -
16*sqrt(3)*(2*f^2*x^7 - 4*f^2*x^6 + 12*f^2*x^5 + 10*f^2*x^4 + 4*f^2*x^3 + 1
2*f^2*x^2 + 8*f^2*x + 8*f^2 - (x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2
+ 4*x + 4)*e^2 + 2*(f*x^7 - 2*f*x^6 + 6*f*x^5 + 5*f*x^4 + 2*f*x^3 + 6*f*x^2
 + 4*f*x + 4*f)*e))/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x
^2 - 64*x + 16)), -1/6*sqrt(6*f*e - 2*sqrt(3)*(f^2 + f*e + e^2) + 3*e^2)*ar
ctan(-1/6*(3*f*x^2 - 6*f*x - 6*(x + 1)*e - sqrt(3)*(f*x^2 + 2*f*x - (x^2 -
4*x - 2)*e + 4*f))*sqrt(-x^3 - 1)*sqrt(6*f*e - 2*sqrt(3)*(f^2 + f*e + e^2)
+ 3*e^2)/(2*f^2*x^3 + 2*f^2 - (x^3 + 1)*e^2 + 2*(f*x^3 + f)*e))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{-\left(x+1\right)\left(x^2-x+1\right)}\,\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.131 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=332

$$-\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}}-\frac{\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\right.}{}$$

```
[Out] -arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/2))*
(b^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)-1/3*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*(1/2*6^
(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2166, 224, 2165, 212}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1
/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt
[a]*b^(2/3))) - (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*
a^(1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 a\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+}}}{12\sqrt{3}\,a^{4/3}b^{4/3}}$$

$$= -\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\cdots}{\left(\left(1\cdots\right.\right.}}}$$

$$= -\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{\cdots}\right.}{\sqrt{a+bx^3}}\right.}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.26, size = 438, normalized size = 1.32

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{i\sqrt[4]{3}\,f\left(\left((-2-i)+\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(\sqrt[3]{b}\,e+\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqr
t[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^
(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*
```

```
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3
)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a
+ b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 43.68, size = 6491, normalized size = 19.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="fricas")
```

```
[Out] [-1/12*(4*sqrt(3)*a^(2/3)*b^(3/2)*e*weierstrassPInverse(0, -4*a/b, x) - a*b
*sqrt((2*sqrt(3)*a^(2/3)*b^(2/3)*f^2 + 2*(sqrt(3)*b*f*e + 3*b*f*e)*a^(1/3)
+ (2*sqrt(3)*b*e^2 + 3*b*e^2)*b^(1/3))/a)*log(-(8*a^2*b^9*f^6*x^24 - 14720*
a^3*b^8*f^6*x^21 + 538112*a^4*b^7*f^6*x^18 - 468992*a^5*b^6*f^6*x^15 + 4032
512*a^6*b^5*f^6*x^12 + 17121280*a^7*b^4*f^6*x^9 + 24805376*a^8*b^3*f^6*x^6
+ 8716288*a^9*b^2*f^6*x^3 + 229376*a^10*b*f^6 + 32*(72*a^2*b^8*f^6*x^22 - 6
768*a^3*b^7*f^6*x^19 + 36936*a^4*b^6*f^6*x^16 + 43776*a^5*b^5*f^6*x^13 + 35
0208*a^6*b^4*f^6*x^10 + 787968*a^7*b^3*f^6*x^7 + 474624*a^8*b^2*f^6*x^4 + 3
6864*a^9*b*f^6*x - 9*(b^10*x^22 - 94*a*b^9*x^19 + 513*a^2*b^8*x^16 + 608*a^
3*b^7*x^13 + 4864*a^4*b^6*x^10 + 10944*a^5*b^5*x^7 + 6592*a^6*b^4*x^4 + 512
```

*a^7*b^3*x)*e^6 + 180*(a*b^9*f^3*x^22 - 94*a^2*b^8*f^3*x^19 + 513*a^3*b^7*f^3*x^16 + 608*a^4*b^6*f^3*x^13 + 4864*a^5*b^5*f^3*x^10 + 10944*a^6*b^4*f^3*x^7 + 6592*a^7*b^3*f^3*x^4 + 512*a^8*b^2*f^3*x)*e^3 - sqrt(3)*(40*a^2*b^8*f^6*x^22 - 4040*a^3*b^7*f^6*x^19 + 17040*a^4*b^6*f^6*x^16 - 39424*a^5*b^5*f^6*x^13 - 229504*a^6*b^4*f^6*x^10 - 430080*a^7*b^3*f^6*x^7 - 281600*a^8*b^2*f^6*x^4 - 20480*a^9*b*f^6*x - (5*b^10*x^22 - 505*a*b^9*x^19 + 2130*a^2*b^8*x^16 - 4928*a^3*b^7*x^13 - 28688*a^4*b^6*x^10 - 53760*a^5*b^5*x^7 - 35200*a^6*b^4*x^4 - 2560*a^7*b^3*x)*e^6 + 20*(5*a*b^9*f^3*x^22 - 505*a^2*b^8*f^3*x^19 + 2130*a^3*b^7*f^3*x^16 - 4928*a^4*b^6*f^3*x^13 - 28688*a^5*b^5*f^3*x^10 - 53760*a^6*b^4*f^3*x^7 - 35200*a^7*b^3*f^3*x^4 - 2560*a^8*b^2*f^3*x)*e^3))*a^(2/3)*b^(1/3) - 8*(24*a^2*b^8*f^6*x^23 - 8616*a^3*b^7*f^6*x^20 + 106560*a^4*b^6*f^6*x^17 - 153600*a^5*b^5*f^6*x^14 - 890880*a^6*b^4*f^6*x^11 - 2760192*a^7*b^3*f^6*x^8 - 2629632*a^8*b^2*f^6*x^5 - 491520*a^9*b*f^6*x^2 - 3*(b^10*x^23 - 359*a*b^9*x^20 + 4440*a^2*b^8*x^17 - 6400*a^3*b^7*x^14 - 37120*a^4*b^6*x^11 - 115008*a^5*b^5*x^8 - 109568*a^6*b^4*x^5 - 20480*a^7*b^3*x^2)*e^6 + 60*(a*b^9*f^3*x^23 - 359*a^2*b^8*f^3*x^20 + 4440*a^3*b^7*f^3*x^17 - 6400*a^4*b^6*f^3*x^14 - 37120*a^5*b^5*f^3*x^11 - 115008*a^6*b^4*f^3*x^8 - 109568*a^7*b^3*f^3*x^5 - 20480*a^8*b^2*f^3*x^2)*e^3 - 2*sqrt(3)*(8*a^2*b^8*f^6*x^23 - 2392*a^3*b^7*f^6*x^20 + 34080*a^4*b^6*f^6*x^17 + 12160*a^5*b^5*f^6*x^14 + 213760*a^6*b^4*f^6*x^11 + 840192*a^7*b^3*f^6*x^8 + 745472*a^8*b^2*f^6*x^5 + 143360*a^9*b*f^6*x^2 - (b^10*x^23 - 299*a*b^9*x^20 + 4260*a^2*b^8*x^17 + 1520*a^3*b^7*x^14 + 26720*a^4*b^6*x^11 + 105024*a^5*b^5*x^8 + 93184*a^6*b^4*x^5 + 17920*a^7*b^3*x^2)*e^6 + 20*(a*b^9*f^3*x^23 - 299*a^2*b^8*f^3*x^20 + 4260*a^3*b^7*f^3*x^17 + 1520*a^4*b^6*f^3*x^14 + 26720*a^5*b^5*f^3*x^11 + 105024*a^6*b^4*f^3*x^8 + 93184*a^7*b^3*f^3*x^5 + 17920*a^8*b^2*f^3*x^2)*e^3))*a^(1/3)*b^(2/3) - 4*sqrt(b*x^3 + a)*((104*a^2*b^7*f^5*x^21 - 16720*a^3*b^6*f^5*x^18 + 158208*a^4*b^5*f^5*x^15 + 41728*a^5*b^4*f^5*x^12 + 1086976*a^6*b^3*f^5*x^9 + 2798592*a^7*b^2*f^5*x^6 + 1138688*a^8*b*f^5*x^3 + 32768*a^9*f^5 - 2*(b^9*x^22 - 764*a*b^8*x^19 + 16860*a^2*b^7*x^16 - 19792*a^3*b^6*x^13 + 42368*a^4*b^5*x^10 + 104448*a^5*b^4*x^7 + 90880*a^6*b^3*x^4 + 7168*a^7*b^2*x)*e^5 - 24*(32*a*b^8*f*x^20 - 1869*a^2*b^7*f*x^17 + 5862*a^3*b^6*f*x^14 - 7280*a^4*b^5*f*x^11 - 1824*a^5*b^4*f*x^8 - 7872*a^6*b^3*f*x^5 - 1408*a^7*b^2*f*x^2)*e^4 + 8*(32*a*b^8*f^2*x^21 - 5425*a^2*b^7*f^2*x^18 + 47184*a^3*b^6*f^2*x^15 - 37256*a^4*b^5*f^2*x^12 - 16064*a^5*b^4*f^2*x^9 + 960*a^6*b^3*f^2*x^6 - 13312*a^7*b^2*f^2*x^3 + 512*a^8*b*f^2)*e^3 + 2*(a*b^8*f^3*x^22 + 424*a^2*b^7*f^3*x^19 - 2256*a^3*b^6*f^3*x^16 + 82592*a^4*b^5*f^3*x^13 + 614336*a^5*b^4*f^3*x^10 + 2178048*a^6*b^3*f^3*x^7 + 1853440*a^7*b^2*f^3*x^4 + 145408*a^8*b*f^3*x)*e^2 - 24*(13*a^2*b^7*f^4*x^20 - 696*a^3*b^6*f^4*x^17 + 3480*a^4*b^5*f^4*x^14 + 14336*a^5*b^4*f^4*x^11 + 104640*a^6*b^3*f^4*x^8 + 144384*a^7*b^2*f^4*x^5 + 30208*a^8*b*f^4*x^2)*e - sqrt(3)*(56*a^2*b^7*f^5*x^21 - 10000*a^3*b^6*f^5*x^18 + 79872*a^4*b^5*f^5*x^15 - 155648*a^5*b^4*f^5*x^12 - 660992*a^6*b^3*f^5*x^9 - 1551360*a^7*b^2*f^5*x^6 - 679936*a^8*b*f^5*x^3 - 16384*a^9*f^5 - (b^9*x^22 - 896*a*b^8*x^19 + 18984*a^2*b^7*x^16 - 31168*a^3*b^6*x^13 - 21184*a^4*b^5*x^10 - 125952*a^5*b^4*x^7 - 104960*a^6*b^3*x^4 - 8192*a^7*b^2*x)*e^5 - 12*(37*a*b^8*f*x^20 - 2154*a^2*b^7*f*

```
x^17 + 6900*a^3*b^6*f*x^14 - 6496*a^4*b^5*f*x^11 + 9600*a^5*b^4*f*x^8 + 729
6*a^6*b^3*f*x^5 + 1792*a^7*b^2*f*x^2)*e^4 + 4*(37*a*b^8*f^2*x^21 - 6260*a^2
*b^7*f^2*x^18 + 54624*a^3*b^6*f^2*x^15 - 40816*a^4*b^5*f^2*x^12 - 2752*a^5*
b^4*f^2*x^9 + 39936*a^6*b^3*f^2*x^6 + 1024*a^7*b^2*f^2*x^3 + 1024*a^8*b*f^2
)*e^3 - 2*(a*b^8*f^3*x^22 - 104*a^2*b^7*f^3*x^19 + 6240*a^3*b^6*f^3*x^16 +
37088*a^4*b^5*f^3*x^13 + 360128*a^5*b^4*f^3*x^10 + 1256448*a^6*b^3*f^3*x^7
+ 1070080*a^7*b^2*f^3*x^4 + 83968*a^8*b*f^3*x)*e^2 - 24*(7*a^2*b^7*f^4*x^20
 - 444*a^3*b^6*f^4*x^17 + 672*a^4*b^5*f^4*x^14 - 11200*a^5*b^4*f^4*x^11 - 5
8944*a^6*b^3*f^4*x^8 - 83712*a^7*b^2*f^4*x^5 - 17408*a^8*b*f^4*x^2)*e))*a^(
2/3)*b^(2/3) - 2*(2*a^2*b^8*f^5*x^22 - 2320*a^3*b^7*f^5*x^19 + 46464*a^4*b^
6*f^5*x^16 - 107840*a^5*b^5*f^5*x^13 - 296576*a^6*b^4*f^5*x^10 - 1173504*a^
7*b^3*f^5*x^7 - 993280*a^8*b^2*f^5*x^4 - 77824*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/
3)*x)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algori
thm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e+f\,x}{\sqrt{b\,x^3+a}\,\left(b^{1/3}\,x-a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] int((e + f*x)/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))), x)
```

3.132 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=336

$$\frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}}+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\,e+\left(1+\sqrt{3}\right)\sqrt[3]{}\right.\cdots}{}$$

```
[Out] arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1/2))*
(b^(1/3)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)+1/3*
(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+
a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(1/2*
6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2166, 224, 2165, 212}

$$\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{a-bx^3}}+\frac{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\,\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6
)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 + Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 \cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a \cdots}}}{12\sqrt{3}\,a^{4/3}b^{4/3}}$$

$$= \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{b}\,e+\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a \cdots}{((\cdots}}}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{\frac{\sqrt[3]{\cdots}}{((1+\cdots}}}$$

$$= \frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\sqrt{a-bx^3}}\right.}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.10, size = 466, normalized size = 1.39

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-i\left(\sqrt[3]{b}\,e-\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x
]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
```

```
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1)))
, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-\sqrt[3]{a}\,\sqrt{a-bx^{3}}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{a-bx^{3}}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\,\sqrt{a-bx^{3}}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{a-bx^{3}}+\sqrt[3]{b}\,x\sqrt{a-bx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(a - b*x*
*3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algo
rithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{e+f\,x}{\sqrt{a-b\,x^3}\,\left(b^{1/3}\,x+a^{1/3}\,\left(\sqrt{3}-1\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] int(-(e + f*x)/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))), x)
```

3.133 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=345

$$\frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}\,e+\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] 1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3
)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e+a^(1/3)*f*(1+3^(1/2)))*(
(a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(b*x^3-a)^(1/2)/(-
a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+arcta
n(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2))*(b^(1/3
)*e+a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2166, 225, 2165, 209}

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{bx^3-a}}\right)\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}f+\sqrt[3]{b}e\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((b^(1/3)*e + (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)
*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a
]*b^(2/3)) + (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e + (1 + Sqrt[3])*a^(1/3)*f)*(a^(1
/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(
1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(
1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3)
- b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx = \frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right.\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-}}}{12\sqrt{3}\,a^{4/3}b^{4/3}}$$

$$= \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}\,e+\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{}}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{}{((}}}$$

$$= \frac{\left(\sqrt[3]{b}\,e+\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{}\right.}{\sqrt{-a+bx^3}}\right.}{\sqrt{3\left(-3+2\sqrt{3}\right)}\sqrt{a}\,b^{2/3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 467, normalized size = 1.35

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}f\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)-i\left(\sqrt[3]{b}e-\left(-1+\sqrt{3}\right)\sqrt[3]{a}f\right)\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((f*(I*(-3 + (2
+ I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3
])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF
[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3
])*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 - I*(b^(1/3)*e - (-1 + Sqrt[3])*a^(1/3
)*f)*Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*
a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[
(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 -
 I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/
```

```
((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 +
(-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
[Out] int((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="maxima")
[Out] -integrate((f*x + e)/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 38.13, size = 6499, normalized size = 18.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="fricas")
[Out] [-1/12*(4*sqrt(3)*a^(2/3)*b^(3/2)*e*weierstrassPInverse(0, 4*a/b, x) - a*b*
sqrt(-(2*sqrt(3)*a^(2/3)*b^(2/3)*f^2 - 2*(sqrt(3)*b*f*e + 3*b*f*e)*a^(1/3)
+ (2*sqrt(3)*b*e^2 + 3*b*e^2)*b^(1/3))/a)*log(-(8*a^2*b^9*f^6*x^24 + 14720*
a^3*b^8*f^6*x^21 + 538112*a^4*b^7*f^6*x^18 + 468992*a^5*b^6*f^6*x^15 + 4032
512*a^6*b^5*f^6*x^12 - 17121280*a^7*b^4*f^6*x^9 + 24805376*a^8*b^3*f^6*x^6
- 8716288*a^9*b^2*f^6*x^3 + 229376*a^10*b*f^6 + 32*(72*a^2*b^8*f^6*x^22 + 6
768*a^3*b^7*f^6*x^19 + 36936*a^4*b^6*f^6*x^16 - 43776*a^5*b^5*f^6*x^13 + 35
0208*a^6*b^4*f^6*x^10 - 787968*a^7*b^3*f^6*x^7 + 474624*a^8*b^2*f^6*x^4 - 3
6864*a^9*b*f^6*x - 9*(b^10*x^22 + 94*a*b^9*x^19 + 513*a^2*b^8*x^16 - 608*a^
3*b^7*x^13 + 4864*a^4*b^6*x^10 - 10944*a^5*b^5*x^7 + 6592*a^6*b^4*x^4 - 512
*a^7*b^3*x)*e^6 - 180*(a*b^9*f^3*x^22 + 94*a^2*b^8*f^3*x^19 + 513*a^3*b^7*f
```

^3*x^16 - 608*a^4*b^6*f^3*x^13 + 4864*a^5*b^5*f^3*x^10 - 10944*a^6*b^4*f^3*x^7 + 6592*a^7*b^3*f^3*x^4 - 512*a^8*b^2*f^3*x)*e^3 - sqrt(3)*(40*a^2*b^8*f^6*x^22 + 4040*a^3*b^7*f^6*x^19 + 17040*a^4*b^6*f^6*x^16 + 39424*a^5*b^5*f^6*x^13 - 229504*a^6*b^4*f^6*x^10 + 430080*a^7*b^3*f^6*x^7 - 281600*a^8*b^2*f^6*x^4 + 20480*a^9*b*f^6*x - (5*b^10*x^22 + 505*a*b^9*x^19 + 2130*a^2*b^8*x^16 + 4928*a^3*b^7*x^13 - 28688*a^4*b^6*x^10 + 53760*a^5*b^5*x^7 - 35200*a^6*b^4*x^4 + 2560*a^7*b^3*x)*e^6 - 20*(5*a*b^9*f^3*x^22 + 505*a^2*b^8*f^3*x^19 + 2130*a^3*b^7*f^3*x^16 + 4928*a^4*b^6*f^3*x^13 - 28688*a^5*b^5*f^3*x^10 + 53760*a^6*b^4*f^3*x^7 - 35200*a^7*b^3*f^3*x^4 + 2560*a^8*b^2*f^3*x)*e^3))*a^(2/3)*b^(1/3) + 8*(24*a^2*b^8*f^6*x^23 + 8616*a^3*b^7*f^6*x^20 + 106560*a^4*b^6*f^6*x^17 + 153600*a^5*b^5*f^6*x^14 - 890880*a^6*b^4*f^6*x^11 + 2760192*a^7*b^3*f^6*x^8 - 2629632*a^8*b^2*f^6*x^5 + 491520*a^9*b*f^6*x^2 - 3*(b^10*x^23 + 359*a*b^9*x^20 + 4440*a^2*b^8*x^17 + 6400*a^3*b^7*x^14 - 37120*a^4*b^6*x^11 + 115008*a^5*b^5*x^8 - 109568*a^6*b^4*x^5 + 20480*a^7*b^3*x^2)*e^6 - 60*(a*b^9*f^3*x^23 + 359*a^2*b^8*f^3*x^20 + 4440*a^3*b^7*f^3*x^17 + 6400*a^4*b^6*f^3*x^14 - 37120*a^5*b^5*f^3*x^11 + 115008*a^6*b^4*f^3*x^8 - 109568*a^7*b^3*f^3*x^5 + 20480*a^8*b^2*f^3*x^2)*e^3 - 2*sqrt(3)*(8*a^2*b^8*f^6*x^23 + 2392*a^3*b^7*f^6*x^20 + 34080*a^4*b^6*f^6*x^17 - 12160*a^5*b^5*f^6*x^14 + 213760*a^6*b^4*f^6*x^11 - 840192*a^7*b^3*f^6*x^8 + 745472*a^8*b^2*f^6*x^5 - 143360*a^9*b*f^6*x^2 - (b^10*x^23 + 299*a*b^9*x^20 + 4260*a^2*b^8*x^17 - 1520*a^3*b^7*x^14 + 26720*a^4*b^6*x^11 - 105024*a^5*b^5*x^8 + 93184*a^6*b^4*x^5 - 17920*a^7*b^3*x^2)*e^6 - 20*(a*b^9*f^3*x^23 + 299*a^2*b^8*f^3*x^20 + 4260*a^3*b^7*f^3*x^17 - 1520*a^4*b^6*f^3*x^14 + 26720*a^5*b^5*f^3*x^11 - 105024*a^6*b^4*f^3*x^8 + 93184*a^7*b^3*f^3*x^5 - 17920*a^8*b^2*f^3*x^2)*e^3))*a^(1/3)*b^(2/3) + 4*sqrt(b*x^3 - a)*((104*a^2*b^7*f^5*x^21 + 16720*a^3*b^6*f^5*x^18 + 158208*a^4*b^5*f^5*x^15 - 41728*a^5*b^4*f^5*x^12 + 1086976*a^6*b^3*f^5*x^9 - 2798592*a^7*b^2*f^5*x^6 + 1138688*a^8*b*f^5*x^3 - 32768*a^9*f^5 - 2*(b^9*x^22 + 764*a*b^8*x^19 + 16860*a^2*b^7*x^16 + 19792*a^3*b^6*x^13 + 42368*a^4*b^5*x^10 - 104448*a^5*b^4*x^7 + 90880*a^6*b^3*x^4 - 7168*a^7*b^2*x)*e^5 + 24*(32*a*b^8*f*x^20 + 1869*a^2*b^7*f*x^17 + 5862*a^3*b^6*f*x^14 + 7280*a^4*b^5*f*x^11 - 1824*a^5*b^4*f*x^8 + 7872*a^6*b^3*f*x^5 - 1408*a^7*b^2*f*x^2)*e^4 - 8*(32*a*b^8*f^2*x^21 + 5425*a^2*b^7*f^2*x^18 + 47184*a^3*b^6*f^2*x^15 + 37256*a^4*b^5*f^2*x^12 - 16064*a^5*b^4*f^2*x^9 - 960*a^6*b^3*f^2*x^6 - 13312*a^7*b^2*f^2*x^3 - 512*a^8*b*f^2)*e^3 - 2*(a*b^8*f^3*x^22 - 424*a^2*b^7*f^3*x^19 - 2256*a^3*b^6*f^3*x^16 - 82592*a^4*b^5*f^3*x^13 + 614336*a^5*b^4*f^3*x^10 - 2178048*a^6*b^3*f^3*x^7 + 1853440*a^7*b^2*f^3*x^4 - 145408*a^8*b*f^3*x)*e^2 - 24*(13*a^2*b^7*f^4*x^20 + 696*a^3*b^6*f^4*x^17 + 3480*a^4*b^5*f^4*x^14 - 14336*a^5*b^4*f^4*x^11 + 104640*a^6*b^3*f^4*x^8 - 144384*a^7*b^2*f^4*x^5 + 30208*a^8*b*f^4*x^2)*e - sqrt(3)*(56*a^2*b^7*f^5*x^21 + 10000*a^3*b^6*f^5*x^18 + 79872*a^4*b^5*f^5*x^15 + 155648*a^5*b^4*f^5*x^12 - 660992*a^6*b^3*f^5*x^9 + 1551360*a^7*b^2*f^5*x^6 - 679936*a^8*b*f^5*x^3 + 16384*a^9*f^5 - (b^9*x^22 + 896*a*b^8*x^19 + 18984*a^2*b^7*x^16 + 31168*a^3*b^6*x^13 - 21184*a^4*b^5*x^10 + 125952*a^5*b^4*x^7 - 104960*a^6*b^3*x^4 + 8192*a^7*b^2*x)*e^5 + 12*(37*a*b^8*f*x^20 + 2154*a^2*b^7*f*x^17 + 6900*a^3*b^6*f*x^14 + 6496*a^4*b^5*f*x^11 + 9600*a^5*b^4*f*x^8 - 729

```
6*a^6*b^3*f*x^5 + 1792*a^7*b^2*f*x^2)*e^4 - 4*(37*a*b^8*f^2*x^21 + 6260*a^2
*b^7*f^2*x^18 + 54624*a^3*b^6*f^2*x^15 + 40816*a^4*b^5*f^2*x^12 - 2752*a^5*
b^4*f^2*x^9 - 39936*a^6*b^3*f^2*x^6 + 1024*a^7*b^2*f^2*x^3 - 1024*a^8*b*f^2
)*e^3 + 2*(a*b^8*f^3*x^22 + 104*a^2*b^7*f^3*x^19 + 6240*a^3*b^6*f^3*x^16 -
37088*a^4*b^5*f^3*x^13 + 360128*a^5*b^4*f^3*x^10 - 1256448*a^6*b^3*f^3*x^7
+ 1070080*a^7*b^2*f^3*x^4 - 83968*a^8*b*f^3*x)*e^2 - 24*(7*a^2*b^7*f^4*x^20
 + 444*a^3*b^6*f^4*x^17 + 672*a^4*b^5*f^4*x^14 + 11200*a^5*b^4*f^4*x^11 - 5
8944*a^6*b^3*f^4*x^8 + 83712*a^7*b^2*f^4*x^5 - 17408*a^8*b*f^4*x^2)*e))*a^(
2/3)*b^(2/3) + 2*(2*a^2*b^8*f^5*x^22 + 2320*a^3*b^7*f^5*x^19 + 46464*a^4*b^
6*f^5*x^16 + 107840*a^5*b^5*f^5*x^13 - 296576*a^6*b^4*f^5*x^10 + 1173504*a^
7*b^3*f^5*x^7 - 993280*a^8*b^2*f^5*x^4 + 77824*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{-\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx-\int \frac{fx}{-\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(e/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(f*x/(-a**(1/3)*sqrt(-a +
b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)
), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e + f*x)/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.134 $\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=345

$$-\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\right.}{}$$

[Out] -1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*(b^(1/3)*e-a^(1/3)*f*(1+3^(1/2)))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*3^(1/4)/a^(1/3)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/2))*(b^(1/3)*e-a^(1/3)*f*(1-3^(1/2)))/b^(2/3)/a^(1/2)/(-9+6*3^(1/2))^(1/2)

Rubi [A]
time = 0.36, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.089$, Rules used = {2166, 225, 2165, 209}

$$-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{-a-bx^3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)}{\sqrt{3\left(2\sqrt{3}-3\right)}\,\sqrt{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -(((b^(1/3)*e - (1 - Sqrt[3])*a^(1/3)*f)*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3*(-3 + 2*Sqrt[3])]*Sqrt[a]*b^(2/3))) - (Sqrt[2 - Sqrt[3]]*(b^(1/3)*e - (1 + Sqrt[3])*a^(1/3)*f)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*a^(1/3)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{e+fx}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\cdots\right.\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}}{12\sqrt{3}\,a^{4/3}b^{4/3}\cdots}$$

$$= -\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{b}\,e-\left(1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\cdots}{3^{3/4}\sqrt[3]{a}\,b^{2/3}\sqrt{-\cdots}}$$

$$= -\frac{\left(\sqrt[3]{b}\,e-\left(1-\sqrt{3}\right)\sqrt[3]{a}\,f\right)\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\cdots}{\sqrt{-a-b\cdots}}\right.}{\sqrt{3\left(-3+2\sqrt{3}\right)}\,\sqrt{a}\,b^{2/3}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.02, size = 441, normalized size = 1.28

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{i\sqrt[4]{3}\,f\left(\left((-2-i)+\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(\sqrt[3]{b}\,e+\left(-1+\sqrt{3}\right)\sqrt[3]{a}\,f\right)\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),
x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*f*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqr
t[I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^
(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[
3])/2])/Sqrt[2] + I*(b^(1/3)*e + (-1 + Sqrt[3])*a^(1/3)*f)*Sqrt[((-2*I)*a^(
1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/
3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 +
2*I)*Sqrt[3]), ArcSin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*
```

```
I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3
)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a
 - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{\left(b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 44.15, size = 6561, normalized size = 19.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/12*(4*sqrt(3)*a^(2/3)*sqrt(-b)*b*e*weierstrassPInverse(0, -4*a/b, x) + a
*b*sqrt(-(2*sqrt(3)*a^(2/3)*b^(2/3)*f^2 + 2*(sqrt(3)*b*f*e + 3*b*f*e)*a^(1/
3) + (2*sqrt(3)*b*e^2 + 3*b*e^2)*b^(1/3))/a)*log(-(8*a^2*b^9*f^6*x^24 - 147
20*a^3*b^8*f^6*x^21 + 538112*a^4*b^7*f^6*x^18 - 468992*a^5*b^6*f^6*x^15 + 4
032512*a^6*b^5*f^6*x^12 + 17121280*a^7*b^4*f^6*x^9 + 24805376*a^8*b^3*f^6*x
^6 + 8716288*a^9*b^2*f^6*x^3 + 229376*a^10*b*f^6 + 32*(72*a^2*b^8*f^6*x^22
- 6768*a^3*b^7*f^6*x^19 + 36936*a^4*b^6*f^6*x^16 + 43776*a^5*b^5*f^6*x^13 +
 350208*a^6*b^4*f^6*x^10 + 787968*a^7*b^3*f^6*x^7 + 474624*a^8*b^2*f^6*x^4
+ 36864*a^9*b*f^6*x - 9*(b^10*x^22 - 94*a*b^9*x^19 + 513*a^2*b^8*x^16 + 608
*a^3*b^7*x^13 + 4864*a^4*b^6*x^10 + 10944*a^5*b^5*x^7 + 6592*a^6*b^4*x^4 +
```

512*a^7*b^3*x)*e^6 + 180*(a*b^9*f^3*x^22 - 94*a^2*b^8*f^3*x^19 + 513*a^3*b^
7*f^3*x^16 + 608*a^4*b^6*f^3*x^13 + 4864*a^5*b^5*f^3*x^10 + 10944*a^6*b^4*f
^3*x^7 + 6592*a^7*b^3*f^3*x^4 + 512*a^8*b^2*f^3*x)*e^3 - sqrt(3)*(40*a^2*b^
8*f^6*x^22 - 4040*a^3*b^7*f^6*x^19 + 17040*a^4*b^6*f^6*x^16 - 39424*a^5*b^5
*f^6*x^13 - 229504*a^6*b^4*f^6*x^10 - 430080*a^7*b^3*f^6*x^7 - 281600*a^8*b
^2*f^6*x^4 - 20480*a^9*b*f^6*x - (5*b^10*x^22 - 505*a*b^9*x^19 + 2130*a^2*b
^8*x^16 - 4928*a^3*b^7*x^13 - 28688*a^4*b^6*x^10 - 53760*a^5*b^5*x^7 - 3520
0*a^6*b^4*x^4 - 2560*a^7*b^3*x)*e^6 + 20*(5*a*b^9*f^3*x^22 - 505*a^2*b^8*f^
3*x^19 + 2130*a^3*b^7*f^3*x^16 - 4928*a^4*b^6*f^3*x^13 - 28688*a^5*b^5*f^3*
x^10 - 53760*a^6*b^4*f^3*x^7 - 35200*a^7*b^3*f^3*x^4 - 2560*a^8*b^2*f^3*x)*
e^3))*a^(2/3)*b^(1/3) - 8*(24*a^2*b^8*f^6*x^23 - 8616*a^3*b^7*f^6*x^20 + 10
6560*a^4*b^6*f^6*x^17 - 153600*a^5*b^5*f^6*x^14 - 890880*a^6*b^4*f^6*x^11 -
2760192*a^7*b^3*f^6*x^8 - 2629632*a^8*b^2*f^6*x^5 - 491520*a^9*b*f^6*x^2 -
3*(b^10*x^23 - 359*a*b^9*x^20 + 4440*a^2*b^8*x^17 - 6400*a^3*b^7*x^14 - 37
120*a^4*b^6*x^11 - 115008*a^5*b^5*x^8 - 109568*a^6*b^4*x^5 - 20480*a^7*b^3*
x^2)*e^6 + 60*(a*b^9*f^3*x^23 - 359*a^2*b^8*f^3*x^20 + 4440*a^3*b^7*f^3*x^1
7 - 6400*a^4*b^6*f^3*x^14 - 37120*a^5*b^5*f^3*x^11 - 115008*a^6*b^4*f^3*x^8
- 109568*a^7*b^3*f^3*x^5 - 20480*a^8*b^2*f^3*x^2)*e^3 - 2*sqrt(3)*(8*a^2*b
^8*f^6*x^23 - 2392*a^3*b^7*f^6*x^20 + 34080*a^4*b^6*f^6*x^17 + 12160*a^5*b^
5*f^6*x^14 + 213760*a^6*b^4*f^6*x^11 + 840192*a^7*b^3*f^6*x^8 + 745472*a^8*
b^2*f^6*x^5 + 143360*a^9*b*f^6*x^2 - (b^10*x^23 - 299*a*b^9*x^20 + 4260*a^2
*b^8*x^17 + 1520*a^3*b^7*x^14 + 26720*a^4*b^6*x^11 + 105024*a^5*b^5*x^8 + 9
3184*a^6*b^4*x^5 + 17920*a^7*b^3*x^2)*e^6 + 20*(a*b^9*f^3*x^23 - 299*a^2*b^
8*f^3*x^20 + 4260*a^3*b^7*f^3*x^17 + 1520*a^4*b^6*f^3*x^14 + 26720*a^5*b^5*
f^3*x^11 + 105024*a^6*b^4*f^3*x^8 + 93184*a^7*b^3*f^3*x^5 + 17920*a^8*b^2*f
^3*x^2)*e^3))*a^(1/3)*b^(2/3) - (b^11*x^24 - 1840*a*b^10*x^21 + 67264*a^2*b
^9*x^18 - 58624*a^3*b^8*x^15 + 504064*a^4*b^7*x^12 + 2140160*a^5*b^6*x^9 +
3100672*a^6*b^5*x^6 + 1089536*a^7*b^4*x^3 + 28672*a^8*b^3)*e^6 + 20*(a*b^10
*f^3*x^24 - 1840*a^2*b^9*f^3*x^21 + 67264*a^3*b^8*f^3*x^18 - 58624*a^4*b^7*
f^3*x^15 + 504064*a^5*b^6*f^3*x^12 + 2140160*a^6*b^5*f^3*x^9 + 3100672*a^7*
b^4*f^3*x^6 + 1089536*a^8*b^3*f^3*x^3 + 28672*a^9*b^2*f^3)*e^3 + 32*sqrt(3)
*(280*a^3*b^8*f^6*x^21 - 9128*a^4*b^7*f^6*x^18 + 20352*a^5*b^6*f^6*x^15 + 5
4080*a^6*b^5*f^6*x^12 + 316160*a^7*b^4*f^6*x^9 + 445440*a^8*b^3*f^6*x^6 + 1
57696*a^9*b^2*f^6*x^3 + 4096*a^10*b*f^6 - (35*a*b^10*x^21 - 1141*a^2*b^9*x^
18 + 2544*a^3*b^8*x^15 + 6760*a^4*b^7*x^12 + 39520*a^5*b^6*x^9 + 55680*a^6*
b^5*x^6 + 19712*a^7*b^4*x^3 + 512*a^8*b^3)*e^6 + 20*(35*a^2*b^9*f^3*x^21 -
1141*a^3*b^8*f^3*x^18 + 2544*a^4*b^7*f^3*x^15 + 6760*a^5*b^6*f^3*x^12 + 395
20*a^6*b^5*f^3*x^9 + 55680*a^7*b^4*f^3*x^6 + 19712*a^8*b^3*f^3*x^3 + 512*a^
9*b^2*f^3)*e^3) - 4*((104*a^2*b^7*f^5*x^21 - 16720*a^3*b^6*f^5*x^18 + 15820
8*a^4*b^5*f^5*x^15 + 41728*a^5*b^4*f^5*x^12 + 1086976*a^6*b^3*f^5*x^9 + 279
8592*a^7*b^2*f^5*x^6 + 1138688*a^8*b*f^5*x^3 + 32768*a^9*f^5 - 2*(b^9*x^22
- 764*a*b^8*x^19 + 16860*a^2*b^7*x^16 - 19792*a^3*b^6*x^13 + 42368*a^4*b^5*
x^10 + 104448*a^5*b^4*x^7 + 90880*a^6*b^3*x^4 + 7168*a^7*b^2*x)*e^5 - 24*(3
2*a*b^8*f*x^20 - 1869*a^2*b^7*f*x^17 + 5862*a^3*b^6*f*x^14 - 7280*a^4*b^5*f
*x^11 - 1824*a^5*b^4*f*x^8 - 7872*a^6*b^3*f*x^5 - 1408*a^7*b^2*f*x^2)*e^4 +

```
8*(32*a*b^8*f^2*x^21 - 5425*a^2*b^7*f^2*x^18 + 47184*a^3*b^6*f^2*x^15 - 37
256*a^4*b^5*f^2*x^12 - 16064*a^5*b^4*f^2*x^9 + 960*a^6*b^3*f^2*x^6 - 13312*
a^7*b^2*f^2*x^3 + 512*a^8*b*f^2)*e^3 + 2*(a*b^8*f^3*x^22 + 424*a^2*b^7*f^3*
x^19 - 2256*a^3*b^6*f^3*x^16 + 82592*a^4*b^5*f^3*x^13 + 614336*a^5*b^4*f^3*
x^10 + 2178048*a^6*b^3*f^3*x^7 + 1853440*a^7*b^2*f^3*x^4 + 145408*a^8*b*f^3
*x)*e^2 - 24*(13*a^2*b^7*f^4*x^20 - 696*a^3*b^6*f^4*x^17 + 3480*a^4*b^5*f^4
*x^14 + 14336*a^5*b^4*f^4*x^11 + 104640*a^6*b^3*f^4*x^8 + 144384*a^7*b^2*f^
4*x^5 + 30208*a^8*b*f^4*x^2)*e - sqrt(3)*(56*a^2*b^7*f^5*x^21 - 10000*a^3*b
^6*f^5*x^18 + 79872*a^4*b^5*f^5*x^15 - 155648*a^5*b^4*f^5*x^12 - 660992*a^6
*b^3*f^5*x^9 - 1551360*a^7*b^2*f^5*x^6 - 679936*a^8*b*f^5*x^3 - 16384*a^9*f
^5 - (b^9*x^22 - 896*a*b^8*x^19 + 18984*a^2*b^7*x^16 - 31168*a^3*b^6*x^13 -
 21184*a^4*b^5*x^10 - 125952*a^5*b^4*x^7 - 1049...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-a - bx^3} \left(-\sqrt{3}\,\sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1
/3)*x)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algor
ithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

3.135 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}+\frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -1/3*arctan((1+x)*(3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(1
+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2-x+1)
/(1+x+3^(1/2))^2)^(1/2)*3^(1/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2166, 224, 2165, 209}

$$\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4)) +
(Sqrt[2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 + x)
/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} + \frac{\left(-22+\left(1+\sqrt{3}\right)^3 \cdots\right.}{-28+\left(1+ \cdots\right.}$$

$$= \frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \cdots$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\cdots\right.}{3^{3/4}\sqrt{\frac{1+x \cdots}{\left(1+\sqrt{3} \cdots\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.33, size = 209, normalized size = 1.54

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[1 + x^3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(108) = 216.
time = 0.79, size = 255, normalized size = 1.88

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(-1-\sqrt{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(-1-3^(1/2))*(3/2-1/2*I*3^(1/
2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1
/2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 58, normalized size = 0.43

$$-\frac{1}{3}\left(\sqrt{3}-3\right)\mathrm{weierstrassPInverse}(0,-4,x)-\frac{1}{6}\cdot 3^{\frac{1}{4}}\sqrt{2}\,\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\left(3\,x^2-\sqrt{3}\,(x^2+2\,x+4)-6\,x\right)}{12\,\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(sqrt(3) - 3)*weierstrassPInverse(0, -4, x) - 1/6*3^(1/4)*sqrt(2)*arct
an(-1/12*3^(1/4)*sqrt(2)*(3*x^2 - sqrt(3)*(x^2 + 2*x + 4) - 6*x)/sqrt(x^3 +
 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)
[Out] \text{Hanged}
```

3.136 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}+\frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -1/3*arctan((1-x)*(3+2*3^(1/2))^(1/2)/(-x^3+1)^(1/2))*2^(1/2)*3^(1/4)+1/3*(
1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*((x^2+x+1
)/(1-x+3^(1/2))^2)^(1/2)*3^(1/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/
2)
```

Rubi [A]
time = 0.16, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2166, 224, 2165, 209}

$$\frac{\sqrt{2}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/3^(3/4)) +
(Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(
1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(3/4)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx = \frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)+6x}{\left(1+\sqrt{3}-x\right)\sqrt{1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} - \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \quad 2S$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{1-x^3}}\right)}{3^{3/4}} + \frac{\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(s\right.}{3^{3/4}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.46, size = 232, normalized size = 1.53

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2\left(1+\sqrt{3}\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[1 - x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 256 vs. 2(124) = 248.
time = 0.73, size = 257, normalized size = 1.69

method	result

default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*
3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2)
)*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-3/2+1/2*I*3^(1/2)-3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1
/2)-3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Fricas [A]
time = 0.13, size = 55, normalized size = 0.36

$$\frac{1}{6}\cdot 3^{\frac{1}{4}}\sqrt{2}\,\arctan\left(-\frac{3^{\frac{1}{4}}\sqrt{2}\,\sqrt{-x^{3}+1}\,\left(3\,x^{2}-\sqrt{3}\,(x^{2}-2\,x+4)+6\,x\right)}{12\,(x^{3}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*3^(1/4)*sqrt(2)*arctan(-1/12*3^(1/4)*sqrt(2)*sqrt(-x^3 + 1)*(3*x^2 - sq
rt(3)*(x^2 - 2*x + 4) + 6*x)/(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^3}-\sqrt{3}\sqrt{1-x^3}-\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(1 - x**3) - sqrt(3)*sqrt(1 - x**3) - sqrt(1 - x**3)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

3.137 $\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}}+\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] -1/3*arctanh((1-x)*(3+2*3^(1/2))^(1/2)/(x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*(
1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2*
2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1
-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2166, 225, 2165, 212}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx = -\frac{\left(-1-\sqrt{3}\right)\int \frac{\left(1+\sqrt{3}\right)\left(-22+\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}-x\right)\sqrt{-1+x^3}}\,dx}{\left(1+\sqrt{3}\right)\left(-28+\left(1+\sqrt{3}\right)^3\right)} - \frac{\left(-22+\left(1+\sqrt{3}\right.\right.}{-28+}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= -\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1-x)}{\sqrt{-1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1-x)\sqrt{\frac{1+}{\left(1-\right.}}}{\sqrt[4]{3}\sqrt{-\frac{}{\left(1\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.26, size = 230, normalized size = 1.40

$$\frac{2i\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{i\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3i+(1+2i)\sqrt{3}+\left(3+(2+i)\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+2\left(1+\sqrt{3}\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)\right)}{\left(3+(2+i)\sqrt{3}\right)\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]
```

```
[Out] ((2*I)*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((I*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]*(3*I + (1 + 2*I)*Sqrt[3] + (3 + (2 + I)*Sqrt[3])*x)*Ellip
ticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + 2*(1 + Sqrt[3])*Sqrt[1 + x + x^2]*Ell
ipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.76, size = 255, normalized size = 1.55

method	result

default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-$ 2(−
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-$ 2(−

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(-1-3^(1/2))*(-3/2-1/2*I*3^
(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)
^(1/2)*3^(1/2)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2
*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 213, normalized size = 1.30

$$\frac{1}{3}\left(\sqrt{3}-3\right)\text{weierstrassPInverse}(0,4,x)+\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8+16\,x^7+112\,x^6+16\,x^5+112\,x^4-224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6+18\,x^5+12\,x^4+40\,x^3-36\,x^2+\sqrt{3}\,(x^6+6\,x^5+24\,x^4-8\,x^3+12\,x^2-24\,x+16)+24\,x-32\right)\sqrt{x^3-1}+64\,x^2+16\,\sqrt{3}\,(x^7+2\,x^6+6\,x^5-5\,x^4+2\,x^3-6\,x^2+4\,x-4)-128\,x+112}{x^8-8\,x^7+16\,x^6+16\,x^5-56\,x^4-32\,x^3+64\,x^2+64\,x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) - 3)*weierstrassPInverse(0, 4, x) + 1/12*3^(1/4)*sqrt(2)*log((
x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 2*3^(1/4)*sqrt(2)*(x^
6 + 18*x^5 + 12*x^4 + 40*x^3 - 36*x^2 + sqrt(3)*(x^6 + 6*x^5 + 24*x^4 - 8*x
```

```
^3 + 12*x^2 - 24*x + 16) + 24*x - 32)*sqrt(x^3 - 1) + 64*x^2 + 16*sqrt(3)*(
x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8
- 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1}-\sqrt{3}\,\sqrt{x^3-1}-\sqrt{x^3-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3**(1/2))/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(x/(x*sqrt(x**3 - 1) - sqrt(3)*sqrt(x**3 - 1) - sqrt(x**3 - 1)), x
)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)
```

```
[Out] \text{Hanged}
```

3.138 $\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}}+\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+\text{[cut off]}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -1/3*arctanh((1+x)*(3+2*3^(1/2))^(1/2)/(-x^3-1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(1/3*6^(1/2)-1/2
*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/
(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2166, 225, 2165, 212}

$$\frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(x+1)}{\sqrt{-x^3-1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/3^(3/4))
+ (2*Sqrt[7/6 - 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx = -\frac{\int \frac{\left(1+\sqrt{3}\right)\left(22-\left(1+\sqrt{3}\right)^3\right)-6x}{\left(1+\sqrt{3}+x\right)\sqrt{-1-x^3}}\,dx}{6\left(3-\sqrt{3}\right)} + \frac{\left(22-\left(1+\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{-1-x^3}}}{28-\left(1+\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\,(1+x)}{\sqrt{-1-x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}-\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x}{\left(1-\sqrt{3}\right.}}}{\sqrt[4]{3}\sqrt{-\frac{}{\left(1-\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 211, normalized size = 1.35

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}} + \frac{2i\left(1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+(2+i)\sqrt{3}}\right)}{\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((2*I)*(1 + Sqrt[3])*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 +
 I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(3 + (2 + I)*Sqrt[3])))/Sqrt[-1 - x^3]
```

Maple [A]
time = 0.70, size = 253, normalized size = 1.62

method	result

default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*I*(-1-3^(1/2))*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^
(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^
(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1
/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^
(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

Fricas [A]
time = 0.12, size = 203, normalized size = 1.30

$$\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\,\log\left(\frac{x^8-16\,x^7+112\,x^6-16\,x^5+112\,x^4+224\,x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18\,x^5+12\,x^4-40\,x^3-36\,x^2+\sqrt{3}\,(x^6-6\,x^5+24\,x^4+8\,x^3+12\,x^2+24\,x+16)-24\,x-32\right)\sqrt{-x^3-1}+64\,x^2-16\,\sqrt{3}\,(x^7-2\,x^6+6\,x^5+5\,x^4+2\,x^3+6\,x^2+4\,x+4)+128\,x+112}{x^8+8\,x^7+16\,x^6-16\,x^5-56\,x^4+32\,x^3+64\,x^2-64\,x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*3^(1/4)*sqrt(2)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x
^3 + 2*3^(1/4)*sqrt(2)*(x^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 + sqrt(3)*(
x^6 - 6*x^5 + 24*x^4 + 8*x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(-x^3 -
 1) + 64*x^2 - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*
x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64
*x^2 - 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\left(x+1+\sqrt{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{1,[2]%%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%%} Error: Bad Argum
ent Va
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.139 $\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=147

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}}+\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -1/3*arctanh((1+x)*(-3+2*3^(1/2))^(1/2)/(x^3+1)^(1/2))*2^(1/2)*3^(1/4)+2/3*
(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/3*6^(1/2)+1/2
*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1
+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2166, 224, 2165, 212}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,(x+1)}{\sqrt{x^3+1}}\right)}{3^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/3^(3/4))
+ (2*Sqrt[7/6 + 2/Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\left(-22+\left(1-\sqrt{3}\right)^3\right)+6x}{\left(1-\sqrt{3}+x\right)\sqrt{1+x^3}}\,dx}{6\left(3+\sqrt{3}\right)} + \frac{\left(-22+\left(1-\sqrt{3}\right)^3\right)\int \frac{1}{\sqrt{1+x^3}}}{-28+\left(1-\sqrt{3}\right)^3}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{}\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,(1+x)}{\sqrt{1+x^3}}\right)}{3^{3/4}} + \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\,(1+x)\sqrt{\frac{1-}{\left(1+\sqrt{}\right.}}}{\sqrt[4]{3}\sqrt{\frac{}{(1}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.41, size = 225, normalized size = 1.53

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(\frac{\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\left(3-(2+i)\sqrt{3}+\left(-3i+(1+2i)\sqrt{3}\right)x\right)F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}-2\left(-1+\sqrt{3}\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)\right)}{\left(-3i+(1+2i)\sqrt{3}\right)\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 + (
-1)^(1/3))]*(3 - (2 + I)*Sqrt[3] + (-3*I + (1 + 2*I)*Sqrt[3])*x)*EllipticF[
ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (
-1)^(2/3)*x)/(1 + (-1)^(1/3))] - 2*(-1 + Sqrt[3])*Sqrt[1 - x + x^2]*Ellipti
cPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/
(1 + (-1)^(1/3))]], (-1)^(1/3)]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[1 + x^3]
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(116) = 232.
time = 0.81, size = 255, normalized size = 1.73

method	result
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} - \frac{2\left(\sqrt{3}-1\right.}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}} + \frac{2\left(1-\sqrt{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(1-3^(1/2))*(3/2-1/2*I*3^(1/2
))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(
1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/
2)*3^(1/2)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^
(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 214, normalized size = 1.46

$$\frac{1}{3}\left(\sqrt{3}+3\right)\mathrm{weierstrassPInverse}(0,-4,x)+\frac{1}{12}\cdot 3^{\frac{1}{4}}\sqrt{2}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+2\cdot 3^{\frac{1}{4}}\sqrt{2}\left(x^6-18x^5+12x^4-40x^3-36x^2-\sqrt{3}\left(x^6-6x^5+24x^4+8x^3+12x^2+24x+16\right)-24x-32\right)\sqrt{x^3+1}+64x^2+16\sqrt{3}\left(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4\right)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + 3)*weierstrassPInverse(0, -4, x) + 1/12*3^(1/4)*sqrt(2)*log(
(x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 2*3^(1/4)*sqrt(2)*(x
```

```
^6 - 18*x^5 + 12*x^4 - 40*x^3 - 36*x^2 - sqrt(3)*(x^6 - 6*x^5 + 24*x^4 + 8*
x^3 + 12*x^2 + 24*x + 16) - 24*x - 32)*sqrt(x^3 + 1) + 64*x^2 + 16*sqrt(3)*
(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8
 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)(x^2-x+1)}\left(x-\sqrt{3}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3**(1/2))/(x**3+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

3.140 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=278

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}}}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}}}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3+a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2166, 224, 2165, 212}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqr
t[a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right)\int}{\left(3+\sqrt{\right.}}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.81, size = 427, normalized size = 1.54

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{i\sqrt[4]{3}\left(\left((-2-i)+\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(-1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b
^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
Sin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="m
axima")
[Out] integrate(x/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 1288, normalized size = 4.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="f
ricas")
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*x
^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 21401
60*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 2*sq
rt(2)*(26*a*b^7*x^21 - 4180*a^2*b^6*x^18 + 39552*a^3*b^5*x^15 + 10432*a^4*b
^4*x^12 + 271744*a^5*b^3*x^9 + 699648*a^6*b^2*x^6 + 284672*a^7*b*x^3 + 8192
*a^8 - (b^7*x^22 - 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^1
3 - 148288*a^4*b^3*x^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7
*x - sqrt(3)*(b^7*x^22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4
*x^13 + 105920*a^4*b^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528
*a^7*x))*a^(2/3)*b^(1/3) - 12*(17*a*b^6*x^20 - 1014*a^2*b^5*x^17 + 2748*a^3
*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 - 53376*a^6*b*x^5 - 11008
*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 +
 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 15168*a^6*b*x^5 + 3200*a^7*x^2))*a^
(1/3)*b^(2/3) - 2*sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*
```

```
x^15 - 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*
a^7*b*x^3 - 2048*a^8))*sqrt(b*x^3 + a)*sqrt(sqrt(3)/a) + 32*(9*b^7*x^22 - 8
46*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10
+ 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 -
505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10
 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(
3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 11
1360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 -
 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^
14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*
x^2))*a^(1/3)*b^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 254
4*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6
+ 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18
 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a
^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)) + 4*b^(7/6)*(sqrt(3) + 3)*weierst
rassPInverse(0, -4*a/b, x))/b^2, 1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)
/a)*arctan(1/12*(2*sqrt(2)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/
a) + sqrt(2)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) + 4*sqr
t(3)*sqrt(2)*a*sqrt(-sqrt(3)/a))/sqrt(b*x^3 + a)) + 2*b^(7/6)*(sqrt(3) + 3)
*weierstrassPInverse(0, -4*a/b, x))/b^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.141 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{a-bx^3}}\,dx$

Optimal. Leaf size=286

$$-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}+\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\cdots\right.}}\cdots}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}\cdots}$$

```
[Out] -1/3*arctanh(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3+a)^(1
/2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+2/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^
(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I
)*(1/3*6^(1/2)+1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2166, 224, 2165, 212}

$$\frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqr
t[a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (2*Sqrt[7/6 + 2/Sqrt[3]]*(a^(1/
3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2
/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x
)^2]*Sqrt[a - b*x^3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx = \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)+6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{a-bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} - \frac{\left(2+\sqrt{3}\right)\int}{\left(3+\sqrt{3}\right.}$$

$$= \frac{2\sqrt{\frac{7}{6}+\frac{2}{\sqrt{3}}}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\right.}}}$$

$$= -\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{2\sqrt{\frac{7}{6}+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.84, size = 454, normalized size = 1.59

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+i\left(-1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"maxima")
[Out] -integrate(x/(sqrt(-b*x^3 + a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 1354, normalized size = 4.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"fricas")
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*x
^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 21401
60*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*(
9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776
*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*
(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 2868
8*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3)
*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3
*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 + 6
1440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 - 1
520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x^
5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 2*sqrt(2)*((b^7*x^22 + 1160*a*b^6*x^1
9 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x^10 + 586752*
a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt(3)*(b^7*x^22 + 632*a*b^
```

```
6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 105920*a^4*b^3*x^10 - 334
848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*sqrt(-b*x^3 + a)*a^(2/3)
*b^(1/3) + 12*(17*a*b^6*x^20 + 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 + 9632
*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a^6*b*x^5 - 11008*a^7*x^2 - 2*sqr
t(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 1038*a^3*b^4*x^14 - 784*a^4*b^3*x^1
1 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3200*a^7*x^2))*sqrt(-b*x^3 + a)*a
^(1/3)*b^(2/3) + 2*(13*a*b^7*x^21 + 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15
- 5216*a^4*b^4*x^12 + 135872*a^5*b^3*x^9 - 349824*a^6*b^2*x^6 + 142336*a^7*
b*x^3 - 4096*a^8 - sqrt(3)*(7*a*b^7*x^21 + 1250*a^2*b^6*x^18 + 9984*a^3*b^5
*x^15 + 19456*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 + 193920*a^6*b^2*x^6 - 84992
*a^7*b*x^3 + 2048*a^8))*sqrt(-b*x^3 + a))*sqrt(sqrt(3)/a) - 32*sqrt(3)*(35*
a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39
520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24
 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*
x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8
)) + 4*sqrt(-b)*b^(2/3)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b, x))/b^2
, -1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*arctan(-1/12*(2*sqrt(2)*sq
rt(-b*x^3 + a)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) - sqrt(2)
*sqrt(-b*x^3 + a)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) -
4*sqrt(3)*sqrt(2)*sqrt(-b*x^3 + a)*a*sqrt(-sqrt(3)/a))/(b*x^3 - a)) - 2*sqr
t(-b)*b^(2/3)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b, x))/b^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-\sqrt[3]{a}\,\sqrt{a-bx^3}+\sqrt{3}\,\sqrt[3]{a}\,\sqrt{a-bx^3}+\sqrt[3]{b}\,x\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
[Out] -Integral(x/(-a**(1/3)*sqrt(a - b*x**3) + sqrt(3)*a**(1/3)*sqrt(a - b*x**3)
 + b**(1/3)*x*sqrt(a - b*x**3)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm=
"giac")
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.142 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=282

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}+\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{si}\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\right.}}}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)-b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(b*x^3-a)^(1/2
))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
2^(1/2)*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*
x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2166, 225, 2165, 209}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\text{ArcSin}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{bx^3-a}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt
[-a + b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) - b^(1/3)*x)*
Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a
+ b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx=-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(-22ab+\left(1-\sqrt{3}\right)^3ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{-a+bx^3}}\,dx}{6\left(3+\sqrt{3}\right)ab^{4/3}}-\frac{\left(2+\sqrt{3}\right.}{\left(3\right.}$$

$$=\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\right.}}}$$

$$=-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{-a+bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}}+\frac{\sqrt{2}\left(\sqrt[3]{\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.22, size = 455, normalized size = 1.61

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(\frac{1}{2}\left(i\left(-3+(2+i)\sqrt{3}\right)\sqrt[3]{a}+\left(3-(2-i)\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{\frac{\left(-i+\sqrt{3}\right)\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)+i\left(-1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{-\frac{i\left(2\sqrt[3]{a}+\left(1-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((I*(-3 + (2 +
I)*Sqrt[3])*a^(1/3) + (3 - (2 - I)*Sqrt[3])*b^(1/3)*x)*Sqrt[((-I + Sqrt[3])
*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*EllipticF[A
rcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])
*a^(1/3))]], (1 + I*Sqrt[3])/2])/2 + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-I)*(2
*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 +
 (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3
)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt
[3])*b^(2/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3
))]*Sqrt[-a + b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-b^{\frac{1}{3}}x+a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] int(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate(x/(sqrt(b*x^3 - a)*(b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 1295, normalized size = 4.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 + 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140
160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*
(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 4377
6*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)
*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 286
88*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x))*a^(2/3
)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^
3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 +
61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 -
1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x
^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2
*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 556
80*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 2*sqrt(b*x^3 - a)*(sqrt(2)*(b
```

```
^7*x^22 + 1160*a*b^6*x^19 + 23232*a^2*b^5*x^16 + 53920*a^3*b^4*x^13 - 14828
8*a^4*b^3*x^10 + 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 + 38912*a^7*x - sqrt
(3)*(b^7*x^22 + 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 + 8416*a^3*b^4*x^13 + 1
05920*a^4*b^3*x^10 - 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 - 22528*a^7*x))*
a^(2/3)*b^(1/3)*sqrt(-sqrt(3)/a) + 12*sqrt(2)*(17*a*b^6*x^20 + 1014*a^2*b^5
*x^17 + 2748*a^3*b^4*x^14 + 9632*a^4*b^3*x^11 - 36096*a^5*b^2*x^8 + 53376*a
^6*b*x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 + 285*a^2*b^5*x^17 + 103
8*a^3*b^4*x^14 - 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 - 15168*a^6*b*x^5 + 3
200*a^7*x^2))*a^(1/3)*b^(2/3)*sqrt(-sqrt(3)/a) + 2*sqrt(2)*(13*a*b^7*x^21 +
 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 - 5216*a^4*b^4*x^12 + 135872*a^5*b^
3*x^9 - 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 - 4096*a^8 - sqrt(3)*(7*a*b^7
*x^21 + 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15 + 19456*a^4*b^4*x^12 - 82624*
a^5*b^3*x^9 + 193920*a^6*b^2*x^6 - 84992*a^7*b*x^3 + 2048*a^8))*sqrt(-sqrt(
3)/a)))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15
+ 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7
*b*x^3 + 4096*a^8)) - 4*b^(7/6)*(sqrt(3) + 3)*weierstrassPInverse(0, 4*a/b,
 x))/b^2, -1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)*arctan(-1/12*sqrt(2
)*(2*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3) - (sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2
/3) - 4*sqrt(3)*a)*sqrt(sqrt(3)/a)/sqrt(b*x^3 - a)) + 2*b^(7/6)*(sqrt(3) +
3)*weierstrassPInverse(0, 4*a/b, x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt{3}\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}\,x\sqrt{-a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-a**(1/3)*sqrt(-a + b*x**3) + sqrt(3)*a**(1/3)*sqrt(-a + b*x**
3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.143 $\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{-a-bx^3}}\,dx$

Optimal. Leaf size=278

$$-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}+\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin\ldots\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\ldots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\ldots\right.}}}$$

```
[Out] -1/3*arctan(a^(1/6)*(a^(1/3)+b^(1/3)*x)*(-3+2*3^(1/2))^(1/2)/(-b*x^3-a)^(1/
2))*2^(1/2)*3^(1/4)/a^(1/6)/b^(2/3)+1/3*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2
^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)*3^(1/4)/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {2166, 225, 2165, 209}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt{2}\,\text{ArcTan}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] -((Sqrt[2]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt
[-a - b*x^3]])/(3^(3/4)*a^(1/6)*b^(2/3))) + (Sqrt[2]*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b
^(1/3)*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(3/4)*b^(2/3)*Sqrt[-((a^(1
/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a
- b*x^3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 2165

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(1 + k)*(e/d), Su
bst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + (1 + k)*d*(x/c))/Sqrt[a + b*x^
3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^
6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3
), 0]
```

Rule 2166

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> Dist[-(6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3))/(c*d*(b*c^3 - 28*a*d
^3)), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/(c*d*(b*c^3 - 28*a*d
^3)), Int[(c*(b*c^3 - 22*a*d^3) + 6*a*d^4*x)/((c + d*x)*Sqrt[a + b*x^3]), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 -
 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3),
0]
```

Rubi steps

$$\int \frac{x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}\,dx = -\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\left(22ab-\left(1-\sqrt{3}\right)^3 ab\right)-6ab^{4/3}x}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{-a-bx^3}}dx}{6\left(3+\sqrt{3}\right)ab^{4/3}} + \frac{\left(2+\sqrt{3}\right.}{\left(3\right.}$$

$$= \frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-}\right.}{3^{3/4}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}}}$$

$$= -\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\,\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{-a-bx^3}}\right)}{3^{3/4}\sqrt[6]{a}\,b^{2/3}} + \frac{\sqrt{2}\left(\sqrt[3]{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.32, size = 430, normalized size = 1.55

$$-\frac{4\sqrt{\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\left(-\frac{i\sqrt[4]{3}\left(\left((-2-i)+\sqrt{3}\right)\sqrt[3]{a}+\left((1+2i)-i\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)}{2\sqrt{2}}+i\left(-1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\sqrt{1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}+\frac{b^{2/3}x^2}{a^{2/3}}}\,\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{-2i\sqrt[3]{a}+\left(i+\sqrt{3}\right)\sqrt[3]{b}\,x}{\left(-3i+\sqrt{3}\right)\sqrt[3]{a}}}\right)\middle|\frac{1}{2}\left(1+i\sqrt{3}\right)\right)\right)}{\left(3-(2-i)\sqrt{3}\right)b^{2/3}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}}\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]
```

```
[Out] (-4*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(((-1/2*I)*3^(1/
4)*(((-2 - I) + Sqrt[3])*a^(1/3) + ((1 + 2*I) - I*Sqrt[3])*b^(1/3)*x)*Sqrt[
I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[((-2*I)*a^(1
/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3]
)/2])/Sqrt[2] + I*(-1 + Sqrt[3])*a^(1/3)*Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3
])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b
^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), Arc
Sin[Sqrt[((-2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/
3))]], (1 + I*Sqrt[3])/2]))/((3 - (2 - I)*Sqrt[3])*b^(2/3)*Sqrt[(a^(1/3) +
(-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}\left(1-\sqrt{3}\right)\right)\sqrt{-b\,x^{3}-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
[Out] int(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
maxima")
[Out] integrate(x/(sqrt(-b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 1348, normalized size = 4.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
fricas")
[Out] [1/12*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(-sqrt(3)/a)*log((b^8*x^24 - 1840*a*b^7*
x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140
160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 - 4*s
qrt(2)*(13*a*b^7*x^21 - 2090*a^2*b^6*x^18 + 19776*a^3*b^5*x^15 + 5216*a^4*b
^4*x^12 + 135872*a^5*b^3*x^9 + 349824*a^6*b^2*x^6 + 142336*a^7*b*x^3 + 4096
*a^8 - sqrt(3)*(7*a*b^7*x^21 - 1250*a^2*b^6*x^18 + 9984*a^3*b^5*x^15 - 1945
6*a^4*b^4*x^12 - 82624*a^5*b^3*x^9 - 193920*a^6*b^2*x^6 - 84992*a^7*b*x^3 -
 2048*a^8))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) + 2*(144*b^7*x^22 - 13536*a*b
^6*x^19 + 73872*a^2*b^5*x^16 + 87552*a^3*b^4*x^13 + 700416*a^4*b^3*x^10 + 1
575936*a^5*b^2*x^7 + 949248*a^6*b*x^4 + 73728*a^7*x + sqrt(2)*(b^7*x^22 - 1
160*a*b^6*x^19 + 23232*a^2*b^5*x^16 - 53920*a^3*b^4*x^13 - 148288*a^4*b^3*x
^10 - 586752*a^5*b^2*x^7 - 496640*a^6*b*x^4 - 38912*a^7*x - sqrt(3)*(b^7*x^
22 - 632*a*b^6*x^19 + 14736*a^2*b^5*x^16 - 8416*a^3*b^4*x^13 + 105920*a^4*b
^3*x^10 + 334848*a^5*b^2*x^7 + 286720*a^6*b*x^4 + 22528*a^7*x))*sqrt(-b*x^3
```

```
 - a)*sqrt(-sqrt(3)/a) - 16*sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2
*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35
200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x
^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 34502
4*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 3*sqrt(2)*(17*a*b^6*x^20
 - 1014*a^2*b^5*x^17 + 2748*a^3*b^4*x^14 - 9632*a^4*b^3*x^11 - 36096*a^5*b^
2*x^8 - 53376*a^6*b*x^5 - 11008*a^7*x^2 - 2*sqrt(3)*(5*a*b^6*x^20 - 285*a^2
*b^5*x^17 + 1038*a^3*b^4*x^14 + 784*a^4*b^3*x^11 + 11424*a^5*b^2*x^8 + 1516
8*a^6*b*x^5 + 3200*a^7*x^2))*sqrt(-b*x^3 - a)*sqrt(-sqrt(3)/a) - 2*sqrt(3)*
(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*
a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/
3)*b^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x
^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7
*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^
3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6
- 40960*a^7*b*x^3 + 4096*a^8)) - 4*sqrt(-b)*b^(2/3)*(sqrt(3) + 3)*weierstra
ssPInverse(0, -4*a/b, x))/b^2, 1/6*(sqrt(2)*a^(1/3)*b^(4/3)*sqrt(sqrt(3)/a)
*arctan(1/12*sqrt(2)*(2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 3*x)*a^(2/3)*b^(1/3)
+ sqrt(-b*x^3 - a)*(sqrt(3)*x^2 + 3*x^2)*a^(1/3)*b^(2/3) + 4*sqrt(3)*sqrt(-
b*x^3 - a)*a)*sqrt(sqrt(3)/a)/(b*x^3 + a)) - 2*sqrt(-b)*b^(2/3)*(sqrt(3) +
3)*weierstrassPInverse(0, -4*a/b, x))/b^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-\sqrt{3}\sqrt[3]{a}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)),
 x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="
giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
[Out] \text{Hanged}
```

3.144 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=317

$$-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+4\sqrt[4]{3}\sqrt{2-\cdots}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/
((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+
x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1
/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(x^3+1
)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.90, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2167, 2138, 551, 585, 95, 211}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\left(1-\sqrt{3}\right)d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{ArcTan}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])) + (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*E
llipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqrt
[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[(
1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+}}$$

$$= -\frac{\left(4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\left(-c+d+\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\right.}{\sqrt{\frac{1+}{\left(1+\sqrt{3}\right.}}}$$

$$= -\frac{4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= -\frac{\left(c-d-\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1}{\left(1+\right.}}}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{\frac{1-x}{\left(1+\sqrt{3}\right.}}}\right.}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.40, size = 214, normalized size = 0.68

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c-\left(1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}\,d}{c+\sqrt[3]{-1}\,d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}\,d}\right)}{d\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c - (1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-
1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]
)/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A]
time = 0.29, size = 275, normalized size = 0.87

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2\left(d\sqrt{3}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(d*3^(1/2)-c+d)/d^2*(3/2-1/2*
I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*(x + sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{(x+1)\left(x^2-x+1\right)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.145 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=329

$$-\frac{\left(c+d+\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{4\sqrt[4]{3}\sqrt{2+}}{}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-
x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.92, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2167, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (4*3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*El
lipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3] -
 x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)/
(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1+\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\right.}}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}\,d\right)^2}{\left(c+d-\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-}{1+\sqrt{3}-}\right.\right.}{\left(c+d-\sqrt{3}\,d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}\,d\right)^2}{\left(c+d-\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-}{1+\sqrt{3}-}\right.\right.}{\left(c+d-\sqrt{3}\,d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{\left(c+d+\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{}{\left(1+\right.}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+}{\left(1+\right.}}}\right.}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.52, size = 235, normalized size = 0.71

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}\,c+\left(3+\sqrt{3}\right)d\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}\,d}{-c+\sqrt[3]{-1}\,d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}\,d}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3
])
```

Maple [A]
time = 0.28, size = 264, normalized size = 0.80

method	result
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3d\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3d\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(c+d+d*3^(1/2))/d^2*3^(1/2)*(I*(x+1/2-1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I
*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx-\int\frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx-\int\left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
[Out] -Integral(-sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x
/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3
) + d*x*sqrt(1 - x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.146 $\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(c+d+\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{4\sqrt[4]{3}\sqrt{2+}}{}$$

```
[Out] -(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*(c+d+d*3^(1/2))*((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/(
(1-x)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x
+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1
/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c+d-d*3^(1/2))/(x^3-1)^(1/2)
/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.66, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2167, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c-\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(c+\sqrt{3}d+d\right)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((c + d + Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcT
anh[(Sqrt[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt
[c + d]*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (4*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*E
llipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, ArcSin[(1 - Sqrt[3]
- x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c + d - Sqrt[3]*d)*Sqrt[(1 - x)
/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(c+\left(1-\sqrt{3}\right)d+\left(c+\left(1-\right.\right.\right.}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\right.}}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}\,d\right)^2}{\left(c+d-\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-}{1+\sqrt{3}-}\right.\right.}{\left(c+d-\sqrt{3}\,d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d+\sqrt{3}\,d\right)^2}{\left(c+d-\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1-\sqrt{3}-}{1+\sqrt{3}-}\right.\right.}{\left(c+d-\sqrt{3}\,d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

$$= -\frac{\left(c+d+\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{}{\left(1+\right.}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+}{\left(1+\right.}}}\right.}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 233, normalized size = 0.72

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}\,c+\left(3+\sqrt{3}\right)d\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}\,d}{-c+\sqrt[3]{-1}\,d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}\,d}\right)}{3d\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (3 + Sqrt[3])*d)*Sqrt[1 + x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^
3])
```

Maple [A]
time = 0.28, size = 273, normalized size = 0.84

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(c+d+d*3^(1/2))/d^2*(-3/2-1
/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/
(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1
/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx-\int\frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int\left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
[Out] -Integral(-sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x
/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1
) + d*x*sqrt(x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.147 $\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=321

$$-\frac{\left(c-\left(1+\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2\cdots}}{\cdots}$$

```
[Out] -(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)
/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*(c-d*(1+3^(1/2)))*((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)-4*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1
+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d*(1-3^(1/2)))/(-x^3
-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.75, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2167, 2138, 551, 585, 95, 211}

$$\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\left(1-\sqrt{3}\right)d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(c-\left(1+\sqrt{3}\right)d\right)\text{ArcTan}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 + Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*Ar
cTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]
*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sq
rt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])) + (4
*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, ArcSin[(1 - Sqr
t[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c - (1 - Sqrt[3])*d)*Sqrt[
(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(-c+\left(1-\sqrt{3}\right)d+\left(-c+\right.\right.}\right.}{\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1}}$$

$$= -\frac{\left(4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(-c+d+\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\right.}{\sqrt{\frac{1+}{\left(1+\sqrt{3}\right.}}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1-}{1+}\right.\right.}{\left(c-\left(1-\sqrt{3}\right)d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= -\frac{\left(c-d-\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1}{\left(1+\right.}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-}{\left(1+\right.}}}\right.}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.51, size = 233, normalized size = 0.73

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(3+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{3d\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (3 + Sqrt[3])*d)*Sqrt[1 - x + x^2
]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^3
])
```

Maple [A]
time = 0.28, size = 266, normalized size = 0.83

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i(x-}}{}\right.}{3d\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i(x-}}{}\right.}{3d\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*(d*3^(1/2)-c+d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
```

```
(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1+\sqrt{3}}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.148 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=358

$$-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1
/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2)
))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1
)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((-1-x)/(1+x-3^(1/2
))^2)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$,
Rules used = {2168, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
```

```
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[1 + x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*
(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[3
])*d)^2/(c - (1 + Sqrt[3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)
], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2
)]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
```

```
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\operatorname{Subst}\left(\int \frac{}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-c\right.\right.}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{}}$$

$$= -\frac{\left(4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\left(-c+d+\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\operatorname{Subst}\left(\int\right.}{\sqrt{-\frac{1}{\left(1-\sqrt{}\right.}}}$$

$$= \frac{4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c-d-\sqrt{3}\,d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= \frac{4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c-d-\sqrt{3}\,d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{c^2+c}}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{7+}}\right.}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{1}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.33, size = 213, normalized size = 0.59

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i\left(c+\left(-1+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
(I*(c + (-1 + Sqrt[3])*d)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (
-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)
])/(c + (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A]
time = 0.27, size = 275, normalized size = 0.77

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-2\left(d\sqrt{3}+\right.$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-2\left(d\sqrt{3}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)+c-d)/d^2*(3/2-1/2*
I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^
3+1)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*
I*3^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^3 + 1)*(x - sqrt(3) + 1)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

```
[Out] \text{Hanged}
```

3.149 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=346

$$-\frac{\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}+\frac{4\sqrt[4]{3}\sqrt{2-}}{}$$

```
[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c
+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-
x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/
((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1
-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)
-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(-x^3+1)^(1
/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2168, 2138, 551, 585, 95, 211}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}d+d\right)^2}{\left(c+\sqrt{3}d+d\right)^2};\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}d+d\right)\text{ArcTan}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcT
an[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sq
rt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*S
qrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])) +
 (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt
[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-((
1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\left(1\right.\right.\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-\right.}}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1}{\left(1-\right.}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x}{\left(1-\sqrt{3}\right.}}}\right.}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.45, size = 235, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}\,c+\left(-3+\sqrt{3}\right)d\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}\,d}{-c+\sqrt[3]{-1}\,d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}\,d}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^
3])
```

Maple [A]
time = 0.26, size = 268, normalized size = 0.77

method	result
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3d\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\right.}}{}\right.}{3d\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I
*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)-c-d)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*
I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I
*3^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{3}}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx-\int \frac{x}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\,dx-\int \left(-\frac{1}{c\sqrt{1-x^3}+dx\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(-x**3+1)**(1/2),x)
[Out] -Integral(sqrt(3)/(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(x/
(c*sqrt(1 - x**3) + d*x*sqrt(1 - x**3)), x) - Integral(-1/(c*sqrt(1 - x**3)
 + d*x*sqrt(1 - x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.150 $\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=342

$$-\frac{\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}}{\sqrt{d}\,\sqrt{c+d}\,\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\,\sqrt{c+d}\,\sqrt{c^2-cd+d^2}\,\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}+\frac{4\sqrt[4]{3}\,\sqrt{2-}\cdots}{\cdots}$$

[Out] -(1-x)*arctan((c^2-c*d+d^2)^(1/2)*((-1+x)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2))*(c+d-d*3^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-4*3^(1/4)*(1-x)*EllipticPi((-1+x-3^(1/2))/(1-x-3^(1/2)),(c+d-d*3^(1/2))^2/(c+d+d*3^(1/2))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)/(c+d+d*3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)

Rubi [A]

time = 0.63, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2168, 2138, 551, 585, 95, 211}

$$\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,\Pi\left(\frac{\left(c-\sqrt{3}\,d+d\right)^2}{\left(c+\sqrt{3}\,d+d\right)^2};\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}\,d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(c-\sqrt{3}\,d+d\right)\text{ArcTan}\left(\frac{\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]

[Out] -(((c + d - Sqrt[3]*d)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*ArcTan[(Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)])/(Sqrt[d]*Sqrt[c + d]*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 - c*d + d^2]*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticPi[(c + d - Sqrt[3]*d)^2/(c + d + Sqrt[3]*d)^2, ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/((c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3])

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(c+\left(1+\sqrt{3}\right)d+\left(c+\right.\right.}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-}}$$

$$= \frac{\left(4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1}}\right.}{\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-\right.}}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= -\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\Pi\left(\frac{\left(c+d-\sqrt{3}\,d\right)^2}{\left(c+d+\sqrt{3}\,d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

$$= -\frac{\left(c+d-\sqrt{3}\,d\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{-\frac{}{\left(1-\right.}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+}{\left(1-\right.}}}\right.}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 233, normalized size = 0.68

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}\,c+\left(-3+\sqrt{3}\right)d\right)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}\,d}{-c+\sqrt[3]{-1}\,d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{c-\sqrt[3]{-1}\,d}\right)}{3d\sqrt{-1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c + (-3 + Sqrt[3])*d)*Sqrt[1 + x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3
)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c - (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x
^3])
```

Maple [A]
time = 0.26, size = 277, normalized size = 0.81

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3
/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2*(d*3^(1/2)-c-d)/d^2*(-3/2-1
/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3
/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/
(x^3-1)^(1/2)/(1+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1
/2*I*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{3}}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \frac{x}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\,dx-\int \left(-\frac{1}{c\sqrt{x^3-1}+dx\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/(d*x+c)/(x**3-1)**(1/2),x)
[Out] -Integral(sqrt(3)/(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(x/
(c*sqrt(x**3 - 1) + d*x*sqrt(x**3 - 1)), x) - Integral(-1/(c*sqrt(x**3 - 1)
 + d*x*sqrt(x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.151 $\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=362

$$-\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{7+4\sqrt{3}+\frac{\left(1+\sqrt{3}+x\right)^2}{\left(1-\sqrt{3}+x\right)^2}}}\right.}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] -(1+x)*arctanh(2*(c^2+c*d+d^2)^(1/2)*((-1-x)/(1+x-3^(1/2))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))/(c-d)^(1/2)/d^(1/2)/(7+4*3^(1/2)+(1+x+3^(1/2))^2/(1+x-3^
(1/2))^2)^(1/2))*(c-d*(1-3^(1/2)))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)/(c-d)^
(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(
1/2)+4*3^(1/4)*(1+x)*EllipticPi((-1-x-3^(1/2))/(1+x-3^(1/2)),(c-d*(1-3^(1/2
)))^2/(c-d*(1+3^(1/2)))^2,2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x-3^(1/2))^2)^(1/2)/(-d*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1
/2))^2)^(1/2)
```

Rubi [A]

time = 0.64, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2168, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}-\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(c-\left(1-\sqrt{3}\right)d\right)\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{\left(x+\sqrt{3}+1\right)^2}{\left(x-\sqrt{3}+1\right)^2}+4\sqrt{3}+7}\sqrt{c-d}}\right)}{\sqrt{d}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] -(((c - (1 - Sqrt[3])*d)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*Ar
cTanh[(2*Sqrt[2 + Sqrt[3]]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1 + x)/(1 - Sqrt[3
] + x)^2)])/(Sqrt[c - d]*Sqrt[d]*Sqrt[7 + 4*Sqrt[3] + (1 + Sqrt[3] + x)^2/(
1 - Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[-((1
```

```
 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3])) - (4*3^(1/4)*Sqrt[2 - Sqrt[3]]
*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticPi[(c - (1 - Sqrt[
3])*d)^2/(c - (1 + Sqrt[3])*d)^2, ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x
)], -7 + 4*Sqrt[3]])/((c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^
2)]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2168

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(-1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
 + Sqrt[3]]*f*(1 - q*x)*(Sqrt[(1 + q*x + q^2*x^2)/(1 - Sqrt[3] - q*x)^2]/(q
*Sqrt[a + b*x^3]*Sqrt[-(1 - q*x)/(1 - Sqrt[3] - q*x)^2])), Subst[Int[1/(((1
```

```
 + Sqrt[3])*d + c*q + ((1 - Sqrt[3])*d + c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 + 4*S
qrt[3] + x^2]), x], x, (1 + Sqrt[3] - q*x)/(-1 + Sqrt[3] + q*x)], x]] /; Fr
eeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 - 3*Sqr
t[3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx = -\frac{\left(4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int \frac{}{\left(-c+\left(1+\sqrt{3}\right)d+\left(-\right.\right.}\right.}{\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-}}$$

$$= -\frac{\left(4\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\left(-c+d+\sqrt{3}\,d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\right)\text{Subst}\left(\int\right.}{\sqrt{-\frac{1+}{\left(1-\sqrt{3}\right.}}}$$

$$= \frac{4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{\left(c-d-\sqrt{3}\,d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= \frac{4\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\Pi\left(\frac{\left(c-\left(1-\sqrt{3}\right)d\right)^2}{\left(c-\left(1+\sqrt{3}\right)d\right)^2};-\sin^{-1}\left(\frac{1+\sqrt{}}{1-\sqrt{}}\right.\right.}{\left(c-d-\sqrt{3}\,d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{\left(c-\left(1-\sqrt{3}\right)d\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\;\tanh^{-1}\left(\frac{2\sqrt{2+\sqrt{3}}\,\sqrt{c^2+\right.}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{7-}}\right.}{\sqrt{c-d}\,\sqrt{d}\,\sqrt{c^2+cd+d^2}\,\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.41, size = 233, normalized size = 0.64

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\left(\sqrt{3}c-\left(-3+\sqrt{3}\right)d\right)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{3d\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*((-3*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
(-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
((-1)^(1/3)*(1 + (-1)^(1/3))*(Sqrt[3]*c - (-3 + Sqrt[3])*d)*Sqrt[1 - x + x^
2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)
*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 - x^
3])
```

Maple [A]
time = 0.25, size = 266, normalized size = 0.73

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\cdots\right.}}{\cdots}\right.}{3d\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\cdots\right.}}{\cdots}\right.}{3d\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*I*(d*3^(1/2)+c-d)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^
```

```
(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^
(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/
2)+c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x-\sqrt{3}+1}{\sqrt{-(x+1)(x^2-x+1)}\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(d*x+c)/(-x**3-1)**(1/2),x)
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [F(-1)]
time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
[Out] \text{Hanged}
```

3.152 $\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=125

$$-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{}\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$,
Rules used = {1846, 272, 65, 213, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\Big(1+$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\Big(2\Big(1$$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\sin^{-1}}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1}}$$

Mathematica [A]
time = 10.29, size = 149, normalized size = 1.19

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{2\tanh^{-1}\left(\sqrt{1+x^3}\right)}{\sqrt{3}} - \frac{2\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left((-1)^{2/3}+x\right)}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3 - (2*ArcTanh[Sqrt[1 + x^3]])/Sqrt[3] - (2*((-
1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3)
 + x))/(1 + (-1)^(1/3)))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1
+ x^3])
```

Maple [A]
time = 0.41, size = 132, normalized size = 1.06

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{\sqrt{3}\,\left(-2\sqrt{\pi}\,\ln\left(\right.\right.}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*arctanh((x^3+1)^(1/2))*(1+3^(
1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 33, normalized size = 0.26

$$\frac{1}{3}\left(\sqrt{3}+1\right)\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) + 1)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInve
rse(0, -4, x)
```

Sympy [A]
time = 2.51, size = 56, normalized size = 0.45

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\,\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{2\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*sqrt(3)*asinh(x**(-3/2))/3 - 2*asinh(x**(-3/2))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B]
time = 0.14, size = 334, normalized size = 2.67

$$-\frac{2\sqrt{3}\,\operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2
))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2) - (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 - (2*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.153 $\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=139

$$-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1846, 272, 65, 212, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 + Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\Big(1 -$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\Big(2\Big($$

$$= -\frac{2}{3}\left(1+\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\Big(\sin}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\,\sqrt{}}$$

Mathematica [A]
time = 10.34, size = 157, normalized size = 1.13

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1-x^3}\right) - \frac{2\tanh^{-1}\left(\sqrt{1-x^3}\right)}{\sqrt{3}} - \frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 - x^3]])/3 - (2*ArcTanh[Sqrt[1 - x^3]])/Sqrt[3] - (2*Sqr
t[(1 - x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*
x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/
3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 - x^3
])
```

Maple [A]
time = 0.43, size = 125, normalized size = 0.90

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}} - x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)+\frac{\sqrt{3}\left(-2\sqrt{\pi}\ \mathrm{l}\right.}{}$
default	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x+\frac{1}{2}\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*arctanh((-x^3+1)^(1/2))*(1+3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [A]
time = 0.10, size = 29, normalized size = 0.21

$$\frac{1}{3}\left(\sqrt{3}+1\right)\log\left(-\frac{x^3+2\sqrt{-x^3+1}-2}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

[Out] 1/3*(sqrt(3) + 1)*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3)

Sympy [A]
time = 4.23, size = 99, normalized size = 0.71

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^3e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}-\frac{2\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}-\frac{2\,\text{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|}>1\\ \frac{2i\,\text{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3**(1/2))/x/(-x**3+1)**(1/2),x)

[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/3)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/3, True)) + sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/3, True))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)

Mupad [B]
time = 3.64, size = 373, normalized size = 2.68

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)^3\left(\sqrt{1-x^3}+1\right)}{x^6}\right)}{3}+\frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\right)}{\sqrt{1-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3^(1/2) - x + 1)/(x*(1 - x^3)^(1/2)),x)

[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)^3*((1 - x^3)^(1/2) + 1))/x^6))/3 + ((x^3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(

```
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.154 $\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=142

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1846, 272, 65, 209, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\text{ArcTan}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\Big(1$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\Big(2\Big($$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\Big(\sin^{-1}}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-}}$$

Mathematica [A]
time = 10.38, size = 150, normalized size = 1.06

$$\frac{2}{3}\left(\tan^{-1}\left(\sqrt{-1+x^3}\right)+\sqrt{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 + x^3]] + Sqrt[3]*ArcTan[Sqrt[-1 + x^3]] - (3*Sqrt[(1 -
x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
(-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3])))/3
```

Maple [A]
time = 0.69, size = 132, normalized size = 0.93

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\,\mathrm{arct}}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2\,\mathrm{arct}}{}$
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+\left(-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^3-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^3-1\right)}\,x}{\sqrt{\mathrm{signum}\,(}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*arctan((x^3-1)^(1/2))*(1+3^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 52, normalized size = 0.37

$$\frac{1}{3}\sqrt{2\sqrt{3}+4}\,\arctan\left(-\frac{\left(x^3-\sqrt{3}\left(x^3-2\right)-2\right)\sqrt{2\sqrt{3}+4}}{4\sqrt{x^3-1}}\right)-2\,\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 - 2) - 2)*sqrt(2*sq
rt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

Sympy [A]
time = 4.22, size = 94, normalized size = 0.66

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^{3}|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}+\sqrt{3}\left(\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^{3}|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) + Piecewise((
2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, True)) + s
qrt(3)*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x - sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B]
time = 2.72, size = 334, normalized size = 2.35

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\sqrt{x^3-1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3^(1/2) - x + 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(
1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*elli
pticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2
)/((3^(1/2)*1i)/2 - 3/2)))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) -
 x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((
```

```
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.155 $\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=136

$$\frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1+3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1846, 272, 65, 210, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1+\sqrt{3}\right)\text{ArcTan}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 + Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1+\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \left(1+\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} + \frac{1}{3}\Big(1$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}\Big(2\Big($$

$$= \frac{2}{3}\left(1+\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\Big(\sin^{-}}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-}}$$

Mathematica [A]
time = 10.38, size = 155, normalized size = 1.14

$$\frac{2}{3}\left(\tan^{-1}\left(\sqrt{-1-x^3}\right)+\sqrt{3}\tan^{-1}\left(\sqrt{-1-x^3}\right)-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 - x^3]] + Sqrt[3]*ArcTan[Sqrt[-1 - x^3]] - (3*((-1)^(1/3
) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
(-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 - x^3])))/3
```

Maple [A]
time = 0.70, size = 125, normalized size = 0.92

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}} - ix\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right) - \frac{i\sqrt{3}\,\left(-2\sqrt{\right.}}{}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2/3*arctan((-x^3-1)^(1/2))*(1+3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [A]
time = 0.13, size = 54, normalized size = 0.40

$$\frac{1}{3}\sqrt{2\sqrt{3}+4}\,\arctan\left(-\frac{\left(x^3-\sqrt{3}\,(x^3+2)+2\right)\sqrt{-x^3-1}\,\sqrt{2\sqrt{3}+4}}{4\,(x^3+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2*sqrt(3) + 4)*arctan(-1/4*(x^3 - sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^3
 - 1)*sqrt(2*sqrt(3) + 4)/(x^3 + 1))
```

Sympy [A]
time = 2.66, size = 61, normalized size = 0.45

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\,i\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) + 2*I*asinh(x**(-3/2))/3 + 2*sqrt(3)*I*asinh(x**(-3/2))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B]
time = 4.45, size = 376, normalized size = 2.76

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)\left(\sqrt{-x^3-1}+1\mathrm{i}\right)^3}{x^6}\right)1\mathrm{i}}{3}+\frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\right)}{\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)*((- x^3 - 1)^(1/2) + 1i)^3)/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
```

```
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.156 $\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=127

$$-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\ldots\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*arctanh((x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(
1+x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/
2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1846, 272, 65, 213, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 + x)*
Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/
(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{1+x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\Big(1 -$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}\Big(2\Big($$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\Big(\sin}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{}}$$

Mathematica [A]
time = 10.19, size = 149, normalized size = 1.17

$$-\frac{2}{3}\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\tanh^{-1}\left(\sqrt{1+x^3}\right)}{\sqrt{3}} - \frac{2\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left((-1)^{2/3}+x\right)}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[1 + x^3]])/3 + (2*ArcTanh[Sqrt[1 + x^3]])/Sqrt[3] - (2*((-
1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3)
 + x))/(1 + (-1)^(1/3)))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1
)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1
+ x^3])
```

Maple [A]
time = 0.26, size = 132, normalized size = 1.04

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}}{3\sqrt{\pi}}+x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)-\frac{\sqrt{3}\,\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}\right.\right.}{}$
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}+\frac{2\left(\sqrt{3}-1\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2\,\mathrm{arctanh}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((x^3+1)^(
1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 33, normalized size = 0.26

$$\frac{1}{3}\left(\sqrt{3}-1\right)\log\left(\frac{x^3+2\sqrt{x^3+1}+2}{x^3}\right)+2\,\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sqrt(3) - 1)*log((x^3 + 2*sqrt(x^3 + 1) + 2)/x^3) + 2*weierstrassPInve
rse(0, -4, x)
```

Sympy [A]
time = 2.53, size = 56, normalized size = 0.44

$$\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2\sqrt{3}\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(x**3+1)**(1/2),x)
```

```
[Out] x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/3))
 - 2*asinh(x**(-3/2))/3 + 2*sqrt(3)*asinh(x**(-3/2))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B]
time = 2.65, size = 334, normalized size = 2.63

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\sqrt{x^3+1}\right)}{3}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] (2*3^(1/2)*atanh((x^3 + 1)^(1/2)))/3 + (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(
1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellip
ticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^
(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1
/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
)/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)
```

3.157 $\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=141

$$-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*arctanh((-x^3+1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1
/2))^2)^(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1846, 272, 65, 212, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*(1 - Sqrt[3])*ArcTanh[Sqrt[1 - x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*(1 - x)*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/
(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x
)^2]*Sqrt[1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{1-x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{1-x^3}}\,dx - \int \frac{1}{\sqrt{1-x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} + \frac{1}{3}\Big(1-$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}\Big(2\Big(1$$

$$= -\frac{2}{3}\left(1-\sqrt{3}\right)\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\Big(\sin^{-1}}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1}}$$

Mathematica [A]
time = 10.37, size = 158, normalized size = 1.12

$$\frac{2}{3}\left(-\tanh^{-1}\left(\sqrt{1-x^3}\right)+\sqrt{3}\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (2*(-ArcTanh[Sqrt[1 - x^3]] + Sqrt[3]*ArcTanh[Sqrt[1 - x^3]] - (3*Sqrt[(1 -
 x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
 (-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 - x^3])))/3
```

Maple [A]
time = 0.28, size = 125, normalized size = 0.89

method	result
meijerg	$\frac{-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}}{3\sqrt{\pi}} - x\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right) - \frac{\sqrt{3}\,\left(-2\sqrt{\pi}\right.}{}$
default	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$
elliptic	$\frac{2i\sqrt{3}\,\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x+\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*(3^(1/2)-1)*arctanh((-x^3+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [A]
time = 0.11, size = 29, normalized size = 0.21

$$\frac{1}{3}\left(\sqrt{3}-1\right)\log\left(-\frac{x^3-2\sqrt{-x^3+1}-2}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

[Out] 1/3*(sqrt(3) - 1)*log(-(x^3 - 2*sqrt(-x^3 + 1) - 2)/x^3)

Sympy [A]
time = 4.23, size = 99, normalized size = 0.70

$$-\frac{x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\sqrt{3}\left(\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^{3}|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^{3}|}>1\\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(-x**3+1)**(1/2),x)
```

```
[Out] -x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi))/(3*gamma(4/
3)) - sqrt(3)*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin
(x**(-3/2))/3, True)) + Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1),
 (2*I*asin(x**(-3/2))/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B]
time = 3.24, size = 373, normalized size = 2.65

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{1-x^3}-1\right)\left(\sqrt{1-x^3}+1\right)^3}{x^6}\right)}{3}+\frac{\sqrt{x^3-1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}\right)}{\sqrt{1-x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((1 - x^3)^(1/2) - 1)*((1 - x^3)^(1/2) + 1)^3)/x^6))/3 + ((x^
3 - 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1
/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(
```

```
x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/
2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)))/(1 - x^3)^(1/2)
```

3.158 $\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=144

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\,\sqrt{-1+x^3}}$$

```
[Out] 2/3*arctan((x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-
x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2)
)^2)^(1/2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1846, 272, 65, 209, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\,\sqrt{x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\text{ArcTan}\left(\sqrt{x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 + x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 - x)*S
qrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(
1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] -
x)^2)]*Sqrt[-1 + x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}-x}{x\sqrt{-1+x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1+x^3}}\,dx - \int \frac{1}{\sqrt{-1+x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(1-\right.$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}\left(2\left(1\right.\right.$$

$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-}}$$

Mathematica [A]
time = 10.31, size = 151, normalized size = 1.05

$$\frac{2}{3}\left(\tan^{-1}\left(\sqrt{-1+x^3}\right)-\sqrt{3}\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{3\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1+x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] - x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 + x^3]] - Sqrt[3]*ArcTan[Sqrt[-1 + x^3]] - (3*Sqrt[(1 -
x)/(1 + (-1)^(1/3))]*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
(-1)^(1/3)])/(Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3])))/3
```

Maple [A]
time = 0.55, size = 132, normalized size = 0.92

method	result
default	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}-\frac{2(}{}$
elliptic	$-\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^{3}-1}}+\frac{2\,\mathrm{a}}{}$
meijerg	$\frac{\sqrt{-\mathrm{signum}\left(x^{3}-1\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^{3}+1}}{2}\right)+\left(-2\ln(2)+3\ln(x)+i\pi\right)\sqrt{\pi}\right)}{3\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(x^{3}-1\right)}}-\frac{\sqrt{-\mathrm{signum}\left(x^{3}-1\right)}}{\sqrt{\mathrm{signum}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*arctan((x^3-1)^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 51, normalized size = 0.35

$$-\frac{1}{3}\sqrt{-2\sqrt{3}+4}\,\arctan\left(\frac{\left(x^{3}+\sqrt{3}\left(x^{3}-2\right)-2\right)\sqrt{-2\sqrt{3}+4}}{4\sqrt{x^{3}-1}}\right)-2\,\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 - 2) - 2)*sqrt(-2*
sqrt(3) + 4)/sqrt(x^3 - 1)) - 2*weierstrassPInverse(0, 4, x)
```

Sympy [A]
time = 4.23, size = 94, normalized size = 0.65

$$\frac{ix\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^3\right)}{3\Gamma\left(\frac{4}{3}\right)} - \sqrt{3}\left(\begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|} > 1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right) + \begin{cases}\frac{2i\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|} > 1\\ -\frac{2\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3**(1/2))/x/(x**3-1)**(1/2),x)
```

```
[Out] I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3)) - sqrt(3)*Pie
cewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/3, Tr
ue)) + Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3
/2))/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x + sqrt(3) - 1)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B]
time = 2.70, size = 334, normalized size = 2.32

$$-\frac{2\sqrt{3}\,\operatorname{atan}\left(\sqrt{x^3-1}\right)}{3} + \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle| -\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\operatorname{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle| -\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 3^(1/2) - 1)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) - (2*3^(1/2)*atan((x^3 - 1)^(1/2)))/3 - (2*((
```

```
3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)
```

3.159 $\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=138

$$\frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*arctan((-x^3-1)^(1/2))*(1-3^(1/2))+2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1
+x-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2
))^2)^(1/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1846, 272, 65, 210, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}\left(1-\sqrt{3}\right)\text{ArcTan}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(1 - Sqrt[3])*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*(1 + x)*S
qrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(
1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] +
x)^2)]*Sqrt[-1 - x^3])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{1-\sqrt{3}+x}{x\sqrt{-1-x^3}}\,dx = \left(1-\sqrt{3}\right)\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} + \frac{1}{3}\Big(1-$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} - \frac{1}{3}\Big(2\Big(1$$

$$= \frac{2}{3}\left(1-\sqrt{3}\right)\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\Big(\sin^{-1}}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-}}$$

Mathematica [A]
time = 10.30, size = 156, normalized size = 1.13

$$\frac{2}{3}\left(\tan^{-1}\left(\sqrt{-1-x^3}\right)-\sqrt{3}\tan^{-1}\left(\sqrt{-1-x^3}\right)-\frac{3\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[3] + x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*(ArcTan[Sqrt[-1 - x^3]] - Sqrt[3]*ArcTan[Sqrt[-1 - x^3]] - (3*((-1)^(1/3
) - x)*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[((-1)^(1/3) - (-1)^(2/3)*x)/(1 +
 (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]],
(-1)^(1/3)])/(Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 - x^3])))/3
```

Maple [A]
time = 0.52, size = 125, normalized size = 0.91

method	result
meijerg	$-\frac{i\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}-ix\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right)+\frac{i\sqrt{3}\,\left(-2\right.}{}$
default	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))-2/3*(3^(1/2)-1)*arctan((-x^3-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [A]
time = 0.13, size = 53, normalized size = 0.38

$$-\frac{1}{3}\sqrt{-2\sqrt{3}+4}\,\arctan\left(\frac{\left(x^3+\sqrt{3}\,(x^3+2)+2\right)\sqrt{-x^3-1}\,\sqrt{-2\sqrt{3}+4}}{4\,(x^3+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-2*sqrt(3) + 4)*arctan(1/4*(x^3 + sqrt(3)*(x^3 + 2) + 2)*sqrt(-x^
3 - 1)*sqrt(-2*sqrt(3) + 4)/(x^3 + 1))
```

Sympy [**A**]
time = 2.58, size = 61, normalized size = 0.44

$$-\frac{ix\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle| x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2\sqrt{3}\,i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{2i\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/x/(-x**3-1)**(1/2),x)
```

```
[Out] -I*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(I*pi))/(3*gamma(4/
3)) - 2*sqrt(3)*I*asinh(x**(-3/2))/3 + 2*I*asinh(x**(-3/2))/3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [**B**]
time = 4.10, size = 376, normalized size = 2.72

$$\frac{\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^3-1}-\mathrm{i}\right)^3\left(\sqrt{-x^3-1}+1\mathrm{i}\right)}{x^6}\right)1\mathrm{i}}{3}+\frac{\sqrt{x^3+1}\left(\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}\right)}{\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] (3^(1/2)*log((((- x^3 - 1)^(1/2) - 1i)^3*((- x^3 - 1)^(1/2) + 1i))/x^6)*1i)
/3 + ((x^3 + 1)^(1/2)*((2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3
^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x +
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2
- 3/2)))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/
2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(
```

```
1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((3^
(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*1i)/2 + 1/2))^(1/2)))/(- x^3 - 1)^(1/2)
```

3.160 $\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=332

$$-\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(1+x)\sqrt{\frac{1-\cdots}{\left(1+\sqrt{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{\cdots}{\left(1+\cdots\right.}}}$$

[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)-2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*(7*2^(1/2)+4*6^(1/2))*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.44, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 210}

$$-\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}+\frac{12\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\Pi\left(97-56\sqrt{3};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{ArcTan}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 + x)*Sqrt[1 + x^3]),x]

[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(Sqrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) - (2*Sqrt[2*(97 + 56*Sqrt[3])]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{2\sqrt{2\left(97+56\sqrt{3}\right)}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\middle|-7-4\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} - \frac{2\sqrt{2\left(97\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 194, normalized size = 0.58

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[1 + x^3]
```

Maple [A]
time = 0.26, size = 240, normalized size = 0.72

method	result
default	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$
elliptic	$\frac{2\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{3\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2
-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*(
(x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(
((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),-3/4+1/4*I*3^(1/2),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x+1)\,(x^2-x+1)}\,\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt((x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 + 1)*(x + 3)), x)
```

Mupad [B]
time = 0.23, size = 207, normalized size = 0.62

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{2\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 + 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(
1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi(- (3^
(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)
*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2
))^(1/2))
```

3.161 $\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=377

$$\frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{1+x \cdots}{\left(1+\sqrt{3} \cdots\right.}}}{13\sqrt[4]{3}\sqrt{\frac{\cdots}{\left(1+ \cdots\right.}}}$$

```
[Out] 3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1-
x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(-x^3+1)^(1/
2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/
2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2
))^2)^(1/2)-2/39*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(1/2)),I*3^(1/2)+2*I)
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*(5*2^(1/2)+2*6^(1/2))*3^(3/4)/(-x^3+1)^(
1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 212}

$$-\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}+\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1
 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(
2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) - (2*Sqrt[2*(37
+ 20*Sqrt[3])]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[Ar
cSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*3^(1/4)*Sqr
t[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]) + (12*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sq
rt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{1-x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{1-x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{\right.}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{\right.}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{\right.}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= -\frac{2\sqrt{2\left(37+20\sqrt{3}\right)}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\middle|-7-4\sqrt{\right.}{13\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt[2]{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{2\sqrt{2\left(37+2\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 195, normalized size = 0.52

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-3+\sqrt[3]{-1}}\right)}{\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[1 - x^3]
```

Maple [A]
time = 0.26, size = 240, normalized size = 0.64

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3+1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2
+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
```

```
*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(-x^3+1)^(1/2)/(5/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*
I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(5/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/
2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 + 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3+1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 + 1)*(x + 3)), x)
```

Mupad [B]
time = 2.74, size = 224, normalized size = 0.59

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{4\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))
- 3*ellipticPi((3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))
^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.162 $\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=373

$$\frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{2\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{2\sqrt{2}(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}F\left(\sin^{-}\right.}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-}{\left(1-\sqrt{3}\right.}}}$$

[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*2^(1/2)*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(4+3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)+3/14*(1-x)*arctanh(1/2*7^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)*7^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-12/13*3^(1/4)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),553/169+304/169*3^(1/2),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.41, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 212}

$$-\frac{2\sqrt{2}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{12\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\Pi\left(\frac{1}{169}\left(553+304\sqrt{3}\right);\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{13\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}+\frac{3(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}}{2\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Int[x/((3 + x)*Sqrt[-1 + x^3]),x]

[Out] (3*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(2*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])]])/(2*Sqrt[7]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3]) - (2*Sqrt[2]*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*(4 + Sqrt[3])*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 + x^3]) + (12*3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticPi[(553 + 304*Sqrt[3])/169, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(13*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1+x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}-x}{(3+x)\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{4+\sqrt{3}}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{\Big(12+ \cdots}{\cdots}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{\Big(12+ \cdots}{\cdots}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3} \cdots}{\cdots}$$

$$= -\frac{2\sqrt{2}\,(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\left(4+\sqrt{3}\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{12\sqrt[4]{3} \cdots}{\cdots}$$

$$= \frac{3(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,\tanh^{-1}\left(\frac{\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt[2]{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{2\sqrt{7}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} - \frac{2\sqrt{2}\,(1- \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 193, normalized size = 0.52

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1+x+x^2}\,\Pi\left(\frac{2\sqrt{3}}{5i+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{-3+\sqrt[3]{-1}}\right)}{\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 +
(-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] + ((3
*I)*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(5*I + Sqrt[3]), ArcSin[Sqrt[(
1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(-3 + (-1)^(1/3))))/Sqrt
[-1 + x^3]
```

Maple [A]
time = 0.25, size = 240, normalized size = 0.64

method	result
default	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\right.}{}$
elliptic	$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}-\frac{3\left(-\frac{3}{2}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^
(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+
1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3/2*(-3/2-1/2*I*3^(1/2))*((-1+x)
/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*Ellipti
cPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),3/8+1/8*I*3^(1/2),((3/2+1/2*I*3^(1/
2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 - 1)*x/(x^4 + 3*x^3 - x - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(x-1)(x^2+x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt((x - 1)*(x**2 + x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(x^3 - 1)*(x + 3)), x)
```

Mupad [B]
time = 2.65, size = 208, normalized size = 0.56

$$-\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(4\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(\frac{3}{8}+\frac{\sqrt{3}\,1\mathrm{i}}{8};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{4\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(4*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*ellipticPi((
3^(1/2)*1i)/8 + 3/8, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1
/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(4*(((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) +
 x^3)^(1/2))
```

3.163 $\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=341

$$-\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}-\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+\cdots}{\left(1-\sqrt{3}\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{-\frac{1}{\left(1-\cdots\right.}}\cdots}$$

```
[Out] -3/26*(1+x)*arctan(1/2*26^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/((x^2-x+1)/(1
+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*26^(1/2)/(-x^3-1)^(
1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-12*3^(1/4)*(1+x)*EllipticPi((-1-x+3^(1/2
))/(1+x+3^(1/2)),97-56*3^(1/2),I*3^(1/2)+2*I)*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)/(-x^3-1)^(1/2)/(1/2*6^(1/2)-1/2*2^(1/2))/((1+x)/(1+x+3^(1/2))^2)^(1/2)
-2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*((x^2-x+1)/
(1+x-3^(1/2))^2)^(1/2)*(2*2^(1/2)+6^(1/2))*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(
1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 210}

$$-\frac{2\sqrt{14+8\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{12\sqrt[4]{3}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\Pi\left(97-56\sqrt{3};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{2-\sqrt{3}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}-\frac{3(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}}{\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (-3*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[13/2]*Sqrt
[(1 + x)/(1 + Sqrt[3] + x)^2])/Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]])/(S
qrt[26]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) - (2*Sqrt[14 + 8*
Sqrt[3]]*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(
1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 +
x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3]) + (12*3^(1/4)*(1 + x)*Sqrt[(1 - x
+ x^2)/(1 + Sqrt[3] + x)^2]*EllipticPi[97 - 56*Sqrt[3], ArcSin[(1 - Sqrt[3]
 + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(Sqrt[2 - Sqrt[3]]*Sqrt[(1 + x)/
(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{x}{(3+x)\sqrt{-1-x^3}}\,dx = -\frac{3\int \frac{1+\sqrt{3}+x}{(3+x)\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}} + \frac{\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{-2+\sqrt{3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{2\sqrt{14+8\sqrt{3}}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}}$$

$$= -\frac{3(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{\frac{13}{2}}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{26}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{-1-x^3}} - \frac{2\sqrt{14+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 196, normalized size = 0.57

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{3i\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}}{3+\sqrt[3]{-1}};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{3+\sqrt[3]{-1}}\right)}{\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 + x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) - (
-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1
+ (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]) +
((3*I)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3])/(3 + (-1)^(1/3)), ArcSin[Sq
rt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(3 + (-1)^(1/3))))/S
qrt[-1 - x^3]
```

Maple [A]
time = 0.25, size = 240, normalized size = 0.70

method	result
default	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3+x)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^
(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/
2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((
```

```
1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-
x^3-1)^(1/2)/(7/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(7/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^3 - 1)*x/(x^4 + 3*x^3 + x + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x+1)(x^2-x+1)}\,(x+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x**3-1)**(1/2),x)
[Out] Integral(x/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3+x)/(-x^3-1)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^3 - 1)*(x + 3)), x)
```

Mupad [B]
time = 2.61, size = 223, normalized size = 0.65

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-3\,\Pi\left(-\frac{3}{4}-\frac{\sqrt{3}\,1\mathrm{i}}{4};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|\,-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{2\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- x^3 - 1)^(1/2)*(x + 3)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(2*ellipticF(asin(((x + 1)/((3^(1/2)
*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - 3*
ellipticPi(- (3^(1/2)*1i)/4 - 3/4, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(2*(- x^3 - 1)^(1/2
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.164 $\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=450

$$\frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(e-f-\right.}{\sqrt[4]{}}$$

[Out] (-c*f+d*e)*(1+x)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)+2/3*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(e-f-f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(-d*3^(1/2)+c-d)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)

Rubi [A]
time = 0.93, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 211}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c^2-2cd-2d^2\right)}+\frac{2\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\left(c-\sqrt{3}d-d\right)}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{ArcTan}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]

[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2 + Sqrt[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*(c - d - Sqrt[3]*d)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) -

```
(4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sq
rt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)^2, A
rcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2*c*d
- 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1+x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{1+x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{2\sqrt{2+\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right.}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.38, size = 211, normalized size = 0.47

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{1+x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[1 + x^3])
```

Maple [A]
time = 0.26, size = 274, normalized size = 0.61

method	result
default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}+\frac{2(-cf+de}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3+1}}-\frac{2(cf-de)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3
^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1
/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/
2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(3/2-1/2*I*3
^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*
I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1
)^(1/2)/(-1+c/d)*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3
^(1/2))/(-1+c/d),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^3 + 1)*(f*x + e)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{(x + 1)\left(x^2 - x + 1\right)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt((x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*(d*x + c)), x)
```

Mupad [B]
time = 0.13, size = 356, normalized size = 0.79

$$\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}-\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}-1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 + 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(x^3 - x*
(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)
*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*
1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i
)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(
c/d - 1)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.165 $\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=474

$$-\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}-\frac{2\sqrt{2+\sqrt{3}}\,\left(e+\right.}{}$$

```
[Out] -(-c*f+d*e)*(1-x)*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)
/d^(1/2)/(c+d)^(1/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(
1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(-x^3+1)^(1/2)/((1-x
)/(1-x+3^(1/2))^2)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2
))/(1-x+3^(1/2)),(c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^
(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(-x^
3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)-2/3*(1-x)*EllipticF((1-x-3^(1/2))/
(1-x+3^(1/2)),I*3^(1/2)+2*I)*(e+f+f*3^(1/2))*(1/2*6^(1/2)+1/2*2^(1/2))*((x^
2+x+1)/(1-x+3^(1/2))^2)^(1/2)*3^(3/4)/(c+d+d*3^(1/2))/(-x^3+1)^(1/2)/((1-x)
/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 1.03, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2169, 224, 2167, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\,\left(c^2+2cd-2d^2\right)}-\frac{2\sqrt{2+\sqrt{3}}\,(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\left(c+\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])) - (2*Sqrt[2 +
Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2
]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/(
3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3]
```

```
) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1
+ Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)^2, A
rcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2*c*d
- 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{1-x^3}}\,dx=\frac{\left(e+f+\sqrt{3}f\right)\int\frac{1}{\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d}+\frac{(de-cf)\int\frac{1+\sqrt{3}-x}{(c+dx)\sqrt{1-x^3}}\,dx}{c+d+\sqrt{3}d}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\right.}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\right.}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\right.}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{2\sqrt{2+\sqrt{3}}\left(e+f+\sqrt{3}f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)\right.}{\sqrt[4]{3}\left(c+d+\sqrt{3}d\right)\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

$$=-\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right.}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)}}}\right.}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.47, size = 233, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{-c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{-c+\sqrt[3]{-1}d}\right)}{3d\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[1 - x^3])
```

Maple [A]
time = 0.26, size = 265, normalized size = 0.56

method	result
default	$-\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3d\sqrt{-x^3+1}}$
elliptic	$-\frac{2if\sqrt{3}\sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x\right.}}{}\right.}{3d\sqrt{-x^3+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/
2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(
-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x+1/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*
```

```
3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3
^(1/2)+c/d),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-\left(x - 1\right)\left(x^{2} + x + 1\right)}\,\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3+1)**(1/2),x)
[Out] Integral((e + f*x)/(sqrt(-(x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*(d*x + c)), x)
```

Mupad [B]
time = 0.09, size = 387, normalized size = 0.82

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(c\,f-d\,e)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\sqrt{1-x^3}\,\left(\frac{c}{d}+1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((1 - x^3)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(c*f - d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3
^(1/2)*1i)/2 + 3/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(1 - x^3)^(1/2)*(c
/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 +
3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(
1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2)
)^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d*(1 - x^3)^(1/
2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.166 $\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=475

$$-\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}}\right)}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}-\frac{2\sqrt{2-\sqrt{3}}\left(e+f\right.}{}$$

```
[Out] -2/3*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2)),2*I-I*3^(1/2))*(e+f+f*3^(1
/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/2)*3^(3/4)/(c
+d+d*3^(1/2))/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)-(-c*f+d*e)*(1-x)
*arctanh((c^2-c*d+d^2)^(1/2)*((1-x)/(1-x+3^(1/2))^2)^(1/2)/d^(1/2)/(c+d)^(1
/2)/((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/d^
(1/2)/(c+d)^(1/2)/(c^2-c*d+d^2)^(1/2)/(x^3-1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)
^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1-x)*EllipticPi((-1+x+3^(1/2))/(1-x+3^(1/2)),(
c+d+d*3^(1/2))^2/(c+d-d*3^(1/2))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))
*((x^2+x+1)/(1-x+3^(1/2))^2)^(1/2)/(c^2+2*c*d-2*d^2)/(x^3-1)^(1/2)/((1-x)/(
1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.88, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 214}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c+\sqrt{3}d+d\right)^2}{\left(c-\sqrt{3}d+d\right)^2};\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c^2+2cd-2d^2\right)}-\frac{2\sqrt{2-\sqrt{3}}(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\left(e+\sqrt{3}f+f\right)F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\left(c+\sqrt{3}d+d\right)}-\frac{(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}(de-cf)\tanh^{-1}\left(\frac{\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c^2-cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{c+d}}\right)}{\sqrt{d}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{x^3-1}\sqrt{c+d}\sqrt{c^2-cd+d^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] -(((d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2]*ArcTanh[(Sqr
t[c^2 - c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2])/(Sqrt[d]*Sqrt[c + d]*
Sqrt[(1 + x + x^2)/(1 + Sqrt[3] - x)^2])])/(Sqrt[d]*Sqrt[c + d]*Sqrt[c^2 -
c*d + d^2]*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])) - (2*Sqrt[2 -
 Sqrt[3]]*(e + f + Sqrt[3]*f)*(1 - x)*Sqrt[(1 + x + x^2)/(1 - Sqrt[3] - x)^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3] - x)], -7 + 4*Sqrt[3]])/
(3^(1/4)*(c + d + Sqrt[3]*d)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqrt[-1 +
```

```
 x^3]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 - x)*Sqrt[(1 + x + x^2
)/(1 + Sqrt[3] - x)^2]*EllipticPi[(c + d + Sqrt[3]*d)^2/(c + d - Sqrt[3]*d)
^2, ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[3] - x)], -7 - 4*Sqrt[3]])/((c^2 + 2
*c*d - 2*d^2)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[-1 + x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1+x^3}}\,dx = \frac{\left(e+f+\sqrt{3}\,f\right)\int \frac{1}{\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}\,d} + \frac{(de-cf)\int \frac{1+\sqrt{3}-x}{(c+dx)\sqrt{-1+x^3}}\,dx}{c+d+\sqrt{3}\,d}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}\,f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.\cdots}{\sqrt[4]{3}\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}\,f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.\cdots}{\sqrt[4]{3}\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}\,f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.\cdots}{\sqrt[4]{3}\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}\cdots}$$

$$= -\frac{2\sqrt{2-\sqrt{3}}\left(e+f+\sqrt{3}\,f\right)(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-\cdots}{1-\sqrt{3}-\cdots}\right.\right.\cdots}{\sqrt[4]{3}\left(c+d+\sqrt{3}\,d\right)\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}\cdots}$$

$$= -\frac{(de-cf)(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\tanh^{-1}\left(\frac{\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-\cdots\right.}}\cdots}{\sqrt{d}\sqrt{c+d}\sqrt{\frac{1+x+x\cdots}{\left(1+\sqrt{3}-\cdots\right.}}\cdots}\right.\cdots}{\sqrt{d}\sqrt{c+d}\sqrt{c^2-cd+d^2}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 231, normalized size = 0.49

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\frac{3f\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{\sqrt[3]{-1}\sqrt{3}\left(1+\sqrt[3]{-1}\right)(-de+cf)\sqrt{1+x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{-c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{-c+\sqrt[3]{-1}d}\right)}{3d\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((3*f*((-1)^(1/3) + x)*Sqrt[((-1)^(1/3) +
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))] +
 ((-1)^(1/3)*Sqrt[3]*(1 + (-1)^(1/3))*(-(d*e) + c*f)*Sqrt[1 + x + x^2]*Elli
pticPi[(I*Sqrt[3]*d)/(-c + (-1)^(1/3)*d), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1
 + (-1)^(1/3))]], (-1)^(1/3)])/(-c + (-1)^(1/3)*d)))/(3*d*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.28, size = 274, normalized size = 0.58

method	result
default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}+\frac{2(-cf}{}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{d\sqrt{x^3-1}}-\frac{2(cf-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f/d*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*
I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(
1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-c*f+d*e)/d^2*(-3/2-1/2*I
*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1
/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3
-1)^(1/2)/(1+c/d)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I
*3^(1/2))/(1+c/d),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{(x - 1)\,(x^2 + x + 1)}\,\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt((x - 1)*(x**2 + x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*(d*x + c)), x)
```

Mupad [B]
time = 2.67, size = 355, normalized size = 0.75

$$-\frac{2\,f\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\,\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(c\,f-d\,e\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\,\left(\frac{c}{d}+1\right)\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(c*f -
d*e)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3
/2)/(c/d + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i
)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(c/d + 1)*(((3^(1/2)*1i)/2 - 1/2)*
((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) +
 1) + x^3)^(1/2)) - (2*f*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2
)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2
 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/
2 - 3/2)))/(d*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)
*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.167 $\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=463

$$\frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right)}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}+\frac{2\sqrt{2-\sqrt{3}}\left(e-f-\right.}{\sqrt[4]{}}$$

```
[Out] 2/3*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/2)),2*I-I*3^(1/2))*(e-f-f*3^(1/
2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1/2)*3^(3/4)/(-d
*3^(1/2)+c-d)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)+(-c*f+d*e)*(1+x
)*arctan((c^2+c*d+d^2)^(1/2)*((1+x)/(1+x+3^(1/2))^2)^(1/2)/(c-d)^(1/2)/d^(1
/2)/((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c
-d)^(1/2)/d^(1/2)/(c^2+c*d+d^2)^(1/2)/(-x^3-1)^(1/2)/((1+x)/(1+x+3^(1/2))^2
)^(1/2)+4*3^(1/4)*(-c*f+d*e)*(1+x)*EllipticPi((-1-x+3^(1/2))/(1+x+3^(1/2)),
(c-d*(1+3^(1/2)))^2/(c-d*(1-3^(1/2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/(c^2-2*c*d-2*d^2)/(-x^3-1)^(1/2)/((
1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.92, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2169, 225, 2167, 2138, 551, 585, 95, 211}

$$-\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}(de-cf)\Pi\left(\frac{\left(c-\left(1+\sqrt{3}\right)d\right)^2}{\left(c-\left(1-\sqrt{3}\right)d\right)^2};\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c^2-2cd-2d^2\right)}+\frac{2\sqrt{2-\sqrt{3}}(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\left(e-\sqrt{3}f-f\right)F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\left(c-\sqrt{3}d-d\right)}+\frac{(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\text{ArcTan}\left(\frac{\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c^2+cd+d^2}}{\sqrt{d}\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{c-d}}\right)(de-cf)}{\sqrt{d}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}\sqrt{c-d}\sqrt{c^2+cd+d^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] ((d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*ArcTan[(Sqrt[c
^2 + c*d + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2])/(Sqrt[c - d]*Sqrt[d]*Sqr
t[(1 - x + x^2)/(1 + Sqrt[3] + x)^2])])/(Sqrt[c - d]*Sqrt[d]*Sqrt[c^2 + c*d
 + d^2]*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3]) + (2*Sqrt[2 - Sqr
t[3]]*(e - f - Sqrt[3]*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1 - Sqrt[3] + x)^2]*E
llipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3] + x)], -7 + 4*Sqrt[3]])/(3^(
1/4)*(c - d - Sqrt[3]*d)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqrt[-1 - x^3
```

```
]) - (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*(d*e - c*f)*(1 + x)*Sqrt[(1 - x + x^2)/(1
 + Sqrt[3] + x)^2]*EllipticPi[(c - (1 + Sqrt[3])*d)^2/(c - (1 - Sqrt[3])*d)
^2, ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/((c^2 - 2
*c*d - 2*d^2)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[-1 - x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
```

```
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2169

```
Int[((e_.) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x
_Symbol] :> With[{q = Rt[b/a, 3]}, Dist[((1 + Sqrt[3])*f - e*q)/((1 + Sqrt[
3])*d - c*q), Int[1/Sqrt[a + b*x^3], x], x] + Dist[(d*e - c*f)/((1 + Sqrt[3
])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sqrt[a + b*x^3]), x], x]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && NeQ[b^2*c^6 - 20*a*b
*c^3*d^3 - 8*a^2*d^6, 0] && NeQ[b^2*e^6 - 20*a*b*e^3*f^3 - 8*a^2*f^6, 0]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt{-1-x^3}}\,dx = \frac{\left(e-\left(1+\sqrt{3}\right)f\right)\int \frac{1}{\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d} - \frac{(de-cf)\int \frac{1+\sqrt{3}+x}{(c+dx)\sqrt{-1-x^3}}\,dx}{c-\left(1+\sqrt{3}\right)d}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{2\sqrt{2-\sqrt{3}}\left(e-f-\sqrt{3}f\right)(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)\right.}{\sqrt[4]{3}\left(c-d-\sqrt{3}d\right)\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

$$= \frac{(de-cf)(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,\tan^{-1}\left(\frac{\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}}{\sqrt{c-d}\sqrt{d}\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}}\right.}{\sqrt{c-d}\sqrt{d}\sqrt{c^2+cd+d^2}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.29, size = 213, normalized size = 0.46

$$\frac{2\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\left(-\frac{f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{\sqrt[3]{-1}-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}}+\frac{i(-de+cf)\sqrt{1-x+x^2}\,\Pi\left(\frac{i\sqrt{3}d}{c+\sqrt[3]{-1}d};\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{c+\sqrt[3]{-1}d}\right)}{d\sqrt{-1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e + f*x)/((c + d*x)*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*(-((f*((-1)^(1/3) - x)*Sqrt[((-1)^(1/3) -
 (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(
1 + (-1)^(1/3))]], (-1)^(1/3)])/Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))])
+ (I*(-(d*e) + c*f)*Sqrt[1 - x + x^2]*EllipticPi[(I*Sqrt[3]*d)/(c + (-1)^(1
/3)*d), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(c
+ (-1)^(1/3)*d)))/(d*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.25, size = 265, normalized size = 0.57

method	result
default	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3d\sqrt{-x^3-1}}$
elliptic	$-\frac{2 i f \sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{i\left(x\right.}}{}\right.}{3d\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f/d*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/
2+1/2*I*3^(1/2)))^(1/2))-2/3*I*(-c*f+d*e)/d^2*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
```

```
2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/
2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+c/d)*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
c/d),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^3 - 1)*(f*x + e)/(d*x^4 + c*x^3 + d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt{-\left(x + 1\right)\left(x^{2} - x + 1\right)}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((e + f*x)/(sqrt(-(x + 1)*(x**2 - x + 1))*(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*(d*x + c)), x)
```

Mupad [B]
time = 0.10, size = 388, normalized size = 0.84

$$\frac{2 f \left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}} - \frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,(c f-d e)\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{c}{d}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{d^2\sqrt{-x^3-1}\,\left(\frac{c}{d}-1\right)\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((- x^3 - 1)^(1/2)*(c + d*x)),x)
```

```
[Out] (2*f*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/(d*(- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3
^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*(c*f - d*e)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(c/d - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^
(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(d^2*(- x^3 - 1)^(1/2)*(c/d - 1
)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.168 $\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right)+\frac{2\sqrt{2+\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

```
[Out] -2/3*e*arctanh((x^3+1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1
/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(
1/2)*3^(3/4)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1846, 272, 65, 213, 12, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\sqrt{x^3+1}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 + (2*Sqrt[2 + Sqrt[3]]*f*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[
3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[
1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{e+fx}{x\sqrt{1+x^3}}\,dx = e\int \frac{1}{x\sqrt{1+x^3}}\,dx + \int \frac{f}{\sqrt{1+x^3}}\,dx$$

$$= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{1+x^3}}\,dx$$

$$= \frac{2\sqrt{2+\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}} + \frac{1}{3}(2e)\text{S}$$

$$= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right) + \frac{2\sqrt{2+\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+}{1+\sqrt{3}+}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}$$

Mathematica [A]
time = 10.24, size = 134, normalized size = 1.12

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1+x^3}\right) - \frac{2f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left((-1)^{2/3}+x\right)}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 + x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 + x^3]])/3 - (2*f*((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (
-1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3) + x))/(1 + (-1)^(1/3)))]*Ellipti
cF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1
 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 + x^3])
```

Maple [A]
time = 0.25, size = 129, normalized size = 1.08

method	result
meijerg	$fx\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right) + \frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$

default	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\mathrm{arcta}}{}$
elliptic	$\frac{2f\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{1+x}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}-\frac{2e\,\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(
1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+
1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((x^3+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 31, normalized size = 0.26

$$\frac{1}{3}\,e\log\left(\frac{x^3-2\sqrt{x^3+1}+2}{x^3}\right)+2\,f\mathrm{weierstrassPInverse}(0,-4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*e*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3) + 2*f*weierstrassPInverse(0, -4,
 x)
```

Sympy [A]
time = 1.45, size = 42, normalized size = 0.35

$$-\frac{2e\,\mathrm{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}+\frac{fx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|x^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3+1)**(1/2),x)
```

```
[Out] -2*e*asinh(x**(-3/2))/3 + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp
_polar(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 + 1)*x), x)
```

Mupad [B]
time = 2.64, size = 207, normalized size = 1.72

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/
2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin(
((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*
1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^
(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1
/2)
```

3.169 $\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=134

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)-\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

```
[Out] -2/3*e*arctanh((-x^3+1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x-3^(1/2))/(1-x+3^(
1/2)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((x^2+x+1)/(1-x+3^(1/2))^2)^
(1/2)*3^(3/4)/(-x^3+1)^(1/2)/((1-x)/(1-x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1846, 272, 65, 212, 12, 224}

$$-\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x+\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x-\sqrt{3}+1}{-x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(-x+\sqrt{3}+1\right)^2}}\sqrt{1-x^3}}-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 - (2*Sqrt[2 + Sqrt[3]]*f*(1 - x)*Sqrt[(1 +
x + x^2)/(1 + Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] - x)/(1 + Sqrt[
3] - x)], -7 - 4*Sqrt[3]])/(3^(1/4)*Sqrt[(1 - x)/(1 + Sqrt[3] - x)^2]*Sqrt[
1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{e+fx}{x\sqrt{1-x^3}}\,dx = e\int \frac{1}{x\sqrt{1-x^3}}\,dx + \int \frac{f}{\sqrt{1-x^3}}\,dx$$
$$= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{1-x^3}}\,dx$$
$$= -\frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}-x}{1+\sqrt{3}-x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}} - \frac{1}{3}(2$$
$$= -\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right) - \frac{2\sqrt{2+\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1+\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{1-x}{\left(1+\sqrt{3}-x\right)^2}}\sqrt{1-x^3}}$$

Mathematica [A]
time = 10.25, size = 140, normalized size = 1.04

$$-\frac{2}{3}e\tanh^{-1}\left(\sqrt{1-x^3}\right) + \frac{2f\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*e*ArcTanh[Sqrt[1 - x^3]])/3 + (2*f*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)
^(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[Ar
cSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-
1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[1 - x^3])
```

Maple [A]
time = 0.25, size = 122, normalized size = 0.91

method	result
meijerg	$fx\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right) + \frac{e\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x)+i\pi)\sqrt{\pi}\right)}{3\sqrt{\pi}}$

default	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x \right.}}{}\right.}{3 \sqrt{-x^{3}+1}}$
elliptic	$-\frac{2 i f \sqrt{3}\, \sqrt{i \left(x+\frac{1}{2}-\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\, \sqrt{\frac{-1+x}{-\frac{3}{2}+\frac{i \sqrt{3}}{2}}}\, \sqrt{-i \left(x+\frac{1}{2}+\frac{i \sqrt{3}}{2}\right) \sqrt{3}}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\, \sqrt{i \left(x \right.}}{}\right.}{3 \sqrt{-x^{3}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*
I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3
/2+1/2*I*3^(1/2)))^(1/2))-2/3*e*arctanh((-x^3+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Fricas [A]
time = 0.10, size = 26, normalized size = 0.19

$$\frac{1}{3}\, e \log \left(-\frac{x^{3}+2 \sqrt{-x^{3}+1}-2}{x^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*e*log(-(x^3 + 2*sqrt(-x^3 + 1) - 2)/x^3)
```

Sympy [A]
time = 1.53, size = 65, normalized size = 0.49

$$e\left(\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for } \frac{1}{|x^3|} > 1 \\ \frac{2i\operatorname{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise} \end{cases}\right) + \frac{fx\Gamma\left(\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{1}{3}, \frac{1}{2} \\ \frac{4}{3} \end{matrix}\middle| x^3 e^{2i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3+1)**(1/2),x)
```

```
[Out] e*Piecewise((-2*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (2*I*asin(x**(-3/2))/
3, True)) + f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*exp_polar(2*I*pi)
)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 + 1)*x), x)
```

Mupad [B]
time = 0.06, size = 223, normalized size = 1.66

$$-\frac{\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\left(\sqrt{3}-3\mathrm{i}\right)\,1\mathrm{i}}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(1 - x^3)^(1/2)),x)
```

```
[Out] -((x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2
)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(as
in((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1
/2)*1i)/2 - 3/2)) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))
*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((1 - x^3)^(1/2
)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.170 $\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=137

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right)-\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

```
[Out] 2/3*e*arctan((x^3-1)^(1/2))-2/3*f*(1-x)*EllipticF((1-x+3^(1/2))/(1-x-3^(1/2
)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2+x+1)/(1-x-3^(1/2))^2)^(1/
2)*3^(3/4)/(x^3-1)^(1/2)/((-1+x)/(1-x-3^(1/2))^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1846, 272, 65, 209, 12, 225}

$$\frac{2}{3}e\text{ArcTan}\left(\sqrt{x^3-1}\right)-\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{x^2+x+1}{\left(-x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{-x+\sqrt{3}+1}{-x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(-x-\sqrt{3}+1\right)^2}}\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 - (2*Sqrt[2 - Sqrt[3]]*f*(1 - x)*Sqrt[(1 + x
 + x^2)/(1 - Sqrt[3] - x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - x)/(1 - Sqrt[3
] - x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 - x)/(1 - Sqrt[3] - x)^2)]*Sqr
t[-1 + x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{e+fx}{x\sqrt{-1+x^3}}\,dx = e\int \frac{1}{x\sqrt{-1+x^3}}\,dx + \int \frac{f}{\sqrt{-1+x^3}}\,dx$$
$$= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{-1+x^3}}\,dx$$
$$= -\frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-x}{1-\sqrt{3}-x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}} + \frac{1}{3}(2$$
$$= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right) - \frac{2\sqrt{2-\sqrt{3}}\,f(1-x)\sqrt{\frac{1+x+x^2}{\left(1-\sqrt{3}-x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}-}{1-\sqrt{3}-}\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{1-x}{\left(1-\sqrt{3}-x\right)^2}}\sqrt{-1+x^3}}$$

Mathematica [A]
time = 10.20, size = 136, normalized size = 0.99

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-1+x^3}\right) + \frac{2f\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\left(\sqrt[3]{-1}+x\right)\sqrt{\frac{\sqrt[3]{-1}+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 + x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 + x^3]])/3 + (2*f*Sqrt[(1 - x)/(1 + (-1)^(1/3))]*((-1)^
(1/3) + x)*Sqrt[((-1)^(1/3) + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*EllipticF[Arc
Sin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1 - (-1
)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 + x^3])
```

Maple [A]
time = 0.26, size = 129, normalized size = 0.94

method	result
meijerg	$\frac{f\sqrt{-\text{signum}\left(x^3-1\right)}\,x\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],x^3\right)}{\sqrt{\text{signum}\left(x^3-1\right)}} + \frac{e\sqrt{-\text{signum}\left(x^3-1\right)}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-x^3+1}}{2}\right)+(\right.}{3\sqrt{\pi}\,\sqrt{\text{signum}\left(x^3-1\right)}}$

default	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\,\mathrm{ar}}{}$
elliptic	$\frac{2f\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{-1+x}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}+\frac{2e\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*f*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*
3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/
2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/
2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((x^3-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 27, normalized size = 0.20

$$\frac{1}{3}\arctan\left(\frac{x^3-2}{2\sqrt{x^3-1}}\right)e+2\,f\mathrm{weierstrassPInverse}(0,4,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(1/2*(x^3 - 2)/sqrt(x^3 - 1))*e + 2*f*weierstrassPInverse(0, 4, x
)
```

Sympy [A]
time = 1.44, size = 60, normalized size = 0.44

$$e\left(\begin{cases}\frac{2i\,\mathrm{acosh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{for}\ \frac{1}{|x^3|}>1\\ -\frac{2\,\mathrm{asin}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3} & \text{otherwise}\end{cases}\right)-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|x^3\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x**3-1)**(1/2),x)
```

```
[Out] e*Piecewise((2*I*acosh(x**(-3/2))/3, 1/Abs(x**3) > 1), (-2*asin(x**(-3/2))/
3, True)) - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3)/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(x^3 - 1)*x), x)
```

Mupad [B]
time = 2.62, size = 207, normalized size = 1.51

$$-\frac{\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\left(\sqrt{3}-3i\right)\,1i}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)\,x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)
*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(f*ellipticF(asin((-(x - 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
) + e*ellipticPi((3^(1/2)*1i)/2 + 3/2, asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))*(-(x - 1)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(3^(1/2) - 3i)*1i)/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/
2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x
^3)^(1/2)
```

3.171 $\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right)+\frac{2\sqrt{2-\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

```
[Out] 2/3*e*arctan((-x^3-1)^(1/2))+2/3*f*(1+x)*EllipticF((1+x+3^(1/2))/(1+x-3^(1/
2)),2*I-I*3^(1/2))*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x-3^(1/2))^2)^(1
/2)*3^(3/4)/(-x^3-1)^(1/2)/((-1-x)/(1+x-3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1846, 272, 65, 210, 12, 225}

$$\frac{2\sqrt{2-\sqrt{3}}\,f(x+1)\sqrt{\frac{x^2-x+1}{\left(x-\sqrt{3}+1\right)^2}}\,F\left(\text{ArcSin}\left(\frac{x+\sqrt{3}+1}{x-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{x+1}{\left(x-\sqrt{3}+1\right)^2}}\sqrt{-x^3-1}}+\frac{2}{3}e\text{ArcTan}\left(\sqrt{-x^3-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 + (2*Sqrt[2 - Sqrt[3]]*f*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 - Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + x)/(1 - Sqrt[3
] + x)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((1 + x)/(1 - Sqrt[3] + x)^2)]*Sqr
t[-1 - x^3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1846

```
Int[(Pq_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[Coeff[Pq,
x, 0], Int[1/(x*Sqrt[a + b*x^n]), x], x] + Int[ExpandToSum[(Pq - Coeff[Pq,
x, 0])/x, x]/Sqrt[a + b*x^n], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGt
Q[n, 0] && NeQ[Coeff[Pq, x, 0], 0]
```

Rubi steps

$$\int \frac{e+fx}{x\sqrt{-1-x^3}}\,dx = e\int \frac{1}{x\sqrt{-1-x^3}}\,dx + \int \frac{f}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{1}{3}e\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, x^3\right) + f\int \frac{1}{\sqrt{-1-x^3}}\,dx$$

$$= \frac{2\sqrt{2-\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}+x}{1-\sqrt{3}+x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}} - \frac{1}{3}(2$$

$$= \frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right) + \frac{2\sqrt{2-\sqrt{3}}\,f(1+x)\sqrt{\frac{1-x+x^2}{\left(1-\sqrt{3}+x\right)^2}}\,F\left(\sin^{-1}\left(\frac{1+\sqrt{3}}{1-\sqrt{3}}\right.\right.}{\sqrt[4]{3}\sqrt{-\frac{1+x}{\left(1-\sqrt{3}+x\right)^2}}\sqrt{-1-x^3}}$$

Mathematica [A]
time = 10.18, size = 138, normalized size = 1.05

$$\frac{2}{3}e\tan^{-1}\left(\sqrt{-1-x^3}\right) - \frac{2f\left(\sqrt[3]{-1}-x\right)\sqrt{\frac{1+x}{1+\sqrt[3]{-1}}}\sqrt{-\frac{(-1)^{2/3}\left((-1)^{2/3}+x\right)}{1+\sqrt[3]{-1}}}\,F\left(\sin^{-1}\left(\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)|\sqrt[3]{-1}\right)}{\sqrt{\frac{1+(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\sqrt{-1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)/(x*Sqrt[-1 - x^3]),x]
```

```
[Out] (2*e*ArcTan[Sqrt[-1 - x^3]])/3 - (2*f*((-1)^(1/3) - x)*Sqrt[(1 + x)/(1 + (-
1)^(1/3))]*Sqrt[-(((-1)^(2/3)*((-1)^(2/3) + x))/(1 + (-1)^(1/3)))]*Elliptic
F[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(Sqrt[(1
+ (-1)^(2/3)*x)/(1 + (-1)^(1/3))]*Sqrt[-1 - x^3])
```

Maple [A]
time = 0.24, size = 122, normalized size = 0.93

method	result
meijerg	$-ifx\,\text{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2}\right],\left[\frac{4}{3}\right],-x^3\right) - \frac{ie\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$

default	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$
elliptic	$-\frac{2if\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{i\left(x-\right.}}{}\right.}{3\sqrt{-x^3-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/x/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*I*f*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*
3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+
1/2*I*3^(1/2)))^(1/2))+2/3*e*arctan((-x^3-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Fricas [A]
time = 0.10, size = 28, normalized size = 0.21

$$\frac{1}{3}\arctan\left(\frac{(x^3+2)\sqrt{-x^3-1}}{2\,(x^3+1)}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(1/2*(x^3 + 2)*sqrt(-x^3 - 1)/(x^3 + 1))*e
```

Sympy [A]
time = 1.61, size = 46, normalized size = 0.35

$$\frac{2ie\operatorname{asinh}\left(\frac{1}{x^{\frac{3}{2}}}\right)}{3}-\frac{ifx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|x^{3}e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x**3-1)**(1/2),x)
```

```
[Out] 2*I*e*asinh(x**(-3/2))/3 - I*f*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), x**3*
exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/x/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)/(sqrt(-x^3 - 1)*x), x)
```

Mupad [B]
time = 2.66, size = 223, normalized size = 1.70

$$\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(f\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)-e\,\Pi\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/(x*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*(f*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)),
 -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - e*ellipticPi((3^(1/2)*1i
)/2 + 3/2, asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 +
 3/2)/((3^(1/2)*1i)/2 - 3/2)))*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^
(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2))/((- x^3 - 1)^(1/2)*(x
^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.172 $\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

```
[Out] -ln(d*x+c)/d+3/2*ln(d*(d*x+2*c)-d*(d^3*x^3+2*c^3)^(1/3))/d-arctan(1/3*(1+2*
(d*x+2*c)/(d^3*x^3+2*c^3)^(1/3))*3^(1/2))*3^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2176}

$$-\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}-\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d
) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*
d)
```

Rule 2176

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)),
x_Symbol] :> Simp[Sqrt[3]*f*(ArcTan[(1 + 2*Rt[b, 3]*((2*c + d*x)/(d*(a + b*
x^3)^(1/3))))/Sqrt[3]]/(Rt[b, 3]*d)), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]
*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[
b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2
*b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d}-\frac{\log(c+dx)}{d}+\frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

Mathematica [A]
time = 1.24, size = 159, normalized size = 1.67

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{4c+2dx+\sqrt[3]{2c^3+d^3x^3}}\right)}{d}+\frac{\log\left(-2c-dx+\sqrt[3]{2c^3+d^3x^3}\right)}{d}-\frac{\log\left(4c^2+4cdx+d^2x^2+(2c+dx)\sqrt[3]{2c^3+d^3x^3}+(2c^3+d^3x^3)^{2/3}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 + d
^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*c^
2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d^3*
x^3)^(2/3)]/(2*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{-dx+c}{(dx+c)\left(d^3x^3+2c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

```
[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")
```

```
[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (residue poly has multiple non-linear facto
rs)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{c}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\right)dx-\int\frac{dx}{c\sqrt[3]{2c^3+d^3x^3}+dx\sqrt[3]{2c^3+d^3x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)
```

```
[Out] -Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/
3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*
x**3)**(1/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{c-d\,x}{{\left(2\,c^3+d^3\,x^3\right)}^{1/3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)
```

```
[Out] int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)
```

3.173 $\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$

Optimal. Leaf size=234

$$\frac{f\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}+\frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2}+\frac{(de-cf)\log((c-dx)(c+dx)^2}{4\sqrt[3]{2}\,cd^2}$$

```
[Out] 1/8*(-c*f+d*e)*ln((-d*x+c)*(d*x+c)^2)*2^(2/3)/c/d^2-1/2*f*ln(-d*x+(d^3*x^3-
c^3)^(1/3))/d^2-3/8*(-c*f+d*e)*ln(d*(-d*x+c)+2^(2/3)*d*(d^3*x^3-c^3)^(1/3))
*2^(2/3)/c/d^2+1/3*f*arctan(1/3*(1+2*d*x/(d^3*x^3-c^3)^(1/3))*3^(1/2))/d^2*
3^(1/2)+1/4*(-c*f+d*e)*arctan(1/3*(1-2^(1/3)*(-d*x+c)/(d^3*x^3-c^3)^(1/3))*
3^(1/2))*3^(1/2)*2^(2/3)/c/d^2
```

Rubi [A]
time = 0.16, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2177, 245, 2174}

$$\frac{\sqrt{3}\,\text{ArcTan}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{d^3x^3-c^3}}}{\sqrt{3}}\right)(de-cf)}{2\sqrt[3]{2}\,cd^2}+\frac{f\,\text{ArcTan}\left(\frac{\frac{2dx}{\sqrt[3]{d^3x^3-c^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d^2}-\frac{3(de-cf)\log\left(2^{2/3}d\sqrt[3]{d^3x^3-c^3}+d(c-dx)\right)}{4\sqrt[3]{2}\,cd^2}-\frac{f\log\left(\sqrt[3]{d^3x^3-c^3}-dx\right)}{2d^2}+\frac{(de-cf)\log\left((c-dx)(c+dx)^2\right)}{4\sqrt[3]{2}\,cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
```

```
[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) + (S
qrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/
Sqrt[3]])/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2
^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e
- c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^
2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + 2*Rt[b, 3]*
(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rule 2174

```
Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[
Sqrt[3]*(ArcTan[(1 - 2^(1/3)*Rt[b, 3]*((c - d*x)/(d*(a + b*x^3)^(1/3))))/Sq
rt[3]]/(2^(4/3)*Rt[b, 3]*c)), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3
)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^
```

```
(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 +
a*d^3, 0]
```

Rule 2177

```
Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3))
, x_Symbol] :> Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)
/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

Rubi steps

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx = \frac{f\int \frac{1}{\sqrt[3]{-c^3+d^3x^3}}\,dx}{d} + \frac{(de-cf)\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx}{d}$$

$$= \frac{f\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2} + \frac{\sqrt{3}\,(de-cf)\tan^{-1}\left(\frac{1-\frac{\sqrt[3]{2}\,(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,cd^2} +$$

Mathematica [**F**]
time = 10.12, size = 0, normalized size = 0.00

$$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]
[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{fx+e}{(dx+c)\left(d^3x^3-c^3\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt[3]{(-c + dx)\,(c^2 + cdx + d^2x^2)}\,(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)
[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")
[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e + f\,x}{{\left(d^3\,x^3 - c^3\right)}^{1/3}\,\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)
[Out] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)
```

3.174 $\int x^2(a+bx)^n\left(c+dx^3\right)\,dx$

Optimal. Leaf size=160

$$\frac{a^2(b^3c-a^3d)\,(a+bx)^{1+n}}{b^6(1+n)}-\frac{a(2b^3c-5a^3d)\,(a+bx)^{2+n}}{b^6(2+n)}+\frac{(b^3c-10a^3d)\,(a+bx)^{3+n}}{b^6(3+n)}+\frac{10a^2d(a+bx)^{4+n}}{b^6(4+n)}-\frac{5ad(}{b}$$

```
[Out] a^2*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*(-5*a^3*d+2*b^3*c)*(b*x+a)^(2+
n)/b^6/(2+n)+(-10*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d*(b*x+a)^(4+
n)/b^6/(4+n)-5*a*d*(b*x+a)^(5+n)/b^6/(5+n)+d*(b*x+a)^(6+n)/b^6/(6+n)
```

Rubi [A]

time = 0.07, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {1634}

$$-\frac{a(2b^3c-5a^3d)\,(a+bx)^{n+2}}{b^6(n+2)}+\frac{(b^3c-10a^3d)\,(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2(b^3c-a^3d)\,(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad(a+bx)^{n+5}}{b^6(n+5)}+\frac{d(a+bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3
*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n
))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a +
 b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\left(c+dx^3\right)\,dx&=\int\left(\frac{(a^2b^3c-a^5d)\,(a+bx)^n}{b^5}+\frac{a(-2b^3c+5a^3d)\,(a+bx)^{1+n}}{b^5}+\frac{(b^3c-10a^3d)\,(}{b^5}\right.\\&=\frac{a^2(b^3c-a^3d)\,(a+bx)^{1+n}}{b^6(1+n)}-\frac{a(2b^3c-5a^3d)\,(a+bx)^{2+n}}{b^6(2+n)}+\frac{(b^3c-10a^3d)\,(a+}{b^6(3+n)}\end{aligned}$$

Mathematica [A]

time = 0.13, size = 133, normalized size = 0.83

$$\frac{(a+bx)^{1+n}\left(\frac{a^2b^3c-a^5d}{1+n}+\frac{a(-2b^3c+5a^3d)(a+bx)}{2+n}+\frac{(b^3c-10a^3d)(a+bx)^2}{3+n}+\frac{10a^2d(a+bx)^3}{4+n}-\frac{5ad(a+bx)^4}{5+n}+\frac{d(a+bx)^5}{6+n}\right)}{b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(
a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a
 + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))
)/b^6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(160) = 320$.
time = 0.21, size = 415, normalized size = 2.59

method	result
norman	$\frac{d x^{6} e^{n \ln(bx+a)}}{6+n} + \frac{(b^{3} c n^{3}+15 b^{3} c n^{2}+20 a^{3} d n+74 b^{3} c n+120 b^{3} c) x^{3} e^{n \ln(bx+a)}}{b^{3}(n^{4}+18 n^{3}+119 n^{2}+342 n+360)} + \frac{d a n x^{5} e^{n \ln(bx+a)}}{b(n^{2}+11 n+30)} - \frac{2 a^{3}(-b^{3} c n^{3}-15 b^{3} c n^{2}-74}{b^{6}(n^{6}+21 n^{5}+175 n^{4}+}$
gosper	$-\frac{(bx+a)^{1+n}(-b^{5} d n^{5} x^{5}-15 b^{5} d n^{4} x^{5}+5 a b^{4} d n^{4} x^{4}-85 b^{5} d n^{3} x^{5}+50 a b^{4} d n^{3} x^{4}-b^{5} c n^{5} x^{2}-225 b^{5} d n^{2} x^{5}-20 a^{2} b^{3} d n^{3} x^{3}+175 a b^{4} d n}{}$
risch	$-\frac{(-b^{6} d n^{5} x^{6}-a b^{5} d n^{5} x^{5}-15 b^{6} d n^{4} x^{6}-10 a b^{5} d n^{4} x^{5}-85 b^{6} d n^{3} x^{6}+5 a^{2} b^{4} d n^{4} x^{4}-35 a b^{5} d n^{3} x^{5}-b^{6} c n^{5} x^{3}-225 b^{6} d n^{2} x^{6}+30 a^{2} b^{4} d n}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] d/(6+n)*x^6*exp(n*ln(b*x+a))+(b^3*c*n^3+15*b^3*c*n^2+20*a^3*d*n+74*b^3*c*n+
120*b^3*c)/b^3/(n^4+18*n^3+119*n^2+342*n+360)*x^3*exp(n*ln(b*x+a))+d*a/b*n/
(n^2+11*n+30)*x^5*exp(n*ln(b*x+a))-2*a^3*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*
n+60*a^3*d-120*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*
exp(n*ln(b*x+a))+2/b^5*n*a^2*(-b^3*c*n^3-15*b^3*c*n^2-74*b^3*c*n+60*a^3*d-1
20*b^3*c)/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)*x*exp(n*ln(b*x+a
))-5*n*a^2/b^2*d/(n^3+15*n^2+74*n+120)*x^4*exp(n*ln(b*x+a))-(-b^3*c*n^3-15*
b^3*c*n^2-74*b^3*c*n+60*a^3*d-120*b^3*c)*a/b^4*n/(n^5+20*n^4+155*n^3+580*n^
2+1044*n+720)*x^2*exp(n*ln(b*x+a))
```

Maxima [A]
time = 0.29, size = 253, normalized size = 1.58

$$\frac{((n^2+3n+2)b^3x^3+(n^2+n)ab^2x^2-2a^2bnx+2a^3)(bx+a)^n c}{(n^3+6n^2+11n+6)b^3} + \frac{((n^5+15n^4+85n^3+225n^2+274n+120)b^6x^6+(n^5+10n^4+35n^3+50n^2+24n)ab^5x^5-5(n^4+6n^3+11n^2+6n)a^2b^4x^4+20(n^3+3n^2+2n)a^3b^3x^3-60(n^2+n)a^4b^2x^2+120a^5bnx-120a^6)(bx+a)^n d}{(n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2
 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5
 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*
```

```
b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d
/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(160) = 320$.
time = 0.52, size = 490, normalized size = 3.06

$$\frac{(2\,a^3b^3cn^3 + 30\,a^3b^3cn^2 + 148\,a^3b^3cn + 240\,a^3b^3c - 120\,a^6d + (b^6dn^5 + 15\,b^6dn^4 + 85\,b^6dn^3 + 225\,b^6dn^2 + 274\,b^6dn + 120\,b^6d)x^6 + (ab^5dn^5 + 10\,ab^5dn^4 + 35\,ab^5dn^3 + 50\,ab^5dn^2 + 24\,ab^5dn)x^5 - 5\,(a^2b^4dn^4 + 6\,a^2b^4dn^3 + 11\,a^2b^4dn^2 + 6\,a^2b^4dn)x^4 + (b^6cn^5 + 18\,b^6cn^4 + 240\,b^6c + (121\,b^6c + 20\,a^3b^3d)n^3 + 12\,(31\,b^6c + 5\,a^3b^3d)n^2 + 4\,(127\,b^6c + 10\,a^3b^3d)n)x^3 + (ab^5cn^5 + 16\,ab^5cn^4 + 89\,ab^5cn^3 + 2\,(97\,ab^5c - 30\,a^4b^2d)n^2 + 60\,(2\,ab^5c - a^4b^2d)n)x^2 - 2\,(a^2b^4cn^4 + 15\,a^2b^4cn^3 + 74\,a^2b^4cn^2 + 60\,(2\,a^2b^4c - a^5bd)n)x)(bx + a)^n}{b^6n^6 + 21\,b^6n^5 + 175\,b^6n^4 + 735\,b^6n^3 + 1624\,b^6n^2 + 1764\,b^6n + 720\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
 - 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
 - a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 6397 vs. $2(144) = 288$.
time = 2.09, size = 6397, normalized size = 39.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*b**
7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11
*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*
a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 6
25*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log
(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b
**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x**2/(60*a*
*5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a
*b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*
x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x
```

5) + 600*a2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 10*a*b**4*c*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*a*b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) + 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5*d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4*b*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 270*a**4*b*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b**4*c*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*d*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*b**5*c*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9

```
*x**3), Eq(n, -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6
*b**8*x**2) - 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**
4*b*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b
*d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/
b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b +
x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 +
 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7*
x + 6*b**8*x**2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x +
6*b**8*x**2) + 12*a*b**4*c*x/(6*a**2*b**6 + 12*...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 835 vs. 2(160) = 320.
time = 3.97, size = 835, normalized size = 5.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
 a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
 a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
 508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

Mupad [B]
time = 3.20, size = 495, normalized size = 3.09

$$(a+b\,x)^n\,\left(\frac{d\,x^6\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}+\frac{2\,a^3\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{x^3\,\left(n^2+3\,n+2\right)\,\left(20\,d\,a^3\,n+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{2\,a^2\,n\,x\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,d\,n\,x^5\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,n\,x^2\,\left(n+1\right)\,\left(-60\,d\,a^3+c\,b^3\,n^3+15\,c\,b^3\,n^2+74\,c\,b^3\,n+120\,c\,b^3\right)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{5\,a^2\,d\,n\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(120*b^
3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*n + 1
624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*b^3*c
- 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 35*n^2 +
10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n
^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^
6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

3.175 $\int x(a+bx)^n (c+dx^3)\, dx$

Optimal. Leaf size=126

$$-\frac{a(b^3c-a^3d)(a+bx)^{1+n}}{b^5(1+n)}+\frac{(b^3c-4a^3d)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\frac{4ad(a+bx)^{4+n}}{b^5(4+n)}+\frac{d(a+bx)^{5+n}}{b^5(5+n)}$$

```
[Out] -a*(-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^5/(1+n)+(-4*a^3*d+b^3*c)*(b*x+a)^(2+n)/b^
5/(2+n)+6*a^2*d*(b*x+a)^(3+n)/b^5/(3+n)-4*a*d*(b*x+a)^(4+n)/b^5/(4+n)+d*(b*
x+a)^(5+n)/b^5/(5+n)
```

Rubi [A]
time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1634}

$$-\frac{a(b^3c-a^3d)(a+bx)^{n+1}}{b^5(n+1)}+\frac{(b^3c-4a^3d)(a+bx)^{n+2}}{b^5(n+2)}+\frac{6a^2d(a+bx)^{n+3}}{b^5(n+3)}-\frac{4ad(a+bx)^{n+4}}{b^5(n+4)}+\frac{d(a+bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)
*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n
)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(
5 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n (c+dx^3)\, dx &= \int \left(\frac{a(-b^3c+a^3d)(a+bx)^n}{b^4}+\frac{(b^3c-4a^3d)(a+bx)^{1+n}}{b^4}+\frac{6a^2d(a+bx)^{2+n}}{b^4}-\right.\\ &= -\frac{a(b^3c-a^3d)(a+bx)^{1+n}}{b^5(1+n)}+\frac{(b^3c-4a^3d)(a+bx)^{2+n}}{b^5(2+n)}+\frac{6a^2d(a+bx)^{3+n}}{b^5(3+n)}-\end{aligned}$$

Mathematica [A]
time = 0.09, size = 104, normalized size = 0.83

$$\frac{(a+bx)^{1+n}\left(\frac{a(-b^3c+a^3d)}{1+n}+\frac{(b^3c-4a^3d)(a+bx)}{2+n}+\frac{6a^2d(a+bx)^2}{3+n}-\frac{4ad(a+bx)^3}{4+n}+\frac{d(a+bx)^4}{5+n}\right)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a
+ b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 +
n) + (d*(a + b*x)^4)/(5 + n)))/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(126) = 252$.
time = 0.24, size = 283, normalized size = 2.25

method	result
gosper	$\frac{(bx+a)^{1+n}(b^4dn^4x^4+10b^4dn^3x^4-4ab^3dn^3x^3+35b^4dn^2x^4-24ab^3dn^2x^3+b^4cn^4x+50b^4dnx^4+12a^2b^2dn^2x^2-44ab^3dnx^3+13b^4}{b^5(n^5+}$
norman	$\frac{dx^5e^{n\ln(bx+a)}}{5+n}+\frac{a^2(-b^3cn^3-12b^3cn^2-47b^3cn+24a^3d-60b^3c)e^{n\ln(bx+a)}}{b^5(n^5+15n^4+85n^3+225n^2+274n+120)}+\frac{(b^3cn^3+12b^3cn^2+12a^3dn+47b^3cn+60b^3c)x^2e^{n\,l}}{b^3(n^4+14n^3+71n^2+154n+120)}$
risch	$\frac{(b^5dn^4x^5+ab^4dn^4x^4+10b^5dn^3x^5+6ab^4dn^3x^4+35b^5dn^2x^5-4a^2b^3dn^3x^3+11ab^4dn^2x^4+b^5cn^4x^2+50b^5dnx^5-12a^2b^3dn^2x^3+a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+35*b^4*d*n^
2*x^4-24*a*b^3*d*n^2*x^3+b^4*c*n^4*x+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-44
*a*b^3*d*n*x^3+13*b^4*c*n^3*x+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-a*b^3*c*n^3-2
4*a*b^3*d*x^3+59*b^4*c*n^2*x-24*a^3*b*d*n*x+24*a^2*b^2*d*x^2-12*a*b^3*c*n^2
+107*b^4*c*n*x-24*a^3*b*d*x-47*a*b^3*c*n+60*b^4*c*x+24*a^4*d-60*a*b^3*c)/b^
5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

Maxima [A]
time = 0.29, size = 184, normalized size = 1.46

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3n+2)b^2}+\frac{((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+12(n^2+n)a^3b^2x^2-24a^4bnx+24a^5)(bx+a)^nd}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n
^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*
b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24
*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*
n + 120)*b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(126) = 252$.
time = 0.44, size = 348, normalized size = 2.76

$$-\frac{(a^2b^3cn^3+12a^2b^3cn^2+47a^2b^3cn+60a^2b^3c-24a^5d-(b^5dn^4+10b^5dn^3+35b^5dn^2+50b^5dn+24b^5d)x^5-(ab^4dn^4+6ab^4dn^3+11ab^4dn^2+6ab^4dn)x^4+4(a^2b^3dn^3+3a^2b^3dn^2+2a^2b^3dn)x^3-(b^5cn^4+13b^5cn^3+60b^5c+(59b^5c+12a^3b^2d)n^2+(107b^5c+12a^3b^2d)n)x^2-(ab^4cn^4+12ab^4cn^3+47ab^4cn^2+12(5ab^4c-2a^4bd)n)x)(bx+a)^n}{b^5n^5+15b^5n^4+85b^5n^3+225b^5n^2+274b^5n+120b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")

[Out] -(a^2*b^3*c*n^3 + 12*a^2*b^3*c*n^2 + 47*a^2*b^3*c*n + 60*a^2*b^3*c - 24*a^5 *d - (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^5 - (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^4 + 4*(a^2 *b^3*d*n^3 + 3*a^2*b^3*d*n^2 + 2*a^2*b^3*d*n)*x^3 - (b^5*c*n^4 + 13*b^5*c*n ^3 + 60*b^5*c + (59*b^5*c + 12*a^3*b^2*d)*n^2 + (107*b^5*c + 12*a^3*b^2*d)* n)*x^2 - (a*b^4*c*n^4 + 12*a*b^4*c*n^3 + 47*a*b^4*c*n^2 + 12*(5*a*b^4*c - 2 *a^4*b*d)*n)*x)*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^ 2 + 274*b^5*n + 120*b^5)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 3704 vs. $2(112) = 224$.
time = 1.30, size = 3704, normalized size = 29.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)

[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b** 9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48 *a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 4 8*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a** 3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3 *b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b** 2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x** 3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7* x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a **4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x* *4) + 48*a*b**3*d*x**3/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x** 3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a **2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a* *2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a **3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/ (6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b** 2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b* *8*x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x* *2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2

```
 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6
*x + 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b
**6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/
(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**
6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x +
 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12
*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a
*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*
b**5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2
*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -
3)), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 +
 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2
*d*x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*
x) + 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*
x) + 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5
+ 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d
*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4
/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2
*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*
n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a
**2*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225
*b**5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n
**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5)
- 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 22
5*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n
/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 12
0*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x*
*3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
274*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*
n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*
n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**
2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*
b**4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5
*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5
 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(126) =

252.
time = 4.88, size = 577, normalized size = 4.58

$$\frac{(bx+a)^n b^5 d n^4 x^5 + (bx+a)^n a b^4 d n^4 x^4 + 10 (bx+a)^n b^5 d n^3 x^5 + 6 (bx+a)^n a b^4 d n^3 x^4 + 35 (bx+a)^n b^5 d n^2 x^5 + (bx+a)^n b^5 c n^4 x^2 - 4 (bx+a)^n a^2 b^3 d n^3 x^3 + 11 (bx+a)^n a b^4 d n^2 x^4 + 50 (bx+a)^n b^5 d n x^5 + (bx+a)^n a b^4 c n^4 x + 13 (bx+a)^n b^5 c n^3 x^2 - 12 (bx+a)^n a^2 b^3 d n^2 x^3 + 6 (bx+a)^n a b^4 d n x^4 + 24 (bx+a)^n b^5 d x^5 + 12 (bx+a)^n a b^4 c n^3 x + 59 (bx+a)^n b^5 c n^2 x^2 + 12 (bx+a)^n a^3 b^2 d n^2 x^2 - 8 (bx+a)^n a^2 b^3 d n x^3 - (bx+a)^n a^2 b^3 c n^3 + 47 (bx+a)^n a b^4 c n^2 x + 107 (bx+a)^n b^5 c n x^2 + 12 (bx+a)^n a^3 b^2 d n x^2 - 12 (bx+a)^n a^2 b^3 c n^2 + 60 (bx+a)^n a b^4 c n x - 24 (bx+a)^n a^4 b d n x + 60 (bx+a)^n b^5 c x^2 - 47 (bx+a)^n a^2 b^3 c n - 60 (bx+a)^n a^2 b^3 c + 24 (bx+a)^n a^5 d}{b^5 n^5 + 15 b^5 n^4 + 85 b^5 n^3 + 225 b^5 n^2 + 274 b^5 n + 120 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^5*d*n^4*x^5 + (b*x + a)^n*a*b^4*d*n^4*x^4 + 10*(b*x + a)^n*b
^5*d*n^3*x^5 + 6*(b*x + a)^n*a*b^4*d*n^3*x^4 + 35*(b*x + a)^n*b^5*d*n^2*x^5
 + (b*x + a)^n*b^5*c*n^4*x^2 - 4*(b*x + a)^n*a^2*b^3*d*n^3*x^3 + 11*(b*x +
a)^n*a*b^4*d*n^2*x^4 + 50*(b*x + a)^n*b^5*d*n*x^5 + (b*x + a)^n*a*b^4*c*n^4
*x + 13*(b*x + a)^n*b^5*c*n^3*x^2 - 12*(b*x + a)^n*a^2*b^3*d*n^2*x^3 + 6*(b
*x + a)^n*a*b^4*d*n*x^4 + 24*(b*x + a)^n*b^5*d*x^5 + 12*(b*x + a)^n*a*b^4*c
*n^3*x + 59*(b*x + a)^n*b^5*c*n^2*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n^2*x^2 -
8*(b*x + a)^n*a^2*b^3*d*n*x^3 - (b*x + a)^n*a^2*b^3*c*n^3 + 47*(b*x + a)^n*
a*b^4*c*n^2*x + 107*(b*x + a)^n*b^5*c*n*x^2 + 12*(b*x + a)^n*a^3*b^2*d*n*x^
2 - 12*(b*x + a)^n*a^2*b^3*c*n^2 + 60*(b*x + a)^n*a*b^4*c*n*x - 24*(b*x + a
)^n*a^4*b*d*n*x + 60*(b*x + a)^n*b^5*c*x^2 - 47*(b*x + a)^n*a^2*b^3*c*n - 6
0*(b*x + a)^n*a^2*b^3*c + 24*(b*x + a)^n*a^5*d)/(b^5*n^5 + 15*b^5*n^4 + 85*
b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
```

Mupad [B]
time = 2.95, size = 363, normalized size = 2.88

$$(a+bx)^n\left(\frac{d\,x^5\,(n^4+10\,n^3+35\,n^2+50\,n+24)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{a^2\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^5\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{x^2\,(n+1)\,(12\,d\,a^3\,n+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^3\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{a\,n\,x\,(-24\,d\,a^3+c\,b^3\,n^3+12\,c\,b^3\,n^2+47\,c\,b^3\,n+60\,c\,b^3)}{b^4\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}+\frac{a\,d\,n\,x^4\,(n^3+6\,n^2+11\,n+6)}{b\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}-\frac{4\,a^2\,d\,n\,x^3\,(n^2+3\,n+2)}{b^2\,(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 +
 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 +
b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 12
0)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b
^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*x*(60
*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274*n + 2
25*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6
))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n*x^3*(3*
n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
```

3.176 $\int (a+bx)^n (c+dx^3)\, dx$

Optimal. Leaf size=94

$$\frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}$$

```
[Out] (-a^3*d+b^3*c)*(b*x+a)^(1+n)/b^4/(1+n)+3*a^2*d*(b*x+a)^(2+n)/b^4/(2+n)-3*a*
d*(b*x+a)^(3+n)/b^4/(3+n)+d*(b*x+a)^(4+n)/b^4/(4+n)
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1864}

$$\frac{(b^3c-a^3d)(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2d(a+bx)^{n+2}}{b^4(n+2)}-\frac{3ad(a+bx)^{n+3}}{b^4(n+3)}+\frac{d(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 +
 n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)
^(4 + n))/(b^4*(4 + n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n (c+dx^3)\, dx &= \int \left(\frac{(b^3c-a^3d)(a+bx)^n}{b^3}+\frac{3a^2d(a+bx)^{1+n}}{b^3}-\frac{3ad(a+bx)^{2+n}}{b^3}+\frac{d(a+bx)^{3+n}}{b^3}\right) dx\\ &= \frac{(b^3c-a^3d)(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2d(a+bx)^{2+n}}{b^4(2+n)}-\frac{3ad(a+bx)^{3+n}}{b^4(3+n)}+\frac{d(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 95, normalized size = 1.01

$$\frac{(a+bx)^{1+n}(-6a^3d+6a^2bd(1+n)x-3ab^2d(2+3n+n^2)x^2+b^3(6+5n+n^2)(c(4+n)+d(1+n)x^3))}{b^4(1+n)(2+n)(3+n)(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3),x]
```

```
[Out] ((a + b*x)^(1 + n)*(-6*a^3*d + 6*a^2*b*d*(1 + n)*x - 3*a*b^2*d*(2 + 3*n + n
^2)*x^2 + b^3*(6 + 5*n + n^2)*(c*(4 + n) + d*(1 + n)*x^3)))/(b^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Maple [A]
time = 0.20, size = 167, normalized size = 1.78

method	result
gosper	$-\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-11b^3dnx^3+9ab^2dnx^2-b^3cn^3-6dx^3b^3-6a^2bdnx+6adx^2b^2-9b^3cn^2-6a^2dxb-2}{b^4(n^4+10n^3+35n^2+50n+24)}$
risch	$-\frac{(-b^4dn^3x^4-ab^3dn^3x^3-6b^4dn^2x^4-3ab^3dn^2x^3-11b^4dnx^4+3a^2b^2dn^2x^2-2ab^3dnx^3-b^4cn^3x-6dx^4b^4+3a^2b^2dnx^2-ab^3cn^3-}{(3+n)(4+n)(2+n)(1+n)b^4}$
norman	$\frac{dx^4\mathrm{e}^{n\ln(bx+a)}}{4+n}+\frac{(b^3cn^3+9b^3cn^2+6a^3dn+26b^3cn+24b^3c)x\,\mathrm{e}^{n\ln(bx+a)}}{b^3(n^4+10n^3+35n^2+50n+24)}+\frac{nadx^3\mathrm{e}^{n\ln(bx+a)}}{b(n^2+7n+12)}-\frac{a(-b^3cn^3-9b^3cn^2-26b^3cn+6a}{b^4(n^4+10n^3+35n^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n
*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^
3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3*c)/b^4/(n^4+10*n^3+35*n^2+50*
n+24)
```

Maxima [A]
time = 0.28, size = 122, normalized size = 1.30

$$\frac{(bx+a)^{n+1}c}{b(n+1)}+\frac{((n^3+6\,n^2+11\,n+6)b^4x^4+(n^3+3\,n^2+2\,n)ab^3x^3-3\,(n^2+n)a^2b^2x^2+6\,a^3bnx-6\,a^4)(bx+a)^nd}{(n^4+10\,n^3+35\,n^2+50\,n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3
+ 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(
b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(94) = 188.
time = 0.35, size = 222, normalized size = 2.36

$$\frac{(ab^3cn^3+9\,ab^3cn^2+26\,ab^3cn+24\,ab^3c-6\,a^4d+(b^4dn^3+6\,b^4dn^2+11\,b^4dn+6\,b^4d)x^4+(ab^3dn^3+3\,ab^3dn^2+2\,ab^3dn)x^3-3\,(a^2b^2dn^2+a^2b^2dn)x^2+(b^4cn^3+9\,b^4cn^2+24\,b^4c+2\,(13\,b^4c+3\,a^3bd)n)x)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d
*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n
^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 +
9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^
4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(83) = 166$.
time = 0.78, size = 1906, normalized size = 20.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c),x)
```

```
[Out] Piecewise((a**n*(c*x + d*x**4/4), Eq(b, 0)), (6*a**3*d*log(a/b + x)/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*log(a
/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*
a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**
2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(n, -4)), (-6*a**3*d*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*
x + 2*b**6*x**2) - 12*a**2*b*d*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2
*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*
b**2*d*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b*
*5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a*b**
4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4
+ 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a
/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*log(a/b + x)/b + d*x**
3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35
*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x
**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*
b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b*
*4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10
*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x
)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a
```

```
*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
 + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(
a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4)
 + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b*
*4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*
x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b*
*4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 5
0*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n*
*3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/
(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d
*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n +
 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(94) = 188.
time = 3.99, size = 361, normalized size = 3.84

$$\frac{(bx+a)^n b^4 d n^3 x^4 + (bx+a)^n a b^3 d n^3 x^3 + 6\,(bx+a)^n b^4 d n^2 x^4 + 3\,(bx+a)^n a b^3 d n^2 x^3 + 11\,(bx+a)^n b^4 d n x^4 + (bx+a)^n b^4 c n^3 x - 3\,(bx+a)^n a^2 b^2 d n^2 x^2 + 2\,(bx+a)^n a b^3 d n x^3 + 6\,(bx+a)^n b^4 d x^4 + (bx+a)^n a b^3 c n^3 + 9\,(bx+a)^n b^4 c n^2 x - 3\,(bx+a)^n a^2 b^2 d n x^2 + 9\,(bx+a)^n a b^3 c n^2 + 26\,(bx+a)^n b^4 c n x + 6\,(bx+a)^n a^3 b d n x + 26\,(bx+a)^n a b^3 c n + 24\,(bx+a)^n b^4 c x + 24\,(bx+a)^n a b^3 c - 6\,(bx+a)^n a^4 d}{b^4 n^4 + 10\,b^4 n^3 + 35\,b^4 n^2 + 50\,b^4 n + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^
4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 +
(b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a
*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x +
 a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n
^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*
a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n
*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

Mupad [B]
time = 2.95, size = 247, normalized size = 2.63

$$(a+b\,x)^n\,\left(\frac{x\,\left(6\,d\,a^3\,b\,n+c\,b^4\,n^3+9\,c\,b^4\,n^2+26\,c\,b^4\,n+24\,c\,b^4\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,\left(-6\,d\,a^3+c\,b^3\,n^3+9\,c\,b^3\,n^2+26\,c\,b^3\,n+24\,c\,b^3\right)}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{d\,x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{3\,a^2\,d\,n\,x^2\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,d\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b*
d*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d +
```

```
9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24
) - (3*a^2*d*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*
d*n*x^3*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))
```

3.177 $\int \frac{(a+bx)^n(c+dx^3)}{x} \, dx$

Optimal. Leaf size=99

$$\frac{a^2d(a+bx)^{1+n}}{b^3(1+n)}-\frac{2ad(a+bx)^{2+n}}{b^3(2+n)}+\frac{d(a+bx)^{3+n}}{b^3(3+n)}-\frac{c(a+bx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)}{a(1+n)}$$

```
[Out] a^2*d*(b*x+a)^(1+n)/b^3/(1+n)-2*a*d*(b*x+a)^(2+n)/b^3/(2+n)+d*(b*x+a)^(3+n)
/b^3/(3+n)-c*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1634, 67}

$$\frac{a^2d(a+bx)^{n+1}}{b^3(n+1)}-\frac{2ad(a+bx)^{n+2}}{b^3(n+2)}+\frac{d(a+bx)^{n+3}}{b^3(n+3)}-\frac{c(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] (a^2*d*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*d*(a + b*x)^(2 + n))/(b^3*(2
 + n)) + (d*(a + b*x)^(3 + n))/(b^3*(3 + n)) - (c*(a + b*x)^(1 + n)*Hyperge
ometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n (c+dx^3)}{x} dx = \int \left(\frac{a^2d(a+bx)^n}{b^2} + \frac{c(a+bx)^n}{x} - \frac{2ad(a+bx)^{1+n}}{b^2} + \frac{d(a+bx)^{2+n}}{b^2}\right) dx$$
$$= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)} - \frac{2ad(a+bx)^{2+n}}{b^3(2+n)} + \frac{d(a+bx)^{3+n}}{b^3(3+n)} + c\int \frac{(a+bx)^n}{x} dx$$
$$= \frac{a^2d(a+bx)^{1+n}}{b^3(1+n)} - \frac{2ad(a+bx)^{2+n}}{b^3(2+n)} + \frac{d(a+bx)^{3+n}}{b^3(3+n)} - \frac{c(a+bx)^{1+n}\,{}_2F_1(1,1+n}{a(1+n)}$$

Mathematica [A]
time = 0.08, size = 94, normalized size = 0.95

$$\frac{(a+bx)^{1+n}\left(ad(2a^2-2ab(1+n)x+b^2(2+3n+n^2)\,x^2)-b^3c(6+5n+n^2)\;{}_2F_1\left(1,1+n;2+n;1+\frac{bx}{a}\right)\right)}{ab^3(1+n)(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3))/x,x]
```

```
[Out] ((a + b*x)^(1 + n)*(a*d*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2)
 - b^3*c*(6 + 5*n + n^2)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a]))/
(a*b^3*(1 + n)*(2 + n)*(3 + n))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n\left(d\,x^3+c\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(83) = 166$.

time = 2.99, size = 741, normalized size = 7.48

$$-\frac{b^{n}cn\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma\left(n+2\right)}-\frac{b^{n}c\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{\Gamma\left(n+2\right)}+d\left(\begin{cases}\frac{a^{n}x^{3}}{3} & \text{for}\: b=0\\ \frac{2a^{2}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}}+\frac{3a^{2}}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}}+\frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}}+\frac{4abx}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}}+\frac{2b^{2}x^{2}\log\left(\frac{a}{b}+x\right)}{2a^{2}b^{3}+4ab^{4}x+2b^{5}x^{2}} & \text{for}\: n=-3\\ -\frac{2a^{2}\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x}-\frac{2a^{2}}{ab^{3}+b^{4}x}-\frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^{3}+b^{4}x}+\frac{b^{2}x^{2}}{ab^{3}+b^{4}x} & \text{for}\: n=-2\\ \frac{a^{2}\log\left(\frac{a}{b}+x\right)}{b^{3}}-\frac{ax}{b^{2}}+\frac{x^{2}}{2b} & \text{for}\: n=-1\\ \frac{2a^{3}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}-\frac{2a^{2}bnx(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}+\frac{ab^{2}n^{2}x^{2}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}+\frac{ab^{2}nx^{2}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}+\frac{b^{3}n^{2}x^{3}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}+\frac{3b^{3}nx^{3}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}}+\frac{2b^{3}x^{3}(a+bx)^{n}}{b^{3}n^{3}+6b^{3}n^{2}+11b^{3}n+6b^{3}} & \text{otherwise}\end{cases}\right)-\frac{bb^{n}cnx\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma\left(n+2\right)}-\frac{bb^{n}cx\left(\frac{a}{b}+x\right)^{n}\Phi\left(1+\frac{bx}{a},1,n+1\right)\Gamma(n+1)}{a\Gamma\left(n+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)/x,x)
```

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(n +
 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) + d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*
b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**
5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/
(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)
/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**
3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b
**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 +
6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**
3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3
*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*
n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) - b*b**n*c*n*x*(a/b + x)**
n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2)) - b*b**n*c*x*
(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*gamma(n + 2))
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x + a)^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x^3+c)\,\left(a+b\,x\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)*(a + b*x)^n)/x, x)
```

3.178 $\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=294

$$\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{\left(b^6c^2-20a^3b^3cd+28a^6d^2\right)(a+bx)^{3+n}}{b^9(3+n)}$$

```
[Out] a^2*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^9/(1+n)-2*a*(-4*a^3*d+b^3*c)*(-a^3*d+b
^3*c)*(b*x+a)^(2+n)/b^9/(2+n)+(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(
3+n)/b^9/(3+n)+4*a^2*d*(-14*a^3*d+5*b^3*c)*(b*x+a)^(4+n)/b^9/(4+n)-10*a*d*(
-7*a^3*d+b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+2*d*(-28*a^3*d+b^3*c)*(b*x+a)^(6+n)
/b^9/(6+n)+28*a^2*d^2*(b*x+a)^(7+n)/b^9/(7+n)-8*a*d^2*(b*x+a)^(8+n)/b^9/(8+
n)+d^2*(b*x+a)^(9+n)/b^9/(9+n)
```

Rubi [A]
time = 0.14, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1634}

$$-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^9(n+2)}-\frac{10ad\left(b^3c-7a^3d\right)(a+bx)^{n+5}}{b^9(n+5)}+\frac{2d\left(b^3c-28a^3d\right)(a+bx)^{n+6}}{b^9(n+6)}+\frac{28a^2d^2(a+bx)^{n+7}}{b^9(n+7)}+\frac{\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^9(n+3)}+\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{n+1}}{b^9(n+1)}+\frac{4a^2d\left(5b^3c-14a^3d\right)(a+bx)^{n+4}}{b^9(n+4)}-\frac{8ad^2(a+bx)^{n+8}}{b^9(n+8)}+\frac{d^2(a+bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a
^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3
*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c
 - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(
a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n)
)/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(
a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)^2\,dx=\int\left(\frac{\left(ab^3c-a^4d\right)^2(a+bx)^n}{b^8}-\frac{2\left(ab^6c^2-5a^4b^3cd+4a^7d^2\right)(a+bx)^{1+n}}{b^8}+\frac{(b^6\right.$$

$$=\frac{a^2\left(b^3c-a^3d\right)^2(a+bx)^{1+n}}{b^9(1+n)}-\frac{2a\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^9(2+n)}+\frac{(b^6c^2}{}$$

Mathematica [A]
time = 0.24, size = 252, normalized size = 0.86

$$\frac{(a+bx)^{1+n}\left(\frac{(ab^3c-a^4d)^2}{1+n}-\frac{2a(b^3c-4a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{(b^6c^2-20a^3b^3cd+28a^6d^2)(a+bx)^2}{3+n}+\frac{4a^2d(5b^3c-14a^3d)(a+bx)^3}{4+n}+\frac{10ad(-b^3c+7a^3d)(a+bx)^4}{5+n}+\frac{2d(b^3c-28a^3d)(a+bx)^5}{6+n}+\frac{28a^2d^2(a+bx)^6}{7+n}-\frac{8ad^2(a+bx)^7}{8+n}+\frac{d^2(a+bx)^8}{9+n}\right)}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b
^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)
*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n)
+ (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*
d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a +
b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n)))/b^9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. 2(294) = 588.
time = 0.25, size = 1565, normalized size = 5.32

method	result	size
gosper	Expression too large to display	1565
risch	Expression too large to display	1799

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7*x^7+546*b
^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+4536*b^8*d^2*n^5*x^8
+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8*c*d*n^7*x^5+22449*b^8
*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*
n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+
9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*
c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*d^2*n
^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660*a*b^7*c*d*n^5*
x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938*b^8*c*d*n^4*x^5+10958
4*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2
*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*
d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5
+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*
a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*
b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5
*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*
x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+7680
0*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b
```

```
^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2
*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c*d*n^
3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^6*c*d*n^2*x^3-1
1780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^8*c^2*n^3*x^2+120960*b
^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c
*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*
n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290
276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b
^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530*a^2*b^6*c
^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+301872*b^8*c^2*n*x^2-4
0320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3*c*d*n^2+166800*a^4*b^
4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x
+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*
d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*
d+120960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4
+723680*n^3+1172700*n^2+1026576*n+362880)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(294) = 588$.
time = 0.30, size = 601, normalized size = 2.04

$$\frac{\left(\left(n^{2}+3n+2\right)b^{3}x^{3}+\left(n^{2}+n\right)ab^{2}x^{2}-2a^{2}bnx+2a^{3}\right)(bx+a)^{n}c^{2}}{\left(n^{3}+6n^{2}+11n+6\right)b^{3}}+\frac{2\left(\left(n^{5}+15n^{4}+85n^{3}+225n^{2}+274n+120\right)b^{6}x^{6}+\left(n^{5}+10n^{4}+35n^{3}+50n^{2}+24n\right)ab^{5}x^{5}-5\left(n^{4}+6n^{3}+11n^{2}+6n\right)a^{2}b^{4}x^{4}+20\left(n^{3}+3n^{2}+2n\right)a^{3}b^{3}x^{3}-60\left(n^{2}+n\right)a^{4}b^{2}x^{2}+120a^{5}bnx-120a^{6}\right)(bx+a)^{n}cd}{\left(n^{6}+21n^{5}+175n^{4}+735n^{3}+1624n^{2}+1764n+720\right)b^{6}}+\frac{\left(\left(n^{8}+36n^{7}+546n^{6}+4536n^{5}+22449n^{4}+67284n^{3}+118124n^{2}+109584n+40320\right)b^{9}x^{9}+\left(n^{8}+28n^{7}+322n^{6}+1960n^{5}+6769n^{4}+13132n^{3}+13068n^{2}+5040n\right)ab^{8}x^{8}-8\left(n^{7}+21n^{6}+175n^{5}+735n^{4}+1624n^{3}+1764n^{2}+720n\right)a^{2}b^{7}x^{7}+56\left(n^{6}+15n^{5}+85n^{4}+225n^{3}+274n^{2}+120n\right)a^{3}b^{6}x^{6}-336\left(n^{5}+10n^{4}+35n^{3}+50n^{2}+24n\right)a^{4}b^{5}x^{5}+1680\left(n^{4}+6n^{3}+11n^{2}+6n\right)a^{5}b^{4}x^{4}-6720\left(n^{3}+3n^{2}+2n\right)a^{6}b^{3}x^{3}+20160\left(n^{2}+n\right)a^{7}b^{2}x^{2}-40320a^{8}bnx+40320a^{9}\right)(bx+a)^{n}d^{2}}{\left(n^{9}+45n^{8}+870n^{7}+9450n^{6}+63273n^{5}+269325n^{4}+723680n^{3}+1172700n^{2}+1026576n+362880\right)b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) +
 ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 +
 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4
+ 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7
35*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n
^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^3 +
50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*x^4
- 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - 4032
0*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^2/((n^9 + 45*n^8 + 870*n^7 + 9450*n^
6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 362880)
*b^9)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1565 vs. $2(294) = 588$.

time = 0.36, size = 1565, normalized size = 5.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3
*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n
^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2
*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*
d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*
a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*
n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 73
5*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b
^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^
9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*
b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 +
 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*
d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*
(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n
^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*
a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*
d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a
^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b
^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 +
 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 +
 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60
*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9
*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a
^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 2
80*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*
b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12
*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b
^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 84
0*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x
^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^
2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*
n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 -
 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n
)*x)*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273
*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*
n + 362880*b^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26746 vs. $2(275) = 550$.

time = 9.82, size = 26746, normalized size = 90.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**3/3 + c*d*x**6/3 + d**2*x**9/9), Eq(b, 0)), (840*a
**8*d**2*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11
*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x*
*5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 2283*a**
8*d**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a
**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2
*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 6720*a**7*b*d**2*x*log(
a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040
*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a*
*2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 17424*a**7*b*d**2*x/(
840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**1
2*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x
**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 23520*a**6*b**2*d**2*x**2*log(a
/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*
a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**
2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 57624*a**6*b**2*d**2*x
**2/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5
*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b*
*15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 10*a**5*b**3*c*d/(840*a**8
*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 67
20*a*b**16*x**7 + 840*b**17*x**8) + 47040*a**5*b**3*d**2*x**3*log(a/b + x)/
(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**
12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*
x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 107408*a**5*b**3*d**2*x**3/(84
0*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*
x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**
6 + 6720*a*b**16*x**7 + 840*b**17*x**8) - 80*a**4*b**4*c*d*x/(840*a**8*b**9
 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 5880
0*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*
b**16*x**7 + 840*b**17*x**8) + 58800*a**4*b**4*d**2*x**4*log(a/b + x)/(840*
a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x*
*3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6
+ 6720*a*b**16*x**7 + 840*b**17*x**8) + 122500*a**4*b**4*d**2*x**4/(840*a**
8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3
+ 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6
720*a*b**16*x**7 + 840*b**17*x**8) - 280*a**3*b**5*c*d*x**2/(840*a**8*b**9
+ 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800
```

```
*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b
**16*x**7 + 840*b**17*x**8) + 47040*a**3*b**5*d**2*x**5*log(a/b + x)/(840*a
**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**
3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 +
 6720*a*b**16*x**7 + 840*b**17*x**8) + 86240*a**3*b**5*d**2*x**5/(840*a**8*
b**9 + 6720*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 +
58800*a**4*b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 672
0*a*b**16*x**7 + 840*b**17*x**8) - 5*a**2*b**6*c**2/(840*a**8*b**9 + 6720*a
**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b*
*13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**
7 + 840*b**17*x**8) - 560*a**2*b**6*c*d*x**3/(840*a**8*b**9 + 6720*a**7*b**
10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**
4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840
*b**17*x**8) + 23520*a**2*b**6*d**2*x**6*log(a/b + x)/(840*a**8*b**9 + 6720
*a**7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*
b**13*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x
**7 + 840*b**17*x**8) + 35280*a**2*b**6*d**2*x**6/(840*a**8*b**9 + 6720*a**
7*b**10*x + 23520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**1
3*x**4 + 47040*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7
+ 840*b**17*x**8) - 40*a*b**7*c**2*x/(840*a**8*b**9 + 6720*a**7*b**10*x + 2
3520*a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 4704
0*a**3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x
**8) - 700*a*b**7*c*d*x**4/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*a**6*
b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**3*b**
14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8) + 672
0*a*b**7*d**2*x**7*log(a/b + x)/(840*a**8*b**9 + 6720*a**7*b**10*x + 23520*
a**6*b**11*x**2 + 47040*a**5*b**12*x**3 + 58800*a**4*b**13*x**4 + 47040*a**
3*b**14*x**5 + 23520*a**2*b**15*x**6 + 6720*a*b**16*x**7 + 840*b**17*x**8)
+ 6720*a*b**7*d**2*x**7/(840*a**8*b**9 + 6720*a...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2660 vs. $2(294) = 588$.
time = 4.66, size = 2660, normalized size = 9.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^9*d^2*n^8*x^9 + (b*x + a)^n*a*b^8*d^2*n^8*x^8 + 36*(b*x + a)
^n*b^9*d^2*n^7*x^9 + 28*(b*x + a)^n*a*b^8*d^2*n^7*x^8 + 546*(b*x + a)^n*b^9
*d^2*n^6*x^9 + 2*(b*x + a)^n*b^9*c*d*n^8*x^6 - 8*(b*x + a)^n*a^2*b^7*d^2*n^
7*x^7 + 322*(b*x + a)^n*a*b^8*d^2*n^6*x^8 + 4536*(b*x + a)^n*b^9*d^2*n^5*x^
9 + 2*(b*x + a)^n*a*b^8*c*d*n^8*x^5 + 78*(b*x + a)^n*b^9*c*d*n^7*x^6 - 168*
(b*x + a)^n*a^2*b^7*d^2*n^6*x^7 + 1960*(b*x + a)^n*a*b^8*d^2*n^5*x^8 + 2244
```

9*(b*x + a)^n*b^9*d^2*n^4*x^9 + 68*(b*x + a)^n*a*b^8*c*d*n^7*x^5 + 1272*(b*x + a)^n*b^9*c*d*n^6*x^6 + 56*(b*x + a)^n*a^3*b^6*d^2*n^6*x^6 - 1400*(b*x + a)^n*a^2*b^7*d^2*n^5*x^7 + 6769*(b*x + a)^n*a*b^8*d^2*n^4*x^8 + 67284*(b*x + a)^n*b^9*d^2*n^3*x^9 + (b*x + a)^n*b^9*c^2*n^8*x^3 - 10*(b*x + a)^n*a^2*b^7*c*d*n^7*x^4 + 932*(b*x + a)^n*a*b^8*c*d*n^6*x^5 + 11268*(b*x + a)^n*b^9*c*d*n^5*x^6 + 840*(b*x + a)^n*a^3*b^6*d^2*n^5*x^6 - 5880*(b*x + a)^n*a^2*b^7*d^2*n^4*x^7 + 13132*(b*x + a)^n*a*b^8*d^2*n^3*x^8 + 118124*(b*x + a)^n*b^9*d^2*n^2*x^9 + (b*x + a)^n*a*b^8*c^2*n^8*x^2 + 42*(b*x + a)^n*b^9*c^2*n^7*x^3 - 300*(b*x + a)^n*a^2*b^7*c*d*n^6*x^4 + 6608*(b*x + a)^n*a*b^8*c*d*n^5*x^5 - 336*(b*x + a)^n*a^4*b^5*d^2*n^5*x^5 + 58938*(b*x + a)^n*b^9*c*d*n^4*x^6 + 4760*(b*x + a)^n*a^3*b^6*d^2*n^4*x^6 - 12992*(b*x + a)^n*a^2*b^7*d^2*n^3*x^7 + 13068*(b*x + a)^n*a*b^8*d^2*n^2*x^8 + 109584*(b*x + a)^n*b^9*d^2*n*x^9 + 40*(b*x + a)^n*a*b^8*c^2*n^7*x^2 + 744*(b*x + a)^n*b^9*c^2*n^6*x^3 + 40*(b*x + a)^n*a^3*b^6*c*d*n^6*x^3 - 3460*(b*x + a)^n*a^2*b^7*c*d*n^5*x^4 + 25898*(b*x + a)^n*a*b^8*c*d*n^4*x^5 - 3360*(b*x + a)^n*a^4*b^5*d^2*n^4*x^5 + 185022*(b*x + a)^n*b^9*c*d*n^3*x^6 + 12600*(b*x + a)^n*a^3*b^6*d^2*n^3*x^6 - 14112*(b*x + a)^n*a^2*b^7*d^2*n^2*x^7 + 5040*(b*x + a)^n*a*b^8*d^2*n*x^8 + 40320*(b*x + a)^n*b^9*d^2*x^9 - 2*(b*x + a)^n*a^2*b^7*c^2*n^7*x + 664*(b*x + a)^n*a*b^8*c^2*n^6*x^2 + 7218*(b*x + a)^n*b^9*c^2*n^5*x^3 + 1080*(b*x + a)^n*a^3*b^6*c*d*n^5*x^3 - 19200*(b*x + a)^n*a^2*b^7*c*d*n^4*x^4 + 1680*(b*x + a)^n*a^5*b^4*d^2*n^4*x^4 + 55532*(b*x + a)^n*a*b^8*c*d*n^3*x^5 - 11760*(b*x + a)^n*a^4*b^5*d^2*n^3*x^5 + 337228*(b*x + a)^n*b^9*c*d*n^2*x^6 + 15344*(b*x + a)^n*a^3*b^6*d^2*n^2*x^6 - 5760*(b*x + a)^n*a^2*b^7*d^2*n*x^7 - 78*(b*x + a)^n*a^2*b^7*c^2*n^6*x + 5890*(b*x + a)^n*a*b^8*c^2*n^5*x^2 - 120*(b*x + a)^n*a^4*b^5*c*d*n^5*x^2 + 41619*(b*x + a)^n*b^9*c^2*n^4*x^3 + 10600*(b*x + a)^n*a^3*b^6*c*d*n^4*x^3 - 52690*(b*x + a)^n*a^2*b^7*c*d*n^3*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n^3*x^4 + 59568*(b*x + a)^n*a*b^8*c*d*n^2*x^5 - 16800*(b*x + a)^n*a^4*b^5*d^2*n^2*x^5 + 322032*(b*x + a)^n*b^9*c*d*n*x^6 + 6720*(b*x + a)^n*a^3*b^6*d^2*n*x^6 + 2*(b*x + a)^n*a^3*b^6*c^2*n^6 - 1250*(b*x + a)^n*a^2*b^7*c^2*n^5*x + 29839*(b*x + a)^n*a*b^8*c^2*n^4*x^2 - 3000*(b*x + a)^n*a^4*b^5*c*d*n^4*x^2 + 144468*(b*x + a)^n*b^9*c^2*n^3*x^3 + 45000*(b*x + a)^n*a^3*b^6*c*d*n^3*x^3 - 6720*(b*x + a)^n*a^6*b^3*d^2*n^3*x^3 - 66900*(b*x + a)^n*a^2*b^7*c*d*n^2*x^4 + 18480*(b*x + a)^n*a^5*b^4*d^2*n^2*x^4 + 24192*(b*x + a)^n*a*b^8*c*d*n*x^5 - 8064*(b*x + a)^n*a^4*b^5*d^2*n*x^5 + 120960*(b*x + a)^n*b^9*c*d*x^6 + 78*(b*x + a)^n*a^3*b^6*c^2*n^5 - 10530*(b*x + a)^n*a^2*b^7*c^2*n^4*x + 240*(b*x + a)^n*a^5*b^4*c*d*n^4*x + 84790*(b*x + a)^n*a*b^8*c^2*n^3*x^2 - 25800*(b*x + a)^n*a^4*b^5*c*d*n^3*x^2 + 290276*(b*x + a)^n*b^9*c^2*n^2*x^3 + 75760*(b*x + a)^n*a^3*b^6*c*d*n^2*x^3 - 20160*(b*x + a)^n*a^6*b^3*d^2*n^2*x^3 - 30240*(b*x + a)^n*a^2*b^7*c*d*n*x^4 + 10080*(b*x + a)^n*a^5*b^4*d^2*n*x^4 + 1250*(b*x + a)^n*a^3*b^6*c^2*n^4 - 49148*(b*x + a)^n*a^2*b^7*c^2*n^3*x + 5760*(b*x + a)^n*a^5*b^4*c*d*n^3*x + 120696*(b*x + a)^n*a*b^8*c^2*n^2*x^2 - 83400*(b*x + a)^n*a^4*b^5*c*d*n^2*x^2 + 20160*(b*x + a)^n*a^7*b^2*d^2*n^2*x^2 + 301872*(b*x + a)^n*b^9*c^2*n*x^3 + 40320*(b*x + a)^n*a^3*b^6*c*d*n*x^3 - 13440*(b*x + a)^n*a^6*b^3*d^2*n*x^3 + 10530*(b*x + a)^n*a^3*b^6*c^2*n^3 - 240*(b*x + a)^n*a^6*b^3*c*d*

n^3 - 120432*(b*x + a)^n*a^2*b^7*c^2*n^2*x + 45840*(b*x + a)^n*a^5*b^4*c*d*
n^2*x + 60480*(b*x + a)^n*a*b^8*c^2*n*x^2 - 60480*(b*x + a)^n*a^4*b^5*c*d*n
x^2 + 20160(b*x + a)^n*a^7*b^2*d^2*n*x^2 + 120960*(b*x + a)^n*b^9*c^2*x^3
 + 49148*(b*x + a)^n*a^3*b^6*c^2*n^2 - 5760*(b*x + a)^n*a^6*b^3*c*d*n^2 - 1
20960*(b*x + a)^n*a^2*b^7*c^2*n*x + 120960*(b*x + a)^n*a^5*b^4*c*d*n*x - 40
320*(b*x + a)^n*a^8*b*d^2*n*x + 120432*(b*x + a)^n*a^3*b^6*c^2*n - 45840*(b
*x + a)^n*a^6*b^3*c*d*n + 120960*(b*x + a)^n*a^3*b^6*c^2 - 120960*(b*x + a)
^n*a^6*b^3*c*d + 40320*(b*x + a)^n*a^9*d^2)/(b^9*n^9 + 45*b^9*n^8 + 870*b^9
*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 117
2700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)

Mupad [B]
time = 3.73, size = 1410, normalized size = 4.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*
n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(1026576*n + 1172700*n^2 + 723680*n^
3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880) +
(2*a^3*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 60216*b^6*c^2*n + 24574
*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*c^
2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^9*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4
 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (x^3*(a + b*x
)^n*(3*n + n^2 + 2)*(60480*b^6*c^2 - 6720*a^6*d^2*n + 60216*b^6*c^2*n + 245
74*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b^6*c^2*n^5 + b^6*
c^2*n^6 + 20160*a^3*b^3*c*d*n + 7640*a^3*b^3*c*d*n^2 + 960*a^3*b^3*c*d*n^3
+ 40*a^3*b^3*c*d*n^4))/(b^6*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*
n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*d*x^6*(
a + b*x)^n*(504*b^3*c + 24*b^3*c*n^2 + b^3*c*n^3 + 28*a^3*d*n + 191*b^3*c*n
)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^3*(1026576*n + 117270
0*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 +
 n^9 + 362880)) - (2*a^2*n*x*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6
0216*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 3
9*b^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 288
0*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(1026576*n + 1172700*n^2 + 7
23680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36
2880)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(20160*a^6*d^2 + 60480*b^6*c^2 + 6021
6*b^6*c^2*n + 24574*b^6*c^2*n^2 + 5265*b^6*c^2*n^3 + 625*b^6*c^2*n^4 + 39*b
^6*c^2*n^5 + b^6*c^2*n^6 - 60480*a^3*b^3*c*d - 22920*a^3*b^3*c*d*n - 2880*a
^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1026576*n + 1172700*n^2 + 7236
80*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 36288
```

```
0)) + (a*d^2*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 +
 322*n^5 + 28*n^6 + n^7 + 5040))/(b*(1026576*n + 1172700*n^2 + 723680*n^3 +
 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (8
*a^2*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6 + 720))/(b^2*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 632
73*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) - (10*a^2*d*n*x^4*(a
+ b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(504*b^3*c - 168*a^3*d + 24*b^3*c*n^2 + b
^3*c*n^3 + 191*b^3*c*n))/(b^5*(1026576*n + 1172700*n^2 + 723680*n^3 + 26932
5*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) + (2*a*d*n
*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(504*b^3*c - 168*a^3*d
 + 24*b^3*c*n^2 + b^3*c*n^3 + 191*b^3*c*n))/(b^4*(1026576*n + 1172700*n^2 +
 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 +
362880))
```

3.179 $\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=248

$$-\frac{a(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^8(2+n)}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^{3+n}}{b^8(3+n)}-\frac{ad(8b^3}{}$$

```
[Out] -a*(-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^8/(1+n)+(-7*a^3*d+b^3*c)*(-a^3*d+b^3*c)
*(b*x+a)^(2+n)/b^8/(2+n)+3*a^2*d*(-7*a^3*d+4*b^3*c)*(b*x+a)^(3+n)/b^8/(3+n)
-a*d*(-35*a^3*d+8*b^3*c)*(b*x+a)^(4+n)/b^8/(4+n)+d*(-35*a^3*d+2*b^3*c)*(b*x
+a)^(5+n)/b^8/(5+n)+21*a^2*d^2*(b*x+a)^(6+n)/b^8/(6+n)-7*a*d^2*(b*x+a)^(7+n
)/b^8/(7+n)+d^2*(b*x+a)^(8+n)/b^8/(8+n)
```

Rubi [A]
time = 0.11, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1634}

$$-\frac{a(b^3c-a^3d)^2 (a+bx)^{n+1}}{b^8(n+1)}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{n+2}}{b^8(n+2)}-\frac{ad(8b^3c-35a^3d)(a+bx)^{n+4}}{b^8(n+4)}+\frac{d(2b^3c-35a^3d)(a+bx)^{n+5}}{b^8(n+5)}+\frac{21a^2d^2(a+bx)^{n+6}}{b^8(n+6)}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^{n+3}}{b^8(n+3)}-\frac{7ad^2(a+bx)^{n+7}}{b^8(n+7)}+\frac{d^2(a+bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*
d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7
*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b
*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^
8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b
*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n \left(c+dx^3\right)^2 \, dx = \int \left(-\frac{a(-b^3c+a^3d)^2 (a+bx)^n}{b^7}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{1+n}}{b^7}-3a\right.$$

$$= -\frac{a(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^8(1+n)}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)^{2+n}}{b^8(2+n)}+\frac{3a^2d(4}{}$$

Mathematica [A]
time = 0.19, size = 211, normalized size = 0.85

$$\frac{(a+bx)^{1+n}\left(-\frac{a(b^3c-a^3d)^2}{1+n}+\frac{(b^3c-7a^3d)(b^3c-a^3d)(a+bx)}{2+n}+\frac{3a^2d(4b^3c-7a^3d)(a+bx)^2}{3+n}+\frac{ad(-8b^3c+35a^3d)(a+bx)^3}{4+n}+\frac{d(2b^3c-35a^3d)(a+bx)^4}{5+n}+\frac{21a^2d^2(a+bx)^5}{6+n}-\frac{7ad^2(a+bx)^6}{7+n}+\frac{d^2(a+bx)^7}{8+n}\right)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-((a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(
b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^
2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c
- 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*
d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n)))/b^8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(248) = 496.
time = 0.25, size = 893, normalized size = 3.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(8+n)*x^8*exp(n*ln(b*x+a))+1/b^7*n*a*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^
6*c^2*n^4-48*a^3*b^3*c*d*n^3+3135*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^
6*c^2*n^2-7008*a^3*b^3*c*d*n+24552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d
+20160*b^6*c^2)/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2
+109584*n+40320)*x*exp(n*ln(b*x+a))+d^2*a/b*n/(n^2+15*n+56)*x^7*exp(n*ln(b*
x+a))-a^2*(b^6*c^2*n^6+33*b^6*c^2*n^5+445*b^6*c^2*n^4-48*a^3*b^3*c*d*n^3+31
35*b^6*c^2*n^3-1008*a^3*b^3*c*d*n^2+12154*b^6*c^2*n^2-7008*a^3*b^3*c*d*n+24
552*b^6*c^2*n+5040*a^6*d^2-16128*a^3*b^3*c*d+20160*b^6*c^2)/b^8/(n^8+36*n^7
+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)*exp(n*ln(b
*x+a))-(-b^6*c^2*n^6-33*b^6*c^2*n^5-24*a^3*b^3*c*d*n^4-445*b^6*c^2*n^4-504*
a^3*b^3*c*d*n^3-3135*b^6*c^2*n^3-3504*a^3*b^3*c*d*n^2-12154*b^6*c^2*n^2+252
0*a^6*d^2*n-8064*a^3*b^3*c*d*n-24552*b^6*c^2*n-20160*b^6*c^2)/b^6/(n^7+35*n
^6+511*n^5+4025*n^4+18424*n^3+48860*n^2+69264*n+40320)*x^2*exp(n*ln(b*x+a))
+2*d*(b^3*c*n^3+21*b^3*c*n^2+21*a^3*d*n+146*b^3*c*n+336*b^3*c)/b^3/(n^4+26*
n^3+251*n^2+1066*n+1680)*x^5*exp(n*ln(b*x+a))-7*n*a^2*d^2/b^2/(n^3+21*n^2+1
46*n+336)*x^6*exp(n*ln(b*x+a))-2*n*a*d*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n
+105*a^3*d-336*b^3*c)/b^4/(n^5+30*n^4+355*n^3+2070*n^2+5944*n+6720)*x^4*exp
(n*ln(b*x+a))+8*(-b^3*c*n^3-21*b^3*c*n^2-146*b^3*c*n+105*a^3*d-336*b^3*c)*a
^2/b^5*d*n/(n^6+33*n^5+445*n^4+3135*n^3+12154*n^2+24552*n+20160)*x^3*exp(n*
ln(b*x+a))
```

Maxima [A]
time = 0.30, size = 474, normalized size = 1.91

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^n c^2}{(n^2+3n+2)b^2}+\frac{2\left((n^4+10n^3+35n^2+50n+24)b^5x^5+(n^4+6n^3+11n^2+6n)ab^4x^4-4(n^3+3n^2+2n)a^2b^3x^3+12(n^2+n)a^3b^2x^2-24a^4bnx+24a^5\right)(bx+a)^n cd}{(n^5+15n^4+85n^3+225n^2+274n+120)b^5}+\frac{\left((n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^8x^8+(n^7+21n^6+175n^5+735n^4+1624n^3+1764n^2+720n)ab^7x^7-7(n^6+15n^5+85n^4+225n^3+274n^2+120n)a^2b^6x^6+42(n^5+10n^4+35n^3+50n^2+24n)a^3b^5x^5-210(n^4+6n^3+11n^2+6n)a^4b^4x^4+840(n^3+3n^2+2n)a^5b^3x^3-2520(n^2+n)a^6b^2x^2+5040a^7bnx-5040a^8\right)(bx+a)^n d^2}{(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320)b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2
+ 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 1313
2*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 27
4*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3
*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2
+ 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8
)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n
^3 + 118124*n^2 + 109584*n + 40320)*b^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(248) = 496$.
time = 0.36, size = 1216, normalized size = 4.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^
6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 +
322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 +
 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 +
175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2
*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*
a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d
^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d +
 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d
 + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*
b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283
*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d
- 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16
*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*
n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(27
2*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*
n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 50
4*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239
*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (1
5289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d
```

```
- 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^
2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33
*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^
4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4
*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x)*(b
*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^
4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 18328 vs. $2(228) = 456$.
time = 6.59, size = 18328, normalized size = 73.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x**2/2 + 2*c*d*x**5/5 + d**2*x**8/8), Eq(b, 0)), (420
*a**7*d**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10
*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**
5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1089*a**7*d**2/(420*a**7*b**8 + 2
940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3
*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) +
2940*a**6*b*d**2*x*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 7203*a**6*b*d**2*x/(420*a
**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3
+ 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b*
*15*x**7) + 8820*a**5*b**2*d**2*x**2*log(a/b + x)/(420*a**7*b**8 + 2940*a**
6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*
x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 20139*a
**5*b**2*d**2*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2
 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2
940*a*b**14*x**6 + 420*b**15*x**7) - 8*a**4*b**3*c*d/(420*a**7*b**8 + 2940*
a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**
12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 1470
0*a**4*b**3*d**2*x**3*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820
*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**
2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 30625*a**4*b**3*d**2*x
**3/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b
**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**
6 + 420*b**15*x**7) - 56*a**3*b**4*c*d*x/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 14700*a**3*b**4*
d**2*x**4*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*
```

```
x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5
 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 26950*a**3*b**4*d**2*x**4/(420*a**
7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 +
14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**1
5*x**7) - 168*a**2*b**5*c*d*x**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a
**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*
b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 8820*a**2*b**5*d**2*x**5
*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14
700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a
*b**14*x**6 + 420*b**15*x**7) + 13230*a**2*b**5*d**2*x**5/(420*a**7*b**8 +
2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**
3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) -
 10*a*b**6*c**2/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 +
14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940
*a*b**14*x**6 + 420*b**15*x**7) - 280*a*b**6*c*d*x**3/(420*a**7*b**8 + 2940
*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b*
*12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 294
0*a*b**6*d**2*x**6*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a*
*5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b
**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) + 2940*a*b**6*d**2*x**6/(42
0*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x*
*3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420
*b**15*x**7) - 70*b**7*c**2*x/(420*a**7*b**8 + 2940*a**6*b**9*x + 8820*a**5
*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 8820*a**2*b**
13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7) - 280*b**7*c*d*x**4/(420*a**7
*b**8 + 2940*a**6*b**9*x + 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 1
4700*a**3*b**12*x**4 + 8820*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15
*x**7) + 420*b**7*d**2*x**7*log(a/b + x)/(420*a**7*b**8 + 2940*a**6*b**9*x
+ 8820*a**5*b**10*x**2 + 14700*a**4*b**11*x**3 + 14700*a**3*b**12*x**4 + 88
20*a**2*b**13*x**5 + 2940*a*b**14*x**6 + 420*b**15*x**7), Eq(n, -8)), (-420
*a**7*d**2*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x*
*2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**
14*x**6) - 1029*a**7*d**2/(60*a**6*b**8 + 360*a**5*b**9*x + 900*a**4*b**10*
x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b
**14*x**6) - 2520*a**6*b*d**2*x*log(a/b + x)/(60*a**6*b**8 + 360*a**5*b**9*
x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4 + 360*
a*b**13*x**5 + 60*b**14*x**6) - 5754*a**6*b*d**2*x/(60*a**6*b**8 + 360*a**5
*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a**2*b**12*x**4
+ 360*a*b**13*x**5 + 60*b**14*x**6) - 6300*a**5...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2034 vs. $2(248) = 496$.

time = 3.76, size = 2034, normalized size = 8.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")

[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x + a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(b*x + a)^n*a^2*b^6*c*d*n^5*x^3 + 2898*(b*x + a)^n*a*b^7*c*d*n^4*x^4 - 210*(b*x + a)^n*a^4*b^4*d^2*n^4*x^4 + 19088*(b*x + a)^n*b^8*c*d*n^3*x^5 + 1470*(b*x + a)^n*a^3*b^5*d^2*n^3*x^5 - 1918*(b*x + a)^n*a^2*b^6*d^2*n^2*x^6 + 720*(b*x + a)^n*a*b^7*d^2*n*x^7 + 5040*(b*x + a)^n*b^8*d^2*x^8 + 33*(b*x + a)^n*a*b^7*c^2*n^6*x + 478*(b*x + a)^n*b^8*c^2*n^5*x^2 + 24*(b*x + a)^n*a^3*b^5*c*d*n^5*x^2 - 1688*(b*x + a)^n*a^2*b^6*c*d*n^4*x^3 + 7496*(b*x + a)^n*a*b^7*c*d*n^3*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n^3*x^4 + 39128*(b*x + a)^n*b^8*c*d*n^2*x^5 + 2100*(b*x + a)^n*a^3*b^5*d^2*n^2*x^5 - 840*(b*x + a)^n*a^2*b^6*d^2*n*x^6 - (b*x + a)^n*a^2*b^6*c^2*n^6 + 445*(b*x + a)^n*a*b^7*c^2*n^5*x + 3580*(b*x + a)^n*b^8*c^2*n^4*x^2 + 528*(b*x + a)^n*a^3*b^5*c*d*n^4*x^2 - 6528*(b*x + a)^n*a^2*b^6*c*d*n^3*x^3 + 840*(b*x + a)^n*a^5*b^3*d^2*n^3*x^3 + 9144*(b*x + a)^n*a*b^7*c*d*n^2*x^4 - 2310*(b*x + a)^n*a^4*b^4*d^2*n^2*x^4 + 40608*(b*x + a)^n*b^8*c*d*n*x^5 + 1008*(b*x + a)^n*a^3*b^5*d^2*n*x^5 - 33*(b*x + a)^n*a^2*b^6*c^2*n^5 + 3135*(b*x + a)^n*a*b^7*c^2*n^4*x - 48*(b*x + a)^n*a^4*b^4*c*d*n^4*x + 15289*(b*x + a)^n*b^8*c^2*n^3*x^2 + 4008*(b*x + a)^n*a^3*b^5*c*d*n^3*x^2 - 10400*(b*x + a)^n*a^2*b^6*c*d*n^2*x^3 + 2520*(b*x + a)^n*a^5*b^3*d^2*n^2*x^3 + 4032*(b*x + a)^n*a*b^7*c*d*n*x^4 - 1260*(b*x + a)^n*a^4*b^4*d^2*n*x^4 + 16128*(b*x + a)^n*b^8*c*d*x^5 - 445*(b*x + a)^n*a^2*b^6*c^2*n^4 + 12154*(b*x + a)^n*a*b^7*c^2*n^3*x - 1008*(b*x + a)^n*a^4*b^4*c*d*n^3*x + 36706*(b*x + a)^n*b^8*c^2*n^2*x^2 + 11568*(b*x + a)^n*a^3*b^5*c*d*n^2*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n^2*x^2 - 5376*(b*x + a)^n*a^2*b^6*c*d*n*x^3 + 1680*(b*x + a)^n*a^5*b^3*d^2*n*x^3 - 3135*(b*x + a)^n*a^2*b^6*c^2*n^3 + 48*(b*x + a)^n*a^5*b^3*c*d*n^3 + 24552*(b*x + a)^n*a*b^7*c^2*n^2*x - 7008*(b*x + a)^n*a^4*b^4*c*d*n^2*x + 44712*(b*x + a)^n*b^8*c^2*n*x^2 + 8064*(b*x + a)^n*a^3*b^5*c*d*n*x^2 - 2520*(b*x + a)^n*a^6*b^2*d^2*n*x^2 - 12154*(b*x + a)^n*a^2*b^6*c^2*n^2 + 1008*(b*x + a)^n*a^5*b^3*c*d*n^2 + 20160*(b*x + a)^n*a*b^7*c^2*n*x - 16128*(b*x + a)^n*a^4*b^4*c*d*n*x +

```
5040*(b*x + a)^n*a^7*b*d^2*n*x + 20160*(b*x + a)^n*b^8*c^2*x^2 - 24552*(b*x
 + a)^n*a^2*b^6*c^2*n + 7008*(b*x + a)^n*a^5*b^3*c*d*n - 20160*(b*x + a)^n*
a^2*b^6*c^2 + 16128*(b*x + a)^n*a^5*b^3*c*d - 5040*(b*x + a)^n*a^8*d^2)/(b^
8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8
*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

Mupad [B]
time = 3.39, size = 1136, normalized size = 4.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 +
 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 453
6*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2 +
20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 44
5*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3
*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584*n + 1
18124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 403
20)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 24552*b^6
*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^
2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 + 504*a^3*b
^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 +
 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)
^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 31
35*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3
*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))
/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^
4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*n + 146*b^3*c*n))/
(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 +
36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 + 67284*n^3
+ 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (7*a^2*d^2*n*x^
6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584
*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8
 + 40320)) + (2*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(336*b^3*c -
 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^4*(109584*n + 1181
24*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)
) - (8*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(336*b^3*c - 105*a^3*d + 21*
b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^5*(109584*n + 118124*n^2 + 67284*n
^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))
```

3.180 $\int (a+bx)^n \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=203

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^7(2+n)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{3+n}}{b^7(3+n)}+\frac{2d\left(b^3c-10a^3d\right)(a+}{b^7(4+n)}$$

```
[Out] (-a^3*d+b^3*c)^2*(b*x+a)^(1+n)/b^7/(1+n)+6*a^2*d*(-a^3*d+b^3*c)*(b*x+a)^(2+
n)/b^7/(2+n)-3*a*d*(-5*a^3*d+2*b^3*c)*(b*x+a)^(3+n)/b^7/(3+n)+2*d*(-10*a^3*
d+b^3*c)*(b*x+a)^(4+n)/b^7/(4+n)+15*a^2*d^2*(b*x+a)^(5+n)/b^7/(5+n)-6*a*d^2
*(b*x+a)^(6+n)/b^7/(6+n)+d^2*(b*x+a)^(7+n)/b^7/(7+n)
```

Rubi [A]
time = 0.08, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1864}

$$\frac{\left(b^3c-a^3d\right)^2 (a+bx)^{n+1}}{b^7(n+1)}-\frac{3ad\left(2b^3c-5a^3d\right)(a+bx)^{n+3}}{b^7(n+3)}+\frac{2d\left(b^3c-10a^3d\right)(a+bx)^{n+4}}{b^7(n+4)}+\frac{15a^2d^2(a+bx)^{n+5}}{b^7(n+5)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{n+2}}{b^7(n+2)}-\frac{6ad^2(a+bx)^{n+6}}{b^7(n+6)}+\frac{d^2(a+bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3
*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)
^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(
4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)
^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^2 \, dx = \int \left(\frac{\left(b^3c-a^3d\right)^2 (a+bx)^n}{b^6}-\frac{6a^2d\left(-b^3c+a^3d\right)(a+bx)^{1+n}}{b^6}+\frac{3ad\left(-2b^3c+5a\right.}{b^6}\right.$$

$$= \frac{\left(b^3c-a^3d\right)^2 (a+bx)^{1+n}}{b^7(1+n)}+\frac{6a^2d\left(b^3c-a^3d\right)(a+bx)^{2+n}}{b^7(2+n)}-\frac{3ad\left(2b^3c-5a^3d\right)(a}{b^7(3+n)}$$

Mathematica [A]
time = 0.18, size = 297, normalized size = 1.46

$$\frac{(a+bx)^{1+n}\left(720a^6d^2-720a^5bd^2(1+n)x+360a^4b^2d^2\left(2+3n+n^2\right)x^2-6ab^5d\left(10+17n+8n^2+n^3\right)x^2\left(c\left(42+13n+n^2\right)+d\left(12+7n+n^2\right)x^3\right)-12a^3b^3d\left(c\left(210+107n+18n^2+n^3\right)+10d\left(6+11n+6n^2+n^3\right)x^3\right)+6a^2b^4d(1+n)x\left(2c\left(210+107n+18n^2+n^3\right)+5d\left(24+26n+9n^2+n^3\right)x^3\right)+b^6\left(180+216n+91n^2+16n^3+n^4\right)\left(c^2\left(28+11n+n^2\right)+2cd\left(7+8n+n^2\right)x^3+d^2\left(4+5n+n^2\right)x^6\right)\right)}{b^7(1+n)(2+n)(3+n)(4+n)(5+n)(6+n)(7+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] ((a + b*x)^(1 + n)*(720*a^6*d^2 - 720*a^5*b*d^2*(1 + n)*x + 360*a^4*b^2*d^2
*(2 + 3*n + n^2)*x^2 - 6*a*b^5*d*(10 + 17*n + 8*n^2 + n^3)*x^2*(c*(42 + 13*
n + n^2) + d*(12 + 7*n + n^2)*x^3) - 12*a^3*b^3*d*(c*(210 + 107*n + 18*n^2
+ n^3) + 10*d*(6 + 11*n + 6*n^2 + n^3)*x^3) + 6*a^2*b^4*d*(1 + n)*x*(2*c*(2
10 + 107*n + 18*n^2 + n^3) + 5*d*(24 + 26*n + 9*n^2 + n^3)*x^3) + b^6*(180
+ 216*n + 91*n^2 + 16*n^3 + n^4)*(c^2*(28 + 11*n + n^2) + 2*c*d*(7 + 8*n +
n^2)*x^3 + d^2*(4 + 5*n + n^2)*x^6)))/(b^7*(1 + n)*(2 + n)*(3 + n)*(4 + n)*
(5 + n)*(6 + n)*(7 + n))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(203) = 406$.
time = 0.25, size = 700, normalized size = 3.45 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(7+n)*x^7*exp(n*ln(b*x+a))+a*(b^6*c^2*n^6+27*b^6*c^2*n^5+295*b^6*c^2*n^
4-12*a^3*b^3*c*d*n^3+1665*b^6*c^2*n^3-216*a^3*b^3*c*d*n^2+5104*b^6*c^2*n^2-
1284*a^3*b^3*c*d*n+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+5040*b^6*c^2
)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)*exp(n*l
n(b*x+a))+d^2*a*n/b/(n^2+13*n+42)*x^6*exp(n*ln(b*x+a))-(-b^6*c^2*n^6-27*b^6
*c^2*n^5-12*a^3*b^3*c*d*n^4-295*b^6*c^2*n^4-216*a^3*b^3*c*d*n^3-1665*b^6*c^
2*n^3-1284*a^3*b^3*c*d*n^2-5104*b^6*c^2*n^2+720*a^6*d^2*n-2520*a^3*b^3*c*d*
n-8028*b^6*c^2*n-5040*b^6*c^2)/b^6/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13
132*n^2+13068*n+5040)*x*exp(n*ln(b*x+a))+2*(b^3*c*n^3+18*b^3*c*n^2+15*a^3*d
*n+107*b^3*c*n+210*b^3*c)*d/b^3/(n^4+22*n^3+179*n^2+638*n+840)*x^4*exp(n*ln
(b*x+a))-6*n*a^2*d^2/b^2/(n^3+18*n^2+107*n+210)*x^5*exp(n*ln(b*x+a))-2*n*d*
a*(-b^3*c*n^3-18*b^3*c*n^2-107*b^3*c*n+60*a^3*d-210*b^3*c)/b^4/(n^5+25*n^4+
245*n^3+1175*n^2+2754*n+2520)*x^3*exp(n*ln(b*x+a))+6*(-b^3*c*n^3-18*b^3*c*n
^2-107*b^3*c*n+60*a^3*d-210*b^3*c)*d*a^2/b^5*n/(n^6+27*n^5+295*n^4+1665*n^3
+5104*n^2+8028*n+5040)*x^2*exp(n*ln(b*x+a))
```

Maxima [**A**]
time = 0.30, size = 359, normalized size = 1.77

$$\frac{(bx+a)^{n+1}c^2}{b(n+1)} + \frac{2\left((n^3+6n^2+11n+6)b^4x^4 + (n^3+3n^2+2n)ab^3x^3 - 3(n^2+n)a^2b^2x^2 + 6a^3bnx - 6a^4\right)(bx+a)^n cd}{(n^4+10n^3+35n^2+50n+24)b^4} + \frac{\left((n^6+21n^5+175n^4+735n^3+1624n^2+1764n+720)b^7x^7 + (n^6+15n^5+85n^4+225n^3+274n^2+120n)ab^6x^6 - 6(n^5+10n^4+35n^3+50n^2+24n)a^2b^5x^5 + 30(n^4+6n^3+11n^2+6n)a^3b^4x^4 - 120(n^3+3n^2+2n)a^4b^3x^3 + 360(n^2+n)a^5b^2x^2 - 720a^6bnx + 720a^7\right)(bx+a)^n d^2}{(n^7+28n^6+322n^5+1960n^4+6769n^3+13132n^2+13068n+5040)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 21*n
```

```
^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 +
 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 +
 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 -
 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*
b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769
*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(203) = 406$.
time = 0.37, size = 893, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
 + 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x)*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11851 vs. $2(187) = 374$.
time = 3.79, size = 11851, normalized size = 58.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d*
*2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200
*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6)
+ 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*
a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x
**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**7 +
 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**
11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*x**2/(60*
a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 9
00*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a**3*b**3*c*d/(6
0*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d
**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4
*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5
+ 60*b**13*x**6) - 12*a**2*b**4*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900
*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*
x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x)/(60*a**6*b**7
+ 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b*
*11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b**4*d**2*x**4/(60
*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +
900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 30*a*b**5*c*d*x**
2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x*
*3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a*b**5*d
**2*x**5*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2
+ 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*
x**6) + 360*a*b**5*d**2*x**5/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**
9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60
*b**13*x**6) - 10*b**6*c**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9
*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*
b**13*x**6) - 40*b**6*c*d*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b
**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 +
60*b**13*x**6) + 60*b**6*d**2*x**6*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b*
*8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 36
0*a*b**12*x**5 + 60*b**13*x**6), Eq(n, -7)), (-60*a**6*d**2*log(a/b + x)/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) - 137*a**6*d**2/(10*a**5*b**7 + 50*a**4*b**
8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12
*x**5) - 300*a**5*b*d**2*x*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 10
0*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) -
```

```
 625*a**5*b*d**2*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 10
0*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 600*a**4*b**2*d**2*x
**2*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*
a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 1100*a**4*b**2*d**2*x*
*2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**
3 + 50*a*b**11*x**4 + 10*b**12*x**5) - a**3*b**3*c*d/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 600*a**3*b**3*d**2*x**3*log(a/b + x)/(10*a**5*b**7 + 50*a**4*
b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b*
*12*x**5) - 900*a**3*b**3*d**2*x**3/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a*
*3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 5*a
**2*b**4*c*d*x/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a*
*2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4*
log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2
*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*x**5) - 300*a**2*b**4*d**2*x**4/(1
0*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 5
0*a*b**11*x**4 + 10*b**12*x**5) - 10*a*b**5*c*d*x**2/(10*a**5*b**7 + 50*a**
4*b**8*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*
b**12*x**5) - 60*a*b**5*d**2*x**5*log(a/b + x)/(10*a**5*b**7 + 50*a**4*b**8
*x + 100*a**3*b**9*x**2 + 100*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12*
x**5) - 2*b**6*c**2/(10*a**5*b**7 + 50*a**4*b**8*x + 100*a**3*b**9*x**2 + 1
00*a**2*b**10*x**3 + 50*a*b**11*x**4 + 10*b**12...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1477 vs. $2(203) = 406$.
time = 4.12, size = 1477, normalized size = 7.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
 120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
```

```
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
 a)^n*a^3*b^4*d^2*n*x^4 + 295*(b*x + a)^n*a*b^6*c^2*n^4 + 1665*(b*x + a)^n*
b^7*c^2*n^3*x + 216*(b*x + a)^n*a^3*b^4*c*d*n^3*x - 1902*(b*x + a)^n*a^2*b^
5*c*d*n^2*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n^2*x^2 + 840*(b*x + a)^n*a*b^6
*c*d*n*x^3 - 240*(b*x + a)^n*a^4*b^3*d^2*n*x^3 + 2520*(b*x + a)^n*b^7*c*d*x
^4 + 1665*(b*x + a)^n*a*b^6*c^2*n^3 - 12*(b*x + a)^n*a^4*b^3*c*d*n^3 + 5104
*(b*x + a)^n*b^7*c^2*n^2*x + 1284*(b*x + a)^n*a^3*b^4*c*d*n^2*x - 1260*(b*x
 + a)^n*a^2*b^5*c*d*n*x^2 + 360*(b*x + a)^n*a^5*b^2*d^2*n*x^2 + 5104*(b*x +
 a)^n*a*b^6*c^2*n^2 - 216*(b*x + a)^n*a^4*b^3*c*d*n^2 + 8028*(b*x + a)^n*b^
7*c^2*n*x + 2520*(b*x + a)^n*a^3*b^4*c*d*n*x - 720*(b*x + a)^n*a^6*b*d^2*n*
x + 8028*(b*x + a)^n*a*b^6*c^2*n - 1284*(b*x + a)^n*a^4*b^3*c*d*n + 5040*(b
*x + a)^n*b^7*c^2*x + 5040*(b*x + a)^n*a*b^6*c^2 - 2520*(b*x + a)^n*a^4*b^3
*c*d + 720*(b*x + a)^n*a^7*d^2)/(b^7*n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*
b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^2 + 13068*b^7*n + 5040*b^7)
```

Mupad [B]
time = 3.19, size = 878, normalized size = 4.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^2*(a + b*x)^n,x)
```

```
[Out] (a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2*
n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 - 2
520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*c*d
*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 +
 n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4
 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c^2*n +
5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*n^5 + b^
7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*d*n^2 + 2
16*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*
n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a + b*x)^n*(11
*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 15*a^3*d*n +
107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 2
8*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 +
 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(50*n + 35*n^2 +
```

```
10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n
^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(210*
b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^4*(13068*n +
 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2
*d*n*x^2*(n + 1)*(a + b*x)^n*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n
^3 + 107*b^3*c*n))/(b^5*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^
5 + 28*n^6 + n^7 + 5040))
```

3.181 $\int \frac{(a+bx)^n(c+dx^3)^2}{x}\,dx$

Optimal. Leaf size=209

$$\frac{a^2d(2b^3c-a^3d)\,(a+bx)^{1+n}}{b^6(1+n)}-\frac{ad(4b^3c-5a^3d)\,(a+bx)^{2+n}}{b^6(2+n)}+\frac{2d(b^3c-5a^3d)\,(a+bx)^{3+n}}{b^6(3+n)}+\frac{10a^2d^2(a+bx)^{4+}}{b^6(4+n)}$$

```
[Out] a^2*d*(-a^3*d+2*b^3*c)*(b*x+a)^(1+n)/b^6/(1+n)-a*d*(-5*a^3*d+4*b^3*c)*(b*x+
a)^(2+n)/b^6/(2+n)+2*d*(-5*a^3*d+b^3*c)*(b*x+a)^(3+n)/b^6/(3+n)+10*a^2*d^2*
(b*x+a)^(4+n)/b^6/(4+n)-5*a*d^2*(b*x+a)^(5+n)/b^6/(5+n)+d^2*(b*x+a)^(6+n)/b
^6/(6+n)-c^2*(b*x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1634, 67}

$$-\frac{ad(4b^3c-5a^3d)\,(a+bx)^{n+2}}{b^6(n+2)}+\frac{2d(b^3c-5a^3d)\,(a+bx)^{n+3}}{b^6(n+3)}+\frac{10a^2d^2(a+bx)^{n+4}}{b^6(n+4)}+\frac{a^2d(2b^3c-a^3d)\,(a+bx)^{n+1}}{b^6(n+1)}-\frac{5ad^2(a+bx)^{n+5}}{b^6(n+5)}+\frac{d^2(a+bx)^{n+6}}{b^6(n+6)}-\frac{c^2(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a^2*d*(2*b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*d*(4*b^3*c -
 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*
x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^(4 + n))/(b^6*(4 + n)) -
(5*a*d^2*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d^2*(a + b*x)^(6 + n))/(b^6*(6
 + n)) - (c^2*(a + b*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b*x
)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{(a+bx)^n (c+dx^3)^2}{x} dx = \int \left(-\frac{a^2d(-2b^3c+a^3d)(a+bx)^n}{b^5} + \frac{c^2(a+bx)^n}{x} + \frac{ad(-4b^3c+5a^3d)(a+bx)^1}{b^5}\right.$$

$$= \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(}{b^6(3+}$$

$$= \frac{a^2d(2b^3c-a^3d)(a+bx)^{1+n}}{b^6(1+n)} - \frac{ad(4b^3c-5a^3d)(a+bx)^{2+n}}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(}{b^6(3+}$$

Mathematica [A]
time = 0.16, size = 188, normalized size = 0.90

$$(a+bx)^{1+n}\left(\frac{a^2d(2b^3c-a^3d)}{b^6(1+n)} + \frac{ad(-4b^3c+5a^3d)(a+bx)}{b^6(2+n)} + \frac{2d(b^3c-5a^3d)(a+bx)^2}{b^6(3+n)} + \frac{10a^2d^2(a+bx)^3}{b^6(4+n)} - \frac{5ad^2(a+bx)^4}{b^6(5+n)} + \frac{d^2(a+bx)^5}{b^6(6+n)} - \frac{c^2\,{}_2F_1\left(1,1+n;2+n;\frac{a+bx}{a}\right)}{a+an}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^2)/x,x]
```

```
[Out] (a + b*x)^(1 + n)*((a^2*d*(2*b^3*c - a^3*d))/(b^6*(1 + n)) + (a*d*(-4*b^3*c
 + 5*a^3*d)*(a + b*x))/(b^6*(2 + n)) + (2*d*(b^3*c - 5*a^3*d)*(a + b*x)^2)/
(b^6*(3 + n)) + (10*a^2*d^2*(a + b*x)^3)/(b^6*(4 + n)) - (5*a*d^2*(a + b*x)
^4)/(b^6*(5 + n)) + (d^2*(a + b*x)^5)/(b^6*(6 + n)) - (c^2*Hypergeometric2F
1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n (d\,x^3+c)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

```
[Out] int((b*x+a)^n*(d*x^3+c)^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4007 vs. $2(187) = 374$.
time = 4.82, size = 4760, normalized size = 22.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**2/x,x)
```

```
[Out] -b**n*c**2*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**2*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 2*c*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x
)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4
*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5
*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log
(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*lo
g(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b +
 x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(
a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b*
*3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n
/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**
n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + d**2*Piecewise
((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b*
*7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1
1*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 6
00*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x*log(a/
b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9
*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60*a**5*b**6 + 30
0*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
+ 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4
*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b
```

```
**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3
*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a
**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*
x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b*
*3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**
9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b + x)/(
60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 +
300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5*b**6 + 300*a**
4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*
b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5),
 Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**
2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12*a**4*b**6 + 48
*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a*
*4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48
*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*x**2
*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**
9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3
*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**
9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3*b**7*x
 + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x**4*log
(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x*
*3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2
*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*log(a/b
+ x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a*
*5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4
*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x
**3) + 270*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9
*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18
*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a**2*b**
7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)/(6*a**
3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*x**4/(6
*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5*x**5/(
6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n, -4)), (
-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90*a**5/(
6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)/(6*a**
2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12*a*b**7
*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7
*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6*a**2*b...
```

<u>Giac</u> [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2*(b*x + a)^n/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x^3+c)^2\,(a+b\,x)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)^2*(a + b*x)^n)/x,x)
```

```
[Out] int(((c + d*x^3)^2*(a + b*x)^n)/x, x)
```

3.182 $\int x^2(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=459

$$\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{12}(1+n)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{12}(2+n)}+\frac{\left(b^3c-a^3d\right)\left(b^6c^2-29a^3b^3cd+55a^6d\right.}{b^{12}(3+n)}$$

```
[Out] a^2*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^12/(1+n)-a*(-11*a^3*d+2*b^3*c)*(-a^3*d
+b^3*c)^2*(b*x+a)^(2+n)/b^12/(2+n)+(-a^3*d+b^3*c)*(55*a^6*d^2-29*a^3*b^3*c*
d+b^6*c^2)*(b*x+a)^(3+n)/b^12/(3+n)+3*a^2*d*(55*a^6*d^2-56*a^3*b^3*c*d+10*b
^6*c^2)*(b*x+a)^(4+n)/b^12/(4+n)-15*a*d*(22*a^6*d^2-14*a^3*b^3*c*d+b^6*c^2)
*(b*x+a)^(5+n)/b^12/(5+n)+3*d*(154*a^6*d^2-56*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^
(6+n)/b^12/(6+n)+42*a^2*d^2*(-11*a^3*d+2*b^3*c)*(b*x+a)^(7+n)/b^12/(7+n)-6*
a*d^2*(-55*a^3*d+4*b^3*c)*(b*x+a)^(8+n)/b^12/(8+n)+3*d^2*(-55*a^3*d+b^3*c)*
(b*x+a)^(9+n)/b^12/(9+n)+55*a^2*d^3*(b*x+a)^(10+n)/b^12/(10+n)-11*a*d^3*(b*
x+a)^(11+n)/b^12/(11+n)+d^3*(b*x+a)^(12+n)/b^12/(12+n)
```

Rubi [A]
time = 0.22, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1634}

$$-\frac{6ad^2\left(4b^3c-55a^3d\right)(a+bx)^{n+8}}{b^{12}(n+8)}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^{n+9}}{b^{12}(n+9)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{12}(n+2)}+\frac{55a^2d^3(a+bx)^{n+10}}{b^{12}(n+10)}+\frac{\left(b^3c-a^3d\right)\left(55a^6d^2-29a^3b^3cd+b^6c^2\right)(a+bx)^{n+3}}{b^{12}(n+3)}-\frac{15ad\left(22a^6d^2-14a^3b^3cd+b^6c^2\right)(a+bx)^{n+5}}{b^{12}(n+5)}+\frac{3d\left(154a^6d^2-56a^3b^3cd+b^6c^2\right)(a+bx)^{n+6}}{b^{12}(n+6)}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^{n+7}}{b^{12}(n+7)}+\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{12}(n+1)}+\frac{3a^2d\left(55a^6d^2-56a^3b^3cd+10b^6c^2\right)(a+bx)^{n+4}}{b^{12}(n+4)}-\frac{11ad^3(a+bx)^{n+11}}{b^{12}(n+11)}+\frac{d^3(a+bx)^{n+12}}{b^{12}(n+12)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11
*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3
*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n
)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))
/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)
^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a
 + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x
)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n)
)/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 +
n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)
^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int x^2(a+bx)^n\left(c+dx^3\right)^3\,dx=\int\left(-\frac{a^2\left(-b^3c+a^3d\right)^3(a+bx)^n}{b^{11}}+\frac{a\left(-b^3c+a^3d\right)^2\left(-2b^3c+11a^3d\right)(a+b$$

$$=\frac{a^2\left(b^3c-a^3d\right)^3(a+bx)^{1+n}}{b^{12}(1+n)}-\frac{a\left(2b^3c-11a^3d\right)\left(b^3c-a^3d\right)^2(a+bx)^{2+n}}{b^{12}(2+n)}+\frac{(b$$

Mathematica [A]
time = 0.37, size = 402, normalized size = 0.88

$$\frac{(a+bx)^{1+n}\left(\frac{a^2\left(b^3c-a^3d\right)^3}{1+n}+\frac{a\left(b^3c-a^3d\right)^2\left(-2b^3c+11a^3d\right)(a+bx)}{2+n}+\frac{\left(b^3c-a^3d\right)\left(b^6c^2-29a^3b^3cd+55a^6d^2\right)(a+bx)^2}{3+n}+\frac{3a^2d\left(10b^6c^2-56a^3b^3cd+55a^6d^2\right)(a+bx)^3}{4+n}-\frac{15ad\left(b^6c^2-14a^3b^3cd+22a^6d^2\right)(a+bx)^4}{5+n}+\frac{3d\left(b^6c^2-56a^3b^3cd+154a^6d^2\right)(a+bx)^5}{6+n}+\frac{42a^2d^2\left(2b^3c-11a^3d\right)(a+bx)^6}{7+n}+\frac{6ad^2\left(-4b^3c+55a^3d\right)(a+bx)^7}{8+n}+\frac{3d^2\left(b^3c-55a^3d\right)(a+bx)^8}{9+n}+\frac{55a^2d^3(a+bx)^9}{10+n}-\frac{11ad^3(a+bx)^{10}}{11+n}+\frac{d^3(a+bx)^{11}}{12+n}\right)}{b^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*
(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a
^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b
^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d
+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a +
 b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3
*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10
 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n)))/b^
12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3779 vs. 2(459) = 918.
time = 0.29, size = 3780, normalized size = 8.24

method	result	size
gosper	Expression too large to display	3780
risch	Expression too large to display	4231

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x+a)^(1+n)*(-b^11*d^3*n^11*x^11-66*b^11*d^3*n^10*x^11+11*a*b^10*d^3*n^1
0*x^10-1925*b^11*d^3*n^9*x^11+605*a*b^10*d^3*n^9*x^10-3*b^11*c*d^2*n^11*x^8
-32670*b^11*d^3*n^8*x^11-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^10*d^3*n^8*x^10-
207*b^11*c*d^2*n^10*x^8-357423*b^11*d^3*n^7*x^11-4950*a^2*b^9*d^3*n^8*x^9+2
```

4*a*b^10*c*d^2*n^10*x^7+199650*a*b^10*d^3*n^7*x^10-6288*b^11*c*d^2*n^9*x^8-
2637558*b^11*d^3*n^6*x^11+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9
+1464*a*b^10*c*d^2*n^9*x^7+1735503*a*b^10*d^3*n^6*x^10-3*b^11*c^2*d*n^11*x^
5-110718*b^11*c*d^2*n^8*x^8-13339535*b^11*d^3*n^5*x^11+35640*a^3*b^8*d^3*n^
7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*a*b^10*c*
d^2*n^8*x^7+9922605*a*b^10*d^3*n^5*x^10-216*b^11*c^2*d*n^10*x^5-1251927*b^1
1*c*d^2*n^7*x^8-45995730*b^11*d^3*n^4*x^11-7920*a^4*b^7*d^3*n^7*x^7+540540*
a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+
15*a*b^10*c^2*d*n^10*x^4+577008*a*b^10*c*d^2*n^7*x^7+37586230*a*b^10*d^3*n^
4*x^10-6855*b^11*c^2*d*n^9*x^5-9512559*b^11*c*d^2*n^6*x^8-105258076*b^11*d^
3*n^3*x^11-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4490640*a^
3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^3*n^4*x^9
+1005*a*b^10*c^2*d*n^9*x^4+5399352*a*b^10*c*d^2*n^6*x^7+92504500*a*b^10*d^3
*n^3*x^10-b^11*c^3*n^11*x^2-126180*b^11*c^2*d*n^8*x^5-49357662*b^11*c*d^2*n
^5*x^8-150917976*b^11*d^3*n^2*x^11+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^
7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8-60*a
^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^9*d^3*n^3
*x^9+29250*a*b^10*c^2*d*n^8*x^4+32905656*a*b^10*c*d^2*n^5*x^7+140289336*a*b
^10*d^3*n^2*x^10-75*b^11*c^3*n^10*x^2-1491309*b^11*c^2*d*n^7*x^5-173991492*
b^11*c*d^2*n^4*x^8-120543840*b^11*d^3*n*x^11+1164240*a^5*b^6*d^3*n^5*x^6-50
40*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*
n^6*x^5+66611160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^
2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^10*c^3*n^10*x+48465
0*a*b^10*c^2*d*n^7*x^4+131616048*a*b^10*c*d^2*n^4*x^7+116915040*a*b^10*d^3*
n*x^10-2492*b^11*c^3*n^9*x^2-11832048*b^11*c^2*d*n^6*x^5-405697080*b^11*c*d
^2*n^3*x^8-39916800*b^11*d^3*x^11-332640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^
6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180
*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*
n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-1129233
60*a^2*b^9*d^3*n*x^9+146*a*b^10*c^3*n^9*x+5033295*a*b^10*c^2*d*n^6*x^4+3390
03552*a*b^10*c*d^2*n^3*x^7+39916800*a*b^10*d^3*x^10-48294*b^11*c^3*n^8*x^2-
63978405*b^11*c^2*d*n^5*x^5-590770944*b^11*c*d^2*n^2*x^8-4989600*a^6*b^5*d^
3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*
a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7
*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c
^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x^6-399168
00*a^2*b^9*d^3*x^9+4692*a*b^10*c^3*n^8*x+33993765*a*b^10*c^2*d*n^5*x^4+5335
48224*a*b^10*c*d^2*n^2*x^7-604581*b^11*c^3*n^7*x^2-234340020*b^11*c^2*d*n^4
*x^5-477740160*b^11*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^
5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360
*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n
*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+39916800*
a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3-47140934
4*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^10*c^3*n^7*x+149923200*a*b^10*c^2*d*n^4*x
^4+457781760*a*b^10*c*d^2*n*x^7-5112891*b^11*c^3*n^6*x^2-565580388*b^11*c^2

```
*d*n^3*x^5-159667200*b^11*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*
b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x
^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^2+4033491
84*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2*d*n^4*x^
3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100*a*b^10*c
^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-848562336
*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^
4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*
d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a^4*b^
7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2*d*n^4*x^
2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^2*b^9*c^2
*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+717481440*a
*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-3
9916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3
*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*n^4*x-376
427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1153 vs. $2(459) = 918$.
time = 0.31, size = 1153, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
 + 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 -
40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9
450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3
62880)*b^9) + ((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755
8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054
```

```
3840*n + 39916800)*b^12*x^12 + (n^11 + 55*n^10 + 1320*n^9 + 18150*n^8 + 157
773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640*
n^2 + 3628800*n)*a*b^11*x^11 - 11*(n^10 + 45*n^9 + 870*n^8 + 9450*n^7 + 632
73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^
2*b^10*x^10 + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^
4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32
2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 +
 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5*
b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6
*x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200
*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9
*b^3*x^3 - 19958400*(n^2 + n)*a^10*b^2*x^2 + 39916800*a^11*b*n*x - 39916800
*a^12)*(b*x + a)^n*d^3/((n^12 + 78*n^11 + 2717*n^10 + 55770*n^9 + 749463*n^
8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888
*n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^12)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3564 vs. $2(459) = 918$.
time = 0.40, size = 3564, normalized size = 7.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^
3*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
)*n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
 20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
 288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
```

608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*
b^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
 - a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^
12*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
 + 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^
3*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3
*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
 - 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
 96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940

```
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75191 vs. $2(439) = 878$.
time = 87.44, size = 75191, normalized size = 163.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x**3/3 + c**2*d*x**6/2 + c*d**2*x**9/3 + d**3*x**12/1
2), Eq(b, 0)), (27720*a**11*d**3*log(a/b + x)/(27720*a**11*b**12 + 304920*a
**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*
a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 914
7600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 +
304920*a*b**22*x**10 + 27720*b**23*x**11) + 83711*a**11*d**3/(27720*a**11*b
**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*
x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b
**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**
2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 304920*a**10*b*d
**3*x*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9
*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*
a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573
800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 2772
0*b**23*x**11) + 893101*a**10*b*d**3*x/(27720*a**11*b**12 + 304920*a**10*b*
*13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b*
*16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a*
*4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*
a*b**22*x**10 + 27720*b**23*x**11) + 1524600*a**9*b**2*d**3*x**2*log(a/b +
x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 45
73800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5
+ 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x
**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +
 4313045*a**9*b**2*d**3*x**2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 15
24600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 +
 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x
**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x*
*10 + 27720*b**23*x**11) - 168*a**8*b**3*c*d**2/(27720*a**11*b**12 + 304920
*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 914760
0*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9
147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9
+ 304920*a*b**22*x**10 + 27720*b**23*x**11) + 4573800*a**8*b**3*d**3*x**3*l
og(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*
```

```
x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b*
*17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**
3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23
*x**11) + 12430935*a**8*b**3*d**3*x**3/(27720*a**11*b**12 + 304920*a**10*b*
*13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b*
*16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a*
*4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*
a*b**22*x**10 + 27720*b**23*x**11) - 1848*a**7*b**4*c*d**2*x/(27720*a**11*b
**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*
x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b
**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**
2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) + 9147600*a**7*b**
4*d**3*x**4*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10*b**13*x + 152460
0*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 128
06640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7
+ 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10
+ 27720*b**23*x**11) + 23718420*a**7*b**4*d**3*x**4/(27720*a**11*b**12 + 30
4920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 91
47600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6
 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x
**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) - 9240*a**6*b**5*c*d**2*x**
2/(27720*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 457
3800*a**8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 +
 12806640*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x*
*8 + 1524600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 27720*b**23*x**11) +
12806640*a**6*b**5*d**3*x**5*log(a/b + x)/(27720*a**11*b**12 + 304920*a**10
*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**8*b**15*x**3 + 9147600*a**7
*b**16*x**4 + 12806640*a**6*b**17*x**5 + 12806640*a**5*b**18*x**6 + 9147600
*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 1524600*a**2*b**21*x**9 + 3049
20*a*b**22*x**10 + 27720*b**23*x**11) + 31376268*a**6*b**5*d**3*x**5/(27720
*a**11*b**12 + 304920*a**10*b**13*x + 1524600*a**9*b**14*x**2 + 4573800*a**
8*b**15*x**3 + 9147600*a**7*b**16*x**4 + 12806640*a**6*b**17*x**5 + 1280664
0*a**5*b**18*x**6 + 9147600*a**4*b**19*x**7 + 4573800*a**3*b**20*x**8 + 152
4600*a**2*b**21*x**9 + 304920*a*b**22*x**10 + 2...
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error:
Bad Argument Value
```

Mupad [B]
time = 7.14, size = 2500, normalized size = 5.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3
*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 47
6049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 +
b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a
^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 19
95840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n
^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d
*n^6))/(b^12*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^
4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2
717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^3*x^12*(a + b*x)^n*(120543840*
n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n
^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800))/(14864
42880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 +
 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11
+ n^12 + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(79833600*b^9*c^3 +
6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b
^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 +
 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 39916800*a^3*b^6*c^2*d*n
 - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2 - 7297920*a^6*b^3*
c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b^3*c*d^2*n^3 + 1008900*
a^3*b^6*c^2*d*n^4 - 20160*a^6*b^3*c*d^2*n^4 + 80700*a^3*b^6*c^2*d*n^5 + 342
0*a^3*b^6*c^2*d*n^6 + 60*a^3*b^6*c^2*d*n^7))/(b^9*(1486442880*n + 193155955
2*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 692
6634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)
) + (3*d*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)*(6
65280*b^6*c^2 - 18480*a^6*d^2*n + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 1
6815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 + 73920*
a^3*b^3*c*d*n + 20272*a^3*b^3*c*d*n^2 + 1848*a^3*b^3*c*d*n^3 + 56*a^3*b^3*c
*d*n^4))/(b^6*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n
^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 +
2717*n^10 + 78*n^11 + n^12 + 479001600)) - (2*a^2*n*x*(a + b*x)^n*(79833600
*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 +
17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9
*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*
b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^
6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 30
26700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^
```

4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6))/(b^11*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (d^2*x^9*(a + b*x)^n*(3960*b^3*c + 99*b^3*c*n^2 + 3*b^3*c*n^3 + 110*a^3*d*n + 1086*b^3*c*n)*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^3*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (a*d^3*n*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800))/(b*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) - (11*a^2*d^3*n*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b^2*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6))/(b^10*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^12 + 479001600)) + (3*a*d*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(110880*a^6*d^2 + 665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c...

3.183 $\int x(a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=396

$$-\frac{a(b^3c-a^3d)^3\,(a+bx)^{1+n}}{b^{11}(1+n)}+\frac{(b^3c-10a^3d)\,(b^3c-a^3d)^2\,(a+bx)^{2+n}}{b^{11}(2+n)}+\frac{9a^2d(2b^3c-5a^3d)\,(b^3c-a^3d)\,(a+bx)^{3+n}}{b^{11}(3+n)}$$

[Out] -a*(-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^11/(1+n)+(-10*a^3*d+b^3*c)*(-a^3*d+b^3*c)^2*(b*x+a)^(2+n)/b^11/(2+n)+9*a^2*d*(-5*a^3*d+2*b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^11/(3+n)-3*a*d*(40*a^6*d^2-35*a^3*b^3*c*d+4*b^6*c^2)*(b*x+a)^(4+n)/b^11/(4+n)+3*d*(70*a^6*d^2-35*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(5+n)/b^11/(5+n)+63*a^2*d^2*(-4*a^3*d+b^3*c)*(b*x+a)^(6+n)/b^11/(6+n)-21*a*d^2*(-10*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^11/(7+n)+3*d^2*(-40*a^3*d+b^3*c)*(b*x+a)^(8+n)/b^11/(8+n)+45*a^2*d^3*(b*x+a)^(9+n)/b^11/(9+n)-10*a*d^3*(b*x+a)^(10+n)/b^11/(10+n)+d^3*(b*x+a)^(11+n)/b^11/(11+n)

Rubi [A]
time = 0.19, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1634}

$$-\frac{21ad^2(b^3c-10a^3d)\,(a+bx)^{n+7}}{b^{11}(n+7)}+\frac{3d^2(b^3c-40a^3d)\,(a+bx)^{n+8}}{b^{11}(n+8)}-\frac{a(b^3c-a^3d)^3\,(a+bx)^{n+1}}{b^{11}(n+1)}+\frac{(b^3c-10a^3d)\,(b^3c-a^3d)^2\,(a+bx)^{n+2}}{b^{11}(n+2)}+\frac{45a^2d^3(a+bx)^{n+9}}{b^{11}(n+9)}-\frac{3ad(40a^6d^2-35a^3b^3cd+4b^6c^2)\,(a+bx)^{n+4}}{b^{11}(n+4)}+\frac{3d(70a^6d^2-35a^3b^3cd+b^6c^2)\,(a+bx)^{n+5}}{b^{11}(n+5)}+\frac{63a^2d^2(b^3c-4a^3d)\,(a+bx)^{n+6}}{b^{11}(n+6)}+\frac{9a^2d(2b^3c-5a^3d)\,(b^3c-a^3d)\,(a+bx)^{n+3}}{b^{11}(n+3)}-\frac{10ad^3(a+bx)^{n+10}}{b^{11}(n+10)}+\frac{d^3(a+bx)^{n+11}}{b^{11}(n+11)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 + n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\int x(a+bx)^n \left(c+dx^3\right)^3 dx = \int \left(\frac{a(-b^3c+a^3d)^3 (a+bx)^n}{b^{10}} + \frac{(b^3c-10a^3d)(b^3c-a^3d)^2 (a+bx)^{1+n}}{b^{10}} + 9a\right.$$

$$= -\frac{a(b^3c-a^3d)^3 (a+bx)^{1+n}}{b^{11}(1+n)} + \frac{(b^3c-10a^3d)(b^3c-a^3d)^2 (a+bx)^{2+n}}{b^{11}(2+n)} + \frac{9a^2c}{}$$

Mathematica [A]
time = 0.32, size = 345, normalized size = 0.87

$$\frac{(a+bx)^{1+n}\left(\frac{a(-b^3c+a^3d)^3}{1+n}+\frac{(b^3c-10a^3d)(b^3c-a^3d)^2(a+bx)}{2+n}+\frac{9a^2d(-b^3c+a^3d)(-2b^3c+5a^3d)(a+bx)^2}{3+n}-\frac{3ad(4b^6c^2-35a^3b^3cd+40a^6d^2)(a+bx)^3}{4+n}+\frac{3d(b^6c^2-35a^3b^3cd+70a^6d^2)(a+bx)^4}{5+n}+\frac{63a^2d^2(b^3c-4a^3d)(a+bx)^5}{6+n}+\frac{21ad^2(-b^3c+10a^3d)(a+bx)^6}{7+n}+\frac{3d^2(b^3c-40a^3d)(a+bx)^7}{8+n}+\frac{45a^2d^3(a+bx)^8}{9+n}-\frac{10ad^3(a+bx)^9}{10+n}+\frac{d^3(a+bx)^{10}}{11+n}\right)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*
(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*
c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40
*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^
2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c -
40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d
^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n)))/b^11
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2971 vs. $2(396) = 792$.
time = 0.28, size = 2972, normalized size = 7.51

method	result	size
gosper	Expression too large to display	2972
risch	Expression too large to display	3409

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (b*x+a)^(1+n)*(b^10*d^3*n^10*x^10+55*b^10*d^3*n^9*x^10-10*a*b^9*d^3*n^9*x^9
+1320*b^10*d^3*n^8*x^10-450*a*b^9*d^3*n^8*x^9+3*b^10*c*d^2*n^10*x^7+18150*b
^10*d^3*n^7*x^10+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^10*c*d
^2*n^9*x^7+157773*b^10*d^3*n^6*x^10+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2
*n^9*x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^10*c*d^2*n^8*x^7+902055*b^10*d^3*n^
5*x^10-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n
^8*x^6-632730*a*b^9*d^3*n^5*x^9+3*b^10*c^2*d*n^10*x^4+62946*b^10*c*d^2*n^7*
x^7+3416930*b^10*d^3*n^4*x^10-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n
```

^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d
^3*n^4*x^9+183*b^10*c^2*d*n^9*x^4+568701*b^10*c*d^2*n^6*x^7+8409500*b^10*d^
3*n^3*x^10+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8
*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*
b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^10*c^2*d*n^8*x^4+3363066
*b^10*c*d^2*n^5*x^7+12753576*b^10*d^3*n^2*x^10+105840*a^4*b^6*d^3*n^5*x^6-6
30*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n
^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c*
d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7
*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3
*n^5*x^5+882000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3
*b^7*d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+106
31160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4
*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+3
3074574*b^10*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5
+2520*a^4*b^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^
2*n^5*x^4-9455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^
2*b^8*c*d^2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^
2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^
3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b
^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184
960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^
4-9408960*a^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c*
d^2*n^3*x^5+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^
5*x^3-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d
*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5
*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+88
90560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3
*x^4-3628800*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8*
c*d^2*n^2*x^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9
*c*d^2*n*x^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1
0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a
^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400
*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31
701600*a^3*b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8
*c*d^2*n*x^5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^
9*c*d^2*x^6+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7*
b^3*d^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960
*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710
680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2
*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b
^9*c^3*n^5-61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^1
0*c^2*d*n*x^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720*
a^6*b^4*c*d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+
77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d*

```
n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b
^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10
*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^
3*d^3*x^3+4974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a
^4*b^6*c^2*d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53
565408*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+5
0312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600
*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+53
65728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x
^2-19978308*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880
*a^7*b^3*c*d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080
*a^3*b^7*c^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3-
14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)/b^11/(n^1
1+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558...
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(396) = 792$.
time = 0.30, size = 953, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
```

```
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2919 vs. $2(396) = 792$.
time = 0.43, size = 2919, normalized size = 7.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^
2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^11*
d^3 - (b^11*d^3*n^10 + 55*b^11*d^3*n^9 + 1320*b^11*d^3*n^8 + 18150*b^11*d^3
*n^7 + 157773*b^11*d^3*n^6 + 902055*b^11*d^3*n^5 + 3416930*b^11*d^3*n^4 + 8
409500*b^11*d^3*n^3 + 12753576*b^11*d^3*n^2 + 10628640*b^11*d^3*n + 3628800
*b^11*d^3)*x^11 - (a*b^10*d^3*n^10 + 45*a*b^10*d^3*n^9 + 870*a*b^10*d^3*n^8
 + 9450*a*b^10*d^3*n^7 + 63273*a*b^10*d^3*n^6 + 269325*a*b^10*d^3*n^5 + 723
680*a*b^10*d^3*n^4 + 1172700*a*b^10*d^3*n^3 + 1026576*a*b^10*d^3*n^2 + 3628
80*a*b^10*d^3*n)*x^10 + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*
b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*
d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d
^3*n)*x^9 - 3*(b^11*c*d^2*n^10 + 58*b^11*c*d^2*n^9 + 4989600*b^11*c*d^2 + 3
*(487*b^11*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^11*c*d^2 + 140*a^3*b^8*d
^3)*n^7 + 21*(9027*b^11*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^11*c*d^2
 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^11*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2
*(5512429*b^11*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^11*c*d^2 + 10
890*a^3*b^8*d^3)*n^2 + 360*(40123*b^11*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*
(a*b^10*c*d^2*n^10 + 51*a*b^10*c*d^2*n^9 + 1104*a*b^10*c*d^2*n^8 + 6*(2209*
a*b^10*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^10*c*d^2 - 240*a^4*b^7*d^
3)*n^6 + 21*(21119*a*b^10*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^10*
c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^10*c*d^2 - 32480*a^4*b^7*d^
3)*n^3 + 360*(5449*a*b^10*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^10*
c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6
 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 3
0*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a
^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(7706
9*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^
5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*
a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^11*c^2*d*n^10 + 61*b^11*c^2*d*
n^9 + 7983360*b^11*c^2*d + 6*(270*b^11*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(
```

```
117*b^11*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^11*c^2*d + 8876*a^3*b^8*
c*d^2)*n^6 + 3*(488231*b^11*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)
*n^5 + 2*(3008035*b^11*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^
4 + 20*(795769*b^11*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 +
 72*(356683*b^11*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*
(78167*b^11*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(19634
3*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^10*c^2*d*n^10 + 57*a*b^10*
c^2*d*n^9 + 1392*a*b^10*c^2*d*n^8 + 6*(3167*a*b^10*c^2*d - 35*a^4*b^7*c*d^2
)*n^7 + 15*(10571*a*b^10*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^10*
c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^10*c^2*d - 327600*a^4*b^7
*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
 + 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
 9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56151 vs. $2(374) = 748$.
time = 37.09, size = 56151, normalized size = 141.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)

[Out] Piecewise((a**n*(c**3*x**2/2 + 3*c**2*d*x**5/5 + 3*c*d**2*x**8/8 + d**3*x**
11/11), Eq(b, 0)), (2520*a**10*d**3*log(a/b + x)/(2520*a**10*b**11 + 25200*
a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**
6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**
3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**
10) + 7381*a**10*d**3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*
b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*
b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*
b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 25200*a**9*b*d**3*x*l
og(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2
 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5
 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8
 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 71290*a**9*b*d**3*x/(2520*a**10
*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x*
*3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x*
*6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 +
 2520*b**21*x**10) + 113400*a**8*b**2*d**3*x**2*log(a/b + x)/(2520*a**10*b*
*11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3
+ 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6
+ 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 25
20*b**21*x**10) + 308205*a**8*b**2*d**3*x**2/(2520*a**10*b**11 + 25200*a**9
*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b*
*15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b*
*18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10)
- 21*a**7*b**3*c*d**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*
b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*
b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*
b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 302400*a**7*b**3*d**3
*x**3*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**
13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**
16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**
19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 784080*a**7*b**3*d**3*x*
*3/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400
*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200
*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*
a*b**20*x**9 + 2520*b**21*x**10) - 210*a**6*b**4*c*d**2*x/(2520*a**10*b**11
 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 5
29200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 3
02400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*
b**21*x**10) + 529200*a**6*b**4*d**3*x**4*log(a/b + x)/(2520*a**10*b**11 +
25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 5292
00*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 3024
00*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**
```

```
21*x**10) + 1296540*a**6*b**4*d**3*x**4/(2520*a**10*b**11 + 25200*a**9*b**1
2*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x
**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x
**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) - 945
*a**5*b**5*c*d**2*x**2/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8
*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5
*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2
*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 635040*a**5*b**5*d**
3*x**5*log(a/b + x)/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b*
*13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b*
*16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b*
*19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 1450008*a**5*b**5*d**3*
x**5/(2520*a**10*b**11 + 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 3024
00*a**7*b**14*x**3 + 529200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 5292
00*a**4*b**17*x**6 + 302400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 2520
0*a*b**20*x**9 + 2520*b**21*x**10) - 6*a**4*b**6*c**2*d/(2520*a**10*b**11 +
 25200*a**9*b**12*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529
200*a**6*b**15*x**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302
400*a**3*b**18*x**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b*
*21*x**10) - 2520*a**4*b**6*c*d**2*x**3/(2520*a**10*b**11 + 25200*a**9*b**1
2*x + 113400*a**8*b**13*x**2 + 302400*a**7*b**14*x**3 + 529200*a**6*b**15*x
**4 + 635040*a**5*b**16*x**5 + 529200*a**4*b**17*x**6 + 302400*a**3*b**18*x
**7 + 113400*a**2*b**19*x**8 + 25200*a*b**20*x**9 + 2520*b**21*x**10) + 529
200*a**4*b**6*d**3*x**6*log(a/b + x)/(2520*a**1...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4934 vs. $2(396) = 792$.
time = 4.12, size = 4934, normalized size = 12.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
 + a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
 a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
```

3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
- 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2
*n^6*x^7 - 15120*(b*x + a)^n*a^4*b^7*d^3*n^6*x^7 + 3363066*(b*x + a)^n*b^11
*c*d^2*n^5*x^8 + 176400*(b*x + a)^n*a^3*b^8*d^3*n^5*x^8 - 672840*(b*x + a)^
n*a^2*b^9*d^3*n^4*x^9 + 1172700*(b*x + a)^n*a*b^10*d^3*n^3*x^10 + 12753576*
(b*x + a)^n*b^11*d^3*n^2*x^11 + (b*x + a)^n*b^11*c^3*n^10*x^2 - 12*(b*x + a
)^n*a^2*b^9*c^2*d*n^9*x^3 + 4176*(b*x + a)^n*a*b^10*c^2*d*n^8*x^4 + 73710*(
b*x + a)^n*b^11*c^2*d*n^7*x^5 + 5040*(b*x + a)^n*a^3*b^8*c*d^2*n^7*x^5 - 17
3250*(b*x + a)^n*a^2*b^9*c*d^2*n^6*x^6 + 5040*(b*x + a)^n*a^5*b^6*d^3*n^6*x
^6 + 1330497*(b*x + a)^n*a*b^10*c*d^2*n^5*x^7 - 126000*(b*x + a)^n*a^4*b^7*
d^3*n^5*x^7 + 13114077*(b*x + a)^n*b^11*c*d^2*n^4*x^8 + 609210*(b*x + a)^n*
a^3*b^8*d^3*n^4*x^8 - 1181240*(b*x + a)^n*a^2*b^9*d^3*n^3*x^9 + 1026576*(b*
x + a)^n*a*b^10*d^3*n^2*x^10 + 10628640*(b*x + a)^n*b^11*d^3*n*x^11 + (b*x
+ a)^n*a*b^10*c^3*n^10*x + 64*(b*x + a)^n*b^11*c^3*n^9*x^2 - 648*(b*x + a)^
n*a^2*b^9*c^2*d*n^8*x^3 + 57006*(b*x + a)^n*a*b^10*c^2*d*n^7*x^4 - 630*(b*x
 + a)^n*a^4*b^7*c*d^2*n^7*x^4 + 703719*(b*x + a)^n*b^11*c^2*d*n^6*x^5 + 798
84*(b*x + a)^n*a^3*b^8*c*d^2*n^6*x^5 - 993069*(b*x + a)^n*a^2*b^9*c*d^2*n^5
x^6 + 75600(b*x + a)^n*a^5*b^6*d^3*n^5*x^6 + 3800598*(b*x + a)^n*a*b^10*c
*d^2*n^4*x^7 - 529200*(b*x + a)^n*a^4*b^7*d^3*n^4*x^7 + 33074574*(b*x + a)^
n*b^11*c*d^2*n^3*x^8 + 1181880*(b*x + a)^n*a^3*b^8*d^3*n^3*x^8 - 1095840*(b
*x + a)^n*a^2*b^9*d^3*n^2*x^9 + 362880*(b*x + a)^n*a*b^10*d^3*n*x^10 + 3628
800*(b*x + a)^n*b^11*d^3*x^11 + 63*(b*x + a)^n*a*b^10*c^3*n^9*x + 1797*(b*x
 + a)^n*b^11*c^3*n^8*x^2 + 36*(b*x + a)^n*a^3*b^8*c^2*d*n^8*x^2 - 14760*(b*
x + a)^n*a^2*b^9*c^2*d*n^7*x^3 + 475695*(b*x + a)^n*a*b^10*c^2*d*n^6*x^4 -
22680*(b*x + a)^n*a^4*b^7*c*d^2*n^6*x^4 + 4394079*(b*x + a)^n*b^11*c^2*d*n^
5*x^5 + 640080*(b*x + a)^n*a^3*b^8*c*d^2*n^5*x^5 - 30240*(b*x + a)^n*a^6*b^
5*d^3*n^5*x^5 - 3355065*(b*x + a)^n*a^2*b^9*c*d^2*n^4*x^6 + 428400*(b*x + a
)^n*a^5*b^6*d^3*n^4*x^6 + 6470388*(b*x + a)^n*a*b^10*c*d^2*n^3*x^7 - 116928
0*(b*x + a)^n*a^4*b^7*d^3*n^3*x^7 + 51177636*(b*x + a)^n*b^11*c*d^2*n^2*x^8
 + 1176120*(b*x + a)^n*a^3*b^8*d^3*n^2*x^8 - 403200*(b*x + a)^n*a^2*b^9*d^3
*n*x^9 - (b*x + a)^n*a^2*b^9*c^3*n^9 + 1734*(b*x + a)^n*a*b^10*c^3*n^8*x +
29076*(b*x + a)^n*b^11*c^3*n^7*x^2 + 1872*(b*x + a)^n*a^3*b^8*c^2*d*n^7*x^2
 - 183744*(b*x + a)^n*a^2*b^9*c^2*d*n^6*x^3 + 2520*(b*x + a)^n*a^5*b^6*c*d^
2*n^6*x^3 + 2491299*(b*x + a)^n*a*b^10*c^2*d*n^5*x^4 - 308700*(b*x + a)^n*a
^4*b^7*c*d^2*n^5*x^4 + 18048210*(b*x + a)^n*b^11*c^2*d*n^4*x^5 + 2758014*(b
*x + a)^n*a^3*b^8*c*d^2*n^4*x^5 - 302400*(b*x + a)^n*a^6*b^5*d^3*n^4*x^5 -

```
6473796*(b*x + a)^n*a^2*b^9*c*d^2*n^3*x^6 + 1134000*(b*x + a)^n*a^5*b^6*d^3
*n^3*x^6 + 5884920*(b*x + a)^n*a*b^10*c*d^2*n^2*x^7 - 1270080*(b*x + a)^n*a
^4*b^7*d^3*n^2*x^7 + 43332840*(b*x + a)^n*b^11*c*d^2*n*x^8 + 453600*(b*x +
a)^n*a^3*b^8*d^3*n*x^8 - 63*(b*x + a)^n*a^2*b^9*c^3*n^8 + 27342*(b*x + a)^n
*a*b^10*c^3*n^7*x - 72*(b*x + a)^n*a^4*b^7*c^2*d*n^7*x + 299271*(b*x + a)^n
*b^11*c^3*n^6*x^2 + 40536*(b*x + a)^n*a^3*b^8*c^2*d*n^6*x^2 - 1351548*(b*x
+ a)^n*a^2*b^9*c^2*d*n^5*x^3 + 83160*(b*x + a)^n*a^5*b^6*c*d^2*n^5*x^3 + 80
83014*(b*x + a)^n*a*b^10*c^2*d*n^4*x^4 - 1965600*(b*x + a)^n*a^4*b^7*c*d^2*
n^4*x^4 + 151200*(b*x + a)^n*a^7*b^4*d^3*n^4*x^4 + 47746140*(b*x + a)^n*b^1
1*c^2*d*n^3*x^5 + 6340320*(b*x + a)^n*a^3*b^8*c*d^2*n^3*x^5 - 1058400*(b*x
+ a)^n*a^6*b^5*d^3*n^3*x^5 - 6449940*(b*x + a)^...
```

Mupad [B]
time = 5.60, size = 2500, normalized size = 6.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^
4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362880
0))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*
n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39
916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334320*b
^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c^3*n^4
 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^9*c^3*n
^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 176402
88*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 +
453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n
^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^
6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133
39535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^1
1 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 + 1814400*a^9*d
^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 17
67087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n
^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d*n - 7484400*a^6*
b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3*c*d^2*n^2 + 26828
64*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460*a^3*b^6*c^2*d*n^4
- 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 + 1836*a^3*b^6*c^2*d*n^6
 + 36*a^3*b^6*c^2*d*n^7))/(b^9*(120543840*n + 150917976*n^2 + 105258076*n^3
 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 192
5*n^9 + 66*n^10 + n^11 + 39916800)) + (a*n*x*(a + b*x)^n*(19958400*b^9*c^3
- 3628800*a^9*d^3 + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9
*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1
```

734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 17640288*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 + 453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^6))/(b^10*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(332640*b^6*c^2 - 10080*a^6*d^2*n + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 + 41580*a^3*b^3*c*d*n + 12558*a^3*b^3*c*d*n^2 + 1260*a^3*b^3*c*d*n^3 + 42*a^3*b^3*c*d*n^4))/(b^6*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*d^2*x^8*(a + b*x)^n*(990*b^3*c + 30*b^3*c*n^2 + b^3*c*n^3 + 30*a^3*d*n + 299*b^3*c*n)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^3*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (a*d^3*n*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(b*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) - (10*a^2*d^3*n*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b^2*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*a*d^2*n*x^7*(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*c*n^3 + 299*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^4*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) - (21*a^2*d^2*n*x^6*(a + b*x)^n*(990*b^3*c - 240*a^3*d + 30*b^3*c*n^2 + b^3*c*n^3 + 299*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^5*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) + (3*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(50400*a^6*d^2 + 332640*b^6*c^2 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3*b^3*c*d - 62790*a^3*b^3*c*d*n - 6300*a^3*b^3*c*d*n^2 - 210*a^3*b^3*c*d*n^3))/(b^7*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39916800)) - (12*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(50400*a^6*d^2 + 332640*b^6*c^2 + 245004*b^6*c^2*n + 74524*b^6*c^2*n^2 + 11985*b^6*c^2*n^3 + 1075*b^6*c^2*n^4 + 51*b^6*c^2*n^5 + b^6*c^2*n^6 - 207900*a^3...

3.184 $\int (a+bx)^n \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=337

$$\frac{\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{10}(1+n)}+\frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{10}(2+n)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{3+n}}{b^{10}(3+n)}+\frac{3d(b^6c^2$$

```
[Out] (-a^3*d+b^3*c)^3*(b*x+a)^(1+n)/b^10/(1+n)+9*a^2*d*(-a^3*d+b^3*c)^2*(b*x+a)^
(2+n)/b^10/(2+n)-9*a*d*(-4*a^3*d+b^3*c)*(-a^3*d+b^3*c)*(b*x+a)^(3+n)/b^10/(
3+n)+3*d*(28*a^6*d^2-20*a^3*b^3*c*d+b^6*c^2)*(b*x+a)^(4+n)/b^10/(4+n)+9*a^2
*d^2*(-14*a^3*d+5*b^3*c)*(b*x+a)^(5+n)/b^10/(5+n)-18*a*d^2*(-7*a^3*d+b^3*c)
*(b*x+a)^(6+n)/b^10/(6+n)+3*d^2*(-28*a^3*d+b^3*c)*(b*x+a)^(7+n)/b^10/(7+n)+
36*a^2*d^3*(b*x+a)^(8+n)/b^10/(8+n)-9*a*d^3*(b*x+a)^(9+n)/b^10/(9+n)+d^3*(b
*x+a)^(10+n)/b^10/(10+n)
```

Rubi [A]
time = 0.15, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{1864\}$

$$-\frac{18ad^2\left(b^3c-7a^3d\right)(a+bx)^{n+6}}{b^{10}(n+6)}+\frac{3d^2\left(b^3c-28a^3d\right)(a+bx)^{n+7}}{b^{10}(n+7)}+\frac{\left(b^3c-a^3d\right)^3(a+bx)^{n+1}}{b^{10}(n+1)}-\frac{9ad\left(b^3c-4a^3d\right)\left(b^3c-a^3d\right)(a+bx)^{n+3}}{b^{10}(n+3)}+\frac{36a^2d^3(a+bx)^{n+8}}{b^{10}(n+8)}+\frac{3d\left(28a^6d^2-20a^3b^3cd+b^6c^2\right)(a+bx)^{n+4}}{b^{10}(n+4)}+\frac{9a^2d^2\left(5b^3c-14a^3d\right)(a+bx)^{n+5}}{b^{10}(n+5)}+\frac{9a^2d\left(b^3c-a^3d\right)^2(a+bx)^{n+2}}{b^{10}(n+2)}-\frac{9ad^3(a+bx)^{n+9}}{b^{10}(n+9)}+\frac{d^3(a+bx)^{n+10}}{b^{10}(n+10)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^
3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c
- a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d
 + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14
*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a
+ b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n
))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^
3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 +
n))
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int (a+bx)^n \left(c+dx^3\right)^3 dx = \int \left(\frac{\left(b^3c-a^3d\right)^3 (a+bx)^n}{b^9} + \frac{9d\left(ab^3c-a^4d\right)^2 (a+bx)^{1+n}}{b^9} + \frac{9ad(b^3c-4a^3d)\,(\cdot}{}\right.$$

$$= \frac{\left(b^3c-a^3d\right)^3 (a+bx)^{1+n}}{b^{10}(1+n)} + \frac{9a^2d\left(b^3c-a^3d\right)^2 (a+bx)^{2+n}}{b^{10}(2+n)} - \frac{9ad(b^3c-4a^3d)\,(b^3}{b^{10}(3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 706 vs. 2(337) = 674.
time = 0.44, size = 706, normalized size = 2.09

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x^3)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-362880*a^9*d^3 + 362880*a^8*b*d^3*(1 + n)*x - 181440*a
^7*b^2*d^3*(2 + 3*n + n^2)*x^2 + 2160*a^6*b^3*d^2*(c*(720 + 242*n + 27*n^2
+ n^3) + 28*d*(6 + 11*n + 6*n^2 + n^3)*x^3) - 2160*a^5*b^4*d^2*(1 + n)*x*(c
*(720 + 242*n + 27*n^2 + n^3) + 7*d*(24 + 26*n + 9*n^2 + n^3)*x^3) + 216*a^
4*b^5*d^2*(2 + 3*n + n^2)*x^2*(5*c*(720 + 242*n + 27*n^2 + n^3) + 14*d*(60
+ 47*n + 12*n^2 + n^3)*x^3) - 9*a*b^8*d*(80 + 146*n + 81*n^2 + 16*n^3 + n^4
)*x^2*(c^2*(3780 + 1968*n + 379*n^2 + 32*n^3 + n^4) + 2*c*d*(1080 + 858*n +
 235*n^2 + 26*n^3 + n^4)*x^3 + d^2*(504 + 450*n + 145*n^2 + 20*n^3 + n^4)*x
^6) - 18*a^3*b^6*d*(c^2*(151200 + 127860*n + 44524*n^2 + 8175*n^3 + 835*n^4
 + 45*n^5 + n^6) + 20*c*d*(4320 + 9372*n + 7144*n^2 + 2475*n^3 + 415*n^4 +
33*n^5 + n^6)*x^3 + 28*d^2*(720 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 2
1*n^5 + n^6)*x^6) + 18*a^2*b^7*d*(1 + n)*x*(c^2*(151200 + 127860*n + 44524*
n^2 + 8175*n^3 + 835*n^4 + 45*n^5 + n^6) + 5*c*d*(17280 + 24528*n + 13420*n
^2 + 3624*n^3 + 511*n^4 + 36*n^5 + n^6)*x^3 + 4*d^2*(5040 + 8028*n + 5104*n
^2 + 1665*n^3 + 295*n^4 + 27*n^5 + n^6)*x^6) + b^9*(12960 + 18612*n + 10404
*n^2 + 2915*n^3 + 435*n^4 + 33*n^5 + n^6)*(c^3*(280 + 138*n + 21*n^2 + n^3)
 + 3*c^2*d*(70 + 87*n + 18*n^2 + n^3)*x^3 + 3*c*d^2*(40 + 54*n + 15*n^2 + n
^3)*x^6 + d^3*(28 + 39*n + 12*n^2 + n^3)*x^9)))/(b^10*(1 + n)*(2 + n)*(3 +
n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n)*(9 + n)*(10 + n))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. 2(337) = 674.
time = 0.26, size = 2280, normalized size = 6.77

method	result	size
gosper	Expression too large to display	2280

risch	Expression too large to display	2665

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^n*(d*x^3+c)^3,x,method=_RETURNVERBOSE)

[Out] -(b*x+a)^(1+n)*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870
*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6
*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63
273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a
*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*
d^3*n^6*x^6-23184*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*
d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*
x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n
^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8
*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*
x^5+88200*a^3*b^6*d^3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3
*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3
*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3
*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c
^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-
41058*b^9*c^2*d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*
a^5*b^4*d^3*n^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+
818496*a^3*b^6*d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*
x^4-940896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n
^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824
*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-68
0400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*
x^6-828*a^2*b^7*c^2*d*n^6*x-1533960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^
3*x^7+96930*a*b^8*c^2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^
7-1469817*b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^
3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^
3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*
c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d
*n^3*x^3-1555200*b^9*c*d^2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^
2*n^4*x+756000*a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^
5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^
7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+15552
00*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*
b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a
^5*b^4*d^3*x^4-1620000*a^4*b^5*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+337392
0*a^3*b^6*c*d^2*n*x^3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+
5614452*a*b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443

```
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a
^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+155
5200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^
2-2655764*b^9*c^3*n^3-2721600*b^9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7
*b^2*d^3*x^2-58320*a^6*b^3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*
b^5*c*d^2*x^2+801432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*
b^8*c^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n
+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-69
99840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-
3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(337) = 674$.
time = 0.31, size = 770, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n
^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x
^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*
a^6*b*n*x + 720*a^7)*(b*x + a)^n*c*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4
+ 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7) + ((n^9 + 45*n^8 + 870*n^7 +
9450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n +
362880)*b^10*x^10 + (n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*
n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a*b^9*x^9 - 9*(n^8 + 28*n^7 + 322*
n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^2*b^8*x^8 + 7
2*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^3*b^7*
x^7 - 504*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^4*b^6*x^6 +
 3024*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^5*b^5*x^5 - 15120*(n^4 + 6*
n^3 + 11*n^2 + 6*n)*a^6*b^4*x^4 + 60480*(n^3 + 3*n^2 + 2*n)*a^7*b^3*x^3 - 1
81440*(n^2 + n)*a^8*b^2*x^2 + 362880*a^9*b*n*x - 362880*a^10)*(b*x + a)^n*d
^3/((n^10 + 55*n^9 + 1320*n^8 + 18150*n^7 + 157773*n^6 + 902055*n^5 + 34169
30*n^4 + 8409500*n^3 + 12753576*n^2 + 10628640*n + 3628800)*b^10)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2313 vs. $2(337) = 674$.
time = 0.45, size = 2313, normalized size = 6.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")

[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 3628800*a*b^9*c^3 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^10*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 + 13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 + 1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 - 270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6 + 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 - 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 120*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(105*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8 + 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(2

```
1548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 192
4*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b
^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2
*d + 1452*a^7*b^3*c*d^2)*n + (b^10*c^3*n^9 + 54*b^10*c^3*n^8 + 3628800*b^10
*c^3 + 6*(211*b^10*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^10*c^3 + 45*a^3*b
^7*c^2*d)*n^6 + 3*(46963*b^10*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^1
0*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^10*c^3 +
200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^10*c^3 + 3196
5*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^10*c^3 + 1890*a^3*
b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x)*(b*x + a)^n/(b^10*n^1
0 + 55*b^10*n^9 + 1320*b^10*n^8 + 18150*b^10*n^7 + 157773*b^10*n^6 + 902055
*b^10*n^5 + 3416930*b^10*n^4 + 8409500*b^10*n^3 + 12753576*b^10*n^2 + 10628
640*b^10*n + 3628800*b^10)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 40536 vs. $2(316) = 632$.
time = 68.29, size = 40536, normalized size = 120.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Piecewise((a**n*(c**3*x + 3*c**2*d*x**4/4 + 3*c*d**2*x**7/7 + d**3*x**10/10
), Eq(b, 0)), (2520*a**9*d**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 7129*a**9*d**3/(2520*a**9*b**
10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 +
317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 +
90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 22680*a**8*
b*d**3*x*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b*
*12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b*
*15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x
**8 + 2520*b**19*x**9) + 61641*a**8*b*d**3*x/(2520*a**9*b**10 + 22680*a**8*
b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**1
4*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17
*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 90720*a**7*b**2*d**3*x**2*l
og(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 +
 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 +
 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520
*b**19*x**9) + 235224*a**7*b**2*d**3*x**2/(2520*a**9*b**10 + 22680*a**8*b**
11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x
**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x*
*7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 30*a**6*b**3*c*d**2/(2520*a**9
```

```
*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**
3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**
6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 211680*
a**6*b**3*d**3*x**3*log(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90
720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317
520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 2268
0*a*b**18*x**8 + 2520*b**19*x**9) + 518616*a**6*b**3*d**3*x**3/(2520*a**9*b
**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3
+ 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6
+ 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 270*a**5*
b**4*c*d**2*x/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2
 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5
 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 25
20*b**19*x**9) + 317520*a**5*b**4*d**3*x**4*log(a/b + x)/(2520*a**9*b**10 +
 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 3175
20*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 9072
0*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 725004*a**5*b**
4*d**3*x**4/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 +
 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 +
 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520
*b**19*x**9) - 1080*a**4*b**5*c*d**2*x**2/(2520*a**9*b**10 + 22680*a**8*b**
11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x
**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x*
*7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 317520*a**4*b**5*d**3*x**5*log
(a/b + x)/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 2
11680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 2
11680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b
**19*x**9) + 661500*a**4*b**5*d**3*x**5/(2520*a**9*b**10 + 22680*a**8*b**11
*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**
4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7
 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 15*a**3*b**6*c**2*d/(2520*a**9*b
**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3
+ 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6
+ 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) - 2520*a**3
*b**6*c*d**2*x**3/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*
x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*
x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8
+ 2520*b**19*x**9) + 211680*a**3*b**6*d**3*x**6*log(a/b + x)/(2520*a**9*b**
10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 +
317520*a**5*b**14*x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 +
90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + 388080*a**3
*b**6*d**3*x**6/(2520*a**9*b**10 + 22680*a**8*b**11*x + 90720*a**7*b**12*x*
*2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*x**4 + 317520*a**4*b**15*x*
*5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x**7 + 22680*a*b**18*x**8 +
2520*b**19*x**9) - 135*a**2*b**7*c**2*d*x/(2520...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3874 vs. $2(337) = 674$.
time = 4.46, size = 3874, normalized size = 11.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^10*d^3*n^9*x^10 + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x +
a)^n*b^10*d^3*n^8*x^10 + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n
*b^10*d^3*n^7*x^10 + 3*(b*x + a)^n*b^10*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b
^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^10*
d^3*n^6*x^10 + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^10*c*d
^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d
^3*n^6*x^9 + 63273*(b*x + a)^n*b^10*d^3*n^5*x^10 + 126*(b*x + a)^n*a*b^9*c*
d^2*n^8*x^6 + 2952*(b*x + a)^n*b^10*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*
d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^
9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^10*d^3*n^4*x^10 + 3*(b*x + a)^n*b^10*c
^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^
9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^10*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a
^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a
)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^10*d^3*n^3*x^10 + 3*(b*x + a)^
n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^10*c^2*d*n^8*x^4 - 666*(b*x + a)^
n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x
+ a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^10*c*d^2*n^5*x^7 + 12600*
(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 1
18124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^10*d^3*n^2*x^10
 + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^10*c^2*d*n^7*x^
4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n
^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*
d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^10*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^
3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x +
a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^10*d^3*n*x^10 + (b*x + a)^n*
b^10*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b
^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^10*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*
a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x
 + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 4284
0*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^10*c*d^2*n^3*x^7
+ 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n
^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^10*d^3*x^
10 + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^10*c^3*n^8*x - 414*(b*x +
 a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(
b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^10*c^2*d*n^5*x^4 +
37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*
```

```
n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*
c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^
n*b^10*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x
+ a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*
b^10*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*
b^8*c^2*d*n^6*x^2 + 212157*(b*x + a)^n*a*b^9*c^2*d*n^5*x^3 - 10800*(b*x + a
)^n*a^4*b^6*c*d^2*n^5*x^3 + 1469817*(b*x + a)^n*b^10*c^2*d*n^4*x^4 + 222750
*(b*x + a)^n*a^3*b^7*c*d^2*n^4*x^4 - 15120*(b*x + a)^n*a^6*b^4*d^3*n^4*x^4
- 683064*(b*x + a)^n*a^2*b^8*c*d^2*n^3*x^5 + 105840*(b*x + a)^n*a^5*b^5*d^3
*n^3*x^5 + 678960*(b*x + a)^n*a*b^9*c*d^2*n^2*x^6 - 138096*(b*x + a)^n*a^4*
b^6*d^3*n^2*x^6 + 4332960*(b*x + a)^n*b^10*c*d^2*n*x^7 + 51840*(b*x + a)^n*
a^3*b^7*d^3*n*x^7 + 1266*(b*x + a)^n*a*b^9*c^3*n^7 + 16884*(b*x + a)^n*b^10
*c^3*n^6*x + 810*(b*x + a)^n*a^3*b^7*c^2*d*n^6*x - 81090*(b*x + a)^n*a^2*b^
8*c^2*d*n^5*x^2 + 1080*(b*x + a)^n*a^5*b^5*c*d^2*n^5*x^2 + 833346*(b*x + a)
^n*a*b^9*c^2*d*n^4*x^3 - 117000*(b*x + a)^n*a^4*b^6*c*d^2*n^4*x^3 + 4371522
*(b*x + a)^n*b^10*c^2*d*n^3*x^4 + 642960*(b*x + a)^n*a^3*b^7*c*d^2*n^3*x^4
- 90720*(b*x + a)^n*a^6*b^4*d^3*n^3*x^4 - 752544*(b*x + a)^n*a^2*b^8*c*d^2*
n^2*x^5 + 151200*(b*x + a)^n*a^5*b^5*d^3*n^2*x^5 + 259200*(b*x + a)^n*a*b^9
*c*d^2*n*x^6 - 60480*(b*x + a)^n*a^4*b^6*d^3*n*x^6 + 1555200*(b*x + a)^n*b^
10*c*d^2*x^7 + 16884*(b*x + a)^n*a*b^9*c^3*n^6 - 18*(b*x + a)^n*a^4*b^6*c^2
*d*n^6 + 140889*(b*x + a)^n*b^10*c^3*n^5*x + 15030*(b*x + a)^n*a^3*b^7*c^2*
d*n^5*x - 474291*(b*x + a)^n*a^2*b^8*c^2*d*n^4*x^2 + 30240*(b*x + a)^n*a^5*
b^5*c*d^2*n^4*x^2 + 1871484*(b*x + a)^n*a*b^9*c^2*d*n^3*x^3 - 540000*(b*x +
 a)^n*a^4*b^6*c*d^2*n^3*x^3 + 60480*(b*x + a)^n*a^7*b^3*d^3*n^3*x^3 + 77424
12*(b*x + a)^n*b^10*c^2*d*n^2*x^4 + 843480*(b*x + a)^n*a^3*b^7*c*d^2*n^2*x^
4 - 166320*(b*x + a)^n*a^6*b^4*d^3*n^2*x^4 - 311040*(b*x + a)^n*a^2*b^8*c*d
^2*n*x^5 + 72576*(b*x + a)^n*a^5*b^5*d^3*n*x^5 + 140889*(b*x + a)^n*a*b^9*c
^3*n^5 - 810*(b*x + a)^n*a^4*b^6*c^2*d*n^5 + 761166*(b*x + a)^n*b^10*c^3*n^
4*x + 147150*(b*x + a)^n*a^3*b^7*c^2*d*n^4*x - 2160*(b*x + a)^n*a^6*b^4*c*d
^2*n^4*x - 1551456*(b*x + a)^n*a^2*b^8*c^2*d*n^3*x^2 + 290520*(b*x + a)^n*a
^5*b^5*c*d^2*n^3*x^2 + 2127960*(b*x + a)^n*a*b^...
```

Mupad [B]
time = 4.34, size = 2001, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*(3628800*a*b^9*c^3 - 362880*a^10*d^3 - 2721600*a^4*b^6*c^2*d +
 1555200*a^7*b^3*c*d^2 + 5753736*a*b^9*c^3*n^2 + 2655764*a*b^9*c^3*n^3 + 76
1166*a*b^9*c^3*n^4 + 140889*a*b^9*c^3*n^5 + 16884*a*b^9*c^3*n^6 + 1266*a*b^
9*c^3*n^7 + 54*a*b^9*c^3*n^8 + a*b^9*c^3*n^9 + 6999840*a*b^9*c^3*n - 230148
0*a^4*b^6*c^2*d*n + 522720*a^7*b^3*c*d^2*n - 801432*a^4*b^6*c^2*d*n^2 + 583
```

20*a^7*b^3*c*d^2*n^2 - 147150*a^4*b^6*c^2*d*n^3 + 2160*a^7*b^3*c*d^2*n^3 - 15030*a^4*b^6*c^2*d*n^4 - 810*a^4*b^6*c^2*d*n^5 - 18*a^4*b^6*c^2*d*n^6))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (x*(a + b*x)^n*(3628800*b^10*c^3 + 6999840*b^10*c^3*n + 5753736*b^10*c^3*n^2 + 2655764*b^10*c^3*n^3 + 761166*b^10*c^3*n^4 + 140889*b^10*c^3*n^5 + 16884*b^10*c^3*n^6 + 1266*b^10*c^3*n^7 + 54*b^10*c^3*n^8 + b^10*c^3*n^9 + 362880*a^9*b*d^3*n + 2721600*a^3*b^7*c^2*d*n - 1555200*a^6*b^4*c*d^2*n + 2301480*a^3*b^7*c^2*d*n^2 - 522720*a^6*b^4*c*d^2*n^2 + 801432*a^3*b^7*c^2*d*n^3 - 58320*a^6*b^4*c*d^2*n^3 + 147150*a^3*b^7*c^2*d*n^4 - 2160*a^6*b^4*c*d^2*n^4 + 15030*a^3*b^7*c^2*d*n^5 + 810*a^3*b^7*c^2*d*n^6 + 18*a^3*b^7*c^2*d*n^7))/(b^10*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (d^3*x^10*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880))/(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800) + (3*d^2*x^7*(a + b*x)^n*(720*b^3*c + 27*b^3*c*n^2 + b^3*c*n^3 + 24*a^3*d*n + 242*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b^3*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (3*d*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(151200*b^6*c^2 - 5040*a^6*d^2*n + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 + 21600*a^3*b^3*c*d*n + 7260*a^3*b^3*c*d*n^2 + 810*a^3*b^3*c*d*n^3 + 30*a^3*b^3*c*d*n^4))/(b^6*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (a*d^3*n*x^9*(a + b*x)^n*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))/(b*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d^3*n*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(b^2*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (3*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d*n*x^2*(n + 1)*(a + b*x)^n*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^8*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) + (3*a*d^2*n*x^6*(a + b*x)^n*(720*b^3*c - 168*a^3*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/

```
(b^4*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 +
157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (18*a^2*d^2
*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(720*b^3*c - 168*a^3
*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n))/(b^5*(10628640*n + 12753576*n
^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320
*n^8 + 55*n^9 + n^10 + 3628800))
```

3.185 $\int \frac{(a+bx)^n (c+dx^3)^3}{x} \, dx$

Optimal. Leaf size=358

$$\frac{a^2 d(3b^6c^2 - 3a^3b^3cd + a^6d^2)\,(a+bx)^{1+n}}{b^9(1+n)} - \frac{ad(6b^6c^2 - 15a^3b^3cd + 8a^6d^2)\,(a+bx)^{2+n}}{b^9(2+n)} + \frac{d(3b^6c^2 - 30a^3b^3cd + 2}{b^9(3+}$$

```
[Out] a^2*d*(a^6*d^2-3*a^3*b^3*c*d+3*b^6*c^2)*(b*x+a)^(1+n)/b^9/(1+n)-a*d*(8*a^6*
d^2-15*a^3*b^3*c*d+6*b^6*c^2)*(b*x+a)^(2+n)/b^9/(2+n)+d*(28*a^6*d^2-30*a^3*
b^3*c*d+3*b^6*c^2)*(b*x+a)^(3+n)/b^9/(3+n)+2*a^2*d^2*(-28*a^3*d+15*b^3*c)*(
b*x+a)^(4+n)/b^9/(4+n)-5*a*d^2*(-14*a^3*d+3*b^3*c)*(b*x+a)^(5+n)/b^9/(5+n)+
d^2*(-56*a^3*d+3*b^3*c)*(b*x+a)^(6+n)/b^9/(6+n)+28*a^2*d^3*(b*x+a)^(7+n)/b^
9/(7+n)-8*a*d^3*(b*x+a)^(8+n)/b^9/(8+n)+d^3*(b*x+a)^(9+n)/b^9/(9+n)-c^3*(b*
x+a)^(1+n)*hypergeom([1, 1+n],[2+n],1+b*x/a)/a/(1+n)
```

Rubi [A]

time = 0.15, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1634, 67}

$$-\frac{5ad^2(3b^3c-14a^3d)\,(a+bx)^{n+5}}{b^9(n+5)} + \frac{d^2(3b^3c-56a^3d)\,(a+bx)^{n+6}}{b^9(n+6)} + \frac{28a^2d^3(a+bx)^{n+7}}{b^9(n+7)} - \frac{ad(8a^6d^2-15a^3b^3cd+6b^6c^2)\,(a+bx)^{n+2}}{b^9(n+2)} + \frac{d(28a^6d^2-30a^3b^3cd+3b^6c^2)\,(a+bx)^{n+3}}{b^9(n+3)} + \frac{2a^2d^2(15b^3c-28a^3d)\,(a+bx)^{n+4}}{b^9(n+4)} + \frac{a^2d(a^6d^2-3a^3b^3cd+3b^6c^2)\,(a+bx)^{n+1}}{b^9(n+1)} - \frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)} + \frac{d^3(a+bx)^{n+9}}{b^9(n+9)} - \frac{c^3(a+bx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{bx}{a}+1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^n*(c + d*x^3)^3)/x,x]
```

```
[Out] (a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2)*(a + b*x)^(1 + n))/(b^9*(1 + n
)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x)^(2 + n))/(b^9*
(2 + n)) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/
(b^9*(3 + n)) + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4
 + n)) - (5*a*d^2*(3*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (
d^2*(3*b^3*c - 56*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^3*(a
+ b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^3*(a + b*x)^(8 + n))/(b^9*(8 + n)) +
 (d^3*(a + b*x)^(9 + n))/(b^9*(9 + n)) - (c^3*(a + b*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, 1 + (b*x)/a])/(a*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
```

```
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^n \left(c+dx^3\right)^3}{x}\,dx &= \int \left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)\,(a+bx)^n}{b^8}+\frac{c^3(a+bx)^n}{x}-\frac{ad(6b^6c^2-15a^3b\right.\\ &= \frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)\,(a+bx)^{1+n}}{b^9(1+n)}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)\,(a+}{b^9(2+n)}\\ &= \frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)\,(a+bx)^{1+n}}{b^9(1+n)}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)\,(a+}{b^9(2+n)}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 332, normalized size = 0.93

$$(a+bx)^{1+n}\left(\frac{a^2d(3b^6c^2-3a^3b^3cd+a^6d^2)}{b^9(1+n)}-\frac{ad(6b^6c^2-15a^3b^3cd+8a^6d^2)\,(a+bx)}{b^9(2+n)}+\frac{d(3b^6c^2-30a^3b^3cd+28a^6d^2)\,(a+bx)^2}{b^9(3+n)}+\frac{2a^2d^2(15b^3c-28a^3d)\,(a+bx)^3}{b^9(4+n)}+\frac{5ad^2(-3b^3c+14a^3d)\,(a+bx)^4}{b^9(5+n)}+\frac{d^2(3b^3c-56a^3d)\,(a+bx)^5}{b^9(6+n)}+\frac{28a^2d^3(a+bx)^6}{b^9(7+n)}-\frac{8ad^3(a+bx)^7}{b^9(8+n)}+\frac{d^3(a+bx)^8}{b^9(9+n)}-\frac{c^3\,{}_2F_1\left(1,1+n;2+n;\frac{a+bx}{a}\right)}{a+an}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^n*(c + d*x^3)^3)/x,x]
[Out] (a + b*x)^(1 + n)*((a^2*d*(3*b^6*c^2 - 3*a^3*b^3*c*d + a^6*d^2))/(b^9*(1 +
n)) - (a*d*(6*b^6*c^2 - 15*a^3*b^3*c*d + 8*a^6*d^2)*(a + b*x))/(b^9*(2 + n)
) + (d*(3*b^6*c^2 - 30*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(b^9*(3 + n))
 + (2*a^2*d^2*(15*b^3*c - 28*a^3*d)*(a + b*x)^3)/(b^9*(4 + n)) + (5*a*d^2*(
-3*b^3*c + 14*a^3*d)*(a + b*x)^4)/(b^9*(5 + n)) + (d^2*(3*b^3*c - 56*a^3*d)
*(a + b*x)^5)/(b^9*(6 + n)) + (28*a^2*d^3*(a + b*x)^6)/(b^9*(7 + n)) - (8*a
*d^3*(a + b*x)^7)/(b^9*(8 + n)) + (d^3*(a + b*x)^8)/(b^9*(9 + n)) - (c^3*Hy
pergeometric2F1[1, 1 + n, 2 + n, (a + b*x)/a])/(a + a*n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n \left(d\,x^3+c\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x^3+c)^3/x,x)
[Out] int((b*x+a)^n*(d*x^3+c)^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x + a)^n/x, x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12492 vs. 2(338) = 676.
time = 14.40, size = 17258, normalized size = 48.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x**3+c)**3/x,x)
```

```
[Out] -b**n*c**3*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/gamma(
n + 2) - b**n*c**3*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/
gamma(n + 2) + 3*c**2*d*Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b
+ x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b
**4*x + 2*b**5*x**2) + 4*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b
**5*x**2) + 4*a*b*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*
log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2
*log(a/b + x)/(a*b**3 + b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/
b + x)/(a*b**3 + b**4*x) + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*l
og(a/b + x)/b**3 - a*x/b**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/
(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/
(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)
**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x
)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b
*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a +
b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b
*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3), True)) + 3*c*d**2*Pi
ecewise((a**n*x**6/6, Eq(b, 0)), (60*a**5*log(a/b + x)/(60*a**5*b**6 + 300*
a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 +
60*b**11*x**5) + 137*a**5/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x
**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*x
```

```
*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a*
*2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*x/(60*a**5*b*
*6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**1
0*x**4 + 60*b**11*x**5) + 600*a**3*b**2*x**2*log(a/b + x)/(60*a**5*b**6 + 3
00*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4
 + 60*b**11*x**5) + 1100*a**3*b**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 6
00*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)
+ 600*a**2*b**3*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**
3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*
a**2*b**3*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a
**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4*log(a/b
 + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*
x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*x**4/(60*a**5*b**6 +
300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**
4 + 60*b**11*x**5) + 60*b**5*x**5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**
7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11
*x**5), Eq(n, -6)), (-60*a**5*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x +
 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 125*a**5/(12*a**4*b*
*6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) -
 240*a**4*b*x*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x*
*2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 440*a**4*b*x/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b*
*2*x**2*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 4
8*a*b**9*x**3 + 12*b**10*x**4) - 540*a**3*b**2*x**2/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b*
*3*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 4
8*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*x**3/(12*a**4*b**6 + 48*a**3
*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 60*a*b**4*x
**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*
b**9*x**3 + 12*b**10*x**4) + 12*b**5*x**5/(12*a**4*b**6 + 48*a**3*b**7*x +
72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*l
og(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
 110*a**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 1
80*a**4*b*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6
*b**9*x**3) + 270*a**4*b*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 +
 6*b**9*x**3) + 180*a**3*b**2*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7
*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*x**2/(6*a**3*b**6 + 18*a
**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*x**3*log(a/b + x)
/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15*a*b**4*
x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 3*b**5
*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3), Eq(n,
-4)), (-60*a**5*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 90
*a**5/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x*log(a/b + x)
/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b*x/(6*a**2*b**6 + 12
*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*x**2*log(a/b + x)/(6*a**2*b**6 + 12
```

```
*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*x**3/(6...
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3/x,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^3*(b*x + a)^n/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^3\,\left(a+b\,x\right)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^3)^3*(a + b*x)^n)/x,x)
[Out] int(((c + d*x^3)^3*(a + b*x)^n)/x, x)
```

3.186 $\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=324

$$\frac{e^2(e+fx)^{1+n}}{bf^3(1+n)}-\frac{2e(e+fx)^{2+n}}{bf^3(2+n)}+\frac{(e+fx)^{3+n}}{bf^3(3+n)}+\frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{5/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}+\frac{a(e+fx)^{1}\cdots}{3\cdots}$$

```
[Out] e^2*(f*x+e)^(1+n)/b/f^3/(1+n)-2*e*(f*x+e)^(2+n)/b/f^3/(2+n)+(f*x+e)^(3+n)/b
/f^3/(3+n)+1/3*a*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^
(1/3)*e-a^(1/3)*f))/b^(5/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)
*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))
/b^(5/3)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*a*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(5/3)
/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A]

time = 0.63, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6857, 70}

$$\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)}+\frac{a(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{3b^{5/3}(n+1)\left(\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f\right)}+\frac{e^2(e+fx)^{n+1}}{bf^3(n+1)}-\frac{2e(e+fx)^{n+2}}{bf^3(n+2)}+\frac{(e+fx)^{n+3}}{bf^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e^2*(e + f*x)^(1 + n))/(b*f^3*(1 + n)) - (2*e*(e + f*x)^(2 + n))/(b*f^3*(2
 + n)) + (e + f*x)^(3 + n)/(b*f^3*(3 + n)) + (a*(e + f*x)^(1 + n)*Hypergeom
etric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*
b^(5/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (a*(e + f*x)^(1 + n)*Hypergeomet
ric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)
*f)])/(3*b^(5/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)*(1 + n)) + (a*(e + f*x)
^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)])/(3*b^(5/3)*(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*(1
+ n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(e+fx)^n}{a+bx^3}\,dx &= \int \left(\frac{e^2(e+fx)^n}{bf^2} - \frac{2e(e+fx)^{1+n}}{bf^2} + \frac{(e+fx)^{2+n}}{bf^2} - \frac{ax^2(e+fx)^n}{b\,(a+bx^3)}\right) dx \\
&= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{b} \\
&= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(e\cdots}{3b^{2/3}\left(-\sqrt[3]{-\cdots}\right.}\right.}{\cdots} \\
&= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} - \frac{a\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{5/3}} - \frac{a\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\cdots}}{3b^{5/3}} \\
&= \frac{e^2(e+fx)^{1+n}}{bf^3(1+n)} - \frac{2e(e+fx)^{2+n}}{bf^3(2+n)} + \frac{(e+fx)^{3+n}}{bf^3(3+n)} + \frac{a(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{\cdots}}{\sqrt[3]{\cdots}}\right.}{3b^{5/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 284, normalized size = 0.88

$$
\frac{(e+fx)^{1+n}\left(\frac{3b^{2/3}e^2}{f^3(1+n)} - \frac{6b^{2/3}e(e+fx)}{f^3(2+n)} + \frac{3b^{2/3}(e+fx)^2}{f^3(3+n)} + \frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} + \frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{\left(\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f\right)(1+n)} + \frac{a\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{\left(\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f\right)(1+n)}\right)}{3b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((3*b^(2/3)*e^2)/(f^3*(1 + n)) - (6*b^(2/3)*e*(e + f*x))
/(f^3*(2 + n)) + (3*b^(2/3)*(e + f*x)^2)/(f^3*(3 + n)) + (a*Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)
*e - a^(1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(
e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/((b^(1/3)*e + (-1)^(1/3)*a^(
1/3)*f)*(1 + n)) + (a*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/((b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)*
(1 + n))))/(3*b^(5/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(f x +e \right)^{n}}{b \,x^{3}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^5*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^5/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(e+f\,x\right)}^n}{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^5*(e + f*x)^n)/(a + b*x^3), x)
```

3.187 $\int \frac{x^4(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=332

$$-\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}-\frac{a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{4/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}+\frac{\sqrt[3]{-1}\,a^{2/3}(e+fx)^{1+n}\,{}_2 \cdots}{3b^{4/3}\left((-1 \cdots\right.}$$

```
[Out] -e*(f*x+e)^(1+n)/b/f^2/(1+n)+(f*x+e)^(2+n)/b/f^2/(2+n)-1/3*a^(2/3)*(f*x+e)^
(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b^(4/
3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*a^(2/3)*(f*x+e)^(1+n)*hyperge
om([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*
f))/b^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(2/3)*a^(2/3)*(
f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3
)*b^(1/3)*e+a^(1/3)*f))/b^(4/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A]
time = 0.62, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6857, 70}

$$-\frac{a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{-1}\,a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{(-1)^{2/3}a^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3b^{4/3}(n+1)\left(\sqrt[3]{a}\,f+\sqrt[3]{-1}\,\sqrt[3]{b}\,e\right)}-\frac{e(e+fx)^{n+1}}{bf^2(n+1)}+\frac{(e+fx)^{n+2}}{bf^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -((e*(e + f*x)^(1 + n))/(b*f^2*(1 + n))) + (e + f*x)^(2 + n)/(b*f^2*(2 + n)
) - (a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1
+ n)) + ((-1)^(1/3)*a^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(
3*b^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*a^(2/3)
*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b^(4/3)*((-1)^(1/3)*b^(1/
3)*e + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^4(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{e(e+fx)^n}{bf}+\frac{(e+fx)^{1+n}}{bf}-\frac{ax(e+fx)^n}{b\left(a+bx^3\right)}\right)dx$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}-\frac{a\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}-\frac{a\int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\right.}\right.}{b}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}+\frac{a^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}}-\frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}{3b^{4/3}}$$

$$= -\frac{e(e+fx)^{1+n}}{bf^2(1+n)}+\frac{(e+fx)^{2+n}}{bf^2(2+n)}-\frac{a^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3b^{4/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+$$

Mathematica [A]
time = 0.54, size = 292, normalized size = 0.88

$$\frac{(e+fx)^{1+n}\left(-\frac{3\sqrt[3]{b}e}{f^2(1+n)}+\frac{3\sqrt[3]{b}(e+fx)}{f^2(2+n)}-\frac{a^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{\sqrt[3]{-1}\,a^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{(-1)^{2/3}a^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{\left(\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f\right)(1+n)}\right)}{3b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*((-3*b^(1/3)*e)/(f^2*(1 + n)) + (3*b^(1/3)*(e + f*x))/(f
^2*(2 + n)) - (a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x
))/(b^(1/3)*e - a^(1/3)*f)])/((b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(1/3
)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))
/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(
1 + n)) + ((-1)^(2/3)*a^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3
)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(((-1)^(1/3)*b^(1
/3)*e + a^(1/3)*f)*(1 + n))))/(3*b^(4/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(f x +e \right)^{n}}{b \,x^{3}+a}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^4*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^4/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(e+f\,x\right)}^n}{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^4*(e + f*x)^n)/(a + b*x^3), x)
```

3.188 $\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx$

Optimal. Leaf size=293

$$\frac{(e+fx)^{1+n}}{bf(1+n)}+\frac{\sqrt[3]{a}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}+\frac{\sqrt[3]{a}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-}{(-1}\right.}{3b\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1}$$

```
[Out] (f*x+e)^(1+n)/b/f/(1+n)+1/3*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],
b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/3)*f))/b/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*a^
(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-
1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/b/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3
*a^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/
((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/b/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A]
time = 0.38, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6857, 70}

$$\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{\sqrt[3]{a}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3b(n+1)\left(\sqrt[3]{a}\,f+\sqrt[3]{-1}\,\sqrt[3]{b}\,e\right)}+\frac{(e+fx)^{n+1}}{bf(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (e + f*x)^(1 + n)/(b*f*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric
2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*b*(b^
(1/3)*e - a^(1/3)*f)*(1 + n)) + (a^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F
1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a
^(1/3)*f)])/(3*b*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (a^(1/3)*(e
+ f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e +
f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*b*((-1)^(1/3)*b^(1/3)*e + a^(
1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{x^3(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{b} - \frac{a(e+fx)^n}{b\,(a+bx^3)}\right)dx$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \frac{(e+fx)^n}{a+bx^3}\,dx}{b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} - \frac{a\int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1\right.}\right.}{b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{(e+f}{-\sqrt[3]{a}-(-1}}{3b}$$
$$= \frac{(e+fx)^{1+n}}{bf(1+n)} + \frac{\sqrt[3]{a}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} + \frac{\sqrt[3]{a}\,(e+fx)^{1+n}\,{}_2I}{3b\left((-\right.}$$

Mathematica [A]
time = 0.24, size = 239, normalized size = 0.82

$$\frac{(e+fx)^{1+n}\left(\frac{3}{f} + \frac{\sqrt[3]{a}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} + \frac{\sqrt[3]{a}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} - \frac{\sqrt[3]{a}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(3/f + (a^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1
/3)*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (a^(1/3)
*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2
/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - (a^(1/3)*
Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/
3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*b*(1 +
n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^3(fx+e)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^3*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^3/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(e+f\,x\right)}^n}{b\,x^3+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^3*(e + f*x)^n)/(a + b*x^3), x)
```

3.189 $\int \frac{x^2(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=253

$$-\frac{(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}-\frac{(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f\right)(1+n)}-\frac{(e$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/b^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/b^(2/3)/(b^(1
/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n
],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/b^(2/3)/(b^(1/3)*e-(-1)
^(2/3)*a^(1/3)*f)/(1+n)
```

Rubi [A]
time = 0.26, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {6857, 70}

$$-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{3b^{2/3}(n+1)\left(\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] -1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*
x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b
^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e + (-1)^(1/3)*a^(1/
3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(
1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*b^(2/3)*(b^(1/3)*e
- (-1)^(2/3)*a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx = \int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}\right.$$

$$= \frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}} + \frac{\int \frac{(e+fx)^n}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\right.}{3b^{2/3}\left(\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f\right)}$$

Mathematica [A]
time = 0.16, size = 213, normalized size = 0.84

$$\frac{(e+fx)^{1+n}\left(-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} - \frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f} - \frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{3b^{2/3}(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)]/(b^(1
/3)*e + (-1)^(1/3)*a^(1/3)*f) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)]/(b^(1/3)*e - (-1)^(2/3)*a^(
1/3)*f)))/(3*b^(2/3)*(1 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2(fx+e)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int(x^2*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x^2/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(e+f\,x\right)}^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x^2*(e + f*x)^n)/(a + b*x^3), x)
```

3.190 $\int \frac{x(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=288

$$\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}-\frac{\sqrt[3]{-1}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}-$$

```
[Out] 1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1/
3)*f))/a^(1/3)/b^(1/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(-1)^(1/3)*(f*x+e)^(
1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3
)*e-a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*
(-1)^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e
)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(1/3)/b^(1/3)/((-1)^(1/3)*b^(1/3)*e+a
^(1/3)*f)/(1+n)
```

Rubi [A]
time = 0.21, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6857, 70}

$$\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{\sqrt[3]{-1}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}-\frac{(-1)^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,(n+1)\left(\sqrt[3]{a}\,f+\sqrt[3]{-1}\,\sqrt[3]{b}\,e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(
b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)
) - ((-1)^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^
(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(1/3)*b^
(1/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) - ((-1)^(2/3)*(e + f*x)^(
1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((
-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(1/3)*b^(1/3)*((-1)^(1/3)*b^(1/3)*e
 + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(-1)^{2/3}(e+fx)^n}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} + \frac{\sqrt[3]{-1}\,(e}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{a}+\right.}\right.$$

$$= -\frac{\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{(e+fx)^n}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3\sqrt[3]{a}\,\sqrt[3]{b}}$$

$$= \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} - \frac{\sqrt[3]{-1}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+\right.}{3\sqrt[3]{a}\,\sqrt[3]{b}\,\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{}\right.}$$

Mathematica [A]
time = 0.19, size = 237, normalized size = 0.82

$$\frac{(e+fx)^{1+n}\left(\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} - \frac{\sqrt[3]{-1}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} - \frac{(-1)^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3\sqrt[3]{a}\,\sqrt[3]{b}\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/
(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f) - ((-1)^(1/3)*Hypergeometr
ic2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e
 - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) - ((-1)^(2/3)*Hypergeome
tric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)
*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(1/3)*b^(1/3)*(
1 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x(fx+e)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n*x/(b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n*x/(b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,(e+f\,x)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(e + f*x)^n)/(a + b*x^3),x)
[Out] int((x*(e + f*x)^n)/(a + b*x^3), x)
```

3.191 $\int \frac{(e+fx)^n}{a+bx^3} dx$

Optimal. Leaf size=263

$$-\frac{(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}-\frac{(e+fx)^{1+n} {}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)(1+n)}+\frac{(e}{}$$

```
[Out] -1/3*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-a^(1
/3)*f))/a^(2/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)-1/3*(f*x+e)^(1+n)*hypergeom([1,
 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f))/a^
(2/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],(-1)^(1/3)*b^(1/3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a
^(2/3)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f)/(1+n)
```

Rubi [A]

time = 0.13, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6857, 70}

$$-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}(e+fx)}{\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{b}e-\sqrt[3]{a}f\right)}-\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}(e+fx)}{(-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}e-\sqrt[3]{a}f\right)}+\frac{(e+fx)^{n+1} {}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\sqrt[3]{b}(e+fx)}{\sqrt[3]{-1}\sqrt[3]{b}e+\sqrt[3]{a}f}\right)}{3a^{2/3}(n+1)\left(\sqrt[3]{a}f+\sqrt[3]{-1}\sqrt[3]{b}e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] -1/3*((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*
x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(2/3)*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) - (
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e
 + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3*a^(2/3)*((-1)^(2/3)*b^(1/3
)*e - a^(1/3)*f)*(1 + n)) + ((e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
2 + n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/
(3*a^(2/3)*((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\int \frac{(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)} - \frac{(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{\right.}\right.$$

$$= -\frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}} - \frac{\int \frac{(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a^{2/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} - \frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2}}{(-1)^{2/}}\right.}{3a^{2/3}\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(}$$

Mathematica [A]
time = 0.13, size = 222, normalized size = 0.84

$$\frac{(e+fx)^{1+n}\left(-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} - \frac{{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} + \frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3a^{2/3}(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(a + b*x^3),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x)
)/(b^(1/3)*e - a^(1/3)*f)]/(b^(1/3)*e - a^(1/3)*f)) - Hypergeometric2F1[1,
1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3
)*f)]/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + Hypergeometric2F1[1, 1 + n, 2 +
n, ((-1)^(1/3)*b^(1/3)*(e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)]/((-1)
^(1/3)*b^(1/3)*e + a^(1/3)*f)))/(3*a^(2/3)*(1 + n))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(b*x^3 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/(b*x**3+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(a + b*x^3),x)
[Out] int((e + f*x)^n/(a + b*x^3), x)
```

3.192 $\int \frac{(e+fx)^n}{x(a+bx^3)}\,dx$

Optimal. Leaf size=300

$$\frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}+\frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3a\left(\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f\right)(1+n)}$$

```
[Out] 1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)
*e-a^(1/3)*f))/a/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hype
rgeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e+(-1)^(1/3)*a^(1/3)*f))/a/(b
^(1/3)*e+(-1)^(1/3)*a^(1/3)*f)/(1+n)+1/3*b^(1/3)*(f*x+e)^(1+n)*hypergeom([1
, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3)*e-(-1)^(2/3)*a^(1/3)*f))/a/(b^(1/3)*e
-(-1)^(2/3)*a^(1/3)*f)/(1+n)-(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],1+f*x/e
)/a/e/(1+n)
```

Rubi [A]
time = 0.45, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6857, 67, 70}

$$\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,f+\sqrt[3]{b}\,e\right)}+\frac{\sqrt[3]{b}\,(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{3a(n+1)\left(\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f\right)}-\frac{(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{fx}{e}+1\right)}{ae(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e +
 f*x))/(b^(1/3)*e - a^(1/3)*f)])/(3*a*(b^(1/3)*e - a^(1/3)*f)*(1 + n)) + (b
^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f
*x))/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f)])/(3*a*(b^(1/3)*e + (-1)^(1/3)*a^(1
/3)*f)*(1 + n)) + (b^(1/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/(3*a*(b^(1/3)
*e - (-1)^(2/3)*a^(1/3)*f)*(1 + n)) - ((e + f*x)^(1 + n)*Hypergeometric2F1[
1, 1 + n, 2 + n, 1 + (f*x)/e])/(a*e*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx = \int \left(\frac{(e+fx)^n}{ax} - \frac{bx^2(e+fx)^n}{a\,(a+bx^3)}\right)\,dx$$

$$= \frac{\int \frac{(e+fx)^n}{x}\,dx}{a} - \frac{b\int \frac{x^2(e+fx)^n}{a+bx^3}\,dx}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{b\int \left(\frac{(e+fx)^n}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(e+fx)^n}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\right.}\right.}{a}$$

$$= -\frac{(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{ae(1+n)} - \frac{\sqrt[3]{b}\,\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a} - \frac{\sqrt[3]{b}\,\int \frac{(e+fx)^n}{-\sqrt[3]{-1}\,\sqrt[3]{a}}}{3a}$$

$$= \frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} + \frac{\sqrt[3]{b}\,(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+\right.}{3a\left(\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\right.}$$

Mathematica [A]
time = 0.29, size = 244, normalized size = 0.81

$$\frac{(e+fx)^{1+n}\left(\frac{\sqrt[3]{b}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e+\sqrt[3]{-1}\,\sqrt[3]{a}\,f} + \frac{\sqrt[3]{b}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-(-1)^{2/3}\sqrt[3]{a}\,f} - \frac{3\,{}_2F_1\left(1,1+n;2+n;1+\frac{fx}{e}\right)}{e}\right)}{3a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*((b^(1/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e
 + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f) + (b^(1/3)*Hyper
```

```
geometric2F1[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e + (-1)^(1/3)*a
^(1/3)*f)])/(b^(1/3)*e + (-1)^(1/3)*a^(1/3)*f) + (b^(1/3)*Hypergeometric2F1
[1, 1 + n, 2 + n, (b^(1/3)*(e + f*x))/(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f)])/
(b^(1/3)*e - (-1)^(2/3)*a^(1/3)*f) - (3*Hypergeometric2F1[1, 1 + n, 2 + n,
1 + (f*x)/e])/e))/(3*a*(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x\,(b\,x^3+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x/(b*x^3+a),x)
```

```
[Out] int((f*x+e)^n/x/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n/(b*x^4 + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e+fx)^n}{x\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x/(b*x**3+a),x)
```

```
[Out] Integral((e + f*x)**n/(x*(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x*(a + b*x^3)),x)
[Out] int((e + f*x)^n/(x*(a + b*x^3)), x)
```

3.193 $\int \frac{(e+fx)^n}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=326

$$-\frac{b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a^{4/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}+\frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e}{(-1)^{2/3}\sqrt[3]{b}\,e-}\right.}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)}$$

```
[Out] -1/3*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/3)*(f*x+e)/(b^(1/3
)*e-a^(1/3)*f))/a^(4/3)/(b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3*(-1)^(1/3)*b^(2/3)*
(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(2/3)*b^(1/3)*(f*x+e)/((-1)^(2/
3)*b^(1/3)*e-a^(1/3)*f))/a^(4/3)/((-1)^(2/3)*b^(1/3)*e-a^(1/3)*f)/(1+n)+1/3
*(-1)^(2/3)*b^(2/3)*(f*x+e)^(1+n)*hypergeom([1, 1+n],[2+n],(-1)^(1/3)*b^(1/
3)*(f*x+e)/((-1)^(1/3)*b^(1/3)*e+a^(1/3)*f))/a^(4/3)/((-1)^(1/3)*b^(1/3)*e+
a^(1/3)*f)/(1+n)+f*(f*x+e)^(1+n)*hypergeom([2, 1+n],[2+n],1+f*x/e)/a/e^2/(1
+n)
```

Rubi [A]

time = 0.48, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6857, 67, 70}

$$-\frac{b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a^{4/3}(n+1)\left((-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)}+\frac{(-1)^{2/3}b^{2/3}(e+fx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{3a^{4/3}(n+1)\left(\sqrt[3]{a}\,f+\sqrt[3]{-1}\,\sqrt[3]{b}\,e\right)}+\frac{f(e+fx)^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{fx}{e}+1\right)}{ae^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] -1/3*(b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)
*(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(a^(4/3)*(b^(1/3)*e - a^(1/3)*f)*(1 +
 n)) + ((-1)^(1/3)*b^(2/3)*(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
+ n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/(3
*a^(4/3)*((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)*(1 + n)) + ((-1)^(2/3)*b^(2/3)*
(e + f*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(e
 + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/(3*a^(4/3)*((-1)^(1/3)*b^(1/3
)*e + a^(1/3)*f)*(1 + n)) + (f*(e + f*x)^(1 + n)*Hypergeometric2F1[2, 1 + n
, 2 + n, 1 + (f*x)/e])/(a*e^2*(1 + n))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^n}{x^2\,(a+bx^3)}\,dx &= \int \left(\frac{(e+fx)^n}{ax^2} - \frac{bx(e+fx)^n}{a\,(a+bx^3)}\right)dx \\
&= \frac{\int \frac{(e+fx)^n}{x^2}\,dx}{a} - \frac{b\int \frac{x(e+fx)^n}{a+bx^3}\,dx}{a} \\
&= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} - \frac{b\int\left(-\frac{(e+fx)^n}{3\sqrt[3]{a}\,\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{(\cdots}{3\sqrt[3]{a}\,\sqrt[3]{b}\cdots}\right.}{\cdots} \\
&= \frac{f(e+fx)^{1+n}\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{ae^2(1+n)} + \frac{b^{2/3}\int \frac{(e+fx)^n}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int \frac{\cdots}{\sqrt[3]{a}\cdots}}{3a^{4/\cdots}} \\
&= -\frac{b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{3a^{4/3}\left(\sqrt[3]{b}\,e-\sqrt[3]{a}\,f\right)(1+n)} + \frac{\sqrt[3]{-1}\,b^{2/3}(e+fx)^{1+n}\,{}_2F_1\left(1,1\cdots\right.}{3a^{4/3}\left((-1)^{2/3}\sqrt[3]{b}\cdots\right.}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 273, normalized size = 0.84

$$\frac{(e+fx)^{1+n}\left(-\frac{b^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} + \frac{\sqrt[3]{-1}\,b^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{(-1)^{2/3}\sqrt[3]{b}\,(e+fx)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f}\right)}{(-1)^{2/3}\sqrt[3]{b}\,e-\sqrt[3]{a}\,f} + \frac{(-1)^{2/3}b^{2/3}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[3]{-1}\,\sqrt[3]{b}\,(e+fx)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f}\right)}{\sqrt[3]{-1}\,\sqrt[3]{b}\,e+\sqrt[3]{a}\,f} + \frac{3\sqrt[3]{a}\,f\,{}_2F_1\left(2,1+n;2+n;1+\frac{fx}{e}\right)}{e^2}\right)}{3a^{4/3}(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e + f*x)^n/(x^2*(a + b*x^3)),x]
```

```
[Out] ((e + f*x)^(1 + n)*(-((b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/3)*
(e + f*x))/(b^(1/3)*e - a^(1/3)*f)])/(b^(1/3)*e - a^(1/3)*f)) + ((-1)^(1/3)
```

```
*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(2/3)*b^(1/3)*(e + f*x))/
((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f)])/((-1)^(2/3)*b^(1/3)*e - a^(1/3)*f) + (
(-1)^(2/3)*b^(2/3)*Hypergeometric2F1[1, 1 + n, 2 + n, ((-1)^(1/3)*b^(1/3)*(
e + f*x))/((-1)^(1/3)*b^(1/3)*e + a^(1/3)*f)])/((-1)^(1/3)*b^(1/3)*e + a^(1
/3)*f) + (3*a^(1/3)*f*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f*x)/e])/e^2)
)/(3*a^(4/3)*(1 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx+e)^n}{x^2\,(b\,x^3+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^n/x^2/(b*x^3+a),x)
[Out] int((f*x+e)^n/x^2/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="fricas")
[Out] integral((f*x + e)^n/(b*x^5 + a*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**n/x**2/(b*x**3+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^n/x^2/(b*x^3+a),x, algorithm="giac")
[Out] integrate((f*x + e)^n/((b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e+f\,x)^n}{x^2\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)^n/(x^2*(a + b*x^3)),x)
[Out] int((e + f*x)^n/(x^2*(a + b*x^3)), x)
```

3.194 $\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx$

Optimal. Leaf size=253

$$-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-\sqrt[3]{a}\,d}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d\right)(2+n)}-\frac{(c+\cdots}{\cdots}$$

```
[Out] -1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c-a^(1
/3)*d))/b^(2/3)/(b^(1/3)*c-a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1,
 2+n],[3+n],b^(1/3)*(d*x+c)/(b^(1/3)*c+(-1)^(1/3)*a^(1/3)*d))/b^(2/3)/(b^(1
/3)*c+(-1)^(1/3)*a^(1/3)*d)/(2+n)-1/3*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n
],b^(1/3)*(d*x+c)/(b^(1/3)*c-(-1)^(2/3)*a^(1/3)*d))/b^(2/3)/(b^(1/3)*c-(-1)
^(2/3)*a^(1/3)*d)/(2+n)
```

Rubi [A]
time = 0.41, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6857, 70}

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-\sqrt[3]{a}\,d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{-1}\,\sqrt[3]{a}\,d+\sqrt[3]{b}\,c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-(-1)^{2/3}\sqrt[3]{a}\,d}\right)}{3b^{2/3}(n+2)\left(\sqrt[3]{b}\,c-(-1)^{2/3}\sqrt[3]{a}\,d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] -1/3*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*
x))/(b^(1/3)*c - a^(1/3)*d)])/(b^(2/3)*(b^(1/3)*c - a^(1/3)*d)*(2 + n)) - (
(c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b
^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c + (-1)^(1/3)*a^(1/
3)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(
1/3)*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)])/(3*b^(2/3)*(b^(1/3)*c
- (-1)^(2/3)*a^(1/3)*d)*(2 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

[n, 0]

Rubi steps

$$\int \frac{x^2(c+dx)^{1+n}}{a+bx^3}\,dx = \int \left(\frac{(c+dx)^{1+n}}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left(-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(c+dx)^{1+n}}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{\vphantom{b}}\right.}$$

$$= \frac{\int \frac{(c+dx)^{1+n}}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{-\sqrt[3]{-1}\,\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}} + \frac{\int \frac{(c+dx)^{1+n}}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^{2/3}}$$

$$= -\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-\sqrt[3]{a}\,d}\right)}{3b^{2/3}\left(\sqrt[3]{b}\,c-\sqrt[3]{a}\,d\right)(2+n)} - \frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\right.}{3b^{2/3}\left(\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\right.}$$

Mathematica [A]
time = 0.31, size = 213, normalized size = 0.84

$$\frac{(c+dx)^{2+n}\left(-\frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-\sqrt[3]{a}\,d}\right)}{\sqrt[3]{b}\,c-\sqrt[3]{a}\,d} - \frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d}\right)}{\sqrt[3]{b}\,c+\sqrt[3]{-1}\,\sqrt[3]{a}\,d} - \frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[3]{b}\,(c+dx)}{\sqrt[3]{b}\,c-(-1)^{2/3}\sqrt[3]{a}\,d}\right)}{\sqrt[3]{b}\,c-(-1)^{2/3}\sqrt[3]{a}\,d}\right)}{3b^{2/3}(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x)^(1 + n))/(a + b*x^3),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)*(c + d*x)
)/(b^(1/3)*c - a^(1/3)*d)]/(b^(1/3)*c - a^(1/3)*d)) - Hypergeometric2F1[1,
2 + n, 3 + n, (b^(1/3)*(c + d*x))/(b^(1/3)*c + (-1)^(1/3)*a^(1/3)*d)]/(b^(1
/3)*c + (-1)^(1/3)*a^(1/3)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/3)
*(c + d*x))/(b^(1/3)*c - (-1)^(2/3)*a^(1/3)*d)]/(b^(1/3)*c - (-1)^(2/3)*a^(
1/3)*d)))/(3*b^(2/3)*(2 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^2(dx+c)^{1+n}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

```
[Out] int(x^2*(d*x+c)^(1+n)/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="fricas")
[Out] integral((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x+c)**(1+n)/(b*x**3+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x+c)^(1+n)/(b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x + c)^(n + 1)*x^2/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(c+d\,x\right)}^{n+1}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3),x)
[Out] int((x^2*(c + d*x)^(n + 1))/(a + b*x^3), x)
```

3.195 $\int \frac{x^m(e+fx)^n}{a+bx^3} \, dx$

Optimal. Leaf size=211

$$\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(1+m)}+\frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;\right.}{3a(1+m}$$

```
[Out] 1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,-b^(1/3)*x/a^(1/3),-f*x/e)/a/(1
+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF1(1+m,1,-n,2+m,(-1)^(1/3)*b^
(1/3)*x/a^(1/3),-f*x/e)/a/(1+m)/((1+f*x/e)^n)+1/3*x^(1+m)*(f*x+e)^n*AppellF
1(1+m,-n,1,2+m,-f*x/e,-(-1)^(2/3)*b^(1/3)*x/a^(1/3))/a/(1+m)/((1+f*x/e)^n)
```

Rubi [A]
time = 0.39, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6857, 140, 138}

$$\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},\frac{\sqrt[3]{-1}\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(m+1)}+\frac{x^{m+1}(e+fx)^n\left(\frac{fx}{e}+1\right)^{-n}F_1\left(m+1;-n,1;m+2;-\frac{fx}{e},-\frac{(-1)^{2/3}\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e + f*x)^n)/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2 + m, -((f*x)/e), -((b^(1/3)
*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*Appel
lF1[1 + m, -n, 1, 2 + m, -((f*x)/e), ((-1)^(1/3)*b^(1/3)*x)/a^(1/3)])/(3*a*
(1 + m)*(1 + (f*x)/e)^n) + (x^(1 + m)*(e + f*x)^n*AppellF1[1 + m, -n, 1, 2
+ m, -((f*x)/e), -(((-1)^(2/3)*b^(1/3)*x)/a^(1/3))])/(3*a*(1 + m)*(1 + (f*x
)/e)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx = \int \left(-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x\right)}-\frac{x^m(e+fx)^n}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2}\right.}\right.$$

$$= -\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}-\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}+\sqrt[3]{-1}\,\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}-\frac{\int \frac{x^m(e+fx)^n}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}$$

$$= -\frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)^n}{-\sqrt[3]{a}-\sqrt[3]{b}\,x}\,dx}{3a^{2/3}}-\frac{\left((e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}\right)\int \frac{x^m\left(1+\frac{fx}{e}\right)}{-\sqrt[3]{a}+\sqrt[3]{-1}}}{3a^{2/3}}$$

$$= \frac{x^{1+m}(e+fx)^n\left(1+\frac{fx}{e}\right)^{-n}F_1\left(1+m;-n,1;2+m;-\frac{fx}{e},-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3a(1+m)}+\frac{x^{1+m}(e+fx)^n\,(}{}$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^m(e+fx)^n}{a+bx^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3),x]
[Out] Integrate[(x^m*(e + f*x)^n)/(a + b*x^3), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^m(fx+e)^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(f*x+e)^n/(b*x^3+a),x)
[Out] int(x^m*(f*x+e)^n/(b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(f*x+e)**n/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(f*x+e)^n/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^n*x^m/(b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^m\,{\left(e+f\,x\right)}^n}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e + f*x)^n)/(a + b*x^3),x)
```

```
[Out] int((x^m*(e + f*x)^n)/(a + b*x^3), x)
```

3.196 $\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$

Optimal. Leaf size=1480

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2a\sqrt[3]{d}\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{b^2c^{2/3}+ab\sqrt[3]{c}\sqrt[3]{d}+a^2d^{2/3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\dots\right.}{}$$

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2*a*d^(1/3)*(d*x^3+c)^(1/2)/b^2/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))+3^(1/4)*a*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)
*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2
*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+2/3*a*d^(1/3)*(c^(1/3)+d^(1/3)*
x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
),I*3^(1/2)+2*I)*(a*d^(1/3)+b*c^(1/3)*(1-3^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/b^3/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2/3*(-a^3*d+b^3*c)*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^3/(-a*d^(1/3)+b*c^(1/3
)*(1+3^(1/2)))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)-c^(1/6)*(c^(1/3)+d^(1/3)*x)*arctanh(1/3*(b^2*c^(2
/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*(1-(d^
(1/3)*x+c^(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(
3/4)/c^(1/6)/b^(1/2)/(b*c^(1/3)-a*d^(1/3))^(1/2)/(7-4*3^(1/2)+(d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2))*(b*c^(1/3)-a
*d^(1/3))^(1/2)*(b^2*c^(2/3)+a*b*c^(1/3)*d^(1/3)+a^2*d^(2/3))^(1/2)*(c^(2/3
)*(1-d^(1/3)*x/c^(1/3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)/b^(5/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))^2)^(1/2)-4*3^(1/4)*c^(1/3)*(-a^3*d+b^3*c)*(c^(1/3)+d^(1
/3)*x)*EllipticPi((-d^(1/3)*x-c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),(-a*d^(1/3)+b*c^(1/3)*(1+3^(1/2)))^2/(-a*d^(1/3)+b*c^(1/3)*(1-3^(1/
2)))^2,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(c^(2/3)*(1-d^(1/3)*x/c^(1/
```

```
3)+d^(2/3)*x^2/c^(2/3))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^2/(2*b^2
*c^(2/3)+2*a*b*c^(1/3)*d^(1/3)-a^2*d^(2/3))/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 1.98, antiderivative size = 1480, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {2173, 267, 2161, 224, 2167, 2138, 551, 585, 95, 214, 1892, 1891}

$$\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\sqrt[3]{d}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}E\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)a}{b^2\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}a+\left(1-\sqrt{3}\right)b\sqrt[3]{c}\right)\sqrt[3]{d}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)a}{\sqrt[4]{3}b^3\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{2\sqrt[3]{d}\sqrt{dx^3+c}\,a}{b^2\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)}-\frac{\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{d^{2/3}a^2+b\sqrt[3]{c}\sqrt[3]{d}a+b^2c^{2/3}}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\tanh^{-1}\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{d^{2/3}a^2+b\sqrt[3]{c}\sqrt[3]{d}a+b^2c^{2/3}}\sqrt{1-\frac{\left(\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}}{\sqrt[4]{3}\sqrt{b}\sqrt[6]{c}\sqrt{b\sqrt[3]{c}-a\sqrt[3]{d}}\sqrt{\frac{\left(\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}\right)^2}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}-4\sqrt{3}+7}}\right)}{b^{5/2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}-\frac{2\sqrt{2+\sqrt{3}}\left(b^3c-a^3d\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{d^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}F\left(\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^3\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}+\frac{4\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{c}\left(b^3c-a^3d\right)\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)\sqrt{\frac{c^{2/3}\left(\frac{d^{2/3}x^2}{c^{2/3}}-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}+1\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\Pi\left(\frac{\left(\left(1+\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2}{\left(\left(1-\sqrt{3}\right)b\sqrt[3]{c}-a\sqrt[3]{d}\right)^2};\text{ArcSin}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{b^2\left(-d^{2/3}a^2+2b\sqrt[3]{c}\sqrt[3]{d}a+2b^2c^{2/3}\right)\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{d}x+\sqrt[3]{c}\right)}{\left(\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}\right)^2}}\sqrt{dx^3+c}}+\frac{2\sqrt{dx^3+c}}{3b}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*a*d^(1/3)*Sqrt[c + d*x^3])/(b^2*((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)) - (c^(1/6)*Sqrt[b*c^(1/3) - a*d^(1/3)]*Sqrt[b^2*c^
(2/3) + a*b*c^(1/3)*d^(1/3) + a^2*d^(2/3)]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2
/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*ArcTanh[(Sqrt[2 - Sqrt[3]]*Sqrt[b^2*c^(2/3) + a*b*c^(1/3
)*d^(1/3) + a^2*d^(2/3)]*Sqrt[1 - ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])/(3^(1/4)*Sqrt[b]*c^(1/6)*Sqrt[b*c^(1/3
) - a*d^(1/3)]*Sqrt[7 - 4*Sqrt[3] + ((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)^2/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2])])/(b^(5/2)*Sqrt[(c^(1/3)*(c^(1/3) +
d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(1
/4)*Sqrt[2 - Sqrt[3]]*a*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^2*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2 + Sqrt[
3]]*a*((1 - Sqrt[3])*b*c^(1/3) + a*d^(1/3))*d^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (2*Sqrt[2 + Sqrt[3]]*(b^3*c - a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^3*((1 + Sqrt[3])*b*c^(1/3) - a*
d^(1/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) + (4*3^(1/4)*Sqrt[2 + Sqrt[3]]*c^(1/3)*(b^3*c -
a^3*d)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)*(1 - (d^(1/3)*x)/c^(1/3) + (d^(2
/3)*x^2)/c^(2/3)))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticPi[((1 +
Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2/((1 - Sqrt[3])*b*c^(1/3) - a*d^(1/3))^2,
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(b^2*(2*b^2*c^(2/3) + 2*a*b*c^(1/3)*d^(1/3) - a^2*d^
```

```
(2/3))*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 1892

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(c*r - (1 - Sqrt[3])*d*s)/r,
 Int[1/Sqrt[a + b*x^3], x], x] + Dist[d/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt
[a + b*x^3], x], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && NeQ[b*c^3 - 2*
(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 2161

```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[-q/((1 + Sqrt[3])*d - c*q), Int[1/Sqrt[a + b*x^3], x],
 x] + Dist[d/((1 + Sqrt[3])*d - c*q), Int[(1 + Sqrt[3] + q*x)/((c + d*x)*Sq
rt[a + b*x^3]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2*c^6 - 20*a*b*c
^3*d^3 - 8*a^2*d^6, 0]
```

Rule 2167

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> With[{q = Simplify[(1 + Sqrt[3])*(f/e)]}, Dist[4*3^(1/4)*Sqrt[2
- Sqrt[3]]*f*(1 + q*x)*(Sqrt[(1 - q*x + q^2*x^2)/(1 + Sqrt[3] + q*x)^2]/(q*
Sqrt[a + b*x^3]*Sqrt[(1 + q*x)/(1 + Sqrt[3] + q*x)^2])), Subst[Int[1/(((1 -
 Sqrt[3])*d - c*q + ((1 + Sqrt[3])*d - c*q)*x)*Sqrt[1 - x^2]*Sqrt[7 - 4*Sqr
t[3] + x^2]), x], x, (-1 + Sqrt[3] - q*x)/(1 + Sqrt[3] + q*x)], x]] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*e^3 - 2*(5 + 3*Sqrt[
3])*a*f^3, 0] && NeQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2173

```
Int[Sqrt[(a_) + (b_.)*(x_)^3]/((c_) + (d_.)*(x_)), x_Symbol] :> Dist[b/d, I
nt[x^2/Sqrt[a + b*x^3], x], x] + (-Dist[(b*c^3 - a*d^3)/d^3, Int[1/((c + d*
```

```
x)*Sqrt[a + b*x^3]), x], x] + Dist[b*(c/d^3), Int[(c - d*x)/Sqrt[a + b*x^3]
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c^3 - a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx = \frac{(ad)\int \frac{a-bx}{\sqrt{c+dx^3}}\,dx}{b^3} + \frac{d\int \frac{x^2}{\sqrt{c+dx^3}}\,dx}{b} - \left(-c+\frac{a^3 d}{b^3}\right)\int \frac{1}{(a+bx)\sqrt{c+dx^3}}\,dx$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{(ad^{2/3})\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{b^2} + \frac{\left(a\left(a+\frac{\left(1-\sqrt{3}\right)b\sqrt[3]{c}}{\sqrt[3]{d}}\right)d\right)\int \frac{1}{\sqrt{c+}}}{b^3}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\,\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b^2}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\,\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b^2}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\,\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{b^2}$$

$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2a\sqrt[3]{d}\,\sqrt{c+dx^3}}{b^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,a\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.41, size = 820, normalized size = 0.55

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x),x]
```

```
[Out] (2*(c + d*x^3 - (3^(3/4)*a^2*d^(2/3)*((-1)^(1/3)*c^(1/3) - d^(1/3)*x)*Sqrt[
(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[(-1)^(1/6) - (I*d^(1
/3)*x)/c^(1/3)]*EllipticF[ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1
+ (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(b^2*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1
/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) + (3^(3/4)*a*c^(1/3)*d^(1/3)*((-1)^(1/3
)*c^(1/3) - d^(1/3)*x)*Sqrt[I + Sqrt[3] - ((2*I)*d^(1/3)*x)/c^(1/3)]*Sqrt[(
I*(1 + (d^(1/3)*x)/c^(1/3)))/(3*I + Sqrt[3])]*((-1 + (-1)^(2/3))*EllipticE[
ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)/(-1 +
(-1)^(1/3))] + EllipticF[ArcSin[Sqrt[(-1)^(1/6) - (I*d^(1/3)*x)/c^(1/3)]/3^
(1/4)], (-1)^(1/3)/(-1 + (-1)^(1/3))]))/(b*Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]) - ((3*I)*b*c^(4/3)*Sqrt[(c^(1/3) + d^(1/
3)*x)/((1 + (-1)^(1/3))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x
^2)/c^(2/3)]*EllipticPi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(
1/3)), ArcSin[Sqrt[(c^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/
3))]], (-1)^(1/3)])/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)) + ((-1)^(1/3)*Sqrt[3
]*(1 + (-1)^(1/3))*a^3*c^(1/3)*d*Sqrt[(c^(1/3) + d^(1/3)*x)/((1 + (-1)^(1/3
))*c^(1/3))]*Sqrt[1 - (d^(1/3)*x)/c^(1/3) + (d^(2/3)*x^2)/c^(2/3)]*Elliptic
Pi[(I*Sqrt[3]*b*c^(1/3))/((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)), ArcSin[Sqrt[(c
^(1/3) + (-1)^(2/3)*d^(1/3)*x)/((1 + (-1)^(1/3))*c^(1/3))]], (-1)^(1/3)])/(
b^2*((-1)^(1/3)*b*c^(1/3) + a*d^(1/3)))))/(3*b*Sqrt[c + d*x^3])
```

Maple [A]
time = 0.64, size = 1126, normalized size = 0.76

method	result	size
default	Expression too large to display	1126
elliptic	Expression too large to display	1126
risch	Expression too large to display	1137

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b-2/3*I*a^2/b^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*
d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*
((x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/
```

```
3)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1
/2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
/d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))
^(1/2),(I*3^(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(
-c*d^2)^(1/3)))^(1/2))+2/3*I*a/b^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d
^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*(
(x-1/d*(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/
2)*d/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)*((-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^
(1/2)/d*(-c*d^2)^(1/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/
2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*
(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2
))+1/d*(-c*d^2)^(1/3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*
I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),(I*3^(1/2)/d*(-
c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2))
)+2/3*I*(a^3*d-b^3*c)/b^4*3^(1/2)/d*(-c*d^2)^(1/3)*(I*(x+1/2/d*(-c*d^2)^(1/
3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2)*((x-1/d*
(-c*d^2)^(1/3))/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/
2)*(-I*(x+1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-
c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*
(-c*d^2)^(1/3)+a/b)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/d*(-c*d^2)^(1/3)-1/2*I
*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^(1/2),I*3^(1/2)/d*(-c*
d^2)^(1/3)/(-1/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)+a/b),(I*3^
(1/2)/d*(-c*d^2)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3
)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(a + b*x), x)
```

3.197 $\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$

Optimal. Leaf size=135

$$\frac{(d^3+e^3x^3)^p \left(1+\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d}\right)^{-p} \left(1-\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)^{-p} F_1\left(p;-p,-p;1+p;-\frac{2(d+ex)}{\left(-3+i\sqrt{3}\right)d},\frac{2(d+ex)}{\left(3+i\sqrt{3}\right)d}\right)}{ep}$$

```
[Out] (e^3*x^3+d^3)^p*AppellF1(p,-p,-p,1+p,-2*(e*x+d)/d/(-3+I*3^(1/2)),2*(e*x+d)/
d/(3+I*3^(1/2)))/e/p/((1+2*(e*x+d)/d/(-3+I*3^(1/2)))^p)/((1-2*(e*x+d)/d/(3+
I*3^(1/2)))^p)
```

Rubi [F]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d^3 + e^3*x^3)^p/(d + e*x),x]
[Out] Defer[Int][(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Rubi steps

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx = \int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$

Mathematica [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(d^3+e^3x^3)^p}{d+ex} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d^3 + e^3*x^3)^p/(d + e*x),x]
[Out] Integrate[(d^3 + e^3*x^3)^p/(d + e*x), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(e^3x^3+d^3)^p}{ex+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

```
[Out] int((e^3*x^3+d^3)^p/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate((x^3*e^3 + d^3)^p/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral((x^3*e^3 + d^3)^p/(x*e + d), x)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 636 vs. 2(102) = 204.
time = 27.23, size = 636, normalized size = 4.71

$$\frac{0^p \log\left(1+\frac{e^3x^3}{d^3}\right)\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)}{4\pi^2 e} + \frac{0^p e^{\frac{i\pi}{3}}\log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)} + \frac{0^p e^{\frac{2i\pi}{3}}\log\left(1-\frac{exe^{\frac{i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} - \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)} + \frac{0^p \log\left(1-\frac{exe^{i\pi}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} + \frac{0^p e^{-\frac{2i\pi}{3}}\log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma^3\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)}{12\pi^2 e\Gamma\left(\frac{4}{3}\right)} + \frac{0^p e^{-\frac{i\pi}{3}}\log\left(1-\frac{exe^{\frac{5i\pi}{3}}}{d}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)}{6\pi^2 e\Gamma\left(\frac{5}{3}\right)} - \frac{d^2e^{3p}px^{3p}\Gamma\left(-\frac{2}{3}\right)\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{5}{3}\right)\Gamma(p)\Gamma\left(\frac{2}{3}-p\right){}_2F_1\left(\begin{matrix}1-p, \frac{2}{3}-p \\ \frac{5}{3}-p\end{matrix}\middle| \frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2e^3x^2\Gamma\left(\frac{5}{3}-p\right)\Gamma(p+1)} - \frac{de^{3p}px^{3p}\Gamma\left(-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right)\Gamma(p)\Gamma\left(\frac{1}{3}-p\right){}_2F_1\left(\begin{matrix}1-p, \frac{1}{3}-p \\ \frac{4}{3}-p\end{matrix}\middle| \frac{d^3e^{i\pi}}{e^3x^3}\right)}{4\pi^2e^2x\Gamma\left(\frac{4}{3}-p\right)\Gamma(p+1)} - \frac{d^{3p}e^2x^3\Gamma^2\left(\frac{1}{3}\right)\Gamma^2\left(\frac{2}{3}\right)\Gamma(p)\Gamma(1-p){}_3F_2\left(\begin{matrix}2,1,1-p \\ 2,2\end{matrix}\middle| \frac{e^3x^3e^{i\pi}}{d^3}\right)}{4\pi^2d^3\Gamma(-p)\Gamma(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e**3*x**3+d**3)**p/(e*x+d),x)
```

```
[Out] 0**p*log(1 + e**3*x**3/d**3)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)/
(4*pi**2*e) + 0**p*exp(I*pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(-1/3)
*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*exp(2*I*
pi/3)*log(1 - e*x*exp_polar(I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**
2*e*gamma(4/3)) - 0**p*log(1 - e*x*exp_polar(I*pi)/d)*gamma(-1/3)*gamma(1/3
)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3)) + 0**p*log(1 - e*x*exp_po
```

```
lar(I*pi)/d)*gamma(1/3)**3*gamma(2/3)**2/(12*pi**2*e*gamma(4/3)) + 0**p*exp
(-2*I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)/d)*gamma(1/3)**3*gamma(2/3)**2/
(12*pi**2*e*gamma(4/3)) + 0**p*exp(-I*pi/3)*log(1 - e*x*exp_polar(5*I*pi/3)
/d)*gamma(-1/3)*gamma(1/3)*gamma(2/3)**2*gamma(4/3)/(6*pi**2*e*gamma(5/3))
- d**2*e**(3*p)*p*x**(3*p)*gamma(-2/3)*gamma(-1/3)*gamma(4/3)*gamma(5/3)*ga
mma(p)*gamma(2/3 - p)*hyper((1 - p, 2/3 - p), (5/3 - p,), d**3*exp_polar(I*
pi)/(e**3*x**3))/(4*pi**2*e**3*x**2*gamma(5/3 - p)*gamma(p + 1)) - d*e**(3*
p)*p*x**(3*p)*gamma(-1/3)*gamma(1/3)*gamma(2/3)*gamma(4/3)*gamma(p)*gamma(1
/3 - p)*hyper((1 - p, 1/3 - p), (4/3 - p,), d**3*exp_polar(I*pi)/(e**3*x**3
))/(4*pi**2*e**2*x*gamma(4/3 - p)*gamma(p + 1)) - d**(3*p)*e**2*x**3*gamma(
1/3)**2*gamma(2/3)**2*gamma(p)*gamma(1 - p)*hyper((2, 1, 1 - p), (2, 2), e*
*3*x**3*exp_polar(I*pi)/d**3)/(4*pi**2*d**3*gamma(-p)*gamma(p + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e^3*x^3+d^3)^p/(e*x+d),x, algorithm="giac")
[Out] integrate((x^3*e^3 + d^3)^p/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d^3+e^3\,x^3)^p}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^3 + e^3*x^3)^p/(d + e*x),x)
[Out] int((d^3 + e^3*x^3)^p/(d + e*x), x)
```

3.198 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=16

$$2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)$$

```
[Out] 2*arctan((1+x)/(x^3+1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2171, 209}

$$2\text{ArcTan}\left(\frac{x+1}{\sqrt{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\\ &= 2\tan^{-1}\left(\frac{1+x}{\sqrt{1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.88, size = 23, normalized size = 1.44

$$2\tan^{-1}\left(\frac{\sqrt{1+x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] 2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.41, size = 1640, normalized size = 102.50

method	result	size
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^2-2x\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{x^3+1}}{x^2+2}\right)$	46
default	Expression too large to display	1640
elliptic	Expression too large to display	1845

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2
))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I
*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))
*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+
1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+
1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(
1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)
/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*
I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/
(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1
/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/
2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1
/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I
*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

```
+I*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(
1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*
(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2
))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I
*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/
(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*
3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/
(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2
*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+I*2
^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(
-1+I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+I*(1/(3/2-
1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2
/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/
2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3+1)^(1/2)/(-1+I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^
(1/2),(-3/2+1/2*I*3^(1/2))/(-1+I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I
*3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3
^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(
-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*
I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+I*2^
(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-
1+I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*(1/
(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*
x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3
/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(
1/2))^(1/2)/(x^3+1)^(1/2)/(-1+I*2^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/
2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+I*2^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-
1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A]
time = 0.48, size = 19, normalized size = 1.19

$$-\arctan\left(\frac{x^2-2\,x}{2\,\sqrt{x^3+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/
(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3
+ 1) + 2*sqrt(x**3 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B]
time = 0.20, size = 273, normalized size = 17.06

$$\frac{\left(3+\sqrt{3}\ 1\mathrm{i}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{1+\sqrt{2}\ 1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-1+\sqrt{2}\ 1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\ 1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)
*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/
2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1),
 asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^
(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)
), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*
1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2))
, -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*(((3^(1/2)*1i)
/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2))^(1/2)
```

3.199 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx$

Optimal. Leaf size=20

$$-2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

```
[Out] -2*arctan((1-x)/(-x^3+1)^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2171, 209}

$$-2\text{ArcTan}\left(\frac{1-x}{\sqrt{1-x^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{1-x}{\sqrt{1-x^3}}\right)\right)\\
&= -2\tan^{-1}\left(\frac{1-x}{\sqrt{1-x^3}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.91, size = 23, normalized size = 1.15

$$-2\tan^{-1}\left(\frac{\sqrt{1-x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] -2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.40, size = 732, normalized size = 36.60 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*
(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*
I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^
(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/
2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(
1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*
3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)
/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)
^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)
*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2
^(1/2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/
2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1
/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
+1)^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/
2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2)),(I*3^(1
/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Fricas [A]
time = 0.50, size = 28, normalized size = 1.40

$$-\arctan\left(\frac{\sqrt{-x^3+1}\,(x^2+2\,x)}{2\,(x^3-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int\frac{x^2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int\left(-\frac{2}{x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2
/(x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(x**2*sqrt(1 -
x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)
```

Mupad [B]
time = 2.83, size = 292, normalized size = 14.60

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1
i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(2^(1/2)*1i + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/
2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + e
llipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2
)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(
(1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/
2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

3.200 $\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=18

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)$$

```
[Out] -2*arctanh((1-x)/(x^3-1)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2171, 212}

$$-2\tanh^{-1}\left(\frac{1-x}{\sqrt{x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\right)\\ &= -2\tanh^{-1}\left(\frac{1-x}{\sqrt{-1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.92, size = 21, normalized size = 1.17

$$2\tanh^{-1}\left(\frac{\sqrt{-1+x^3}}{1+x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] 2*ArcTanh[Sqrt[-1 + x^3]/(1 + x + x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.36, size = 1656, normalized size = 92.00

method	result	size
trager	$-\ln\left(\frac{-x^2+2\sqrt{x^3-1}-2x}{x^2+2}\right)$	30
default	Expression too large to display	1656
elliptic	Expression too large to display	1865

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-3*(-1/(-3/2-1/2*I*3^(1/2))+1/(-
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2)
)-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/
2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I
*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/
2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(-1/(
-3/2-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*
x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+
1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))
^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2))
)^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*
3^(1/2)))^(1/2))*3^(1/2)+3*I*2^(1/2)*(-1/(-3/2-1/2*I*3^(1/2))+1/(-3/2-1/2*I
*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(
3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3
^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+
1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(-I*2
^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(-1/(-3/
2-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1
```

```
/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2
*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1
/2)/(x^3-1)^(1/2)/(-I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(
1/2),(3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(
1/2)))^(1/2))*3^(1/2)-3*(-1/(-3/2-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^
(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I
/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(
((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(-1/(-3/2-1/2*I*3^(1/2))+1/(-
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2)
)-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/
2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*
2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2
))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3
*I*2^(1/2)*(-1/(-3/2-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2
-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2)
)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)+1)*EllipticPi(((-1+x)/(-3/2
-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),((3/2+1/2*I*3^(1/2
))/(3/2-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*(-1/(-3/2-1/2*I*3^(1/2))+1/(-3/2-1/2
*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I
/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+1/2*I
*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(I*2^(1/2)
+1)*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(I*2
^(1/2)+1),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="maxima")
[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A]
time = 0.51, size = 25, normalized size = 1.39

$$\log\left(\frac{x^2 + 2\,x + 2\,\sqrt{x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log((x^2 + 2*x + 2*sqrt(x^3 - 1))/(x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2x}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int \frac{x^2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int \left(-\frac{2}{x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2
/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3
 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B]
time = 2.77, size = 276, normalized size = 15.33

$$-\frac{\left(3+\sqrt{3}\,1i\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-1+\sqrt{2}\,1i};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)),x)
```

```
[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/
2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^
(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i +
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(
1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)
```

3.201 $\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=18

$$2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)$$

```
[Out] 2*arctanh((1+x)/(-x^3-1)^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2171, 212}

$$2\tanh^{-1}\left(\frac{x+1}{\sqrt{-x^3-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2171

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_)^3]), x_Symbol] :> Dist[-g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 +
 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[
b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\\ &= 2\tanh^{-1}\left(\frac{1+x}{\sqrt{-1-x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.92, size = 25, normalized size = 1.39

$$-2\tanh^{-1}\left(\frac{\sqrt{-1-x^3}}{1-x+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] -2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 724, normalized size = 40.22 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/
2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticP
i(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1
/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^
(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+
1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3
/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/
2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1/2*I*3
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/
2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*
x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1
/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)
))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Fricas [A]
time = 0.37, size = 28, normalized size = 1.56

$$\log\left(-\frac{x^2 - 2\,x - 2\,\sqrt{-x^3 - 1}}{x^2 + 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \frac{x^2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx - \int \left(-\frac{2}{x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**
2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-
x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

Mupad [B]
time = 0.11, size = 289, normalized size = 16.06

$$\frac{\left(3+\sqrt{3}\,1\mathrm{i}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\left(-\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)+\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-1+\sqrt{2}\,1\mathrm{i}};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)
```

```
[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1
i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipt
icPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/((- x^3
 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) -
((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))
```

3.202 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=30

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}$$

```
[Out] 2*arctan((1+x)*(1+d)^(1/2)/(x^3+1)^(1/2))/(1+d)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2170, 210}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}}\,dx &= -\left(4\text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2}\,dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [A]
time = 1.62, size = 37, normalized size = 1.23

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1+d}\sqrt{1+x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.38, size = 4397, normalized size = 146.57

method	result	size
default	Expression too large to display	4397
elliptic	Expression too large to display	4602

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(3/2-1/2*I*3^(1/2))*((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)
))^(1/2)/(x^3+1)^(1/2)*EllipticF(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1
/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/
2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(
-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*
I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-
1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
,((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2+1/2*I/(d^2-4*d-8)^(
1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3
^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2
)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1
/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Elliptic
Pi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^
2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1
/2)+3/2*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*
I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(
1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3
^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*Ellip
ticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*
(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*d+I*(
1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2)
```

```
)*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(
-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3
^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1
+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-
8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)+6/(d^2
-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/
2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1
/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1
/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2))
*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1/2*
d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
*d-2*I/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1
/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^
(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/
2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*
d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2
))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/
2)))^(1/2))*d*3^(1/2)-3*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/
2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2
*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2-4*d
-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2)
)/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2
)))^(1/2))-1/2*I*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(
-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/
2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d+1/2*(d^2-4*d-8)^(1/
2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1+1
/2*d+1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/
2))*d*3^(1/2)+12/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1
/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/
2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3
^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1
/2*(d^2-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/
2*I*3^(1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1
/2*I*3^(1/2)))^(1/2))-1/2*I*(1/(3/2-1/2*I*3^(1/2))+1/(3/2-1/2*I*3^(1/2))*x)
^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/2/(-3/2+1/2*I*3^(1/2))
+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1+1/2*d-1/2*(d^2
-4*d-8)^(1/2))*EllipticPi(((1+x)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(
1/2))/(-1+1/2*d-1/2*(d^2-4*d-8)^(1/2)),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^
(1/2)))^(1/2))*d*3^(1/2)+3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))+1/(3/
2-1/2*I*3^(1/2))*x)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2)
)-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.38, size = 181, normalized size = 6.03

$$\left[-\frac{\sqrt{-d-1}\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-d^2+4\,\sqrt{x^3+1}\,((d+2)x-x^2+d)\sqrt{-d-1}-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,(d+1)},-\frac{\arctan\left(-\frac{\sqrt{x^3+1}\,((d+2)x-x^2+d)\sqrt{d+1}}{2\,((d+1)x^3+d+1)}\right)}{\sqrt{d+1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d +
 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/
((d + 1)*x^3 + d + 1))/sqrt(d + 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \frac{x^2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\,dx-\int \left(-\frac{2}{dx\sqrt{x^3+1}+d\sqrt{x^3+1}+x^2\sqrt{x^3+1}+2\sqrt{x^3+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1)
+ 2*sqrt(x**3 + 1)), x) - Integral(x**2/(d*x*sqrt(x**3 + 1) + d*sqrt(x**3 +
 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(d*x*sqrt(x
**3 + 1) + d*sqrt(x**3 + 1) + x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B]
time = 2.82, size = 632, normalized size = 21.07

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{\sqrt{d^2-4\,d-8}}{2}-\frac{d}{2}+1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\left(d-(d-2)\left(\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)+4\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}\,\sqrt{d^2-4\,d-8}\,\left(\frac{\sqrt{d^2-4\,d-8}}{2}-\frac{d}{2}+1\right)}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\left(d-(d-2)\left(\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}\right)+4\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}\,\sqrt{d^2-4\,d-8}\,\left(\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3
/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/
2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 +
3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(x^3 - x*(((
3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*(
(3^(1/2)*1i)/2 + 1/2))^(1/2) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)
/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((
3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3
^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)
)*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1
i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*
1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 +
1)) - (2*((3^(1/2)*1i)/2 + 3/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2
 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x
+ 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(d/
2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 + (d
^2 - 4*d - 8)^(1/2)/2) + 4))/((x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i
)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2
 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1))
```

3.203 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}} \, dx$

Optimal. Leaf size=38

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

```
[Out] -2*arctan((1-x)*(1-d)^(1/2)/(-x^3+1)^(1/2))/(1-d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2170, 210}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}} \, dx &= 4\text{Subst}\left(\int \frac{1}{-2-(2-2d)x^2} \, dx, x, \frac{1-x}{\sqrt{1-x^3}}\right) \\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}
\end{aligned}$$

Mathematica [A]
time = 1.71, size = 37, normalized size = 0.97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-1+d}\sqrt{1-x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 1908, normalized size = 50.21

method	result	size
default	Expression too large to display	1908
elliptic	Expression too large to display	1919

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((-1+x)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3
^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*
(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1
/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))
*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/
2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2
)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2
)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/
2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1
/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/
2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1
/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2
)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-
```

```
1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2+4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3
^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1
/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2
*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1
/2))-1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*
(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-
8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*
3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1
/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/
2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1
/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4
*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)
,I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/
2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2
*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(1
/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/
2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1
/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/
2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))*d-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/
2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/2*I*3^(1/2))*x)^(
1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1
/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(
1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1
/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2+4*d-8)^(1/2)*3^(1/2
)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(-1/(-3/2+1/2*I*3^(1/2))+1/(-3/2+1/
2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)
/(-1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d+1
/2*(d^2+4*d-8)^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.39, size = 191, normalized size = 5.03

$$\left[\frac{\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-4\,\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{d-1}-d^2+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,\sqrt{d-1}},-\frac{\sqrt{-d+1}\,\arctan\left(-\frac{\sqrt{-x^3+1}\,((d-2)x-x^2-d)\,\sqrt{-d+1}}{2\,((d-1)x^3-d+1)}\right)}{d-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(
(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/sqrt(d -
1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d
+ 1)/((d - 1)*x^3 - d + 1))/(d - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx-\int\frac{x^2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\,dx-\int\left(-\frac{2}{dx\sqrt{1-x^3}-d\sqrt{1-x^3}+x^2\sqrt{1-x^3}+2\sqrt{1-x^3}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3)
 + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x
**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(
1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B]
time = 0.14, size = 677, normalized size = 17.82

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\left(d+(d+2)\left(\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}\right)-4\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}\,\left(\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}+1\right)\sqrt{d^2+4\,d-8}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}\right)\sqrt{x^3-1}\,\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1i}{2}}\right)\left(d+(d+2)\left(\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}\right)-4\right)}{\sqrt{1-x^3}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}\,\left(\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}+1\right)\sqrt{d^2+4\,d-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/(
(3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/
2)))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((
(3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) + (2*((3^(1
/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*
(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(
d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/2 -
 (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2 - 1/2)*((
3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1
) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/
2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/
2 + 3/2)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/
2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d +
 2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((1 - x^3)^(1/2)*(((3^(1/2)*1i)/2
 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
+ 1/2) + 1) + x^3)^(1/2)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8
)^(1/2))
```

3.204 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx$

Optimal. Leaf size=36

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

[Out] -2*arctanh((1-x)*(1-d)^(1/2)/(x^3-1)^(1/2))/(1-d)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2170, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}}\,dx &= 4\text{Subst}\left(\int \frac{1}{-2-(-2+2d)x^2}\,dx, x, \frac{1-x}{\sqrt{-1+x^3}}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{1-d}\,(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}
\end{aligned}$$

Mathematica [A]
time = 1.68, size = 35, normalized size = 0.97

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-1+d}\sqrt{-1+x^3}}{1+x+x^2}\right)}{\sqrt{-1+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]
[Out] (2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)])/Sqrt[-1 + d]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.37, size = 4437, normalized size = 123.25

method	result	size
default	Expression too large to display	4437
elliptic	Expression too large to display	4646

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-3/2-1/2*I*3^(1/2))*((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3
^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2
)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(-1/(-3/2
-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/
2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*
I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/
2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1
/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((
3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2-1/2*I*(-1/(-3/2-1/2*I*3^
(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/
2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2)
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1
)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1
/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)-3/2*(-1/(-3/2-1/2*I*3^(1/2)
)+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3
^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1
/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/
2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))
^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1
/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d+2*I/(d^2+4*d-8)^(1/2)*(-1/(-3/2-1/2*I*3^
(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/
```

```
2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2)
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1
)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1
/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I
*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d*3^(1/2)+6/(d^2+4*d-8)^(1/2)*(-1/(-3
/2-1/2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+
1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/
2*I*3^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(
1/2)/(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2
-1/2*I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*d-I*(-1/(-3/2-1/2*I*3^(1/2
))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*
3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+
1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1
/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2))
)^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(
1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(-1/(-3/2-1/2*I*3^(1/2))+1/(-3/
2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-
1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/2+
1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+1/
2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2),(
3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(3/
2-1/2*I*3^(1/2)))^(1/2))-1/2*I/(d^2+4*d-8)^(1/2)*(-1/(-3/2-1/2*I*3^(1/2))+1
/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1
/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/
(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/
(1+1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1
/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2)
)/(3/2-1/2*I*3^(1/2)))^(1/2))*d^2*3^(1/2)-12/(d^2+4*d-8)^(1/2)*(-1/(-3/2-1/
2*I*3^(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(
3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3
^(1/2))*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/
(x^3-1)^(1/2)/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*
I*3^(1/2)))^(1/2),(3/2+1/2*I*3^(1/2))/(1+1/2*d-1/2*(d^2+4*d-8)^(1/2)),((3/2
+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(-1/(-3/2-1/2*I*3^(1/2))+1/(-
3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2)
)-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/2/(3/
2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)/(1+
1/2*d+1/2*(d^2+4*d-8)^(1/2))*EllipticPi(((-1+x)/(-3/2-1/2*I*3^(1/2)))^(1/2)
,(3/2+1/2*I*3^(1/2))/(1+1/2*d+1/2*(d^2+4*d-8)^(1/2)),((3/2+1/2*I*3^(1/2))/(
3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3/2/(d^2+4*d-8)^(1/2)*(-1/(-3/2-1/2*I*3^
(1/2))+1/(-3/2-1/2*I*3^(1/2))*x)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/
2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2)
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^...
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for
more de
```

Fricas [A]
time = 0.43, size = 187, normalized size = 5.19

$$\left[-\frac{\sqrt{-d+1}\log\left(-\frac{2\,(3\,d-4)x^3-x^4-(d^2-2\,d+4)x^2-d^2+4\,\sqrt{x^3-1}\,((d-2)x-x^2-d)\sqrt{-d+1}+2\,(d^2-2\,d)x-4\,d+4}{2\,dx^3+x^4+(d^2-2\,d+4)x^2+d^2-2\,(d^2-2\,d)x-4\,d+4}\right)}{2\,(d-1)},\ -\frac{\arctan\left(-\frac{\sqrt{x^3-1}\,((d-2)x-x^2-d)\sqrt{d-1}}{2\,((d-1)x^3-d+1)}\right)}{\sqrt{d-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2
+ 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*
d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d +
 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/
((d - 1)*x^3 - d + 1))/sqrt(d - 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\,dx-\int\left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)
```

```
[Out] -Integral(-2*x/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1)
 + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(x**3 - 1) - d*sqrt(x**3
- 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(d*x*sqrt(
x**3 - 1) - d*sqrt(x**3 - 1) + x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)
```

Mupad [B]
time = 2.80, size = 629, normalized size = 17.47

$$\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}}+\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\left(d+\left(d+2\right)\left(\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}\right)-4\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}\left(\frac{d}{2}-\frac{\sqrt{d^2+4\,d-8}}{2}+1\right)\sqrt{d^2+4\,d-8}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}+1};\mathrm{asin}\left(\sqrt{-\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\right)\left(d+\left(d+2\right)\left(\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}\right)-4\right)}{\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-1\right)x+\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}\left(\frac{d}{2}+\frac{\sqrt{d^2+4\,d-8}}{2}+1\right)\sqrt{d^2+4\,d-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)
```

```
[Out] (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/
2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x -
1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin((-(x - 1)/((3^(1/2)*1i)/2 +
 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*
1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1
i)/2 + 1/2) + 1) + x^3)^(1/2) + (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1
i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(
1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi
(((3^(1/2)*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)
/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 -
3/2))*(d + (d + 2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
 1/2) + 1) + x^3)^(1/2)*(d/2 - (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)
^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*
1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/
2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2
)/(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2
))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d + (d + 2)*(d/
2 + (4*d + d^2 - 8)^(1/2)/2) - 4))/((((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2
)*(d/2 + (4*d + d^2 - 8)^(1/2)/2 + 1)*(4*d + d^2 - 8)^(1/2))
```

3.205 $\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}$$

```
[Out] 2*arctanh((1+x)*(1+d)^(1/2)/(-x^3-1)^(1/2))/(1+d)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2170, 213}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d+1}\,(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2170

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + 2*h*(x/g))/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}}\,dx &= -\left(4\text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2}\,dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}\,(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}\end{aligned}$$

Mathematica [A]
time = 1.68, size = 39, normalized size = 1.22

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{1+d}\sqrt{-1-x^3}}{1-x+x^2}\right)}{\sqrt{1+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]
[Out] (-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)])/Sqrt[1 + d]
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.33, size = 1888, normalized size = 59.00

method	result	size
default	Expression too large to display	1888
elliptic	Expression too large to display	1897

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((1+x)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1
/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^
(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/
2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2
))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/
2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)
+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/
2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(
d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^
(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/
(3/2+1/2*I*3^(1/2)))^(1/2))*d-4/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-
1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1
/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2
)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/
2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)
),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I*3^(1/2)*(I*3^(1/2)*x-1/2*I
*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(
-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2
*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3
^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2)),(I*
```

```
3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-8/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1
/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*3^(1/2))
*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*
3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)
^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-1/3*I/(d^2-4*d-8)^(1/2)*3^(1
/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2
*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/
(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(
x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*
(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d^2-1/3*I*3^(1/2)
*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/2+1/2*I*
3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/
2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1
/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^
2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+4/3*I/(d^2-4*d-8)^
(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+
1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3
-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^
(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+
1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))*d+2/3*I
*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))+1/(3/
2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(
1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)
*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d
+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+8/3*I/(d^2-4
*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(
1/2))+1/(3/2+1/2*I*3^(1/2))*x)^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)
/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2))*EllipticPi(
1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(1/2+1/2*I*3^
(1/2)+1/2*d+1/2*(d^2-4*d-8)^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

time = 0.38, size = 185, normalized size = 5.78

$$\left[\frac{\log\left(-\frac{2\,(3\,d+4)x^3-x^4-(d^2+2\,d+4)x^2-4\,\sqrt{-x^3-1}\,((d+2)x-x^2+d)\,\sqrt{d+1}-d^2-2\,(d^2+2\,d)x+4\,d+4}{2\,dx^3+x^4+(d^2+2\,d+4)x^2+d^2+2\,(d^2+2\,d)x+4\,d+4}\right)}{2\,\sqrt{d+1}}, -\frac{\sqrt{-d-1}\,\arctan\left(-\frac{\sqrt{-x^3-1}\,((d+2)x-x^2+d)\,\sqrt{-d-1}}{2\,((d+1)x^3+d+1)}\right)}{d+1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(
(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^
3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d +
1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d
 - 1)/((d + 1)*x^3 + d + 1))/(d + 1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \frac{x^2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\,dx-\int \left(-\frac{2}{dx\sqrt{-x^3-1}+d\sqrt{-x^3-1}+x^2\sqrt{-x^3-1}+2\sqrt{-x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
```

```
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 -
1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-
x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*
x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3
 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)
```

Mupad [B]
time = 0.12, size = 680, normalized size = 21.25

$$-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\Pi\left(\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{\sqrt{d^2-4\,d-8}}{2}-\frac{d}{2}+1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)\left(d-(d-2)\left(\frac{d}{2}-\frac{\sqrt{d^2-4\,d-8}}{2}\right)+4\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}\,\sqrt{d^2-4\,d-8}\,\left(\frac{\sqrt{d^2-4\,d-8}}{2}-\frac{d}{2}+1\right)}-\frac{2\left(\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\sqrt{x^3+1}\,\sqrt{\frac{x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\sqrt{\frac{\frac{1}{2}-x+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\,\Pi\left(-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}-1};\mathrm{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}}\right)\middle|-\frac{\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}{-\frac{3}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}}\right)\left(d-(d-2)\left(\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}\right)+4\right)}{\sqrt{-x^3-1}\,\sqrt{x^3+\left(-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-1\right)x-\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}\,\sqrt{d^2-4\,d-8}\,\left(\frac{d}{2}+\frac{\sqrt{d^2-4\,d-8}}{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
```

```
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3
^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2
)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((
3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2
)))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1
/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(
1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)
/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 -
x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((
d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1
/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2)*(d/2 - (
d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2
 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1)) -
 (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^
(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)
*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2
 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2
+ 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*(d - (d - 2
)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*(((3^(1
/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(
1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)
/2 - 1))
```

3.206 $\int (d+ex)^3 \sqrt{a+cx^4}\, dx$

Optimal. Leaf size=355

$$\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}+\frac{3ad^2e\tanh^{-1}\Big(}{4\sqrt{c}}$$

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3/4*a*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/
c^(1/2)+3/4*d^2*e*x^2*(c*x^4+a)^(1/2)+1/15*d*x*(9*e^2*x^2+5*d^2)*(c*x^4+a)^
(1/2)+6/5*a*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-6/5*a^(5/
4)*d*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1
/2)+1/15*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(
c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))
*(9*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^
2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1899, 1191, 1212, 226, 1210, 1262, 655, 201, 223, 212}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(9\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{6a^{5/4}de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{15}dx\sqrt{a+cx^4}\left(5d^2+9e^2x^2\right)+\frac{3}{4}d^2ex^2\sqrt{a+cx^4}+\frac{3ad^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^3\left(a+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (3*d^2*e*x^2*Sqrt[a + c*x^4])/4 + (6*a*d*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*
(Sqrt[a] + Sqrt[c]*x^2)) + (d*x*(5*d^2 + 9*e^2*x^2)*Sqrt[a + c*x^4])/15 + (
e^3*(a + c*x^4)^(3/2))/(6*c) + (3*a*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*
x^4]])/(4*Sqrt[c]) - (6*a^(5/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*d*(5*Sqrt[c]*d^2 + 9*Sqrt[a]*e^2)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int (d+ex)^3\sqrt{a+cx^4}\,dx = \int \left(\left(d^3+3de^2x^2\right)\sqrt{a+cx^4}+x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\right)dx$$

$$= \int \left(d^3+3de^2x^2\right)\sqrt{a+cx^4}\,dx + \int x\left(3d^2e+e^3x^2\right)\sqrt{a+cx^4}\,dx$$

$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{15}\int \frac{10ad^3+18ade^2x^2}{\sqrt{a+cx^4}}\,dx + \frac{1}{2}\text{Subst}\left(\int (3d^2e$$

$$= \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3\left(a+cx^4\right)^{3/2}}{6c} + \frac{1}{2}\left(3d^2e\right)\text{Subst}\left(\int \sqrt{a+cx^2}$$

$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3}{}$$

$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3}{}$$

$$= \frac{3}{4}d^2ex^2\sqrt{a+cx^4} + \frac{6ade^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{15}dx\left(5d^2+9e^2x^2\right)\sqrt{a+cx^4} + \frac{e^3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.11, size = 186, normalized size = 0.52

$$\frac{\sqrt{a+cx^4}\left(2ae^3\sqrt{1+\frac{cx^4}{a}}+9cd^2ex^2\sqrt{1+\frac{cx^4}{a}}+2ce^3x^4\sqrt{1+\frac{cx^4}{a}}+9\sqrt{a}\,\sqrt{c}\,d^2e\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+12cd^3x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+12cde^2x^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{12c\sqrt{1+\frac{cx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(2*a*e^3*Sqrt[1 + (c*x^4)/a] + 9*c*d^2*e*x^2*Sqrt[1 + (c*x
^4)/a] + 2*c*e^3*x^4*Sqrt[1 + (c*x^4)/a] + 9*Sqrt[a]*Sqrt[c]*d^2*e*ArcSinh[
(Sqrt[c]*x^2)/Sqrt[a]] + 12*c*d^3*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)] + 12*c*d*e^2*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)]))
/(12*c*Sqrt[1 + (c*x^4)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 269, normalized size = 0.76

method	result
default	$\frac{e^3(c\,x^4+a)^{\frac{3}{2}}}{6c}+3d\,e^2\left(\frac{x^3\sqrt{c\,x^4+a}}{5}+\frac{2ia^{\frac{3}{2}}\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{Ellipt}\right.}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}\right.$
elliptic	$\frac{e^3x^4\sqrt{c\,x^4+a}}{6}+\frac{3d\,e^2x^3\sqrt{c\,x^4+a}}{5}+\frac{3d^2e\,x^2\sqrt{c\,x^4+a}}{4}+\frac{d^3x\sqrt{c\,x^4+a}}{3}+\frac{a\,e^3\sqrt{c\,x^4+a}}{6c}+\frac{2a\,d^3}{}$
risch	$\frac{(10e^3x^4c+36d\,e^2x^3c+45c\,d^2e\,x^2+20c\,d^3x+10a\,e^3)\sqrt{c\,x^4+a}}{60c}+\frac{6ia^{\frac{3}{2}}d\,e^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}}{5\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*e^3*(c*x^4+a)^(3/2)/c+3*d*e^2*(1/5*x^3*(c*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I
/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2
)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2)
,I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+3*d^2*e*(1/4*x^2*(c*x^4+a)^(
1/2)+1/4*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2)))+d^3*(1/3*x*(c*x^4+a)^(1
/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^
(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1
/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(x*e + d)^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 2.30, size = 175, normalized size = 0.49

$$\frac{\sqrt{a}\, d^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{c x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{3\sqrt{a}\, d^2 e x^2 \sqrt{1 + \frac{c x^4}{a}}}{4} + \frac{3\sqrt{a}\, d e^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{c x^4 e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)} + \frac{3 a d^2 e \operatorname{asinh}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{4\sqrt{c}} + e^3 \left(\begin{cases} \frac{\sqrt{a}\, x^4}{4} & \text{for}\ c = 0 \\ \frac{\left(a + c x^4\right)^{\frac{3}{2}}}{6c} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**3*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + 3*sqrt(a)*d**2*e*x**2*sqrt(1 + c*x**4/a)/4 + 3*sqrt(a)
*d*e**2*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*gamma(7/4)) + 3*a*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(4*sqrt(c)) + e**
3*Piecewise((sqrt(a)*x**4/4, Eq(c, 0)), ((a + c*x**4)**(3/2)/(6*c), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(x*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+a}\,\left(d+e\,x\right)^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^3,x)
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^3, x)
```

3.207 $\int (d+ex)^2 \sqrt{a+cx^4}\, dx$

Optimal. Leaf size=326

$$\frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{2a^{5/4}e^2}{}$$

```
[Out] 1/2*a*d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*e*x^2*(c*x^4+a
)^(1/2)+1/15*x*(3*e^2*x^2+5*d^2)*(c*x^4+a)^(1/2)+2/5*a*e^2*x*(c*x^4+a)^(1/2
)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-2/5*a^(5/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(
1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/15*a^(3/4)*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*ar
ctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)+5*d^2*c^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^
(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1899, 281, 201, 223, 212, 1191, 1212, 226, 1210}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+5\sqrt{c}\,d^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{a+cx^4}}-\frac{2a^{5/4}e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{a+cx^4}}+\frac{1}{15}x\sqrt{a+cx^4}\left(5d^2+3e^2x^2\right)+\frac{1}{2}dex^2\sqrt{a+cx^4}+\frac{ade\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*x^2*Sqrt[a + c*x^4])/2 + (2*a*e^2*x*Sqrt[a + c*x^4])/(5*Sqrt[c]*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(5*d^2 + 3*e^2*x^2)*Sqrt[a + c*x^4])/15 + (a*d*e*Ar
cTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) - (2*a^(5/4)*e^2*(Sqrt[a]
 + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[a + c*x^4]) + (a^(3/4)*(5*S
qrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[
a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(15*c^(
3/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int (d+ex)^2\sqrt{a+cx^4}\,dx = \int \left(2dex\sqrt{a+cx^4} + \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\right) dx$$

$$= (2de)\int x\sqrt{a+cx^4}\,dx + \int \left(d^2+e^2x^2\right)\sqrt{a+cx^4}\,dx$$

$$= \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{15}\int \frac{10ad^2+6ae^2x^2}{\sqrt{a+cx^4}}\,dx + (de)\text{Subst}\left(\int \sqrt{a+\right.$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{1}{2}(ade)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\right.$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} - \frac{2a^{5/}}{}$$

$$= \frac{1}{2}dex^2\sqrt{a+cx^4} + \frac{2ae^2x\sqrt{a+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{1}{15}x\left(5d^2+3e^2x^2\right)\sqrt{a+cx^4} + \frac{ade}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 9.01, size = 146, normalized size = 0.45

$$\frac{\sqrt{a+cx^4}\left(6\sqrt{c}\,d^2x\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)+e\left(3d\left(\sqrt{c}\,x^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)\right)+2\sqrt{c}\,ex^3\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)\right)\right)}{6\sqrt{c}\,\sqrt{1+\frac{cx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*Sqrt[a + c*x^4],x]

[Out] (Sqrt[a + c*x^4]*(6*Sqrt[c]*d^2*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*x^
4)/a)] + e*(3*d*(Sqrt[c]*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*ArcSinh[(Sqrt[c]
```

*x^2)/Sqrt[a]]) + 2*Sqrt[c]*e*x^3*Hypergeometric2F1[-1/2, 3/4, 7/4, -((c*x^4)/a)])))/(6*Sqrt[c]*Sqrt[1 + (c*x^4)/a])

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 248, normalized size = 0.76

method	result
default	$e^{2} \left(\frac{x^{3} \sqrt{c\,x^{4}+a}}{5}+\frac{2 i a^{\frac{3}{2}} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\mathrm{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right.}{5 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}\right.$
elliptic	$\frac{e^{2} x^{3} \sqrt{c\,x^{4}+a}}{5}+\frac{d e\,x^{2} \sqrt{c\,x^{4}+a}}{2}+\frac{d^{2} x \sqrt{c\,x^{4}+a}}{3}+\frac{2 a\,d^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \cdots\right.}{3 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}$
risch	$\frac{x \left(6 e^{2} x^{2}+15 d e x+10 d^{2}\right) \sqrt{c\,x^{4}+a}}{30}+\frac{2 i a^{\frac{3}{2}} e^{2} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \mathrm{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{5 \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}-\frac{2 i a^{\frac{3}{2}} e^{2} \cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] e^2*(1/5*x^3*(c*x^4+a)^(1/2)+2/5*I*a^(3/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+2*d*e*(1/4*x^2*(c*x^4+a)^(1/2)+1/4*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2)))+d^2*(1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + a)*(x*e + d)^2, x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.02, size = 138, normalized size = 0.42

$$\frac{\sqrt{a}\,d^2x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,dex^2\sqrt{1+\frac{cx^4}{a}}}{2}+\frac{\sqrt{a}\,e^2x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{ade\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d**2*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)
/a)/(4*gamma(5/4)) + sqrt(a)*d*e*x**2*sqrt(1 + c*x**4/a)/2 + sqrt(a)*e**2*x
**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*gamm
a(7/4)) + a*d*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{c\,x^4+a}\,\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x)^2,x)
```

```
[Out] int((a + c*x^4)^(1/2)*(d + e*x)^2, x)
```

3.208 $\int (d+ex)\sqrt{a+cx^4}\,dx$

Optimal. Leaf size=158

$$\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\right.}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/4*a*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/3*d*x*(c*x^4+a)^(1/2
)+1/4*e*x^2*(c*x^4+a)^(1/2)+1/3*a^(3/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))
^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x
/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1899, 201, 226, 281, 223, 212}

$$\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (d*x*Sqrt[a + c*x^4])/3 + (e*x^2*Sqrt[a + c*x^4])/4 + (a*e*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[a + c*x^4]])/(4*Sqrt[c]) + (a^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sq
rt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(3*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int (d+ex)\sqrt{a+cx^4}\,dx &= \int \left(d\sqrt{a+cx^4}+ex\sqrt{a+cx^4}\right)dx \\
&= d\int \sqrt{a+cx^4}\,dx+e\int x\sqrt{a+cx^4}\,dx \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{3}(2ad)\int \frac{1}{\sqrt{a+cx^4}}\,dx+\frac{1}{2}e\text{Subst}\left(\int \sqrt{a+cx^2}\,dx,x,x^2\right) \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\Big(\cdots}{3\sqrt[4]{c}\sqrt{a+cx^4}} \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\Big(\cdots}{3\sqrt[4]{c}\sqrt{a+cx^4}} \\
&= \frac{1}{3}dx\sqrt{a+cx^4}+\frac{1}{4}ex^2\sqrt{a+cx^4}+\frac{ae\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{4\sqrt{c}}+\frac{a^{3/4}d\left(\sqrt{a}+\sqrt{c}\cdots\right.}{\cdots}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.28, size = 109, normalized size = 0.69

$$\frac{\sqrt{a+cx^4}\left(\sqrt{c}\,ex^2\sqrt{1+\frac{cx^4}{a}}+\sqrt{a}\,e\sinh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+4\sqrt{c}\,dx\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{cx^4}{a}\right)\right)}{4\sqrt{c}\sqrt{1+\frac{cx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*Sqrt[a + c*x^4],x]
```

```
[Out] (Sqrt[a + c*x^4]*(Sqrt[c]*e*x^2*Sqrt[1 + (c*x^4)/a] + Sqrt[a]*e*ArcSinh[(Sq
rt[c]*x^2)/Sqrt[a]] + 4*Sqrt[c]*d*x*Hypergeometric2F1[-1/2, 1/4, 5/4, -((c*
x^4)/a)]))/(4*Sqrt[c]*Sqrt[1 + (c*x^4)/a])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 129, normalized size = 0.82

method	result
risch	$\frac{x(3ex+4d)\sqrt{c\,x^4+a}}{12}+\frac{ae\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{}}{\sqrt{}}}\right.}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}$
default	$e\left(\frac{x^2\sqrt{c\,x^4+a}}{4}+\frac{a\ln\left(x^2\sqrt{c}+\sqrt{c\,x^4+a}\right)}{4\sqrt{c}}\right)+d\left(\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x}}\right.$
elliptic	$\frac{e\,x^2\sqrt{c\,x^4+a}}{4}+\frac{dx\sqrt{c\,x^4+a}}{3}+\frac{2ad\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}+\frac{ae\ln\left(2x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(1/4*x^2*(c*x^4+a)^(1/2)+1/4*a/c^(1/2)*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2)))+d
*(1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2
)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I
/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(x*e + d), x)
```

Fricas [A]
time = 0.12, size = 95, normalized size = 0.60

$$\frac{16\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}\operatorname{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x},-1\right)+3\,a\sqrt{c}\,e\log\left(-2\,cx^{4}-2\,\sqrt{cx^{4}+a}\,\sqrt{c}\,x^{2}-a\right)+2\,\sqrt{cx^{4}+a}\,(3\,cx^{2}e+4\,cdx)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(16*c^(3/2)*d*(-a/c)^(3/4)*ellipticF((-a/c)^(1/4)/x, -1) + 3*a*sqrt(c)
*e*log(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a) + 2*sqrt(c*x^4 + a)*(3
*c*x^2*e + 4*c*d*x))/c
```

Sympy [C] Result contains complex when optimal does not.
time = 1.69, size = 88, normalized size = 0.56

$$\frac{\sqrt{a}\,dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^{4}e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{a}\,ex^{2}\sqrt{1+\frac{cx^{4}}{a}}}{4}+\frac{ae\operatorname{asinh}\left(\frac{\sqrt{c}\,x^{2}}{\sqrt{a}}\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*d*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)
/(4*gamma(5/4)) + sqrt(a)*e*x**2*sqrt(1 + c*x**4/a)/4 + a*e*asinh(sqrt(c)*x
**2/sqrt(a))/(4*sqrt(c))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)*(x*e + d), x)
```

<u>**Mupad**</u> [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^4+a}\,\left(d+e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)*(d + e*x),x)
[Out] int((a + c*x^4)^(1/2)*(d + e*x), x)
```

3.209 $\int \sqrt{a+cx^4}\, dx$

Optimal. Leaf size=105

$$\frac{1}{3}x\sqrt{a+cx^4}+\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+1/3*a^(3/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {201, 226}

$$\frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{1}{3}x\sqrt{a+cx^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/3 + (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(
Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(3
*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \sqrt{a+cx^4}\,dx = \frac{1}{3}x\sqrt{a+cx^4} + \frac{1}{3}(2a)\int \frac{1}{\sqrt{a+cx^4}}\,dx$$
$$= \frac{1}{3}x\sqrt{a+cx^4} + \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.73, size = 89, normalized size = 0.85

$$\frac{x(a+cx^4) - \frac{2ia\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) - ((2*I)*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*S
qrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 85, normalized size = 0.81

method	result	size
default	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
risch	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85
elliptic	$\frac{x\sqrt{c\,x^4+a}}{3}+\frac{2a\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{3\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)+2/3*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*
x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a
^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

Fricas [A]
time = 0.10, size = 40, normalized size = 0.38

$$\frac{2}{3}\sqrt{c}\left(-\frac{a}{c}\right)^{\frac{3}{4}}\text{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x},-1\right)+\frac{1}{3}\sqrt{cx^4+a}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c)*(-a/c)^(3/4)*ellipticF((-a/c)^(1/4)/x, -1) + 1/3*sqrt(c*x^4 + a
)*x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 37, normalized size = 0.35

$$\frac{\sqrt{a}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), c*x**4*exp_polar(I*pi)/a)/(
4*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a), x)
```

Mupad [B]
time = 2.64, size = 37, normalized size = 0.35

$$\frac{x\,\sqrt{c\,x^4+a}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{\frac{c\,x^4}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2),x)
```

```
[Out] (x*(a + c*x^4)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -(c*x^4)/a))/((c*x^4)/a +
1)^(1/2)
```

3.210 $\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx$

Optimal. Leaf size=730

$$\frac{\sqrt{a+cx^4}}{2e}-\frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3}+\frac{\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-$$

```
[Out] 1/2*d^2*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*arctan(x*(-a*e
^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/e^3-1/2*arctanh((
c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))*(a*e^4+c*d^4)^(1/2)/e
^3+1/2*(c*x^4+a)^(1/2)/e-d*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/
2))+a^(1/4)*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*
x^4+a)^(1/2)+1/2*c^(1/4)*d*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^
2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2
))^2)^(1/2)/a^(1/4)/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(a*e^
4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*
c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*
(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(
1/4)/d/e^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*a^(1/4)*c^(1/4)*d*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^4
/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1743, 1223, 1212, 226, 1210, 1231, 1721, 1262, 749, 858, 223, 212, 739}

$$-\frac{\sqrt{-ae^4-cd^4}\,\text{ArcTan}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\frac{\sqrt{c}d^2}{\sqrt{a}}+e^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2e^4\sqrt{a+cx^4}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(ae^4+cd^4\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}e^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{c}d^2-\sqrt{a}e^2\right)\left(ae^4+cd^4\right)\Pi\left(\frac{\left(\sqrt{c}d^2+\sqrt{a}e^2\right)^2}{4\sqrt{a}\sqrt{c}d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\,de^4\sqrt{a+cx^4}\left(\sqrt{a}e^2+\sqrt{c}d^2\right)}+\frac{\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{e^2\sqrt{a+cx^4}}+\frac{\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2e^3}-\frac{\sqrt{ae^4+cd^4}\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2e^3}-\frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt{a+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] Sqrt[a + c*x^4]/(2*e) - (Sqrt[c]*d*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[
c]*x^2)) - (Sqrt[-(c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*S
qrt[a + c*x^4])])/(2*e^3) + (Sqrt[c]*d^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x
^4]])/(2*e^3) - (Sqrt[c*d^4 + a*e^4]*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^
```

```
4 + a*e^4]*Sqrt[a + c*x^4])])/(2*e^3) + (a^(1/4)*c^(1/4)*d*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(
1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) - (a^(1/4)*c^(1/4)*d*((Sqrt[c
]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*e^4*Sqrt[a
 + c*x^4]) + (c^(1/4)*d*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c
*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1
/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sqrt[c
]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*S
qrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^
(1/4)*d*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
```

```
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1743

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[x*((a + c*x^4)^p/(d^
2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{d+ex}\,dx = d\int \frac{\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx - e\int \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}\,dx$$

$$= \left(d\left(a+\frac{cd^4}{e^4}\right)\right)\int \frac{1}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx - \frac{d\int \frac{cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{e^4} - \frac{1}{2}e\text{Subst}\left(\int \frac{\sqrt{a+}}{d^2-}\right.$$

$$= \frac{\sqrt{a+cx^4}}{2e} + \frac{(\sqrt{a}\sqrt{c}\,d)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{e^2} + \frac{\text{Subst}\left(\int \frac{-ae^2-cd^2x}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)}{2e} + \cdots$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\cdots}{\cdots}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,\cdots}{\cdots}$$

$$= \frac{\sqrt{a+cx^4}}{2e} - \frac{\sqrt{c}\,dx\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2e^3} + \frac{\sqrt{c}\,d^2\,\text{ta}\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.84, size = 421, normalized size = 0.58

$$\frac{-2\sqrt{a}\,c^{3/4}d^2e^2\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+2c^{3/4}d^2\left(i\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(-2\sqrt[4]{-1}\,\sqrt[4]{a}\,(cd^4+ae^4)\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)+\sqrt[4]{c}\,de\left(e^2(a+cx^4)-2\sqrt{-cd^4-ae^4}\,\sqrt{a+cx^4}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)-\sqrt{c}\,d^2\sqrt{a+cx^4}\,\log\left(-\sqrt{c}\,x^2+\sqrt{a+cx^4}\right)\right)\right)}{2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt[4]{c}\,de^4\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x),x]
```

```
[Out] (-2*Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh[Sqrt[(I
*Sqrt[c])/Sqrt[a]]*x], -1] + 2*c^(3/4)*d^2*(I*Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sq
rt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + S
qrt[(I*Sqrt[c])/Sqrt[a]]*(-2*(-1)^(1/4)*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[1 + (c
*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/
4)*x)/a^(1/4)], -1] + c^(1/4)*d*e*(e^2*(a + c*x^4) - 2*Sqrt[-(c*d^4) - a*e^
4]*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/S
qrt[-(c*d^4) - a*e^4]] - Sqrt[c]*d^2*Sqrt[a + c*x^4]*Log[-(Sqrt[c]*x^2) + S
qrt[a + c*x^4]])))/(2*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/4)*d*e^4*Sqrt[a + c*x^
4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.25, size = 404, normalized size = 0.55

method	result
default	$\frac{\sqrt{c x^4+a}}{2e} - \frac{c d^3 \sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c x^4+a}} + \frac{\sqrt{c}\, d^2 \ln\left(2x^2\sqrt{c}+2\sqrt{c x^4+a}\right.}{2e^3}$
elliptic	$\frac{\sqrt{c x^4+a}}{2e} - \frac{c d^3 \sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c x^4+a}} + \frac{\sqrt{c}\, d^2 \ln\left(2x^2\sqrt{c}+2\sqrt{c x^4+a}\right.}{2e^3}$

risch	$\frac{\sqrt{c x^4+a}}{2e} - \frac{i\sqrt{c}\, d\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}, i\right)}{e^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c x^4+a}} + \frac{i\sqrt{c}\, d\sqrt{a}\,\sqrt{1-\frac{i}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+a)^(1/2)/e-c*d^3/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*
(I/a^(1/2)*c^(1/2))^(1/2),I)+1/2*c^(1/2)*d^2/e^3*ln(2*x^2*c^(1/2)+2*(c*x^4+
a)^(1/2))-I*c^(1/2)*d/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+(
a*e^4+c*d^4)/e^5*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a
)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)
*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1
/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(x*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + a)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^4}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**4)/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + a)/(x*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+a}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c*x^4)^(1/2)/(d + e*x), x)
```

3.211 $\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx$

Optimal. Leaf size=1221

$$\frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)}+\frac{x\sqrt{a+cx^4}}{d^2-e^2x^2}+\frac{\sqrt{-cd^4-ae^4}\,\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2de^3}-\frac{(cd^4-ae^4)\tan\cdots}{2de^3\sqrt{\cdots}}$$

```
[Out] -d*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))*c^(1/2)/e^3-1/2*(-a*e^4+c*d^4)*arct
an(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/d/e^3/(-a*e^4-c*d^4)^(1/2)+1
/2*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))*(-a*e^4-c*d^4)^(1/2)/
d/e^3+c*d^3*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/
e^3/(a*e^4+c*d^4)^(1/2)-d*(c*x^4+a)^(1/2)/e/(-e^2*x^2+d^2)+x*(c*x^4+a)^(1/2
)/(-e^2*x^2+d^2)+2*x*c^(1/2)*(c*x^4+a)^(1/2)/e^2/(a^(1/2)+x^2*c^(1/2))-2*a^
(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/
4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/e^2/(c*x^4+a)^(
1/2)-1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^
(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(
-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(
1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(c*x^4+a)^(1/2)-1/2*c^
(1/4)*(a*e^4+c*d^4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan
(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e
^4/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*(a*e^4+c*d^4)*(cos(2*arcta
n(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(
sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^
(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d^2/e^4/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)+1/4*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/
4)*x/a^(1/4))),1/2*2^(1/2))*(e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))
*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/e^4/(c*x^4+a)^(1/2)+3/4*
a^(1/4)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(
1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a
^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/
2))^2)^(1/2)/e^4/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 1.29, antiderivative size = 1221, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.790$, Rules used = {2184, 1241, 1212, 226, 1210, 1231, 1721, 1262, 747, 858, 223, 212, 739, 1350, 1223}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (2*Sqrt[c]*x*Sqrt[a + c*x^4])/(e^2*(Sqrt[a] + Sqrt[c]*x^2)) - (d*Sqrt[a + c
*x^4])/(e*(d^2 - e^2*x^2)) + (x*Sqrt[a + c*x^4])/(d^2 - e^2*x^2) + (Sqrt[-(
c*d^4) - a*e^4]*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(
2*d*e^3) - ((c*d^4 - a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a +
 c*x^4])])/(2*d*e^3*Sqrt[-(c*d^4) - a*e^4]) - (Sqrt[c]*d*ArcTanh[(Sqrt[c]*x
^2)/Sqrt[a + c*x^4]])/e^3 + (c*d^3*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4
+ a*e^4]*Sqrt[a + c*x^4])])/(e^3*Sqrt[c*d^4 + a*e^4]) - (2*a^(1/4)*c^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(e^2*Sqrt[a + c*x^4]) + (3*a^(1/4)*
c^(1/4)*((Sqrt[c]*d^2)/Sqrt[a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x
^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2
])/(4*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*e^4*Sqrt[a + c*x^4]) + (c^(1/4)*(
Sqrt[c]*d^2 + Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/
4)*e^4*Sqrt[a + c*x^4]) - (c^(1/4)*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)*
Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], 1/2])/(2*a^(1/4)*e^4*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
 + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt
[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/
4)*d^2*e^4*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(c*d^4 + a*e^4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d^2*e^4*(Sqrt[c]*d^2 + Sqrt[a]*
e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1223

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[-(e
^2)^(-1), Int[(c*d - c*e*x^2)*(a + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 + a
```

```
*e^2)/e^2, Int[(a + c*x^4)^(p - 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1241

```
Int[Sqrt[(a_) + (c_.)*(x_)^4]/((d_) + (e_.)*(x_)^2)^2, x_Symbol] :> Simp[x*
(Sqrt[a + c*x^4]/(2*d*(d + e*x^2))), x] + (Dist[c/(2*d*e^2), Int[(d - e*x^2
)/Sqrt[a + c*x^4], x], x] - Dist[(c*d^2 - a*e^2)/(2*d*e^2), Int[1/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]) /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^
2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 2184

```
Int[((c_) + (d_.)*(x_)^(n_.))^(q_)*((a_) + (b_.)*(x_)^(nn_.))^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^nn)^p, (c/(c^2 - d^2*x^(2*n)) - d*(x^n/(
```

```
c^2 - d^2*x^(2*n))))^(-q), x], x] /; FreeQ[{a, b, c, d, n, nn, p}, x] &&  !
IntegerQ[p] && ILtQ[q, 0] && IGtQ[Log[2, nn/n], 0]
```

Rubi steps

$$\int \frac{\sqrt{a+cx^4}}{(d+ex)^2}\,dx = \int \left(\frac{d^2\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2} - \frac{2dex\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2} + \frac{e^2x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\right)dx$$

$$= d^2\int \frac{\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx - (2de)\int \frac{x\sqrt{a+cx^4}}{\left(d^2-e^2x^2\right)^2}\,dx + e^2\int \frac{x^2\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx$$

$$= \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + \frac{1}{2}\left(a-\frac{cd^4}{e^4}\right)\int \frac{1}{\left(d^2-e^2x^2\right)\sqrt{a+cx^4}}\,dx + \frac{c\int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2e^4} - (de)\mathrm{S}$$

$$= -\frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{2\left(d^2-e^2x^2\right)} + d^2\int \frac{\sqrt{a+cx^4}}{\left(-d^2+e^2x^2\right)^2}\,dx + \frac{1}{2}\left(\sqrt{a}\left(\sqrt{a}-\frac{\sqrt{c}\,d^2}{e^2}\right)\right)$$

$$= \frac{\sqrt{c}\,x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}}{de\sqrt{a+cx^4}}\right.}{4de^3\sqrt{-cd^4-ae^4}}$$

$$= \frac{\sqrt{c}\,x\sqrt{a+cx^4}}{2e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} - \frac{\left(cd^4-ae^4\right)\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}}{de\sqrt{a+cx^4}}\right.}{4de^3\sqrt{-cd^4-ae^4}}$$

$$= \frac{2\sqrt{c}\,x\sqrt{a+cx^4}}{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{d\sqrt{a+cx^4}}{e\left(d^2-e^2x^2\right)} + \frac{x\sqrt{a+cx^4}}{d^2-e^2x^2} + \frac{\sqrt{-cd^4-ae^4}\tan^{-1}\left(\frac{\sqrt{-cd^4-a}}{de\sqrt{a+cx}}\right.}{2de^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.51, size = 394, normalized size = 0.32

$$\frac{-\frac{e^3\left(a+cx^4\right)}{d+ex} - \frac{2cd^3e\sqrt{a+cx^4}\tan^{-1}\left(\frac{\sqrt{c}\left(d^2-e^2x^2\right)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}} - 2ia\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,e^2\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right) - \frac{2\sqrt{c}\left(i\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}} + 2\sqrt[4]{-1}\sqrt[4]{a}\,c^{3/4}d^2\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right) + \sqrt{c}\,de\sqrt{a+cx^4}\log\left(-\sqrt{c}\,x^2+\sqrt{a+cx^4}\right)}{e^4\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c*x^4]/(d + e*x)^2,x]
```

```
[Out] (-((e^3*(a + c*x^4))/(d + e*x)) - (2*c*d^3*e*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c
]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*
```

```
d^4) - a*e^4] - (2*I)*a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*e^2*Sqrt[1 + (c*x^4)/a]*E
llipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - (2*Sqrt[c]*(I*Sqrt[c
]*d^2 + Sqrt[a]*e^2)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c
])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/
4)*d^2*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin
[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1] + Sqrt[c]*d*e*Sqrt[a + c*x^4]*Log[-(S
qrt[c]*x^2) + Sqrt[a + c*x^4]])/(e^4*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 402, normalized size = 0.33

method	result
default	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\sqrt{c}\,d\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+\right.}{e^3}$
elliptic	$-\frac{\sqrt{c\,x^4+a}}{e(ex+d)}+\frac{2c\,d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{e^4\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{\sqrt{c}\,d\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+\right.}{e^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+a)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e*(c*x^4+a)^(1/2)/(e*x+d)+2*c*d^2/e^4/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(
1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ell
ipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-c^(1/2)*d/e^3*ln(2*x^2*c^(1/2)+2*(c*x
^4+a)^(1/2))+2*I*c^(1/2)/e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)
*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(Ellipt
icF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
)-2*c*d^3/e^5*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(
c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^
```

```
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*El
lipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)
*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + a)/(x*e + d)^2, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + cx^4}}{(d + ex)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+a)**(1/2)/(e*x+d)**2,x)
[Out] Integral(sqrt(a + c*x**4)/(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+a)^(1/2)/(e*x+d)^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + a)/(x*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+a}}{{\left(d+e\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^(1/2)/(d + e*x)^2,x)
[Out] int((a + c*x^4)^(1/2)/(d + e*x)^2, x)
```

3.212 $\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{e^3\sqrt{a+cx^4}}{2c}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x\right.}}}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 3/2*d^2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*e^3*(c*x^4+a)^(1
/2)/c+3*d*e^2*x*(c*x^4+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-3*a^(1/4)*d*e
^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4
)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^
(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/
2*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(3*e^2*a^(1/2)
+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/
2)/a^(1/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1899, 1212, 226, 1210, 1262, 655, 223, 212}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(3\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,c^{3/4}\sqrt{a+cx^4}}-\frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{e^3\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d*e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a]
 + Sqrt[c]*x^2)) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt
[c]) - (3*a^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] +
 Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sq
rt[a + c*x^4]) + (d*(Sqrt[c]*d^2 + 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a
^(1/4)], 1/2])/(2*a^(1/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{d^3+3de^2x^2}{\sqrt{a+cx^4}} + \frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\right) dx$$

$$= \int \frac{d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx + \int \frac{x(3d^2e+e^3x^2)}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{3d^2e+e^3x}{\sqrt{a+cx^2}}\,dx, x, x^2\right) - \frac{\left(3\sqrt{a}\,de^2\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d\left(d^2 + \frac{3\sqrt{a}\,e^2}{\sqrt{c}}\right)\right.$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(\right.}{c^{3/4}\sqrt{a+cx^4}}$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(\right.}{c^{3/4}\sqrt{a+cx^4}}$$

$$= \frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3de^2x\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} - \frac{3\sqrt[4]{a}\,de^2\left(\sqrt{a}+\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.10, size = 157, normalized size = 0.53

$$\frac{e^3\sqrt{a+cx^4}}{2c} + \frac{3d^2e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d^3x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{de^2x^3\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/Sqrt[a + c*x^4],x]
```

```
[Out] (e^3*Sqrt[a + c*x^4])/(2*c) + (3*d^2*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4
]])/(2*Sqrt[c]) + (d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/
4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeo
metric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 218, normalized size = 0.74

method	result

default	$\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+\frac{3 i d\,e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \left(\operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)-\operatorname{EllipticE}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}$
elliptic	$\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+\frac{d^{3} \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}}+\frac{3 d^{2} e \ln\left(2 x^{2} \sqrt{c}+2 \sqrt{c\,x^{4}+a}\right)}{2 \sqrt{c}}$
risch	$\frac{e^{3} \sqrt{c\,x^{4}+a}}{2 c}+\frac{3 i d\,e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \sqrt{1+\frac{i \sqrt{c}\, x^{2}}{\sqrt{a}}}\, \operatorname{EllipticF}\left(x \sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}, i\right)}{\sqrt{\frac{i \sqrt{c}}{\sqrt{a}}}\, \sqrt{c\,x^{4}+a}\, \sqrt{c}}-\frac{3 i d\,e^{2} \sqrt{a}\, \sqrt{1-\frac{i \sqrt{c}}{\sqrt{c}}}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^3*(c*x^4+a)^(1/2)/c+3*I*d*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/
a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/
c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^
(1/2))^(1/2),I))+3/2*d^2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d^3/(I/a
^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*
x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/sqrt(c*x^4 + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [A]
time = 1.99, size = 141, normalized size = 0.48

$$e^3 \left(\begin{cases} \frac{x^4}{4\sqrt{a}} & \text{for } c = 0 \\ \frac{\sqrt{a+cx^4}}{2c} & \text{otherwise} \end{cases} \right) + \frac{3d^2 e \operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}} + \frac{d^3 x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)} + \frac{3de^2 x^3 \Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix} \middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(1/2),x)
```

```
[Out] e**3*Piecewise((x**4/(4*sqrt(a)), Eq(c, 0)), (sqrt(a + c*x**4)/(2*c), True)
) + 3*d**2*e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d**3*x*gamma(1/4)*hy
per((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) +
3*d*e**2*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a
)/(4*sqrt(a)*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)^3/(a + c*x^4)^(1/2), x)
```

3.213 $\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=263

$$\frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{a}}\right.\right.}{c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] d*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+e^2*x*(c*x^4+a)^(1/2)/c^(1
/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4
)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4
)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*(e^2+d^2*c^(1/2)/a^(1/2))*
((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1899, 281, 223, 212, 1212, 226, 1210}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\frac{\sqrt{c}\,d^2}{\sqrt{a}}+e^2\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}}+\frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (e^2*x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) + (d*e*ArcTanh[(S
qrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] - (a^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a + c*x^4]) + (a^(1/4)*((Sqrt[c]*d^2)/Sqrt[
a] + e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{2dex}{\sqrt{a+cx^4}} + \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\right) dx$$

$$= (2de)\int \frac{x}{\sqrt{a+cx^4}}\,dx + \int \frac{d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx$$

$$= (de)\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, x^2\right) - \frac{\left(\sqrt{a}\,e^2\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}} + \left(d^2+\frac{\sqrt{a}\,e^2}{\sqrt{c}}\right)\int \frac{}{\sqrt{a}}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{c^{3/4}\sqrt{a+cx^4}}$$

$$= \frac{e^2x\sqrt{a+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} - \frac{\sqrt[4]{a}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\right.}}}{c^{3/4}\sqrt{a+c}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 133, normalized size = 0.51

$$\frac{de\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{\sqrt{c}} + \frac{d^2x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}} + \frac{e^2x^3\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{cx^4}{a}\right)}{3\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/Sqrt[a + c*x^4],x]
```

```
[Out] (d*e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/Sqrt[c] + (d^2*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4] + (e
^2*x^3*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((c*x^4)/a)])/
(3*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 197, normalized size = 0.75

method	result
default	$\frac{ie^2\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\text{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\text{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}} + \frac{de\ln\left(x^2\sqrt{c}+\sqrt{\right.}}{\sqrt{c}}$

elliptic	$\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}+\frac{de\ln\left(2x^2\sqrt{c}+2\sqrt{c\,x^4+a}\right)}{\sqrt{c}}+\frac{ie^2\sqrt{a}\,\sqrt{1-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*e^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2
)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+d*e*ln(x^2*c^
(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c
^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF
(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/sqrt(c*x^4 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.65, size = 105, normalized size = 0.40

$$\frac{de\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{c}}+\frac{d^2x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}+\frac{e^2x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] d*e*asinh(sqrt(c)*x**2/sqrt(a))/sqrt(c) + d**2*x*gamma(1/4)*hyper((1/4, 1/2
), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4)) + e**2*x**3*gam
ma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamm
a(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)^2/(a + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=121

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}+\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*e*arctanh(x^2*c^(1/2)/(c*x^4+a)^(1/2))/c^(1/2)+1/2*d*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1899, 226, 281, 223, 212}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
```

```
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{d+ex}{\sqrt{a+cx^4}}\,dx = \int \left(\frac{d}{\sqrt{a+cx^4}}+\frac{ex}{\sqrt{a+cx^4}}\right)dx$$

$$= d\int \frac{1}{\sqrt{a+cx^4}}\,dx + e\int \frac{x}{\sqrt{a+cx^4}}\,dx$$

$$= \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\right.$$

$$= \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}} + \frac{1}{2}e\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx\right.$$

$$= \frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 79, normalized size = 0.65

$$\frac{e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}} + \frac{dx\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/Sqrt[a + c*x^4],x]
```

```
[Out] (e*ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]])/(2*Sqrt[c]) + (d*x*Sqrt[1 + (c*x
^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/Sqrt[a + c*x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 96, normalized size = 0.79

method	result	size
default	$\frac{e\ln\left(x^{2}\sqrt{c}+\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	96
elliptic	$\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{e\ln\left(2x^{2}\sqrt{c}+2\sqrt{c\,x^{4}+a}\right)}{2\sqrt{c}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)+d/(I/a^(1/2)*c^(1/2))^(1/2)*(
1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1
/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/sqrt(c*x^4 + a), x)
```

Fricas [A]
time = 0.12, size = 72, normalized size = 0.60

$$\frac{4\,c^{\frac{3}{2}}d\left(-\frac{a}{c}\right)^{\frac{3}{4}}\mathrm{ellipticF}\left(\frac{\left(-\frac{a}{c}\right)^{\frac{1}{4}}}{x},-1\right)+a\sqrt{c}\,e\log\left(-2\,cx^{4}-2\,\sqrt{cx^{4}+a}\,\sqrt{c}\,x^{2}-a\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*c^(3/2)*d*(-a/c)^(3/4)*ellipticF((-a/c)^(1/4)/x, -1) + a*sqrt(c)*e*l
og(-2*c*x^4 - 2*sqrt(c*x^4 + a)*sqrt(c)*x^2 - a))/(a*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 61, normalized size = 0.50

$$\frac{e\operatorname{asinh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}}+\frac{dx\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(1/2),x)
```

```
[Out] e*asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c)) + d*x*gamma(1/4)*hyper((1/4, 1/2)
, (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/sqrt(c*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d+e\,x}{\sqrt{c\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^(1/2),x)
```

```
[Out] int((d + e*x)/(a + c*x^4)^(1/2), x)
```

3.215 $\int \frac{1}{\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^4+a)
^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {226}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(1/4)*Sqrt[a + c*x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+cx^4}}\,dx = \frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
 -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 70, normalized size = 0.80

method	result	size
default	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70
elliptic	$\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^
(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [A]
time = 0.15, size = 28, normalized size = 0.32

$$-\frac{\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}\text{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a)*(-c/a)^(3/4)*ellipticF(x*(-c/a)^(1/4), -1)/c
```

Sympy [C] Result contains complex when optimal does not.
time = 0.35, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*sqrt(a)
*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Mupad [B]
time = 2.63, size = 37, normalized size = 0.42

$$\frac{x\sqrt{\frac{c\,x^4}{a}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\sqrt{c\,x^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(1/2),x)
```

```
[Out] (x*((c*x^4)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(1/2)
```

3.216 $\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=405

$$\frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{}}$$

```
[Out] 1/2*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^(1/
2)-1/2*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*
e^4+c*d^4)^(1/2)+1/2*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/c
os(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),
1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2
)/a^(1/4)/(e^2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*(cos(2*arctan(c^(1/
4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c
^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^
4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(e^2*a^(1/2)+d^2*c^(1
/2))/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1739, 1231, 226, 1721, 1262, 739, 212}

$$\frac{e\text{ArcTan}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-ae^4-cd^4}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\sqrt{c}\,d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{c}\,d\sqrt{a+cx^4}\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\sqrt{ae^4+cd^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (e*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*Sqrt[-(c*d^
4) - a*e^4]) - (e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a +
 c*x^4])])/(2*Sqrt[c*d^4 + a*e^4]) + (c^(1/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(
1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4]) - ((Sq
rt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a]
+ Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[
c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*c^(1/4)*d*(Sqr
t[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\sqrt{a+cx^4}}\,dx = d\int \frac{1}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx - e\int \frac{x}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{1}{(d^2-e^2x)\sqrt{a+cx^2}}\,dx, x, x^2\right)\right) + \frac{(\sqrt{c}\,d)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{\sqrt{c}\,d^2+\sqrt{a}\,e^2} +$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} + \frac{\sqrt[4]{c}\,d(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,F\Big(2\tan}{2\sqrt[4]{a}\,(\sqrt{c}\,d^2+\sqrt{a}\,e^2)\sqrt{a+cx^4}}$$

$$= \frac{e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{2\sqrt{-cd^4-ae^4}} - \frac{e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{a+cx^4}}\right)}{2\sqrt{cd^4+ae^4}} + \frac{\sqrt[4]{c}\,d(\sqrt{a}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 200, normalized size = 0.49

$$\frac{\sqrt{1+\frac{cx^4}{a}}\left(-2\sqrt[4]{-1}\,\sqrt[4]{a}\,\sqrt{1+\frac{cd^4}{ae^4}}\,e\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)+\sqrt[4]{c}\,d\log\left(\frac{-d^2+e^2x^2}{cd^2x^2+ae^2\left(1+\sqrt{1+\frac{cd^4}{ae^4}}\sqrt{1+\frac{cx^4}{a}}\right)}\right)\right)}{2\sqrt[4]{c}\,d\sqrt{1+\frac{cd^4}{ae^4}}\,e\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*Sqrt[a + c*x^4]),x]
```

```
[Out] (Sqrt[1 + (c*x^4)/a]*(-2*(-1)^(1/4)*a^(1/4)*Sqrt[1 + (c*d^4)/(a*e^4)]*e*Ell
ipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4
)], -1] + c^(1/4)*d*Log[(-d^2 + e^2*x^2)/(c*d^2*x^2 + a*e^2*(1 + Sqrt[1 + (
c*d^4)/(a*e^4)]*Sqrt[1 + (c*x^4)/a]))]))/(2*c^(1/4)*d*Sqrt[1 + (c*d^4)/(a*e
^4)]*e*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.26, size = 169, normalized size = 0.42

method	result

default	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$
elliptic	$\frac{-\frac{\operatorname{arctanh}\left(\frac{\frac{2c\,x^2d^2}{e^2}+2a}{2\sqrt{\frac{c\,d^4}{e^4}+a}\,\sqrt{c\,x^4+a}}\right)}{2\sqrt{\frac{c\,d^4}{e^4}+a}}+\frac{e\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticPi}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},-\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2},\frac{\sqrt{-\frac{i\sqrt{c}}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,d\sqrt{c\,x^4+a}}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+
a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1
/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x
*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^
(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+cx^4}\,(d+ex)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)), x)
```

3.217 $\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=610

$$-\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)}+\frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{a+cx^4}}\right)}{(-cd^4-ae^4)^{3/2}}-\frac{cd^3e\tanh^{-1}\left(\frac{\cdots}{\sqrt{cd^4+\cdots}}\right)}{(cd^4+a\cdots}$$

```
[Out] -c*d^3*e*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-c*d^3*e*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)-e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)+e^2*x*c^(1/
2)*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*c^(1/4)*e^2*
(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))
*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/
2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)
+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4
)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(e^2*a^(
1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/2*c^(3/4)*d^2*(cos(2*arctan(c^(1/4)*x/a
^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2)
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2*c^(1/2
))/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {1741, 1756, 12, 1262, 739, 212, 1729, 1210, 1723, 226, 1721}

$$-\frac{c^{3/4}d^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\,\sqrt{c}\,d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+cx^4}\,\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)(ae^4+cd^4)}-\frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}\,(ae^4+cd^4)}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt{a+cx^4}\,\left(\sqrt{a}\,e^2+\sqrt{c}\,d^2\right)}-\frac{cd^3e\text{ArcTan}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{\left(-ae^4-cd^4\right)^{3/2}}-\frac{e^3\sqrt{a+cx^4}}{(d+ex)(ae^4+cd^4)}+\frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)(ae^4+cd^4)}-\frac{cd^3e\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\,\sqrt{ae^4+cd^4}}\right)}{\left(ae^4+cd^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x))) + (Sqrt[c]*e^2*x*Sqrt[
a + c*x^4])/((c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (c*d^3*e*ArcTan[(Sq
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(-(c*d^4) - a*e^4)^(3/2) -
(c*d^3*e*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(c*d^4 + a*e^4)^(3/2) - (a^(1/4)*c^(1/4)*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], 1/2])/((c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
```

```
4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[a + c*x^4
]) - (c^(3/4)*d^2*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^
2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a
^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
```

```
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1741

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[
d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} - \frac{c\int \frac{-d^3+d^2ex-de^2x^2-e^3x^3}{(d+ex)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} - \frac{c\int \frac{2d^3ex}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4} - \frac{c\int \frac{-d^4-2d^2e^2x^2+e^4x^4}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{cd^4+ae^4}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\int \frac{cd^4e^2+\sqrt{a}\sqrt{c}\,d^2e^4+\left(2cd^2e^4-e^4\left(cd^2+\sqrt{a}\sqrt{c}\,e^2\right)\right)x^2}{(d^2-e^2x^2)\sqrt{a+cx^4}}\,dx}{e^2(cd^4+ae^4)} -$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,\sqrt[4]{c}\,e^2\left(\sqrt{a}+\sqrt{c}\,x\right.}{}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-}}{de\sqrt{a+c}}\right.}{(-cd^4-ae^4)^{3/2}}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{(cd^4+ae^4)(d+ex)} + \frac{\sqrt{c}\,e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{cd^3e\tan^{-1}\left(\frac{\sqrt{-cd^4-}}{de\sqrt{a+c}}\right.}{(-cd^4-ae^4)^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.95, size = 448, normalized size = 0.73

$$-\frac{\sqrt{a}\sqrt{c}\,e^2\sqrt{-cd^4-ae^4}\,(d+ex)\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+i\sqrt{c}\left(\sqrt{c}\,d^2+i\sqrt{a}\,e^2\right)\sqrt{-cd^4-ae^4}\,(d+ex)\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)-\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(e^3\sqrt{-cd^4-ae^4}\,(a+cx^4)-2cd^3e(d+ex)\sqrt{a+cx^4}\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)+2\sqrt[4]{-1}\,\sqrt[4]{a}\,c^{3/4}d^2\sqrt{-cd^4-ae^4}\,(d+ex)\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,(-cd^4-ae^4)^{3/2}(d+ex)\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*Sqrt[a + c*x^4]),x]
```

```
[Out] -((Sqrt[a]*Sqrt[c]*e^2*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]
*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + I*Sqrt[c]*(Sqrt[c]
*d^2 + I*Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*(d + e*x)*Sqrt[1 + (c*x^4)/a]*
EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - Sqrt[(I*Sqrt[c])/Sq
rt[a]]*(e^3*Sqrt[-(c*d^4) - a*e^4]*(a + c*x^4) - 2*c*d^3*e*(d + e*x)*Sqrt[a
 + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d
^4) - a*e^4]] + 2*(-1)^(1/4)*a^(1/4)*c^(3/4)*d^2*Sqrt[-(c*d^4) - a*e^4]*(d
+ e*x)*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin
[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*(-(c*d^4
) - a*e^4)^(3/2)*(d + e*x)*Sqrt[a + c*x^4]))
```

Maple [C] Result contains complex when optimal does not.
time = 0.83, size = 421, normalized size = 0.69

method	result
default	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}}{\sqrt{a}}}}{}$
elliptic	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)\left(ex+d\right)}-\frac{c\,d^{2}\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{\left(e^{4}a+d^{4}c\right)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}+\frac{ie^{2}\sqrt{c}\,\sqrt{a}\,\sqrt{1-\frac{i\sqrt{c}}{\sqrt{a}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)-c*d^2/(a*e^4+c*d^4)/(I/a^(1/2)*c
^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/
2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+I*e^2*c^(1/2)/(
a*e^4+c*d^4)*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1
/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))+2*c*d^3/(a*e^4
+c*d^4)/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^
4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2
)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*Ellipt
icPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(
1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)^2), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(c*x**4+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(c*x^4+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2),x)
```

```
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^2), x)
```

3.218 $\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx$

Optimal. Leaf size=659

$$-\frac{e^3\sqrt{a+cx^4}}{2\left(cd^4+ae^4\right)(d+ex)^2}-\frac{3cd^3e^3\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2(d+ex)}+\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(cd^4+ae^4\right)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{3cd^2e\left(cd^4-ae^4\right)\tan^{-1}\left(\cdots\right.}{2\left(-cd^4-ae^4\cdots\right.}$$

```
[Out] 3/2*c*d^2*e*(-a*e^4+c*d^4)*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2
))/(-a*e^4-c*d^4)^(5/2)-3/2*c*d^2*e*(-a*e^4+c*d^4)*arctanh((c*d^2*x^2+a*e^2
)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2))/(a*e^4+c*d^4)^(5/2)-1/2*e^3*(c*x^4+a
)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2
/(e*x+d)+3*c^(3/2)*d^3*e^2*x*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(a^(1/2)+x^2*c
^(1/2))-3*a^(1/4)*c^(5/4)*d^3*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2
)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4)
)),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(
1/2)/(a*e^4+c*d^4)^2/(c*x^4+a)^(1/2)+1/2*c^(3/4)*d*(cos(2*arctan(c^(1/4)*x/
a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(
c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+
x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-3/4*c^(3/4)*d*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticPi(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2
/d^2/e^2/a^(1/2)/c^(1/2),1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))^2*(a^(1/2)
+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^
4)^2/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {1741, 1753, 1756, 12, 1262, 739, 212, 1729, 1210, 1723, 226, 1721}

$$\frac{c^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)}-\frac{3\sqrt[4]{a}\,c^{5/4}d^3e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{a+cx^4}\left(ae^4+cd^4\right)^2}-\frac{3c^{3/4}d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)^2\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\sqrt{c}\,d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+cx^4}\left(ae^4+cd^4\right)^2}+\frac{3cd^2e\left(cd^4-ae^4\right)\text{ArcTan}\left(\frac{x\sqrt{-ae^4-cd^4}}{de\sqrt{a+cx^4}}\right)}{2\left(-ae^4-cd^4\right)^{5/2}}+\frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(ae^4+cd^4\right)^2}-\frac{e^3\sqrt{a+cx^4}}{2(d+ex)^2\left(ae^4+cd^4\right)}-\frac{3cd^3e^3\sqrt{a+cx^4}}{(d+ex)\left(ae^4+cd^4\right)^2}-\frac{3cd^2e\left(cd^4-ae^4\right)\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{a+cx^4}\sqrt{ae^4+cd^4}}\right)}{2\left(ae^4+cd^4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*(e^3*Sqrt[a + c*x^4])/((c*d^4 + a*e^4)*(d + e*x)^2) - (3*c*d^3*e^3*Sqr
t[a + c*x^4])/((c*d^4 + a*e^4)^2*(d + e*x)) + (3*c^(3/2)*d^3*e^2*x*Sqrt[a +
 c*x^4])/((c*d^4 + a*e^4)^2*(Sqrt[a] + Sqrt[c]*x^2)) + (3*c*d^2*e*(c*d^4 -
a*e^4)*ArcTan[(Sqrt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^
4) - a*e^4)^(5/2)) - (3*c*d^2*e*(c*d^4 - a*e^4)*ArcTanh[(a*e^2 + c*d^2*x^2)
/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])])/(2*(c*d^4 + a*e^4)^(5/2)) - (3*a^(
1/4)*c^(5/4)*d^3*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sq
```

```
rt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/((c*d^4 + a*e^
4)^2*Sqrt[a + c*x^4]) + (c^(3/4)*d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)
/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/
(2*a^(1/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (3*c^(3/4)*d*(Sqrt[c]*d^2 - S
qrt[a]*e^2)^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x
^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2)
, 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*(c*d^4 + a*e^4)^2*Sqrt[a
+ c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1723

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]
), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q) - a*B*(e + d*q
))/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] + Dist[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x],
 x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2
- a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1729

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Dist
[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x^2
)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2]
 && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1741

```
Int[((d_) + (e_.)*(x_))^(q_)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[e
^3*(d + e*x)^(q + 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist
[c/((q + 1)*(c*d^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[
d^3*(q + 1) - d^2*e*(q + 1)*x + d*e^2*(q + 1)*x^2 - e^3*(q + 3)*x^3, x], x]
, x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^4 + a*e^4, 0] && ILtQ[q, -1]
```

Rule 1753

```
Int[((Px_)*((d_) + (e_.)*(x_))^(q_))/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :
> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D =
Coeff[Px, x, 3]}, Simp[(-(d^3*D - C*d^2*e + B*d*e^2 - A*e^3))*(d + e*x)^(q
+ 1)*(Sqrt[a + c*x^4]/((q + 1)*(c*d^4 + a*e^4))), x] + Dist[1/((q + 1)*(c*d
^4 + a*e^4)), Int[((d + e*x)^(q + 1)/Sqrt[a + c*x^4])*Simp[(q + 1)*(a*e*(d^
2*D - C*d*e + B*e^2) + A*d*(c*d^2)) - (e*(q + 1)*(A*c*d^2 + a*e*(d*D - C*e)
) - B*d*(c*d^2*(q + 1)))*x + (q + 1)*(D*e*(a*e^2) + c*d*(C*d^2 - e*(B*d - A
*e)))*x^2 + c*(q + 3)*(d^3*D - C*d^2*e + B*d*e^2 - A*e^3)*x^3, x], x], x]]
/; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px, x], 3] && NeQ[c*
```

```
d^4 + a*e^4, 0] && LtQ[q, -1]
```

Rule 1756

```
Int[(Px_)/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Wit
h[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2], D = Coeff
[Px, x, 3]}, Int[(x*(B*d - A*e + (d*D - C*e)*x^2))/((d^2 - e^2*x^2)*Sqrt[a
+ c*x^4]), x] + Int[(A*d + (C*d - B*e)*x^2 - D*e*x^4)/((d^2 - e^2*x^2)*Sqrt
[a + c*x^4]), x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x] && LeQ[Expon[Px
, x], 3] && NeQ[c*d^4 + a*e^4, 0]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\sqrt{a+cx^4}}\,dx = -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{c\int \frac{-2d^3+2d^2ex-2de^2x^2}{(d+ex)^2\sqrt{a+cx^4}}\,dx}{2\,(cd^4+ae^4)}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} + \frac{c\int \frac{2d^2(cd^4-2ae^4)-2de(2cd^4-ae^4)}{(d+ex)\sqrt{a+}}}{2\,(cd^4+a}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} + \frac{c\int \frac{(-2d^2e(cd^4-2ae^4)-2d^2e(2cd^4-}{(d^2-e^2x^2)\sqrt{a+cx^4}}}{2\,(cd^4+ae^4)^2}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} - \frac{\int \frac{-6\sqrt{a}\,c^{3/2}d^5e^4-2cd^3e^2(cd^4-2a}{}}{}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right.}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right.}$$

$$= -\frac{e^3\sqrt{a+cx^4}}{2\,(cd^4+ae^4)\,(d+ex)^2} - \frac{3cd^3e^3\sqrt{a+cx^4}}{(cd^4+ae^4)^2\,(d+ex)} + \frac{3c^{3/2}d^3e^2x\sqrt{a+cx^4}}{(cd^4+ae^4)^2\left(\sqrt{a}+\sqrt{c}\,x^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.

time = 11.41, size = 614, normalized size = 0.93

$$-\frac{\frac{cd^4e^3\sqrt{a+cx^4}}{(d+ex)^2}+\frac{ae^7\sqrt{a+cx^4}}{(d+ex)^2}+\frac{6cd^3e^3\sqrt{a+cx^4}}{d+ex}-\frac{6c^2d^6e\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}+\frac{6acd^2e^5\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)}{\sqrt{-cd^4-ae^4}}+\frac{6ia\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,cd^3e^2\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{a+cx^4}}+\frac{2icd\left(-2cd^4-3i\sqrt{a}\,\sqrt{c}\,d^2e^2+ae^4\right)\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{a+cx^4}}+\frac{6\sqrt[4]{-1}\,\sqrt[4]{a}\,c^{7/4}d^5\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a+cx^4}}-\frac{6\sqrt[4]{-1}\,a^{5/4}c^{3/4}de^4\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{a+cx^4}}}{2\left(cd^4+ae^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*Sqrt[a + c*x^4]),x]
```

```
[Out] -1/2*((c*d^4*e^3*Sqrt[a + c*x^4])/(d + e*x)^2 + (a*e^7*Sqrt[a + c*x^4])/(d
+ e*x)^2 + (6*c*d^3*e^3*Sqrt[a + c*x^4])/(d + e*x) - (6*c^2*d^6*e*ArcTan[(S
qrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt
[-(c*d^4) - a*e^4] + (6*a*c*d^2*e^5*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*S
qrt[a + c*x^4])/Sqrt[-(c*d^4) - a*e^4]])/Sqrt[-(c*d^4) - a*e^4] + ((6*I)*a*
Sqrt[(I*Sqrt[c])/Sqrt[a]]*c*d^3*e^2*Sqrt[1 + (c*x^4)/a]*EllipticE[I*ArcSinh
[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[a + c*x^4] + ((2*I)*c*d*(-2*c*d^4
- (3*I)*Sqrt[a]*Sqrt[c]*d^2*e^2 + a*e^4)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*Ar
cSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a
+ c*x^4]) + (6*(-1)^(1/4)*a^(1/4)*c^(7/4)*d^5*Sqrt[1 + (c*x^4)/a]*EllipticP
i[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1
])/Sqrt[a + c*x^4] - (6*(-1)^(1/4)*a^(5/4)*c^(3/4)*d*e^4*Sqrt[1 + (c*x^4)/a
]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSin[((-1)^(3/4)*c^(1/4)*x)/a
^(1/4)], -1])/Sqrt[a + c*x^4])/(c*d^4 + a*e^4)^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 483, normalized size = 0.73

method	result
default	$-\frac{e^3\sqrt{c\,x^4+a}}{2(e^4a+d^4c)(ex+d)^2}-\frac{3c\,d^3e^3\sqrt{c\,x^4+a}}{(e^4a+d^4c)^2(ex+d)}+\frac{cd(e^4a-2d^4c)\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\right.}{(e^4a+d^4c)^2\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$

elliptic	$-\frac{e^{3}\sqrt{c\,x^{4}+a}}{2\left(e^{4}a+d^{4}c\right)\left(ex+d\right)^{2}}-\frac{3c\,d^{3}e^{3}\sqrt{c\,x^{4}+a}}{\left(e^{4}a+d^{4}c\right)^{2}\left(ex+d\right)}+\frac{cd\left(e^{4}a-2d^{4}c\right)\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},\right.}{\left(e^{4}a+d^{4}c\right)^{2}\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d)^3/(c*x^4+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)/(e*x+d)^2-3*c*d^3*e^3*(c*x^4+a)^(1/2)/(a*e^4+c*d^4)^2/(e*x+d)+c*d*(a*e^4-2*c*d^4)/(a*e^4+c*d^4)^2/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)+3*I*d^3*e^2*c^(3/2)/(a*e^4+c*d^4)^2*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I))-3*c*d^2*(a*e^4-c*d^4)/(a*e^4+c*d^4)^2/e*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)^3), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="fricas")

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + cx^4}\,(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(c*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**4)*(d + e*x)**3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(c*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^4 + a)*(x*e + d)^3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{c\,x^4+a}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3),x)
[Out] int(1/((a + c*x^4)^(1/2)*(d + e*x)^3), x)
```

3.219 $\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=298

$$-\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\Big(2\,\mathrm{ta}}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}$$

```
[Out] 1/2*(-a*e^3+c*x*(3*d*e^2*x^2+3*d^2*e*x+d^3))/a/c/(c*x^4+a)^(1/2)-3/2*d*e^2*
x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1/2)+x^2*c^(1/2))+3/2*d*e^2*(cos(2*arctan(c
^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(
2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/
(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*d*(cos(2
*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Ellip
ticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(-3*e^2*a^(1/2)+d^2*c^(1
/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4
)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1868, 1212, 226, 1210}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{c}\,d^2-3\sqrt{a}\,e^2\right)F\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{3de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\mathrm{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}-\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}}-\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-3*d*e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a*e^3
 - c*x*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2))/(2*a*c*Sqrt[a + c*x^4]) + (3*d*e^2*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4])
 + (d*(Sqrt[c]*d^2 - 3*Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])
/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1868

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[(a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^q
, x])*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(d+ex)^3}{(a+cx^4)^{3/2}}\,dx = -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} - \frac{\int \frac{-d^3+3de^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}$$

$$= -\frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} + \frac{(3de^2)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\sqrt{c}} + \frac{\left(d\left(d^2-\frac{3\sqrt{a}\,e^2}{\sqrt{c}}\right)\right)\int \ldots}{2a}$$

$$= -\frac{3de^2x\sqrt{a+cx^4}}{2a\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{ae^3-cx(d^3+3d^2ex+3de^2x^2)}{2ac\sqrt{a+cx^4}} + \frac{3de^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\ldots}}{2\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 126, normalized size = 0.42

$$\frac{-ae^3+cd^3x+3cd^2ex^2+cd^3x\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2cde^2x^3\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)}{2ac\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + c*x^4)^(3/2),x]
```

```
[Out] (-(a*e^3) + c*d^3*x + 3*c*d^2*e*x^2 + c*d^3*x*Sqrt[1 + (c*x^4)/a]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)] + 2*c*d*e^2*x^3*Sqrt[1 + (c*x^4)/a]*H
ypergeometric2F1[3/4, 3/2, 7/4, -((c*x^4)/a)])/(2*a*c*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 261, normalized size = 0.88

method	result
elliptic	$-\frac{2c\left(-\frac{3d\,e^2x^3}{4ac}-\frac{3d^2e\,x^2}{4ca}-\frac{d^3x}{4ac}+\frac{e^3}{4c^2}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^3\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{3id\,e^2\sqrt{1-}}{}$
default	$-\frac{e^3}{2c\sqrt{c\,x^4+a}}+3d\,e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{E}\right.}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^3/c/(c*x^4+a)^(1/2)+3*d*e^2*(1/2*x^3/a/((x^4+a/c)*c)^(1/2)-1/2*I/a^(
1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)
*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2))^(1/2),I)))+3/2*d^2*e/(c*x^4+a)^(
1/2)*x^2/a+d^3*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)
*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^
(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/(c*x^4 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^3}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**3/(a + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^3/(c*x^4 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^3}{(c\,x^4+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + c*x^4)^(3/2),x)
```

```
[Out] int((d + e*x)^3/(a + c*x^4)^(3/2), x)
```

3.220 $\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=270

$$\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}-\frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\,(\sqrt{a}+\sqrt{c}\,x^2)}+\frac{e^2(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{(\sqrt{}}{}$$

```
[Out] 1/2*x*(e*x+d)^2/a/(c*x^4+a)^(1/2)-1/2*e^2*x*(c*x^4+a)^(1/2)/a/c^(1/2)/(a^(1
/2)+x^2*c^(1/2))+1/2*e^2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3
/4)/c^(3/4)/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/
cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4)))
,1/2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/c^(3/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1869, 1212, 226, 1210}

$$\frac{(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,(\sqrt{c}\,d^2-\sqrt{a}\,e^2)\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}c^{3/4}\sqrt{a+cx^4}}+\frac{e^2(\sqrt{a}+\sqrt{c}\,x^2)\sqrt{\frac{a+cx^4}{(\sqrt{a}+\sqrt{c}\,x^2)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}+\frac{x(d+ex)^2}{2a\sqrt{a+cx^4}}-\frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\,(\sqrt{a}+\sqrt{c}\,x^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x)^2)/(2*a*Sqrt[a + c*x^4]) - (e^2*x*Sqrt[a + c*x^4])/(2*a*Sqrt[c
]*(Sqrt[a] + Sqrt[c]*x^2)) + (e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(
2*a^(3/4)*c^(3/4)*Sqrt[a + c*x^4]) + ((Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a]
+ Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*c^(3/4)*Sqrt[a + c*x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^2}{(a+cx^4)^{3/2}}\,dx &= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{\int \frac{-d^2+e^2x^2}{\sqrt{a+cx^4}}\,dx}{2a}\\
&= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} + \frac{e^2\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+cx^4}}\,dx}{2\sqrt{a}\,\sqrt{c}} + \frac{\left(d^2-\frac{\sqrt{a}\,e^2}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}\\
&= \frac{x(d+ex)^2}{2a\sqrt{a+cx^4}} - \frac{e^2x\sqrt{a+cx^4}}{2a\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{e^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\Big(2}{2a^{3/4}c^{3/4}\sqrt{a+cx^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.05, size = 108, normalized size = 0.40

$$\frac{x\left(3d(d+2ex)+3d^2\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)+2e^2x^2\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{cx^4}{a}\right)\right)}{6a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(3*d*(d + 2*e*x) + 3*d^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2,
 5/4, -((c*x^4)/a)] + 2*e^2*x^2*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[3/4,
3/2, 7/4, -((c*x^4)/a)]))/(6*a*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 239, normalized size = 0.89

method	result
elliptic	$-\frac{2c\left(-\frac{e^2x^3}{4ac}-\frac{de\,x^2}{2ca}-\frac{d^2x}{4ac}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}-\frac{ie^2\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{}$
default	$e^2\left(\frac{x^3}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}-\frac{i\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\left(\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)-\mathrm{EllipticE}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)\right)}{2\sqrt{a}\,\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}\,\sqrt{c}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2*(1/2*x^3/a/((x^4+a/c)*c)^(1/2)-1/2*I/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(
1/2)/c^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/
2)*c^(1/2))^(1/2),I)))+d*e/(c*x^4+a)^(1/2)*x^2/a+d^2*(1/2*x/a/((x^4+a/c)*c)
^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))
^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/(c*x^4 + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex)^2}{(a+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral((d + e*x)**2/(a + c*x**4)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)^2/(c*x^4 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x)^2}{(c\,x^4+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + c*x^4)^(3/2),x)
```

```
[Out] int((d + e*x)^2/(a + c*x^4)^(3/2), x)
```

3.221 $\int \frac{d+ex}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=114

$$\frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*x*(e*x+d)/a/(c*x^4+a)^(1/2)+1/4*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^
(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1869, 12, 226}

$$\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x(d+ex)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + c*x^4)^(3/2),x]
```

```
[Out] (x*(d + e*x))/(2*a*Sqrt[a + c*x^4]) + (d*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
1/2])/(4*a^(5/4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1869

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(-x)*Pq*((a + b
*x^n)^(p + 1)/(a*n*(p + 1))), x] + Dist[1/(a*n*(p + 1)), Int[ExpandToSum[n*
```

```
(p + 1)*Pq + D[x*Pq, x], x]*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x]
 && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[Expon[Pq, x], n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex}{\left(a+cx^4\right)^{3/2}}\,dx &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{\int \frac{d}{\sqrt{a+cx^4}}\,dx}{2a}\\ &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{d\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}\\ &= \frac{x(d+ex)}{2a\sqrt{a+cx^4}}+\frac{d\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 59, normalized size = 0.52

$$\frac{x\left(d+ex+d\sqrt{1+\frac{cx^4}{a}}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)\right)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + c*x^4)^(3/2),x]
[Out] (x*(d + e*x + d*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x
^4)/a)]))/(2*a*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 115, normalized size = 1.01

method	result	size
default	$\frac{e\,x^2}{2\sqrt{c\,x^4+a}\;a}+d\left(\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}\right)$	115

elliptic	$-\frac{2c\left(-\frac{e\,x^{2}}{4ca}-\frac{dx}{4ac}\right)}{\sqrt{\left(x^{4}+\frac{a}{c}\right)c}}+\frac{d\sqrt{1-\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^{2}}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^{4}+a}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e/(c*x^4+a)^(1/2)*x^2/a+d*(1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)
*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(
1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/(c*x^4 + a)^(3/2), x)
```

Fricas [A]
time = 0.10, size = 76, normalized size = 0.67

$$-\frac{(cdx^{4}+ad)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}\mathrm{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)-\sqrt{cx^{4}+a}\,(cx^{2}e+cdx)}{2\,(ac^{2}x^{4}+a^{2}c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x^4 + a*d)*sqrt(a)*(-c/a)^(3/4)*ellipticF(x*(-c/a)^(1/4), -1) -
sqrt(c*x^4 + a)*(c*x^2*e + c*d*x))/(a*c^2*x^4 + a^2*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.02, size = 61, normalized size = 0.54

$$\frac{dx\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{cx^{4}e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}+\frac{ex^{2}}{2a^{\frac{3}{2}}\sqrt{1+\frac{cx^{4}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] d*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3
/2)*gamma(5/4)) + e*x**2/(2*a**(3/2)*sqrt(1 + c*x**4/a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + d)/(c*x^4 + a)^(3/2), x)
```

Mupad [B]
time = 2.88, size = 57, normalized size = 0.50

$$\frac{e\,x^2}{2\,a\,\sqrt{c\,x^4+a}}+\frac{d\,x\,{\left(\frac{c\,x^4}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{{\left(c\,x^4+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + c*x^4)^(3/2),x)
```

```
[Out] (e*x^2)/(2*a*(a + c*x^4)^(1/2)) + (d*x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4
, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4)^(3/2)
```

3.222 $\int \frac{1}{(a+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=108

$$\frac{x}{2a\sqrt{a+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

```
[Out] 1/2*x/a/(c*x^4+a)^(1/2)+1/4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(
2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2
*2^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a
^(5/4)/c^(1/4)/(c*x^4+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {205, 226}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}+\frac{x}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*x^4)^(-3/2),x]
```

```
[Out] x/(2*a*Sqrt[a + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a
] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/
4)*c^(1/4)*Sqrt[a + c*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rubi steps

$$\int \frac{1}{(a+cx^4)^{3/2}}\,dx = \frac{x}{2a\sqrt{a+cx^4}} + \frac{\int \frac{1}{\sqrt{a+cx^4}}\,dx}{2a}$$
$$= \frac{x}{2a\sqrt{a+cx^4}} + \frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\sqrt[4]{c}\,\sqrt{a+cx^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.19, size = 55, normalized size = 0.51

$$\frac{x + x\sqrt{1+\frac{cx^4}{a}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{cx^4}{a}\right)}{2a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*x^4)^(-3/2),x]
```

```
[Out] (x + x*Sqrt[1 + (c*x^4)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((c*x^4)/a)])/
(2*a*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 94, normalized size = 0.87

method	result	size
default	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}} + \frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94
elliptic	$\frac{x}{2a\sqrt{\left(x^4+\frac{a}{c}\right)c}} + \frac{\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}},i\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,\sqrt{c\,x^4+a}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x/a/((x^4+a/c)*c)^(1/2)+1/2/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^
(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(
x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Fricas [A]
time = 0.08, size = 63, normalized size = 0.58

$$-\frac{(cx^4+a)\sqrt{a}\left(-\frac{c}{a}\right)^{\frac{3}{4}}\text{ellipticF}\left(x\left(-\frac{c}{a}\right)^{\frac{1}{4}},-1\right)-\sqrt{cx^4+a}\,cx}{2\left(ac^2x^4+a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^4 + a)*sqrt(a)*(-c/a)^(3/4)*ellipticF(x*(-c/a)^(1/4), -1) - sqrt
(c*x^4 + a)*c*x)/(a*c^2*x^4 + a^2*c)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.39, size = 36, normalized size = 0.33

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4a^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+a)**(3/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/(4*a**(3/2
)*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + a)^(-3/2), x)
```

Mupad [B]
time = 2.66, size = 37, normalized size = 0.34

$$\frac{x\left(\frac{c\,x^4}{a}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{c\,x^4}{a}\right)}{\left(c\,x^4+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + c*x^4)^(3/2),x)
```

```
[Out] (x*((c*x^4)/a + 1)^(3/2)*hypergeom([1/4, 3/2], 5/4, -(c*x^4)/a))/(a + c*x^4
)^(3/2)
```

3.223 $\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=818

$$\frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}}+\frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}}-\frac{\sqrt{c}\,de^2x\sqrt{a+cx^4}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-a}}{de\sqrt{a+c}}\right.}{2\left(-cd^4-ae^4\right)^{3/}}$$

```
[Out] -1/2*e^5*arctan(x*(-a*e^4-c*d^4)^(1/2)/d/e/(c*x^4+a)^(1/2))/(-a*e^4-c*d^4)^
(3/2)-1/2*e^5*arctanh((c*d^2*x^2+a*e^2)/(a*e^4+c*d^4)^(1/2)/(c*x^4+a)^(1/2)
)/(a*e^4+c*d^4)^(3/2)+1/2*e*(-c*d^2*x^2+a*e^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1
/2)+1/2*c*d*x*(e^2*x^2+d^2)/a/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)-1/2*d*e^2*x*c^(
1/2)*(c*x^4+a)^(1/2)/a/(a*e^4+c*d^4)/(a^(1/2)+x^2*c^(1/2))+1/2*c^(1/4)*d*e^
2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)
))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(a*e^4+c*d^4)/(c*x^
4+a)^(1/2)+1/4*c^(1/4)*d*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^
(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(1/2)
+x^2*c^(1/2))^2)^(1/2)/a^(5/4)/(a*e^4+c*d^4)/(c*x^4+a)^(1/2)+1/2*c^(1/4)*d*
e^4*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x^2*c
^(1/2))*((c*x^4+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/(a*e^4+c*d^4)/(e^
2*a^(1/2)+d^2*c^(1/2))/(c*x^4+a)^(1/2)-1/4*e^4*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(c^(
1/4)*x/a^(1/4))),1/4*(e^2*a^(1/2)+d^2*c^(1/2))^2/d^2/e^2/a^(1/2)/c^(1/2),1/
2*2^(1/2))*(-e^2*a^(1/2)+d^2*c^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(1/4)/c^(1/4)/d/(a*e^4+c*d^4)/(e^2*a^(1/2)+d^2
*c^(1/2))/(c*x^4+a)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {1743, 1236, 1193, 1212, 226, 1210, 1231, 1721, 1262, 755, 12, 739, 212}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{-cd^4-ae^4}\,x}{de\sqrt{cx^4+a}}\right)e^5}{2\left(-cd^4-ae^4\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{ae^2+cd^2x^2}{\sqrt{cd^4+ae^4}\sqrt{cx^4+a}}\right)e^5}{2\left(cd^4+ae^4\right)^{3/2}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{2\sqrt[4]{a}\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\frac{\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)^2}{4\sqrt{a}\sqrt{c}\,d^2e^2};2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^4}{4\sqrt[4]{a}\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\left(cd^4+ae^4\right)\sqrt{cx^4+a}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e^2}{2a^{3/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}-\frac{\sqrt{c}\,dx\sqrt{cx^4+a}\,e^2}{2a\left(cd^4+ae^4\right)\left(\sqrt{c}\,x^2+\sqrt{a}\right)}+\frac{\left(ae^2-cd^2x^2\right)e}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}+\frac{\sqrt[4]{c}\,d\left(\sqrt{c}\,d^2-\sqrt{a}\,e^2\right)\left(\sqrt{c}\,x^2+\sqrt{a}\right)\sqrt{\frac{cx^4+a}{\left(\sqrt{c}\,x^2+\sqrt{a}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}\left(cd^4+ae^4\right)\sqrt{cx^4+a}}+\frac{cdx\left(d^2+e^2x^2\right)}{2a\left(cd^4+ae^4\right)\sqrt{cx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] (e*(a*e^2 - c*d^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c*d*x*(d^2
 + e^2*x^2))/(2*a*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) - (Sqrt[c]*d*e^2*x*Sqrt[
a + c*x^4])/(2*a*(c*d^4 + a*e^4)*(Sqrt[a] + Sqrt[c]*x^2)) - (e^5*ArcTan[(Sq
```

```
rt[-(c*d^4) - a*e^4]*x)/(d*e*Sqrt[a + c*x^4])])/(2*(-(c*d^4) - a*e^4)^(3/2)
) - (e^5*ArcTanh[(a*e^2 + c*d^2*x^2)/(Sqrt[c*d^4 + a*e^4]*Sqrt[a + c*x^4])]
)/(2*(c*d^4 + a*e^4)^(3/2)) + (c^(1/4)*d*e^2*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4
)], 1/2])/(2*a^(3/4)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*(Sqrt[c]
*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqr
t[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(5/4)*(c*d
^4 + a*e^4)*Sqrt[a + c*x^4]) + (c^(1/4)*d*e^4*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], 1/2])/(2*a^(1/4)*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a +
c*x^4]) - (e^4*(Sqrt[c]*d^2 - Sqrt[a]*e^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticPi[(Sqrt[c]*d^2 + Sqrt[a]*e^2)^
2/(4*Sqrt[a]*Sqrt[c]*d^2*e^2), 2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c^(1/4)*d*(Sqrt[c]*d^2 + Sqrt[a]*e^2)*(c*d^4 + a*e^4)*Sqrt[a + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1236

```
Int[((a_) + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symbol] :> Dist[1/(
c*d^2 + a*e^2), Int[(c*d - c*e*x^2)*(a + c*x^4)^p, x], x] + Dist[e^2/(c*d^2
 + a*e^2), Int[(a + c*x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1743

```
Int[((a_) + (c_.)*(x_)^4)^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] :> Dist[d, I
nt[(a + c*x^4)^p/(d^2 - e^2*x^2), x], x] - Dist[e, Int[x*((a + c*x^4)^p/(d^
2 - e^2*x^2)), x], x] /; FreeQ[{a, c, d, e}, x] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+cx^4\right)^{3/2}}\,dx = d\int \frac{1}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx - e\int \frac{x}{\left(d^2-e^2x^2\right)\left(a+cx^4\right)^{3/2}}\,dx$$

$$= -\left(\frac{1}{2}e\text{Subst}\left(\int \frac{1}{\left(d^2-e^2x\right)\left(a+cx^2\right)^{3/2}}\,dx, x, x^2\right)\right) + \frac{d\int \frac{cd^2+ce^2x^2}{\left(a+cx^4\right)^{3/2}}\,dx}{cd^4+ae^4} + \frac{(d}{}$$

$$= \frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{d\int \frac{-cd^2+ce^2x^2}{\sqrt{a+cx^4}}\,dx}{2a\left(cd^4+ae^4\right)} -$$

$$= \frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{e^5\tan^{-1}\left(\frac{\sqrt{-cd^4-}}{de\sqrt{a+}}\right.}{2\left(-cd^4-ae^4\right)^{3}}$$

$$= \frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}\,de^2x\sqrt{a+}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}\right.}$$

$$= \frac{e(ae^2-cd^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} + \frac{cdx(d^2+e^2x^2)}{2a\left(cd^4+ae^4\right)\sqrt{a+cx^4}} - \frac{\sqrt{c}\,de^2x\sqrt{a+}}{2a\left(cd^4+ae^4\right)\left(\sqrt{a}\right.}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.64, size = 455, normalized size = 0.56

$$-\frac{-\sqrt{a}\,c^{3/4}d^2e^2\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+c^{3/4}d^2\left(-i\sqrt{c}\,d^2+\sqrt{a}\,e^2\right)\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\left(\sqrt[4]{c}\,d\left(\sqrt{-cd^4-ae^4}\left(ae^3+cdx(d^2-dex+e^2x^2)\right)+2ae^5\sqrt{a+cx^4}\tan^{-1}\left(\frac{\sqrt{c}\,(d^2-e^2x^2)+e^2\sqrt{a+cx^4}}{\sqrt{-cd^4-ae^4}}\right)\right)-2\sqrt[4]{-1}\,a^{5/4}e^4\sqrt{-cd^4-ae^4}\sqrt{1+\frac{cx^4}{a}}\,\Pi\left(\frac{i\sqrt{a}\,e^2}{\sqrt{c}\,d^2};\sin^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\middle|-1\right)\right)}{2a\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt[4]{c}\,d\,(-cd^4-ae^4)^{3/2}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*(-(Sqrt[a]*c^(3/4)*d^2*e^2*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*
EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]) + c^(3/4)*d^2*((-I)*
Sqrt[c]*d^2 + Sqrt[a]*e^2)*Sqrt[-(c*d^4) - a*e^4]*Sqrt[1 + (c*x^4)/a]*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] + Sqrt[(I*Sqrt[c])/Sqrt[a]
]*(c^(1/4)*d*(Sqrt[-(c*d^4) - a*e^4]*(a*e^3 + c*d*x*(d^2 - d*e*x + e^2*x^2)
) + 2*a*e^5*Sqrt[a + c*x^4]*ArcTan[(Sqrt[c]*(d^2 - e^2*x^2) + e^2*Sqrt[a +
c*x^4])/Sqrt[-(c*d^4) - a*e^4]]) - 2*(-1)^(1/4)*a^(5/4)*e^4*Sqrt[-(c*d^4) -
 a*e^4]*Sqrt[1 + (c*x^4)/a]*EllipticPi[(I*Sqrt[a]*e^2)/(Sqrt[c]*d^2), ArcSi
n[((-1)^(3/4)*c^(1/4)*x)/a^(1/4)], -1]))/(a*Sqrt[(I*Sqrt[c])/Sqrt[a]]*c^(1/
4)*d*(-(c*d^4) - a*e^4)^(3/2)*Sqrt[a + c*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 496, normalized size = 0.61

method	result
default	$-\frac{2c\left(-\frac{d\,e^2x^3}{4a\left(e^4a+d^4c\right)}+\frac{d^2e\,x^2}{4a\left(e^4a+d^4c\right)}-\frac{d^3x}{4a\left(e^4a+d^4c\right)}-\frac{e^3}{4\left(e^4a+d^4c\right)c}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i}{\ldots}}\right.}{2a(e^4a+d^4c)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}$
elliptic	$-\frac{2c\left(-\frac{d\,e^2x^3}{4a\left(e^4a+d^4c\right)}+\frac{d^2e\,x^2}{4a\left(e^4a+d^4c\right)}-\frac{d^3x}{4a\left(e^4a+d^4c\right)}-\frac{e^3}{4\left(e^4a+d^4c\right)c}\right)}{\sqrt{\left(x^4+\frac{a}{c}\right)c}}+\frac{d^3c\sqrt{1-\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{c}\,x^2}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\frac{i}{\ldots}}\right.}{2a(e^4a+d^4c)\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{c\,x^4+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(c*x^4+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*c*(-1/4/a*d*e^2/(a*e^4+c*d^4)*x^3+1/4/a*d^2*e/(a*e^4+c*d^4)*x^2-1/4*d^3/
a/(a*e^4+c*d^4)*x-1/4*e^3/(a*e^4+c*d^4)/c)/((x^4+a/c)*c)^(1/2)+1/2*d^3/a*c/
(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+
I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2)
)^(1/2),I)-1/2*I*c^(1/2)/a^(1/2)*d*e^2/(a*e^4+c*d^4)/(I/a^(1/2)*c^(1/2))^(1
/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+
a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c
^(1/2))^(1/2),I))+e^3/(a*e^4+c*d^4)*(-1/2/(c*d^4/e^4+a)^(1/2)*arctanh(1/2*(
2*c*x^2*d^2/e^2+2*a)/(c*d^4/e^4+a)^(1/2)/(c*x^4+a)^(1/2))+1/(I/a^(1/2)*c^(1
/2))^(1/2)/d*e*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1
/2)/(c*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*c^(1/2))^(1/2),-I*a^(1/2)/c^(1/
2)/d^2*e^2,(-I/a^(1/2)*c^(1/2))^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(x*e + d)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + cx^{4}\right)^{\frac{3}{2}} \left(d + ex\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x**4+a)**(3/2),x)
```

```
[Out] Integral(1/((a + c*x**4)**(3/2)*(d + e*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(c*x^4+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + a)^(3/2)*(x*e + d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(c\,x^4+a\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^(3/2)*(d + e*x)),x)
```

```
[Out] int(1/((a + c*x^4)^(3/2)*(d + e*x)), x)
```

3.224 $\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)(1+n)}-\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\right.}{4b^{3/4}\left(\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\,d\right)(1+n)}$$

```
[Out] -1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hyperge
om([1, 1+n],[2+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(1+n)-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(1+n)*hypergeom([1, 1+n],[2+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(1+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A]
time = 0.57, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {6857, 845, 70}

$$-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}\,c\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d\right)}-\frac{(c+dx)^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(n+1)\left(\sqrt[4]{-a}\,d+\sqrt[4]{b}\,c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] -1/4*((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*
x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]
*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/
4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqr
t[-Sqrt[-a]]*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c - (-a)^(1/4)*d)*(1 + n)) - ((c + d*x)^(1 + n)*Hypergeometric2F1[1, 1 + n,
 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4
)*c + (-a)^(1/4)*d)*(1 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3(c+dx)^n}{a+bx^4}\,dx = \int \left(\frac{x(c+dx)^n}{2\left(-\sqrt{-a}\,\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^n}{2\left(\sqrt{-a}\,\sqrt{b}+bx^2\right)}\right)dx$$

$$= \frac{1}{2}\int \frac{x(c+dx)^n}{-\sqrt{-a}\,\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^n}{\sqrt{-a}\,\sqrt{b}+bx^2}\,dx$$

$$= \frac{1}{2}\int \left(-\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x\right)}+\frac{(c+dx)^n}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x\right)}\right)dx+\frac{1}{2}\int \Bigg(-$$

$$= -\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^n}{\sqrt[4]{-a}+\sqrt[4]{b}}}{4b^{3/4}}$$

$$= -\frac{(c+dx)^{1+n}\,{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)(1+n)}-\frac{(c+dx)^{1+n}\,{}_2F_1\Bigg(1,1+n;2+}{4b^{3/4}\left(\sqrt[4]{b}\,c+\sqrt{-}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.35, size = 274, normalized size = 0.79

$$\frac{(c+dx)^{1+n}\left(-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-i\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c-i\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+i\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c+i\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,1+n;2+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^n)/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 1 + n, 2 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(1 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(d x +c \right)^{n}}{b \,x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^n/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**n/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^n/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^n*x^3/(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c+d\,x\right)}^n}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x)^n)/(a + b*x^4),x)
```

```
[Out] int((x^3*(c + d*x)^n)/(a + b*x^4), x)
```

3.225 $\int \frac{x^3(c+dx)^{1+n}}{a+bx^4} \, dx$

Optimal. Leaf size=349

$$-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\,d\right)(2+n)}$$

```
[Out] -1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c-(-a)
^(1/4)*d))/b^(3/4)/(b^(1/4)*c-(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hyperge
om([1, 2+n],[3+n],b^(1/4)*(d*x+c)/(b^(1/4)*c+(-a)^(1/4)*d))/b^(3/4)/(b^(1/4
)*c+(-a)^(1/4)*d)/(2+n)-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*
(d*x+c)/(b^(1/4)*c-d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c-d*(-(-a
)^(1/2))^(1/2))-1/4*(d*x+c)^(2+n)*hypergeom([1, 2+n],[3+n],b^(1/4)*(d*x+c)/
(b^(1/4)*c+d*(-(-a)^(1/2))^(1/2)))/b^(3/4)/(2+n)/(b^(1/4)*c+d*(-(-a)^(1/2))
^(1/2))
```

Rubi [A]
time = 0.46, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {6857, 845, 70}

$$-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt{-\sqrt{-a}}\,d+\sqrt[4]{b}\,c\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d\right)}-\frac{(c+dx)^{n+2}\,{}_2F_1\left(1,n+2;n+3;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(n+2)\left(\sqrt[4]{-a}\,d+\sqrt[4]{b}\,c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] -1/4*((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*
x))/(b^(1/4)*c - Sqrt[-Sqrt[-a]]*d)])/(b^(3/4)*(b^(1/4)*c - Sqrt[-Sqrt[-a]]
*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/
4)*(c + d*x))/(b^(1/4)*c + Sqrt[-Sqrt[-a]]*d)])/(4*b^(3/4)*(b^(1/4)*c + Sqr
t[-Sqrt[-a]]*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n, 3
 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4)*
c - (-a)^(1/4)*d)*(2 + n)) - ((c + d*x)^(2 + n)*Hypergeometric2F1[1, 2 + n,
 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)])/(4*b^(3/4)*(b^(1/4
)*c + (-a)^(1/4)*d)*(2 + n))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
```

```
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{x^3(c+dx)^{1+n}}{a+bx^4}\,dx = \int \left(\frac{x(c+dx)^{1+n}}{2\left(-\sqrt{-a}\,\sqrt{b}+bx^2\right)}+\frac{x(c+dx)^{1+n}}{2\left(\sqrt{-a}\,\sqrt{b}+bx^2\right)}\right)dx$$

$$=\frac{1}{2}\int \frac{x(c+dx)^{1+n}}{-\sqrt{-a}\,\sqrt{b}+bx^2}\,dx+\frac{1}{2}\int \frac{x(c+dx)^{1+n}}{\sqrt{-a}\,\sqrt{b}+bx^2}\,dx$$

$$=\frac{1}{2}\int \left(-\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x\right)}+\frac{(c+dx)^{1+n}}{2b^{3/4}\left(\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x\right)}\right)dx+\frac{1}{2}\int \Bigg(-$$

$$=-\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}-\frac{\int \frac{(c+dx)^{1+n}}{\sqrt[4]{-a}-\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1+n}}{\sqrt{-\sqrt{-a}}+\sqrt[4]{b}\,x}\,dx}{4b^{3/4}}+\frac{\int \frac{(c+dx)^{1}}{\sqrt[4]{-a}+}}{4b^{3/}}$$

$$=-\frac{(c+dx)^{2+n}\,{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d}\right)}{4b^{3/4}\left(\sqrt[4]{b}\,c-\sqrt{-\sqrt{-a}}\,d\right)(2+n)}-\frac{(c+dx)^{2+n}\,{}_2F_1\Bigg(1,2+n;}{4b^{3/4}\left(\sqrt[4]{b}\,c+\sqrt{-\right.}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.24, size = 274, normalized size = 0.79

$$\frac{(c+dx)^{2+n}\left(-\frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c-\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c-i\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c-i\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+i\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c+i\sqrt[4]{-a}\,d}-\frac{{}_2F_1\left(1,2+n;3+n;\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{\sqrt[4]{b}\,c+\sqrt[4]{-a}\,d}\right)}{4b^{3/4}(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x)^(1 + n))/(a + b*x^4),x]
```

```
[Out] ((c + d*x)^(2 + n)*(-(Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x)
)/(b^(1/4)*c - (-a)^(1/4)*d)]/(b^(1/4)*c - (-a)^(1/4)*d)) - Hypergeometric2
F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c - I*(-a)^(1/4)*d)]/(b^(1
/4)*c - I*(-a)^(1/4)*d) - Hypergeometric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c +
d*x))/(b^(1/4)*c + I*(-a)^(1/4)*d)]/(b^(1/4)*c + I*(-a)^(1/4)*d) - Hypergeo
metric2F1[1, 2 + n, 3 + n, (b^(1/4)*(c + d*x))/(b^(1/4)*c + (-a)^(1/4)*d)]/
(b^(1/4)*c + (-a)^(1/4)*d)))/(4*b^(3/4)*(2 + n))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(d x +c \right)^{1+n}}{b \,x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

```
[Out] int(x^3*(d*x+c)^(1+n)/(b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x+c)**(1+n)/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x+c)^(1+n)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(n + 1)*x^3/(b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c+d\,x\right)}^{n+1}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4),x)
```

```
[Out] int((x^3*(c + d*x)^(n + 1))/(a + b*x^4), x)
```

3.226 $\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx$

Optimal. Leaf size=1605

$$-\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\,(d^2-2ce)}\,x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right)}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cde\sqrt{d^2-4ce}\right)}}+\frac{e^2\tan^{-1}\left(\frac{\sqrt{\ }}{\ }\right)}{\sqrt{2}\sqrt{d^2-4ce}}$$

```
[Out] 1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/
4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x^2*b^(1/2))*(d-(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(1/2))^2
)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*
c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)+1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*x/a
^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))^2/e^2
/a^(1/2)/b^(1/2)/(d-(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*b^(1/2)
)*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a)/(a^(1
/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(d-(-4*c*e+d^2)^(1/2))/(-4*c*e+d^
2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e-d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)
^(1/2)-1/2*b^(1/4)*e*(cos(2*arctan(b^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x^2*b^(1/2))*(d+(-4*c*e+d^2)^(1/2))*((b*x^4+a)/(a^(1/2)+x^2*b^(
1/2))^2)^(1/2)/a^(1/4)/(-4*c*e+d^2)^(1/2)/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e
+d*(-4*c*e+d^2)^(1/2)))/(b*x^4+a)^(1/2)-1/2*e*(cos(2*arctan(b^(1/4)*x/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x/a^(1/4)))*EllipticPi(sin(2*arctan(b^(1
/4)*x/a^(1/4))),1/4*(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2))
)^2/e^2/a^(1/2)/b^(1/2)/(d+(-4*c*e+d^2)^(1/2))^2,1/2*2^(1/2))*(a^(1/2)+x^2*
b^(1/2))*(2*e^2*a^(1/2)-b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))*((b*x^4+a
)/(a^(1/2)+x^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/(-4*c*e+d^2)^(1/2)/(d+(-4*
c*e+d^2)^(1/2))/(2*e^2*a^(1/2)+b^(1/2)*(d^2-2*c*e+d*(-4*c*e+d^2)^(1/2)))/(b
*x^4+a)^(1/2)-1/2*e^2*arctanh(1/4*(4*a*e^2+b*x^2*(d-(-4*c*e+d^2)^(1/2))^2)*
2^(1/2)/(b*x^4+a)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+
d^2)*(-4*c*e+d^2)^(1/2))^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2
*e+2*b*c^2*e^2+2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)+1/2*e^2*a
rctanh(1/4*(4*a*e^2+b*x^2*(d+(-4*c*e+d^2)^(1/2))^2)*2^(1/2)/(b*x^4+a)^(1/2)
/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2)
)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(b*d^4-4*b*c*d^2*e+2*b*c^2*e^2+2*a*e^4+
b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)-1/2*e^2*arctan(x*2^(1/2)*(-b*d^4
+4*b*c*d^2*e-2*b*c^2*e^2-2*a*e^4-b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)
/e/(d+(-4*c*e+d^2)^(1/2))/(b*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a
*e^4-b*(d^4-4*c*d^2*e+2*c^2*e^2+d^3*(-4*c*e+d^2)^(1/2)-2*c*d*e*(-4*c*e+d^2)
```

^(1/2)))^(1/2)+1/2*e^2*arctan(x*2^(1/2)*(-b*d^4+4*b*c*d^2*e-2*b*c^2*e^2-2*a*e^4+b*d*(-2*c*e+d^2)*(-4*c*e+d^2)^(1/2))^(1/2)/e/(d-(-4*c*e+d^2)^(1/2))/(b*x^4+a)^(1/2))*2^(1/2)/(-4*c*e+d^2)^(1/2)/(-2*a*e^4-b*(d^4-4*c*d^2*e+2*c^2*e^2-d^3*(-4*c*e+d^2)^(1/2)+2*c*d*e*(-4*c*e+d^2)^(1/2)))^(1/2)

Rubi [A]

time = 7.06, antiderivative size = 1605, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6860, 1739, 1231, 226, 1721, 1262, 739, 212}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bced^2-b\sqrt{d^2-4ce}\,(d^2-2ce)\,d-2ae^4-2bc^2e^2}\,x}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4+\sqrt{d^2-4ce}\,d^3-4ced^2-2ce\sqrt{d^2-4ce}\,d+2c^2e^2\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bced^2+b\sqrt{d^2-4ce}\,(d^2-2ce)\,d-2ae^4-2bc^2e^2}\,x}{e\left(d-\sqrt{d^2-4ce}\right)\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-\sqrt{d^2-4ce}\,d^3-4ced^2+2ce\sqrt{d^2-4ce}\,d+2c^2e^2\right)}}-\frac{\tanh^{-1}\left(\frac{4ae^2+b\left(d-\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2-b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}}+\frac{\tanh^{-1}\left(\frac{4ae^2+b\left(d+\sqrt{d^2-4ce}\right)^2x^2}{2\sqrt{2}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}\sqrt{bx^4+a}}\right)e^2}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{bd^4-4bced^2+b\sqrt{d^2-4ce}\,(d^2-2ce)\,d+2ae^4+2bc^2e^2}}+\frac{\sqrt[4]{b}\left(d-\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}\left(d+\sqrt{d^2-4ce}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt{d^2-4ce}\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}+\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)^2}{4\sqrt{a}\sqrt{b}e^2\left(d-\sqrt{d^2-4ce}\right)^2};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2-\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}-\frac{\left(2\sqrt{a}e^2-\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)^2}{4\sqrt{a}\sqrt{b}e^2\left(d+\sqrt{d^2-4ce}\right)^2};2\text{ArcTan}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)e}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{d^2-4ce}\left(d+\sqrt{d^2-4ce}\right)\left(2\sqrt{a}e^2+\sqrt{b}\left(d^2+\sqrt{d^2-4ce}\,d-2ce\right)\right)\sqrt{bx^4+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] -((e^2*ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4
- b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d + Sqrt[d^2 - 4*c*e])*Sqrt[a
 + b*x^4])]])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e
+ 2*c^2*e^2 + d^3*Sqrt[d^2 - 4*c*e] - 2*c*d*e*Sqrt[d^2 - 4*c*e])]])) + (e^2*
ArcTan[(Sqrt[2]*Sqrt[-(b*d^4) + 4*b*c*d^2*e - 2*b*c^2*e^2 - 2*a*e^4 + b*d*S
qrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*x)/(e*(d - Sqrt[d^2 - 4*c*e])*Sqrt[a + b*x^
4])]])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[-2*a*e^4 - b*(d^4 - 4*c*d^2*e + 2*c^2
*e^2 - d^3*Sqrt[d^2 - 4*c*e] + 2*c*d*e*Sqrt[d^2 - 4*c*e])]]) - (e^2*ArcTanh[
(4*a*e^2 + b*(d - Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d
^2*e + 2*b*c^2*e^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a
+ b*x^4])]])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e
^2 + 2*a*e^4 - b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]]) + (e^2*ArcTanh[(4*a*e^
2 + b*(d + Sqrt[d^2 - 4*c*e])^2*x^2)/(2*Sqrt[2]*Sqrt[b*d^4 - 4*b*c*d^2*e +
2*b*c^2*e^2 + 2*a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]*Sqrt[a + b*x^4
]])]])/(Sqrt[2]*Sqrt[d^2 - 4*c*e]*Sqrt[b*d^4 - 4*b*c*d^2*e + 2*b*c^2*e^2 + 2*
a*e^4 + b*d*Sqrt[d^2 - 4*c*e]*(d^2 - 2*c*e)]]) + (b^(1/4)*e*(d - Sqrt[d^2 -
4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]
*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e
]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*
x^4]) - (b^(1/4)*e*(d + Sqrt[d^2 - 4*c*e])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(2*a^(1/4)*Sqrt[d^2 - 4*c*e]*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
+ d*Sqrt[d^2 - 4*c*e]))*Sqrt[a + b*x^4]) + (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2
 - 2*c*e - d*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(
Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e
- d*Sqrt[d^2 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d - Sqrt[d^2 - 4*c*e])^2)
, 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]
*(d - Sqrt[d^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e - d*Sqrt[d^2
 - 4*c*e]))*Sqrt[a + b*x^4]) - (e*(2*Sqrt[a]*e^2 - Sqrt[b]*(d^2 - 2*c*e + d
*Sqrt[d^2 - 4*c*e]))*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sq
```

```
rt[b]*x^2)^2]*EllipticPi[(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2
 - 4*c*e]))^2/(4*Sqrt[a]*Sqrt[b]*e^2*(d + Sqrt[d^2 - 4*c*e])^2), 2*ArcTan[(
b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*Sqrt[d^2 - 4*c*e]*(d + Sqrt[d
^2 - 4*c*e])*(2*Sqrt[a]*e^2 + Sqrt[b]*(d^2 - 2*c*e + d*Sqrt[d^2 - 4*c*e]))*
Sqrt[a + b*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1231

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 1721

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*((a + c*x^4)/(a*(A + B*x^2)^2))]/(
4*d*e*A*q*Sqrt[a + c*x^4]))*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)],
2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rule 1739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[d,
 Int[1/((d^2 - e^2*x^2)*Sqrt[a + c*x^4]), x], x] - Dist[e, Int[x/((d^2 - e^
2*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{(c+dx+ex^2)\sqrt{a+bx^4}}\,dx = \int \left(\frac{2e}{\sqrt{d^2-4ce}\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}} - \frac{}{\sqrt{d^2-4ce}\left(d+\right.}\right.$$

$$= \frac{(2e)\int \frac{1}{\left(d-\sqrt{d^2-4ce}+2ex\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} - \frac{(2e)\int \frac{1}{\left(d+\sqrt{d^2-4ce}+2ex\right)\sqrt{a}}}{\sqrt{d^2-4ce}}$$

$$= -\frac{(4e^2)\int \frac{x}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x^2\right)\sqrt{a+bx^4}}\,dx}{\sqrt{d^2-4ce}} + \frac{(4e^2)\int \frac{}{\left(\left(d+\sqrt{d^2-4c}\right.\right.}}{\sqrt{d^2}}$$

$$= -\frac{(2e^2)\,\mathrm{Subst}\left(\int \frac{1}{\left(\left(d-\sqrt{d^2-4ce}\right)^2-4e^2x\right)\sqrt{a+bx^2}}\,dx, x, x^2\right)}{\sqrt{d^2-4ce}} + \frac{(2e^2)\,\mathrm{Su}}{}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\,(d^2}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right.}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cd\right.}}$$

$$= -\frac{e^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-bd^4+4bcd^2e-2bc^2e^2-2ae^4-bd\sqrt{d^2-4ce}\,(d^2}{e\left(d+\sqrt{d^2-4ce}\right)\sqrt{a+bx^4}}\right.}{\sqrt{2}\sqrt{d^2-4ce}\sqrt{-2ae^4-b\left(d^4-4cd^2e+2c^2e^2+d^3\sqrt{d^2-4ce}-2cd\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 10.97, size = 455, normalized size = 0.28

$$-\frac{i\sqrt{1+\frac{bx^4}{a}}\left(\left(-d^2+\sqrt{d^4-4cd^2e}\right)\Pi\left(\frac{2i\sqrt{a}\,e^2}{\sqrt{b}\left(d^2-2ce+\sqrt{d^4-4cd^2e}\right)};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)+\left(d^2+\sqrt{d^4-4cd^2e}\right)\Pi\left(-\frac{2i\sqrt{a}\,e^2}{\sqrt{b}\left(-d^2+2ce+\sqrt{d^4-4cd^2e}\right)};i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,x\right)\middle|-1\right)\right)}{2\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,c\sqrt{d^4-4cd^2e}\,\sqrt{a+bx^4}}+\sqrt{b}\,d\text{RootSum}\left[a^2e^2-2a\sqrt{b}\,d^2\#1+4a\sqrt{b}\,ce\#1+4bc^2\#1^2-2ae^2\#1^2+2\sqrt{b}\,d^2\#1^3-4\sqrt{b}\,ce\#1^3+e^2\#1^4\&,\frac{\log\left(-\sqrt{b}\,x^2+\sqrt{a+bx^4}-\#1\right)\#1}{-a\sqrt{b}\,d^2+2a\sqrt{b}\,ce+4bc^2\#1-2ae^2\#1+3\sqrt{b}\,d^2\#1^2-6\sqrt{b}\,ce\#1^2+2e^2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + d*x + e*x^2)*Sqrt[a + b*x^4]),x]
```

```
[Out] ((-1/2*I)*Sqrt[1 + (b*x^4)/a]*((-d^2 + Sqrt[d^4 - 4*c*d^2*e])*EllipticPi[((
2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(d^2 - 2*c*e + Sqrt[d^4 - 4*c*d^2*e])), I*ArcSin
h[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1] + (d^2 + Sqrt[d^4 - 4*c*d^2*e])*Ellipti
cPi[((-2*I)*Sqrt[a]*e^2)/(Sqrt[b]*(-d^2 + 2*c*e + Sqrt[d^4 - 4*c*d^2*e])),
I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[a]]*x], -1]))/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*c*
Sqrt[d^4 - 4*c*d^2*e]*Sqrt[a + b*x^4]) + Sqrt[b]*d*RootSum[a^2*e^2 - 2*a*Sq
rt[b]*d^2*#1 + 4*a*Sqrt[b]*c*e*#1 + 4*b*c^2*#1^2 - 2*a*e^2*#1^2 + 2*Sqrt[b]
*d^2*#1^3 - 4*Sqrt[b]*c*e*#1^3 + e^2*#1^4 & , (Log[-(Sqrt[b]*x^2) + Sqrt[a
+ b*x^4] - #1]*#1)/(-(a*Sqrt[b]*d^2) + 2*a*Sqrt[b]*c*e + 4*b*c^2*#1 - 2*a*e
^2*#1 + 3*Sqrt[b]*d^2*#1^2 - 6*Sqrt[b]*c*e*#1^2 + 2*e^2*#1^3) & ]
```

Maple [C] Result contains complex when optimal does not.
time = 0.32, size = 1153, normalized size = 0.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2
*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e
+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d^2-1/2/(1/2*b/e^4
*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^
(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(
1/2*b/e^4*d^4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*
c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^
4-1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2+b/e^3*c*d*(-4*c*e+d^2)^(1/
2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^
(1/2))^(1/2)*e/(-d+(-4*c*e+d^2)^(1/2))*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I
/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2)
)^(1/2),-4*I*a^(1/2)/b^(1/2)*e^2/(-d+(-4*c*e+d^2)^(1/2))^2,(-I/a^(1/2)*b^(1
/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))+1/2/(-4*c*e+d^2)^(1/2)/(1/2*b/e^4*d^4
+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2
)+b/e^2*c^2+a)^(1/2)*arctanh(1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^
(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+
a)^(1/2)*b*x^2/e^2*d^2+1/2/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-
2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/
```

```
2)*b*x^2/e^2*d*(-4*c*e+d^2)^(1/2)-1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^
2)^(1/2)-2*b/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x
^4+a)^(1/2)*b*x^2/e*c+1/(1/2*b/e^4*d^4+1/2*b/e^4*d^3*(-4*c*e+d^2)^(1/2)-2*b
/e^3*c*d^2-b/e^3*c*d*(-4*c*e+d^2)^(1/2)+b/e^2*c^2+a)^(1/2)/(b*x^4+a)^(1/2)*
a)-2/(-4*c*e+d^2)^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)/(d+(-4*c*e+d^2)^(1/2))*e*
(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(
1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),-4*I*a^(1/2)/b^(1/2)/(d+(-4*c*e
+d^2)^(1/2))^2*e^2,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a)*(x^2*e + d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^4}\,(c+dx+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d*x+c)/(b*x**4+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x + e*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d*x+c)/(b*x^4+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a)*(x^2*e + d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{b\,x^4+a}\,\left(e\,x^2+d\,x+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)),x)
[Out] int(1/((a + b*x^4)^(1/2)*(c + d*x + e*x^2)), x)
```

3.227 $\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a^3cx^{1+m}\sqrt{c(a+bx^2)^2}}{(1+m)(a+bx^2)}+\frac{3a^2bcx^{3+m}\sqrt{c(a+bx^2)^2}}{(3+m)(a+bx^2)}+\frac{3ab^2cx^{5+m}\sqrt{c(a+bx^2)^2}}{(5+m)(a+bx^2)}+\frac{b^3cx^{7+m}\sqrt{c(a+bx^2)^2}}{(7+m)(a+bx^2)}$$

```
[Out] a^3*c*x^(1+m)*(c*(b*x^2+a)^2)^(1/2)/(1+m)/(b*x^2+a)+3*a^2*b*c*x^(3+m)*(c*(b
*x^2+a)^2)^(1/2)/(3+m)/(b*x^2+a)+3*a*b^2*c*x^(5+m)*(c*(b*x^2+a)^2)^(1/2)/(5
+m)/(b*x^2+a)+b^3*c*x^(7+m)*(c*(b*x^2+a)^2)^(1/2)/(7+m)/(b*x^2+a)
```

Rubi [A]

time = 0.05, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1973, 276}

$$\frac{a^3cx^{m+1}\sqrt{c(a+bx^2)^2}}{(m+1)(a+bx^2)}+\frac{3a^2bcx^{m+3}\sqrt{c(a+bx^2)^2}}{(m+3)(a+bx^2)}+\frac{b^3cx^{m+7}\sqrt{c(a+bx^2)^2}}{(m+7)(a+bx^2)}+\frac{3ab^2cx^{m+5}\sqrt{c(a+bx^2)^2}}{(m+5)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^(1 + m)*Sqrt[c*(a + b*x^2)^2])/((1 + m)*(a + b*x^2)) + (3*a^2*b*c*
x^(3 + m)*Sqrt[c*(a + b*x^2)^2])/((3 + m)*(a + b*x^2)) + (3*a*b^2*c*x^(5 +
m)*Sqrt[c*(a + b*x^2)^2])/((5 + m)*(a + b*x^2)) + (b^3*c*x^(7 + m)*Sqrt[c*(
a + b*x^2)^2])/((7 + m)*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx = \int x^m (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int x^m (abc+b^2cx^2)^3 dx}{b^2c(abc+b^2cx^2)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int (a^3b^3c^3x^m + 3a^2b^4c^3x^{2+m} + 3ab^5c^3x^{4+m} + b^6c^3x^{6+m}}{b^2c(abc+b^2cx^2)}$$
$$= \frac{a^3cx^{1+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(1+m)(a+bx^2)} + \frac{3a^2bcx^{3+m}\sqrt{a^2c+2abcx^2+b^2cx^4}}{(3+m)(a+bx^2)} + \frac{3ab^2}{}$$

Mathematica [A]
time = 0.09, size = 132, normalized size = 0.82

$$\frac{x^{1+m}\left(c(a+bx^2)^2\right)^{3/2}(a^3(105+71m+15m^2+m^3)+3a^2b(35+47m+13m^2+m^3)x^2+3ab^2(21+31m+11m^2+m^3)x^4+b^3(15+23m+9m^2+m^3)x^6)}{(1+m)(3+m)(5+m)(7+m)(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2
*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4
+ b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a +
 b*x^2)^3)
```

Maple [A]
time = 0.01, size = 200, normalized size = 1.24

method	result
gosper	$\frac{x^{1+m}(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m^3+1}{(7+m)(5+m)(3+m)(1+m)(b\,x^2+a)^3}$
risch	$\frac{c\sqrt{c\,(b\,x^2+a)^2}\,(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63}{(b\,x^2+a)(7+m)(5+m)(3+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^
2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x
^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(c*(b
*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 119, normalized size = 0.74

$$\frac{\left((m^3+9\,m^2+23\,m+15)b^3c^{\frac{3}{2}}x^7+3\,(m^3+11\,m^2+31\,m+21)ab^2c^{\frac{3}{2}}x^5+3\,(m^3+13\,m^2+47\,m+35)a^2bc^{\frac{3}{2}}x^3+(m^3+15\,m^2+71\,m+105)a^3c^{\frac{3}{2}}x\right)x^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*c^(3/2)*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a
*b^2*c^(3/2)*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*c^(3/2)*x^3 + (m^3 +
15*m^2 + 71*m + 105)*a^3*c^(3/2)*x)*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 10
5)
```

Fricas [A]
time = 0.35, size = 233, normalized size = 1.45

$$\frac{((b^3cm^3+9\,b^3cm^2+23\,b^3cm+15\,b^3c)x^7+3\,(ab^2cm^3+11\,ab^2cm^2+31\,ab^2cm+21\,ab^2c)x^5+3\,(a^2bcm^3+13\,a^2bcm^2+47\,a^2bcm+35\,a^2bc)x^3+(a^3cm^3+15\,a^3cm^2+71\,a^3cm+105\,a^3c)x)\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,x^m}{am^4+16\,am^3+86\,am^2+(bm^4+16\,bm^3+86\,bm^2+176\,bm+105\,b)x^2+176\,am+105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 1
1*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*
c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3
*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*
a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 17
6*a*m + 105*a)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(153) = 306.
time = 3.37, size = 355, normalized size = 2.20

$$\frac{(b^3m^3x^7x^m\mathrm{sgn}(bx^2+a)+9\,b^3m^2x^7x^m\mathrm{sgn}(bx^2+a)+3\,ab^2m^3x^5x^m\mathrm{sgn}(bx^2+a)+23\,b^3mx^7x^m\mathrm{sgn}(bx^2+a)+33\,ab^2m^2x^5x^m\mathrm{sgn}(bx^2+a)+15\,b^3x^7x^m\mathrm{sgn}(bx^2+a)+3\,a^2bm^3x^3x^m\mathrm{sgn}(bx^2+a)+93\,ab^2mx^5x^m\mathrm{sgn}(bx^2+a)+39\,a^2bm^2x^3x^m\mathrm{sgn}(bx^2+a)+63\,ab^2x^5x^m\mathrm{sgn}(bx^2+a)+a^3m^3xx^m\mathrm{sgn}(bx^2+a)+141\,a^2bmx^3x^m\mathrm{sgn}(bx^2+a)+15\,a^3m^2xx^m\mathrm{sgn}(bx^2+a)+105\,a^2bx^3x^m\mathrm{sgn}(bx^2+a)+71\,a^3mxx^m\mathrm{sgn}(bx^2+a)+105\,a^3xx^m\mathrm{sgn}(bx^2+a))c^{\frac{3}{2}}}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^
2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m
^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3
*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*
x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*
x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2
+ a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 1
05*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B]
time = 2.97, size = 234, normalized size = 1.45

$$\frac{x^m\left(\frac{3\,a^2\,c\,x^3\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{b^2\,c\,x^7\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{3\,a\,b\,c\,x^5\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a^3\,c\,x\,\sqrt{c\,{\left(b\,x^2+a\right)}^2}\,\left(m^3+15\,m^2+71\,m+105\right)}{b\,\left(m^4+16\,m^3+86\,m^2+176\,m+105\right)}\right)}{\frac{a}{b}+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] (x^m*((3*a^2*c*x^3*(c*(a + b*x^2)^2)^(1/2)*(47*m + 13*m^2 + m^3 + 35))/(176
*m + 86*m^2 + 16*m^3 + m^4 + 105) + (b^2*c*x^7*(c*(a + b*x^2)^2)^(1/2)*(23*
m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4 + 105) + (3*a*b*c*x^5
*(c*(a + b*x^2)^2)^(1/2)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m
^3 + m^4 + 105) + (a^3*c*x*(c*(a + b*x^2)^2)^(1/2)*(71*m + 15*m^2 + m^3 + 1
05))/(b*(176*m + 86*m^2 + 16*m^3 + m^4 + 105))))/(a/b + x^2)
```

3.228 $\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal. Leaf size=143

$$\frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}$$

```
[Out] 1/6*a^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/8*a^2*b*c*x^8*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/10*a*b^2*c*x^10*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/12*b
^3*c*x^12*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 272, 45}

$$\frac{a^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3a^2bcx^8\sqrt{c\left(a+bx^2\right)^2}}{8\left(a+bx^2\right)}+\frac{b^3cx^{12}\sqrt{c\left(a+bx^2\right)^2}}{12\left(a+bx^2\right)}+\frac{3ab^2cx^{10}\sqrt{c\left(a+bx^2\right)^2}}{10\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(a + b*x^2)) + (3*a^2*b*c*x^8*Sqrt[c*(
a + b*x^2)^2])/(8*(a + b*x^2)) + (3*a*b^2*c*x^10*Sqrt[c*(a + b*x^2)^2])/(10
*(a + b*x^2)) + (b^3*c*x^12*Sqrt[c*(a + b*x^2)^2])/(12*(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^5\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\,dx$$
$$=\frac{1}{2}\text{Subst}\left(\int x^2\left(a^2c+2abcx+b^2cx^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\text{Subst}\left(\int\left(\frac{a^2(abc+b^2cx)^3}{b^2}-\frac{2a(abc+b^2cx)^4}{b^3c}+\frac{(abc+b^2cx)^5}{b^4c^2}\right)dx\right.}{2b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^2c(a+bx^2)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}-\frac{ac(a+bx^2)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.44

$$\frac{x^6\left(c(a+bx^2)^2\right)^{3/2}\left(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6\right)}{120\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3
*x^6))/(120*(a + b*x^2)^3)
```

Maple [A]
time = 0.06, size = 60, normalized size = 0.42

method	result
gosper	$\frac{x^6(10b^3x^6+36ab^2x^4+45a^2bx^2+20a^3)\left(c(bx^2+a)^2\right)^{\frac{3}{2}}}{120(bx^2+a)^3}$
default	$\frac{x^6(10b^3x^6+36ab^2x^4+45a^2bx^2+20a^3)\left(c(bx^2+a)^2\right)^{\frac{3}{2}}}{120(bx^2+a)^3}$
trager	$\frac{cx^6(10b^3x^6+36ab^2x^4+45a^2bx^2+20a^3)\sqrt{b^2cx^4+2abcx^2+a^2c}}{120bx^2+120a}$
risch	$\frac{a^3cx^6\sqrt{c(bx^2+a)^2}}{6bx^2+6a}+\frac{3a^2bcx^8\sqrt{c(bx^2+a)^2}}{8(bx^2+a)}+\frac{3ab^2cx^{10}\sqrt{c(bx^2+a)^2}}{10(bx^2+a)}+\frac{b^3cx^{12}\sqrt{c(bx^2+a)^2}}{12bx^2+12a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*(c*(b*x^2+a)^2)^(3/
2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 136, normalized size = 0.95

$$\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}a^2x^2}{8\,b^2}+\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}a^3}{8\,b^3}+\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{5}{2}}x^2}{12\,b^2c}-\frac{7\,(b^2cx^4+2\,abcx^2+a^2c)^{\frac{5}{2}}a}{60\,b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 +
2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)
^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3*c)
```

Fricas [A]
time = 0.33, size = 74, normalized size = 0.52

$$\frac{(10\,b^3cx^{12}+36\,ab^2cx^{10}+45\,a^2bcx^8+20\,a^3cx^6)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{120\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqr
t(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**5*(c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 6.01, size = 72, normalized size = 0.50

$$\frac{1}{120}\left(10\,b^3x^{12}\mathrm{sgn}\left(bx^2+a\right)+36\,ab^2x^{10}\mathrm{sgn}\left(bx^2+a\right)+45\,a^2bx^8\mathrm{sgn}\left(bx^2+a\right)+20\,a^3x^6\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b
*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(c\left(b\,x^2+a\right)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^5*(c*(a + b*x^2)^2)^(3/2), x)
```

3.229 $\int x^4\left(c(a+bx^2)^2\right)^{3/2}\,dx$

Optimal. Leaf size=143

$$\frac{a^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}+\frac{b^3cx^{11}\sqrt{c\left(a+bx^2\right)^2}}{11\left(a+bx^2\right)}$$

```
[Out] 1/5*a^3*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/7*a^2*b*c*x^7*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+1/3*a*b^2*c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/11*b^3
*c*x^11*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1973, 276}

$$\frac{a^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{b^3cx^{11}\sqrt{c\left(a+bx^2\right)^2}}{11\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{c\left(a+bx^2\right)^2}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a^2*b*c*x^7*Sqrt[c*(
a + b*x^2)^2])/(7*(a + b*x^2)) + (a*b^2*c*x^9*Sqrt[c*(a + b*x^2)^2])/(3*(a
+ b*x^2)) + (b^3*c*x^11*Sqrt[c*(a + b*x^2)^2])/(11*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx=\int x^4\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\,dx$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^4\left(abc+b^2cx^2\right)^3\,dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int\left(a^3b^3c^3x^4+3a^2b^4c^3x^6+3ab^5c^3x^8+b^6c^3x^{10}\right)dx}{b^2c\left(abc+b^2cx^2\right)}$$
$$=\frac{a^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\left(a+bx^2\right)}+\frac{3a^2bcx^7\sqrt{a^2c+2abcx^2+b^2cx^4}}{7\left(a+bx^2\right)}+\frac{ab^2cx^9\sqrt{a^2}}{3}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.44

$$\frac{x^5\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6))/(1155*(a + b*x^2)^3)
```

Maple [A]
time = 0.06, size = 60, normalized size = 0.42

method	result
gosper	$\frac{x^{5}\left(105b^{3}x^{6}+385a\,b^{2}x^{4}+495a^{2}b\,x^{2}+231a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$
default	$\frac{x^{5}\left(105b^{3}x^{6}+385a\,b^{2}x^{4}+495a^{2}b\,x^{2}+231a^{3}\right)\left(c\left(b\,x^{2}+a\right)^{2}\right)^{\frac{3}{2}}}{1155\left(b\,x^{2}+a\right)^{3}}$
trager	$\frac{c\,x^{5}\left(105b^{3}x^{6}+385a\,b^{2}x^{4}+495a^{2}b\,x^{2}+231a^{3}\right)\sqrt{b^{2}c\,x^{4}+2abc\,x^{2}+a^{2}c}}{1155b\,x^{2}+1155a}$
risch	$\frac{a^{3}c\,x^{5}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{5b\,x^{2}+5a}+\frac{3a^{2}bc\,x^{7}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{7\left(b\,x^{2}+a\right)}+\frac{a\,b^{2}c\,x^{9}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{3b\,x^{2}+3a}+\frac{b^{3}c\,x^{11}\sqrt{c\left(b\,x^{2}+a\right)^{2}}}{11b\,x^{2}+11a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*(c*(b*x^2+a)^2
)^(3/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.33

$$\frac{1}{11}\,b^{3}c^{\frac{3}{2}}x^{11}+\frac{1}{3}\,ab^{2}c^{\frac{3}{2}}x^{9}+\frac{3}{7}\,a^{2}bc^{\frac{3}{2}}x^{7}+\frac{1}{5}\,a^{3}c^{\frac{3}{2}}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5
*a^3*c^(3/2)*x^5
```

Fricas [A]
time = 0.33, size = 74, normalized size = 0.52

$$\frac{\left(105\,b^{3}cx^{11}+385\,ab^{2}cx^{9}+495\,a^{2}bcx^{7}+231\,a^{3}cx^{5}\right)\sqrt{b^{2}cx^{4}+2\,abcx^{2}+a^{2}c}}{1155\,(bx^{2}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)
*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}\left(c\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**4*(c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 5.31, size = 72, normalized size = 0.50

$$\frac{1}{1155}\left(105\,b^{3}x^{11}\mathrm{sgn}\left(bx^{2}+a\right)+385\,ab^{2}x^{9}\mathrm{sgn}\left(bx^{2}+a\right)+495\,a^{2}bx^{7}\mathrm{sgn}\left(bx^{2}+a\right)+231\,a^{3}x^{5}\mathrm{sgn}\left(bx^{2}+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^
2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int(x^4*(c*(a + b*x^2)^2)^(3/2), x)
```

3.230 $\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal. Leaf size=66

$$-\frac{ac(a+bx^2)^3\sqrt{c(a+bx^2)^2}}{8b^2}+\frac{c(a+bx^2)^4\sqrt{c(a+bx^2)^2}}{10b^2}$$

```
[Out] -1/8*a*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b^2+1/10*c*(b*x^2+a)^4*(c*(b*x^2
+a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 272, 45}

$$\frac{c(a+bx^2)^4\sqrt{c(a+bx^2)^2}}{10b^2}-\frac{ac(a+bx^2)^3\sqrt{c(a+bx^2)^2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] -1/8*(a*c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/b^2 + (c*(a + b*x^2)^4*Sqrt[
c*(a + b*x^2)^2])/(10*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(c\left(a+bx^2\right)^2\right)^{3/2}\,dx &= \int x^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x\left(a^2c+2abcx+b^2cx^2\right)^{3/2}\,dx,x,x^2\right)\\ &= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}-\frac{a\text{Subst}\left(\int\left(a^2c+2abcx+b^2cx^2\right)^{3/2}\,dx,x,x^2\right)}{2b}\\ &= -\frac{a\left(a+bx^2\right)\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{8b^2}+\frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{5/2}}{10b^2c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.95

$$\frac{x^4\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*
x^6))/(40*(a + b*x^2)^3)
```

Maple [A]
time = 0.05, size = 60, normalized size = 0.91

method	result
gosper	$\frac{x^4\left(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^2+a\right)^3}$
default	$\frac{x^4\left(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{40\left(b\,x^2+a\right)^3}$
trager	$\frac{c\,x^4\left(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3\right)\sqrt{b^2c\,x^4+2abc\,x^2+a^2c}}{40b\,x^2+40a}$
risch	$\frac{c\sqrt{c\left(b\,x^2+a\right)^2}\,b^3x^{10}}{10b\,x^2+10a}+\frac{3c\sqrt{c\left(b\,x^2+a\right)^2}\,a\,b^2x^8}{8\left(b\,x^2+a\right)}+\frac{c\sqrt{c\left(b\,x^2+a\right)^2}\,a^2b\,x^6}{2b\,x^2+2a}+\frac{c\sqrt{c\left(b\,x^2+a\right)^2}\,a^3x^4}{4b\,x^2+4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*(c*(b*x^2+a)^2)^(3/2)
/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 98, normalized size = 1.48

$$-\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}ax^2}{8\,b}-\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{3}{2}}a^2}{8\,b^2}+\frac{(b^2cx^4+2\,abcx^2+a^2c)^{\frac{5}{2}}}{10\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a*x^2/b - 1/8*(b^2*c*x^4 + 2*a
*b*c*x^2 + a^2*c)^(3/2)*a^2/b^2 + 1/10*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5
/2)/(b^2*c)
```

Fricas [A]
time = 0.32, size = 74, normalized size = 1.12

$$\frac{(4\,b^3cx^{10}+15\,ab^2cx^8+20\,a^2bcx^6+10\,a^3cx^4)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{40\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b
^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**3*(c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 5.80, size = 48, normalized size = 0.73

$$\frac{1}{40}\left(4\,b^3x^{10}+15\,ab^2x^8+20\,a^2bx^6+10\,a^3x^4\right)c^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*c^(3/2)*sgn(b*
x^2 + a)
```

Mupad [B]
time = 2.83, size = 50, normalized size = 0.76

$$\frac{(-a^2 + 3\,a\,b\,x^2 + 4\,b^2\,x^4)\,\left(c\,a^2 + 2\,c\,a\,b\,x^2 + c\,b^2\,x^4\right)^{3/2}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] ((4*b^2*x^4 - a^2 + 3*a*b*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(40
*b^2)
```

3.231 $\int x^2\left(c(a+bx^2)^2\right)^{3/2}\,dx$

Optimal. Leaf size=143

$$\frac{a^3cx^3\sqrt{c\,(a+bx^2)^2}}{3\,(a+bx^2)}+\frac{3a^2bcx^5\sqrt{c\,(a+bx^2)^2}}{5\,(a+bx^2)}+\frac{3ab^2cx^7\sqrt{c\,(a+bx^2)^2}}{7\,(a+bx^2)}+\frac{b^3cx^9\sqrt{c\,(a+bx^2)^2}}{9\,(a+bx^2)}$$

```
[Out] 1/3*a^3*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/5*a^2*b*c*x^5*(c*(b*x^2+a)^
2)^(1/2)/(b*x^2+a)+3/7*a*b^2*c*x^7*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/9*b^3*
c*x^9*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]

time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {1973, 276}

$$\frac{a^3cx^3\sqrt{c\,(a+bx^2)^2}}{3\,(a+bx^2)}+\frac{3a^2bcx^5\sqrt{c\,(a+bx^2)^2}}{5\,(a+bx^2)}+\frac{b^3cx^9\sqrt{c\,(a+bx^2)^2}}{9\,(a+bx^2)}+\frac{3ab^2cx^7\sqrt{c\,(a+bx^2)^2}}{7\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x^3*Sqrt[c*(a + b*x^2)^2])/(3*(a + b*x^2)) + (3*a^2*b*c*x^5*Sqrt[c*(
a + b*x^2)^2])/(5*(a + b*x^2)) + (3*a*b^2*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(
a + b*x^2)) + (b^3*c*x^9*Sqrt[c*(a + b*x^2)^2])/(9*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^2\left(c(a+bx^2)^2\right)^{3/2}\,dx=\int x^2\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\,dx$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int x^2(abc+b^2cx^2)^3\,dx}{b^2c\,(abc+b^2cx^2)}$$
$$=\frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int (a^3b^3c^3x^2+3a^2b^4c^3x^4+3ab^5c^3x^6+b^6c^3x^8)\,dx}{b^2c\,(abc+b^2cx^2)}$$
$$=\frac{a^3cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{3\,(a+bx^2)}+\frac{3a^2bcx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5\,(a+bx^2)}+\frac{3ab^2cx^7\sqrt{c}}{}$$

Mathematica [**A**]
time = 0.01, size = 63, normalized size = 0.44

$$\frac{\left(c(a+bx^2)^2\right)^{3/2}(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9)}{315\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*
b^3*x^9))/(315*(a + b*x^2)^3)
```

Maple [**A**]
time = 0.06, size = 60, normalized size = 0.42

method	result
gosper	$\frac{x^3(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3)\left(c(b\,x^2+a)^2\right)^{\frac{3}{2}}}{315(b\,x^2+a)^3}$
default	$\frac{x^3(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3)\left(c(b\,x^2+a)^2\right)^{\frac{3}{2}}}{315(b\,x^2+a)^3}$
trager	$\frac{c\,x^3(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3)\sqrt{b^2c\,x^4+2abc\,x^2+a^2c}}{315b\,x^2+315a}$
risch	$\frac{a^3c\,x^3\sqrt{c\,(b\,x^2+a)^2}}{3b\,x^2+3a}+\frac{3a^2bc\,x^5\sqrt{c\,(b\,x^2+a)^2}}{5(b\,x^2+a)}+\frac{3a\,b^2c\,x^7\sqrt{c\,(b\,x^2+a)^2}}{7(b\,x^2+a)}+\frac{b^3c\,x^9\sqrt{c\,(b\,x^2+a)^2}}{9b\,x^2+9a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*(c*(b*x^2+a)^2)^
(3/2)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 47, normalized size = 0.33

$$\frac{1}{9}\,b^3c^{\frac{3}{2}}x^9+\frac{3}{7}\,ab^2c^{\frac{3}{2}}x^7+\frac{3}{5}\,a^2bc^{\frac{3}{2}}x^5+\frac{1}{3}\,a^3c^{\frac{3}{2}}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a
^3*c^(3/2)*x^3
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.52

$$\frac{\left(35\,b^3cx^9+135\,ab^2cx^7+189\,a^2bcx^5+105\,a^3cx^3\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{315\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sq
rt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 5.08, size = 72, normalized size = 0.50

$$\frac{1}{315}\left(35\,b^3x^9\mathrm{sgn}\left(bx^2+a\right)+135\,ab^2x^7\mathrm{sgn}\left(bx^2+a\right)+189\,a^2bx^5\mathrm{sgn}\left(bx^2+a\right)+105\,a^3x^3\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b
*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c \left(b x^2+a\right)^2\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^2)^(3/2),x)
[Out] int(x^2*(c*(a + b*x^2)^2)^(3/2), x)
```

3.232 $\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

[Out] 1/8*c*(b*x^2+a)^3*(c*(b*x^2+a)^2)^(1/2)/b

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1605, 15, 30}

$$\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^2}}{8b}$$

Antiderivative was successfully verified.

[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]

[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(c(a+bx^2)^2\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int (cx^2)^{3/2}\,dx, x, a+bx^2\right)}{2b}$$
$$= \frac{\left(c\sqrt{c\,(a+bx^2)^2}\right)\text{Subst}\left(\int x^3\,dx, x, a+bx^2\right)}{2b\,(a+bx^2)}$$
$$= \frac{c(a+bx^2)^3\,\sqrt{c\,(a+bx^2)^2}}{8b}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.91

$$\frac{(a+bx^2)\left(c(a+bx^2)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [A]
time = 0.05, size = 26, normalized size = 0.81

method	result	size
default	$\frac{\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(b\,x^2+a\right)}{8b}$	26
risch	$\frac{c\left(b\,x^2+a\right)^3\sqrt{c\left(b\,x^2+a\right)^2}}{8b}$	29
gosper	$\frac{x^2\left(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{8\left(b\,x^2+a\right)^3}$	59
trager	$\frac{c\,x^2\left(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3\right)\sqrt{b^2c\,x^4+2abc\,x^2+a^2c}}{8b\,x^2+8a}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*(b*x^2+a)^2)^(3/2)*(b*x^2+a)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(28) = 56$.

time = 0.28, size = 60, normalized size = 1.88

$$\frac{1}{8}\left(b^{2} c x^{4}+2\, a b c x^{2}+a^{2} c\right)^{\frac{3}{2}} x^{2}+\frac{\left(b^{2} c x^{4}+2\, a b c x^{2}+a^{2} c\right)^{\frac{3}{2}} a}{8\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*x^2 + 1/8*(b^2*c*x^4 + 2*a*b*c*
x^2 + a^2*c)^(3/2)*a/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.
time = 0.33, size = 73, normalized size = 2.28

$$\frac{\left(b^{3} c x^{8}+4\, a b^{2} c x^{6}+6\, a^{2} b c x^{4}+4\, a^{3} c x^{2}\right) \sqrt{b^{2} c x^{4}+2\, a b c x^{2}+a^{2} c}}{8\, \left(b x^{2}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^
4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a+b x^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral(x*(c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.17, size = 25, normalized size = 0.78

$$\frac{\left(b x^{2}+a\right)^{4} c^{\frac{3}{2}} \mathrm{sgn}\left(b x^{2}+a\right)}{8\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b
```

Mupad [B]
time = 2.84, size = 40, normalized size = 1.25

$$\frac{(b^2 x^2 + a b) (c a^2 + 2 c a b x^2 + c b^2 x^4)^{3/2}}{8 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^2)^(3/2),x)
[Out] ((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^(3/2))/(8*b^2)
```

3.233 $\int \left(c(a+bx^2)^2\right)^{3/2} dx$

Optimal. Leaf size=135

$$\frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}$$

```
[Out] a^3*c*x*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^2*b*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(
b*x^2+a)+3/5*a*b^2*c*x^5*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/7*b^3*c*x^7*(c*(
b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.02, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1973, 200}

$$\frac{a^3cx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{a^2bcx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^7\sqrt{c\left(a+bx^2\right)^2}}{7\left(a+bx^2\right)}+\frac{3ab^2cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] (a^3*c*x*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) + (a^2*b*c*x^3*Sqrt[c*(a + b*x^
2)^2])/(a + b*x^2) + (3*a*b^2*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
+ (b^3*c*x^7*Sqrt[c*(a + b*x^2)^2])/(7*(a + b*x^2))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^2\right)^{3/2} dx = \int \left(a^2c+2abcx^2+b^2cx^4\right)^{3/2} dx$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\int \left(2abc+2b^2cx^2\right)^3 dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}\int \left(8a^3b^3c^3+24a^2b^4c^3x^2+24ab^5c^3x^4+8b^6c^3x^6\right) dx}{\left(2abc+2b^2cx^2\right)^3}$$
$$= \frac{a^3x\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{a^2bx^3\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{\left(a+bx^2\right)^3} + \frac{3ab^2x^5\left(a^2c+\right.}{5\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.45

$$\frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^
7))/(35*(a + b*x^2)^3)
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.43

method	result	si
gosper	$\frac{x\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{35\left(b\,x^2+a\right)^3}$	58
default	$\frac{x\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}{35\left(b\,x^2+a\right)^3}$	58
trager	$\frac{cx\left(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3\right)\sqrt{b^2c\,x^4+2abc\,x^2+a^2c}}{35b\,x^2+35a}$	70
risch	$\frac{a^3cx\sqrt{c\left(b\,x^2+a\right)^2}}{b\,x^2+a}+\frac{a^2bc\,x^3\sqrt{c\left(b\,x^2+a\right)^2}}{b\,x^2+a}+\frac{3a\,b^2c\,x^5\sqrt{c\left(b\,x^2+a\right)^2}}{5\left(b\,x^2+a\right)}+\frac{b^3c\,x^7\sqrt{c\left(b\,x^2+a\right)^2}}{7b\,x^2+7a}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*(c*(b*x^2+a)^2)^(3/2)/(
b*x^2+a)^3
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.32

$$\frac{1}{7}\,b^3c^{\frac{3}{2}}x^7+\frac{3}{5}\,ab^2c^{\frac{3}{2}}x^5+a^2bc^{\frac{3}{2}}x^3+a^3c^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/
2)*x
```

Fricas [A]
time = 0.32, size = 72, normalized size = 0.53

$$\frac{(5\,b^3cx^7+21\,ab^2cx^5+35\,a^2bcx^3+35\,a^3cx)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{35\,(bx^2+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\big(a+bx^2\big)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 3.55, size = 46, normalized size = 0.34

$$\frac{1}{35}\,(5\,b^3x^7+21\,ab^2x^5+35\,a^2bx^3+35\,a^3x)\,c^{\frac{3}{2}}\mathrm{sgn}\big(bx^2+a\big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2
 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c \left(b x^2+a\right)^2\right)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2), x)
```

3.234 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x} dx$

Optimal. Leaf size=139

$$\frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}\log(x)}{a+bx^2}$$

```
[Out] 3/2*a^2*b*c*x^2*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3/4*a*b^2*c*x^4*(c*(b*x^2+a
)^2)^(1/2)/(b*x^2+a)+1/6*b^3*c*x^6*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+a^3*c*ln
(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 272, 45}

$$\frac{a^3c\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{3a^2bcx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^6\sqrt{c\left(a+bx^2\right)^2}}{6\left(a+bx^2\right)}+\frac{3ab^2cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*c*x^2*Sqrt[c*(a + b*x^2)^2])/(2*(a + b*x^2)) + (3*a*b^2*c*x^4*Sqrt
[c*(a + b*x^2)^2])/(4*(a + b*x^2)) + (b^3*c*x^6*Sqrt[c*(a + b*x^2)^2])/(6*(
a + b*x^2)) + (a^3*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x}\,dx &= \int \frac{(a^2c+2abcx^2+b^2cx^4)^{3/2}}{x}\,dx\\ &= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{(abc+b^2cx^2)^3}{x}\,dx}{b^2c\,(abc+b^2cx^2)}\\ &= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\mathrm{Subst}\left(\int \frac{(abc+b^2cx)^3}{x}\,dx,x,x^2\right)}{2b^2c\,(abc+b^2cx^2)}\\ &= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\mathrm{Subst}\left(\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x}+3ab^5c^3x+b^6c^3x^2\right)\,dx,x,x^2\right)}{2b^2c\,(abc+b^2cx^2)}\\ &= \frac{3a^2bcx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\,(a+bx^2)}+\frac{3ab^2cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4\,(a+bx^2)}+\frac{b^3cx^6\sqrt{a^2c}}{6}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.45

$$\frac{\left(c(a+bx^2)^2\right)^{3/2}(bx^2(18a^2+9abx^2+2b^2x^4)+12a^3\log(x))}{12\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^2)^(3/2)*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*L
og[x]))/(12*(a + b*x^2)^3)
```

Maple [A]
time = 0.04, size = 59, normalized size = 0.42

method	result	size
default	$\frac{\left(c\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}\left(2b^3x^6+9a\,b^2x^4+18a^2b\,x^2+12a^3\ln(x)\right)}{12\left(b\,x^2+a\right)^3}$	59
risch	$\frac{c\sqrt{c\,(b\,x^2+a)^2}\;b\left(\frac{1}{6}b^2x^6+\frac{3}{4}ab\,x^4+\frac{3}{2}a^2x^2\right)}{b\,x^2+a}+\frac{a^3c\ln(x)\sqrt{c\,(b\,x^2+a)^2}}{b\,x^2+a}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(c*(b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x)
)/(b*x^2+a)^3
```

Maxima [A]
time = 0.28, size = 171, normalized size = 1.23

$$\frac{1}{2}\,(-1)^{2\,b^2cx^2+2\,abc}\,a^3c^{\frac{3}{2}}\log\left(2\,b^2cx^2+2\,abc\right)-\frac{1}{2}\,(-1)^{2\,abcx^2+2\,a^2c}\,a^3c^{\frac{3}{2}}\log\left(2\,abc+\frac{2\,a^2c}{x^2}\right)+\frac{1}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,abcx^2+\frac{3}{4}\,\sqrt{b^2cx^4+2\,abcx^2+a^2c}\,a^2c+\frac{1}{6}\,\left(b^2cx^4+2\,abcx^2+a^2c\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 1
/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 1/
4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 + 2*
a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)
```

Fricas [A]
time = 0.33, size = 73, normalized size = 0.53

$$\frac{\left(2\,b^3cx^6+9\,ab^2cx^4+18\,a^2bcx^2+12\,a^3c\log\left(x\right)\right)\sqrt{b^2cx^4+2\,abcx^2+a^2c}}{12\,\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(
b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)
```

Giac [A]
time = 3.37, size = 73, normalized size = 0.53

$$\frac{1}{12}\,\left(2\,b^3x^6\mathrm{sgn}\left(bx^2+a\right)+9\,ab^2x^4\mathrm{sgn}\left(bx^2+a\right)+18\,a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+6\,a^3\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*
sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x, x)
```

3.235 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=134

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}$$

```
[Out] -a^3*c*(c*(b*x^2+a)^2)^(1/2)/x/(b*x^2+a)+3*a^2*b*c*x*(c*(b*x^2+a)^2)^(1/2)/
(b*x^2+a)+a*b^2*c*x^3*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+1/5*b^3*c*x^5*(c*(b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.03, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1973, 276}

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}+\frac{3a^2bcx\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^5\sqrt{c\left(a+bx^2\right)^2}}{5\left(a+bx^2\right)}+\frac{ab^2cx^3\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
```

```
[Out] -((a^3*c*Sqrt[c*(a + b*x^2)^2])/(x*(a + b*x^2))) + (3*a^2*b*c*x*Sqrt[c*(a +
 b*x^2)^2])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt[c*(a + b*x^2)^2])/(a + b*x^2) +
 (b^3*c*x^5*Sqrt[c*(a + b*x^2)^2])/(5*(a + b*x^2))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2}\,dx = \int \frac{(a^2c+2abcx^2+b^2cx^4)^{3/2}}{x^2}\,dx$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{(abc+b^2cx^2)^3}{x^2}\,dx}{b^2c\,(abc+b^2cx^2)}$$
$$= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \left(3a^2b^4c^3+\frac{a^3b^3c^3}{x^2}+3ab^5c^3x^2+b^6c^3x^4\right)\,dx}{b^2c\,(abc+b^2cx^2)}$$
$$= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x\,(a+bx^2)} + \frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2} + \frac{ab^2cx^3\sqrt{a^2c+}}{a+}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.46

$$\frac{\left(c(a+bx^2)^2\right)^{3/2}(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6)}{5x\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]
[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(
5*x*(a + b*x^2)^3)
```

Maple [A]
time = 0.04, size = 60, normalized size = 0.45

method	result	size
gosper	$-\frac{(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3)\left(c(b\,x^2+a)^2\right)^{\frac{3}{2}}}{5x(b\,x^2+a)^3}$	60
default	$-\frac{(-b^3x^6-5a\,b^2x^4-15a^2b\,x^2+5a^3)\left(c(b\,x^2+a)^2\right)^{\frac{3}{2}}}{5x(b\,x^2+a)^3}$	60
risch	$\frac{c\sqrt{c\,(b\,x^2+a)^2}\;b(\frac{1}{5}b^2x^5+ab\,x^3+3a^2x)}{b\,x^2+a}-\frac{a^3c\sqrt{c\,(b\,x^2+a)^2}}{x(b\,x^2+a)}$	79
trager	$\frac{c(b^3x^5+b^3x^4+5a\,b^2x^3+b^3x^3+5a\,b^2x^2+b^3x^2+15a^2bx+5a\,b^2x+b^3x+5a^3)(-1+x)\sqrt{b^2c\,x^4+2abc\,x^2+a^2c}}{5x(b\,x^2+a)}$	114

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*(c*(b*x^2+a)^2)^(3/2)/x/(b*x
^2+a)^3
```

Maxima [A]
time = 0.30, size = 48, normalized size = 0.36

$$\frac{b^3c^{\frac{3}{2}}x^6 + 5\,ab^2c^{\frac{3}{2}}x^4 + 15\,a^2bc^{\frac{3}{2}}x^2 - 5\,a^3c^{\frac{3}{2}}}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*c^(3/2)*x^6 + 5*a*b^2*c^(3/2)*x^4 + 15*a^2*b*c^(3/2)*x^2 - 5*a^3*c
^(3/2))/x
```

Fricas [A]
time = 0.33, size = 72, normalized size = 0.54

$$\frac{(b^3cx^6 + 5\,ab^2cx^4 + 15\,a^2bcx^2 - 5\,a^3c)\sqrt{b^2cx^4 + 2\,abcx^2 + a^2c}}{5\,(bx^3 + ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 +
 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a + bx^2)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)
```

Giac [A]
time = 3.57, size = 69, normalized size = 0.51

$$\frac{1}{5}\left(b^3x^5\mathrm{sgn}\left(bx^2 + a\right) + 5\,ab^2x^3\mathrm{sgn}\left(bx^2 + a\right) + 15\,a^2bx\mathrm{sgn}\left(bx^2 + a\right) - \frac{5\,a^3\mathrm{sgn}(bx^2 + a)}{x}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b
*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^2, x)
```

3.236 $\int \frac{\left(c\left(a+bx^2\right)^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3a^2bc\sqrt{c\left(a+bx^2\right)^2}\,\log(x)}{a+bx^2}$$

```
[Out] -1/2*a^3*c*(c*(b*x^2+a)^2)^(1/2)/x^2/(b*x^2+a)+3/2*a*b^2*c*x^2*(c*(b*x^2+a)
^2)^(1/2)/(b*x^2+a)+1/4*b^3*c*x^4*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)+3*a^2*b*c
*ln(x)*(c*(b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.04, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 272, 45}

$$-\frac{a^3c\sqrt{c\left(a+bx^2\right)^2}}{2x^2\left(a+bx^2\right)}+\frac{3a^2bc\log(x)\sqrt{c\left(a+bx^2\right)^2}}{a+bx^2}+\frac{b^3cx^4\sqrt{c\left(a+bx^2\right)^2}}{4\left(a+bx^2\right)}+\frac{3ab^2cx^2\sqrt{c\left(a+bx^2\right)^2}}{2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*c*Sqrt[c*(a + b*x^2)^2])/(x^2*(a + b*x^2)) + (3*a*b^2*c*x^2*Sqrt[
c*(a + b*x^2)^2])/(2*(a + b*x^2)) + (b^3*c*x^4*Sqrt[c*(a + b*x^2)^2])/(4*(a
 + b*x^2)) + (3*a^2*b*c*Sqrt[c*(a + b*x^2)^2]*Log[x])/(a + b*x^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3}\,dx &= \int \frac{(a^2c+2abcx^2+b^2cx^4)^{3/2}}{x^3}\,dx\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\int \frac{(abc+b^2cx^2)^3}{x^3}\,dx}{b^2c\,(abc+b^2cx^2)}\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\mathrm{Subst}\left(\int \frac{(abc+b^2cx)^3}{x^2}\,dx,x,x^2\right)}{2b^2c\,(abc+b^2cx^2)}\\
&= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4}\,\mathrm{Subst}\left(\int \left(3ab^5c^3+\frac{a^3b^3c^3}{x^2}+\frac{3a^2b^4c^3}{x}+b^6c^3x\right)dx,x,x^2\right)}{2b^2c\,(abc+b^2cx^2)}\\
&= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2\,(a+bx^2)}+\frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2\,(a+bx^2)}+\frac{b^3cx^4\sqrt{a^2c+}}{4\,(a}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 65, normalized size = 0.46

$$
-\frac{\left(c(a+bx^2)^2\right)^{3/2}(2a^3-6ab^2x^4-b^3x^6-12a^2bx^2\log(x))}{4x^2\,(a+bx^2)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3,x]
```

```
[Out] -1/4*((c*(a + b*x^2)^2)^(3/2)*(2*a^3 - 6*a*b^2*x^4 - b^3*x^6 - 12*a^2*b*x^2
*Log[x]))/(x^2*(a + b*x^2)^3)
```

Maple [A]
time = 0.05, size = 61, normalized size = 0.44

method	result	size
default	$\frac{\left(c(b\,x^2+a)^2\right)^{\frac{3}{2}}(b^3x^6+6a\,b^2x^4+12a^2b\ln(x)x^2-2a^3)}{4(b\,x^2+a)^3x^2}$	61
risch	$\frac{c\sqrt{c\,(b\,x^2+a)^2}\;b(b\,x^2+3a)^2}{4b\,x^2+4a}-\frac{a^3c\sqrt{c\,(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{3a^2bc\ln(x)\sqrt{c\,(b\,x^2+a)^2}}{b\,x^2+a}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(c*(b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*a^2*b*ln(x)*x^2-2*a^3)/(b
*x^2+a)^3/x^2
```

Maxima [A]
time = 0.28, size = 176, normalized size = 1.26

$$\frac{3}{2}(-1)^{2b^2cx^2+2abc}a^2bc^{\frac{3}{2}}\log\left(2b^2cx^2+2abc\right)-\frac{3}{2}(-1)^{2abcx^2+2a^2c}a^2bc^{\frac{3}{2}}\log\left(2abc+\frac{2a^2c}{x^2}\right)+\frac{3}{4}\sqrt{b^2cx^4+2abcx^2+a^2c}\,b^2cx^2+\frac{9}{4}\sqrt{b^2cx^4+2abcx^2+a^2c}\,abc-\frac{\left(b^2cx^4+2abcx^2+a^2c\right)^{\frac{3}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) -
 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2)
+ 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x^4
+ 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)/
x^2
```

Fricas [A]
time = 0.34, size = 76, normalized size = 0.54

$$\frac{\left(b^3cx^6+6ab^2cx^4+12a^2bcx^2\log\left(x\right)-2a^3c\right)\sqrt{b^2cx^4+2abcx^2+a^2c}}{4\left(bx^4+ax^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*
c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(c\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)
```

Giac [A]
time = 5.78, size = 91, normalized size = 0.65

$$\frac{1}{4}\left(b^3x^4\mathrm{sgn}\left(bx^2+a\right)+6ab^2x^2\mathrm{sgn}\left(bx^2+a\right)+6a^2b\log\left(x^2\right)\mathrm{sgn}\left(bx^2+a\right)-\frac{2\left(3a^2bx^2\mathrm{sgn}\left(bx^2+a\right)+a^3\mathrm{sgn}\left(bx^2+a\right)\right)}{x^2}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)
*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*
c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,{\left(b\,x^2+a\right)}^2\right)^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^2)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^2)^(3/2)/x^3, x)
```

3.237 $\int x^2 \left(c(a+bx^2)^3\right)^{3/2} \, dx$

Optimal. Leaf size=253

$$\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] 7/128*a^3*c*x^3*(c*(b*x^2+a)^3)^(1/2)+21/1024*a^5*c*x*(c*(b*x^2+a)^3)^(1/2)
/b/(b*x^2+a)+21/512*a^4*c*x^3*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+21/320*a^2*c*
x^3*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+3/40*a*c*x^3*(b*x^2+a)^2*(c*(b*x^2+a)^3
)^(1/2)+1/12*c*x^3*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-21/1024*a^(9/2)*c*arcs
inh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/b^(3/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A]
time = 0.09, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1973, 285, 327, 221}

$$-\frac{21a^{9/2}c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{1024b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)}+\frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)}+\frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3]
)/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x
^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a
 + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*
x^2)^3])/12 - (21*a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*x)/Sqrt[
a]])/(1024*b^(3/2)*(1 + (b*x^2)/a)^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^2\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2(a+bx^2)^{9/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(3ac\sqrt{c\left(a+bx^2\right)^3}\right)\int x^2(a+bx^2)^{7/2}\,dx}{4\left(a+bx^2\right)^{3/2}}$$

$$= \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(21a^2c\sqrt{}\right.}{}$$

$$= \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{12}cx^3$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{3}{40}acx^3\left(a+bx\right.$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} + \frac{21}{320}a^2cx^3\left(a+bx^2\right)\sqrt{c\left(a\right.}$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} +$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} +$$

$$= \frac{7}{128}a^3cx^3\sqrt{c\left(a+bx^2\right)^3} + \frac{21a^5cx\sqrt{c\left(a+bx^2\right)^3}}{1024b\left(a+bx^2\right)} + \frac{21a^4cx^3\sqrt{c\left(a+bx^2\right)^3}}{512\left(a+bx^2\right)} +$$

Mathematica [A]
time = 0.15, size = 134, normalized size = 0.53

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}\,x\sqrt{a+bx^2}\left(315a^5+4910a^4bx^2+11432a^3b^2x^4+12144a^2b^3x^6+6272ab^4x^8+1280b^5x^{10}\right)+315a^6\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{15360b^{3/2}\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(315*a^5 + 4910*a^4*b*x
^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10
```

```
) + 315*a^6*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(15360*b^(3/2)*(a + b*x^2
)^(9/2))
```

Maple [A]
time = 0.06, size = 236, normalized size = 0.93

method	result
risch	$\frac{x\left(1280b^{5}x^{10}+6272b^{4}a\,x^{8}+12144a^{2}b^{3}x^{6}+11432b^{2}a^{3}x^{4}+4910b\,a^{4}x^{2}+315a^{5}\right)c\sqrt{c\left(b\,x^{2}+a\right)^{3}}}{15360\left(b\,x^{2}+a\right)b}-\frac{21a^{6}\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^{2}\ldots}\right.}{\ldots}$
default	$\frac{\left(c\left(b\,x^{2}+a\right)^{3}\right)^{\frac{3}{2}}\left(1280x^{7}\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}b^{3}\sqrt{bc}+3712\sqrt{bc}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}a\,b^{2}x^{5}+3440\sqrt{bc}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}a^{2}b\,x^{3}+840\sqrt{bc}\,\left(bc\,x^{2}\ldots\right.\right.}{15360b\left(b\,x^{2}+a\right)^{3}\left(c\left(b\,x\ldots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15360*(c*(b*x^2+a)^3)^(3/2)/b*(1280*x^7*(b*c*x^2+a*c)^(5/2)*b^3*(b*c)^(1/
2)+3712*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b^2*x^5+3440*(b*c)^(1/2)*(b*c*x^2
+a*c)^(5/2)*a^2*b*x^3+840*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^3*x-210*(b*c)^(
1/2)*(b*c*x^2+a*c)^(3/2)*a^4*c*x-315*(b*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^5*c^
2*x-315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^6*c^3)/(b
*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)
```

Fricas [A]
time = 0.41, size = 433, normalized size = 1.71

$$\left[\frac{315\left(a^{6}bcx^{2}+a^{7}c\right)\sqrt{\frac{c}{b}}\log\left(-\frac{2b^{2}cx^{4}+3abcx^{2}+a^{2}c-2\sqrt{b^{3}cx^{6}+3ab^{2}cx^{4}+3a^{2}bcx^{2}+a^{3}c}\,bx\sqrt{\frac{c}{b}}}{bx^{2}+a}\right)+2\left(1280b^{5}cx^{11}+6272ab^{4}cx^{9}+12144a^{2}b^{3}cx^{7}+11432a^{3}b^{2}cx^{5}+4910a^{4}bcx^{3}+315a^{5}cx\right)\sqrt{b^{3}cx^{6}+3ab^{2}cx^{4}+3a^{2}bcx^{2}+a^{3}c}}{30720\left(b^{2}x^{2}+ab\right)},\ \frac{315\left(a^{6}bcx^{2}+a^{7}c\right)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^{3}cx^{6}+3ab^{2}cx^{4}+3a^{2}bcx^{2}+a^{3}c}\,bx\sqrt{-\frac{c}{b}}}{b^{2}cx^{4}+2abcx^{2}+a^{2}c}\right)+\left(1280b^{5}cx^{11}+6272ab^{4}cx^{9}+12144a^{2}b^{3}cx^{7}+11432a^{3}b^{2}cx^{5}+4910a^{4}bcx^{3}+315a^{5}cx\right)\sqrt{b^{3}cx^{6}+3ab^{2}cx^{4}+3a^{2}bcx^{2}+a^{3}c}}{15360\left(b^{2}x^{2}+ab\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*
```

```
sqrt(c/b))/(b*x^2 + a)) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2
*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3
*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(
315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4
+ 3*a^2*b*c*x^2 + a^3*c)*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c))
+ (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2
*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3
*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c\left(a + bx^2\right)^3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral(x**2*(c*(a + b*x**2)**3)**(3/2), x)
```

Giac [A]
time = 4.76, size = 177, normalized size = 0.70

$$\frac{1}{15360}\left(\frac{315\,a^6c\log\left(\left|-\sqrt{bc}\,x + \sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}\,b} + \left(\frac{315\,a^5\mathrm{sgn}\left(bx^2+a\right)}{b} + 2\left(2455\,a^4\mathrm{sgn}\left(bx^2+a\right) + 4\left(1429\,a^3b\mathrm{sgn}\left(bx^2+a\right) + 2\left(759\,a^2b^2\mathrm{sgn}\left(bx^2+a\right) + 8\left(10\,b^4x^2\mathrm{sgn}\left(bx^2+a\right) + 49\,ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a)
+ 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*
x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*x^2)*x^2)*sqrt(b*c*
x^2 + a*c)*x)*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] int(x^2*(c*(a + b*x^2)^3)^(3/2), x)
```

3.238 $\int x\left(c(a+bx^2)^3\right)^{3/2}\,dx$

Optimal. Leaf size=32

$$\frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}$$

```
[Out] 1/11*c*(b*x^2+a)^4*(c*(b*x^2+a)^3)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1605, 15, 30}

$$\frac{c(a+bx^2)^4\sqrt{c\,(a+bx^2)^3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\left(cx^3\right)^{3/2}\,dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\text{Subst}\left(\int x^{9/2}\,dx,x,a+bx^2\right)}{2b\left(a+bx^2\right)^{3/2}}$$
$$=\frac{c\left(a+bx^2\right)^4\sqrt{c\left(a+bx^2\right)^3}}{11b}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.91

$$\frac{\left(a+bx^2\right)\left(c\left(a+bx^2\right)^3\right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(28) = 56.
time = 0.04, size = 138, normalized size = 4.31

method	result	si
gosper	$\frac{\left(b\,x^2+a\right)\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}}{11b}$	2
risch	$\frac{c\sqrt{c\left(b\,x^2+a\right)^3}\,\left(b^5x^{10}+5b^4a\,x^8+10a^2b^3x^6+10b^2a^3x^4+5b\,a^4x^2+a^5\right)}{11\left(b\,x^2+a\right)b}$	8
trager	$\frac{c\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4\right)\sqrt{b^3c\,x^6+3a\,b^2c\,x^4+3a^2bc\,x^2+a^3c}}{11b}$	8
default	$-\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(-5x^6\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^3-15\left(bc\,x^2+ac\right)^{\frac{5}{2}}a\,b^2x^4-15\left(bc\,x^2+ac\right)^{\frac{5}{2}}a^2b\,x^2+6\left(bc\,x^2+ac\right)^{\frac{5}{2}}a^3-11a^3\left(c\left(b\,x^2+a\right)\right)^{\frac{5}{2}}\right)}{55b\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/55*(c*(b*x^2+a)^3)^(3/2)/b*(-5*x^6*(b*c*x^2+a*c)^(5/2)*b^3-15*(b*c*x^2+a
*c)^(5/2)*a*b^2*x^4-15*(b*c*x^2+a*c)^(5/2)*a^2*b*x^2+6*(b*c*x^2+a*c)^(5/2)*
a^3-11*a^3*(c*(b*x^2+a))^(5/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.
time = 0.29, size = 70, normalized size = 2.19

$$\frac{\left(b^4c^{\frac{3}{2}}x^8 + 4\,ab^3c^{\frac{3}{2}}x^6 + 6\,a^2b^2c^{\frac{3}{2}}x^4 + 4\,a^3bc^{\frac{3}{2}}x^2 + a^4c^{\frac{3}{2}}\right)(bx^2 + a)^{\frac{3}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/11*(b^4*c^(3/2)*x^8 + 4*a*b^3*c^(3/2)*x^6 + 6*a^2*b^2*c^(3/2)*x^4 + 4*a^3
*b*c^(3/2)*x^2 + a^4*c^(3/2))*(b*x^2 + a)^(3/2)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(28) = 56$.
time = 0.36, size = 87, normalized size = 2.72

$$\frac{(b^4cx^8 + 4\,ab^3cx^6 + 6\,a^2b^2cx^4 + 4\,a^3bcx^2 + a^4c)\sqrt{b^3cx^6 + 3\,ab^2cx^4 + 3\,a^2bcx^2 + a^3c}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*
sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\left(a + bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral(x*(c*(a + b*x**2)**3)**(3/2), x)
```

Giac [A]
time = 4.30, size = 28, normalized size = 0.88

$$\frac{(bcx^2 + ac)^{\frac{11}{2}}\operatorname{sgn}(bx^2 + a)}{11\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)
```

Mupad [**B**]
time = 2.71, size = 62, normalized size = 1.94

$$\sqrt{c\,{\left(b\,x^2+a\right)}^3}\,\left(\frac{a^4\,c}{11\,b}+\frac{4\,a^3\,c\,x^2}{11}+\frac{b^3\,c\,x^8}{11}+\frac{6\,a^2\,b\,c\,x^4}{11}+\frac{4\,a\,b^2\,c\,x^6}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^3)^(1/2)*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/11
 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)
```

3.239 $\int \left(c(a+bx^2)^3\right)^{3/2}\,dx$

Optimal. Leaf size=207

$$\frac{21}{128}a^3cx\sqrt{c\,(a+bx^2)^3}+\frac{63a^4cx\sqrt{c\,(a+bx^2)^3}}{256\,(a+bx^2)}+\frac{21}{160}a^2cx(a+bx^2)\,\sqrt{c\,(a+bx^2)^3}+\frac{9}{80}acx(a+bx^2)^2\,\sqrt{c\,(}$$

```
[Out] 21/128*a^3*c*x*(c*(b*x^2+a)^3)^(1/2)+63/256*a^4*c*x*(c*(b*x^2+a)^3)^(1/2)/(
b*x^2+a)+21/160*a^2*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/80*a*c*x*(b*x^2+a
)^2*(c*(b*x^2+a)^3)^(1/2)+1/10*c*x*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)+63/256
*a^(7/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/
2)/b^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1973, 201, 221}

$$\frac{63a^{7/2}c\sqrt{c\,(a+bx^2)^3}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256\sqrt{b}\,\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{63a^4cx\sqrt{c\,(a+bx^2)^3}}{256\,(a+bx^2)}+\frac{21}{128}a^3cx\sqrt{c\,(a+bx^2)^3}+\frac{21}{160}a^2cx(a+bx^2)\,\sqrt{c\,(a+bx^2)^3}+\frac{9}{80}acx(a+bx^2)^2\,\sqrt{c\,(a+bx^2)^3}+\frac{1}{10}cx(a+bx^2)^3\,\sqrt{c\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2),x]
```

```
[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])
/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (
9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c
*(a + b*x^2)^3])/10 + (63*a^(7/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[(Sqrt[b]*
x)/Sqrt[a]])/(256*Sqrt[b]*(1 + (b*x^2)/a)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \left(c\left(a+bx^2\right)^3\right)^{3/2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{9/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(9ac\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}\,dx}{10\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3} + \frac{\left(63a^2c\sqrt{c\left(\right.}\right.}{}$$

$$= \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} + \frac{1}{10}cx(a+$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{80}acx\left(a+bx^2\right)^2\sqrt{}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right.}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right.}$$

$$= \frac{21}{128}a^3cx\sqrt{c\left(a+bx^2\right)^3} + \frac{63a^4cx\sqrt{c\left(a+bx^2\right)^3}}{256\left(a+bx^2\right)} + \frac{21}{160}a^2cx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right.}$$

Mathematica [A]
time = 0.14, size = 123, normalized size = 0.59

$$\frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}\left(\sqrt{b}\,x\sqrt{a+bx^2}\left(965a^4+1490a^3bx^2+1368a^2b^2x^4+656ab^3x^6+128b^4x^8\right)-315a^5\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{1280\sqrt{b}\,\left(a+bx^2\right)^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^3)^(3/2),x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(965*a^4 + 1490*a^3*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) - 315*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]))/(1280*Sqrt[b]*(a + b*x^2)^(9/2))

Maple [A]
time = 0.04, size = 205, normalized size = 0.99

method	result
risch	$\frac{x\left(128b^4x^8+656a\,b^3x^6+1368a^2b^2x^4+1490a^3b\,x^2+965a^4\right)c\sqrt{c\left(b\,x^2+a\right)^3}}{1280b\,x^2+1280a}+\frac{63a^5\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c\,(b\ldots}}{256\sqrt{bc}\ (b\,x^2+\ldots}$
default	$\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(128x^5\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2\sqrt{bc}+400\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\ ab\,x^3+440\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\ a^2x+210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\ \ldots\right.}{1280\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c\sqrt{bc}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(b*x^2+a)^3)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/1280*(c*(b*x^2+a)^3)^(3/2)*(128*x^5*(b*c*x^2+a*c)^(5/2)*b^2*(b*c)^(1/2)+400*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*a*b*x^3+440*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*a^2*x+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*a^3*c*x+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*a^4*c^2*x+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^5*c^3)/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)

Fricas [A]
time = 0.39, size = 402, normalized size = 1.94

$$\left[\frac{315\,(a^5bcx^2+a^6c)\sqrt{\frac{c}{b}}\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{\frac{c}{b}}}{bx^2+a}\right)+2\,(128\,b^4cx^9+656\,ab^3cx^7+1368\,a^2b^2cx^5+1490\,a^3bcx^3+965\,a^4cx)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{2560\,(bx^2+a)},\ -\frac{315\,(a^5bcx^2+a^6c)\sqrt{-\frac{c}{b}}\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,bx\sqrt{-\frac{c}{b}}}{b^2cx^4+2\,abcx^2+a^2c}\right)-(128\,b^4cx^9+656\,ab^3cx^7+1368\,a^2b^2cx^5+1490\,a^3bcx^3+965\,a^4cx)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{1280\,(bx^2+a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")

```
[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^
2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*s
qrt(c/b))/(b*x^2 + a)) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*
c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(
-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*b*x*sq
rt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*
x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 +
 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\left(a+bx^2\right)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2),x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2), x)
```

Giac [A]
time = 3.28, size = 153, normalized size = 0.74

$$-\frac{1}{1280}\left(\frac{315\,a^5c\log\left(\left|-\sqrt{bc}\,x+\sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}\left(bx^2+a\right)}{\sqrt{bc}}-\left(965\,a^4\mathrm{sgn}\left(bx^2+a\right)+2\left(745\,a^3b\mathrm{sgn}\left(bx^2+a\right)+4\left(171\,a^2b^2\mathrm{sgn}\left(bx^2+a\right)+2\left(8\,b^4x^2\mathrm{sgn}\left(bx^2+a\right)+41\,ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 +
 a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(
171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x
^2 + a))*x^2)*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2), x)
```

3.240 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x} \, dx$

Optimal. Leaf size=192

$$\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}+\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}$$

```
[Out] 1/3*a^3*c*(c*(b*x^2+a)^3)^(1/2)+a^4*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+a)+1/5*a
^2*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+1/7*a*c*(b*x^2+a)^2*(c*(b*x^2+a)^3)^(1
/2)+1/9*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)-a^3*c*arctanh((1+b*x^2/a)^(1/2)
)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1973, 272, 52, 65, 214}

$$\frac{a^4c\sqrt{c\left(a+bx^2\right)^3}}{a+bx^2}+\frac{1}{3}a^3c\sqrt{c\left(a+bx^2\right)^3}-\frac{a^3c\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}+\frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2)
 + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*
(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^3*c*Sqrt
[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])/(1 + (b*x^2)/a)^(3/2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\,(a+bx^2)^3}\right)\int\frac{(a+bx^2)^{9/2}}{x}\,dx}{(a+bx^2)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\,(a+bx^2)^3}\right)\mathrm{Subst}\left(\int\frac{(a+bx)^{9/2}}{x}\,dx,x,x^2\right)}{2\,(a+bx^2)^{3/2}}$$

$$= \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\,(a+bx^2)^3} + \frac{\left(ac\sqrt{c\,(a+bx^2)^3}\right)\mathrm{Subst}\left(\int\frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)}{2\,(a+bx^2)^{3/2}}$$

$$= \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\,(a+bx^2)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{c\,(a+bx^2)^3} + \frac{\left(a^2c\sqrt{c\,(a+bx\right.}}{}$$

$$= \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\,(a+bx^2)^3} + \frac{1}{9}c\left(a+bx^2\right)^3\sqrt{}$$

$$= \frac{1}{3}a^3c\sqrt{c\,(a+bx^2)^3} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} + \frac{1}{7}ac\left(a+bx^2\right)^2\sqrt{c\,(a+bx}$$

$$= \frac{1}{3}a^3c\sqrt{c\,(a+bx^2)^3} + \frac{a^4c\sqrt{c\,(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} + \frac{1}{7}ac$$

$$= \frac{1}{3}a^3c\sqrt{c\,(a+bx^2)^3} + \frac{a^4c\sqrt{c\,(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} + \frac{1}{7}ac$$

$$= \frac{1}{3}a^3c\sqrt{c\,(a+bx^2)^3} + \frac{a^4c\sqrt{c\,(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c\left(a+bx^2\right)\sqrt{c\,(a+bx^2)^3} + \frac{1}{7}ac$$

Mathematica [A]
time = 0.11, size = 111, normalized size = 0.58

$$\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}\,(563a^4+506a^3bx^2+408a^2b^2x^4+185ab^3x^6+35b^4x^8)-315a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{315\,(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^
2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))
```

Maple [A]
time = 0.05, size = 221, normalized size = 1.15

method	result
default	$-\frac{\left(c\left(b\,x^{2}+a\right)^{3}\right)^{\frac{3}{2}}\left(-35\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}b^{2}x^{4}-115\sqrt{ac}\,\left(bc\,x^{2}+ac\right)^{\frac{5}{2}}ab\,x^{2}+315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^{2}+ac}}{x}\right)a^{5}c^{3}+46\cdots\right.}{315\left(b\,x^{2}+a\right)^{3}\left(c\left(b\,x^{2}+a\right)\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(c*(b*x^2+a)^3)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-1
15*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b*x^2+315*ln(2*((a*c)^(1/2)*(b*c*x^2+a
*c)^(1/2)+a*c)/x)*a^5*c^3+46*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*c)^
(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2
-189*a^2*(c*(b*x^2+a))^(5/2)*(a*c)^(1/2))/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c
/(a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)
```

Fricas [A]
time = 0.41, size = 391, normalized size = 2.04

$$\left[\frac{315\,(a^4bcx^2+a^5c)\sqrt{ac}\,\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{ac}}{bx^4+ax^2}\right)+2\,(35\,b^4cx^8+185\,ab^3cx^6+408\,a^2b^2cx^4+506\,a^3bcx^2+563\,a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{630\,(bx^2+a)},\ \frac{315\,(a^4bcx^2+a^5c)\sqrt{-ac}\,\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{-ac}}{b^2cx^4+2\,abcx^2+a^2c}\right)+(35\,b^4cx^8+185\,ab^3cx^6+408\,a^2b^2cx^4+506\,a^3bcx^2+563\,a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{315\,(bx^2+a)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 +
 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a
*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x
^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*
c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*ar
```

```
ctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^
2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2
*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 +
3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)
```

Giac [A]
time = 3.89, size = 185, normalized size = 0.96

$$\frac{1}{315}\left(\frac{315\,a^5\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{-ac}}+\frac{315\,\sqrt{bcx^2+ac}\,a^4c^{44}\mathrm{sgn}(bx^2+a)+105\,(bcx^2+ac)^{\frac{3}{2}}a^3c^{43}\mathrm{sgn}(bx^2+a)+63\,(bcx^2+ac)^{\frac{5}{2}}a^2c^{42}\mathrm{sgn}(bx^2+a)+45\,(bcx^2+ac)^{\frac{7}{2}}ac^{41}\mathrm{sgn}(bx^2+a)+35\,(bcx^2+ac)^{\frac{9}{2}}c^{40}\mathrm{sgn}(bx^2+a)}{c^{45}}\right)c^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-
a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^44*sgn(b*x^2 + a) + 105*(b*c*x^2 + a*
c)^(3/2)*a^3*c^43*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^42*sgn(b*
x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^41*sgn(b*x^2 + a) + 35*(b*c*x^2 + a
*c)^(9/2)*c^40*sgn(b*x^2 + a))/c^45)*c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x, x)
```

3.241 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=208

$$\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c(c$$

```
[Out] 105/64*a^2*b*c*x*(c*(b*x^2+a)^3)^(1/2)+315/128*a^3*b*c*x*(c*(b*x^2+a)^3)^(1
/2)/(b*x^2+a)+21/16*a*b*c*x*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/8*b*c*x*(b*x^
2+a)^2*(c*(b*x^2+a)^3)^(1/2)-c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x+315/128*
a^(5/2)*c*arcsinh(x*b^(1/2)/a^(1/2))*b^(1/2)*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2
/a)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1973, 283, 201, 221}

$$\frac{315a^{5/2}\sqrt{b}\,c\sqrt{c\left(a+bx^2\right)^3}\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)}+\frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3}+\frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{x}+\frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2
)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16
 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[
c*(a + b*x^2)^3])/x + (315*a^(5/2)*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcSinh[
(Sqrt[b]*x)/Sqrt[a]])/(128*(1 + (b*x^2)/a)^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\left(a+bx^2\right)^3}\right)\int \frac{\left(a+bx^2\right)^{9/2}}{x^2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= -\frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(9bc\sqrt{c\left(a+bx^2\right)^3}\right)\int \left(a+bx^2\right)^{7/2}\,dx}{\left(a+bx^2\right)^{3/2}}$$

$$= \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3\sqrt{c\left(a+bx^2\right)^3}}{x} + \frac{\left(63abc\sqrt{c\left(a+\right.}\right.}{8\left(\right.}$$

$$= \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3} - \frac{c(a+bx^2)^3}{}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3} + \frac{9}{8}bcx\left(a+bx^2\right)^2\sqrt{}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

$$= \frac{105}{64}a^2bcx\sqrt{c\left(a+bx^2\right)^3} + \frac{315a^3bcx\sqrt{c\left(a+bx^2\right)^3}}{128\left(a+bx^2\right)} + \frac{21}{16}abcx\left(a+bx^2\right)\sqrt{c\left(a+\right.}$$

Mathematica [A]
time = 0.14, size = 121, normalized size = 0.58

$$-\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}\,(128a^4-325a^3bx^2-210a^2b^2x^4-88ab^3x^6-16b^4x^8)+315a^4\sqrt{b}\,x\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{128x\,(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2,x]
```

```
[Out] -1/128*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(128*a^4 - 325*a^3*b*x^2 -
 210*a^2*b^2*x^4 - 88*a*b^3*x^6 - 16*b^4*x^8) + 315*a^4*Sqrt[b]*x*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]]))/(x*(a + b*x^2)^(9/2))
```

Maple [A]
time = 0.05, size = 215, normalized size = 1.03

method	result
risch	$-\frac{\left(-16b^4x^8-88a\,b^3x^6-210a^2b^2x^4-325a^3b\,x^2+128a^4\right)c\sqrt{c\left(b\,x^2+a\right)^3}}{128\left(b\,x^2+a\right)x}+\frac{315b\,a^4\ln\left(\frac{bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+ac}\right)c\sqrt{c\left(b\,x\right.}}{128\sqrt{bc}\,\left(b\,x^2+a\right)^2}$
default	$-\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(-16\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,b^2x^4-56\left(bc\,x^2+ac\right)^{\frac{5}{2}}\sqrt{bc}\,ab\,x^2-210\left(bc\,x^2+ac\right)^{\frac{3}{2}}\sqrt{bc}\,a^2bc\,x^2-315\sqrt{bc\,x^2+ac}\right.}{128\left(b\,x^2+a\right)^3\left(c\left(b\,x^2+a\right)\right)^{\frac{3}{2}}c\sqrt{bc}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(c*(b*x^2+a)^3)^(3/2)*(-16*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*b^2*x^4-5
6*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*a*b*x^2-210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/
2)*a^2*b*c*x^2-315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*a^3*b*c^2*x^2-315*ln((b*
c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^4*b*c^3*x+128*(b*c*x^2+
a*c)^(5/2)*(b*c)^(1/2)*a^2)/(b*x^2+a)^3/(c*(b*x^2+a))^(3/2)/c/(b*c)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)
```

Fricas [A]
time = 0.38, size = 396, normalized size = 1.90

$$\left[\frac{315\,(a^4bcx^3+a^5cx)\sqrt{bc}\,\log\left(-\frac{2\,b^2cx^4+3\,abcx^2+a^2c+2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{bc}\,x}{bx^2+a}\right)+2\,(16\,b^4cx^8+88\,ab^3cx^6+210\,a^2b^2cx^4+325\,a^3bcx^2-128\,a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{256\,(bx^3+ax)},\ -\frac{315\,(a^4bcx^3+a^5cx)\sqrt{-bc}\,\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{-bc}\,x}{b^2cx^4+2\,abcx^2+a^2c}\right)-(16\,b^4cx^8+88\,ab^3cx^6+210\,a^2b^2cx^4+325\,a^3bcx^2-128\,a^4c)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{128\,(bx^3+ax)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(b*c)*log(-(2*b^2*c*x^4 + 3*a*b*c*x
^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt
(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^
4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c
*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*sqrt(-b*c
)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-b*c)
*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 21
0*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x
^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**2, x)
```

Giac [A]

time = 4.64, size = 185, normalized size = 0.89

$$\frac{1}{256}\left(\frac{512\sqrt{bc}\,a^5c\mathrm{sgn}(bx^2+a)}{\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac}-315\sqrt{bc}\,a^4\log\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2\right)\mathrm{sgn}\left(bx^2+a\right)+2\left(325\,a^3b\mathrm{sgn}\left(bx^2+a\right)+2\left(105\,a^2b^2\mathrm{sgn}\left(bx^2+a\right)+4\left(2\,b^4x^2\mathrm{sgn}\left(bx^2+a\right)+11\,ab^3\mathrm{sgn}\left(bx^2+a\right)\right)x^2\right)x^2\right)\sqrt{bcx^2+ac}\,x\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/256*(512*sqrt(b*c)*a^5*c*sgn(b*x^2 + a)/((sqrt(b*c)*x - sqrt(b*c*x^2 + a*
c))^2 - a*c) - 315*sqrt(b*c)*a^4*log((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2)
*sgn(b*x^2 + a) + 2*(325*a^3*b*sgn(b*x^2 + a) + 2*(105*a^2*b^2*sgn(b*x^2 +
a) + 4*(2*b^4*x^2*sgn(b*x^2 + a) + 11*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(
b*c*x^2 + a*c)*x)*c
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,{\left(b\,x^2+a\right)}^3\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^2,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^2, x)
```

3.242 $\int \frac{\left(c\left(a+bx^2\right)^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=202

$$\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}+\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

```
[Out] 3/2*a^2*b*c*(c*(b*x^2+a)^3)^(1/2)+9/2*a^3*b*c*(c*(b*x^2+a)^3)^(1/2)/(b*x^2+
a)+9/10*a*b*c*(b*x^2+a)*(c*(b*x^2+a)^3)^(1/2)+9/14*b*c*(b*x^2+a)^2*(c*(b*x^
2+a)^3)^(1/2)-1/2*c*(b*x^2+a)^3*(c*(b*x^2+a)^3)^(1/2)/x^2-9/2*a^2*b*c*arcta
nh((1+b*x^2/a)^(1/2))*(c*(b*x^2+a)^3)^(1/2)/(1+b*x^2/a)^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1973, 272, 43, 52, 65, 214}

$$\frac{9a^3bc\sqrt{c\left(a+bx^2\right)^3}}{2\left(a+bx^2\right)}+\frac{3}{2}a^2bc\sqrt{c\left(a+bx^2\right)^3}-\frac{9a^2bc\sqrt{c\left(a+bx^2\right)^3}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}}+\frac{9}{10}abc\left(a+bx^2\right)\sqrt{c\left(a+bx^2\right)^3}-\frac{c\left(a+bx^2\right)^3\sqrt{c\left(a+bx^2\right)^3}}{2x^2}+\frac{9}{14}bc\left(a+bx^2\right)^2\sqrt{c\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*
(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a +
 b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3
])/(2*x^2) - (9*a^2*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[1 + (b*x^2)/a]])
/(2*(1 + (b*x^2)/a)^(3/2))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\,(a+bx^2)^3}\right)\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx}{(a+bx^2)^{3/2}}$$

$$= \frac{\left(c\sqrt{c\,(a+bx^2)^3}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2}\,dx,x,x^2\right)}{2\,(a+bx^2)^{3/2}}$$

$$= -\frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{\left(9bc\sqrt{c\,(a+bx^2)^3}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{7/2}}{x}\,dx,x,x^2\right)}{4\,(a+bx^2)^{3/2}}$$

$$= \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c\,(a+bx^2)^3}}{2x^2} + \frac{\left(9abc\sqrt{c\,(a+bx^2\right.}}{}$$

$$= \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c}}{2x^2}$$

$$= \frac{3}{2}a^2bc\sqrt{c\,(a+bx^2)^3} + \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2\sqrt{c\,(a+}$$

$$= \frac{3}{2}a^2bc\sqrt{c\,(a+bx^2)^3} + \frac{9a^3bc\sqrt{c\,(a+bx^2)^3}}{2\,(a+bx^2)} + \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} +$$

$$= \frac{3}{2}a^2bc\sqrt{c\,(a+bx^2)^3} + \frac{9a^3bc\sqrt{c\,(a+bx^2)^3}}{2\,(a+bx^2)} + \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} +$$

$$= \frac{3}{2}a^2bc\sqrt{c\,(a+bx^2)^3} + \frac{9a^3bc\sqrt{c\,(a+bx^2)^3}}{2\,(a+bx^2)} + \frac{9}{10}abc(a+bx^2)\sqrt{c\,(a+bx^2)^3} +$$

Mathematica [A]
time = 0.12, size = 118, normalized size = 0.58

$$-\frac{\left(c(a+bx^2)^3\right)^{3/2}\left(\sqrt{a+bx^2}\,(35a^4-388a^3bx^2-156a^2b^2x^4-58ab^3x^6-10b^4x^8)+315a^{7/2}bx^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{70x^2\,(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] -1/70*((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(35*a^4 - 388*a^3*b*x^2 - 1
56*a^2*b^2*x^4 - 58*a*b^3*x^6 - 10*b^4*x^8) + 315*a^(7/2)*b*x^2*ArcTanh[Sqr
t[a + b*x^2]/Sqrt[a]]))/(x^2*(a + b*x^2)^(9/2))
```

Maple [A]
time = 0.07, size = 238, normalized size = 1.18

method	result
risch	$-\frac{a^4 c\sqrt{c\left(b\,x^2+a\right)^3}}{2(b\,x^2+a)x^2}+\frac{\left(\frac{b^4 x^6\sqrt{bc\,x^2+ac}}{7c}+\frac{29b^3 a\,x^4\sqrt{bc\,x^2+ac}}{35c}+\frac{78b^2a^2x^2\sqrt{bc\,x^2+ac}}{35c}-\frac{156b\,a^3\sqrt{bc\,x^2}}{35c}\right.}{}$
default	$\frac{\left(c\left(b\,x^2+a\right)^3\right)^{\frac{3}{2}}\left(10\sqrt{ac}\,\left(bc\,x^2+ac\right)^{\frac{5}{2}}b^2x^4-315\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)a^4b\,c^3x^2+42ab\left(c\left(b\,x^2+a\right)\right)^{\frac{5}{2}}x^2\sqrt{ac}-4\sqrt{}\right.}{70(b\,x^2+a)^3(c(b\,x^2+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/70*(c*(b*x^2+a)^3)^(3/2)*(10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-315*
ln(2*((a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)+a*c)/x)*a^4*b*c^3*x^2+42*a*b*(c*(b*x^
2+a))^(5/2)*x^2*(a*c)^(1/2)-4*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b*x^2+105*(
b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*b*c*x^2+315*(b*c*x^2+a*c)^(1/2)*(a*c)^(1
/2)*a^3*b*c^2*x^2-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/(c*(b
*x^2+a))^(3/2)/c/x^2/(a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)
```

Fricas [A]
time = 0.37, size = 411, normalized size = 2.03

$$\left[\frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{ac}\,\log\left(-\frac{b^2cx^4+3\,abcx^2+2\,a^2c-2\,\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{ac}}{bx^4+ax^2}\right)+2\left(10\,b^4cx^8+58\,ab^3cx^6+156\,a^2b^2cx^4+388\,a^3bcx^2-35\,a^4c\right)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{140\left(bx^4+ax^2\right)},\ \frac{315\left(a^3b^2cx^4+a^4bcx^2\right)\sqrt{-ac}\,\arctan\left(\frac{\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}\,\sqrt{-ac}}{b^2cx^4+2\,abcx^2+a^2c}\right)+\left(10\,b^4cx^8+58\,ab^3cx^6+156\,a^2b^2cx^4+388\,a^3bcx^2-35\,a^4c\right)\sqrt{b^3cx^6+3\,ab^2cx^4+3\,a^2bcx^2+a^3c}}{70\left(bx^4+ax^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b
*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c
)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*
b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*
a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x
^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*
c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3
*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3
*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a+bx^2)^3\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)
```

```
[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x**3, x)
```

Giac [A]
time = 5.48, size = 204, normalized size = 1.01

$$\frac{\left(\frac{315\,a^4b^2c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)\mathrm{sgn}(bx^2+a)}{\sqrt{-ac}}-\frac{35\,\sqrt{bcx^2+ac}\,a^4b\,\mathrm{sgn}(bx^2+a)}{x^2}+\frac{2\left(140\,\sqrt{bcx^2+ac}\,a^3b^2c^{21}\mathrm{sgn}(bx^2+a)+35\,(bcx^2+ac)^{\frac{3}{2}}a^2b^2c^{20}\mathrm{sgn}(bx^2+a)+14\,(bcx^2+ac)^{\frac{5}{2}}ab^2c^{19}\mathrm{sgn}(bx^2+a)+5\,(bcx^2+ac)^{\frac{7}{2}}b^2c^{18}\mathrm{sgn}(bx^2+a)\right)}{c^{21}}\right)c}{70\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/s
qrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt(b
*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2*b^
2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 + a)
+ 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21)*c/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,{\left(b\,x^2+a\right)}^3\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^3)^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^3)^(3/2)/x^3, x)
```

3.243 $\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=77

$$-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\sqrt{a}\,c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{1+\frac{bx^2}{a}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] -c*x*(c/(b*x^2+a))^(1/2)/b+c*arcsinh(x*b^(1/2)/a^(1/2))*a^(1/2)*(c/(b*x^2+a
))^(1/2)*(1+b*x^2/a)^(1/2)/b^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 294, 221}

$$\frac{\sqrt{a}\,c\sqrt{\frac{bx^2}{a}+1}\,\sqrt{\frac{c}{a+bx^2}}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (Sqrt[a]*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b
*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(\frac{c}{a+bx^2}\right)^{3/2}dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{x^2}{\left(a+bx^2\right)^{3/2}}\,dx\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b}+\frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 64, normalized size = 0.83

$$
-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{b}\,x+\sqrt{a+bx^2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)}{b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c/(a + b*x^2))^(3/2),x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)]*(Sqrt[b]*x + Sqrt[a + b*x^2]*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]]))/b^(3/2))
```

Maple [A]
time = 0.03, size = 59, normalized size = 0.77

method	result	size
default	$\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}\left(b\,x^2+a\right)\left(-x\,b^{\frac{3}{2}}+\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)b\sqrt{b\,x^2+a}\right)}{b^{\frac{5}{2}}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c/(b*x^2+a))^(3/2)*(b*x^2+a)*(-x*b^(3/2)+ln(b^(1/2)*x+(b*x^2+a)^(1/2))*b*(
b*x^2+a)^(1/2))/b^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*(c/(b*x^2 + a))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 141, normalized size = 1.83

$$\left[-\frac{2\,cx\sqrt{\frac{c}{bx^2+a}} - c\sqrt{\frac{c}{b}}\,\log\left(-2\,bcx^2 - ac - 2\,(b^2x^3 + abx)\sqrt{\frac{c}{bx^2+a}}\,\sqrt{\frac{c}{b}}\right)}{2\,b}, -\frac{cx\sqrt{\frac{c}{bx^2+a}} + c\sqrt{-\frac{c}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\,\sqrt{-\frac{c}{b}}}{c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^
2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a))
 + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Integral(x**2*(c/(a + b*x**2))**(3/2), x)
```

Giac [A]
time = 5.72, size = 71, normalized size = 0.92

$$-\left(\frac{cx\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}\,b} + \frac{c\log\left(\left|-\sqrt{bc}\,x + \sqrt{bcx^2+ac}\right|\right)\mathrm{sgn}(bx^2+a)}{\sqrt{bc}\,b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -(c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b) + c*log(abs(-sqrt(b*c)*x + sqr
t(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b))*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\frac{c}{b\,x^2+a}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c/(a + b*x^2))^(3/2),x)
```

```
[Out] int(x^2*(c/(a + b*x^2))^(3/2), x)
```

3.244 $\int x\left(\frac{c}{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

[Out] -c*(c/(b*x^2+a))^(1/2)/b

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1605, 15, 30}

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] Int[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(\frac{c}{a+bx^2}\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(\frac{c}{x}\right)^{3/2}dx,x,a+bx^2\right)}{2b}$$
$$=\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{x^{3/2}}dx,x,a+bx^2\right)}{2b}$$
$$=-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]
[Out] -((c*Sqrt[c/(a + b*x^2)])/b)
```

Maple [A]
time = 0.05, size = 26, normalized size = 1.24

method	result	size
trager	$-\frac{c\sqrt{\frac{c}{b\,x^2+a}}}{b}$	20
gosper	$-\frac{(b\,x^2+a)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{b}$	26
derivativedivides	$-\frac{(b\,x^2+a)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{b}$	26
default	$-\frac{(b\,x^2+a)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -(b*x^2+a)/b*(c/(b*x^2+a))^(3/2)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Fricas [A]
time = 0.37, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] -c*sqrt(c/(b*x^2 + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.
time = 0.30, size = 42, normalized size = 2.00

$$\begin{cases} -\frac{a\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{b} - x^2\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((-a*(c/(a + b*x**2))**(3/2)/b - x**2*(c/(a + b*x**2))**(3/2), Ne(
b, 0)), (x**2*(c/a)**(3/2)/2, True))
```

Giac [A]
time = 4.64, size = 28, normalized size = 1.33

$$-\frac{c^2\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)
```

Mupad [B]
time = 2.64, size = 19, normalized size = 0.90

$$-\frac{c\,\sqrt{\frac{c}{b\,x^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c/(a + b*x^2))^(3/2),x)
```

```
[Out] -(c*(c/(a + b*x^2))^(1/2))/b
```

3.245 $\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

[Out] c*x*(c/(b*x^2+a))^(1/2)/a

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1973, 197}

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2),x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{(a+bx^2)^{3/2}}\,dx\\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2),x]
```

```
[Out] (c*x*Sqrt[c/(a + b*x^2)])/a
```

Maple [A]
time = 0.02, size = 26, normalized size = 1.24

method	result	size
trager	$\frac{cx\sqrt{\frac{c}{b\,x^2+a}}}{a}$	20
gosper	$\frac{(b\,x^2+a)x\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a}$	26
default	$\frac{(b\,x^2+a)x\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*x/a*(c/(b*x^2+a))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c/(b*x^2 + a))^(3/2), x)
```

Fricas [A]
time = 0.37, size = 19, normalized size = 0.90

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")
```

```
[Out] c*x*sqrt(c/(b*x^2 + a))/a
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.
time = 0.33, size = 46, normalized size = 2.19

$$\begin{cases} x\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} + \frac{bx^3\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a \neq 0 \\ -\frac{x\left(\frac{c}{bx^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2),x)
```

```
[Out] Piecewise((x*(c/(a + b*x**2))**(3/2) + b*x**3*(c/(a + b*x**2))**(3/2)/a, Ne
(a, 0)), (-x*(c/(b*x**2))**(3/2)/2, True))
```

Giac [A]
time = 4.79, size = 28, normalized size = 1.33

$$\frac{c^2 x \operatorname{sgn}(bx^2 + a)}{\sqrt{bcx^2 + ac}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")
```

```
[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)
```

Mupad [B]
time = 2.74, size = 19, normalized size = 0.90

$$\frac{c\,x\,\sqrt{\frac{c}{b\,x^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2),x)
```

```
[Out] (c*x*(c/(a + b*x^2))^(1/2))/a
```

3.246 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=71

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{1+\frac{bx^2}{a}}\,\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{a}$$

```
[Out] c*(c/(b*x^2+a))^(1/2)/a-c*arctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1
+b*x^2/a)^(1/2)/a
```

Rubi [A]
time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {1973, 272, 53, 65, 214}

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{bx^2}{a}+1}\,\sqrt{\frac{c}{a+bx^2}}\,\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)])/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcT
anh[Sqrt[1 + (b*x^2)/a]])/a
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x}\,dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\int \frac{1}{x\,(a+bx^2)^{3/2}}\,dx \\
&= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^2\right) \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right)}{2a} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}+\frac{\left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{ab} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a}-\frac{c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 0.83

$$\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}-\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]
```

```
[Out] (c*Sqrt[c/(a + b*x^2)]*(Sqrt[a] - Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]))/a^(3/2)
```

Maple [A]
time = 0.03, size = 64, normalized size = 0.90

method	result	size
default	$-\frac{\left(\frac{c}{b\,x^{2}+a}\right)^{\frac{3}{2}} \left(b\,x^{2}+a\right) \left(\ln \left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right) a \sqrt{b\,x^{2}+a}-a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*x^
2+a)^(1/2)-a^(3/2))/a^(5/2)
```

Maxima [A]
time = 0.48, size = 80, normalized size = 1.13

$$\frac{1}{2}\,c\left(\frac{c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a}+\frac{2\sqrt{\frac{c}{bx^2+a}}}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + s
qrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)
```

Fricas [A]
time = 0.36, size = 138, normalized size = 1.94

$$\left[\frac{c\sqrt{\frac{c}{a}}\,\log\left(-\frac{bcx^2+2\,ac-2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\,\sqrt{\frac{c}{a}}}{x^2}\right)+2\,c\sqrt{\frac{c}{bx^2+a}}}{2\,a},\ \frac{c\sqrt{-\frac{c}{a}}\,\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\,\sqrt{-\frac{c}{a}}}{c}\right)+c\sqrt{\frac{c}{bx^2+a}}}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 +
 a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*s
qrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)
```

Giac [**A**]
time = 3.53, size = 59, normalized size = 0.83

$$c\left(\frac{c\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a}+\frac{c}{\sqrt{bcx^2+ac}\,a}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")
```

```
[Out] c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x^
2 + a*c)*a))*sgn(b*x^2 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{c}{b\,x^2+a}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x, x)
```

3.247 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

```
[Out] -c*(c/(b*x^2+a))^(1/2)/a/x-2*b*c*x*(c/(b*x^2+a))^(1/2)/a^2
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1973, 277, 197}

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)])/a^2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2}\,dx = \left(c\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\int \frac{1}{x^2\,(a+bx^2)^{3/2}}\,dx$$

$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\,\sqrt{a+bx^2}\right)\int \frac{1}{(a+bx^2)^{3/2}}\,dx}{a}$$

$$= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.67

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}\,(a+2bx^2)}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]
[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))
```

Maple [A]
time = 0.04, size = 37, normalized size = 0.77

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(2b\,x^2+a)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a^2x}$	37
default	$-\frac{(b\,x^2+a)\,(2b\,x^2+a)\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}}{a^2x}$	37
trager	$-\frac{(ac+bc)\,(2b\,x^2+a)\,\sqrt{\frac{c}{b\,x^2+a}}}{a^2(a+b)x}$	42
risch	$-\frac{(b\,x^2+a)c\sqrt{\frac{c}{b\,x^2+a}}}{a^2x} - \frac{bcx\sqrt{\frac{c}{b\,x^2+a}}}{a^2}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x^2,x,method=_RETURNVERBOSE)
[Out] -(b*x^2+a)*(2*b*x^2+a)*(c/(b*x^2+a))^(3/2)/a^2/x
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.96

$$-\frac{2\,b^2c^{\frac{3}{2}}x^4 + 3\,abc^{\frac{3}{2}}x^2 + a^2c^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(2*b^2*c^(3/2)*x^4 + 3*a*b*c^(3/2)*x^2 + a^2*c^(3/2))/((b*x^2 + a)^(3/2)*a
^2*x)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.67

$$-\frac{(2\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**2,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**2, x)
```

Giac [A]
time = 3.94, size = 81, normalized size = 1.69

$$-\left(\frac{bcx\mathrm{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}\,a^2}-\frac{2\,\sqrt{bc}\,c\mathrm{sgn}(bx^2+a)}{\left(\left(\sqrt{bc}\,x-\sqrt{bcx^2+ac}\right)^2-ac\right)a}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2
+ a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a))*c
```

Mupad [B]
time = 2.83, size = 54, normalized size = 1.12

$$-\frac{\left(\frac{b\,c}{a}+\frac{2\,b^2\,c\,x^2}{a^2}\right)\,\sqrt{\frac{c}{b\,x^2+a}}\,\left(\frac{a}{b}+x^2\right)}{b\,x^3+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^2,x)
```

```
[Out] -(((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^(1/2)*(a/b + x^2))/(a*x + b
*x^3)
```

3.248 $\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}+\frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{1+\frac{bx^2}{a}}\tanh^{-1}\left(\sqrt{1+\frac{bx^2}{a}}\right)}{2a^2}$$

```
[Out] -3/2*b*c*(c/(b*x^2+a))^(1/2)/a^2-1/2*c*(c/(b*x^2+a))^(1/2)/a/x^2+3/2*b*c*ar
ctanh((1+b*x^2/a)^(1/2))*(c/(b*x^2+a))^(1/2)*(1+b*x^2/a)^(1/2)/a^2
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1973, 272, 44, 53, 65, 214}

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2}+\frac{3bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2a^2}-\frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*c*Sqrt[c/(a + b*x^2)])/(2*a^2) - (c*Sqrt[c/(a + b*x^2)])/(2*a*x^2) +
(3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]]
)/(2*a^2)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3}\,dx = \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\int \frac{1}{x^3\left(a+bx^2\right)^{3/2}}\,dx$$

$$= \frac{1}{2}\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} + \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, x^2\right)}{2a}$$

$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2} - \frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+\cdots}}\right.}{4a^2}$$

$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2} - \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\cdots\right.}{2a^2}$$

$$= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}\left(a+bx^2\right)}{2a^2x^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\cdots\right.}{2a^{5/2}}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.75

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}\left(\sqrt{a}\,(a+3bx^2)-3bx^2\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2a^{5/2}x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[c/(a + b*x^2)]*(Sqrt[a]*(a + 3*b*x^2) - 3*b*x^2*Sqrt[a + b*x^2
]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(a^(5/2)*x^2)
```

Maple [A]
time = 0.05, size = 81, normalized size = 0.78

method	result
default	$\frac{\left(\frac{c}{b\,x^2+a}\right)^{\frac{3}{2}}(b\,x^2+a)\left(3\sqrt{b\,x^2+a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)a b\,x^2-3a^{\frac{3}{2}}b\,x^2-a^{\frac{5}{2}}\right)}{2a^{\frac{7}{2}}x^2}$
risch	$-\frac{(b\,x^2+a)c\sqrt{\frac{c}{b\,x^2+a}}}{2a^2x^2}+\left(\frac{3b\ln\left(\frac{2ac+2\sqrt{ac}\,\sqrt{bc\,x^2+ac}}{x}\right)}{2a^2\sqrt{ac}}-\frac{b\sqrt{bc\left(x-\frac{\sqrt{-ab}}{b}\right)^2+2c\sqrt{-ab}\,\left(x-\right.}}{2a^2c\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(b*x^2+a))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c/(b*x^2+a))^(3/2)*(b*x^2+a)*(3*(b*x^2+a)^(1/2)*ln(2*(a^(1/2)*(b*x^2+a
)^(1/2)+a)/x)*a*b*x^2-3*a^(3/2)*b*x^2-a^(5/2))/a^(7/2)/x^2
```

Maxima [A]
time = 0.49, size = 121, normalized size = 1.16

$$-\frac{1}{4}\,bc\left(\frac{2\,c\sqrt{\frac{c}{bx^2+a}}}{a^2c-\frac{a^3c}{bx^2+a}}+\frac{3\,c\log\left(\frac{a\sqrt{\frac{c}{bx^2+a}}-\sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^2}+\frac{4\sqrt{\frac{c}{bx^2+a}}}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*b*c*(2*c*sqrt(c/(b*x^2 + a))/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*log((a*
sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sqrt
(a*c)*a^2) + 4*sqrt(c/(b*x^2 + a))/a^2)
```

Fricas [A]
time = 0.37, size = 175, normalized size = 1.68

$$\left[\frac{3\,bcx^2\sqrt{\frac{c}{a}}\,\log\left(-\frac{bcx^2+2\,ac+2\,(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\,\sqrt{\frac{c}{a}}}{x^2}\right)-2\,(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{4\,a^2x^2},\,-\frac{3\,bcx^2\sqrt{-\frac{c}{a}}\,\arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\,\sqrt{-\frac{c}{a}}}{c}\right)+(3\,bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*c*x^2*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*sqrt(c/
(b*x^2 + a))*sqrt(c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^
2*x^2), -1/2*(3*b*c*x^2*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/
c) + (3*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a)))/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

Giac [A]
time = 4.81, size = 103, normalized size = 0.99

$$-\frac{1}{2}\,c\left(\frac{3\,bc\arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^2}+\frac{2\,abc^2-3\,(bcx^2+ac)bc}{\left(\sqrt{bcx^2+ac}\,ac-(bcx^2+ac)^{\frac{3}{2}}\right)a^2}\right)\mathrm{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*c*(3*b*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a^2) + (2*
a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((sqrt(b*c*x^2 + a*c)*a*c - (b*c*x^2 + a*c
)^(3/2))*a^2))*sgn(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c/(a + b*x^2))^(3/2)/x^3,x)
```

```
[Out] int((c/(a + b*x^2))^(3/2)/x^3, x)
```

3.249 $\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=138

$$-\frac{2a^3\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^4}+\frac{6a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^4}-\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{5b^4}+\frac{2\left(c\sqrt{a\right.}}{}$$

```
[Out] -2/7*a^3*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+6/11*a^2*(b*x^2+a)^2*(c*(b
*x^2+a)^(1/2))^(3/2)/b^4-2/5*a*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^4+2/
19*(b*x^2+a)^4*(c*(b*x^2+a)^(1/2))^(3/2)/b^4
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1973, 272, 45}

$$-\frac{2a^3(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4}+\frac{6a^2(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4}+\frac{2(a+bx^2)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4}-\frac{2a(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a^3*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^4) + (6*a^2*(c*Sqrt[a +
 b*x^2])^(3/2)*(a + b*x^2)^2)/(11*b^4) - (2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a
+ b*x^2)^3)/(5*b^4) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)^4)/(19*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^7\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^7(a+bx^2)^{3/4}\,dx}{\sqrt[4]{a+bx^2}}$$

$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^3(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a^3(a+bx)^{3/4}}{b^3}+\frac{3a^2(a+bx)^{7/4}}{b^3}-\frac{3a(a+bx)^{11/4}}{b^3}+\frac{(a+bx)^{15/}}{b^3}\right.\right.}{2\sqrt[4]{a+bx^2}}$$

$$=-\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b^4}+\frac{6a^2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{5/2}}{11b^4}-\frac{2ac\sqrt{c\sqrt{}}}{}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 0.46

$$\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\left(-128a^3+224a^2bx^2-308ab^2x^4+385b^3x^6\right)}{7315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*
b^2*x^4 + 385*b^3*x^6))/(7315*b^4)
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.42

method	result	size
gosper	$-\frac{2(b\,x^2+a)(-385b^3x^6+308a\,b^2x^4-224a^2b\,x^2+128a^3)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7315b^4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*
x^2+a)^(1/2))^(3/2)/b^4
```

Maxima [A]
time = 0.27, size = 85, normalized size = 0.62

$$-\frac{2\left(1045\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^3c^6-1995\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}a^2c^4+1463\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}ac^2-385\left(\sqrt{bx^2+a}\,c\right)^{\frac{19}{2}}\right)}{7315\,b^4c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1045*(sqrt(b*x^2 + a)*c)^(7/2)*a^3*c^6 - 1995*(sqrt(b*x^2 + a)*c)^
(11/2)*a^2*c^4 + 1463*(sqrt(b*x^2 + a)*c)^(15/2)*a*c^2 - 385*(sqrt(b*x^2 +
a)*c)^(19/2))/(b^4*c^8)
```

Fricas [A]
time = 0.38, size = 75, normalized size = 0.54

$$\frac{2\,(385\,b^4cx^8+77\,ab^3cx^6-84\,a^2b^2cx^4+96\,a^3bcx^2-128\,a^4c)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{7315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2
- 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4
```

Sympy [A]
time = 12.49, size = 144, normalized size = 1.04

$$\begin{cases} -\frac{256a^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^4}+\frac{192a^3x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7315b^3}-\frac{24a^2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1045b^2}+\frac{2ax^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{95b}+\frac{2x^8\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{19} & \text{for } b\neq 0 \\ \frac{x^8\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-256*a**4*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**4) + 192*a**3*x**
2*(c*sqrt(a + b*x**2))**(3/2)/(7315*b**3) - 24*a**2*x**4*(c*sqrt(a + b*x**2
))**(3/2)/(1045*b**2) + 2*a*x**6*(c*sqrt(a + b*x**2))**(3/2)/(95*b) + 2*x**
8*(c*sqrt(a + b*x**2))**(3/2)/19, Ne(b, 0)), (x**8*(sqrt(a)*c)**(3/2)/8, Tr
ue))
```

Giac [A]
time = 4.80, size = 137, normalized size = 0.99

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{19\left(77\,(bx^2+a)^{\frac{15}{4}}-315\,(bx^2+a)^{\frac{11}{4}}a+495\,(bx^2+a)^{\frac{7}{4}}a^2-385\,(bx^2+a)^{\frac{3}{4}}a^3\right)a}{b^3}+\frac{1155\,(bx^2+a)^{\frac{19}{4}}-5852\,(bx^2+a)^{\frac{15}{4}}a+11970\,(bx^2+a)^{\frac{11}{4}}a^2-12540\,(bx^2+a)^{\frac{7}{4}}a^3+7315\,(bx^2+a)^{\frac{3}{4}}a^4}{b^3}\right)}{21945\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495
*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2 +
a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 1254
0*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3)/b
```

Mupad [B]
time = 2.96, size = 109, normalized size = 0.79

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^8\,\sqrt{b\,x^2+a}}{19}-\frac{256\,a^4\,c\,\sqrt{b\,x^2+a}}{7315\,b^4}+\frac{2\,a\,c\,x^6\,\sqrt{b\,x^2+a}}{95\,b}-\frac{24\,a^2\,c\,x^4\,\sqrt{b\,x^2+a}}{1045\,b^2}+\frac{192\,a^3\,c\,x^2\,\sqrt{b\,x^2+a}}{7315\,b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^8*(a + b*x^2)^(1/2))/19 - (256*a^4*c*(a
 + b*x^2)^(1/2))/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^(1/2))/(95*b) - (24*a^
2*c*x^4*(a + b*x^2)^(1/2))/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^(1/2))/(
7315*b^3))
```

3.250 $\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{2a^2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^3}-\frac{4a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^3}+\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^3}{15b^3}$$

```
[Out] 2/7*a^2*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^3-4/11*a*(b*x^2+a)^2*(c*(b*x^
2+a)^(1/2))^(3/2)/b^3+2/15*(b*x^2+a)^3*(c*(b*x^2+a)^(1/2))^(3/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {1973, 272, 45}

$$\frac{2a^2(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3}+\frac{2(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3}-\frac{4a(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a^2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^3) - (4*a*(c*Sqrt[a + b*
x^2])^(3/2)*(a + b*x^2)^2)/(11*b^3) + (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x
^2)^3)/(15*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^5\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^5(a+bx^2)^{3/4}\,dx}{\sqrt[4]{a+bx^2}}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x^2(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(\frac{a^2(a+bx)^{3/4}}{b^2}-\frac{2a(a+bx)^{7/4}}{b^2}+\frac{(a+bx)^{11/4}}{b^2}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}\\
&= \frac{2a^2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b^3}-\frac{4ac\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{5/2}}{11b^3}+\frac{2c\sqrt{c\sqrt{a+
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.51

$$\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)\,(32a^2-56abx^2+77b^2x^4)}{1155b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4)
)/(1155*b^3)
```

Maple [A]
time = 0.01, size = 47, normalized size = 0.46

method	result	size
gosper	$\frac{2(b\,x^2+a)(77b^2x^4-56ab\,x^2+32a^2)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{1155b^3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*(c*(b*x^2+a)^(1/2))^(3/2)/b
^3
```

Maxima [A]
time = 0.27, size = 64, normalized size = 0.63

$$\frac{2\left(165\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}a^2c^4-210\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}ac^2+77\left(\sqrt{bx^2+a}\,c\right)^{\frac{15}{2}}\right)}{1155\,b^3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(165*(sqrt(b*x^2 + a)*c)^(7/2)*a^2*c^4 - 210*(sqrt(b*x^2 + a)*c)^(11
/2)*a*c^2 + 77*(sqrt(b*x^2 + a)*c)^(15/2))/(b^3*c^6)
```

Fricas [A]
time = 0.38, size = 63, normalized size = 0.62

$$\frac{2\,(77\,b^3cx^6+21\,ab^2cx^4-24\,a^2bcx^2+32\,a^3c)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{1155\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x
^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3
```

Sympy [A]
time = 6.99, size = 116, normalized size = 1.14

$$\begin{cases} \frac{64a^3\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{1155b^3}-\frac{16a^2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{385b^2}+\frac{2ax^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{55b}+\frac{2x^6\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{15} & \text{for } b\neq 0 \\ \frac{x^6\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((64*a**3*(c*sqrt(a + b*x**2))**(3/2)/(1155*b**3) - 16*a**2*x**2*(
c*sqrt(a + b*x**2))**(3/2)/(385*b**2) + 2*a*x**4*(c*sqrt(a + b*x**2))**(3/2
)/(55*b) + 2*x**6*(c*sqrt(a + b*x**2))**(3/2)/15, Ne(b, 0)), (x**6*(sqrt(a)
*c)**(3/2)/6, True))
```

Giac [A]
time = 5.29, size = 109, normalized size = 1.07

$$\frac{2\,c^{\frac{3}{2}}\left(\frac{5\left(21\,(bx^2+a)^{\frac{11}{4}}-66\,(bx^2+a)^{\frac{7}{4}}a+77\,(bx^2+a)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77\,(bx^2+a)^{\frac{15}{4}}-315\,(bx^2+a)^{\frac{11}{4}}a+495\,(bx^2+a)^{\frac{7}{4}}a^2-385\,(bx^2+a)^{\frac{3}{4}}a^3}{b^2}\right)}{1155\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x
^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*
a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b
```

Mupad [B]
time = 2.90, size = 88, normalized size = 0.86

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^6\,\sqrt{b\,x^2+a}}{15}+\frac{64\,a^3\,c\,\sqrt{b\,x^2+a}}{1155\,b^3}+\frac{2\,a\,c\,x^4\,\sqrt{b\,x^2+a}}{55\,b}-\frac{16\,a^2\,c\,x^2\,\sqrt{b\,x^2+a}}{385\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^6*(a + b*x^2)^(1/2))/15 + (64*a^3*c*(a
+ b*x^2)^(1/2))/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^(1/2))/(55*b) - (16*a^2
*c*x^2*(a + b*x^2)^(1/2))/(385*b^2))
```

3.251 $\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=66

$$-\frac{2a\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)}{7b^2}+\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)^2}{11b^2}$$

```
[Out] -2/7*a*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2+2/11*(b*x^2+a)^2*(c*(b*x^2+a
)^(1/2))^(3/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1973, 272, 45}

$$\frac{2(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^2}-\frac{2a(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (-2*a*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b^2) + (2*(c*Sqrt[a + b*x^2
])^(3/2)*(a + b*x^2)^2)/(11*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^3\left(c\sqrt{a+bx^2}\right)^{3/2}dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^3(a+bx^2)^{3/4}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int x(a+bx)^{3/4}\,dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int\left(-\frac{a(a+bx)^{3/4}}{b}+\frac{(a+bx)^{7/4}}{b}\right)dx,x,x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= -\frac{2ac\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b^2}+\frac{2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{5/2}}{11b^2}$$

Mathematica [A]
time = 0.03, size = 41, normalized size = 0.62

$$\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}(a+bx^2)(-4a+7bx^2)}{77b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)
```

Maple [A]
time = 0.01, size = 36, normalized size = 0.55

method	result	size
gosper	$-\frac{2(b\,x^2+a)(-7b\,x^2+4a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{77b^2}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*(c*(b*x^2+a)^(1/2))^(3/2)/b^2
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.65

$$-\frac{2\left(11\left(\sqrt{bx^2+a}\,c\right)^{\frac{7}{2}}ac^2-7\left(\sqrt{bx^2+a}\,c\right)^{\frac{11}{2}}\right)}{77\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(
b^2*c^4)
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.77

$$\frac{2\,(7\,b^2cx^4+3\,abcx^2-4\,a^2c)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{77\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2
+ a)*c)/b^2
```

Sympy [A]
time = 3.50, size = 87, normalized size = 1.32

$$\begin{cases} -\frac{8a^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b^2}+\frac{6ax^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{77b}+\frac{2x^4\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{11} & \text{for } b\neq 0\\ \frac{x^4\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((-8*a**2*(c*sqrt(a + b*x**2))**(3/2)/(77*b**2) + 6*a*x**2*(c*sqrt
(a + b*x**2))**(3/2)/(77*b) + 2*x**4*(c*sqrt(a + b*x**2))**(3/2)/11, Ne(b,
0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))
```

Giac [A]
time = 4.55, size = 81, normalized size = 1.23

$$\frac{2\left(\frac{11\left(3\,(bx^2+a)^{\frac{7}{4}}-7\,(bx^2+a)^{\frac{3}{4}}a\right)a}{b}+\frac{21\,(bx^2+a)^{\frac{11}{4}}-66\,(bx^2+a)^{\frac{7}{4}}a+77\,(bx^2+a)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 +
a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)/b
```

Mupad [B]
time = 2.89, size = 67, normalized size = 1.02

$$\sqrt{c\,\sqrt{b\,x^2+a}}\,\left(\frac{2\,c\,x^4\,\sqrt{b\,x^2+a}}{11}-\frac{8\,a^2\,c\,\sqrt{b\,x^2+a}}{77\,b^2}+\frac{6\,a\,c\,x^2\,\sqrt{b\,x^2+a}}{77\,b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a +
 b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))
```

3.252 $\int x\left(c\sqrt{a+bx^2}\right)^{3/2}dx$

Optimal. Leaf size=36

$$\frac{2c\sqrt{c\sqrt{a+bx^2}}\,(a+bx^2)^{3/2}}{7b}$$

```
[Out] 2/7*c*(b*x^2+a)^(3/2)*(c*(b*x^2+a)^(1/2))^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1605, 15, 30}

$$\frac{2c(a+bx^2)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$
\begin{aligned}
\int x\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx &= \frac{\mathrm{Subst}\left(\int \left(c\sqrt{x}\right)^{3/2}\,dx, x, a+bx^2\right)}{2b} \\
&= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\mathrm{Subst}\left(\int x^{3/4}\,dx, x, a+bx^2\right)}{2b\sqrt[4]{a+bx^2}} \\
&= \frac{2c\sqrt{c\sqrt{a+bx^2}}\,\left(a+bx^2\right)^{3/2}}{7b}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.86

$$\frac{2\left(c\sqrt{a+bx^2}\right)^{3/2}\left(a+bx^2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)
```

Maple [A]
time = 0.04, size = 26, normalized size = 0.72

method	result	size
gosper	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
derivativedivides	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26
default	$\frac{2(b\,x^2+a)\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{7b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(b*x^2+a)^(1/2))^(3/2),x,method=_RETURNVERBOSE)
[Out] 2/7*(b*x^2+a)*(c*(b*x^2+a)^(1/2))^(3/2)/b
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.69

$$\frac{2\,\left(bx^2+a\right)\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b
```

Fricas [A]
time = 0.38, size = 37, normalized size = 1.03

$$\frac{2\left(bcx^2+ac\right)\sqrt{bx^2+a}\,\sqrt{\sqrt{bx^2+a}\,c}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b
```

Sympy [A]
time = 1.70, size = 58, normalized size = 1.61

$$\begin{cases} \frac{2a\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7b} + \frac{2x^2\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{7} & \text{for } b \neq 0 \\ \frac{x^2\left(\sqrt{a}\,c\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Piecewise((2*a*(c*sqrt(a + b*x**2))**(3/2)/(7*b) + 2*x**2*(c*sqrt(a + b*x**
2))**(3/2)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))
```

Giac [A]
time = 4.10, size = 17, normalized size = 0.47

$$\frac{2\left(bx^2+a\right)^{\frac{7}{4}}c^{\frac{3}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b
```

Mupad [B]
time = 2.77, size = 28, normalized size = 0.78

$$\frac{2\,c\,{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{c\,\sqrt{b\,x^2+a}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)
[Out] (2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)
```

3.253 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=117

$$\frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/3*(c*(b*x^2+a)^(1/2))^(3/2)+arctan((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))
^(3/2)/(1+b*x^2/a)^(3/4)-arctanh((1+b*x^2/a)^(1/4))*(c*(b*x^2+a)^(1/2))^(3/
2)/(1+b*x^2/a)^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1973, 272, 52, 65, 304, 209, 212}

$$\frac{\text{ArcTan}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{2}{3}\left(c\sqrt{a+bx^2}\right)^{3/2}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] (2*(c*Sqrt[a + b*x^2])^(3/2))/3 + ((c*Sqrt[a + b*x^2])^(3/2)*ArcTan[(1 + (b
*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4) - ((c*Sqrt[a + b*x^2])^(3/2)*ArcTanh
[(1 + (b*x^2)/a)^(1/4)])/(1 + (b*x^2)/a)^(3/4)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{(a+bx^2)^{3/4}}{x}\,dx}{\sqrt[4]{a+bx^2}}$$
$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^{3/4}}{x}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^2\right.}{2\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(2ac\sqrt{c\sqrt{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+\right.}{b\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} - \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+\right.}{\sqrt[4]{a+bx^2}}$$
$$= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} - \frac{a^{3/4}c}{}$$

Mathematica [A]
time = 0.07, size = 96, normalized size = 0.82

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2(a+bx^2)^{3/4}+3a^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)-3a^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\right)}{3\left(a+bx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x
^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a +
 b*x^2)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x,x)
```

Maxima [A]
time = 0.51, size = 118, normalized size = 1.01

$$\frac{3\,ac^4\left(\frac{2\,\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}}{\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}-\left(ac^2\right)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}\,c}+\left(ac^2\right)^{\frac{1}{4}}}\right)}{\left(ac^2\right)^{\frac{1}{4}}}\right)+4\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}c^2}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4)
 + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) +
 (a*c^2)^(1/4)))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)
```

Giac [A]
time = 3.19, size = 190, normalized size = 1.62

$$-\frac{1}{12}\left(6\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}+2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)+6\sqrt{2}\left(-a\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-a\right)^{\frac{1}{4}}-2\left(bx^2+a\right)^{\frac{1}{4}}\right)}{2\left(-a\right)^{\frac{1}{4}}}\right)-3\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(\sqrt{2}\left(bx^2+a\right)^{\frac{1}{4}}\left(-a\right)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)+3\sqrt{2}\left(-a\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\left(bx^2+a\right)^{\frac{1}{4}}\left(-a\right)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)-8\left(bx^2+a\right)^{\frac{3}{4}}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x
^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt
(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*lo
g(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sq
rt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a
) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4))*c^(3/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)
```

3.254 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}+\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tan^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}-\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{1+\frac{bx^2}{a}}\right)}{4a\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -1/2*(c*(b*x^2+a)^(1/2))^(3/2)/x^2+3/4*b*arctan((1+b*x^2/a)^(1/4))*(c*(b*x^
2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)-3/4*b*arctanh((1+b*x^2/a)^(1/4))*(c*(
b*x^2+a)^(1/2))^(3/2)/a/(1+b*x^2/a)^(3/4)
```

Rubi [A]
time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1973, 272, 43, 65, 304, 209, 212}

$$\frac{3b\text{ArcTan}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)\left(c\sqrt{a+bx^2}\right)^{3/2}}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{2x^2}-\frac{3b\left(c\sqrt{a+bx^2}\right)^{3/2}\tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -1/2*(c*Sqrt[a + b*x^2])^(3/2)/x^2 + (3*b*(c*Sqrt[a + b*x^2])^(3/2)*ArcTan[
(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4)) - (3*b*(c*Sqrt[a + b*x^
2])^(3/2)*ArcTanh[(1 + (b*x^2)/a)^(1/4)])/(4*a*(1 + (b*x^2)/a)^(3/4))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{(a+bx^2)^{3/4}}{x^3}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2}\,dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}}\,dx, x, x^2\right.}{8\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{\left(3c\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}}\,dx, x, \sqrt[4]{a+b\right.}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} - \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}-x^2}\,dx, x, \sqrt[4]{a+\right.}{4\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2x^2} + \frac{3bc\sqrt{c\sqrt{a+bx^2}}\,\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\,\sqrt[4]{a+bx^2}} - \frac{3bc\sqrt{c\sqrt{}}}{}$$

__Mathematica__ [A]
time = 0.11, size = 107, normalized size = 0.80

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\left(2\sqrt[4]{a}\,(a+bx^2)^{3/4} - 3bx^2\tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) + 3bx^2\tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\right)}{4\sqrt[4]{a}\,x^2\,(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]
```

```
[Out] -1/4*((c*Sqrt[a + b*x^2])^(3/2)*(2*a^(1/4)*(a + b*x^2)^(3/4) - 3*b*x^2*ArcT
an[(a + b*x^2)^(1/4)/a^(1/4)] + 3*b*x^2*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])
)/(a^(1/4)*x^2*(a + b*x^2)^(3/4))
```

__Maple__ [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x)
```

Maxima [A]

time = 0.49, size = 138, normalized size = 1.04

$$\frac{\left(3\,c^4\left(\frac{2\arctan\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}}{(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}}+\frac{\log\left(\frac{\sqrt{\sqrt{bx^2+a}\,c}-(ac^2)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+a}\,c}+(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}}\right)-\frac{4\left(\sqrt{bx^2+a}\,c\right)^{\frac{3}{2}}c^4}{(bx^2+a)c^2-ac^2}\right)b}{8\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) +
 log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (
a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2 + a
)*c^2 - a*c^2))*b/c^2
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)
```

Giac [A]

time = 3.13, size = 212, normalized size = 1.59

$$\frac{\left(\frac{6\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}+2\,(bx^2+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{6\sqrt{2}\,b^2\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\,(-a)^{\frac{1}{4}}-2\,(bx^2+a)^{\frac{1}{4}}\right)}{2\,(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}}+\frac{3\sqrt{2}\,(-a)^{\frac{3}{4}}b^2\log\left(\sqrt{2}\,(bx^2+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)}{a}+\frac{3\sqrt{2}\,b^2\log\left(-\sqrt{2}\,(bx^2+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}\right)}{(-a)^{\frac{1}{4}}}-\frac{8\,(bx^2+a)^{\frac{3}{4}}b}{x^2}\right)c^{\frac{3}{2}}}{16\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^
(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*
(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(
3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(
-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x
^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)
```

3.255 $\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=152

$$\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)}+\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\right.\right.}{15b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/15*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/b+2/9*x^3*(c*(b*x^2+a)^(1/2))^(3/2)-4/15
*a^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/b/(b*x^2+a)+4/15*a^(3/2)*(cos(1/2*arctan(x
*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(si
n(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/b^(3/2)
/(1+b*x^2/a)^(3/4)
```

Rubi [A]

time = 0.04, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1973, 285, 327, 233, 202}

$$\frac{4a^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{4a^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b\left(a+bx^2\right)}+\frac{2ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b}+\frac{2}{9}x^3\left(c\sqrt{a+bx^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b) + (2*x^3*(c*Sqrt[a + b*x^2])^(3/2)
)/9 - (4*a^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(15*b*(a + b*x^2)) + (4*a^(3/2)*(
c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(15*b
^(3/2)*(1 + (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int x^2\left(c\sqrt{a+bx^2}\right)^{3/2}\,dx=\frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int x^2(a+bx^2)^{3/4}\,dx}{\sqrt[4]{a+bx^2}}$$

$$=\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}+\frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right)\int\frac{x^2}{\sqrt[4]{a+bx^2}}\,dx}{3\sqrt[4]{a+bx^2}}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a}\right.}}{1}$$

$$=\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}-\frac{\left(2a^2c\sqrt{c\sqrt{a}\right.}}{}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}$$

$$=-\frac{4a^2cx\sqrt{c\sqrt{a+bx^2}}}{15b\sqrt{a+bx^2}}+\frac{2acx\sqrt{c\sqrt{a+bx^2}}\sqrt{a+bx^2}}{15b}+\frac{2}{9}cx^3\sqrt{c\sqrt{a+bx^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.58, size = 68, normalized size = 0.45

$$\frac{2x\left(c\sqrt{a+bx^2}\right)^{3/2}\left(a+bx^2-\frac{a\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*Sqrt[a + b*x^2])^(3/2),x]
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2,
 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
[Out] int(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c\sqrt{a + bx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Integral(x**2*(c*sqrt(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c\,\sqrt{b\,x^2 + a}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(x^2*(c*(a + b*x^2)^(1/2))^(3/2), x)
```

3.256 $\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx$

Optimal. Leaf size=119

$$\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] 2/5*x*(c*(b*x^2+a)^(1/2))^(3/2)+6/5*a*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)
-6/5*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/
a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*(c*
(b*x^2+a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/b^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1973, 201, 233, 202}

$$-\frac{6\sqrt{a}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{2}{5}x\left(c\sqrt{a+bx^2}\right)^{3/2}+\frac{6ax\left(c\sqrt{a+bx^2}\right)^{3/2}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (2*x*(c*Sqrt[a + b*x^2])^(3/2))/5 + (6*a*x*(c*Sqrt[a + b*x^2])^(3/2))/(5*(a
 + b*x^2)) - (6*Sqrt[a]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]
*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(1 + (b*x^2)/a)^(3/4))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \left(c\sqrt{a+bx^2}\right)^{3/2} dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int (a+bx^2)^{3/4}\, dx}{\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\, dx}{5\sqrt[4]{a+bx^2}}$$

$$= \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} + \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\, dx}{5\sqrt{a+bx^2}}$$

$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} - \frac{\left(3ac\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\cdots}\right.}{5\sqrt{a+b\cdots}}$$

$$= \frac{6acx\sqrt{c\sqrt{a+bx^2}}}{5\sqrt{a+bx^2}} + \frac{2}{5}cx\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2} - \frac{6a^{3/2}c\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\cdots}}{5\sqrt{b}\,\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.17, size = 52, normalized size = 0.44

$$\frac{x\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2),x]
```

```
[Out] (x*(c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)
])/(1 + (b*x^2)/a)^(3/4)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sqrt{a + bx^{2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2),x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c\,\sqrt{b\,x^2+a}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2),x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2), x)
```

3.257 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2} dx$

Optimal. Leaf size=115

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -(c*(b*x^2+a)^(1/2))^(3/2)/x+3*b*x*(c*(b*x^2+a)^(1/2))^(3/2)/(b*x^2+a)-3*(c
os(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)
))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)*(c*(b*x^2+
a)^(1/2))^(3/2)/(1+b*x^2/a)^(3/4)/a^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1973, 283, 233, 202}

$$-\frac{3\sqrt{b}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x}+\frac{3bx\left(c\sqrt{a+bx^2}\right)^{3/2}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] -((c*Sqrt[a + b*x^2])^(3/2)/x) + (3*b*x*(c*Sqrt[a + b*x^2])^(3/2))/(a + b*x
^2) - (3*Sqrt[b]*(c*Sqrt[a + b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(Sqrt[a]*(1 + (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
```

```
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} + \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} - \frac{\left(3bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1+\frac{bx^2}{a}}\right.}{2\sqrt{a+bx^2}}$$

$$= \frac{3bcx\sqrt{c\sqrt{a+bx^2}}}{\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{x} - \frac{3\sqrt{a}\,\sqrt{b}\,c\sqrt{c\sqrt{a+bx^2}}\,\sqrt[4]{1}}{\sqrt{a}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.72, size = 55, normalized size = 0.48

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{2};-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^2,x]
```

```
[Out] -(((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a
)])/(x*(1 + (b*x^2)/a)^(3/4)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{b\,x^{2}+a}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)*c/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^{2}}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**2,x)
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2,x)
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^2, x)
```

3.258 $\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=154

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3}-\frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax}+\frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)}-\frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|\right.}{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

```
[Out] -1/3*(c*(b*x^2+a)^(1/2))^(3/2)/x^3-1/2*b*(c*(b*x^2+a)^(1/2))^(3/2)/a/x+1/2*
b^2*x*(c*(b*x^2+a)^(1/2))^(3/2)/a/(b*x^2+a)-1/2*b^(3/2)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*(c*(b*x^2+a)^(1/2))^(3/2)/a^(3/2)/(
1+b*x^2/a)^(3/4)
```

Rubi [A]
time = 0.04, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1973, 283, 331, 233, 202}

$$-\frac{b^{3/2}\left(c\sqrt{a+bx^2}\right)^{3/2}E\left(\frac{1}{2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}}+\frac{b^2x\left(c\sqrt{a+bx^2}\right)^{3/2}}{2a\left(a+bx^2\right)}-\frac{b\left(c\sqrt{a+bx^2}\right)^{3/2}}{2ax}-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^2])^(3/2)/x^3 - (b*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*x)
+ (b^2*x*(c*Sqrt[a + b*x^2])^(3/2))/(2*a*(a + b*x^2)) - (b^(3/2)*(c*Sqrt[a
+ b*x^2])^(3/2)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*a^(3/2)*(1
+ (b*x^2)/a)^(3/4))
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx}{\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3} + \frac{\left(bc\sqrt{c\sqrt{a+bx^2}}\right)\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx}{2\sqrt[4]{a+bx^2}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax} + \frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\right.}{4a\sqrt[4]{a}}$$

$$= -\frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax} + \frac{\left(b^2c\sqrt{c\sqrt{a+bx^2}}\right.}{}$$

$$= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}$$

$$= \frac{b^2cx\sqrt{c\sqrt{a+bx^2}}}{2a\sqrt{a+bx^2}} - \frac{c\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{3x^3} - \frac{bc\sqrt{c\sqrt{a+bx^2}}\,\sqrt{a+bx^2}}{2ax}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 57, normalized size = 0.37

$$-\frac{\left(c\sqrt{a+bx^2}\right)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^4,x]
```

```
[Out] -1/3*((c*Sqrt[a + b*x^2])^(3/2)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^
2)/a)])/(x^3*(1 + (b*x^2)/a)^(3/4))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{b\,x^2+a}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Shoul
dn't happen
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**4,x)
```

```
[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((sqrt(b*x^2 + a)*c)^(3/2)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,\sqrt{b\,x^2+a}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4,x)
```

```
[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^4, x)
```

3.259 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -1/8*(a-b)^2*arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))-1/4*(a+b-2*x)*(
-a*b+(a+b)*x-x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1976, 626, 635, 210}

$$-\frac{1}{8}(a-b)^2\text{ArcTan}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -1/4*((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2]) - ((a - b)^2*ArcTan[(a
+ b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(b-x)(-a+x)}\,dx &= \int \sqrt{-ab+(a+b)x-x^2}\,dx\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{8}(a-b)^2\int \frac{1}{\sqrt{-ab+(a+b)x-x^2}\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}+\frac{1}{4}(a-b)^2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,\right.\\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)}}\right.\end{aligned}$$

Mathematica [A]
time = 0.11, size = 76, normalized size = 1.07

$$\frac{1}{4}\sqrt{(a-x)(-b+x)}\left(-a-b+2x-\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{-a+x}}\right)}{\sqrt{b-x}\sqrt{-a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
[Out] (Sqrt[(a - x)*(-b + x)]*(-a - b + 2*x - ((a - b)^2*ArcTan[Sqrt[b - x]/Sqrt[
-a + x]])/(Sqrt[b - x]*Sqrt[-a + x])))/4
```

Maple [A]
time = 0.26, size = 68, normalized size = 0.96

method	result
default	$-\frac{(a+b-2x)\sqrt{-ab+(a+b)x-x^2}}{4}-\frac{\left(4ab-(a+b)^2\right)\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)}{8}$
risch	$\frac{(b-x)(a-x)(a+b-2x)}{4\sqrt{-(-b+x)(-a+x)}}-\frac{\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right)ab}{4}+\frac{\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)x-x^2}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/8*(4*a*b-(a+b)^2)*arctan((x-1/2*a
-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.37, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)}{2\,(ab-(a+b)x+x^{2})}\right)-\frac{1}{4}\,\sqrt{-ab+(a+b)x-x^{2}}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{(-a+x)\,(b-x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

Giac [A]
time = 5.45, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}(-a+b)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-\left(a-x\right)\,\left(b-x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int((-(a - x)*(b - x))^(1/2), x)
```

3.260 $\int \sqrt{(1-x^2)(3+x^2)}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(x,1/3*I*3^(1/2))*3^(1
/2)+1/3*x*(-x^4-2*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1976, 1105, 1194, 538, 435, 430}

$$\frac{4F\left(\text{ArcSin}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\text{ArcSin}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{-x^4-2x^2+3}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt[3] + (4*E
llipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{(1-x^2)\,(3+x^2)}\,dx &= \int \sqrt{3-2x^2-x^4}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{1}{3}\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}+\frac{2}{3}\int \frac{6-2x^2}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2}{3}\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx+8\int \frac{1}{\sqrt{2-2x^2}\,\sqrt{6+2x^2}}\,dx\\ &= \frac{1}{3}x\sqrt{3-2x^2-x^4}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.07, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(x\sqrt{3-2x^2-x^4}-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]], -3] - (4*I
)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(44) = 88$.
time = 0.04, size = 114, normalized size = 2.38

method	result
default	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\text{EllipticF}\left(\cdots\right.\right.}{3\sqrt{-x^4-2x\cdots}}$
elliptic	$\frac{x\sqrt{-x^4-2x^2+3}}{3}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\text{EllipticF}\left(\cdots\right.\right.}{3\sqrt{-x^4-2x\cdots}}$
risch	$-\frac{x(x^2-1)(x^2+3)}{3\sqrt{-(x^2-1)(x^2+3)}}+\frac{2\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\left(\text{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)-\text{EllipticE}\left(x,\frac{i\sqrt{3}}{3}\right)\right)}{3\sqrt{-x^4-2x^2+3}}+\frac{2\sqrt{-x^2\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3
)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4
-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-EllipticE(x,1/3*I*3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [A]
time = 0.10, size = 24, normalized size = 0.50

$$\frac{\sqrt{-x^4-2\,x^2+3}\,(x^2+2)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-x^4 - 2*x^2 + 3)*(x^2 + 2)/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(1-x^2)(x^2+3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x**2+1)*(x**2+3))**(1/2),x)
[Out] Integral(sqrt((1 - x**2)*(x**2 + 3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\left(x^2-1\right)\,\left(x^2+3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2 - 1)*(x^2 + 3))^(1/2),x)
[Out] int((-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.261 $\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

```
[Out] -arctan(1/2*(a+b-2*x)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1976, 635, 210}

$$-\text{ArcTan}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(b-x)(-a+x)}}\,dx = \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)$$
$$= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 1.72

$$-\frac{2\sqrt{b-x}\,\sqrt{-a+x}\,\tan^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{-a+x}}\right)}{\sqrt{(b-x)(-a+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
[Out] (-2*Sqrt[b - x]*Sqrt[-a + x]*ArcTan[Sqrt[b - x]/Sqrt[-a + x]])/Sqrt[(b - x)
*(-a + x)]
```

Maple [A]
time = 0.24, size = 28, normalized size = 0.88

method	result	size
default	$\arctan\left(\frac{x-\frac{a}{2}-\frac{b}{2}}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan((x-1/2*a-1/2*b)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b-a>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.40, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}\,(a+b-2\,x)}{2\,(ab-(a+b)x+x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(-a+x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.
time = 4.79, size = 61, normalized size = 1.91

$$\frac{1}{8}\left(a^2-2\,ab+b^2\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}(-a+b)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}\,(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{-\left(a-x\right)\,\left(b-x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(a - x)*(b - x))^(1/2),x)
```

```
[Out] int(1/(-(a - x)*(b - x))^(1/2), x)
```

3.262 $\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1976, 1109, 430}

$$\frac{F\left(\text{ArcSin}(x)\,|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(1-x^2)(3+x^2)}}\,dx = \int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$$
$$= 2\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx$$
$$= \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.02, size = 18, normalized size = 1.50

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x^2)*(3 + x^2)],x]
```

```
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.
time = 0.02, size = 43, normalized size = 3.58

method	result	size
default	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43
elliptic	$\frac{\sqrt{-x^2+1}\,\sqrt{3x^2+9}\,\mathrm{EllipticF}\left(x,\frac{i\sqrt{3}}{3}\right)}{3\sqrt{-x^4-2x^2+3}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x^2+1)*(x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Fricas [A]
time = 0.09, size = 8, normalized size = 0.67

$$\frac{1}{3}\sqrt{3}\,\text{ellipticF}\left(x,-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*ellipticF(x, -1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{(1-x^2)\,(x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x**2+1)*(x**2+3))**(1/2),x)
[Out] Integral(1/sqrt((1 - x**2)*(x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-x^2+1)*(x^2+3))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(x^2 + 3)*(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int\frac{1}{\sqrt{-\left(x^2-1\right)\,\left(x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2),x)
[Out] int(1/(-(x^2 - 1)*(x^2 + 3))^(1/2), x)
```

3.263 $\int x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx$

Optimal. Leaf size=244

$$\frac{(11b^2c^2-2abcd-a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^2d^3}-\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{8bd^3}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx}{6bd^2e}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(a^2*
d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1
/2)/e^(1/2))*e^(1/2)/b^(5/2)/d^(7/2)+1/16*(-a^2*d^2-2*a*b*c*d+11*b^2*c^2)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^3-1/8*(a*d+3*b*c)*(d*x^2+c)^2*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^3
```

Rubi [A]
time = 0.30, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 474, 466, 393, 214}

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3}-\frac{\sqrt{e}\,(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}-\frac{(c+dx^2)^2(ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^3}+\frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6bd^2e}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c +
d*x^2))/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)
/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTa
nh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5
/2)*d^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
+ p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(-ae+cx^2)^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)\text{Subst}\left(\int \frac{x^2(-3(2a^2d^2e^2-(bce-ade)^2)+6bc^2dex^2)}{(be-dx^2)^3}\,dx, x\right.}{6bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)^3}{6bd^2e} + \frac{(bc-ad)\text{Subst}}{}$$

$$= \frac{(11b^2c^2-2abcd-a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+d}{8bd^3}$$

$$= \frac{(11b^2c^2-2abcd-a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+d}{8bd^3}$$

Mathematica [A]
time = 1.64, size = 198, normalized size = 0.81

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-b\sqrt{d}\,(c+dx^2)\,(3a^2d^2-2abd(-2c+dx^2)+b^2(-15c^2+10cdx^2-8d^2x^4)) - \frac{3(bc-ad)^{3/2}(5b^2c^2+2abcd+a^2d^2)\sqrt{\frac{b(c+dx^2)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{\sqrt{a+bx^2}}\right)}{48b^3d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*
a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c -
 a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a
*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/
(48*b^3*d^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(216) = 432.
time = 0.10, size = 527, normalized size = 2.16

method	result

risch	$-\frac{(-8b^2d^2x^4-2ab\,d^2x^2+10b^2cd\,x^2+3a^2d^2+4abcd-15b^2c^2)(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{48b^2d^3}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(}\right.}{32b^2\sqrt{de}}\right.}{}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(-12\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^2ab\,d^2\sqrt{bd}-36\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-12*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x
^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*b*d*x^2+2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^
2+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d
+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
a^2*d^2*(b*d)^(1/2)-24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1
/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/((d*x^2+c)*
(b*x^2+a))^(1/2)/d^3/b^2/(b*d)^(1/2)
```

Maxima [A]

time = 0.51, size = 391, normalized size = 1.60

$$\frac{1}{96}\left(\frac{2\left(3\,(11\,b^3c^3d^2-13\,ab^2c^2d^3+a^2bcd^4+a^3d^5)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}-8\,(5\,b^4c^3d-3\,ab^3c^2d^2-3\,a^2b^2cd^3+a^3bd^4)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}+3\,(5\,b^5c^3-3\,ab^4c^2d-a^2b^3cd^2-a^3b^2d^3)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^5d^3-\frac{3\,(bx^2+a)b^4d^4}{dx^2+c}+\frac{3\,(bx^2+a)^2b^3d^5}{(dx^2+c)^2}-\frac{(bx^2+a)^3b^2d^6}{(dx^2+c)^3}}+\frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^2d^3}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*(11*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 + a^2*b*c*d^4 + a^3*d^5)*((b*
x^2 + a)/(d*x^2 + c))^(5/2) - 8*(5*b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*
c*d^3 + a^3*b*d^4)*((b*x^2 + a)/(d*x^2 + c))^(3/2) + 3*(5*b^5*c^3 - 3*a*b^4
*c^2*d - a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c)))/(b^5*d
^3 - 3*(b*x^2 + a)*b^4*d^4/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^3*d^5/(d*x^2 + c
)^2 - (b*x^2 + a)^3*b^2*d^6/(d*x^2 + c)^3) + 3*(5*b^3*c^3 - 3*a*b^2*c^2*d -
 a^2*b*c*d^2 - a^3*d^3)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(
d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b^2*d^3))*e^(1/2)
```

Fricas [A]
time = 0.37, size = 528, normalized size = 2.16

$$\left[-\frac{3\left(5b^{3}c^{3}-3ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{bd}\,e^{\frac{1}{2}}\log\left(8b^{2}d^{2}x^{4}+b^{2}c^{2}+6abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{2}+4\left(2bd^{2}x^{4}+bc^{2}+acd+\left(3bcd+ad^{2}\right)x^{2}\right)\sqrt{bd}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)-4\left(8b^{3}d^{4}x^{6}+15b^{3}c^{3}d-4ab^{2}c^{2}d^{2}-3a^{2}bcd^{3}-2\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}+\left(5b^{3}c^{2}d^{2}-2ab^{2}cd^{3}-3a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\,e^{\frac{1}{2}}}{192\,b^{3}d^{4}},\ \frac{3\left(5b^{3}c^{3}-3ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^{2}+bc+ad\right)\sqrt{-bd}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{2\left(b^{2}dx^{2}+abd\right)}\right)e^{\frac{1}{2}}+2\left(8b^{3}d^{4}x^{6}+15b^{3}c^{3}d-4ab^{2}c^{2}d^{2}-3a^{2}bcd^{3}-2\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}+\left(5b^{3}c^{2}d^{2}-2ab^{2}cd^{3}-3a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\,e^{\frac{1}{2}}}{96\,b^{3}d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*e^
(1/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*
d^2)*x^2 + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d
)*sqrt((b*x^2 + a)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 + 15*b^3*c^3*d - 4*a*b^
2*c^2*d^2 - 3*a^2*b*c*d^3 - 2*(b^3*c*d^3 - a*b^2*d^4)*x^4 + (5*b^3*c^2*d^2
- 2*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/
(b^3*d^4), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x^
2 + c))/(b^2*d*x^2 + a*b*d))*e^(1/2) + 2*(8*b^3*d^4*x^6 + 15*b^3*c^3*d - 4*
a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 - 2*(b^3*c*d^3 - a*b^2*d^4)*x^4 + (5*b^3*c^2*
d^2 - 2*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/
2))/(b^3*d^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.27, size = 211, normalized size = 0.86

$$\frac{1}{96}\left(2\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\left(2x^{2}\left(\frac{4x^{2}}{d}-\frac{5b^{2}cd-abd^{2}}{b^{2}d^{3}}\right)+\frac{15b^{2}c^{2}-4abcd-3a^{2}d^{2}}{b^{2}d^{3}}\right)+\frac{3\left(5b^{3}c^{3}-3ab^{2}c^{2}d-a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(\left|-bc-ad-2\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)\sqrt{bd}\right|\right)}{\sqrt{bd}\,b^{2}d^{3}}\right)e^{\frac{1}{2}}\mathrm{sgn}\left(dx^{2}+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*x^2*(4*x^2/d - (5*b^2*c*
d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b^2*d^3)) +
 3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log(abs(-b*c - a*d -
 2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*sqrt(b*d)))/(s
qrt(b*d)*b^2*d^3))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.264 $\int x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx$

Optimal. Leaf size=161

$$-\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8bd^2}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4d^2}+\frac{(bc-ad)(3bc+ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(\cdots}{\cdots}}}{\sqrt{b}\cdots}\right.}{8b^{3/2}d^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
(1/2)/e^(1/2))*e^(1/2)/b^(3/2)/d^(5/2)-1/8*(-a*d+5*b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b/d^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^
2
```

Rubi [A]
time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 466, 393, 214}

$$\frac{\sqrt{e}\,(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^2}-\frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/8*((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(b*d^2)
+ (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*
(3*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
Sqrt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=((bc-ad)e)\text{Subst}\left(\int\frac{x^2(-ae+cx^2)}{(be-dx^2)^3}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$=\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2}-\frac{((bc-ad)e)\text{Subst}\left(\int\frac{(bc-ad)e+4cdx^2}{(be-dx^2)^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right.}{4d^2}$$

$$=-\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2}+\frac{((bc-ad)(3b}{}$$

$$=-\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8bd^2}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4d^2}+\frac{(bc-ad)(3bc}{}$$

Mathematica [A]
time = 0.89, size = 161, normalized size = 1.00

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\,(c+dx^2)\,(-3bc+ad+2bdx^2)+(3b^2c^2-2abcd-a^2d^2)\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{8b^{3/2}d^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*
x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[c
+ d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(8*
b^(3/2)*d^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(137) = 274$.
time = 0.08, size = 342, normalized size = 2.12

method	result
risch	$\frac{(2bd\,x^2+ad-3bc)(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8b\,d^2}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a^2}{16b\sqrt{deb}}-\frac{\ln\left(\frac{1}{2}ade\right.}{}\right.}{}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(4\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,bd\,x^2-d^2\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+a}}{2\sqrt{bd}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*b*d*x^2-d^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+2*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*
d+3*b^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/d^2/b/(b*d)^(1/2)
```

Maxima [A]

time = 0.52, size = 253, normalized size = 1.57

$$\frac{1}{16}\left(\frac{2\left(\left(5\,b^2c^2d-6\,abcd^2+a^2d^3\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-\left(3\,b^3c^2-2\,ab^2cd-a^2bd^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^3d^2-\frac{2\,(bx^2+a)b^2d^3}{dx^2+c}+\frac{(bx^2+a)^2bd^4}{(dx^2+c)^2}}-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,bd^2}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*((5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*((b*x^2 + a)/(d*x^2 + c))^(3
/2) - (3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))/
(b^3*d^2 - 2*(b*x^2 + a)*b^2*d^3/(d*x^2 + c) + (b*x^2 + a)^2*b*d^4/(d*x^2 +
 c)^2) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((d*sqrt((b*x^2 + a)/(d*x^2 +
 c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)
*b*d^2))*e^(1/2)
```

Fricas [A]
time = 0.39, size = 394, normalized size = 2.45

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{bd}\,e^{\frac{1}{2}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)-4\,(2\,b^2d^3x^4-3\,b^2c^2d+abcd^2-(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{32\,b^2d^3},\ -\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)e^{\frac{1}{2}}-2\,(2\,b^2d^3x^4-3\,b^2c^2d+abcd^2-(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{16\,b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*e^(1/2)*log(8*b^2*d^2*x
^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d^2
*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d
*x^2 + c))) - 4*(2*b^2*d^3*x^4 - 3*b^2*c^2*d + a*b*c*d^2 - (b^2*c*d^2 - a*b
*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(b^2*d^3), -1/16*((3*b^2*
c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sq
rt(-b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d))*e^(1/2) - 2*(2*
b^2*d^3*x^4 - 3*b^2*c^2*d + a*b*c*d^2 - (b^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*
x^2 + a)/(d*x^2 + c))*e^(1/2))/(b^2*d^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

[Out] Timed out

Giac [A]
time = 7.74, size = 155, normalized size = 0.96

$$\frac{1}{16}\left(2\sqrt{bdx^4+bcx^2+adx^2+ac}\left(\frac{2x^2}{d}-\frac{3bc-ad}{bd^2}\right)-\frac{(3b^2c^2-2abcd-a^2d^2)\log\left(\left|-bc-ad-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)\sqrt{bd}\right|\right)}{\sqrt{bd}\,bd^2}\right)e^{\frac{1}{2}}\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*x^2/d - (3*b*c - a*d)/(b
*d^2)) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log(abs(-b*c - a*d - 2*(sqrt(b*d
)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*sqrt(b*d)))/(sqrt(b*d)*b*d
^2))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.265 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2d}-\frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/
2))*e^(1/2)/d^(3/2)/b^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1981, 1979, 294, 214}

$$\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d}-\frac{\sqrt{e}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*d) - ((b*c - a*d)*Sqrt[e
]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
2*Sqrt[b]*d^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=((bc-ad)e)\text{Subst}\left(\int\frac{x^2}{\left(be-dx^2\right)^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$
$$=\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{((bc-ad)e)\text{Subst}\left(\int\frac{1}{be-dx^2}\,dx,x,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$
$$=\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{2d}-\frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}\,d^{3/2}}$$

Mathematica [A]
time = 0.37, size = 132, normalized size = 1.28

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}\left(c+dx^2\right)-(bc-ad)\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{b}\,d^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d*
x^2) - (b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[
b]*Sqrt[c + d*x^2])]))/(2*Sqrt[b]*d^(3/2)*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(83) = 166$.
time = 0.06, size = 200, normalized size = 1.94

method	result
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad}}{}\right.\right.}{4\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,d\sqrt{bd}}$
risch	$\frac{(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{2d}+\frac{\left(\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a}{4\sqrt{deb}}-\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{}\right.}{}\right.}{b\,x^2+c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(a*ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*d-b*ln(1/2*(2
*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(
1/2))*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^
2+a))^(1/2)/d/(b*d)^(1/2)
```

Maxima [A]
time = 0.49, size = 138, normalized size = 1.34

$$\frac{1}{4}\left(\frac{2\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{bd-\frac{(bx^2+a)d^2}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,d}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b*d - (b*x^2 + a)*d^2/(d*
x^2 + c)) + (b*c - a*d)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(
d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*d))*e^(1/2)
```

Fricas [A]
time = 0.36, size = 292, normalized size = 2.83

$$\left[-\frac{(bc-ad)\sqrt{bd}\,e^{\frac{1}{2}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)-4\,(bd^2x^2+bcd)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{8\,bd^2},\ \frac{(bc-ad)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)e^{\frac{1}{2}}+2\,(bd^2x^2+bcd)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{4\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(b*d)*e^(1/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d +
(3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))) - 4*(b*d^2*
x^2 + b*c*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(b*d^2), 1/4*((b*c - a*
d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a
)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d))*e^(1/2) + 2*(b*d^2*x^2 + b*c*d)*sqrt((b
*x^2 + a)/(d*x^2 + c))*e^(1/2))/(b*d^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.35, size = 123, normalized size = 1.19

$$\frac{1}{4}\left(\frac{2\sqrt{bdx^4+bcx^2+adx^2+ac}}{d}+\frac{(bc-ad)\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{bd^2}\right)e^{\frac{1}{2}}\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)/d + (b*c - a*d)*sqrt(b*d)*lo
g(abs(-2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*b*d - sq
rt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.266 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a}\,\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\,\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{\sqrt{d}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*a^(1/2)*e^(
1/2)/c^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*b^(1/2)*e^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1981, 1980, 492, 214}

$$\frac{\sqrt{b}\,\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\,\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b
*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/Sqrt[d]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= (ae)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right) + (be)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\cdots}\right.$$

$$= -\frac{\sqrt{a}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}} + \frac{\sqrt{b}\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

Mathematica [A]
time = 0.47, size = 147, normalized size = 1.31

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(-\sqrt{a}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(Sqrt[a]*Sqrt[d]*ArcTa
nh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[b]*Sqrt[c]*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[c]*Sqr
t[d]*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.
time = 0.06, size = 179, normalized size = 1.60

method	result
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(b\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}-a\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\right.\right.}{2\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,\sqrt{bd}\,\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)-a
*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*
c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [A]
time = 0.52, size = 140, normalized size = 1.25

$$\frac{1}{2}\left(\frac{a\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}}-\frac{b\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a
)/(d*x^2 + c)) + sqrt(a*c)))/sqrt(a*c) - b*log((d*sqrt((b*x^2 + a)/(d*x^2 +
 c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/sqrt(b*d))
*e^(1/2)
```

Fricas [A]
time = 0.46, size = 815, normalized size = 7.28

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(b/d)*e^(1/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 +
8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d^2
 + a*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(b/d)) + 1/4*sqrt(a/c)*e^(
1/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^
2*c*d)*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2
)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(a/c))/x^4), -1/2*sqrt(-b/d)*arctan(1/2
*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-b/d)/(b^2*x^2
+ a*b))*e^(1/2) + 1/4*sqrt(a/c)*e^(1/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4
+ 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(a/c
))/x^4), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 +
a)/(d*x^2 + c))*sqrt(-a/c)/(a*b*x^2 + a^2))*e^(1/2) + 1/4*sqrt(b/d)*e^(1/2)
*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*
x^2 + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt((b
*x^2 + a)/(d*x^2 + c))*sqrt(b/d)), 1/2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-a/c)/(a*b*x^2 + a^2))*e^(1/
2) - 1/2*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*
x^2 + c))*sqrt(-b/d)/(b^2*x^2 + a*b))*e^(1/2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

3.267 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} dx$

Optimal. Leaf size=127

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{(bc-ad)\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/
2))*e^(1/2)/c^(3/2)/a^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/
(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1981, 1980, 294, 214}

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c*(a - (c*(a + b*x^2))/(
c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^2}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}\,c^{3/2}}$$

Mathematica [A]
time = 0.62, size = 137, normalized size = 1.08

$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\,(c+dx^2)+(bc-ad)x^2\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{a}\,c^{3/2}x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(c
 + d*x^2) + (b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2
])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(3/2)*x^2*Sqrt[a + b*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(107) = 214$.
time = 0.09, size = 326, normalized size = 2.57

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{2c\,x^2}+\frac{\left(\frac{\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}}{x^2}\right)ad}{4c\sqrt{ace}}-\frac{\ln\left(\frac{2ace+(ade+bc}{}\right.}{}\right.}{}$
default	$-\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(-2bd\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^4\sqrt{ac}-a^2\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2}}{x^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*x^4*(a*c)^(1/2)-a^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d*c*x^2+c^2*ln((a*d*x^2+b*c*x^2+2*
(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b*a*x^2-2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*
c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/a/x^2/(a*c)^(1/2)
```

Maxima [**A**]
time = 0.51, size = 138, normalized size = 1.09

$$\frac{1}{4}\left(\frac{2\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{ac-\frac{(bx^2+a)c^2}{dx^2+c}}+\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,c}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*c - (b*x^2 + a)*c^2/(d*
x^2 + c)) + (b*c - a*d)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(
c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*c))*e^(1/2)
```

Fricas [A]
time = 0.45, size = 314, normalized size = 2.47

$$\left[-\frac{\sqrt{ac}\,(bc-ad)x^2e^{\frac{1}{2}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{x^4}\right)+4\,(acdx^2+ac^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{8\,ac^2x^2},\ \frac{\sqrt{-ac}\,(bc-ad)x^2\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{-ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(abcx^2+a^2c)}\right)e^{\frac{1}{2}}-2\,(acdx^2+ac^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{4\,ac^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a*c)*(b*c - a*d)*x^2e^(1/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^
2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c*d + a*d^2)*x^4 + 2
*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)))/x^
4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(a*c^2*x^
2), 1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sq
rt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2 + a^2*c))*e^(1/2) - 2*(a*
c*d*x^2 + a*c^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(a*c^2*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(103) = 206.
time = 3.36, size = 213, normalized size = 1.68

$$\frac{1}{2}\left(\frac{(bc-ad)\arctan\left(-\frac{\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,c}+\frac{\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bc+\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)ad+2\,\sqrt{bd}\,ac}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^2-ac\right)c}\right)e^{\frac{1}{2}}\mathrm{sgn}(dx^2+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*((b*c - a*d)*arctan(-(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2
+ a*c))/sqrt(-a*c))/(sqrt(-a*c)*c) + ((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x
^2 + a*d*x^2 + a*c))*b*c + (sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^
```

```
2 + a*c))*a*d + 2*sqrt(b*d)*a*c)/(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2
+ a*d*x^2 + a*c))^2 - a*c)*c))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

3.268 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=208

$$-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}+\frac{(bc-ad)(bc+3ad)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

```
[Out] 1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
(1/2)/e^(1/2))*e^(1/2)/a^(3/2)/c^(5/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^2/(a-c*(b*x^2+a)/(d*x^2+c))^2+1/8*(-5*a*d+b*c)*(-a*d+b*c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]

time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 466, 393, 214}

$$\frac{\sqrt{e}\,(3ad+bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/4*((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c^2*(a - (c*(a + b*
x^2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c
+ d*x^2)])/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c
+ 3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[
a]*Sqrt[e])])/(8*a^(3/2)*c^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(be-dx^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-(bc-ad)e+4cdx^2}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+b}{c+dx}}\right.}{4c^2}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{((bc-ad)(bc+}{}$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)(bc+}{}$$

Mathematica [A]
time = 1.18, size = 174, normalized size = 0.84

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-bcx^2+3adx^2)+(b^2c^2+2abcd-3a^2d^2)x^4\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*
c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(184) = 368.
time = 0.09, size = 559, normalized size = 2.69

method	result

risch	$-\frac{(d\,x^2+c)\left(-3ad\,x^2+bc\,x^2+2ac\right)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8c^2x^4a}+\frac{\left(-\frac{3a\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{deb\,x^4+(ade+bce)\,x^2+a}}{x^2}\right)}{16c^2\sqrt{ace}}\right.}{}$
default	$-\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(10b\,d^2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6a\sqrt{ac}+2b^2d\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(10*b*d^2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)+2*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*x^6*c*(a*c)^(1/2)+3*a^3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^2*c*x^4-2*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d*b*a^2*c^2*x^4-c^3*l
n((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)
/x^2)*b^2*a*x^4+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1
/2)+8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b*c*d*x^4+2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)-10*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*
b*c*x^2*(a*c)^(1/2)+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/
((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/a^2/x^4/(a*c)^(1/2)
```

Maxima [A]
time = 0.50, size = 249, normalized size = 1.20

$$-\frac{1}{16}\left(\frac{2\left(\left(b^2c^3-6\,abc^2d+5\,a^2cd^2\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}+\left(ab^2c^2+2\,a^2bcd-3\,a^3d^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{a^3c^2-\frac{2\,(bx^2+a)a^2c^3}{dx^2+c}+\frac{(bx^2+a)^2ac^4}{(dx^2+c)^2}}+\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,ac^2}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*((b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*((b*x^2 + a)/(d*x^2 + c))^(
3/2) + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))
/(a^3*c^2 - 2*(b*x^2 + a)*a^2*c^3/(d*x^2 + c) + (b*x^2 + a)^2*a*c^4/(d*x^2
+ c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((c*sqrt((b*x^2 + a)/(d*x^2
+ c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c
)*a*c^2))*e^(1/2)
```

Fricas [**A**]
time = 0.66, size = 416, normalized size = 2.00

$$\left[-\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\sqrt{ac}\,x^4e^{\frac{1}{2}}\log\left(\frac{\left(b^2c^2+6\,abcd+a^2d^2\right)x^4+8\,a^2c^2+8\,\left(abc^2+a^2cd\right)x^2-4\,\left(\left(bcd+ad^2\right)x^4+2\,ac^2+\left(bc^2+3\,acd\right)x^2\right)\sqrt{ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{x^4}\right)+4\left(2\,a^2c^3+\left(abc^2d-3\,a^2cd^2\right)x^4+\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{32\,a^2c^3x^4},\ -\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\sqrt{-ac}\,x^4\arctan\left(\frac{\left(\left(bc+ad\right)x^2+2\,ac\right)\sqrt{-ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\left(abcx^2+a^2c\right)}\right)e^{\frac{1}{2}}+2\left(2\,a^2c^3+\left(abc^2d-3\,a^2cd^2\right)x^4+\left(abc^3-a^2c^2d\right)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{1}{2}}}{16\,a^2c^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(a*c)*x^4*e^(1/2)*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*(
(b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^
2 + a)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 + (a*b*c^2*d - 3*a^2*c*d^2)*x^4 +
(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(a^2*c^3*
x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((
b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2
 + a^2*c))*e^(1/2) + 2*(2*a^2*c^3 + (a*b*c^2*d - 3*a^2*c*d^2)*x^4 + (a*b*c^
3 - a^2*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2))/(a^2*c^3*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(181) = 362.
time = 5.84, size = 481, normalized size = 2.31

$$-\frac{1}{8}\left(\frac{\left(b^2c^2+2\,abcd-3\,a^2d^2\right)\arctan\left(-\frac{\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,ac^2}-\frac{\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^3b^2c^2+\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)ab^2c^3+2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^3abcd+10\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)a^2bc^2d-3\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^3a^2d^2+5\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)a^3cd^2+8\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^2\sqrt{bd}\,abc^2+8\sqrt{bd}\,a^3c^2d}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^2-ac\right)^2ac^2}\right)e^{\frac{1}{2}}\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/8*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(-(sqrt(b*d)*x^2 - sqrt(b*d*x
^4 + b*c*x^2 + a*d*x^2 + a*c))/sqrt(-a*c))/(sqrt(-a*c)*a*c^2) - ((sqrt(b*d)
*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*b^2*c^2 + (sqrt(b*d)*x^2
- sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a*b^2*c^3 + 2*(sqrt(b*d)*x^2 - s
qrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a*b*c*d + 10*(sqrt(b*d)*x^2 - sqr
t(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^2*b*c^2*d - 3*(sqrt(b*d)*x^2 - sqrt
(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a^2*d^2 + 5*(sqrt(b*d)*x^2 - sqrt(b*
```

```
d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^3*c*d^2 + 8*(sqrt(b*d)*x^2 - sqrt(b*d*x
^4 + b*c*x^2 + a*d*x^2 + a*c))^2*sqrt(b*d)*a*b*c^2 + 8*sqrt(b*d)*a^3*c^2*d)
/(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^2 - a*c)^2*a*c
^2))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

3.269 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$

Optimal. Leaf size=318

$$\frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}-\frac{(bc-ad)\left(b^2c^2+2abcd-11a^2d^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}+\frac{(bc-ad)^3e^2\left(\frac{e(a+b}{c+d}\right.}{6ac^2\left(ae-\frac{ce(a+b}{c+d}\right.}$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3-1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(c^(1/2)
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*e^(1/2)/a^(5/2)/c^(7/2)+1/8
*(-a*d+b*c)^2*(3*a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^3/(a-c*(b*x^2+a
)/(d*x^2+c))^2-1/16*(-a*d+b*c)*(-11*a^2*d^2+2*a*b*c*d+b^2*c^2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^3/(a-c*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 474, 466, 393, 214}

$$-\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{\sqrt{e}\left(5a^2d^2+2abcd+b^2c^2\right)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}+\frac{(3ad+bc)(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(8*a*c^3*(a
 - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11
*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(16*a^2*c^3*(a - (c*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/
(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 +
 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^2(be-dx^2)^2}{(-ae+cx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^2(-3(2b^2c^2e^2-(bce-ade)^2)+6acd^2ex^2)}{(-ae+cx^2)^3}\,dx, \right.}{6ac^2}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} - \frac{(bc-ad)\text{Subst}}{}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(b^2c^2+2abcd-11a^2d^2)\sqrt{\frac{e(a+b}{c+dx}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

$$= \frac{(bc-ad)^2(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(b^2c^2+2abcd-11a^2d^2)\sqrt{\frac{e(a+b}{c+dx}}}{16a^2c^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Mathematica [A]
time = 1.50, size = 222, normalized size = 0.70

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}\,(3b^2c^2x^4-2abcx^2(c-2dx^2)+a^2(-8c^2+10cdx^2-15d^2x^4))-3(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3)\,x^6\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(
-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x
^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(290) = 580$.
time = 0.11, size = 849, normalized size = 2.67

method	result
risch	$-\frac{(dx^2+c)(15d^2a^2x^4-4abcdx^4-3b^2c^2x^4-10a^2dcx^2+2abc^2x^2+8a^2c^2)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{48c^3x^6a^2}+\frac{\left(\frac{5a\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\sqrt{}}{}\right.}{32c}\right.}{}$
default	$\frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}(dx^2+c)\left(66bd^3\sqrt{bdx^4+adx^2+bcx^2+ac}\,x^8a^2\sqrt{ac}+24b^2d^2\sqrt{bdx^4+adx^2+bcx^2+ac}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(66*b*d^3*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*x^8*a^2*(a*c)^(1/2)+24*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*x^8*a*c*(a*c)^(1/2)+6*b^3*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^8
*c^2*(a*c)^(1/2)+15*a^4*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^3*c*x^6-9*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*d^2*b*a^3*c^2*x^6-3*ln((a
*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2
)*d*b^2*a^2*c^3*x^6-3*c^4*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^3*a*x^6+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*d^3*a^3*x^6*(a*c)^(1/2)+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*b
*a^2*c*x^6*(a*c)^(1/2)+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*d*a*c^2*x
^6*(a*c)^(1/2)+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^3*c^3*x^6*(a*c)^(1/2
)-66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d^2*a^2*x^4*(a*c)^(1/2)-24*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*b*a*c*x^4*(a*c)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(3/2)*b^2*c^2*x^4*(a*c)^(1/2)+36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3
/2)*d*a^2*c*x^2*(a*c)^(1/2)+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*a*c^2*
x^2*(a*c)^(1/2)-16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a^2*c^2*(a*c)^(1/2))
/((d*x^2+c)*(b*x^2+a))^(1/2)/c^4/a^3/x^6/(a*c)^(1/2)
```

Maxima [A]

time = 0.54, size = 387, normalized size = 1.22

$$-\frac{1}{96}\left(\frac{2\left(3\left(b^3c^5+ab^2c^4d-13a^2bc^3d^2+11a^3c^2d^3\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}-8\left(ab^3c^4-3a^2b^2c^3d-3a^3bc^2d^2+5a^4cd^3\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-3\left(a^2b^3c^3+a^3b^2c^2d+3a^4bcd^2-5a^5d^3\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{a^5c^3-\frac{3(bx^2+a)a^4c^4}{dx^2+c}+\frac{3(bx^2+a)^2a^3c^5}{(dx^2+c)^2}-\frac{(bx^2+a)^3a^2c^6}{(dx^2+c)^3}}-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^2c^3}\right)e^{\frac{1}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/96*(2*(3*(b^3*c^5 + a*b^2*c^4*d - 13*a^2*b*c^3*d^2 + 11*a^3*c^2*d^3)*((b
*x^2 + a)/(d*x^2 + c))^(5/2) - 8*(a*b^3*c^4 - 3*a^2*b^2*c^3*d - 3*a^3*b*c^2
```

```
*d^2 + 5*a^4*c*d^3)*((b*x^2 + a)/(d*x^2 + c))^(3/2) - 3*(a^2*b^3*c^3 + a^3*
b^2*c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c)))/(a^5*
c^3 - 3*(b*x^2 + a)*a^4*c^4/(d*x^2 + c) + 3*(b*x^2 + a)^2*a^3*c^5/(d*x^2 +
c)^2 - (b*x^2 + a)^3*a^2*c^6/(d*x^2 + c)^3) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/
(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a^2*c^3))*e^(1/2)
```

Fricas [A]
time = 1.24, size = 554, normalized size = 1.74

$$\left[-\frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{ac}\,x^{6}e^{\frac{1}{2}}\log\left(\frac{(b^{2}c^{2}+6abcd+a^{2}d^{2})x^{4}+8a^{2}c^{2}+8(abc^{2}+a^{2}cd)x^{2}+4\left((bcd+ad^{2})x^{4}+2ac^{2}+(bc^{2}+3acd)x^{2}\right)\sqrt{ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{x^{4}}\right)+4\left(8a^{3}c^{4}-(3ab^{2}c^{3}d+4a^{2}bc^{2}d^{2}-15a^{3}cd^{3})x^{6}-(3ab^{2}c^{4}+2a^{2}bc^{3}d-5a^{3}c^{2}d^{2})x^{4}+2(a^{2}bc^{4}-a^{3}c^{3}d)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\,e^{\frac{1}{2}}}{192a^{3}c^{4}x^{6}},\ \frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{-ac}\,x^{6}\arctan\left(\frac{((bc+ad)x^{2}+2ac)\sqrt{-ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{2(abcx^{2}+a^{2}c)}\right)e^{\frac{1}{2}}-2\left(8a^{3}c^{4}-(3ab^{2}c^{3}d+4a^{2}bc^{2}d^{2}-15a^{3}cd^{3})x^{6}-(3ab^{2}c^{4}+2a^{2}bc^{3}d-5a^{3}c^{2}d^{2})x^{4}+2(a^{2}bc^{4}-a^{3}c^{3}d)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\,e^{\frac{1}{2}}}{96a^{3}c^{4}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(a*c)*x^
6*e^(1/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
 + a^2*c*d)*x^2 + 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)
*sqrt(a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)))/x^4) + 4*(8*a^3*c^4 - (3*a*b^2*c^
3*d + 4*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^6 - (3*a*b^2*c^4 + 2*a^2*b*c^3*d -
5*a^3*c^2*d^2)*x^4 + 2*(a^2*b*c^4 - a^3*c^3*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2
 + c))*e^(1/2))/(a^3*c^4*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d
^2 - 5*a^3*d^3)*sqrt(-a*c)*x^6*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a
*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2 + a^2*c))*e^(1/2) - 2*(8*a^3*c
^4 - (3*a*b^2*c^3*d + 4*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^6 - (3*a*b^2*c^4 +
2*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^4 + 2*(a^2*b*c^4 - a^3*c^3*d)*x^2)*sqrt((b
*x^2 + a)/(d*x^2 + c))*e^(1/2))/(a^3*c^4*x^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. 2(287) = 574.
time = 6.96, size = 871, normalized size = 2.74

$$\frac{1}{48}\left(\frac{3(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3})\arctan\left(-\frac{\sqrt{bd}x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^{2}c^{3}}-\ldots\right)$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(-(sqrt(b
*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))/sqrt(-a*c))/(sqrt(-a*c)*
a^2*c^3) - (3*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^5*b
^3*c^3 - 8*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a*b^
3*c^4 - 3*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^2*b^3
*c^5 + 3*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^5*a*b^2*
c^2*d - 72*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a^2*
b^2*c^3*d - 51*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^
3*b^2*c^4*d + 9*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^5
*a^2*b*c*d^2 - 24*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
^3*a^3*b*c^2*d^2 - 105*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 +
a*c))*a^4*b*c^3*d^2 - 15*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2
+ a*c))^5*a^3*d^3 + 40*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 +
a*c))^3*a^4*c*d^3 - 33*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 +
a*c))*a^5*c^2*d^3 - 48*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 +
a*c))^2*sqrt(b*d)*a^2*b^2*c^4 - 144*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2
 + a*d*x^2 + a*c))^2*sqrt(b*d)*a^3*b*c^3*d - 16*sqrt(b*d)*a^4*b*c^4*d - 48*
sqrt(b*d)*a^5*c^3*d^2)/(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2
+ a*c))^2 - a*c)^3*a^2*c^3))*e^(1/2)*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

3.270 $\int x^4 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal. Leaf size=357

$$\frac{(8b^2c^2-3abcd-2a^2d^2)\,x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15b^2d^2}-\frac{(4bc-ad)x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{15bd^2}+\frac{x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5d}$$

```
[Out] 1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d
^2-1/15*(-a*d+4*b*c)*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^2+1/5*x^
3*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d+1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)-1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^(5/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 489, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,(-2a^2d^2-3abcd+8b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{5/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}+\frac{x(-2a^2d^2-3abcd+8b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15b^2d^2}+\frac{c^{3/2}(4bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{5/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{x(c+dx^2)\,(4bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15bd^2}+\frac{x^3(c+dx^2)\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
15*b^2*d^2) - ((4*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(15*b*d^2) + (x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d) -
 (Sqrt[c]*(8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(5/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(4*b*c - a*d)*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(15*b*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
```

```
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^4\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$=\frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{5d}-\frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\left(3ac+(4bc-ad)x^2\right)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{5d\sqrt{a+bx^2}}$$

$$=-\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2}+\frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{5d}+\frac{\left(\sqrt{\frac{e(a+}{c+}}\right.}{}$$

$$=-\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2}+\frac{x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{5d}+\frac{\left(ac(4bc-\right.}{}$$

$$=\frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{15b^2d^2}-\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2}-$$

$$=\frac{\left(8b^2c^2-3abcd-2a^2d^2\right)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{15b^2d^2}-\frac{(4bc-ad)x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{15bd^2}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.58, size = 255, normalized size = 0.71

$$\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,dx\left(a+bx^2\right)\left(c+dx^2\right)\left(-4bc+ad+3bdx^2\right)+ic\left(-8b^2c^2+3abcd+2a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic\left(-8b^2c^2+7abcd+a^2d^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{15b\sqrt{\frac{b}{a}}\,d^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(
-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt[1
```

```
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(
b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqrt[b/a
]*d^3*(a + b*x^2))
```

Maple [A]
time = 0.08, size = 552, normalized size = 1.55

method	result
risch	$\frac{x\left(3bd\,x^{2}+ad-4bc\right)\left(d\,x^{2}+c\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{15d^{2}b}-\frac{\left(-\frac{2\left(2a^{2}d^{2}+3abcd-8b^{2}c^{2}\right)ace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^{4}+ade\,x^{2}+bce\,x^{2}}}\right.}{}$
default	$\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+4\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-4\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+\sqrt{\frac{b\,x^{2}+}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(3*(-b/a)^(1/2)*b^2*d^3*x^7+4*
(-b/a)^(1/2)*a*b*d^3*x^5-(-b/a)^(1/2)*b^2*c*d^2*x^5+(-b/a)^(1/2)*a^2*d^3*x^
3-4*(-b/a)^(1/2)*b^2*c^2*d*x^3+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+7*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-8*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^2*c^3-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2)
,(a*d/b/c)^(1/2))*a^2*c*d^2-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+8*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+(-b/a)^(1/2
)*a^2*c*d^2*x-4*(-b/a)^(1/2)*a*b*c^2*d*x)/((d*x^2+c)*(b*x^2+a))^(1/2)/d^3/b
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(x^4*sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^4*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.271 $\int x^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx$

Optimal. Leaf size=266

$$-\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd}+\frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3d}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1\right.}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -1/3*(-a*d+2*b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d+1/3*x*(d*x^2+c)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*
x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+1/3*(-a*d+2
*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1986, 489, 545, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d}-\frac{x(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/3*((2*b*c - a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(b*d) + (x*Sqrt[(e
*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*d) + (Sqrt[c]*(2*b*c - a*d)*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{a+bx^2}}$$

$$= \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{ac+(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}d}{3d\sqrt{a+bx^2}}$$

$$= \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{\left(ac\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{3d\sqrt{a+bx^2}}$$

$$= -\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd} + \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} - \frac{c^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\Big(}{3d^{3/2}\sqrt{\frac{c}{a}}}$$

$$= -\frac{(2bc-ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3bd} + \frac{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d} + \frac{\sqrt{c}(2bc-ad)\sqrt{\frac{e(a+}{c+}}}{3bd^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.06, size = 208, normalized size = 0.78

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2) - ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) + 2ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{3\sqrt{\frac{b}{a}}d^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) -
 I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*S
qrt[b/a]*d^2*(a + b*x^2))
```

Maple [A]
time = 0.05, size = 356, normalized size = 1.34

method	result
risch	$\frac{x(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{3d} - \frac{\left(\frac{2(ad-2bc)ace\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{deb\,x^4+ade\,x^2+bce\,x^2+ace}\,(ade+bce+e(ac}\right.}{}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-2ac\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*b*d^2*x^5+(-b/a)^
(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3-2*a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*a*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)*a*c*d*x)/((d*x^2+c)*(b*x^2+a))^(1/2)/d^2
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(x^2*sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^2*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.272 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\, dx$

Optimal. Leaf size=194

$$x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1\right.}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ell
ipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(b*x^2
+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1986, 433, 429, 506, 422}

$$\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx &= \frac{\left(\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} \\
&= \frac{\left(a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\,\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} + \frac{\left(b\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+d\cdots\right.}{\sqrt{\cdots}} \\
&= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}} + \frac{\sqrt{c}\,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}} - \frac{\left(c\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\cdots\right.}{\cdots} \\
&= x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}} - \frac{\sqrt{c}\,\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\,\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}} + \frac{\sqrt{c}\,\sqrt{\frac{e\left(a+bx^{\cdots}\right.}{c+dx^2}}\cdots}{\sqrt{\cdots}}
\end{aligned}
$$

Mathematica [A]
time = 0.85, size = 86, normalized size = 0.44

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqr
t[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

Maple [A]
time = 0.04, size = 184, normalized size = 0.95

method	result
default	$\frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\,(d x^2+c)\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\left(a\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\,\mathrm{Ellipti}\right.}{\sqrt{(d x^2+c)(b x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/((d*x
^2+c)*(b*x^2+a))^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.273 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx$

Optimal. Leaf size=239

$$\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{cx}-\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{b\sqrt{c}\,\sqrt{\frac{e(a}{c+}}}{}$$

```
[Out] d*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/c/x+b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(
1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 21, 433, 429, 506, 422}

$$\frac{b\sqrt{c}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c - (Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(c*x) - (Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]) + (b*Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ellip
ticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,d}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{\left(b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{\sqrt{a+bx^2}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} + \frac{b\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

$$= \frac{dx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{cx} - \frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\tan^{-1}\left(\frac{\sqrt{d}}{\sqrt{c}}\right.\right.}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A]
time = 1.12, size = 111, normalized size = 0.46

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)\left(-\frac{1}{x}+\frac{b\sqrt{1+\frac{bx^2}{a}}E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}(a+bx^2)\sqrt{1+\frac{dx^2}{c}}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a + b*
x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

Maple [A]
time = 0.05, size = 194, normalized size = 0.81

method	result
default	$\frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\,(d x^2+c)\left(-\sqrt{-\frac{b}{a}}\,bd x^4+bc\sqrt{\frac{b x^2+a}{a}}\,\sqrt{\frac{d x^2+c}{c}}\,x\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-\sqrt{-\frac{b}{a}}\,ad x^2-\sqrt{-\frac{b}{a}}\,b\right.}{\sqrt{(d x^2+c)(b x^2+a)}\,cx\sqrt{-\frac{b}{a}}\,\sqrt{bd x^4+ad x^2+bc x^2+ac}}$
risch	$-\frac{(d x^2+c)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{cx}+\frac{b\left(-\frac{2dace\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(x\sqrt{\right.\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb x^4+ade x^2+bce x^2+ace}\,(ade+bce+e(ad-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-(-b/a)^(1/2)*b*d*x^4+b*c*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
-(-b/a)^(1/2)*a*d*x^2-(-b/a)^(1/2)*b*c*x^2-(-b/a)^(1/2)*a*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/c/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt((b*x^2 + a)/(d*x^2 + c))/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2, x)
```

3.274 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx$

Optimal. Leaf size=321

$$\frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3}-\frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3ac^2x}-\frac{\sqrt{d}\,(bc-2ad)\sqrt{}}{}$$

```
[Out] 1/3*d*(-2*a*d+b*c)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2-1/3*(d*x^2+c)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/c/x^3-1/3*(-2*a*d+b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c^2/x-1/3*(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1
/3*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/
c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$-\frac{\sqrt{d}\,(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{b\sqrt{d}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2}-\frac{(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2x}-\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] (d*(b*c - 2*a*d)*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*a*c^2) - (Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c*x^3) - ((b*c - 2*a*d)*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*a*c^2*x) - (Sqrt[d]*(b*c - 2*a*d)*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
- (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (b*Sq
rt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(3*a*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx &= \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}} \\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-2ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}} \, d}{3c\sqrt{a+bx^2}} \\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3ac^2x} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{} \\
&= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3ac^2x} - \frac{\left(bd\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{} \\
&= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(}{3ac^2x} \\
&= \frac{d(bc-2ad)x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3ac^2} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3cx^3} - \frac{(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(}{3ac^2x}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.63, size = 238, normalized size = 0.74

$$
-\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(bcx^2+a(c-2dx^2)) - ibc(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right) + ibc(-bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)\middle|\frac{ad}{bc}\right)\right)}{3bc^2x^3(a+bx^2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c
 + d*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*
```

```
c)] + I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c^2*x^3*(a + b*x^2))
```

Maple [A]
time = 0.06, size = 443, normalized size = 1.38

method	result
risch	$-\frac{(d x^2+c)(-2ad x^2+bc x^2+ac)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{3c^2x^3a} - \frac{bd\left(-\frac{2(2ad-bc)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{deb x^4+ade x^2+bce}}\right.}{}$
default	$-\frac{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}(d x^2+c)\left(-2\sqrt{-\frac{b}{a}}\,ab d^2x^6+\sqrt{-\frac{b}{a}}\,b^2cd x^6-bd\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\,\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*(-b/a)^(1/2)*a*b*d^2*x^6+(
-b/a)^(1/2)*b^2*c*d*x^6-b*d*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x^3*a*c+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+2*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*
c*d*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))*b^2*c^2*x^3-2*(-b/a)^(1/2)*a^2*d^2*x^4+(-b/a)^(1/2)*b^2*c^2*
x^4-(-b/a)^(1/2)*a^2*c*d*x^2+2*(-b/a)^(1/2)*a*b*c^2*x^2+(-b/a)^(1/2)*a^2*c^
2)/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/x^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] e^(1/2)*integrate(sqrt((b*x^2 + a)/(d*x^2 + c))/x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4, x)
```

3.275 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6} dx$

Optimal. Leaf size=424

$$-\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15a^2c^3}-\frac{\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5}-\frac{(bc-4ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{15ac^2x^3}$$

```
[Out] -1/15*d*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^
2/c^3-1/5*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/x^5-1/15*(-4*a*d+b*c)*(
d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^2/x^3+1/15*(-8*a^2*d^2+3*a*b*c*d
+2*b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c^3/x+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(
x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*b*(-4*a*
d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^2/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$\frac{\sqrt{d}\,(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{b\sqrt{d}\,(bc-4ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{dx(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15a^2c^3}+\frac{(c+dx^2)\,(-8a^2d^2+3abcd+2b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15a^2c^3x}-\frac{(c+dx^2)\,(bc-4ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{15ac^2x^3}-\frac{(c+dx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(d*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(a^2*c^3) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c*x^5
) - ((b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(15*a*c^2
*x^3) + ((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)]*(c + d*x^2))/(15*a^2*c^3*x) + (Sqrt[d]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d
^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(15*a^2*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
- (b*Sqrt[d]*(b*c - 4*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^2*c^(3/2)*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{a+bx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} + \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{bc-4ad-3bdx^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{15ac^2x^3} - \frac{\left(\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd -}{}$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} - \frac{(bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{15ac^2x^3} + \frac{(2b^2c^2+3abcd -}{}$$

$$= -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} - \frac{(bc-4ad}{}$$

$$= -\frac{d(2b^2c^2+3abcd-8a^2d^2)\,x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{15a^2c^3} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5cx^5} - \frac{(bc-4ad}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.77, size = 302, normalized size = 0.71

$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,(a+bx^2)(c+dx^2)\left(-2b^2c^2x^4+abcx^2(c-3dx^2)+a^2(3c^2-4cdx^2+8d^2x^4)\right)+ibc(-2b^2c^2-3abcd+8a^2d^2)\,x^5\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-2ibc(-b^2c^2-abcd+2a^2d^2)\,x^5\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{15a^2\sqrt{\frac{b}{a}}\,c^3x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)
*(-2*b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d^2
*x^4)) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)
*b*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*Sqrt[b/a]*c^
3*x^5*(a + b*x^2))
```

Maple [A]
time = 0.07, size = 708, normalized size = 1.67

method	result
risch	$-\frac{(dx^2+c)(8d^2a^2x^4-3abcdx^4-2b^2c^2x^4-4a^2dcx^2+abc^2x^2+3a^2c^2)\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{15c^3x^5a^2}+\frac{bd\left(-\frac{2(8a^2d^2-3abcd-2b^2c^2)ace\sqrt{1+\frac{bx^2}{a}}}{}\right.}{}$
default	$-\frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)\left(8\sqrt{-\frac{b}{a}}\,a^2bd^3x^8-3\sqrt{-\frac{b}{a}}\,ab^2cd^2x^8-2\sqrt{-\frac{b}{a}}\,b^3c^2dx^8+4\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(8*(-b/a)^(1/2)*a^2*b*d^3*x^8
-3*(-b/a)^(1/2)*a*b^2*c*d^2*x^8-2*(-b/a)^(1/2)*b^3*c^2*d*x^8+4*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b
*c*d^2*x^5-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a*b^2*c^2*d*x^5-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*x^5-8*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*b*c*d
^2*x^5+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*a*b^2*c^2*d*x^5+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^3*c^3*x^5+8*(-b/a)^(1/2)*a^3*d^3
*x^6+(-b/a)^(1/2)*a^2*b*c*d^2*x^6-4*(-b/a)^(1/2)*a*b^2*c^2*d*x^6-2*(-b/a)^(
1/2)*b^3*c^3*x^6+4*(-b/a)^(1/2)*a^3*c*d^2*x^4-3*(-b/a)^(1/2)*a^2*b*c^2*d*x^
4-(-b/a)^(1/2)*a*b^2*c^3*x^4-(-b/a)^(1/2)*a^3*c^2*d*x^2+4*(-b/a)^(1/2)*a^2*
b*c^3*x^2+3*(-b/a)^(1/2)*a^3*c^3)/((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/x^5/a^2/(
-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
[Out] e^(1/2)*integrate(sqrt((b*x^2 + a)/(d*x^2 + c))/x^6, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt((b*x^2 + a)/(d*x^2 + c))*e^(1/2)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6, x)
```

3.276 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=282

$$\frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}+\frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{48bd^4}-\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24d^4}$$

```
[Out] 1/6*(e*(b*x^2+a)/(d*x^2+c))^(5/2)*(d*x^2+c)^3/b/d^2/e-1/16*(-a*d+b*c)*(-a^2
*d^2-10*a*b*c*d+35*b^2*c^2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))
^(1/2)/b^(1/2)/e^(1/2))/b^(3/2)/d^(9/2)+c^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/d^4+1/48*(-5*a^2*d^2-50*a*b*c*d+79*b^2*c^2)*e*(d*x^2+c)*(e*(b*x
^2+a)/(d*x^2+c))^(1/2)/b/d^4-1/24*(a*d+11*b*c)*e*(d*x^2+c)^2*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)/d^4
```

Rubi [A]
time = 0.35, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1981, 1980, 474, 466, 1171, 396, 214}

$$\frac{e(c+dx^2)\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{48bd^4}-\frac{e^{3/2}(bc-ad)\left(-a^2d^2-10abcd+35b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}+\frac{c^2e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4}-\frac{e(c+dx^2)^2(ad+11bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24d^4}+\frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6bd^2e}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^4 + ((79*b^2*c^2 -
50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4
8*b*d^4) - ((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^
2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*
e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(3/2)*d^(9
/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(-ae+cx^2)^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4\left(-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2\right)}{(be-dx^2)^3}\,dx,\right.}{6bd^2}$$

$$= -\frac{(11bc+ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24d^4} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}(c+dx^2)^3}{6bd^2e} - \frac{(bc-ad)}{}$$

$$= \frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e\,(a+bx}{c+dx^2}}}{24d^4}$$

$$= \frac{c^2(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx}{48bd^4}$$

$$= \frac{c^2(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2-50abcd-5a^2d^2)\,e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx}{48bd^4}$$

Mathematica [A]
time = 4.64, size = 294, normalized size = 1.04

$$\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}\,\sqrt{bc-ad}\,(3a^3d^2(c+dx^2)+a^2bd(-100c^2-35cdx^2+17d^2x^4)+b^3x^2(105c^3+35c^2dx^2-14cd^2x^4+8d^3x^6)+ab^2(105c^3-65c^2dx^2-52cd^2x^4+22d^3x^6))-3(bc-ad)^2\,(35b^2c^2-10abcd-a^2d^2)\,\sqrt{a+bx^2}\,\sqrt{\frac{b\,(c+dx^2)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{48b^2d^{9/2}\sqrt{bc-ad}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[b*c - a*d]*(3*a^3*d^2*
(c + d*x^2) + a^2*b*d*(-100*c^2 - 35*c*d*x^2 + 17*d^2*x^4) + b^3*x^2*(105*c
^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6) + a*b^2*(105*c^3 - 65*c^2*d*x
^2 - 52*c*d^2*x^4 + 22*d^3*x^6)) - 3*(b*c - a*d)^2*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt
[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^2*d^(9/2)*Sqrt[b*c - a*d]*(a
+ b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(252) = 504$.

time = 0.15, size = 1027, normalized size = 3.64

method	result
risch	$\frac{\left(8 b^{2} d^{2} x^{4}+14 a b\, d^{2} x^{2}-22 b^{2} c d\, x^{2}+3 a^{2} d^{2}-52 a b c d+57 b^{2} c^{2}\right) \left(d\, x^{2}+c \right) e \sqrt{\frac{e \left(b\, x^{2}+a \right)}{d\, x^{2}+c}}}{48 b\, d^{4}}+\frac{\left(-\frac{\ln \left(\frac{\frac{1}{2} a d e+\frac{1}{2} b c e+d e b\, x^{2}}{\sqrt{d e b}}+\sqrt{d e b\, x^{4}+\cdots}\right)}{32 d b \sqrt{d e \cdots}}\cdots\right.}{\cdots}$
default	$\frac{\left(12 \sqrt{b d\, x^{4}+a d\, x^{2}+b c\, x^{2}+a c}\, \sqrt{b d}\, a b\, d^{3} x^{4}-60 \sqrt{b d\, x^{4}+a d\, x^{2}+b c\, x^{2}+a c}\, \sqrt{b d}\, b^{2} c\, d^{2} x^{4}-3 \ln \left(\frac{2 b d\, x^{2} \cdots}{\cdots}\right.\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*d^3*x^4-60*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d^2*x^4-3*ln(1/2*(2*b*d*
x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*a^3*d^4*x^2-27*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c*d^3*x^2+135*ln(1/2*(2*b*d*x^2+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c^2
*d^2*x^2-105*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^
(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^3*d*x^2+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(3/2)*(b*d)^(1/2)*b*d^2*x^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/
2)*a^2*d^3*x^2-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c*d^
2*x^2+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^2*d*x^2-3*ln
(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/
(b*d)^(1/2))*a^3*c*d^3-27*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c^2*d^2+135*ln(1/2*(2*b*d*x^
2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a
*b^2*c^3*d-105*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d
)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^3*c^4+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2
)*(b*d)^(1/2)*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*c
*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c^2*d+114*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c^3-96*(b*d)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*a*b*c^2*d+96*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^
2*c^3)/d^4/b*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/((d*x^2+c)
*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [A]

time = 0.52, size = 426, normalized size = 1.51

$$\frac{1}{96}\left(\frac{2\left(3\left(29\,b^3c^3d^2-51\,ab^2c^2d^3+23\,a^2bcd^4-a^3d^5\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}-8\left(17\,b^4c^3d-27\,ab^3c^2d^2+9\,a^2b^2cd^3+a^3bd^4\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}+3\left(19\,b^5c^3-29\,ab^4c^2d+9\,a^2b^3cd^2+a^3b^2d^3\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^4d^4-\frac{3\,(bx^2+a)b^3d^5}{dx^2+c}+\frac{3\,(bx^2+a)^2b^2d^6}{(dx^2+c)^2}-\frac{(bx^2+a)^3bd^7}{(dx^2+c)^3}}+\frac{96\,(bc^3-ac^2d)\sqrt{\frac{bx^2+a}{dx^2+c}}}{d^4}+\frac{3\,(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,bd^4}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*(29*b^3*c^3*d^2 - 51*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4 - a^3*d^5)*(
(b*x^2 + a)/(d*x^2 + c))^(5/2) - 8*(17*b^4*c^3*d - 27*a*b^3*c^2*d^2 + 9*a^2
*b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)/(d*x^2 + c))^(3/2) + 3*(19*b^5*c^3 - 2
9*a*b^4*c^2*d + 9*a^2*b^3*c*d^2 + a^3*b^2*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c)
))/(b^4*d^4 - 3*(b*x^2 + a)*b^3*d^5/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^2*d^6/(
d*x^2 + c)^2 - (b*x^2 + a)^3*b*d^7/(d*x^2 + c)^3) + 96*(b*c^3 - a*c^2*d)*sq
rt((b*x^2 + a)/(d*x^2 + c))/d^4 + 3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(d*sqrt(
(b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b*d^4))*e^(3/2)
```

Fricas [A]
time = 0.58, size = 526, normalized size = 1.87

$$\left[\frac{3\,(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\sqrt{bd}\,e^{\frac{3}{2}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)+4\,(8\,b^3d^4x^6+105\,b^3c^3d-100\,ab^2c^2d^2+3\,a^2bcd^3-14\,(b^3cd^3-ab^2d^4)x^4+(35\,b^3c^2d^2-38\,ab^2cd^3+3\,a^2bd^4)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{192\,b^2d^5},\ \frac{3\,(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)e^{\frac{3}{2}}+2\,(8\,b^3d^4x^6+105\,b^3c^3d-100\,ab^2c^2d^2+3\,a^2bcd^3-14\,(b^3cd^3-ab^2d^4)x^4+(35\,b^3c^2d^2-38\,ab^2cd^3+3\,a^2bd^4)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{96\,b^2d^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)
*e^(3/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a
*b*d^2)*x^2 - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(
b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))) + 4*(8*b^3*d^4*x^6 + 105*b^3*c^3*d - 10
0*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 14*(b^3*c*d^3 - a*b^2*d^4)*x^4 + (35*b^3*
c^2*d^2 - 38*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*
e^(3/2))/(b^2*d^5), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 +
a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x
^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d))*e^(3/2) + 2*(8*b^3*d^4*x^6 + 105*
b^3*c^3*d - 100*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - 14*(b^3*c*d^3 - a*b^2*d^4)*
x^4 + (35*b^3*c^2*d^2 - 38*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)
/(d*x^2 + c))*e^(3/2))/(b^2*d^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.81, size = 420, normalized size = 1.49

$$\frac{1}{96}\left(2\sqrt{bdx^4+bcx^2+adx^2+ac}\left(2\left(\frac{4\,bx^2\mathrm{sgn}(dx^2+c)}{d^2}-\frac{11\,b^3cd^{10}\mathrm{sgn}(dx^2+c)-7\,ab^2d^{11}\mathrm{sgn}(dx^2+c)}{b^2d^{13}}\right)x^2+\frac{57\,b^3c^2d^9\mathrm{sgn}(dx^2+c)-52\,ab^2cd^{10}\mathrm{sgn}(dx^2+c)+3\,a^2bd^{11}\mathrm{sgn}(dx^2+c)}{b^2d^{13}}\right)+\frac{96\,(b^2c^4\mathrm{sgn}(dx^2+c)-2\,abc^3d\mathrm{sgn}(dx^2+c)+a^2c^2d^2\mathrm{sgn}(dx^2+c))}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)d+\sqrt{bd}\,c\right)d^4}+\frac{3\left(35\,\sqrt{bd}\,b^3c^3\mathrm{sgn}(dx^2+c)-45\,\sqrt{bd}\,ab^2c^2d\mathrm{sgn}(dx^2+c)+9\,\sqrt{bd}\,a^2bcd^2\mathrm{sgn}(dx^2+c)+\sqrt{bd}\,a^3d^3\mathrm{sgn}(dx^2+c)\right)\log\left(\left|-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{b^2d^5}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*(4*b*x^2*sgn(d*x^2 + c)/
d^2 - (11*b^3*c*d^10*sgn(d*x^2 + c) - 7*a*b^2*d^11*sgn(d*x^2 + c))/(b^2*d^1
3))*x^2 + (57*b^3*c^2*d^9*sgn(d*x^2 + c) - 52*a*b^2*c*d^10*sgn(d*x^2 + c) +
 3*a^2*b*d^11*sgn(d*x^2 + c))/(b^2*d^13)) + 96*(b^2*c^4*sgn(d*x^2 + c) - 2*
a*b*c^3*d*sgn(d*x^2 + c) + a^2*c^2*d^2*sgn(d*x^2 + c))/(((sqrt(b*d)*x^2 - s
qrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*d + sqrt(b*d)*c)*d^4) + 3*(35*sqrt(
b*d)*b^3*c^3*sgn(d*x^2 + c) - 45*sqrt(b*d)*a*b^2*c^2*d*sgn(d*x^2 + c) + 9*s
qrt(b*d)*a^2*b*c*d^2*sgn(d*x^2 + c) + sqrt(b*d)*a^3*d^3*sgn(d*x^2 + c))*log
(abs(-2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*b*d - sqr
t(b*d)*b*c - sqrt(b*d)*a*d))/(b^2*d^5))*e^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.277 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=199

$$-\frac{c(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}-\frac{(9bc-5ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8d^3}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4d^3}+\frac{3(bc-ad)}{}$$

[Out] 3/8*(-a*d+b*c)*(-a*d+5*b*c)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/d^(7/2)/b^(1/2)-c*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3-1/8*(-5*a*d+9*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3+1/4*b*e*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3

Rubi [A]
time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 466, 1171, 396, 214}

$$\frac{3e^{3/2}(bc-ad)(5bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{be(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3}-\frac{e(c+dx^2)(9bc-5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3}-\frac{ce(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^3) - ((9*b*c - 5*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*d^3) + (b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^3) + (3*(b*c - a*d)*(5*b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*Sqrt[b]*d^(7/2))

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = ((bc-ad)e)\text{Subst}\left(\int\frac{x^4(-ae+cx^2)}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{be\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4d^3} + \frac{((bc-ad)e)\text{Subst}\left(\int\frac{-b(bc-ad)e^2-4d(bc-ad)ex^2-4cd^2x^4}{(be-dx^2)^2}\right.}{4d^3}$$

$$= -\frac{(9bc-5ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4d^3} - \frac{(bc-ac}{}$$

$$= -\frac{c(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc-5ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e\,(a}{c}}}{}$$

$$= -\frac{c(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc-5ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8d^3} + \frac{be\sqrt{\frac{e\,(a}{c}}}{}$$

Mathematica [**A**]
time = 2.25, size = 177, normalized size = 0.89

$$\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx^2}\,(ad(13c+5dx^2)+b(-15c^2-5cdx^2+2d^2x^4))+3(5b^2c^2-6abcd+a^2d^2)\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)\right)}{8\sqrt{b}\,d^{7/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*
(13*c + 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*(5*b^2*c^2 - 6*
a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[
b]*Sqrt[c + d*x^2])]))/(8*Sqrt[b]*d^(7/2)*Sqrt[a + b*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(173) = 346.
time = 0.12, size = 679, normalized size = 3.41

method	result

risch	$\frac{(2bd\,x^2+5ad-7bc)\,(d\,x^2+c)\,e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8d^3}+\frac{\left(\frac{3\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a^2}{16d\sqrt{deb}}-\frac{9\ln\left(\right.}{}\right.}{}$
default	$\frac{\left(4\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b\,d^2x^4+3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d^2*x^4+3*ln(1/2*
(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)
^(1/2))*a^2*d^3*x^2-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b*c*d^2*x^2+15*ln(1/2*(2*b*d*x^2+2*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c
^2*d*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d^2*x^2-10*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c*d*x^2+3*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^
2*c*d^2-18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1
/2)+a*d+b*c)/(b*d)^(1/2))*a*b*c^2*d+15*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c^3+10*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*c*d-14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(b*d)^(1/2)*b*c^2+16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d-
16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2)/d^3*(d*x^2+c)*(e*(b*x^2+a
)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [A]
time = 0.52, size = 282, normalized size = 1.42

$$\frac{1}{16}\left(\frac{2\left((9\,b^2c^2d-14\,abcd^2+5\,a^2d^3)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-(7\,b^3c^2-10\,ab^2cd+3\,a^2bd^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^2d^3-\frac{2\,(bx^2+a)bd^4}{dx^2+c}+\frac{(bx^2+a)^2d^5}{(dx^2+c)^2}}-\frac{16\,(bc^2-acd)\sqrt{\frac{bx^2+a}{dx^2+c}}}{d^3}-\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,d^3}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*((9*b^2*c^2*d - 14*a*b*c*d^2 + 5*a^2*d^3)*((b*x^2 + a)/(d*x^2 + c))
^(3/2) - (7*b^3*c^2 - 10*a*b^2*c*d + 3*a^2*b*d^2)*sqrt((b*x^2 + a)/(d*x^2 +
 c)))/(b^2*d^3 - 2*(b*x^2 + a)*b*d^4/(d*x^2 + c) + (b*x^2 + a)^2*d^5/(d*x^2
 + c)^2) - 16*(b*c^2 - a*c*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/d^3 - 3*(5*b^2*
```

```
c^2 - 6*a*b*c*d + a^2*d^2)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d)
)/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*d^3))*e^(3/2)
```

Fricas [**A**]
time = 0.53, size = 396, normalized size = 1.99

$$\left[\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)\sqrt{bd}\,e^{\frac{3}{2}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)+4\,(2\,b^2d^3x^4-15\,b^2c^2d+13\,abcd^2-5\,(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{32\,bd^4},\ -\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)e^{\frac{3}{2}}-2\,(2\,b^2d^3x^4-15\,b^2c^2d+13\,abcd^2-5\,(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{16\,bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*sqrt(b*d)*e^(3/2)*log(8*b^2*d^2*
x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^
2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(
d*x^2 + c))) + 4*(2*b^2*d^3*x^4 - 15*b^2*c^2*d + 13*a*b*c*d^2 - 5*(b^2*c*d^
2 - a*b*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(b*d^4), -1/16*(3*
(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d))*e^(3/2)
- 2*(2*b^2*d^3*x^4 - 15*b^2*c^2*d + 13*a*b*c*d^2 - 5*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(b*d^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.64, size = 321, normalized size = 1.61

$$\frac{1}{16}\left(2\sqrt{bdx^4+bcx^2+adx^2+ac}\left(\frac{2\,bx^2\mathrm{sgn}(dx^2+c)}{d^2}-\frac{7\,b^2cd^5\mathrm{sgn}(dx^2+c)-5\,abd^6\mathrm{sgn}(dx^2+c)}{bd^8}\right)-\frac{16\,(b^2c^3\mathrm{sgn}(dx^2+c)-2\,abc^2d\mathrm{sgn}(dx^2+c)+a^2cd^2\mathrm{sgn}(dx^2+c))}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)d+\sqrt{bd}\,c\right)d^3}-\frac{3\left(5\sqrt{bd}\,b^2c^2\mathrm{sgn}(dx^2+c)-6\sqrt{bd}\,abcd\mathrm{sgn}(dx^2+c)+\sqrt{bd}\,a^2d^2\mathrm{sgn}(dx^2+c)\right)\log\left(\left|-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{bd^4}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*b*x^2*sgn(d*x^2 + c)/d^2
 - (7*b^2*c*d^5*sgn(d*x^2 + c) - 5*a*b*d^6*sgn(d*x^2 + c))/(b*d^8)) - 16*(b
^2*c^3*sgn(d*x^2 + c) - 2*a*b*c^2*d*sgn(d*x^2 + c) + a^2*c*d^2*sgn(d*x^2 +
c))/(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*d + sqrt(b*
d)*c)*d^3) - 3*(5*sqrt(b*d)*b^2*c^2*sgn(d*x^2 + c) - 6*sqrt(b*d)*a*b*c*d*sg
n(d*x^2 + c) + sqrt(b*d)*a^2*d^2*sgn(d*x^2 + c))*log(abs(-2*(sqrt(b*d)*x^2
```

```
- sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*
a*d))/(b*d^4))*e^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.278 $\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=141

$$\frac{3(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d}-\frac{3\sqrt{b}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

```
[Out] 1/2*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)/d-3/2*(-a*d+b*c)*e^(3/2)*arctan
h(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))*b^(1/2)/d^(5/2)+3/
2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1981, 1979, 294, 327, 214}

$$-\frac{3\sqrt{b}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2}+\frac{(c+dx^2)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*d^2) + (((e*(a + b*x
^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2
*d^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = ((bc-ad)e)\text{Subst}\left(\int\frac{x^4}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{(3(bc-ad)e)\text{Subst}\left(\int\frac{x^2}{be-dx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{2d}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{(3b(bc-ad)e^2)\,\text{Subst}\Big(}{}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}(c+dx^2)}{2d} - \frac{3\sqrt{b}\,(bc-ad)e^{3/2}\tanh}{2d}$$

Mathematica [A]

time = 1.40, size = 136, normalized size = 0.96

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\,\sqrt{a+bx^2}\,(3bc-2ad+bdx^2)-3\sqrt{b}\,(bc-ad)\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)\right)}{2d^{5/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[d]*Sqrt[a + b*x^2]*(3*b*c - 2*a*
d + b*d*x^2) - 3*Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[
a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(2*d^(5/2)*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(117) = 234$.
time = 0.10, size = 432, normalized size = 3.06

method	result
default	$\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}{2\sqrt{bd}}\right.\right.}{}$
risch	$\frac{(d\,x^2+c)be\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{2d^2}+\frac{\left(\frac{3b\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a}{4d\sqrt{deb}}-\frac{3b^2\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*a*b*d^2*x^2-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c*d*x^2+2*(b*d)^(1/2
)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*d*x^2+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d-3*ln
(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/
(b*d)^(1/2))*b^2*c^2+2*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c-
4*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d+4*(b*d)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*b*c)/d^2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(b*d)^(1/2)/(
(d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)
```

Maxima [A]

time = 0.52, size = 177, normalized size = 1.26

$$\frac{1}{4}\left(\frac{2\,(b^2c-abd)\sqrt{\frac{bx^2+a}{dx^2+c}}}{bd^2-\frac{(bx^2+a)d^3}{dx^2+c}}+\frac{3\,(bc-ad)b\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,d^2}+\frac{4\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{d^2}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^2*c - a*b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b*d^2 - (b*x^2 + a)*d
^3/(d*x^2 + c)) + 3*(b*c - a*d)*b*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sq
rt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*d^2) + 4
*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/d^2)*e^(3/2)
```

Fricas [A]
time = 0.45, size = 303, normalized size = 2.15

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{b}{d}}\,e^{\frac{3}{2}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bd^3x^4+bc^2d+acd^2+(3\,bcd^2+ad^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{\frac{b}{d}}\right)-4\,(bdx^2+3\,bc-2\,ad)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{8\,d^2},\ \frac{3\,(bc-ad)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{-\frac{b}{d}}}{2\,(b^2x^2+ab)}\right)e^{\frac{3}{2}}+2\,(bdx^2+3\,bc-2\,ad)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{4\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(b/d)*e^(3/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*
c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*
d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(b/d)) - 4
*(b*d*x^2 + 3*b*c - 2*a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/d^2, 1/4*
(3*(b*c - a*d)*sqrt(-b/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 +
a)/(d*x^2 + c))*sqrt(-b/d)/(b^2*x^2 + a*b))*e^(3/2) + 2*(b*d*x^2 + 3*b*c -
2*a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/d^2]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(113) = 226.
time = 5.49, size = 247, normalized size = 1.75

$$\frac{1}{4}\left(\frac{2\sqrt{bdx^4+bcx^2+adx^2+ac}\,b\mathrm{sgn}(dx^2+c)}{d^2}+\frac{4\left(b^2c^2\mathrm{sgn}(dx^2+c)-2\,abcd\mathrm{sgn}(dx^2+c)+a^2d^2\mathrm{sgn}(dx^2+c)\right)}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)d+\sqrt{bd}\,c\right)d^2}+\frac{3\left(\sqrt{bd}\,b^2c\mathrm{sgn}(dx^2+c)-\sqrt{bd}\,abd\mathrm{sgn}(dx^2+c)\right)\log\left(\left|-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{bd^3}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*b*sgn(d*x^2 + c)/d^2 + 4*(b^
2*c^2*sgn(d*x^2 + c) - 2*a*b*c*d*sgn(d*x^2 + c) + a^2*d^2*sgn(d*x^2 + c))/(
((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*d + sqrt(b*d)*c)
*d^2) + 3*(sqrt(b*d)*b^2*c*sgn(d*x^2 + c) - sqrt(b*d)*a*b*d*sgn(d*x^2 + c))
*log(abs(-2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*b*d -
 sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^3))*e^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.279 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=151

$$-\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

```
[Out] -a^(3/2)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1
/2))/c^(3/2)+b^(3/2)*e^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/d^(3/2)-(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d
```

Rubi [A]
time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 490, 536, 214}

$$-\frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}+\frac{b^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) - (a^(3/2)*e^(3/
2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/
c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
```

```
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-abe^2+(bc+ad)ex^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right.}{cd}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(a^2e^2)\,\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{c} + \frac{(b^2e^2)}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\tanh^{-}}{}$$

Mathematica [A]
time = 1.89, size = 187, normalized size = 1.24

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-a^{3/2}d^{3/2}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{c}\left(\sqrt{d}(-bc+ad)\sqrt{a+bx^2}+b^{3/2}c\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)\right)}{c^{3/2}d^{3/2}\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(a^(3/2)*d^(3/2)*Sqrt[c + d*x^2]*Arc
Tanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[c]*(Sqrt[
d]*(-(b*c) + a*d)*Sqrt[a + b*x^2] + b^(3/2)*c*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])))/(c^(3/2)*d^(3/2)*Sqrt[a
+ b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(121) = 242.
time = 0.08, size = 401, normalized size = 2.66

method	result
default	$\frac{\left(\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,b^2cd\,x^2-\sqrt{bd}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4}\ldots}{\ldots}\right.\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c*d*x^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*d^2*x^2+
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*c*d+2*(b*d)^(1/2)
*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d-2*(b*d)^(1/2)*(a*c)^(1/2)*((d*
x^2+c)*(b*x^2+a))^(1/2)*b*c)/c/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(a
*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [A]

time = 0.50, size = 185, normalized size = 1.23

$$\frac{1}{2}\left(\frac{a^2\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,c}-\frac{b^2\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,d}-\frac{2\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{cd}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 +
 a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*c) - b^2*log((d*sqrt((b*x^2 + a)/
(d*x^2 + c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(s
qrt(b*d)*d) - 2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(c*d))*e^(3/2)
```

Fricas [A]
time = 0.73, size = 983, normalized size = 6.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b/d)*e^(3/2)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*
c*d^2 + a*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(b/d)) + a*d*sqrt(a/c
)*e^(3/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2
 + a^2*c*d)*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d
)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(a/c))/x^4) - 4*(b*c - a*d)*sqrt((
b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(c*d), -1/4*(2*b*c*sqrt(-b/d)*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-b/d)/(b^2*x^2 +
a*b))*e^(3/2) - a*d*sqrt(a/c)*e^(3/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 +
2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(a/c))
/x^4) + 4*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(c*d), 1/4*(2*
a*d*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2
 + c))*sqrt(-a/c)/(a*b*x^2 + a^2))*e^(3/2) + b*c*sqrt(b/d)*e^(3/2)*log(8*b^
2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(
2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt((b*x^2 + a)
/(d*x^2 + c))*sqrt(b/d)) - 4*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3
/2))/(c*d), 1/2*(a*d*sqrt(-a/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((
```

```
b*x^2 + a)/(d*x^2 + c))*sqrt(-a/c)/(a*b*x^2 + a^2))*e^(3/2) - b*c*sqrt(-b/d
)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-b/
d)/(b^2*x^2 + a*b))*e^(3/2) - 2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*e
^(3/2))/(c*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

3.280 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$\frac{3(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}-\frac{3\sqrt{a}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2c^{5/2}}$$

```
[Out] 1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/c/(a-c*(b*x^2+a)/(d*x^2+c))-3/
2*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/
e^(1/2))*a^(1/2)/c^(5/2)+3/2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 294, 327, 214}

$$-\frac{3\sqrt{a}\,e^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{a}\,\sqrt{e}}\right)}{2c^{5/2}}+\frac{3e(bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{2c^2}+\frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*c^2) + ((b*c - a*d)*
((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))
) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^4}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc-ad)e)\text{Subst}\left(\int \frac{x^2}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)}{2c}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3a(bc-ad)e^2)\,\text{Subst}\left(\int \frac{1}{-ae+\cdots}\right.}{2c^2}$$

$$= \frac{3(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a}\,(bc-ad)e^{3/2}\tanh^{-1}\left(\cdots\right.}{2c^{5/2}}$$

Mathematica [A]
time = 4.02, size = 146, normalized size = 0.88

$$\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\,\sqrt{a+bx^2}\,(2bcx^2-a(c+3dx^2))-3\sqrt{a}\,(bc-ad)x^2\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)\right)}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*
x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(141) = 282$.
time = 0.13, size = 641, normalized size = 3.88

method	result

risch	$-\frac{a(dx^2+c)e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c^2x^2}+\frac{\left(\frac{3a^2\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}}{x^2}\right)d}{4c^2\sqrt{ace}}-\frac{3a\ln\left(\frac{2ac\cdots}{\cdots}\right)\cdots}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\left(-2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{ac}\,b\,d^2x^6-3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}+2\cdots}{x^2}\right)\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*d^2*x^6-3*ln((a*
d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)
*a^2*c*d^2*x^4+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^2*d*x^4-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*a*d^2*x^4-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*
c*d*x^4-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)+2*a*c)/x^2)*a^2*c^2*d*x^2+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^3*x^2+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*d*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c
)^(1/2)*a*c*d*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c^2*x
^2+4*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d*x^2-4*(a*c)^(1/2)*((d*x^
2+c)*(b*x^2+a))^(1/2)*b*c^2*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(a*c)^(1/2)/x^2/c^3/(b*x^
2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [A]
time = 0.54, size = 177, normalized size = 1.07

$$\frac{1}{4}\left(\frac{2\left(abc-a^2d\right)\sqrt{\frac{bx^2+a}{dx^2+c}}}{ac^2-\frac{(bx^2+a)c^3}{dx^2+c}}+\frac{3\,(bc-ad)a\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,c^2}+\frac{4\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{c^2}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(a*b*c - a^2*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*c^2 - (b*x^2 + a)*c
^3/(d*x^2 + c)) + 3*(b*c - a*d)*a*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sq
```

```
rt(a*c))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*c^2) + 4
*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/c^2)*e^(3/2)
```

Fricas [A]
time = 0.66, size = 330, normalized size = 2.00

$$\left[-\frac{3\,(bc-ad)x^2\sqrt{\frac{a}{c}}\,e^{\frac{3}{2}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,\left((bc^2d+acd^2)x^4+2\,ac^3+(bc^3+3\,ac^2d)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{\frac{a}{c}}}{x^4}\right)-4\,((2\,bc-3\,ad)x^2-ac)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{8\,c^2x^2},\ \frac{3\,(bc-ad)x^2\sqrt{-\frac{a}{c}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{-\frac{a}{c}}}{2\,(abx^2+a^2)}\right)e^{\frac{3}{2}}+2\,((2\,bc-3\,ad)x^2-ac)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{4\,c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*x^2*sqrt(a/c)*e^(3/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*
d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c^2*d + a*c*d^2)*x
^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(
a/c))/x^4) - 4*((2*b*c - 3*a*d)*x^2 - a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^
(3/2))/(c^2*x^2), 1/4*(3*(b*c - a*d)*x^2*sqrt(-a/c)*arctan(1/2*((b*c + a*d)
*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-a/c)/(a*b*x^2 + a^2))*e^(
3/2) + 2*((2*b*c - 3*a*d)*x^2 - a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))
/(c^2*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

3.281 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=256

$$-\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3}-\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{3(bc-5}{}$$

```
[Out] -3/8*(-5*a*d+b*c)*(-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^(1/2)/e^(1/2))/c^(7/2)/a^(1/2)-d*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2
+c))^(1/2)/c^3-1/4*a*(-a*d+b*c)^2*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/(a*
e-c*e*(b*x^2+a)/(d*x^2+c))^2+1/8*(-9*a*d+5*b*c)*(-a*d+b*c)*e^2*(e*(b*x^2+a)
/(d*x^2+c))^(1/2)/c^3/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.24, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 466, 1171, 396, 214}

$$-\frac{3e^{3/2}(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}\,c^{7/2}}-\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{de(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^3) - (a*(b*c - a*d)
^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^3*(a*e - (c*e*(a + b*x^2))/(
c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)])/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*
(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sq
rt[a]*Sqrt[e])])/(8*Sqrt[a]*c^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(be-dx^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)e)\text{Subst}\left(\int \frac{-a(bc-ad)e^2-4c(bc-ad)ex^2+4c^2dx^4}{(-ae+cx^2)^2}\,dx\right.}{4c^3}$$

$$= -\frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\text{Su}}{}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2}{8c^3\left(ae-\frac{ce(a-}{c+}\right.}$$

$$= -\frac{d(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc-ad)^2e^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{(5bc-9ad)(bc-ad)e^2}{8c^3\left(ae-\frac{ce(a-}{c+}\right.}$$

Mathematica [A]
time = 4.47, size = 186, normalized size = 0.73

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\,(bcx^2(5c+13dx^2)+a(2c^2-5cdx^2-15d^2x^4))+3(b^2c^2-6abcd+5a^2d^2)x^4\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{8\sqrt{a}\,c^{7/2}x^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] -1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*
(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^
2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*
x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(230) = 460.
time = 0.15, size = 1042, normalized size = 4.07

method	result
risch	$-\frac{(d x^2+c)(-7ad x^2+5bc x^2+2ac)e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{8c^3x^4}+\frac{\left(-\frac{15\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\sqrt{deb x^4+(ade+bce)x^2 -}}{x^2}\right.}{16c^3\sqrt{ace}}\right.}{}$
default	$-\frac{\left(18\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,ab d^3x^8-6\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,b^2c d^2x^8+15\ln\left(\frac{ad}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b*d^3*x^8-6*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^2*c*d^2*x^8+15*ln((a*d*x^2+b
*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*c*
d^3*x^6-18*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)+2*a*c)/x^2)*a^2*b*c^2*d^2*x^6+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b^2*c^3*d*x^6+18*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*d^3*x^6+26*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(a*c)^(1/2)*a*b*c*d^2*x^6-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(a*c)^(1/2)*b^2*c^2*d*x^6+15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*c^2*d^2*x^4-18*ln((a*d*x^2+b*c*x
^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*b*c^3*
d*x^4+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)+2*a*c)/x^2)*a*b^2*c^4*x^4-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
1/2)*a*d^2*x^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*b*c*d*x^4+
18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*c*d^2*x^4+8*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b*c^2*d*x^4-6*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^2*c^3*x^4-16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a
))^(1/2)*a^2*c*d^2*x^4+16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c^2*d
*x^4-14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*c*d*x^2+6*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*b*c^2*x^2+4*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(3/2)*(a*c)^(1/2)*a*c^2)/a*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/
(a*c)^(1/2)/x^4/c^4/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Maxima [A]

time = 0.52, size = 282, normalized size = 1.10

$$-\frac{1}{16}\left(\frac{2\left(\left(5\,b^2c^3-14\,abc^2d+9\,a^2cd^2\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-\left(3\,ab^2c^2-10\,a^2bcd+7\,a^3d^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{a^2c^3-\frac{2\,(bx^2+a)ac^4}{dx^2+c}+\frac{(bx^2+a)^2c^5}{(dx^2+c)^2}}+\frac{16\,(bcd-ad^2)\sqrt{\frac{bx^2+a}{dx^2+c}}}{c^3}-\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,c^3}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*(2*((5*b^2*c^3 - 14*a*b*c^2*d + 9*a^2*c*d^2)*((b*x^2 + a)/(d*x^2 + c)
)^(3/2) - (3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*sqrt((b*x^2 + a)/(d*x^2
+ c)))/(a^2*c^3 - 2*(b*x^2 + a)*a*c^4/(d*x^2 + c) + (b*x^2 + a)^2*c^5/(d*x^
2 + c)^2) + 16*(b*c*d - a*d^2)*sqrt((b*x^2 + a)/(d*x^2 + c))/c^3 - 3*(b^2*c
^2 - 6*a*b*c*d + 5*a^2*d^2)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c
))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*c^3))*e^(3/2)
```

Fricas [A]
time = 1.23, size = 422, normalized size = 1.65

$$\left[\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)\sqrt{ac}\,x^4e^{\frac{3}{2}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{x^4}\right)-4\,(2\,a^2c^3+(13\,abc^2d-15\,a^2cd^2)x^4+5\,(abc^3-a^2c^2d)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{32\,ac^4x^4},\ \frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)\sqrt{-ac}\,x^4\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{-ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(abcx^2+a^2c)}\right)e^{\frac{3}{2}}-2\,(2\,a^2c^3+(13\,abc^2d-15\,a^2cd^2)x^4+5\,(abc^3-a^2c^2d)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{16\,ac^4x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*sqrt(a*c)*x^4*e^(3/2)*log(((b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*
((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x
^2 + a)/(d*x^2 + c)))/x^4) - 4*(2*a^2*c^3 + (13*a*b*c^2*d - 15*a^2*c*d^2)*x
^4 + 5*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(a
*c^4*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*sqrt(-a*c)*x^4*arctan(
1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b
*c*x^2 + a^2*c))*e^(3/2) - 2*(2*a^2*c^3 + (13*a*b*c^2*d - 15*a^2*c*d^2)*x^4
 + 5*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(a*c
^4*x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

3.282 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=366

$$\frac{d^2(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^4}+\frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}+\frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}-\frac{(bc-ad)\,(}{}$$

```
[Out] 1/6*(-a*d+b*c)^3*e^2*(e*(b*x^2+a)/(d*x^2+c))^(5/2)/a/c^2/(a*e-c*e*(b*x^2+a)
/(d*x^2+c))^3+1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*e^(3/2)*arct
anh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/c^(9/2)+
d^2*(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4+1/24*(-a*d+b*c)^2*(11*a*
d+b*c)*e^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^
2-1/48*(-a*d+b*c)*(-79*a^2*d^2+50*a*b*c*d+5*b^2*c^2)*e^2*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^4/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.37, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1981, 1980, 474, 466, 1171, 396, 214}

$$-\frac{e^2(-79a^2d^2+50abcd+5b^2c^2)\,(bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{48ac^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{e^{3/2}(-35a^2d^2+10abcd+b^2c^2)\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}+\frac{d^2e(bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^4}+\frac{e^3(11ad+bc)(bc-ad)^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}+\frac{e^2(bc-ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^4 + ((b*c - a*d)^3*
e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/
(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)
*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)
])/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2
 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(
c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{x^4(be-dx^2)^2}{(-ae+cx^2)^4}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)\text{Subst}\left(\int \frac{x^4(-6b^2c^2e^2+5(bce-ade)^2+6acd^2ex^2)}{(-ae+cx^2)^3}\,dx, x, \sqrt{\cdots}\right)}{6ac^2}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\text{Subst}\cdots}{\cdots}$$

$$= \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{24c^4\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)\,(5b^2c^2\cdots}{\cdots}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\cdots}}{24c^4\left(ae-\frac{ce(a+bx^2}{c+dx^2}\cdots\right.}$$

$$= \frac{d^2(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc-ad)^3e^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc-ad)^2(bc+11ad)e^3\sqrt{\cdots}}{24c^4\left(ae-\frac{ce(a+bx^2}{c+dx^2}\cdots\right.}$$

Mathematica [A]
time = 4.94, size = 245, normalized size = 0.67

$$\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(-\sqrt{a}\,\sqrt{c}\,\sqrt{a+bx^2}\,(3b^2c^2x^4(c+dx^2)+2abcx^2(7c^2-19cdx^2-50d^2x^4)+a^2(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6))+3(b^3c^3+9ab^2c^2d-45a^2bcd^2+35a^3d^3)\,x^6\sqrt{c+dx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)\right)}{48a^{3/2}c^{9/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3*
b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a
^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a
*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*x^6*S
qrt[a + b*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1497 vs. $2(336) = 672$.
time = 0.15, size = 1498, normalized size = 4.09

method	result
risch	$-\frac{(d\,x^2+c)\left(57d^2a^2x^4-52abcd\,x^4+3b^2c^2x^4-22a^2dc\,x^2+14ab\,c^2x^2+8a^2c^2\right)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{48c^4x^6a}+\frac{\left(35a^2\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{a\ldots}}{}\right)\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b^2*c*d^3*x^10-
216*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*b*c*d^3*x^8+138*(b*
d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b^2*c^2*d^2*x^8-72*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*b*c*d^2*x^6-42*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*b*c^2*d^2*x^6+66*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*a*b^2*c^3*d*x^6-96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))
^(1/2)*a^2*b*c^2*d^2*x^6-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)
*a*b*c^2*d*x^4+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^3-1
74*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*b*d^4*x^10+6*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^3*c^2*d^2*x^10+135*ln((a*d*x^2+b
*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*b*
c^2*d^3*x^8-27*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+2*a*c)/x^2)*a^2*b^2*c^3*d^2*x^8-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b^3*c^4*d*x^8+12*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^3*c^3*d*x^8+135*ln((a*d*x^2+b*
c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*b*c
^3*d^2*x^6-27*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)+2*a*c)/x^2)*a^2*b^2*c^4*d*x^6-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2
)*(a*c)^(1/2)*b^2*c^2*d*x^6-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(
1/2)*a^3*c*d^3*x^6+96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^3*c*d^3*x^6
+114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c*d^2*x^4-44*(b*d*
```

x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^2*d*x^2+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*b*c^3*x^2-105*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^4*c*d^4*x^8-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^3*d^4*x^8-105*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^4*c^2*d^3*x^6-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b^3*c^5*x^6+174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*d^3*x^6+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^3*c^4*x^6-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*b^2*c^3*x^4)/a^2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(a*c)^(1/2)/x^6/c^5/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)

Maxima [A]
time = 0.52, size = 425, normalized size = 1.16

$$\frac{1}{96}\left(\frac{2\left(3\left(b^3c^5-23\,ab^2c^4d+51\,a^2bc^3d^2-29\,a^3c^2d^3\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}+8\left(ab^3c^4+9\,a^2b^2c^3d-27\,a^3bc^2d^2+17\,a^4cd^3\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-3\left(a^2b^3c^3+9\,a^3b^2c^2d-29\,a^4bcd^2+19\,a^5d^3\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{a^4c^4-\frac{3\,(bx^2+a)a^3c^5}{dx^2+c}+\frac{3\,(bx^2+a)^2a^2c^6}{(dx^2+c)^2}-\frac{(bx^2+a)^3ac^7}{(dx^2+c)^3}}+\frac{96\left(bcd^2-ad^3\right)\sqrt{\frac{bx^2+a}{dx^2+c}}}{c^4}-\frac{3\left(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3\right)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,ac^4}\right)e^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")

[Out] 1/96*(2*(3*(b^3*c^5 - 23*a*b^2*c^4*d + 51*a^2*b*c^3*d^2 - 29*a^3*c^2*d^3)*((b*x^2 + a)/(d*x^2 + c))^(5/2) + 8*(a*b^3*c^4 + 9*a^2*b^2*c^3*d - 27*a^3*b*c^2*d^2 + 17*a^4*c*d^3)*((b*x^2 + a)/(d*x^2 + c))^(3/2) - 3*(a^2*b^3*c^3 + 9*a^3*b^2*c^2*d - 29*a^4*b*c*d^2 + 19*a^5*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c)))/(a^4*c^4 - 3*(b*x^2 + a)*a^3*c^5/(d*x^2 + c) + 3*(b*x^2 + a)^2*a^2*c^6/(d*x^2 + c)^2 - (b*x^2 + a)^3*a*c^7/(d*x^2 + c)^3) + 96*(b*c*d^2 - a*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c))/c^4 - 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a*c^4))*e^(3/2)

Fricas [A]
time = 4.46, size = 552, normalized size = 1.51

$$\left[\frac{3\left(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3\right)\sqrt{ac}\,x^6e^{\frac{3}{2}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2+4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{x^4}\right)-4\left(8\,a^3c^4+(3\,ab^2c^3d-100\,a^2bc^2d^2+105\,a^3cd^3)x^6+(3\,ab^2c^4-38\,a^2bc^3d+35\,a^3c^2d^2)x^4+14\,(a^2bc^4-a^3c^3d)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{192\,a^2c^5x^6},\ -\frac{3\left(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3\right)\sqrt{-ac}\,x^6\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{-ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(abcx^2+a^2c)}\right)e^{\frac{3}{2}}+2\left(8\,a^3c^4+(3\,ab^2c^3d-100\,a^2bc^2d^2+105\,a^3cd^3)x^6+(3\,ab^2c^4-38\,a^2bc^3d+35\,a^3c^2d^2)x^4+14\,(a^2bc^4-a^3c^3d)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\,e^{\frac{3}{2}}}{96\,a^2c^5x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")

[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*sqrt(a*c)*x^6*e^(3/2)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^4 + (3*a*b^2*c^3*d - 100*a^2*b*c^2*d^2 + 105*a^3*c*d^3)*x^6 + (3*a*b^2*c^4 - 38*a^2*b*c^3*d + 35*a^3*c^2*d^2)*x^4 + 14*(a^2*b*c^4 - a^3*c^3*d)*x^2)*sqrt((b*x^2 +

```
a)/(d*x^2 + c))*e^(3/2))/(a^2*c^5*x^6), -1/96*(3*(b^3*c^3 + 9*a*b^2*c^2*d -
 45*a^2*b*c*d^2 + 35*a^3*d^3)*sqrt(-a*c)*x^6*arctan(1/2*((b*c + a*d)*x^2 +
2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2 + a^2*c))*e^(3/2
) + 2*(8*a^3*c^4 + (3*a*b^2*c^3*d - 100*a^2*b*c^2*d^2 + 105*a^3*c*d^3)*x^6
+ (3*a*b^2*c^4 - 38*a^2*b*c^3*d + 35*a^3*c^2*d^2)*x^4 + 14*(a^2*b*c^4 - a^3
*c^3*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*e^(3/2))/(a^2*c^5*x^6)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

3.283 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=391

$$-\frac{\left(16ac-\frac{16bc^2}{d}-\frac{a^2d}{b}\right)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}-\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx}{5d^3}$$

```
[Out] -1/5*(16*a*c-16*b*c^2/d-a^2*d/b)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x^
3*(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d-1/5*(-7*a*d+8*b*c)*e*x*(d*x^2+c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^3+6/5*b*e*x^3*(d*x^2+c)*(e*(b*x^2+a)/(d*x
^2+c))^(1/2)/d^2+1/5*c^(3/2)*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*
(a^2*d^2-16*a*b*c*d+16*b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/b/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1986, 478, 595, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,e(a^2d^2-16abcd+16b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5bd^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex\left(-\frac{a^2d}{b}+16ac-\frac{16bc^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}+\frac{c^{3/2}e(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(c+dx^2)(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^3}+\frac{6bex^3(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5d^2}-\frac{ex^3(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/5*((16*a*c - (16*b*c^2)/d - (a^2*d)/b)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)])/d^2 - (e*x^3*(a + b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d - ((8*b
*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^3) + (6
*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*d^2) - (Sqrt[c]*
(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b*d^(7/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*(8*b*c - 7*a*d)*e*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(5*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{\left(e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int\frac{x^4(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} + \frac{\left(e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int\frac{x^2\sqrt{a+bx^2}\,(}{\sqrt{c+dx}}}{d\sqrt{a+bx^2}}$$

$$= -\frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} + \frac{6bex^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5d^2} + \frac{\left(e\sqrt{\frac{e\,(a+}{c+d}}\right.}{}$$

$$= -\frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5d^3} + \frac{6bex}{}$$

$$= -\frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} - \frac{(8bc-7ad)ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5d^3} + \frac{6bex}{}$$

$$= \frac{(16b^2c^2-16abcd+a^2d^2)\,ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5bd^3} - \frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} - \frac{(8}{}$$

$$= \frac{(16b^2c^2-16abcd+a^2d^2)\,ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5bd^3} - \frac{ex^3(a+bx^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{d} - \frac{(8}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.76, size = 290, normalized size = 0.74

$$\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,dx(a^2d(7c+2dx^2)+b^2x^2(-8c^2-2cdx^2+d^2x^4)+ab(-8c^2+5cdx^2+3d^2x^4))-ic(16b^2c^2-16abcd+a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+8ic(2b^2c^2-3abcd+a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{5\sqrt{\frac{b}{a}}\,d^4\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a^2*d*(7*c + 2*d*x^2)
+ b^2*x^2*(-8*c^2 - 2*c*d*x^2 + d^2*x^4) + a*b*(-8*c^2 + 5*c*d*x^2 + 3*d^2*
```

```
x^4)) - I*c*(16*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^
2*c^2 - 3*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. $2(427) = 854$.
time = 0.11, size = 933, normalized size = 2.39

method	result
risch	$\frac{x(bd\,x^2+2ad-3bc)(d\,x^2+c)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{5d^3}+\frac{\left(-\frac{2(a^2d^2-11abcd+11b^2c^2)ace\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{deb\,x^4+ade\,x^2+bce\,x\cdots}}\right.}{\cdots}$
default	$\frac{\left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{\frac{3}{2}}(d\,x^2+c)\left(\sqrt{-\frac{b}{a}}\sqrt{(d\,x^2+c)(b\,x^2+a)}\,b^2d^3x^7+3\sqrt{-\frac{b}{a}}\sqrt{(d\,x^2+c)(b\,x^2+a)}\,abd^3x^5-2\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((-b/a)^(1/2)*((d*x^2+c)*(b*x^2
+a))^(1/2)*b^2*d^3*x^7+3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^3*x
^5-2*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c*d^2*x^5+5*(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d^2*x^3-5*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c^2*d*x^3+2*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a)
)^(1/2)*a^2*d^3*x^3-3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*d*x^
3-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d^2+24*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^
2+a))^(1/2)*a*b*c^2*d-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^3+((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d^2-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)
*a*b*c^2*d+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^3+5*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*c*d^2*x-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(-b/a)^(1/2)*a*b*c^2*d*x+2*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)
```

```
*a^2*c*d^2*x-3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c^2*d*x)/d^4/(b
*x^2+a)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] e^(3/2)*integrate(x^4*((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(x^4*((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.284 $\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=310

$$-\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}+\frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2}+\frac{\sqrt{c}\,(8bc-7ad}{}$$

```
[Out] -1/3*(-7*a*d+8*b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^2-e*x*(b*x^2+a)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)/d+4/3*b*e*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1
/2)/d^2+1/3*(-7*a*d+8*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4
*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 478, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,e(4bc-3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{\sqrt{c}\,e(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{4bex(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(8bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2}-\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/3*((8*b*c - 7*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d^2 - (e*x*(a
+ b*x^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/d + (4*b*e*x*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2))/(3*d^2) + (Sqrt[c]*(8*b*c - 7*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (Sqrt[c]*(4*b*c
- 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{x^2(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{a+bx^2}\,(a+4\cdots}{\sqrt{c+dx^2}\cdots}}{d\sqrt{a+bx^2}\cdots}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} + \frac{\left(e\sqrt{\frac{e(a+bx\cdots}{c+dx^2\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{3d^2} - \frac{\left(b(8bc-7ad)\cdots\right.}{\cdots}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+\cdots}{c+d\cdots}}}{3\cdots}$$

$$= -\frac{(8bc-7ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3d^2} - \frac{ex(a+bx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d} + \frac{4bex\sqrt{\frac{e(a+\cdots}{c+d\cdots}}}{3\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.09, size = 235, normalized size = 0.76

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,dx(a+bx^2)(3ad-b(4c+dx^2))+ibc(-8bc+7ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)+i(8b^2c^2-11abcd+3a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)\right)}{3\sqrt{\frac{b}{a}}\,d^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/3*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*d
 - b*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2 - 1
1*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^3*(a + b*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 737 vs. $2(350) = 700$.

time = 0.08, size = 738, normalized size = 2.38

method	result
default	$-\frac{\left(\frac{e(b x^2+a)}{d x^2+c}\right)^{\frac{3}{2}} (d x^2+c)\left(-\sqrt{-\frac{b}{a}}\,\sqrt{(d x^2+c)(b x^2+a)}\, b^2 d^2 x^5+3\sqrt{bd\, x^4+ad\, x^2+bc\, x^2+ac}\,\sqrt{-\frac{b}{a}}\, ab\cdots\right.}{\cdots}$
risch	$\frac{bex(d x^2+c)\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{3d^2}+\frac{\left(-\frac{2(4d^2ab-5b^2cd)ace\sqrt{1+\frac{b x^2}{a}}\,\sqrt{1+\frac{d x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{E}\cdots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\, x^4+ade\, x^2+bce\, x^2+ace}\,(ade+bce\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(-(-b/a)^(1/2)*((d*x^2+c)*(b*x
^2+a))^(1/2)*b^2*d^2*x^5+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)
*a*b*d^2*x^3-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3
-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^2*x^3-(-b/a)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*b^2*c*d*x^3-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^
2+11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/
b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d-8*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+
a))^(1/2)*b^2*c^2-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d+8*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d
*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/
a)^(1/2)*a^2*d^2*x-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c
*d*x-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x)/(b*x^2+a)^2/d^3/(-
b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(x^2*((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.285 $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=262

$$-\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}-\frac{(2bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1\right.}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -(-a*d+b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d+(-a*d+2*b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c/d-(-a*d+2*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b
*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/d^(3
/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1986, 424, 545, 429, 506, 422}

$$\frac{b\sqrt{c}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{ex(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}+\frac{ex(2bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b*c - a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d)) + ((2*b*c - a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d) - ((2*b*c - a*d)*e*Sqrt[(e*
(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/
(a*d)])/(Sqrt[c]*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*Sqrt[c
]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, 1 - (b*c)/(a*d)])/(d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+d\ldots}}}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{\left(abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+\ldots}}}{d\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{b\sqrt{c}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\ldots}{d^{3/2}\ldots}$$

$$= -\frac{(bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(2bc-ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{(2bc-ad)e\sqrt{\frac{e(a+\ldots}{c+\ldots}}\ldots}{\sqrt{c}\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.99, size = 206, normalized size = 0.79

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc+ad)\left(\sqrt{\frac{b}{a}}dx(a+bx^2)-2ibc\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)\right)}{\sqrt{\frac{b}{a}}cd^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-
(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqr
t[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqrt[b/
a]*c*d^2*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 527, normalized size = 2.01

method	result
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{3}-\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*a*b*d^2*x^3-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)
*b^2*c*d*x^3+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(
1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d-2*((b*x^2+a)/a)^(
1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+
c)*(b*x^2+a))^(1/2)*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d+2*((
b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1
/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(-b/a)^(1/2)*a^2*d^2*x-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a
*b*c*d*x)/(b*x^2+a)^2/d^2/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.286 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=307

$$-\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx}-\frac{(bc-2ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}+\frac{(bc-2ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx}+\frac{(bc-2ad)}{}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x-(-2*a*d+b*c)*e*x*(e*(b*x^
2+a)/(d*x^2+c))^(1/2)/c^2+(-2*a*d+b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/c^2/d/x+(-2*a*d+b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*e*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^
2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\frac{e(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{e(c+dx^2)(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2dx}-\frac{ex(bc-2ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^2}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x)) - ((b*c - 2*a*
d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/c^2 + ((b*c - 2*a*d)*e*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(c^2*d*x) + ((b*c - 2*a*d)*e*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*e*Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^2(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx} - \frac{\left(e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,\sqrt{c+dx^2}\right)\int \frac{a(bc-2ad)-abdx^2}{x^2\sqrt{a+bx^2}\,\sqrt{c+dx^2}}}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{\left(e\sqrt{\frac{e\,(a+bx^2}{c+dx^2}}\right.}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{c^2dx} + \frac{\left(abe\sqrt{\frac{e\,(a+b}{c+dx}}\right.}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e\,(a+bx}{c+dx^2}}}{c^2dx}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx} - \frac{(bc-2ad)ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{c^2} + \frac{(bc-2ad)e\sqrt{\frac{e\,(a+bx}{c+dx^2}}}{c^2dx}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.38, size = 228, normalized size = 0.74

$$-\frac{e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,d(a+bx^2)\,(ac-bcx^2+2adx^2)+ibc(-bc+2ad)x\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ibc(-bc+ad)x\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}\,c^2dx\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c*
x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c) + a
*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))
```

Maple [A]
time = 0.07, size = 670, normalized size = 2.18

method	result
default	$-\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{4}-\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\cdots}\right.}{\cdots}$
risch	$-\frac{a\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{c^{2}x}+\frac{\left(-\frac{2a^{2}bdce\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(x\cdots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,(ade+bce+e(a\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(-b/a)^(1/2)*a*b*d^2*x^4-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2
)*b^2*c*d*x^4+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^2*x^4+((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*(
(d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2
*c^2*x-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x+((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*
x^2+a))^(1/2)*b^2*c^2*x+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^
2*d^2*x^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+(-b/
a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^2*x^2+(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*a*b*c*d*x^2+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d
)/(b*x^2+a)^2/c^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2, x)
```

3.287 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4} dx$

Optimal. Leaf size=383

$$-\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3}+\frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}+\frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3}-\frac{(7bc-8}{}$$

[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^3+1/3*d*(-8*a*d+7*b*c)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3+1/3*(-4*a*d+3*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^3-1/3*(-8*a*d+7*b*c)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/x+1/3*b*(-4*a*d+3*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)

Rubi [A]

time = 0.34, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\frac{be(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{d}\,e(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{e(c+dx^2)(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3x}+\frac{dex(7bc-8ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3}+\frac{e(c+dx^2)(3bc-4ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^2dx^3}-\frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3}$$

Antiderivative was successfully verified.

[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]

[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^3)) + (d*(7*b*c - 8*a*d)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(3*c^3) + ((3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^2*d*x^3) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(3*c^3*x) - (Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*c^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (b*(3*b*c - 4*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*c^(3/2)*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^4(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(3bc-4ad)+b(2bc-3ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} + \frac{\left(e\sqrt{\frac{e(a+bx^2}{c+dx^2}}\right.}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{}}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{3c^2dx^3} - \frac{(7bc-8ad)e\sqrt{}}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+}{c+d}}}{3c^2dx^3}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^3} + \frac{d(7bc-8ad)ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{3c^3} + \frac{(3bc-4ad)e\sqrt{\frac{e(a+}{c+d}}}{3c^2dx^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.81, size = 275, normalized size = 0.72

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}\left(b^2cx^4(4c+7dx^2)+a^2(c^2-4cdx^2-8d^2x^4)+abx^2(5c^2+3cdx^2-8d^2x^4)\right)+ibc(-7bc+8ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-4ibc(-bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{3\sqrt{\frac{b}{a}}\,c^3x^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(b^2*c*x^4*(4*c + 7*d*x^2
) + a^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*x^2*(5*c^2 + 3*c*d*x^2 - 8*d^2*
x^4))) + I*b*c*(-7*b*c + 8*a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x
], (a*d)/(b*c)]))/(3*Sqrt[b/a]*c^3*x^3*(a + b*x^2))
```

Maple [A]

time = 0.08, size = 791, normalized size = 2.07

method	result
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(3\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{6}-3\sqrt{bd\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\cdots}\right.}{\cdots}$
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-5ad\,x^{2}+4bc\,x^{2}+ac\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{3c^{3}x^{3}}-\frac{\left(-\frac{10d^{2}a^{2}bce\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^{4}+ade\,x^{2}+bce\,x\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/
a)^(1/2)*b^2*c*d*x^6+5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^2*x^6
-4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c*d*x^6+4*((b*x^2+a)/a)^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)
*(b*x^2+a))^(1/2)*a*b*c*d*x^3-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*
x^3-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x^3+7*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b
*x^2+a))^(1/2)*b^2*c^2*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/
2)*a^2*d^2*x^4-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x
^4+5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^2*x^4-4*(-b/a)^(1/2)*((
d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*x^4+4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^
(1/2)*a^2*c*d*x^2-5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c^2*x^2-(-
b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c^2)/(b*x^2+a)^2/c^3/x^3/(-b/a)^
(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
[Out] e^(3/2)*integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)/x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
[Out] integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x)
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4, x)
```

3.288 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=480

$$-\frac{(bc-ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx^5}+\frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5ac^4}+\frac{(5bc-6ad)e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+}{5c^2dx^5}$$

```
[Out] -(-a*d+b*c)*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c/d/x^5+1/5*d*(16*a^2*d^2-16*a*
b*c*d+b^2*c^2)*e*x*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4+1/5*(-6*a*d+5*b*c)*e
*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^2/d/x^5-1/5*(-8*a*d+7*b*c)*e*(d*
x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(16*a^2*d^2-16*a*b*c*d+b^2
*c^2)*e*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^4/x-1/5*(16*a^2*d^2-16*
a*b*c*d+b^2*c^2)*e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/a/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*b*(-8*a*d+7*b*c)*
e*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/c^(
5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$-\frac{\sqrt{d}\,e(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{e(c+dx^2)\,(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5ac^4x}+\frac{dex(16a^2d^2-16abcd+b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5ac^4}-\frac{b\sqrt{d}\,e(7bc-8ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}}-\frac{e(c+dx^2)\,(7bc-8ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5c^3x^3}+\frac{e(c+dx^2)\,(5bc-6ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{5c^2dx^5}-\frac{e(bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{cdx^5}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c*d*x^5)) + (d*(b^2*c^
2 - 16*a*b*c*d + 16*a^2*d^2)*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(5*a*c^
4) + ((5*b*c - 6*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(5*c
^2*d*x^5) - ((7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2
))/(5*c^3*x^3) - ((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*(c + d*x^2))/(5*a*c^4*x) - (Sqrt[d]*(b^2*c^2 - 16*a*b*c*d +
16*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]) - (b*Sqrt[d]*(7*b*c - 8*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Ell
ipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*a*c^(5/2)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{(a+bx^2)^{3/2}}{x^6(c+dx^2)^{3/2}}\,dx}{\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} - \frac{\left(e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\right)\int \frac{a(5bc-6ad)+b(4bc-5ad)x^2}{x^6\sqrt{a+bx^2}\sqrt{c+dx^2}}}{cd\sqrt{a+bx^2}}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} + \frac{\left(e\sqrt{\frac{e(a+bx^2}{c+dx^2}}\right.}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{}}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{}}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{(5bc-6ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{5c^2dx^5} - \frac{(7bc-8ad)e\sqrt{}}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6}{}$$

$$= -\frac{(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cdx^5} + \frac{d(b^2c^2-16abcd+16a^2d^2)\,ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{5ac^4} + \frac{(5bc-6}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.34, size = 357, normalized size = 0.74

$$-\frac{\sqrt{\frac{b}{a}}\,e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,(b^3c^2x^6(c+dx^2)+ab^2cx^4(3c^2-8cdx^2-16d^2x^4)+a^2bx^2(3c^3-11c^2dx^2-8cd^2x^4+16d^3x^6)+a^3(c^3-2c^2dx^2+8cd^2x^4+16d^3x^6))+ibc(b^2c^2-16abcd+16a^2d^2)\,x^5\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ibc(b^2c^2-9abcd+8a^2d^2)\,x^5\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{5bc^4x^5\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(b^3*c^2*x^6
*(c + d*x^2) + a*b^2*c*x^4*(3*c^2 - 8*c*d*x^2 - 16*d^2*x^4) + a^2*b*x^2*(3*
c^3 - 11*c^2*d*x^2 - 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*(c^3 - 2*c^2*d*x^2 + 8
*c*d^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x^5*S
qrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c
^4*x^5*(a + b*x^2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(512) = 1024$.
time = 0.09, size = 1197, normalized size = 2.49 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(5*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(-b/a)^(1/2)*a^2*b*d^3*x^8-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(
-b/a)^(1/2)*a*b^2*c*d^2*x^8+11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2
*b*d^3*x^8-11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^2*c*d^2*x^8+(-b/
a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^3*c^2*d*x^8+8*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x
^2+a))^(1/2)*a^2*b*c*d^2*x^5-9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elli
pticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^2*c^2
*d*x^5+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^3*c^3*x^5-16*((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*
(b*x^2+a))^(1/2)*a^2*b*c*d^2*x^5+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^
2*c^2*d*x^5-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^3*c^3*x^5+5*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^3*d^3*x^6-5*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(-b/a)^(1/2)*a^2*b*c*d^2*x^6+11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a)
)^(1/2)*a^3*d^3*x^6-3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*b*c*d^2*
x^6-8*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^2*c^2*d*x^6+(-b/a)^(1/2)
*((d*x^2+c)*(b*x^2+a))^(1/2)*b^3*c^3*x^6+8*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a
))^(1/2)*a^3*c*d^2*x^4-11*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*b*c^
2*d*x^4+3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^2*c^3*x^4-2*(-b/a)^(
1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^3*c^2*d*x^2+3*(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*a^2*b*c^3*x^2+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^3*c
^3)/a/(b*x^2+a)^2/c^4/x^5/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] e^(3/2)*integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)/x^6, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(((b*x^2 + a)/(d*x^2 + c))^(3/2)*e^(3/2)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6, x)
```

3.289 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\,(1+x^2)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)$$

```
[Out] -arctan(((-x^2+1)/(x^2+1))^(1/2))+1/2*(x^2+1)*((-x^2+1)/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1981, 1979, 294, 210}

$$\frac{1}{2}\sqrt{\frac{1-x^2}{x^2+1}}\,(x^2+1)-\text{ArcTan}\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx &= -\left(2\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)\right)\\ &= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\,(1+x^2)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^2}{1+x^2}}\right)\\ &= \frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\,(1+x^2)-\tan^{-1}\left(\sqrt{\frac{1-x^2}{1+x^2}}\right)\end{aligned}$$

Mathematica [A]
time = 0.10, size = 85, normalized size = 1.67

$$\frac{\sqrt{\frac{1-x^2}{1+x^2}}\left(\sqrt{1-x^2}\,(1+x^2)-2\sqrt{1+x^2}\,\tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\right)\right)}{2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(Sqrt[1 - x^2]*(1 + x^2) - 2*Sqrt[1 + x^2]*ArcTa
n[Sqrt[1 - x^2]/Sqrt[1 + x^2]]))/(2*Sqrt[1 - x^2])
```

Maple [A]
time = 0.25, size = 52, normalized size = 1.02

method	result
default	$\frac{\sqrt{-\frac{x^2-1}{x^2+1}}\,(x^2+1)\left(\arcsin(x^2)+\sqrt{-x^4+1}\right)}{2\sqrt{-(x^2+1)(x^2-1)}}$
risch	$\frac{(x^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}}{2}-\frac{\arcsin(x^2)\sqrt{-\frac{x^2-1}{x^2+1}}\,\sqrt{-(x^2+1)(x^2-1)}}{2(x^2-1)}$
trager	$\left(\frac{x^2}{2}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2+1}}+\frac{\text{RootOf}(\_Z^2+1)\ln\left(\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}\,x^2+\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}+x^2\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*(arcsin(x^2)+(-x^4+1)^(1/2))/(-(x^2+1)
*(x^2-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

Fricas [A]
time = 0.37, size = 55, normalized size = 1.08

$$\frac{1}{2}\left(x^{2}+1\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}-\arctan\left(\frac{\left(x^{2}+1\right)\sqrt{-\frac{x^{2}-1}{x^{2}+1}}-1}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1
)/(x^2 + 1)) - 1)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\frac{(x-1)(x+1)}{x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(x - 1)*(x + 1)/(x**2 + 1)), x)
```

Giac [A]
time = 5.55, size = 18, normalized size = 0.35

$$\frac{1}{2}\sqrt{-x^{4}+1}+\frac{1}{2}\arcsin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

Mupad [B]
time = 2.67, size = 55, normalized size = 1.08

$$-\operatorname{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right)-\frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
```

```
[Out] - atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2 -
 1)/(x^2 + 1) - 1)
```

3.290 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal. Leaf size=72

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\,(7+5x^2)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}$$

```
[Out] -37/175*arctan(1/7*35^(1/2)*((-7*x^2+5)/(5*x^2+7))^(1/2))*35^(1/2)+1/10*(5*
x^2+7)*((-7*x^2+5)/(5*x^2+7))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1981, 1979, 294, 210}

$$\frac{1}{10}\sqrt{\frac{5-7x^2}{5x^2+7}}\,(5x^2+7)-\frac{37\text{ArcTan}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
```

```
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx &= -\left(74\text{Subst}\left(\int \frac{x^2}{(-7-5x^2)^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\right)\\ &= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\,(7+5x^2)+\frac{37}{5}\text{Subst}\left(\int \frac{1}{-7-5x^2}\,dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}}\right)\\ &= \frac{1}{10}\sqrt{\frac{5-7x^2}{7+5x^2}}\,(7+5x^2)-\frac{37\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{7+5x^2}}\right)}{5\sqrt{35}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 119, normalized size = 1.65

$$\frac{\sqrt{\frac{5-7x^2}{7+5x^2}}\left(35\sqrt{5-7x^2}\,(7+5x^2)+148\sqrt{35}\sqrt{7+5x^2}\,\tan^{-1}\left(\frac{\sqrt{5}\sqrt{5-7x^2}}{\sqrt{74}-\sqrt{7}\sqrt{7+5x^2}}\right)\right)}{350\sqrt{5-7x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

```
[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(35*Sqrt[5 - 7*x^2]*(7 + 5*x^2) + 148*Sqrt[3
5]*Sqrt[7 + 5*x^2]*ArcTan[(Sqrt[5]*Sqrt[5 - 7*x^2])/(Sqrt[74] - Sqrt[7]*Sqr
t[7 + 5*x^2])]))/(350*Sqrt[5 - 7*x^2])
```

Maple [A]
time = 0.21, size = 78, normalized size = 1.08

method	result

default	$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\,(5x^2+7)\left(37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-(7x^2-5)(5x^2+7)}}$
risch	$\frac{(5x^2+7)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{10}-\frac{37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}\,\sqrt{-(7x^2-5)(5x^2+7)}}{350(7x^2-5)}$
trager	$7\left(\frac{x^2}{14}+\frac{1}{10}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}+\frac{37\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^2+175\sqrt{-\frac{7x^2-5}{5x^2+7}}\,x^2-12\,\mathrm{RootOf}\left(\_Z^2\right.\right.}{350}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2+
12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

Maxima [A]
time = 0.49, size = 76, normalized size = 1.06

$$-\frac{37}{175}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7x^2-5}{5x^2+7}}\right)-\frac{37\sqrt{-\frac{7x^2-5}{5x^2+7}}}{5\left(\frac{5(7x^2-5)}{5x^2+7}-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

```
[Out] -37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/5
*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

Fricas [A]
time = 0.40, size = 77, normalized size = 1.07

$$\frac{1}{10}(5x^2+7)\sqrt{-\frac{7x^2-5}{5x^2+7}}-\frac{37}{350}\sqrt{35}\arctan\left(\frac{\sqrt{35}(35x^2+12)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{35(7x^2-5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/
35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-\frac{7x^2-5}{5x^2+7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

```
[Out] Integral(x*sqrt(-(7*x**2 - 5)/(5*x**2 + 7)), x)
```

Giac [A]
time = 5.38, size = 30, normalized size = 0.42

$$\frac{37}{350}\,\sqrt{35}\,\arcsin\left(\frac{35}{37}\,x^2+\frac{12}{37}\right)+\frac{1}{10}\,\sqrt{-35\,x^4-24\,x^2+35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

```
[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35
)
```

Mupad [B]
time = 0.21, size = 88, normalized size = 1.22

$$-\frac{37\,\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{7}\right)}{175}-\frac{37\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{1225\,\left(\frac{5\,x^2-\frac{25}{7}}{5\,x^2+7}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
```

```
[Out] - (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))/
175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(1225*
((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

3.291 $\int x^2 \sqrt{\frac{1-x^3}{1+x^3}}\, dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\left(1+x^3\right)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

```
[Out] -2/3*arctan(((-x^3+1)/(x^3+1))^(1/2))+1/3*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1981, 1979, 294, 210}

$$\frac{1}{3}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)-\frac{2}{3}\text{ArcTan}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3
)]])/3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx &= -\left(\frac{4}{3}\text{Subst}\left(\int\frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)\\ &= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\,(1+x^3)+\frac{2}{3}\text{Subst}\left(\int\frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\\ &= \frac{1}{3}\sqrt{\frac{1-x^3}{1+x^3}}\,(1+x^3)-\frac{2}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.16, size = 85, normalized size = 1.60

$$\frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}\,(1+x^3)-2\sqrt{1+x^3}\,\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{1+x^3}}\right)\right)}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(1 + x^3) - 2*Sqrt[1 + x^3]*ArcTa
n[Sqrt[1 - x^3]/Sqrt[1 + x^3]]))/(3*Sqrt[1 - x^3])
```

Maple [A]
time = 0.23, size = 68, normalized size = 1.28

method	result
risch	$\frac{(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}}{3}-\frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-(x^3-1)(x^3+1)}}{3(x^3-1)}$
trager	$\left(\frac{x^3}{3}+\frac{1}{3}\right)\sqrt{-\frac{x^3-1}{x^3+1}}+\frac{\text{RootOf}(\_Z^2+1)\ln\left(\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/
2)*(-(x^3-1)*(x^3+1))^(1/2)/(x^3-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A]
time = 0.35, size = 55, normalized size = 1.04

$$\frac{1}{3}\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-\frac{2}{3}\arctan\left(\frac{\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1}{x^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3
 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{-\frac{(x-1)\left(x^{2}+x+1\right)}{x^{3}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x**2 + x + 1)/(x**3 + 1)), x)
```

Giac [A]
time = 5.52, size = 22, normalized size = 0.42

$$\frac{1}{3}\left(\sqrt{-x^{6}+1}+\arcsin\left(x^{3}\right)\right)\operatorname{sgn}\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

Mupad [B]
time = 2.67, size = 56, normalized size = 1.06

$$-\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}-\frac{2\,\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\,\left(x^3-1\right)}{x^3+1}-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] - (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

3.292 $\int x^8 \sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal. Leaf size=113

$$\frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}\,(1+x^3)-\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}\,(1+x^3)^2-\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3-\frac{1}{3}\tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)$$

```
[Out] -1/9*((-x^3+1)/(x^3+1))^(3/2)*(x^3+1)^3-1/3*arctan(((-x^3+1)/(x^3+1))^(1/2)
)+1/2*(x^3+1)*((-x^3+1)/(x^3+1))^(1/2)-1/6*(x^3+1)^2*((-x^3+1)/(x^3+1))^(1/
2)
```

Rubi [A]
time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1981, 1980, 474, 466, 393, 210}

$$-\frac{1}{3}\text{ArcTan}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)-\frac{1}{9}\left(\frac{1-x^3}{x^3+1}\right)^{3/2}\left(x^3+1\right)^3-\frac{1}{6}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)^2+\frac{1}{2}\sqrt{\frac{1-x^3}{x^3+1}}\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x
^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
```

```
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = -\left(\frac{4}{3}\text{Subst}\left(\int \frac{x^2(-1+x^2)^2}{(-1-x^2)^4}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right)$$

$$= -\frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{2}{9}\text{Subst}\left(\int \frac{x^2(6-6x^2)}{(-1-x^2)^3}\,dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)$$

$$= -\frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 + \frac{1}{18}\text{Subst}\left(\int \frac{12-24x^2}{(-1-x^2)^2}\,dx, x,\right.$$

$$= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 + \frac{1}{3}\text{Subst}\left(\right.$$

$$= \frac{1}{2}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3) - \frac{1}{6}\sqrt{\frac{1-x^3}{1+x^3}}(1+x^3)^2 - \frac{1}{9}\left(\frac{1-x^3}{1+x^3}\right)^{3/2}(1+x^3)^3 - \frac{1}{3}\tan^{-1}\left(\right.$$

Mathematica [A]
time = 0.39, size = 95, normalized size = 0.84

$$\frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}\,(4+x^3-x^6+2x^9)-6\sqrt{1+x^3}\,\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{1+x^3}}\right)\right)}{18\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(4 + x^3 - x^6 + 2*x^9) - 6*Sqrt[
1 + x^3]*ArcTan[Sqrt[1 - x^3]/Sqrt[1 + x^3]]))/(18*Sqrt[1 - x^3])
```

Maple [A]
time = 0.20, size = 80, normalized size = 0.71

method	result
risch	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18} - \frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\sqrt{-(x^3-1)(x^3+1)}}{6(x^3-1)}$
trager	$\frac{(x^3+1)(2x^6-3x^3+4)\sqrt{-\frac{x^3-1}{x^3+1}}}{18} + \frac{\text{RootOf}(\_Z^2+1)\ln\left(\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\text{RootOf}(\_Z^2+1)\sqrt{-\frac{x}{x}}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(x^3+1)*(2*x^6-3*x^3+4)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(x^3)*(-(x^
3-1)/(x^3+1))^(1/2)*(-(x^3-1)*(x^3+1))^(1/2)/(x^3-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Fricas [A]
time = 0.34, size = 65, normalized size = 0.58

$$\frac{1}{18}\left(2\,x^{9}-x^{6}+x^{3}+4\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-\frac{1}{3}\,\arctan\left(\frac{\left(x^{3}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}-1}{x^{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
[Out] 1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
+ 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
[Out] integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Mupad [B]
time = 2.66, size = 101, normalized size = 0.89

$$\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9}-\frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}+\frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

```
[Out] (2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3 +
 (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2))
/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

3.293 $\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}}\, dx$

Optimal. Leaf size=106

$$-\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\left(7+5x^5\right)^2+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}$$

```
[Out] 2257/30625*arctan(1/7*35^(1/2)*((-7*x^5+5)/(5*x^5+7))^(1/2))*35^(1/2)-27/35
0*(5*x^5+7)*((-7*x^5+5)/(5*x^5+7))^(1/2)+1/250*(5*x^5+7)^2*((-7*x^5+5)/(5*x
^5+7))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1981, 1980, 466, 393, 210}

$$\frac{2257\text{ArcTan}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}+\frac{1}{250}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)^2-\frac{27}{350}\sqrt{\frac{5-7x^5}{5x^5+7}}\left(5x^5+7\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7
+ 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 +
 5*x^5)]])/(875*Sqrt[35])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx &= -\left(\frac{148}{5}\text{Subst}\left(\int \frac{x^2(-5+7x^2)}{(-7-5x^2)^3}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right)\\ &= \frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\,(7+5x^5)^2+\frac{37}{125}\text{Subst}\left(\int \frac{-74+140x^2}{(-7-5x^2)^2}\,dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\\ &= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\,(7+5x^5)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\,(7+5x^5)^2-\frac{2257}{875}\text{Subst}\left(\int \frac{1}{-7-5x^2\ldots}\right.\\ &= -\frac{27}{350}\sqrt{\frac{5-7x^5}{7+5x^5}}\,(7+5x^5)+\frac{1}{250}\sqrt{\frac{5-7x^5}{7+5x^5}}\,(7+5x^5)^2+\frac{2257\tan^{-1}\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-\ldots}{7+\ldots}}\right.}{875\sqrt{35}}\end{aligned}$$

Mathematica [A]

time = 0.63, size = 106, normalized size = 1.00

$$\frac{\sqrt{\frac{5-7x^5}{7+5x^5}}\left(35\sqrt{5-7x^5}\,(-602-185x^5+175x^{10})+4514\sqrt{35}\,\sqrt{7+5x^5}\,\tan^{-1}\left(\frac{\sqrt{\frac{25}{7}-5x^5}}{\sqrt{7+5x^5}}\right)\right)}{61250\sqrt{5-7x^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

```
[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(35*Sqrt[5 - 7*x^5]*(-602 - 185*x^5 + 175*x^
10) + 4514*Sqrt[35]*Sqrt[7 + 5*x^5]*ArcTan[Sqrt[25/7 - 5*x^5]/Sqrt[7 + 5*x^
5]]))/(61250*Sqrt[5 - 7*x^5])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.42, size = 114, normalized size = 1.08

method	result
trager	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750}+\frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5+175\sqrt{-\frac{7x^5-5}{5x^5+7}}\,x^5+12\,\mathrm{RootOf}\left(\_\right.\right.}{61250}$
risch	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750}+\frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5-12\,\mathrm{RootOf}\left(\_Z^2+35\right)+35\sqrt{-35}\right.}{61250(7x^5-5)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1750*(5*x^5+7)*(35*x^5-86)*(-(7*x^5-5)/(5*x^5+7))^(1/2)+2257/61250*RootOf
(_Z^2+35)*ln(35*RootOf(_Z^2+35)*x^5+175*(-(7*x^5-5)/(5*x^5+7))^(1/2)*x^5+12
*RootOf(_Z^2+35)+245*(-(7*x^5-5)/(5*x^5+7))^(1/2))
```

Maxima [A]
time = 0.48, size = 121, normalized size = 1.14

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)-\frac{37\left(675\left(-\frac{7x^5-5}{5x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7x^5-5}{5x^5+7}}\right)}{875\left(\frac{25(7x^5-5)^2}{(5x^5+7)^2}-\frac{70(7x^5-5)}{5x^5+7}+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

```
[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) - 3
7/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^5
+ 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)
```

Fricas [A]
time = 0.38, size = 82, normalized size = 0.77

$$\frac{1}{1750}\left(175\,x^{10}-185\,x^{5}-602\right)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}+\frac{2257}{61250}\,\sqrt{35}\,\arctan\left(\frac{\sqrt{35}\,\left(35\,x^{5}+12\right)\sqrt{-\frac{7\,x^{5}-5}{5\,x^{5}+7}}}{35\,\left(7\,x^{5}-5\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/612
50*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7
))/(7*x^5 - 5))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3656 deep
```

Giac [A]
time = 4.36, size = 47, normalized size = 0.44

$$\frac{1}{61250}\left(35\,\sqrt{-35\,x^{10}-24\,x^{5}+35}\,\left(35\,x^{5}-86\right)-2257\,\sqrt{35}\,\arcsin\left(\frac{35}{37}\,x^{5}+\frac{12}{37}\right)\right)\mathrm{sgn}\left(5\,x^{5}+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

```
[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcs
in(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

Mupad [B]
time = 2.99, size = 134, normalized size = 1.26

$$\frac{2257\,\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{7}\right)}{30625}-\frac{43\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{4375}-\frac{37\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^5\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{12250}+\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,x^{10}\,\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{350}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

```
[Out] (2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))/
30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/4375
 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12250
 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

3.294 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} \, dx$

Optimal. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\,\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

```
[Out] 1/2*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))*(-x^2/(-x^2+1))^(1/2)*(x^2-1)^(1/2)/x
*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1986, 15, 455, 65, 209}

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\,\sqrt{x^2-1}\,\text{ArcTan}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqr
t[2]*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx &= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\,\sqrt{-1+x^2}\right)\int \frac{x}{\sqrt{-1+x^2}\,(1+x^2)}\,dx}{x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\,\sqrt{-1+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx, x, x^2\right)}{2x}\\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}}\,\sqrt{-1+x^2}\right)\operatorname{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x^2}\right)}{x}\\
&= \frac{\sqrt{-\frac{x^2}{1-x^2}}\,\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 49, normalized size = 0.94

$$
\frac{\sqrt{\frac{x^2}{-1+x^2}}\,\sqrt{-1+x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[2]*x)

Maple [A]
time = 0.20, size = 42, normalized size = 0.81

method	result	size
default	$\frac{\sqrt{\frac{x^2}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^2-1}\,\sqrt{2}}{2}\right)}{2x}$	42
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+2\right)x^3+4x^2\sqrt{\frac{x^2}{x^2-1}}-3\,\mathrm{RootOf}\left(\_Z^2+2\right)x-4\sqrt{\frac{x^2}{x^2-1}}}{x\left(x^2+1\right)}\right)}{4}$	74

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)

[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)

Fricas [A]
time = 0.34, size = 32, normalized size = 0.62

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.97, size = 40, normalized size = 0.77

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\operatorname{sgn}\left(x^2-1\right)\operatorname{sgn}(x)+\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}i\sqrt{2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqr
t(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

3.295 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\,\sqrt{-1+a+(1+a)x^2}\,\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

[Out] 1/2*arctan(1/2*(-1+a+(1+a)*x^2)^(1/2)*2^(1/2))*(-x^2/(1-a-(1+a)*x^2))^(1/2)
*(-1+a+(1+a)*x^2)^(1/2)/x*2^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1986, 15, 455, 65, 211}

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\,\sqrt{(a+1)x^2+a-1}\,\text{ArcTan}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]

[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[
-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\int \frac{x}{(1+x^2)\sqrt{-1+a+(1-}}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a}}\right.}{2x}$$

$$= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\right)\text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}}\,dx\right.}{(1+a)x}$$

$$= \frac{\sqrt{-\frac{x^2}{1-a-(1+a)x^2}}\sqrt{-1+a+(1+a)x^2}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)}}{\sqrt{2}}\right.}{\sqrt{2}\,x}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.96

$$\frac{\sqrt{-1+a+x^2+ax^2}\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

```
[Out] (Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-1
 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Maple [A]
time = 0.20, size = 60, normalized size = 0.88

method	result	size
default	$\frac{\sqrt{\frac{x^{2}}{a\,x^{2}+x^{2}+a-1}}\,\sqrt{a\,x^{2}+x^{2}+a-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^{2}+x^{2}+a-1}\,\sqrt{2}}{2}\right)}{2x}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*
(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

Fricas [A]
time = 0.36, size = 42, normalized size = 0.62

$$\frac{1}{4}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left((a+1)x^{2}+a-3\right)\sqrt{\frac{x^{2}}{(a+1)x^{2}+a-1}}}{4\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
+ a - 1))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x^{2}}{ax^{2}+a+x^{2}-1}}}{x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

Giac [A]
time = 3.03, size = 61, normalized size = 0.90

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 +
 a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn
(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
```

```
[Out] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

3.296 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=281

$$\frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^3d^2e}-\frac{(3bc+5ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^2d^2e}-\frac{\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+d}{6bd(bc}$$

```
[Out] 1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(d^(1/2)*(e*(b*x^2+a)/
(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(7/2)/d^(5/2)/e^(1/2)+1/16*(5*a^2*d^2+2
*a*b*c*d+b^2*c^2)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d^2/e-1/24*(5
*a*d+3*b*c)*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/d^2/e-1/6*(d*x^2+
c)^3*(a-c*(b*x^2+a)/(d*x^2+c))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/d/(-a*d+b*c)
/e
```

Rubi [A]
time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 424, 393, 205, 214}

$$\frac{(bc-ad)\,(5a^2d^2+2abcd+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}+\frac{(c+dx^2)\,(5a^2d^2+2abcd+b^2c^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{16b^3d^2e}-\frac{(c+dx^2)^2\,(5ad+3bc)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{24b^2d^2e}-\frac{(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{6bde(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x
^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*
d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{(-ae+cx^2)^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)e}{(be-dx^2)^3}\right.}{6bd}$$

$$= -\frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^3\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}{6bd(bc-ad)e} +$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2}{24b^2d^2e}$$

$$= \frac{(b^2c^2+2abcd+5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^3d^2e} - \frac{(3bc+5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2}{24b^2d^2e}$$

Mathematica [A]
time = 1.54, size = 224, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\left(\sqrt{d}\,\sqrt{a+bx^2}\,\sqrt{\frac{b(c+dx^2)}{bc-ad}}\,(15a^2d^2-2abd(2c+5dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4))+3\sqrt{bc-ad}\,(b^2c^2+2abcd+5a^2d^2)\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)\right)}{48b^3d^{5/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]
*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^
4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*
Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(253) = 506$.
time = 0.09, size = 527, normalized size = 1.88

method	result

risch	$\frac{(8b^2d^2x^4-10ab\,d^2x^2+2b^2cd\,x^2+15a^2d^2-4abcd-3b^2c^2)(b\,x^2+a)}{48b^3d^2\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{5d\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\cdots}\right)}{32b^3\sqrt{deb}}\cdots\right.}{\cdots}$
default	$\frac{(b\,x^2+a)\left(-36\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^2abd^2\sqrt{bd}-12\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^2cb^2d\sqrt{bd}-15\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(b*x^2+a)*(-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(
1/2)-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)-15*d^3*
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*a^3+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+3*ln(1/2*(2*b*d*x^2+2*(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*
d+3*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)
^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)-24*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*
(b*x^2+a))^(1/2)/b^3/d^2/(b*d)^(1/2)
```

Maxima [A]
time = 0.52, size = 387, normalized size = 1.38

$$\frac{1}{96}\left(\frac{2\left(3\left(b^3c^3d^2+ab^2c^2d^3+3a^2bcd^4-5a^3d^5\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}+8\left(b^4c^3d-3ab^3c^2d^2-3a^2b^2cd^3+5a^3bd^4\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-3\left(b^5c^3+ab^4c^2d-13a^2b^3cd^2+11a^3b^2d^3\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^6d^2-\frac{3(bx^2+a)b^5d^3}{dx^2+c}+\frac{3(bx^2+a)^2b^4d^4}{(dx^2+c)^2}-\frac{(bx^2+a)^3b^3d^5}{(dx^2+c)^3}}-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^3d^2}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*(b^3*c^3*d^2 + a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - 5*a^3*d^5)*((b*x^
2 + a)/(d*x^2 + c))^(5/2) + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*c*d^
3 + 5*a^3*b*d^4)*((b*x^2 + a)/(d*x^2 + c))^(3/2) - 3*(b^5*c^3 + a*b^4*c^2*d
 - 13*a^2*b^3*c*d^2 + 11*a^3*b^2*d^3)*sqrt((b*x^2 + a)/(d*x^2 + c)))/(b^6*d
^2 - 3*(b*x^2 + a)*b^5*d^3/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^4*d^4/(d*x^2 + c
)^2 - (b*x^2 + a)^3*b^3*d^5/(d*x^2 + c)^3) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(
d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b^3*d^2))*e^(-1/2)
```

Fricas [A]
time = 0.39, size = 520, normalized size = 1.85

$$\left[-\frac{\left(3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{bd}\log\left(8b^{2}d^{2}x^{4}+b^{2}c^{2}+6abcd+a^{2}d^{2}+8\left(b^{2}cd+abd^{2}\right)x^{2}-4\left(2bd^{2}x^{4}+bc^{2}+acd+\left(3bcd+ad^{2}\right)x^{2}\right)\sqrt{bd}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)-4\left(8b^{3}d^{4}x^{6}-3b^{3}c^{3}d-4ab^{2}c^{2}d^{2}+15a^{2}bcd^{3}+10\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}-\left(b^{3}c^{2}d^{2}+14ab^{2}cd^{3}-15a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{1}{2}\right)}}{192\,b^{4}d^{3}},\ -\frac{\left(3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^{2}+bc+ad\right)\sqrt{-bd}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{2\left(b^{2}dx^{2}+abd\right)}\right)-2\left(8b^{3}d^{4}x^{6}-3b^{3}c^{3}d-4ab^{2}c^{2}d^{2}+15a^{2}bcd^{3}+10\left(b^{3}cd^{3}-ab^{2}d^{4}\right)x^{4}-\left(b^{3}c^{2}d^{2}+14ab^{2}cd^{3}-15a^{2}bd^{4}\right)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{1}{2}\right)}}{96\,b^{4}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2
 - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((
b*x^2 + a)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2
 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^
2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(b^4*d
^3), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b*d
)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x^2 + c
))/(b^2*d*x^2 + a*b*d)) - 2*(8*b^3*d^4*x^6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2
+ 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^2
*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(b^4*d^
3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.76, size = 210, normalized size = 0.75

$$\frac{\left(2\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\left(2x^{2}\left(\frac{4x^{2}}{b}+\frac{b^{2}cd-5abd^{2}}{b^{3}d^{2}}\right)-\frac{3b^{2}c^{2}+4abcd-15a^{2}d^{2}}{b^{3}d^{2}}\right)-\frac{3\left(b^{3}c^{3}+ab^{2}c^{2}d+3a^{2}bcd^{2}-5a^{3}d^{3}\right)\log\left(\left|-bc-ad-2\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)\sqrt{bd}\right|\right)}{\sqrt{bd}\,b^{3}d^{2}}\right)e^{\left(-\frac{1}{2}\right)}}{96\,\mathrm{sgn}\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*x^2*(4*x^2/b + (b^2*c*d
- 5*a*b*d^2)/(b^3*d^2)) - (3*b^2*c^2 + 4*a*b*c*d - 15*a^2*d^2)/(b^3*d^2)) -
 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-b*c - a*d -
 2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*sqrt(b*d)))/(s
qrt(b*d)*b^3*d^2))*e^(-1/2)/sgn(d*x^2 + c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.297 $\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=169

$$-\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8b^2de}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4bde}-\frac{(bc-ad)(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

```
[Out] -1/8*(-a*d+b*c)*(3*a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(5/2)/d^(3/2)/e^(1/2)-1/8*(3*a*d+b*c)*(d*x^2+c)*(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/b^2/d/e+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
b/d/e
```

Rubi [A]

time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 393, 205, 214}

$$-\frac{(bc-ad)(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}-\frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] -1/8*((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(b^2*d*e
) + (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a
*d)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt
[b]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{-ae+cx^2}{(be-dx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4bde} - \frac{((bc-ad)(bc+3ad)e)\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+}{c+}}\right.}{4bd}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4bde} - \frac{((bc-ad)(bc+}{}$$

$$= -\frac{(bc+3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4bde} - \frac{(bc-ad)(bc+}{}$$

Mathematica [A]

time = 0.92, size = 161, normalized size = 0.95

$$\frac{\sqrt{b}\,\sqrt{d}\,(a+bx^2)\sqrt{c+dx^2}\,(-3ad+b(c+2dx^2))-(b^2c^2+2abcd-3a^2d^2)\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^2}}{\sqrt{b}\,\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*(a + b*x^2)*Sqrt[c + d*x^2]*(-3*a*d + b*(c + 2*d*x^2)) - (
b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(3/2)*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(145) = 290$.
time = 0.07, size = 341, normalized size = 2.02

method	result
risch	$-\frac{(-2bd\,x^2+3ad-bc)(b\,x^2+a)}{8b^2d\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(\frac{3d\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a^2}{16b^2\sqrt{deb}}-\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+de\right.}{\sqrt{deb}}\right.}{}\right.}{}$
default	$-\frac{(b\,x^2+a)\left(-4\sqrt{bd}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,bd\,x^2-3d^2\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}}{2\sqrt{bd}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(b*x^2+a)*(-4*(b*d)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*d*x^2
-3*d^2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+
a*d+b*c)/(b*d)^(1/2))*a^2+2*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c*b*d+ln(1/2*(2*b*d*x^2+2*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c^2+6
*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d-2*(b*d)^(1/2)*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(
b*x^2+a))^(1/2)/b^2/d/(b*d)^(1/2)
```

Maxima [A]

time = 0.52, size = 249, normalized size = 1.47

$$\frac{1}{16}\left(\frac{2\left((b^2c^2d+2\,abcd^2-3\,a^2d^3)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{b^4d-\frac{2\,(bx^2+a)b^3d^2}{dx^2+c}+\frac{(bx^2+a)^2b^2d^3}{(dx^2+c)^2}}+\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^2d}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*((b*x^2 + a)/(d*x^2 + c))^(3
/2) + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))/
(b^4*d - 2*(b*x^2 + a)*b^3*d^2/(d*x^2 + c) + (b*x^2 + a)^2*b^2*d^3/(d*x^2 +
 c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((d*sqrt((b*x^2 + a)/(d*x^2 +
 c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)
*b^2*d))*e^(-1/2)
```

Fricas [**A**]
time = 0.37, size = 388, normalized size = 2.30

$$\left[-\frac{\left((b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2+4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)-4\,(2\,b^2d^3x^4+b^2c^2d-3\,abcd^2+3\,(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{32\,b^3d^2},\ \frac{\left((b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)+2\,(2\,b^2d^3x^4+b^2c^2d-3\,abcd^2+3\,(b^2cd^2-abd^3)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{16\,b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2
*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^4 + b
*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d*x^2 + c
))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*
x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(b^3*d^2), 1/16*((b^2*c^2 + 2*
a*b*c*d - 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*
d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d)) + 2*(2*b^2*d^3*x^4 +
b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*x^2 + a)/(d*
x^2 + c)))*e^(-1/2)/(b^3*d^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

[Out] Timed out

Giac [A]
time = 5.55, size = 153, normalized size = 0.91

$$\frac{\left(2\sqrt{bdx^4+bcx^2+adx^2+ac}\left(\frac{2x^2}{b}+\frac{bc-3ad}{b^2d}\right)+\frac{(b^2c^2+2abcd-3a^2d^2)\log\left(\left|-bc-ad-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)\sqrt{bd}\right|\right)}{\sqrt{bd}\,b^2d}\right)e^{\left(-\frac{1}{2}\right)}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)*(2*x^2/b + (b*c - 3*a*d)/(b
^2*d)) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log(abs(-b*c - a*d - 2*(sqrt(b*d
)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*sqrt(b*d)))/(sqrt(b*d)*b^2
*d))*e^(-1/2)/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.298 $\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=106

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2be}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))/b^(3/2)/d^(1/2)/e^(1/2)+1/2*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b/e
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1981, 1979, 205, 214}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}+\frac{(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcT
anh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3
/2)*Sqrt[d]*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(be-dx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2be} + \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}\\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{2be} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}
\end{aligned}$$

Mathematica [A]
time = 0.40, size = 136, normalized size = 1.28

$$\frac{\sqrt{b}\sqrt{d}\,(a+bx^2)(c+dx^2)+(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

[Out] (Sqrt[b]*Sqrt[d]*(a + b*x^2)*(c + d*x^2) + (b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(2*b^(3/2)*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(86) = 172$.

time = 0.06, size = 200, normalized size = 1.89

method	result
default	$\frac{(b x^2+a)\left(-d\ln\left(\frac{2bd x^2+2\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a+c\ln\left(\frac{2bd x^2+2\sqrt{bd x^4+ad x^2+bc x^2}}{2\sqrt{bd}}\right.\right.}{4\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\sqrt{(d x^2+c)(b x^2+a)}\,b\sqrt{bd}}$
risch	$\frac{b x^2+a}{2b\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(-\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb x^2}{\sqrt{deb}}+\sqrt{deb x^4+(ade+bce)x^2+ace}\right)ad}{4b\sqrt{deb}}+\frac{\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb x^2}{\sqrt{deb}}+\sqrt{de}\right.}{}\right.}{\sqrt{\frac{e(b x^2+a)}{d x^2+c}}(d x^2+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(b*x^2+a)*(-d*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a+c*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/b/(b*d)^(1/2)

Maxima [A]

time = 0.51, size = 138, normalized size = 1.30

$$\frac{1}{4}\left(\frac{2\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{b^2-\frac{(bx^2+a)bd}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2 - (b*x^2 + a)*b*d/(d*x^2 + c)) - (b*c - a*d)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b))*e^(-1/2)

Fricas [A]
time = 0.37, size = 288, normalized size = 2.72

$$\left[-\frac{\left((bc-ad)\sqrt{bd}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,bd^2x^4+bc^2+acd+(3\,bcd+ad^2)x^2)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)-4\,(bd^2x^2+bcd)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,b^2d},\ -\frac{\left((bc-ad)\sqrt{-bd}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(b^2dx^2+abd)}\right)-2\,(bd^2x^2+bcd)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{4\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*c - a*d)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d
 + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))) - 4*(b*d^2*x^2 + b*
c*d)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(b^2*d), -1/4*((b*c - a*d)*sqr
t(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x
^2 + c))/(b^2*d*x^2 + a*b*d)) - 2*(b*d^2*x^2 + b*c*d)*sqrt((b*x^2 + a)/(d*x
^2 + c)))*e^(-1/2)/(b^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.34, size = 126, normalized size = 1.19

$$\frac{\left(\frac{2\sqrt{bdx^4+bcx^2+adx^2+ac}}{b}-\frac{(bc-ad)\sqrt{bd}\log\left(\left|-2\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{b^2d}\right)e^{\left(-\frac{1}{2}\right)}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)/b - (b*c - a*d)*sqrt(b*d)*lo
g(abs(-2*(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*b*d - sq
rt(b*d)*b*c - sqrt(b*d)*a*d))/(b^2*d))*e^(-1/2)/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.299 $\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

```
[Out] -arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))*c^(1/2)/a^(
1/2)/e^(1/2)+arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
*d^(1/2)/b^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1981, 1980, 400, 214}

$$\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqr
t[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\ &= c\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)+d\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\ &= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.46, size = 147, normalized size = 1.31

$$\frac{\sqrt{a+bx^2}\left(-\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-(Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt
[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/
(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[a]*Sqrt[b]*Sqrt[(e*(a + b*x^2))/(c + d*x
^2)]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.
time = 0.05, size = 179, normalized size = 1.60

method	result
default	$\frac{(b\,x^2+a)\left(d\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}-c\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+ac}}{x^2}\right.\right.}{2\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,\sqrt{bd}\,\sqrt{ac}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*x^2+a)*(d*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b
*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)-c*ln((a*d*x^2+b*c*x^2+2*(a*c)^(
1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2))/(e*(b*x^2
+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

Maxima [A]
time = 0.52, size = 140, normalized size = 1.25

$$\frac{1}{2}\left(\frac{c\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}}-\frac{d\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(c*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a
)/(d*x^2 + c)) + sqrt(a*c)))/sqrt(a*c) - d*log((d*sqrt((b*x^2 + a)/(d*x^2 +
 c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/sqrt(b*d))
*e^(-1/2)
```

Fricas [A]
time = 0.46, size = 821, normalized size = 7.33

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

[Out] [1/4*(sqrt(d/b)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(d/b)) + sqrt(c/a)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(c/a))/x^4))*e^(-1/2), -1/4*(2*sqrt(-d/b)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-d/b)/(b*d*x^2 + a*d)) - sqrt(c/a)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(c/a))/x^4))*e^(-1/2), 1/4*(2*sqrt(-c/a)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-c/a)/(b*c*x^2 + a*c)) + sqrt(d/b)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(d/b)))*e^(-1/2), 1/2*(sqrt(-c/a)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-c/a)/(b*c*x^2 + a*c)) - sqrt(-d/b)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-d/b)/(b*d*x^2 + a*d)))*e^(-1/2)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sa

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.300 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=130

$$\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

```
[Out] 1/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))/a^(3/2)/c^(1/2)/e^(1/2)+1/2*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a/(
a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1981, 1980, 205, 214}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*a*(a*e - (c*e*(a + b*x^2
))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx &= ((bc-ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)\\
&= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2a}\\
&= \frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}
\end{aligned}$$

Mathematica [A]
time = 0.66, size = 143, normalized size = 1.10

$$\frac{-\sqrt{a}\sqrt{c}\,(a+bx^2)(c+dx^2)+(bc-ad)x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{c}\,x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c]*(a + b*x^2)*(c + d*x^2)) + (b*c - a*d)*x^2*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^
2])])/(2*a^(3/2)*Sqrt[c]*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(110) = 220$.
time = 0.08, size = 326, normalized size = 2.51

method	result
risch	$-\frac{b\,x^{2}+a}{2 a\,x^{2} \sqrt{\frac{e(b\,x^{2}+a)}{d\,x^{2}+c}}}+\frac{\left(-\frac{\ln\left(\frac{2 a c e+(a d e+b c e) x^{2}+2 \sqrt{a c e}\,\sqrt{d e b\,x^{4}+(a d e+b c e)\,x^{2}+a c e}}{x^{2}}\right) d}{4 \sqrt{a c e}}+\frac{\ln\left(\frac{2 a c e+(a d e+b c e}{}\right.}{}\right.}{\sqrt{\frac{e(b\,x^{2}+a)}{d\,x^{2}+c}}}$
default	$-\frac{(b\,x^{2}+a)\left(-2 b d \sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,x^{4} \sqrt{a c}+a^{2} \ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2 \sqrt{a c}\,\sqrt{b d\,x^{4}+a d\,x^{2}+b c\,x^{2}+}}{x^{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x^2+a)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)+
a^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2
*a*c)/x^2)*d*c*x^2-c^2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b*a*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1
/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/(e*(b*x^2+a)/(d*x^2+
c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/c/x^2/(a*c)^(1/2)
```

Maxima [A]
time = 0.50, size = 138, normalized size = 1.06

$$\frac{1}{4}\left(\frac{2\,(bc-ad)\sqrt{\frac{bx^2+a}{dx^2+c}}}{a^2-\frac{(bx^2+a)ac}{dx^2+c}}-\frac{(bc-ad)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*(b*c - a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a^2 - (b*x^2 + a)*a*c/(d*x^2 + c)) - (b*c - a*d)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a))*e^(-1/2)

Fricas [A]
time = 0.44, size = 310, normalized size = 2.38

$$\left[-\frac{\left(\sqrt{ac}\,(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{x^4}\right)+4\,(acdx^2+ac^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,a^2cx^2},\ -\frac{\left(\sqrt{-ac}\,(bc-ad)x^2\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{-ac}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\,(abcx^2+a^2c)}\right)+2\,(acdx^2+ac^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{1}{2}\right)}}{4\,a^2cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(a*c)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(a^2*c*x^2), -1/4*(sqrt(-a*c)*(b*c - a*d)*x^2*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2 + a^2*c)) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(a^2*c*x^2)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

Giac [A]
time = 6.01, size = 216, normalized size = 1.66

$$-\frac{\left(\frac{(bc-ad)\arctan\left(-\frac{\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a}-\frac{\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)bc+\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)ad+2\,\sqrt{bd}\,ac}{\left(\left(\sqrt{bd}\,x^2-\sqrt{bdx^4+bcx^2+adx^2+ac}\right)^2-ac\right)a}\right)e^{\left(-\frac{1}{2}\right)}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] -1/2*((b*c - a*d)*arctan(-(sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))/sqrt(-a*c))/(sqrt(-a*c)*a) - ((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*

```
x^2 + a*d*x^2 + a*c))*b*c + (sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x
^2 + a*c))*a*d + 2*sqrt(b*d)*a*c)/(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2
 + a*d*x^2 + a*c))^2 - a*c)*a))*e^(-1/2)/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.301 $\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx$

Optimal. Leaf size=218

$$-\frac{(bc-ad)^2 e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}-\frac{(bc-ad)(3bc+ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{(bc-ad)(3bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

```
[Out] -1/8*(-a*d+b*c)*(a*d+3*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(5/2)/c^(3/2)/e^(1/2)-1/4*(-a*d+b*c)^2*e*(e*(b*x^2+a)/(d*
x^2+c))^(1/2)/a/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))^2-1/8*(-a*d+b*c)*(a*d+3*b*c
)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/c/(a*e-c*e*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.17, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 393, 205, 214}

$$-\frac{(ad+3bc)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}-\frac{(ad+3bc)(bc-ad)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}-\frac{e(bc-ad)^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -1/4*((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a*c*(a*e - (c*e*(
a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)])/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c -
a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{be-dx^2}{(-ae+cx^2)^3}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc-ad)(3bc+ad)e)\text{Subst}\left(\int \frac{1}{(-ae+cx^2)^2}\,dx, x, \sqrt{\right.}{4ac}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{((bc-ad)(3b}{}$$

$$= -\frac{(bc-ad)^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc-ad)(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(bc-ad)(3bc}{}$$

Mathematica [A]
time = 1.23, size = 173, normalized size = 0.79

$$\frac{\sqrt{a}\sqrt{c}\,(a+bx^2)\sqrt{c+dx^2}\,(3bcx^2-a(2c+dx^2))-(3b^2c^2-2abcd-a^2d^2)\,x^4\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
- (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sq
rt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(
a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(194) = 388$.
time = 0.09, size = 559, normalized size = 2.56

method	result

risch	$-\frac{(b x^2+a)(a d x^2-3 b c x^2+2 a c)}{8 a^2 x^4 c \sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(\frac{\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}}{x^2}\right)d^2}{16c\sqrt{ace}}+\frac{\ln\left(\frac{2ace+\cdots}{}\right)}{}\right.}{}$
default	$-\frac{(b x^2+a)\left(2bd^2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6 a\sqrt{ac}+10b^2 d\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6 c\sqrt{ac}-a^3\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(b*x^2+a)*(2*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1
/2)+10*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)-a^3*ln((
a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^
2)*d^2*c*x^4-2*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)+2*a*c)/x^2)*d*b*a^2*c^2*x^4+3*c^3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^2*a*x^4+2*(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)+8*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(a*c)^(1/2)*a*b*c*d*x^4+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b
^2*c^2*x^4*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^
(1/2)-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*c*x^2*(a*c)^(1/2)+4*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
((d*x^2+c)*(b*x^2+a))^(1/2)/a^3/c^2/x^4/(a*c)^(1/2)
```

Maxima [A]
time = 0.55, size = 252, normalized size = 1.16

$$\frac{1}{16}\left(\frac{2\left(\left(3\,b^2c^3-2\,abc^2d-a^2cd^2\right)\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-\left(5\,ab^2c^2-6\,a^2bcd+a^3d^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)}{a^4c-\frac{2\,(bx^2+a)a^3c^2}{dx^2+c}+\frac{(bx^2+a)^2a^2c^3}{(dx^2+c)^2}}+\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^2c}\right)e^{\left(-\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2)*((b*x^2 + a)/(d*x^2 + c))^(3
/2) - (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))/
(a^4*c - 2*(b*x^2 + a)*a^3*c^2/(d*x^2 + c) + (b*x^2 + a)^2*a^2*c^3/(d*x^2 +
 c)^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((c*sqrt((b*x^2 + a)/(d*x^2 +
```

```
 c)) - sqrt(a*c))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)
*a^2*c))*e^(-1/2)
```

Fricas [A]
time = 0.66, size = 420, normalized size = 1.93

$$\left[-\frac{\left({\left(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2}\right)}\sqrt{ac}\,x^{4}\log\left(\frac{{\left(b^{2}c^{2}+6\,abcd+a^{2}d^{2}\right)}x^{4}+8\,a^{2}c^{2}+8\,{\left(abc^{2}+a^{2}cd\right)}x^{2}+4\,{\left({\left(bcd+ad^{2}\right)}x^{4}+2\,ac^{2}+{\left(bc^{2}+3\,acd\right)}x^{2}\right)}\sqrt{ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{x^{4}}\right)+4\,{\left(2\,a^{2}c^{3}-{\left(3\,abc^{2}d-a^{2}cd^{2}\right)}x^{4}-3\,{\left(abc^{3}-a^{2}c^{2}d\right)}x^{2}\right)}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{1}{2}\right)}}{32\,a^{3}c^{2}x^{4}},\ \frac{\left({\left(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2}\right)}\sqrt{-ac}\,x^{4}\arctan\left(\frac{{\left({\left(bc+ad\right)}x^{2}+2\,ac\right)}\sqrt{-ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{2\,{\left(abcx^{2}+a^{2}c\right)}}\right)-2\,{\left(2\,a^{2}c^{3}-{\left(3\,abc^{2}d-a^{2}cd^{2}\right)}x^{4}-3\,{\left(abc^{3}-a^{2}c^{2}d\right)}x^{2}\right)}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{1}{2}\right)}}{16\,a^{3}c^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c)*x^4*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 + 4*((b*c*d +
 a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^2 + a)/(
d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4 - 3*(a*b*c
^3 - a^2*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(a^3*c^2*x^4),
 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c)*x^4*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))/(a*b*c*x^2 + a^2
*c)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*
d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-1/2)/(a^3*c^2*x^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(194) = 388$.
time = 3.53, size = 486, normalized size = 2.23

$$\frac{\left(\frac{{\left(3\,b^{2}c^{2}-2\,abcd-a^{2}d^{2}\right)}\arctan\left(-\frac{\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}\,a^{2}c}-\frac{3\,{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}^{3}b^{2}c^{2}-5\,{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}ab^{2}c^{3}-2\,{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}^{3}abcd-10\,{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}a^{2}bc^{2}d-{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}^{3}a^{2}d^{2}-{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}a^{3}cd^{2}-8\,\sqrt{bd}\,a^{2}bc^{3}-8\,{\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}^{2}\sqrt{bd}\,a^{2}cd}{{\left({\left(\sqrt{bd}\,x^{2}-\sqrt{bdx^{4}+bcx^{2}+adx^{2}+ac}\right)}^{2}-ac\right)}^{2}a^{2}c}\right)e^{\left(-\frac{1}{2}\right)}}{8\,\mathrm{sgn}\left(dx^{2}+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(-(sqrt(b*d)*x^2 - sqrt(b*d*x^
4 + b*c*x^2 + a*d*x^2 + a*c))/sqrt(-a*c))/(sqrt(-a*c)*a^2*c) - (3*(sqrt(b*d
)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*b^2*c^2 - 5*(sqrt(b*d)*x
^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a*b^2*c^3 - 2*(sqrt(b*d)*x^2
- sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a*b*c*d - 10*(sqrt(b*d)*x^2 -
```

```
sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^2*b*c^2*d - (sqrt(b*d)*x^2 - sqr
t(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^3*a^2*d^2 - (sqrt(b*d)*x^2 - sqrt(b*d
*x^4 + b*c*x^2 + a*d*x^2 + a*c))*a^3*c*d^2 - 8*sqrt(b*d)*a^2*b*c^3 - 8*(sqr
t(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^2*sqrt(b*d)*a^2*c*d)/
(((sqrt(b*d)*x^2 - sqrt(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))^2 - a*c)^2*a^2*
c))*e^(-1/2)/sgn(d*x^2 + c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.302 $\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=403

$$\frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(2b^2c^2+3abcd-8a^2d^2)\,x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\,(2b^2c^2+3abcd-8a^2d^2)\cdots}{15b^3d^{3/2}\sqrt{\frac{c(a+b\cdots}{a(c+d\cdots}}}$$

```
[Out] 1/15*(-4*a*d+b*c)*x*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*x^3*(
b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c
^2)*x*(b*x^2+a)/b^3/d/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/15*c^(3/2)*
(-4*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^2/d^(3/2)/(d*x^2+c)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15*(-8*a^2*
d^2+3*a*b*c*d+2*b^2*c^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^
3/d^(3/2)/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))
^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.269$, Rules used = {1986, 489, 596, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,(a+bx^2)\,(-8a^2d^2+3abcd+2b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^3d^{3/2}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{x(a+bx^2)\,(-8a^2d^2+3abcd+2b^2c^2)}{15b^3d\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}(a+bx^2)\,(bc-4ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2d^{3/2}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)\,(bc-4ad)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] ((b*c - 4*a*d)*x*(a + b*x^2))/(15*b^2*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
+ (x^3*(a + b*x^2))/(5*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b^2*c^2 +
 3*a*b*c*d - 8*a^2*d^2)*x*(a + b*x^2))/(15*b^3*d*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*(c + d*x^2)) + (Sqrt[c]*(2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*(a + b*
x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^3*d^(3/
2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2)) - (c^(3/2)*(b*c - 4*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]
*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{x^2(3ac+(-bc+4ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-ac(bc-4ad)+(-2b^2c^2-3abcd+8a^2 \cdots}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(bc-4ad)\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\cdots}}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx \cdots}}$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)\,x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \cdots$$

$$= \frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{x^3(a+bx^2)}{5b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2b^2c^2+3abcd-8a^2d^2)\,x(a+bx^2)}{15b^3d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.59, size = 258, normalized size = 0.64

$$\frac{-\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)(4ad-b(c+3dx^2)) - ic(-2b^2c^2-3abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right) + 2ic(-b^2c^2-abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)\middle|\frac{ad}{bc}\right)}{15a^2\left(\frac{b}{a}\right)^{5/2}d^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))

Maple [A]
time = 0.06, size = 554, normalized size = 1.37

method	result
risch	$-\frac{x\left(-3bd\,x^{2}+4ad-bc\right)\left(b\,x^{2}+a\right)}{15d\,b^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(-\frac{2\left(8a^{2}d^{2}-3abcd-2b^{2}c^{2}\right)ace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}\right.}{}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-4\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+4\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+4\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/15*(b*x^2+a)*(-3*(-b/a)^(1/2)*b^2*d^3*x^7+(-b/a)^(1/2)*a*b*d^3*x^5-4*(-b/a)^(1/2)*b^2*c*d^2*x^5+4*(-b/a)^(1/2)*a^2*d^3*x^3-(-b/a)^(1/2)*b^2*c^2*d*x^3+4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*c*d^2+3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c^2*d+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^3+4*(-b/a)^(1/2)*a^2*c*d^2*x-(-b/a)^(1/2)*a*b*c^2*d*x)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/b^2/d^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(x^4/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*e^(-1/2)/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.303 $\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=312

$$\frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\,(bc-2ad)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{c}{3b\sqrt{}}$$

```
[Out] 1/3*x*(b*x^2+a)/b/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-2*a*d+b*c)*x*(b*x^2+a
)/b^2/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(b*x^2+a)*(1/(1+d
*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1
/2),(1-b*c/a/d)^(1/2))/b/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/
a/d)^(1/2))*c^(1/2)/b^2/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1986, 489, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,(a+bx^2)\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}(a+bx^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)\,(bc-2ad)}{3b^2\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (x*(a + b*x^2))/(3*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b*c - 2*a*d)*x*
(a + b*x^2))/(3*b^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[
c]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(3*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan
[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{ac+(-bc+2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left((-bc+2ad)\sqrt{a\right.}}{3b\sqrt{\right.}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{c^{3/2}(a+bx^2)F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.\right.}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+b}{c+dx}}}$$

$$= \frac{x(a+bx^2)}{3b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc-2ad)x(a+bx^2)}{3b^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}(bc-2ad)(a+bx^2)E\left(\tan\right.}{3b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a}{c}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.05, size = 212, normalized size = 0.68

$$\frac{\sqrt{\frac{b}{a}}\,dx(a+bx^2)(c+dx^2)+ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{3b\sqrt{\frac{b}{a}}\,d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/(c +
 d*x^2)]*(c + d*x^2))
```

Maple [A]
time = 0.05, size = 358, normalized size = 1.15

method	result
risch	$\frac{x(b\,x^{2}+a)}{3b\sqrt{\frac{e(b\,x^{2}+a)}{d\,x^{2}+c}}}-\frac{\left(-\frac{2(2ad-bc)ace\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,(ade+bce+e(ad-bc))}\right.}{}$
default	$\frac{(b\,x^{2}+a)\left(\sqrt{-\frac{b}{a}}\,b\,d^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a\,d^{2}x^{3}+\sqrt{-\frac{b}{a}}\,bcd\,x^{3}+ac\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{b\,x^{2}+}{a}}\right.}{3\sqrt{\frac{e(b\,x^{2}+a)}{d\,x^{2}+c}}\,\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)*((-b/a)^(1/2)*b*d^2*x^5+(-b/a)^(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b
*c*d*x^3+a*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+(-b/a)^(1/2)
*a*c*d*x)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/b/(-b/a
)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(x^2/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(-1/2)/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.304 $\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=252

$$\frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\right.}{a\sqrt{d}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2}{c+dx^2}}}$$

```
[Out] d*x*(b*x^2+a)/b/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))/a/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*c^(1/2)*d^(1/2)/b/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(
d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1986, 433, 429, 506, 422}

$$\frac{c^{3/2}(a+bx^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)}{b\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (d*x*(a + b*x^2))/(b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt
[c]*Sqrt[d]*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*(c + d*x^2)) + (c^(3/2)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{\left(c\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}(a+bx^2)\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\left(cd\sqrt{a\right.}}{b\sqrt{\frac{e}{}}}$$

$$= \frac{dx(a+bx^2)}{b\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}(}{a\sqrt{d}}$$

Mathematica [A]
time = 0.85, size = 86, normalized size = 0.34

$$\frac{\sqrt{\frac{a+bx^2}{a}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[
-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

Maple [A]
time = 0.04, size = 127, normalized size = 0.50

method	result	size
default	$\frac{(b\,x^2+a)c\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\sqrt{(d\,x^2+c)(b\,x^2+a)}\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2
)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/sqrt((b*x^2 + a)/(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

3.305 $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=289

$$-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\,\sqrt{d}\,(\ldots}{a\sqrt{\frac{c(\ldots}{a(\ldots}}}$$

```
[Out] (-b*x^2-a)/a/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*x*(b*x^2+a)/a/(d*x^2+c)/(e*(
b*x^2+a)/(d*x^2+c))^(1/2)-(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d
^(1/2)/a/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)+(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)
/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a/(d*x^2+c)/(
c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 21, 433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)}{a\,(c+dx^2)\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])) + (d*x*(a + b*x^2))/
(a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d]*(a + b
*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
 + (Sqrt[c]*Sqrt[d]*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(a*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)]*(c + d*x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} + \frac{\left(bd\sqrt{a+bx^2}\right.}{a\sqrt{\frac{e(}{}}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\right.\right.}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+b}{c+dx}}}$$

$$= -\frac{a+bx^2}{ax\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{dx(a+bx^2)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{d}(a+bx^2)E\left(\tan^{-1}\left(\right.\right.}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+b}{c+dx}}}$$

Mathematica [A]
time = 1.10, size = 111, normalized size = 0.38

$$\frac{(a+bx^2)\left(-\frac{1}{x}+\frac{d\sqrt{1+\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{1+\frac{bx^2}{a}}\,(c+dx^2)}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] ((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)
]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)])
```

Maple [A]
time = 0.05, size = 297, normalized size = 1.03

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{-\frac{b}{a}}\,bd\,x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adx+bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,x\,\mathrm{EllipticF}\left(x\right.\right.}{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)\,(b\,x^2+a)}\,ax}$
risch	$-\frac{b\,x^2+a}{ax\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{d\left(-\frac{2bace\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^4+ade\,x^2+bce\,x^2+ace}\,(ade+bce+e(ad-bc))}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*((-b/a)^(1/2)*b*d*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*x+b*c*((b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*((b*x^2+a)/a
)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))+(-b
/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/(e*(b*x^2+a)/(d*x^
2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/(x^2*sqrt((b*x^2 + a)/(d*x^2 + c))), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/(x^2*sqrt((b*x^2 + a)/(d*x^2 + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.306 $\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal. Leaf size=375

$$\frac{-a-bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{d}\,(2bc-ad)(a+bx^2)\,E\Big(\tan^{-}}{3a^2\sqrt{c}\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a-}{c+}}}$$

```
[Out] 1/3*(-b*x^2-a)/a/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+
a)/a^2/c/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2
/c/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*c)*(b*x^2+a)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)/a^2/(d*x^2+c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^
2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*b*(b*x^2+a)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c
/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^2/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/
(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 372, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 486, 597, 545, 429, 506, 422}

$$-\frac{b\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{d}\,(a+bx^2)(2bc-ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(a+bx^2)(2bc-ad)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)(2bc-ad)}{3a^2c(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -1/3*(a + b*x^2)/(a*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((2*b*c - a*d)
*(a + b*x^2))/(3*a^2*c*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(2*b*c - a
*d)*x*(a + b*x^2))/(3*a^2*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
+ (Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (b*Sqrt[c]*Sqrt[d]*(a + b*x^2
)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{a+bx^2}}\,dx}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-2bc+ad-bdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{abcd+bd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+}}}{3a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{b}{}$$

$$= -\frac{a+bx^2}{3ax^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(2bc-ad)(a+bx^2)}{3a^2cx\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(2bc-ad)x(a+bx^2)}{3a^2c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.46, size = 238, normalized size = 0.63

$$\frac{-\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(-2bcx^2+a(c+dx^2))-ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ibc(-bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{3a^2\sqrt{\frac{b}{a}}cx^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] (-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c*
(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcS
inh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/
(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Maple [A]
time = 0.05, size = 444, normalized size = 1.18

method	result
risch	$-\frac{(b x^2+a)(a d x^2-2bc x^2+ac)}{3a^2x^3c\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}-\frac{bd\left(\frac{2(ad-2bc)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{El}\ldots\right.}{\sqrt{-\frac{b}{a}}\sqrt{deb x^4+ade x^2+bce x^2+ace}\,(ade+\ldots}\right.}{\ldots}$
default	$-\frac{(b x^2+a)\left(\sqrt{-\frac{b}{a}}\,ab d^2x^6-2\sqrt{-\frac{b}{a}}\,b^2cd x^6+2bd\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac-2\sqrt{\frac{b x^2+a}{a}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*((-b/a)^(1/2)*a*b*d^2*x^6-2*(-b/a)^(1/2)*b^2*c*d*x^6+2*b*d*(
(b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(
1/2))*x^3*a*c-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+2*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+
(-b/a)^(1/2)*a^2*d^2*x^4-2*(-b/a)^(1/2)*b^2*c^2*x^4+2*(-b/a)^(1/2)*a^2*c*d*
x^2-(-b/a)^(1/2)*a*b*c^2*x^2+(-b/a)^(1/2)*a^2*c^2)/(e*(b*x^2+a)/(d*x^2+c))^
(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/x^3/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] e^(-1/2)*integrate(1/(x^4*sqrt((b*x^2 + a)/(d*x^2 + c))), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-1/2)/(x^4*sqrt((b*x^2 + a)/(d*x^2 + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

3.307 $\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=354

$$-\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^4de^2}-\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^3d(bc-ad)e^2}-\frac{}{b(b\cdots}$$

[Out] -1/16*(-a*d+b*c)*(b^2*c^2+5*a*d*(-7*a*d+2*b*c))*arctanh(d^(1/2)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(9/2)/d^(3/2)/e^(3/2)-a^2*(d*x^2+c)^3
/b/(-a*d+b*c)^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16*(b^2*c^2+5*a*d*(-7*a*d
+2*b*c))*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^4/d/e^2-1/24*(b^2*c^2+5*
a*d*(-7*a*d+2*b*c))*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^3/d/(-a*d+b
c)/e^2+1/6(7*a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)^3*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)/b^2/d/(-a*d+b*c)^2/e^2

Rubi [A]
time = 0.36, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 1980, 473, 393, 205, 214}

$$\frac{(c+dx^2)^3(7a^2d^2-2abcd+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6b^2de^2(bc-ad)^2}-\frac{a^2(c+dx^2)^3}{be(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(c+dx^2)^2\left(\frac{5a(2bc-7ad)}{b^2}+\frac{c^2}{d}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24be^2(bc-ad)}-\frac{(bc-ad)(5ad(2bc-7ad)+b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}-\frac{(c+dx^2)(5ad(2bc-7ad)+b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^4de^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] -1/16*((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*
(c + d*x^2))/(b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*
x^2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^2*c^2 -
 2*a*b*c*d + 7*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6
*b^2*d*(b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*
ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16
*b^(9/2)*d^(3/2)*e^(3/2))

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{(-ae+cx^2)^2}{x^2\,(be-dx^2)^4}\,dx, x, \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\right)$$

$$= -\frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}} + \frac{(bc-ad)\text{Subst}\left(\int \frac{-a(2bc-7ad)e^2+bc^2ex^2}{(be-dx^2)^4}\,dx, x, \sqrt{\frac{e\,(a+}{c+}}\right.}{b}$$

$$= -\frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-2abcd+7a^2d^2)\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^3}{6b^2d(bc-ad)^2e^2} - \frac{(}{}$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{24b^3d(bc-ad)e^2} - \frac{a^2(c+dx^2)^3}{b(bc-ad)^2e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}} +$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e\,(a}{c+}}}{24b^3d(bc-ad)}$$

$$= -\frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{16b^4de^2} - \frac{(b^2c^2+5ad(2bc-7ad))\sqrt{\frac{e\,(a}{c+}}}{24b^3d(bc-ad)}$$

Mathematica [A]
time = 4.62, size = 247, normalized size = 0.70

$$\frac{\sqrt{d}\sqrt{\frac{b\,(c+dx^2)}{bc-ad}}\,(105a^3d^2+5a^2bd(-20c+7dx^2)+ab^2(3c^2-38cdx^2-14d^2x^4)+b^3x^2(3c^2+14cdx^2+8d^2x^4)) - 3\sqrt{bc-ad}\,(b^2c^2+10abcd-35a^2d^2)\sqrt{a+bx^2}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right)}{48b^4d^{3/2}e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\sqrt{\frac{b\,(c+dx^2)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
+ 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*
```

```
c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^
2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*
b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c -
 a*d)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(324) = 648$.

time = 0.15, size = 1027, normalized size = 2.90

method	result
risch	$\frac{(8b^2d^2x^4-22ab\,d^2x^2+14b^2cd\,x^2+57a^2d^2-52abcd+3b^2c^2)(b\,x^2+a)}{48d\,b^4e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}} + \frac{\left(-\frac{35d^2\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bc}\right.}{32b^4\sqrt{deb}}\right.}{}$
default	$\frac{\left(-60\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,ab^2d^2x^4+12\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^3cd\,x^4-105\ln\left(\frac{2bc}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*(-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b^2*d^2*x^4+12*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^3*c*d*x^4-105*ln(1/2*(2*b
*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/
2))*a^3*b*d^3*x^2+135*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b^2*c*d^2*x^2-27*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*
b^3*c^2*d*x^2-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^4*c^3*x^2+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(3/2)*(b*d)^(1/2)*b^2*d*x^2+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)*a^2*b*d^2*x^2-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b^
2*c*d*x^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^3*c^2*x^2-105
*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*
c)/(b*d)^(1/2))*a^4*d^3+135*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3*b*c*d^2-27*ln(1/2*(2*b*d*x^2
+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^
2*b^2*c^2*d-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)
^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^3*c^3+96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)
^(1/2)*a^3*d^2-96*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a^2*b*c*d+16*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)^(1/2)*a*b*d+114*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(b*d)^(1/2)*a^3*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*
(b*d)^(1/2)*a^2*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b
```

^2*c^2)/d/b^4*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)

Maxima [A]
time = 0.52, size = 428, normalized size = 1.21

$$\frac{1}{96}\left(\frac{2\left(48a^2b^4cd-48a^3b^3d^2+\frac{3\left(b^3c^3d^2+9ab^2c^2d^3-45a^2bcd^4+35a^3d^5\right)\left(bx^2+a\right)^3}{\left(dx^2+c\right)^3}-\frac{8\left(b^4c^3d+9ab^3c^2d^2-45a^2b^2cd^3+35a^3bd^4\right)\left(bx^2+a\right)^2}{\left(dx^2+c\right)^2}-\frac{3\left(b^5c^3-23ab^4c^2d+99a^2b^3cd^2-77a^3b^2d^3\right)\left(bx^2+a\right)}{dx^2+c}\right)}{b^4d^4\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{7}{2}}-3b^5d^3\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}+3b^6d^2\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-b^7d\sqrt{\frac{bx^2+a}{dx^2+c}}}+\frac{3\left(b^3c^3+9ab^2c^2d-45a^2bcd^2+35a^3d^3\right)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^4d}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/96*(2*(48*a^2*b^4*c*d - 48*a^3*b^3*d^2 + 3*(b^3*c^3*d^2 + 9*a*b^2*c^2*d^3 - 45*a^2*b*c*d^4 + 35*a^3*d^5)*(b*x^2 + a)^3/(d*x^2 + c)^3 - 8*(b^4*c^3*d + 9*a*b^3*c^2*d^2 - 45*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*(b*x^2 + a)^2/(d*x^2 + c)^2 - 3*(b^5*c^3 - 23*a*b^4*c^2*d + 99*a^2*b^3*c*d^2 - 77*a^3*b^2*d^3)*(b*x^2 + a)/(d*x^2 + c))/(b^4*d^4*((b*x^2 + a)/(d*x^2 + c))^(7/2) - 3*b^5*d^3*((b*x^2 + a)/(d*x^2 + c))^(5/2) + 3*b^6*d^2*((b*x^2 + a)/(d*x^2 + c))^(3/2) - b^7*d*sqrt((b*x^2 + a)/(d*x^2 + c))) + 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b^4*d))*e^(-3/2)

Fricas [A]
time = 0.64, size = 750, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(b^6*d^2*x^2 + a*b^5*d^2), 1/96*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*d)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*

```
b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt(
(b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(b^6*d^2*x^2 + a*b^5*d^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.308 $\int \frac{x^3}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}{8b^3e^2}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)^2}{4b^2e^2}+\frac{3(bc-5ad)(bc-ad)\,\mathrm{t}\cdots}{8b\cdots}$$

```
[Out] 3/8*(-5*a*d+b*c)*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b
^(1/2)/e^(1/2))/b^(7/2)/e^(3/2)/d^(1/2)+a*(-a*d+b*c)/b^3/e/(e*(b*x^2+a)/(d*
x^2+c))^(1/2)+1/8*(-7*a*d+3*b*c)*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^
3/e^2+1/4*(d*x^2+c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^2/e^2
```

Rubi [A]

time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 467, 464, 214}

$$\frac{3(bc-5ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}\,e^{3/2}}+\frac{(c+dx^2)(3bc-7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^3e^2}+\frac{a(bc-ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d
)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])
])/(8*b^(7/2)*Sqrt[d]*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{-ae+cx^2}{x^2\left(be-dx^2\right)^3}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} - \frac{1}{4}((bc-ad)e)\text{Subst}\left(\int \frac{\frac{4a}{b}-\frac{3(bc-ad)x^2}{b^2e}}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx\right.}{c+dx}}\right.$$

$$= \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2} + \frac{1}{8}((bc-ad)e)\text{Sub}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2}$$

$$= \frac{a(bc-ad)}{b^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)}{8b^3e^2} + \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(c+dx^2\right)^2}{4b^2e^2}$$

Mathematica [A]
time = 2.21, size = 179, normalized size = 0.89

$$\frac{\sqrt{b}\sqrt{d}\sqrt{c+dx^2}\left(-15a^2d+ab(13c-5dx^2)+b^2x^2(5c+2dx^2)\right)+3(b^2c^2-6abcd+5a^2d^2)\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{7/2}\sqrt{d}\,e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[d]*Sqrt[c + d*x^2]*(-15*a^2*d + a*b*(13*c - 5*d*x^2) + b^2*x^
2*(5*c + 2*d*x^2)) + 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Sqrt[a + b*x^2]*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(7/2)*Sqrt
[d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(176) = 352$.
time = 0.11, size = 679, normalized size = 3.36

method	result
risch	$-\frac{(-2bd\,x^2+7ad-5bc)(b\,x^2+a)}{8b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(\frac{15\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a^2d^2}{16b^3\sqrt{deb}}-\frac{9\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bc}{\sqrt{d}}\right.}{}\right.}{}$
default	$-\frac{\left(-4\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2d\,x^4-15\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*d*x^4-15*ln(1
/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b
*d)^(1/2))*a^2*b*d^2*x^2+18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c*d*x^2-3*ln(1/2*(2*b*d*x^
2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b
^3*c^2*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*d*x^2-10*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*x^2-15*ln(1/2*(2*b*d*
x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*a^3*d^2+18*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(
1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*b*c*d-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*b^2*c^2+14*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d-10*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(b*d)^(1/2)*a*b*c+16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*
d-16*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c)/b^3*(b*x^2+a)/(b*d)^(1/
2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [A]

time = 0.51, size = 281, normalized size = 1.39

$$\frac{1}{16}\left(\frac{2\left(8\,ab^3c-8\,a^2b^2d-\frac{3\,(b^2c^2d-6\,abcd^2+5\,a^2d^3)(bx^2+a)^2}{(dx^2+c)^2}+\frac{5\,(b^3c^2-6\,ab^2cd+5\,a^2bd^2)(bx^2+a)}{dx^2+c}\right)}{b^3d^2\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{5}{2}}-2\,b^4d\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}+b^5\sqrt{\frac{bx^2+a}{dx^2+c}}}-\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^3}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(8*a*b^3*c - 8*a^2*b^2*d - 3*(b^2*c^2*d - 6*a*b*c*d^2 + 5*a^2*d^3)*
(b*x^2 + a)^2/(d*x^2 + c)^2 + 5*(b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*(b*x^
```

```
2 + a)/(d*x^2 + c))/(b^3*d^2*((b*x^2 + a)/(d*x^2 + c))^(5/2) - 2*b^4*d*((b*
x^2 + a)/(d*x^2 + c))^(3/2) + b^5*sqrt((b*x^2 + a)/(d*x^2 + c))) - 3*(b^2*c
^2 - 6*a*b*c*d + 5*a^2*d^2)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d
))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b^3))*e^(-3/2)
```

Fricas [A]
time = 0.59, size = 554, normalized size = 2.74

$$\left[\frac{\left(3\left(ab^2c^2-6a^2bcd+5a^3d^2+\left(b^3c^2-6ab^2cd+5a^2bd^2\right)x^2\right)\sqrt{bd}\log\left(8b^2d^2x^4+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^2+4\left(2bd^2x^4+bc^2+acd+\left(3bcd+ad^2\right)x^2\right)\sqrt{bd}\sqrt{\frac{bx^2+a}{dx^2+c}}\right)+4\left(2b^3d^3x^6+13ab^2c^2d-15a^2bcd^2+\left(7b^3cd^2-5ab^2d^3\right)x^4+\left(5b^3c^2d+8ab^2cd^2-15a^2bd^3\right)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{32\left(b^5dx^2+ab^4d\right)},-\frac{\left(3\left(ab^2c^2-6a^2bcd+5a^3d^2+\left(b^3c^2-6ab^2cd+5a^2bd^2\right)x^2\right)\sqrt{-bd}\arctan\left(\frac{\left(2bdx^2+bc+ad\right)\sqrt{-bd}\sqrt{\frac{bx^2+a}{dx^2+c}}}{2\left(b^2dx^2+abd\right)}\right)-2\left(2b^3d^3x^6+13ab^2c^2d-15a^2bcd^2+\left(7b^3cd^2-5ab^2d^3\right)x^4+\left(5b^3c^2d+8ab^2cd^2-15a^2bd^3\right)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{16\left(b^5dx^2+ab^4d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d +
a*d^2)*x^2)*sqrt(b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))) + 4*(2*b^3*d^3*x^6 + 1
3*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c
^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^
(-3/2)/(b^5*d*x^2 + a*b^4*d), -1/16*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2
 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*
x^2 + b*c + a*d)*sqrt(-b*d)*sqrt((b*x^2 + a)/(d*x^2 + c))/(b^2*d*x^2 + a*b*
d)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5
*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b
*x^2 + a)/(d*x^2 + c))*e^(-3/2)/(b^5*d*x^2 + a*b^4*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
```

```
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

<u>**Mupad**</u> **[F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.309 $\int \frac{x}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=146

$$-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2
))*d^(1/2)/b^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(d*x^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1981, 1979, 296, 331, 214}

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\,\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/
(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*
e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\left(be-dx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{x^2(be-dx^2)}\,dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)}{2b}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3d(bc-ad))\text{Subst}\left(\int \frac{1}{be-dx^2}\,dx, x, \sqrt{\frac{e \ldots}{\ldots}}\right.}{2b^2e}$$

$$= -\frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+b \ldots}{c+d \ldots}}}{\sqrt{b}\sqrt{e}}\right.}{2b^{5/2}e^{3/2}}$$

Mathematica [A]
time = 1.41, size = 138, normalized size = 0.95

$$\frac{\sqrt{b}\sqrt{c+dx^2}\left(-2bc+3ad+bdx^2\right)+3\sqrt{d}\,(bc-ad)\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[c + d*x^2]*(-2*b*c + 3*a*d + b*d*x^2) + 3*Sqrt[d]*(b*c - a*d)
*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2]
)])/(2*b^(5/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(122) = 244$.
time = 0.10, size = 432, normalized size = 2.96

method	result

default	$-\frac{\left(3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+}}{2\sqrt{bd}}\right.\right.}{}$
risch	$\frac{(b\,x^2+a)\,d}{2b^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{3d^2\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{deb\,x^4+(ade+bce)\,x^2+ace}\right)a}{4b^2\sqrt{deb}}+\frac{3d\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+deb\,x^2}{\sqrt{deb}}+\sqrt{\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)
+a*d+b*c)/(b*d)^(1/2))*a*b*d^2*x^2-3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b^2*c*d*x^2-2*(b*d)^(1/
2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*d*x^2+3*d^2*ln(1/2*(2*b*d*x^2+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2-3*l
n(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)
/(b*d)^(1/2))*a*c*b*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d
-4*(b*d)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d+4*(b*d)^(1/2)*((d*x^2+c)*(b*
x^2+a))^(1/2)*b*c)/b^2*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d
*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [A]
time = 0.50, size = 180, normalized size = 1.23

$$\frac{1}{4}\left(\frac{2\left(2\,b^2c-2\,abd-\frac{3\,(bcd-ad^2)\,(bx^2+a)}{dx^2+c}\right)}{b^2d\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-b^3\sqrt{\frac{bx^2+a}{dx^2+c}}}-\frac{3\,(bcd-ad^2)\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b^2}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(2*b^2*c - 2*a*b*d - 3*(b*c*d - a*d^2)*(b*x^2 + a)/(d*x^2 + c))/(b^2
*d*((b*x^2 + a)/(d*x^2 + c))^(3/2) - b^3*sqrt((b*x^2 + a)/(d*x^2 + c))) - 3
*(b*c*d - a*d^2)*log((d*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(b*d))/(d*sqrt(
(b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(sqrt(b*d)*b^2))*e^(-3/2)
```

Fricas [A]
time = 0.52, size = 403, normalized size = 2.76

$$\left[-\frac{\left(3\,(abc-a^2d+(b^2c-abd)x^2)\sqrt{\frac{d}{b}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b^2cd+abd^2)x^2-4\,(2\,b^2d^2x^4+b^2c^2+abcd+(3\,b^2cd+abd^2)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{\frac{d}{b}}\right)-4\,(bd^2x^4-2\,bc^2+3\,acd-(bcd-3\,ad^2)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,(b^3x^2+ab^2)},\ -\frac{\left(3\,(abc-a^2d+(b^2c-abd)x^2)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{-\frac{d}{b}}}{2\,(bdx^2+ad)}\right)-2\,(bd^2x^4-2\,bc^2+3\,acd-(bcd-3\,ad^2)x^2)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{4\,(b^3x^2+ab^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(d/b)*log(8*b^2*d^2*x^4
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b^2*d^2*
x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^
2 + c))*sqrt(d/b)) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*a*d^2)*x
^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(b^3*x^2 + a*b^2), -1/4*(3*(a*b
*c - a^2*d + (b^2*c - a*b*d)*x^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x^2 + b*c +
a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-d/b)/(b*d*x^2 + a*d)) - 2*(b*d^2*x
^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*a*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c
)))*e^(-3/2)/(b^3*x^2 + a*b^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.310 $\int \frac{1}{x\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=152

$$\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

```
[Out] -c^(3/2)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(
3/2)/e^(3/2)+d^(3/2)*arctanh(d^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/b^(1/2)/
e^(1/2))/b^(3/2)/e^(3/2)+(-a*d+b*c)/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 491, 536, 214}

$$-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}+\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}+\frac{bc-ad}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*e^(3
/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])])/(b^(3/2)*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
```

```
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)\left(be-dx^2\right)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{(bc-ad)\text{Subst}\left(\int \frac{-(bc+ad)e+cdx^2}{(-ae+cx^2)(be-dx^2)}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{abe}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{c^2\text{Subst}\left(\int \frac{1}{-ae+cx^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)}{ae} + \frac{d^2\text{Subst}\left(\int \frac{1}{be-dx^2}\right.}{}$$

$$= \frac{bc-ad}{abe\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(}{c}}}{\sqrt{b}}\right.}{b^{3/2}e^{3/2}}$$

Mathematica [A]
time = 1.70, size = 189, normalized size = 1.24

$$\frac{-b^{3/2}c^{3/2}\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\left(\sqrt{b}\,(bc-ad)\sqrt{c+dx^2}+ad^{3/2}\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{a^{3/2}b^{3/2}e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(b^(3/2)*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[
a]*Sqrt[c + d*x^2])]) + Sqrt[a]*(Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2] + a*d^
(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d
*x^2])]))/(a^(3/2)*b^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(122) = 244.
time = 0.07, size = 401, normalized size = 2.64

method	result

default	$\frac{\left(\ln\left(\frac{2bd\,x^2+2\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,ab\,d^2x^2-\sqrt{bd}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{bd\,x^4+}}{x^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*
d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*a*b*d^2*x^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^2*c^2*x^2+
ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c
)/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^
(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^2-2*(b*d)^(1/2)
*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*d+2*(b*d)^(1/2)*(a*c)^(1/2)*((d*
x^2+c)*(b*x^2+a))^(1/2)*b*c)/a/b*(b*x^2+a)/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^2+
c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [A]
time = 0.52, size = 185, normalized size = 1.22

$$\frac{1}{2}\left(\frac{c^2\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a}-\frac{d^2\log\left(\frac{d\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{bd}}{d\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{bd}}\right)}{\sqrt{bd}\,b}+\frac{2\,(bc-ad)}{ab\sqrt{\frac{bx^2+a}{dx^2+c}}}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*sqrt((b*x^2 +
 a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a) - d^2*log((d*sqrt((b*x^2 + a)/
(d*x^2 + c)) - sqrt(b*d))/(d*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(b*d)))/(s
qrt(b*d)*b) + 2*(b*c - a*d)/(a*b*sqrt((b*x^2 + a)/(d*x^2 + c))))*e^(-3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(110) = 220.
time = 0.87, size = 1177, normalized size = 7.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*d*x^2 + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d
 + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c
*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(d/b)) +
(b^2*c*x^2 + a*b*c)*sqrt(c/a)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*
a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^
2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(c/a))/x^4
) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*
e^(-3/2)/(a*b^2*x^2 + a^2*b), -1/4*(2*(a*b*d*x^2 + a^2*d)*sqrt(-d/b)*arctan
(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-d/b)/(b*d*
x^2 + a*d)) - (b^2*c*x^2 + a*b*c)*sqrt(c/a)*log(((b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*
x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c))*
sqrt(c/a))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*x^2 + a)/
(d*x^2 + c)))*e^(-3/2)/(a*b^2*x^2 + a^2*b), 1/4*(2*(b^2*c*x^2 + a*b*c)*sqrt
(-c/a)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*s
qrt(-c/a)/(b*c*x^2 + a*c)) + (a*b*d*x^2 + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^
4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^
2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*x^2 + a)/(d*
x^2 + c))*sqrt(d/b)) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*x^2
+ a)/(d*x^2 + c)))*e^(-3/2)/(a*b^2*x^2 + a^2*b), 1/2*((b^2*c*x^2 + a*b*c)*s
qrt(-c/a)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)
)*sqrt(-c/a)/(b*c*x^2 + a*c)) - (a*b*d*x^2 + a^2*d)*sqrt(-d/b)*arctan(1/2*(
2*b*d*x^2 + b*c + a*d)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-d/b)/(b*d*x^2 +
a*d)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c
)))*e^(-3/2)/(a*b^2*x^2 + a^2*b)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.311 $\int \frac{1}{x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}+\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

```
[Out] 3/2*(-a*d+b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2
))*c^(1/2)/a^(5/2)/e^(3/2)-3/2*(-a*d+b*c)/a^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/2*(-a*d+b*c)/a/(a*e-c*e*(b*x^2+a)/(d*x^2+c))/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 296, 331, 214}

$$\frac{3\sqrt{c}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}-\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/
(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2)
)) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = ((bc-ad)e)\text{Subst}\left(\int \frac{1}{x^2\left(-ae+cx^2\right)^2}\,dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3(bc-ad))\text{Subst}\left(\int \frac{1}{x^2(-ae+cx^2)}\,dx, x, \sqrt{\frac{e\,(a}{c}}\right.}{2a}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{(3c(bc-ad))\text{Subst}\left(\right.}{}$$

$$= -\frac{3(bc-ad)}{2a^2e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{3\sqrt{c}\,(bc-ad)\tanh^{-}}{2a}$$

Mathematica [A]
time = 2.12, size = 148, normalized size = 0.87

$$\frac{-\sqrt{a}\,\sqrt{c+dx^2}\,(3bcx^2+a(c-2dx^2))+3\sqrt{c}\,(bc-ad)x^2\sqrt{a+bx^2}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx^2}}{\sqrt{a}\,\sqrt{c+dx^2}}\right)}{2a^{5/2}ex^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
- a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[
c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d
*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(146) = 292.
time = 0.12, size = 641, normalized size = 3.77

method	result

risch	$-\frac{c(b x^2+a)}{2a^2x^2e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(-\frac{3c\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\sqrt{deb x^4+(ade+bce)x^2+ace}}{x^2}\right)d}{4a\sqrt{ace}}+\frac{3c^2\ln\left(\frac{2ace+(a\ldots}{}\right.}{}\right.}{}$
default	$-\frac{\left(-2\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,b^2dx^6+3\ln\left(\frac{ad x^2+bc x^2+2\sqrt{ac}\sqrt{bd x^4+ad x^2+bc x^2+ac}+2ac}{x^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^2*d*x^6+3*ln((a*
d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)
*a^2*b*c*d*x^4-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)+2*a*c)/x^2)*a*b^2*c^2*x^4-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*a*b*d*x^4-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^
2*c*x^4+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)+2*a*c)/x^2)*a^3*c*d*x^2-3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^2*b*c^2*x^2+2*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*b*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c
)^(1/2)*a^2*d*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b*c*x
^2-4*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d*x^2+4*(a*c)^(1/2)*((d*x^
2+c)*(b*x^2+a))^(1/2)*a*b*c*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*a)*(b*x^2+a)/(a*c)^(1/2)/x^2/a^3/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+
c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [A]
time = 0.50, size = 178, normalized size = 1.05

$$-\frac{1}{4}\left(\frac{3\,(bc-ad)c\log\left(\frac{c\sqrt{\frac{bx^2+a}{dx^2+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^2+a}{dx^2+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^2}-\frac{2\left(2\,abc-2\,a^2d-\frac{3\,(bc^2-acd)\,(bx^2+a)}{dx^2+c}\right)}{a^2c\left(\frac{bx^2+a}{dx^2+c}\right)^{\frac{3}{2}}-a^3\sqrt{\frac{bx^2+a}{dx^2+c}}}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*(b*c - a*d)*c*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c))/(c*
sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a^2) - 2*(2*a*b*c -
2*a^2*d - 3*(b*c^2 - a*c*d)*(b*x^2 + a)/(d*x^2 + c))/(a^2*c*((b*x^2 + a)/(d
*x^2 + c))^(3/2) - a^3*sqrt((b*x^2 + a)/(d*x^2 + c))))*e^(-3/2)
```

Fricas [**A**]
time = 1.03, size = 433, normalized size = 2.55

$$\left[-\frac{\left(3\left((b^2c-abd)x^4+(abc-a^2d)x^2\right)\sqrt{\frac{c}{a}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a^2cd)x^2-4\,\left((abcd+a^2d^2)x^4+2\,a^2c^2+(abc^2+3\,a^2cd)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{\frac{c}{a}}}{x^4}\right)+4\left((3\,bcd-2\,ad^2)x^4+ac^2+(3\,bc^2-acd)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,(a^2bx^4+a^3x^2)},-\frac{\left(3\left((b^2c-abd)x^4+(abc-a^2d)x^2\right)\sqrt{-\frac{c}{a}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bx^2+a}{dx^2+c}}\sqrt{-\frac{c}{a}}}{2\,(bcx^2+ac)}\right)+2\left((3\,bcd-2\,ad^2)x^4+ac^2+(3\,bc^2-acd)x^2\right)\sqrt{\frac{bx^2+a}{dx^2+c}}\right)e^{\left(-\frac{3}{2}\right)}}{4\,(a^2bx^4+a^3x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*((b^2*c - a*b*d)*x^4 + (a*b*c - a^2*d)*x^2)*sqrt(c/a)*log(((b^2*c^
2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((
a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*x^2
 + a)/(d*x^2 + c))*sqrt(c/a))/x^4) + 4*((3*b*c*d - 2*a*d^2)*x^4 + a*c^2 + (
3*b*c^2 - a*c*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(a^2*b*x^4 +
a^3*x^2), -1/4*(3*((b^2*c - a*b*d)*x^4 + (a*b*c - a^2*d)*x^2)*sqrt(-c/a)*ar
ctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*x^2 + a)/(d*x^2 + c))*sqrt(-c/a)
/(b*c*x^2 + a*c)) + 2*((3*b*c*d - 2*a*d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*
x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(a^2*b*x^4 + a^3*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.312 $\int \frac{1}{x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=255

$$\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}-\frac{3(bc-ad)(5bc-ad)\,\mathrm{t}}{8a}$$

```
[Out] -3/8*(-a*d+b*c)*(-a*d+5*b*c)*arctanh(c^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/
a^(1/2)/e^(1/2))/a^(7/2)/e^(3/2)/c^(1/2)+b*(-a*d+b*c)/a^3/e/(e*(b*x^2+a)/(d
*x^2+c))^(1/2)-1/4*(-a*d+b*c)^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)/a^2/(a*e-c*e*
(b*x^2+a)/(d*x^2+c))^2-1/8*(-3*a*d+7*b*c)*(-a*d+b*c)*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)/a^3/(a*e^2-c*e^2*(b*x^2+a)/(d*x^2+c))
```

Rubi [A]
time = 0.25, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1981, 1980, 467, 464, 214}

$$-\frac{3(5bc-ad)(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}\,e^{3/2}}-\frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}+\frac{b(bc-ad)}{a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*
Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x
^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c -
a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]
)/(8*a^(7/2)*Sqrt[c]*e^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx = ((bc-ad)e)\text{Subst}\left(\int \frac{be-dx^2}{x^2\left(-ae+cx^2\right)^3}dx, x, \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{1}{4}((bc-ad)e)\text{Subst}\left(\int \frac{\frac{4b}{a}+\frac{3(bc-ad)x^2}{a^2e}}{x^2\left(-ae+cx^2\right)^2}dx, x, \sqrt{\frac{e(}{c}}\right.$$

$$= -\frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)} + \frac{1}{8}((bc-ad)e)$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e\left(a+bx\right.}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}$$

$$= \frac{b(bc-ad)}{a^3e\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} - \frac{(bc-ad)^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4a^2\left(ae-\frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(7bc-3ad)(bc-ad)\sqrt{\frac{e\left(a+bx\right.}{c+dx^2}}}{8a^3\left(ae^2-\frac{ce^2(a+bx^2)}{c+dx^2}\right)}$$

Mathematica [**A**]
time = 4.43, size = 189, normalized size = 0.74

$$\frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}\left(15b^2cx^4+abx^2(5c-13dx^2)-a^2(2c+5dx^2)\right)-3(5b^2c^2-6abcd+a^2d^2)x^4\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{7/2}\sqrt{c}\,ex^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\sqrt{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) -
 a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*
x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(7/
2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. $2(229) = 458$.
time = 0.14, size = 1042, normalized size = 4.09

method	result
risch	$-\frac{(b x^2+a)(5ad x^2-7bc x^2+2ac)}{8a^3x^4e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(-\frac{3\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\sqrt{deb x^4+(ade+bce)x^2+ace}}{x^2}\right)d^2}{16a\sqrt{ace}}+\frac{9\ln\left(\frac{2}{}\right.}{}\right.}{}$
default	$-\frac{\left(-6\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,ab^2d^2x^8+18\sqrt{bd x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,b^3cd x^8+3\ln\left(\frac{ad x}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b^2*d^2*x^8+18*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b^3*c*d*x^8+3*ln((a*d*x^2+b
*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*b*
c*d^2*x^6-18*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)+2*a*c)/x^2)*a^2*b^2*c^2*d*x^6+15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b^3*c^3*x^6-12*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*b*d^2*x^6+26*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b^2*c*d*x^6+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(a*c)^(1/2)*b^3*c^2*x^6+3*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^4*c*d^2*x^4-18*ln((a*d*x^2+b*c*x^2
+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a^3*b*c^2*d*
x^4+15*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)+2*a*c)/x^2)*a^2*b^2*c^3*x^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(
1/2)*a*b*d*x^4-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*b^2*c*x^4
-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^3*d^2*x^4+8*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*a^2*b*c*d*x^4+18*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(a*c)^(1/2)*a*b^2*c^2*x^4+16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+
a))^(1/2)*a^2*b*c*d*x^4-16*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b^2*c^
2*x^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*d*x^2-14*(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*b*c*x^2+4*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c)/c*(b*x^2+a)/(a*c)^(1/2)/x^4/a^4/((d*x^2+c
)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Maxima [A]

time = 0.54, size = 281, normalized size = 1.10

$$\frac{1}{16}\left(\frac{2\left(8\,a^{2}b^{2}c-8\,a^{3}bd+\frac{3\,(5\,b^{2}c^{3}-6\,abc^{2}d+a^{2}cd^{2})(bx^{2}+a)^{2}}{(dx^{2}+c)^{2}}-\frac{5\,(5\,ab^{2}c^{2}-6\,a^{2}bcd+a^{3}d^{2})(bx^{2}+a)}{dx^{2}+c}\right)}{a^{3}c^{2}\left(\frac{bx^{2}+a}{dx^{2}+c}\right)^{\frac{5}{2}}-2\,a^{4}c\left(\frac{bx^{2}+a}{dx^{2}+c}\right)^{\frac{3}{2}}+a^{5}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}+\frac{3\,(5\,b^{2}c^{2}-6\,abcd+a^{2}d^{2})\log\left(\frac{c\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}-\sqrt{ac}}{c\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}+\sqrt{ac}}\right)}{\sqrt{ac}\,a^{3}}\right)e^{\left(-\frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(8*a^2*b^2*c - 8*a^3*b*d + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2)*
(b*x^2 + a)^2/(d*x^2 + c)^2 - 5*(5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*(b*x^
2 + a)/(d*x^2 + c))/(a^3*c^2*((b*x^2 + a)/(d*x^2 + c))^(5/2) - 2*a^4*c*((b*
x^2 + a)/(d*x^2 + c))^(3/2) + a^5*sqrt((b*x^2 + a)/(d*x^2 + c))) + 3*(5*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*log((c*sqrt((b*x^2 + a)/(d*x^2 + c)) - sqrt(a*c
))/(c*sqrt((b*x^2 + a)/(d*x^2 + c)) + sqrt(a*c)))/(sqrt(a*c)*a^3))*e^(-3/2)
```

Fricas [**A**]
time = 1.86, size = 584, normalized size = 2.29

$$\left[\frac{\left(3\left((5\,b^{3}c^{2}-6\,ab^{2}cd+a^{2}bd^{2})x^{6}+(5\,ab^{2}c^{2}-6\,a^{2}bcd+a^{3}d^{2})x^{4}\right)\sqrt{ac}\,\log\left(\frac{(b^{2}c^{2}+6\,abcd+a^{2}d^{2})x^{4}+8\,a^{2}c^{2}+8\,(abc^{2}+a^{2}cd)x^{2}-4\,((bcd+ad^{2})x^{4}+2\,ac^{2}+(bc^{2}+3\,acd)x^{2})\sqrt{ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{x^{4}}\right)+4\,\left((15\,ab^{2}c^{2}d-13\,a^{2}bcd^{2})x^{6}-2\,a^{3}c^{3}+(15\,ab^{2}c^{3}-8\,a^{2}bc^{2}d-5\,a^{3}cd^{2})x^{4}+(5\,a^{2}bc^{3}-7\,a^{3}c^{2}d)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{3}{2}\right)}}{32\,(a^{4}bcx^{6}+a^{5}cx^{4})},\ \frac{\left(3\left((5\,b^{3}c^{2}-6\,ab^{2}cd+a^{2}bd^{2})x^{6}+(5\,ab^{2}c^{2}-6\,a^{2}bcd+a^{3}d^{2})x^{4}\right)\sqrt{-ac}\,\arctan\left(\frac{((bc+ad)x^{2}+2\,ac)\sqrt{-ac}\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}}{2\,(abcx^{2}+a^{2}c)}\right)+2\,\left((15\,ab^{2}c^{2}d-13\,a^{2}bcd^{2})x^{6}-2\,a^{3}c^{3}+(15\,ab^{2}c^{3}-8\,a^{2}bc^{2}d-5\,a^{3}cd^{2})x^{4}+(5\,a^{2}bc^{3}-7\,a^{3}c^{2}d)x^{2}\right)\sqrt{\frac{bx^{2}+a}{dx^{2}+c}}\right)e^{\left(-\frac{3}{2}\right)}}{16\,(a^{4}bcx^{6}+a^{5}cx^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*
b*c*d + a^3*d^2)*x^4)*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 +
8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 +
(b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c)*sqrt((b*x^2 + a)/(d*x^2 + c)))/x^4) + 4*((
15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*
c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*x^2 + a
)/(d*x^2 + c)))*e^(-3/2)/(a^4*b*c*x^6 + a^5*c*x^4), 1/16*(3*((5*b^3*c^2 - 6
*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*x^4)*sq
rt(-a*c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*c)*sqrt((b*x^2 + a)/(
d*x^2 + c))/(a*b*c*x^2 + a^2*c)) + 2*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6
 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*
c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*x^2 + a)/(d*x^2 + c)))*e^(-3/2)/(a^4*b*c*x^
6 + a^5*c*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.313 $\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=453

$$\frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}}{}$$

```
[Out] 1/5*(-8*a*d+7*b*c)*x*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+6/5*d*x^
3*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5*(16*a^2*d^2-16*a*b*c*d+
b^2*c^2)*x*(b*x^2+a)/b^4/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x^3*(d*x
^2+c)/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*c^(3/2)*(-8*a*d+7*b*c)*(b*x^2+a
)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/b^3/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/5*(16*a^2*d^2-16*a*b*c*d+b^2*c
^2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)/b^4/e/(d*x^2+c)/d^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 453, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1986, 478, 595, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,(a+bx^2)\,(16a^2d^2-16abcd+b^2c^2)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^4\sqrt{d}\,e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)\,(16a^2d^2-16abcd+b^2c^2)}{5b^4e\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{c^{3/2}(a+bx^2)\,(7bc-8ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5b^3\sqrt{d}\,e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(a+bx^2)\,(7bc-8ad)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b*c - 8*a*d)*x*(a + b*x^2))/(5*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
 + (6*d*x^3*(a + b*x^2))/(5*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((b^
2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*x*(a + b*x^2))/(5*b^4*e*Sqrt[(e*(a + b*x^2
))/(c + d*x^2)]*(c + d*x^2)) - (x^3*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]) - (Sqrt[c]*(b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(a + b*x^2)*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^4*Sqrt[d]*e*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d
*x^2)) - (c^(3/2)*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], 1 - (b*c)/(a*d)])/(5*b^3*Sqrt[d]*e*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
```

```
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^4(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{c+dx^2}\,(3c+6dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{x^2(3c(5bc-6ad)+3d(7bc-8ad)x^2)}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{3acd(7\cdots}{\cdots}}{15b^3de\sqrt{\cdots}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(ac(7bc-8ad)\sqrt{a\cdots}\right.}{5b^3e\sqrt{\cdots}}$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \cdots$$

$$= \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2c^2-16abcd+16a^2d^2)\,x(a+bx^2)}{5b^4e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.73, size = 271, normalized size = 0.60

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}\,dx(c+dx^2)\left(-8a^2d+ab(7c-2dx^2)+b^2x^2(2c+dx^2)\right)-ic(b^2c^2-16abcd+16a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)+ic(b^2c^2-9abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{5b^3\sqrt{\frac{b}{a}}\,de^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d + a
*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d + 1
```

6*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])
)/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))

Maple [A]
time = 0.13, size = 936, normalized size = 2.07

method	result
risch	$-\frac{x(-bdx^2+3ad-2bc)(bx^2+a)}{5b^3e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}+\frac{\left(-\frac{2(11a^2d^2-11abcd+b^2c^2)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{debx^4+adex^2+bcex^2+ac\cdots}}\right.}{\cdots}$
default	$-\frac{(bx^2+a)\left(-\sqrt{-\frac{b}{a}}\sqrt{(dx^2+c)(bx^2+a)}\,b^2d^3x^7+2\sqrt{-\frac{b}{a}}\sqrt{(dx^2+c)(bx^2+a)}\,abd^3x^5-3\sqrt{-\frac{b}{a}}\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/5*(b*x^2+a)*(-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*d^3*x^7+2*(-b
/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^3*x^5-3*(-b/a)^(1/2)*((d*x^2+c)
*(b*x^2+a))^(1/2)*b^2*c*d^2*x^5+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a
)^(1/2)*a^2*d^3*x^3-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*
c*d^2*x^3+3*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^3*x^3-2*(-b/a)^(
1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*d*x^3+8*((b*x^2+a)/a)^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a
))^(1/2)*a^2*c*d^2-9*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c^2*d+((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((
d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^3-16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2
*c*d^2+16*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c^2*d-((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b
*x^2+a))^(1/2)*b^2*c^3+5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a
^2*c*d^2*x-5*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c^2*d*x+3
(-b/a)^(1/2)((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d^2*x-2*(-b/a)^(1/2)*((d*x^

```
2+c)*(b*x^2+a))^(1/2)*a*b*c^2*d*x)/b^3/d/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x
^2+c)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate(x^4/((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate(x^4*e^(-3/2)/((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
[Out] int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.314 $\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=378

$$\frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,(7bc-8ad)\,(a+bx^2)\,E\Big(\tan^{}}{3b^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+b}{c+dx}}}$$

```
[Out] 4/3*d*x*(b*x^2+a)/b^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*d*(-8*a*d+7*b*c)*
x*(b*x^2+a)/b^3/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-x*(d*x^2+c)/b/e/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*c^(3/2)*(-4*a*d+3*b*c)*(b*x^2+a)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))/a/b^2/e/(d*x^2+c)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-8*a*d+7*b*c)*(b*x^2+a)*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/b^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 478, 542, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,(7bc-8ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^3e\,(c+dx^2)\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}(a+bx^2)\,(3bc-4ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab^2\sqrt{d}\,e\,(c+dx^2)\,\sqrt{\frac{c\,(a+bx^2)}{a\,(c+dx^2)}}\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)\,(7bc-8ad)}{3b^3e\,(c+dx^2)\,\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}+\frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}-\frac{x(c+dx^2)}{be\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (4*d*x*(a + b*x^2))/(3*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(7*b*c
 - 8*a*d)*x*(a + b*x^2))/(3*b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*
x^2)) - (x*(c + d*x^2))/(b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (Sqrt[c]*
Sqrt[d]*(7*b*c - 8*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(3*b^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a
 + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*(3*b*c - 4*a*d)*(a + b*x^2)
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a*b^2*Sqrt[d]*
e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(
c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{x^2(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= -\frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{\sqrt{c+dx^2}(c+4dx^2)}{\sqrt{a+bx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{c(3bc-4ad)+d(7bc-8ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(d(7bc-8ad)\sqrt{a+bx^2}\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+}}}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^{3/2}(3bc-4}{3ab^2\sqrt{d}\,e}$$

$$= \frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(7bc-8ad)x(a+bx^2)}{3b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{x(c+dx^2)}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{d}(7bc}{3b^3e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.10, size = 219, normalized size = 0.58

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)(-3bc+4ad+bdx^2)+ic(-7bc+8ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-4ic(-bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{3a^2\left(\frac{b}{a}\right)^{5/2}e^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*d
 + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

Maple [A]
time = 0.08, size = 643, normalized size = 1.70

method	result
default	$\frac{(bx^2+a)\left(\sqrt{-\frac{b}{a}}\sqrt{(dx^2+c)(bx^2+a)}\,bd^2x^5+3\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{-\frac{b}{a}}\,ad^2x^3-3\sqrt{bdx^4\cdots}\right.}{\cdots}$
risch	$\frac{dx(bx^2+a)}{3b^2e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}-\frac{\left(-\frac{2\left(5d^2ab-4b^2cd\right)ace\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\cdots\right.}{\sqrt{-\frac{b}{a}}\sqrt{debx^4+adex^2+bcex^2+ace}\,(ade+bce+e(ad-\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^2+a)*((-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b*d^2*x^5+3*(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^3-3*(b*d*x^4+a*d*x^2+b*c*
x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1
/2)*a*d^2*x^3+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c*d*x^3+4*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*(
(d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d-4*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2
-8*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/
c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d+7*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^
(1/2)*b*c^2+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-3*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x+(-b/a)^(1/2)*((d*x^2
+c)*(b*x^2+a))^(1/2)*a*c*d*x)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2
/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(x^2/((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*e^(-3/2)/((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.315 $\int \frac{1}{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=327

$$\frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\,\sqrt{d}\,(bc-2ad)\,(a+bx^2)\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}$$

```
[Out] (-a*d+b*c)*x/a/b/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-d*(-2*a*d+b*c)*x*(b*x^2+a)
/a/b^2/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(1-b*c/a/d)^(1/2))*d^(1/2)/a/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+(-2*a*d+b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/
2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a
/d)^(1/2))*c^(1/2)*d^(1/2)/a/b^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2
)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1986, 424, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,(bc-2ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2e\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{c^{3/2}\sqrt{d}\,(a+bx^2)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{abe\,(c+dx^2)\,\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)\,(bc-2ad)}{ab^2e\,(c+dx^2)\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{x(bc-ad)}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(b*c - 2*a*d
)*x*(a + b*x^2))/(a*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) +
(Sqrt[c]*Sqrt[d]*(b*c - 2*a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(a*b^2*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*b*e*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}} - \frac{\left(d(bc-2ad)\sqrt{a+\right.}{abe\sqrt{\right.}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{d}(a+bx^2)F\left(\tan^{-1}\left(\right.\right.}{abe\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx}{c+dx^2}}}$$

$$= \frac{(bc-ad)x}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(bc-2ad)x(a+bx^2)}{ab^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{\sqrt{c}\sqrt{d}(bc-2ad)(a+bx^2)E\left(\right.}{ab^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(}{c}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.00, size = 203, normalized size = 0.62

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(bc-ad)\left(\sqrt{\frac{b}{a}}x(c+dx^2)-ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)\right)}{a^2\left(\frac{b}{a}\right)^{3/2}e^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (
b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/2)*e
^2*(a + b*x^2))
```

Maple [A]
time = 0.04, size = 514, normalized size = 1.57

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3-\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+\sqrt{\frac{b}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^3-(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+((b*x^2+a)/a)^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*
(b*x^2+a))^(1/2)*a*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2-2*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((
d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2+(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+
c)^2/a/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(((b*x^2 + a)/(d*x^2 + c))^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/((b*x^2 + a)/(d*x^2 + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

```
[Out] int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

3.316 $\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=380

$$\frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}-\frac{\sqrt{c}\,\sqrt{d}\,(2bc-ad)(a+bx^2)E\Big(\mathrm{t}}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a-}{c+}}}$$

```
[Out] (-a*d+b*c)/a/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)/a^2
/b/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+d*(-a*d+2*b*c)*x*(b*x^2+a)/a^2/b/e/(d*
x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+c^(3/2)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*d^(1/2)/a^2/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2
+a)/(d*x^2+c))^(1/2)-(-a*d+2*b*c)*(b*x^2+a)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c
^(1/2)*d^(1/2)/a^2/b/e/(d*x^2+c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(e*(b*x^2+
a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{d}\,(a+bx^2)F\left(\mathrm{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2e(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)(2bc-ad)E\left(\mathrm{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)(2bc-ad)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{dx(a+bx^2)(2bc-ad)}{a^2be(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((2*b*c - a*d)*(a
 + b*x^2))/(a^2*b*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (d*(2*b*c - a*d)
*x*(a + b*x^2))/(a^2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (
Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*(a + b*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], 1 - (b*c)/(a*d)])/(a^2*b*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) + (c^(3/2)*Sqrt[d]*(a + b*x^2)*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a^2*e*Sqrt[(c*(a +
 b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^2(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(2bc-ad)-bcdx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{abc^2d+bcd(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\left(cd\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx}}}{ae\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} + \frac{c^{3/2}\sqrt{}}{a^2e}$$

$$= \frac{bc-ad}{abex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(2bc-ad)(a+bx^2)}{a^2bex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{d(2bc-ad)x(a+bx^2)}{a^2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)} - \frac{\sqrt{c}\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.30, size = 223, normalized size = 0.59

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)(ac+2bcx^2-adx^2)+ic(-2bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-2ic(-bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\right)}{a^2\sqrt{\frac{b}{a}}e^2x(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x^
2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*x
*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

Maple [A]
time = 0.07, size = 654, normalized size = 1.72

method	result
default	$\frac{(b x^2+a)\left(\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\;a\,d^2x^4-\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\;bcd\,x^4-\sqrt{-\cdots}\right.}{\cdots}$
risch	$-\frac{c(b\,x^2+a)}{a^2xe\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{2b\,c^2dae\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\cdots}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^4+ade\,x^2+bce\,x^2+ace}\;(ade+bce+e(ad-bc\cdots}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*((-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*d^2*x^4-(-b/a
)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*d*x^4-(-b/a)^(1/2)*((d*x^2+
c)*(b*x^2+a))^(1/2)*b*c*d*x^4+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ell
ipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d*x-
2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c
)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2*x-((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(
1/2)*a*c*d*x+2*c^2*b*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*x+(-b/a)^(1/2)*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*d*x^2-(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*b*c^2*x^2-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*c*d*x^
2-(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b*c^2*x^2-(-b/a)^(1/2)*((d*x^2+c
)*(b*x^2+a))^(1/2)*a*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^2/(-b
/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/x
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] e^(-3/2)*integrate(1/(x^2*((b*x^2 + a)/(d*x^2 + c))^(3/2)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/(x^2*((b*x^2 + a)/(d*x^2 + c))^(3/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.317 $\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} \, dx$

Optimal. Leaf size=444

$$\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(4bc-3ad)\,(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(8bc-7ad)\,(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{d(8bc-7ad)x(a+bx^2)}{3a^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+dx^2)}+\frac{\sqrt{c}\,\sqrt{}}{}$$

```
[Out] (-a*d+b*c)/a/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^
2+a)/a^2/b/e/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)
/a^3/e/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*d*(-7*a*d+8*b*c)*x*(b*x^2+a)/a^3
/e/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/3*(-7*a*d+8*b*c)*(b*x^2+a)*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*b*c)*(b*x^2+a)*
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)/a^3/e/(d*x^2+c)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1986, 479, 597, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,(4bc-3ad)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{\sqrt{c}\,\sqrt{d}\,(a+bx^2)\,(8bc-7ad)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\,(c+dx^2)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{(a+bx^2)\,(8bc-7ad)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{dx(a+bx^2)\,(8bc-7ad)}{3a^3e\,(c+dx^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}-\frac{(a+bx^2)\,(4bc-3ad)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}+\frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (b*c - a*d)/(a*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((4*b*c - 3*a*d
)*(a + b*x^2))/(3*a^2*b*e*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((8*b*c
- 7*a*d)*(a + b*x^2))/(3*a^3*e*x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (d*(8
*b*c - 7*a*d)*x*(a + b*x^2))/(3*a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c
+ d*x^2)) + (Sqrt[c]*Sqrt[d]*(8*b*c - 7*a*d)*(a + b*x^2)*EllipticE[ArcTan[(
Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)) - (Sqrt[c]*Sqrt[d
]*(4*b*c - 3*a*d)*(a + b*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^3*e*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)]*(c + d*x^2))
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}\int \frac{(c+dx^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx}{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+bx^2}\int \frac{-c(4bc-3ad)-d(3bc-2ad)x^2}{x^4\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{abe\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{\sqrt{a+bx^2}\int \frac{-bc^2(8bc-7ad)-bcd(4bc-\cdots}{x^2\sqrt{a+bx^2}\sqrt{c+\cdots}}}{3a^2bce\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+\cdots}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\sqrt{a+b\cdots}}{3a\cdots}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{\left(bd(8bc\cdots\right.}{\cdots}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc\cdots}{3a^3e\sqrt{\cdots}}$$

$$= \frac{bc-ad}{abex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(4bc-3ad)(a+bx^2)}{3a^2bex^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(8bc-7ad)(a+bx^2)}{3a^3ex\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{d(8bc\cdots}{3a^3e\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.84, size = 266, normalized size = 0.60

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}\,(c+dx^2)\,(-8b^2cx^4+a^2(c+4dx^2)+ab(-4cx^2+7dx^4))-ibc(-8bc+7ad)x^3\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)-i(8b^2c^2-11abcd+3a^2d^2)\,x^3\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{b}{a}}\,x\right)|\frac{ad}{bc}\right)\right)}{3a^3\sqrt{\frac{b}{a}}\,e^2x^3\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4 +
 a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*x^
3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3
*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

Maple [A]
time = 0.09, size = 866, normalized size = 1.95

method	result
default	$-\frac{(b x^2+a)\left(3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-3\sqrt{bd\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+4\right.}{}$
risch	$-\frac{(b\,x^2+a)\,(4ad\,x^2-5bc\,x^2+ac)}{3a^3x^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(-\frac{8d^2a^2bce\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{ceb}}\right)-\text{Elliptic}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{deb\,x^4+ade\,x^2+bce\,x^2+ace}\,(ade+bce}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*
x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6+4*(-b/a)
^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*d^2*x^6-5*(-b/a)^(1/2)*((d*x^2+c)*(b
*x^2+a))^(1/2)*b^2*c*d*x^6-3*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^2*x^3
+11*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b
/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x^3-8*((b*x^2+a)/a)^(1/2)*((
d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x
^2+a))^(1/2)*b^2*c^2*x^3-7*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*a*b*c*d*x^3+8
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*x^3+3*(-b/a)^(1/2)*(b*d*x^4+a*d
```

```
*x^2+b*c*x^2+a*c)^(1/2)*a*b*c*d*x^4-3*(-b/a)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2
+a*c)^(1/2)*b^2*c^2*x^4+4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*d^2*
x^4-5*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*b^2*c^2*x^4+5*(-b/a)^(1/2)*(
(d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c*d*x^2-4*(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))
^(1/2)*a*b*c^2*x^2+(-b/a)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*a^2*c^2)/(e*(b*
x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] e^(-3/2)*integrate(1/(x^4*((b*x^2 + a)/(d*x^2 + c))^(3/2)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^(-3/2)/(x^4*((b*x^2 + a)/(d*x^2 + c))^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(\frac{e (b x^2+a)}{d x^2+c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

3.318 $\int x^5 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=216

$$-\frac{\left(b^2+4abc-8a^2c^2\right)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^2d^3}-\frac{(b+4ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^3}+\frac{\left(c+dx^2\right)^3\left(\frac{b+a\cdots}{c\cdots}\right.}{6ad^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/a/d^3+1/16*b*(8*a^2*c^2+4
*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d^3-
1/16*(-8*a^2*c^2+4*a*b*c+b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a
^2/d^3-1/8*(4*a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3
```

Rubi [A]
time = 0.29, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1985, 1981, 1980, 474, 466, 393, 214}

$$\frac{\left(8a^2c^2-b(4ac+b)\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3}+\frac{b\left(8a^2c^2+4abc+b^2\right)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}+\frac{\left(c+dx^2\right)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{6ad^3}-\frac{(4ac+b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((8*a^2*c^2 - b*(b + 4*a*c))*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)])/(16*a^2*d^3) - ((b + 4*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)])/(8*a*d^3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(3/
2))/(6*a*d^3) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(5/2)*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int x^5\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^5\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{x^5\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{x^2\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}}$$

$$= \frac{x^2(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{6ad^2} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Sub}}{6a}$$

$$= -\frac{(3b+8ac)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{24a^2d^3} + \frac{x^2(c+dx^2)\sqrt{a+\frac{b}{c+}}}{6ad}$$

$$= \frac{(b^2+4abc+8a^2c^2)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)(c+dx^2)\sqrt{a+\frac{b}{c+d}}}{24a^2d^3}$$

$$= \frac{(b^2+4abc+8a^2c^2)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)(c+dx^2)\sqrt{a+\frac{b}{c+d}}}{24a^2d^3}$$

$$= \frac{(b^2+4abc+8a^2c^2)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)(c+dx^2)\sqrt{a+\frac{b}{c+d}}}{24a^2d^3}$$

$$= \frac{(b^2+4abc+8a^2c^2)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{16a^2d^3} - \frac{(3b+8ac)(c+dx^2)\sqrt{a+\frac{b}{c+d}}}{24a^2d^3}$$

Mathematica [A]
time = 0.24, size = 145, normalized size = 0.67

$$\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(-3b^2+2ab(-5c+dx^2)+8a^2(c^2-cdx^2+d^2x^4))+3b(b^2+4abc+8a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*
(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*
a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2
)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(196) = 392$.
time = 0.09, size = 533, normalized size = 2.47

method	result
risch	$\frac{(8d^2a^2x^4-8a^2cdx^2+2abdx^2+8a^2c^2-10abc-3b^2)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48d^3a^2}+\frac{\left(\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd}\right.}{4d^2\sqrt{ad^2}}\right.}{}$
default	$\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,(dx^2+c)\left(-48\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,x^2ca^2d\sqrt{ad^2}-12\sqrt{ad^2x^4+2acdx^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-48*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)-12*(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)+24*ln(1/2*(2*a*d^2
*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2
)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b
^2*c*a*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(1/2)
-36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)+3*l
n(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(
a*d^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 328, normalized size = 1.52

$$-\frac{3\,(8\,a^2bc^2-4\,ab^2c-b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^2-ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,(8\,a^4bc^2+4\,a^3b^2c+a^2b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5d^3-\frac{3\,(adx^2+ac+b)a^4d^3}{dx^2+c}+\frac{3\,(adx^2+ac+b)^2a^3d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3a^2d^3}{(dx^2+c)^3}\right)}-\frac{(8\,a^2c^2+4\,abc+b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{5}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^
(5/2) - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3
*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c
+ b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 + c)^3)
 - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/
2)*d^3)
```

Fricas [**A**]
time = 0.39, size = 423, normalized size = 1.96

$$\left[\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(8\,a^3d^3x^6+2\,a^2bd^2x^4+8\,a^3c^3-10\,a^2bc^2-3\,ab^2c-(8\,a^2bc+3\,ab^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192\,a^3d^3},\ -\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\sqrt{-a}\,\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(8\,a^3d^3x^6+2\,a^2bd^2x^4+8\,a^3c^3-10\,a^2bc^2-3\,ab^2c-(8\,a^2bc+3\,ab^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96\,a^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2
*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c +
b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) +
4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c -
 (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d
^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^
2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 +
a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c
^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(a^3*d^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [**A**]
time = 4.23, size = 219, normalized size = 1.01

$$\frac{1}{96}\left(2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{4\,a^2cd^3-abd^3}{a^2d^5}\right)+\frac{8\,a^2c^2d^2-10\,abcd^2-3\,b^2d^2}{a^2d^5}\right)-\frac{3\,(8\,a^2bc^2+4\,ab^2c+b^3)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{5}{2}}d^2|d|}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 - 3
*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(-2*a*c*d -
 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)
)*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d^2*abs(d)))*sgn(d*x^2 + c)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{a + \frac{b}{d\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(1/2), x)
```

3.319 $\int x^3 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8ad^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2}-\frac{b(b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

```
[Out] -1/8*b*(4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)
/d^2+1/8*(-4*a*c+b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/4*(
d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 466, 393, 214}

$$-\frac{b(4ac+b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a*d^2) +
 ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) - (b*(b + 4*
a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2)*d^2
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int x^3\sqrt{a+\frac{b}{c+dx^2}}\,dx=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+a\,(c+dx^2)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\,(c+dx^2)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\,(c+dx^2)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x\sqrt{b+ac+adx}}{\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{b+a\,(c+dx^2)}}$$

$$=\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4ad^2}-\frac{\left((b+4ac)\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right.}{8ad\sqrt{b+}}$$

$$=-\frac{(b+4ac)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2)}{4ad^2}$$

$$=-\frac{(b+4ac)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2)}{4ad^2}$$

$$=-\frac{(b+4ac)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2)}{4ad^2}$$

$$=-\frac{(b+4ac)\,(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8ad^2}+\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2)}{4ad^2}$$

Mathematica [A]
time = 0.14, size = 107, normalized size = 0.76

$$\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(b-2ac+2adx^2)}{8ad^2}-\frac{b(b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a*d*x^2))
/(8*a*d^2) - (b*(b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/S
qrt[a]])/(8*a^(3/2)*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(125) = 250$.
time = 0.06, size = 353, normalized size = 2.50

method	result
risch	$-\frac{(-2ad\,x^2+2ac-b)(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8d^2a}+\frac{\left(-\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{c^2a+bc+(2acd+bd)\,x^2+a\,d^2x^4}\right)c}{4d\sqrt{a\,d^2}}\right.}{}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-4\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{a\,d^2}\,ad\,x^2+4\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(-4*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2+4*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a*b*c*d+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2)*a*c+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d-2*(a*d^2*x^4+2
*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/a/(a*d^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 218, normalized size = 1.55

$$-\frac{(4\,abc-b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc+ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3d^2-\frac{2\,(adx^2+ac+b)a^2d^2}{dx^2+c}+\frac{(adx^2+ac+b)^2ad^2}{(dx^2+c)^2}\right)}+\frac{(4\,ac+b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c
+ a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2 - 2*(a*d*x^2 + a*c
```

```
 + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) + 1/
16*(4*a*c + b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sq
rt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^2)
```

Fricas [A]
time = 0.38, size = 325, normalized size = 2.30

$$\left[\frac{(4\,abc+b^2)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(2\,a^2d^2x^4+abdx^2-2\,a^2c^2+abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,a^2d^2},\ \frac{(4\,abc+b^2)\sqrt{-a}\,\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)+2\,(2\,a^2d^2x^4+abdx^2-2\,a^2c^2+abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,a^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c +
 a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2
+ b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 +
a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*
d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqr
t(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*
(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^2*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A]
time = 3.54, size = 159, normalized size = 1.13

$$\frac{1}{16}\left(2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{d}-\frac{2\,acd-bd}{ad^3}\right)+\frac{(4\,abc+b^2)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{3}{2}}d|d|}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2
*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d)
 - b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{a + \frac{b}{d\,x^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(1/2), x)
```

3.320 $\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=69

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

```
[Out] 1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/d/a^(1/2)+1/2*(d*x^2+c)*(a+b/(
d*x^2+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1605, 248, 43, 65, 214}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*
x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{a+\frac{b}{c+dx^2}}\,dx &= \frac{\operatorname{Subst}\left(\int \sqrt{a+\frac{b}{x}}\,dx, x, c+dx^2\right)}{2d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{c+dx^2}\right)}{2d} \\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} - \frac{b\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4d} \\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2d} \\
&= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 85, normalized size = 1.23

$$\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(57) = 114.
time = 0.09, size = 180, normalized size = 2.61

method	result
derivativedivides	$\frac{\sqrt{\frac{a(dx^2+c)+b}{dx^2+c}}\,(dx^2+c)\left(2\sqrt{a(dx^2+c)^2+b(dx^2+c)}\,\sqrt{a}+b\ln\left(\frac{2\sqrt{a(dx^2+c)^2+b(dx^2}}{2\sqrt{a}}\right.\right.}{4d\sqrt{(dx^2+c)(a(dx^2+c)+b)}\,\sqrt{a}}$
risch	$\frac{(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2d}+\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right)\sqrt{\frac{adx^2+}{dx^2}}}{4\sqrt{ad^2}\,(adx^2+ac+b)}$
default	$\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,(dx^2+c)\left(b\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}+bd}{2\sqrt{ad^2}}\right)a\right.}{4\sqrt{(dx^2+c)(adx^2+ac+b)}\,d\sqrt{ad^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/d/(a*d^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

time = 0.50, size = 126, normalized size = 1.83

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/(
d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)
```

Fricas [A]
time = 0.38, size = 267, normalized size = 3.87

$$\left[\frac{\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,ad},-\frac{\sqrt{-a}\,b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*
c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
a*b)) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(57) = 114.
time = 4.43, size = 127, normalized size = 1.84

$$-\frac{1}{4}\left(\frac{b\log\left(\left|-8\,a^{\frac{3}{2}}cd-8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a|d|-4\,\sqrt{a}\,bd\right|\right)}{\sqrt{a}\,|d|}-\frac{2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{d}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(b*log(abs(-8*a^(3/2)*c*d - 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - 4*sqrt(a)*b*d))/(sqrt(a)*abs(d
)) - 2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 +
 c)
```

Mupad [B]
time = 3.01, size = 120, normalized size = 1.74

$$\frac{\sqrt{\frac{b\,\left(d\,x^2+c\right)+a\,{\left(d\,x^2+c\right)}^2}{{\left(d\,x^2+c\right)}^2}}\,\left(d\,x^2+c\right)\,\left(\frac{b\,\ln\left(\frac{\frac{b}{2}+a\,\left(d\,x^2+c\right)+\sqrt{a}\,\sqrt{b\,\left(d\,x^2+c\right)+a\,{\left(d\,x^2+c\right)}^2}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b\,\left(d\,x^2+c\right)+a\,{\left(d\,x^2+c\right)}^2}}+2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*lo
g((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))/a
^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)
```

3.321 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x} dx$

Optimal. Leaf size=96

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{b+ac}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{c}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))*a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1985, 1981, 1980, 492, 214}

$$\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b +
a*c]*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]
)/Sqrt[c]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
```

```
, 2*n - 1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{\left((-b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x\right.}{2\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+dx^2}\right)}{\sqrt{b+a(c+dx^2)}} - \frac{\left((-b-a\right.}{}$$

$$= -\frac{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{c}\sqrt{b+a(c+dx^2)}} + \frac{\left(a\sqrt{c+\right.}{}$$

$$= \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+a(c+dx^2)}} - \frac{\sqrt{b+ac}\sqrt{c+dx^2}}{}$$

Mathematica [A]
time = 0.14, size = 102, normalized size = 1.06

$$-\frac{\sqrt{-b-ac}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}} + \sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]
```

```
[Out] -((Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sq
rt[-b - a*c]])/Sqrt[c]) + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]/Sqrt[a]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(80) = 160$.
time = 0.05, size = 235, normalized size = 2.45

method	result
default	$-\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+2 \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+c^{2} a+b c}\,\sqrt{a\,d^{2}}+b d}{2 \sqrt{a\,d^{2}}}\right) a c d+\sqrt{c^{2}\cdots}\right.}{2 \sqrt{\left(d\,x^{2}+c\right)\left(a d\,x^{2}+\cdots\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c
^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(
a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 159, normalized size = 1.66

$$\frac{(ac+b)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{1}{2}\sqrt{a}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)
*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a
*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(80) = 160$.
time = 0.42, size = 927, normalized size = 9.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*
b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2
 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c
^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*
c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt
((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(
-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*
sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16
*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^
2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d
*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c
)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c +
 b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^
2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqr
t(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*
d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^
2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 +
2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(
a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x, x)
```

3.322 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx$

Optimal. Leaf size=104

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{3/2}\sqrt{b+ac}}$$

```
[Out] 1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^
(3/2)/(a*c+b)^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 294, 214}

$$\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{3/2}\sqrt{ac+b}}-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) + (b*d*Arc
Tanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(
3/2)*Sqrt[b + a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^3\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} - \frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{}{x\sqrt{c+dx}\ \cdots}\right.}{4c\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} - \frac{\left(bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\ \cdots\right.}{2c\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2cx^2} + \frac{bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+\cdots}}{\sqrt{c}\sqrt{b+a(c-\cdots}}\right.}{2c^{3/2}\sqrt{b+ac}\sqrt{b+a(c+dx^2)}}$$

Mathematica [**A**]
time = 0.21, size = 110, normalized size = 1.06

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2cx^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{3/2}\sqrt{-b-ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3,x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^2) - (b*d*Arc
Tan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(
3/2)*Sqrt[-b - a*c])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(88) = 176$.
time = 0.08, size = 454, normalized size = 4.37

method	result
risch	$-\frac{(d\,x^2+c)\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}}{2c\,x^2}+\frac{bd\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc+(2acd+bd)\,x^2+a\,d^2x^4}}{x^2}\right.}{4c\sqrt{c^2a+bc}\,(a\,d\,x^2+ac+b)}$
default	$-\frac{\sqrt{\frac{a\,d\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2a\,d^2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,x^4\sqrt{c^2a+bc}-\ln\left(\frac{2acd\,x^2+bd\,x^2+2c}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*a*b*c^2*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b
*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^2*c
*d*x^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^2/(a*c+b)/x^2/(a*c^2+b*c)^(1/2)
```

Maxima [A]
time = 0.50, size = 156, normalized size = 1.50

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2+bc-\frac{(adx^2+ac+b)c^2}{dx^2+c}\right)}-\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^2 + b*c - (a*d*x^2 + a*
c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a
*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(88) = 176$.
time = 0.39, size = 433, normalized size = 4.16

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\,(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,(ac^3+bc^2)x^2},-\frac{\sqrt{-ac^2-bc}\,bdx^2\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,(ac^3+bc^2)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x
^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3
+ b*c^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(88) = 176.
time = 5.76, size = 281, normalized size = 2.70

$$-\frac{1}{2}\left(\frac{bd\arctan\left(-\frac{\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}\,c}+\frac{2\,a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}\,bc|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)bd}{\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)c}\right)\mathrm{sgn}\left(dx^2+c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*c^
2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 -
 sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*sgn(d*
x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^3,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^3, x)
```

3.323 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx$

Optimal. Leaf size=174

$$\frac{(5b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^2(b+ac)x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^2x^4}-\frac{b(3b+4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac}{c+}}}{\sqrt{b+a}}\right.}{8c^{5/2}(b+ac)^{3/2}}$$

```
[Out] -1/8*b*(4*a*c+3*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(5/2)/(a*c+b)^(3/2)+1/8*(4*a*c+5*b)*d*(d*x^2+c)*((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2/x^4
```

Rubi [A]
time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 466, 393, 214}

$$-\frac{bd^2(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{5/2}(ac+b)^{3/2}}+\frac{d(4ac+5b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac+b)}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] ((5*b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^2*
(b + a*c)*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c
^2*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)])/Sqrt[b + a*c]])/(8*c^(5/2)*(b + a*c)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
```

```
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^5\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4c(b+ac)x^4} - \frac{\left((3b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}}\right.}{8c(b+ac)\sqrt{}}$$

$$= \frac{(3b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4c(b+ac)x^4} +$$

$$= \frac{(3b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4c(b+ac)x^4} +$$

$$= \frac{(3b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{4c(b+ac)x^4} -$$

Mathematica [A]
time = 0.27, size = 152, normalized size = 0.87

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(b(2c-3dx^2)+2ac(c-dx^2))}{8c^2(b+ac)x^4} - \frac{b(3b+4ac)d^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{5/2}(-b-ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(2*c - 3*d*x^2)
+ 2*a*c*(c - d*x^2)))/(c^2*(b + a*c)*x^4) - (b*(3*b + 4*a*c)*d^2*ArcTan[(Sq
```

```
rt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(5/2)*(-
b - a*c)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(154) = 308$.

time = 0.09, size = 923, normalized size = 5.30

method	result
risch	$-\frac{(d\,x^{2}+c)\left(-2acd\,x^{2}-3bd\,x^{2}+2c^{2}a+2bc\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{8c^{2}x^{4}(ac+b)}+\frac{\left(-\frac{d^{2}b\ln\left(\frac{2c^{2}a+2bc+(2acd+bd)x^{2}+2\sqrt{c^{2}a+bc}\,\sqrt{c^{2}a+bc}}{x^{2}}\right.}{4c(ac+b)\sqrt{c^{2}a+bc}}\right.}{}$
default	$-\frac{\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(12a^{2}d^{3}\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+c^{2}a+bc}\;x^{6}c(c^{2}a+bc)^{\frac{3}{2}}+4\ln\left(\frac{2acd\,x^{2}+bd\,x^{2}+2c^{2}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)+4*ln((2*a*c*d*x^
2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)+2*b*c)/x^2)*a^3*b*c^5*d^2*x^4+10*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)+11*ln((2*a*c*d*x^2+b*d*x^2+2
*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+
2*b*c)/x^2)*a^2*b^2*c^4*d^2*x^4+20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)+10*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*
a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c
)/x^2)*a*b^3*c^3*d^2*x^4+28*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*a*c*d^2*b*x^4*(a*c^2+b*c)^(3/2)+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2
+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^4
*c^2*d^2*x^4+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4
*(a*c^2+b*c)^(3/2)-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d
*x^2*(a*c^2+b*c)^(3/2)-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b
*d*x^2*(a*c^2+b*c)^(3/2)+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*
(a*c^2+b*c)^(3/2)*a*c^2+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(
a*c^2+b*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^3/(a*c+b)^2/x^4/(
a*c^2+b*c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(154) = 308$.

time = 0.51, size = 322, normalized size = 1.85

$$\frac{(4\,abc+3\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(ac^3+bc^2)\sqrt{(ac+b)c}}-\frac{(4\,abc^2+5\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+7\,ab^2c+3\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2+\frac{(ac^5+bc^4)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*(4*a*b*c + 3*b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - s
qrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)
*c)))/((a*c^3 + b*c^2)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + 5*b^2*c)*d^2*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 7*a*b^2*c + 3*b^3)
*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^
2*c^3 + b^3*c^2 + (a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(
a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Fricas [A]
time = 0.49, size = 577, normalized size = 3.32

$$\left[\frac{(4\,abc+3\,b^2)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\,(2\,a^2c^5-(2\,a^2c^3+5\,abc^2+3\,b^2c)d^2x^4+4\,abc^4+2\,b^2c^3-(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,(a^2c^5+2\,abc^4+b^2c^3)x^4},\ \frac{(4\,abc+3\,b^2)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)-2\,(2\,a^2c^5-(2\,a^2c^3+5\,abc^2+3\,b^2c)d^2x^4+4\,abc^4+2\,b^2c^3-(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,(a^2c^5+2\,abc^4+b^2c^3)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*
c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*
b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)
*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/
x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4
 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-
a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 +
(a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 +
 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. 2(154) = 308.
time = 4.45, size = 713, normalized size = 4.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2)*
sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d
^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*
c^3*d*abs(d) + 24*a^(5/2)*b*c^4*d*abs(d) + 36*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 8*(sqrt(a*d^2)
*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*c
^2*d*abs(d) + 24*a^(3/2)*b^2*c^3*d*abs(d) - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 25*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2
+ 8*sqrt(a)*b^3*c^2*d*abs(d) - 3*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*
d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2
*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a*c^3 + b*c^2)*(a
*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 +
b*c))^2 + b*c)^2))*sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^5,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^5, x)
```

3.324 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx$

Optimal. Leaf size=265

$$-\frac{\left(11b^2+20abc+8a^2c^2\right)d^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16c^3(b+ac)^2x^2}+\frac{(3b+4ac)d\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3(b+ac)x^4}-\frac{\left(c+dx^2\right)}{6c^2}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c^2/(a*c+b)/x^6+1/16*b*(
8*a^2*c^2+12*a*b*c+5*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(
1/2)/(a*c+b)^(1/2))/c^(7/2)/(a*c+b)^(5/2)-1/16*(8*a^2*c^2+20*a*b*c+11*b^2)*
d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)^2/x^2+1/8*(4*a*
c+3*b)*d*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/(a*c+b)/x^4
```

Rubi [A]

time = 0.36, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1981, 1980, 474, 466, 393, 214}

$$\frac{bd^3\left(8a^2c^2+12abc+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{7/2}(ac+b)^{5/2}}-\frac{d^2\left(8a^2c^2+20abc+11b^2\right)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16c^3x^2(ac+b)^2}+\frac{d(4ac+3b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^4(ac+b)}-\frac{\left(c+dx^2\right)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{6c^2x^6(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7,x]
```

```
[Out] -1/16*((11*b^2 + 20*a*b*c + 8*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)])/(c^3*(b + a*c)^2*x^2) + ((3*b + 4*a*c)*d*(c + d*x^2)^2*S
qrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*(b + a*c)*x^4) - ((c + d*x^2)^
3*((b + a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(6*c^2*(b + a*c)*x^6) + (b*(5*b^
2 + 12*a*b*c + 8*a^2*c^2)*d^3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)])/Sqrt[b + a*c]])/(16*c^(7/2)*(b + a*c)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a\left(c+dx^2\right)}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^7\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^4\sqrt{c+dx}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{-\frac{1}{2}(5b+4a \cdots}{x^3\sqrt{c+dx}\,\sqrt{\cdots}} \cdots\right.}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{\left(\sqrt{c+dx^2} \cdots\right.}{\cdots}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b \cdots}{\cdots}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b \cdots}{\cdots}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b \cdots}{\cdots}$$

$$= -\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b \cdots}{\cdots}$$

Mathematica [A]
time = 0.42, size = 216, normalized size = 0.82

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2\left(c^2-cdx^2+d^2x^4\right)+2abc\left(8c^2-9cdx^2+13d^2x^4\right)+b^2\left(8c^2-10cdx^2+15d^2x^4\right)\right)}{48c^3(b+ac)^2x^6} - \frac{b\left(5b^2+12abc+8a^2c^2\right)d^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{16c^{7/2}(-b-ac)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7,x]

[Out] -1/48*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^2 - c*d*x^2 + d^2*x^4) + 2*a*b*c*(8*c^2 - 9*c*d*x^2 + 13*d^2*x^4) + b^2*(8*c^2 - 10*c*d*x^2 + 15*d^2*x^4)))/(c^3*(b + a*c)^2*x^6) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(16*c^(7/2)*(-b - a*c)^(5/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(241) = 482$.

time = 0.11, size = 1518, normalized size = 5.73

method	result
risch	$-\frac{(d\,x^2+c)\,(8a^2c^2d^2x^4+26ac\,d^2b\,x^4-8a^2c^3d\,x^2+15b^2d^2x^4-18ab\,c^2d\,x^2+8a^2c^4-10b^2cd\,x^2+16ab\,c^3+8b^2c^2)\,\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{48c^3x^6(ac+b)^2}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)

[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-24*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^5*b*c^8*d^3*x^6-96*a^3*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^8*c^2*(a*c^2+b*c)^(5/2)-108*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^4*b^2*c^7*d^3*x^6-156*a^2*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^8*c*b*(a*c^2+b*c)^(5/2)-195*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^3*b^3*c^6*d^3*x^6-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a^3*c^3*d^3*x^6*(a*c^2+b*c)^(5/2)-66*a*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^8*b^2*(a*c^2+b*c)^(5/2)-177*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b^4*c^5*d^3*x^6-324*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a^2*c^2*d^3*b*x^6*(a*c^2+b*c)^(5/2)-81*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^5*c^4*d^3*x^6-252*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*c*d^3*b^2*x^6*(a*c^2+b*c)^(5/2)-15*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^6*c^3*d^3*x^6+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(5/2)-66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)

^(1/2)*b^3*d^3*x^6*(a*c^2+b*c)^(5/2)+156*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d^2*b*x^4*(a*c^2+b*c)^(5/2)-48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^3*d*x^2+66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b^2*d^2*x^4*(a*c^2+b*c)^(5/2)-84*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^2*d*x^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^4-36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c*d*x^2+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^3+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c^4/(a*c+b)^3/x^6/(a*c^2+b*c)^(5/2)

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(241) = 482$.
time = 0.55, size = 557, normalized size = 2.10

$$-\frac{(8a^2bc^2+12ab^2c+5b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32(a^2c^5+2abc^4+b^2c^3)\sqrt{(ac+b)c}}-\frac{3(8a^2bc^4+20ab^2c^3+11b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8(6a^3bc^4+18a^2b^2c^3+17ab^3c^2+5b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3(8a^4bc^4+28a^3b^2c^3+37a^2b^3c^2+22ab^4c+5b^5)d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5c^8+5a^4bc^7+10a^3b^2c^6+10a^2b^3c^5+5ab^4c^4+b^5c^3-\frac{(a^2c^8+2abc^7+b^2c^6)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3(a^3c^8+3a^2bc^7+3ab^2c^6+b^3c^5)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{3(a^4c^8+4a^3bc^7+6a^2b^2c^6+4ab^3c^5+b^4c^4)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt((a*c + b)*c))
 - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*c +
b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2 + 5*
b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 28*a^
3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^5 +
 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b
)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*
x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 +
 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Fricas [**A**]
time = 0.64, size = 755, normalized size = 2.85

$$\left[\frac{3(8a^2bc^2+12ab^2c+5b^3)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2+4((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)dx^2+2bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4(8a^3c^7+(8a^3c^4+34a^2bc^3+41ab^2c^2+15b^3c)d^3x^6+24a^2bc^6+24ab^2c^5+8b^3c^4+(8a^2bc^4+13ab^2c^3+5b^3c^2)d^2x^4-2(a^2bc^5+2ab^2c^4+b^3c^3)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192(a^3c^7+3a^2bc^6+3ab^2c^5+b^3c^4)x^6},\ -\frac{3(8a^2bc^2+12ab^2c+5b^3)\sqrt{-ac^2-bc}\,d^3x^6\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2(8a^3c^7+(8a^3c^4+34a^2bc^3+41ab^2c^2+15b^3c)d^3x^6+24a^2bc^6+24ab^2c^5+8b^3c^4+(8a^2bc^4+13ab^2c^3+5b^3c^2)d^2x^4-2(a^2bc^5+2ab^2c^4+b^3c^3)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96(a^3c^7+3a^2bc^6+3ab^2c^5+b^3c^4)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(
((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 +
 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3
```

```
 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*
b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*
a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4
+ b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*
b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5
*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 +
2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 +
2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 +
 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*
c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(241) = 482$.
time = 4.43, size = 1414, normalized size = 5.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2
- b*c))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11/2)
*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*a^(9/2
)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b
*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(7/2)*b*c^
5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(
```

```
a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d^3 + 1116*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*b^2*c^5*d^3 + 480*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^2*a^(5/2)*b^2*c^4*d^2*abs(d) + 544*a^(5/2)*b^3*c^5*d^2*ab
s(d) + 24*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
 + b*c))^5*a^2*b*c^2*d^3 - 96*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b^3*c^4*d^3 + 144*(
sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*
a^(3/2)*b^3*c^3*d^2*abs(d) + 256*a^(3/2)*b^4*c^4*d^2*abs(d) + 36*(sqrt(a*d^
2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^5*a*b^2*c*d
^3 - 136*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2
+ b*c))^3*a*b^3*c^2*d^3 + 270*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x
^2 + b*d*x^2 + a*c^2 + b*c))*a*b^4*c^3*d^3 + 48*sqrt(a)*b^5*c^3*d^2*abs(d)
+ 15*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^5*b^3*d^3 - 40*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^
2 + a*c^2 + b*c))^3*b^4*c*d^3 + 33*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*
c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^5*c^2*d^3)/((a^2*c^5 + 2*a*b*c^4 + b^2*
c^3)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))^2 + b*c)^3))*sgn(d*x^2 + c)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2 + c}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^7,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^7, x)
```

3.325 $\int x^4 \sqrt{a+\frac{b}{c+dx^2}}\, dx$

Optimal. Leaf size=368

$$-\frac{(2b^2+7abc-3a^2c^2)\,x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15a^2d^2}+\frac{(b-3ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15ad^2}+\frac{x^3(c+dx^2)\sqrt{\frac{b+ac+}{c+d}}}{5d}$$

```
[Out] -1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^
2+1/15*(-3*a*c+b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*x
^3*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/15*c^(3/2)*(-3*a*c+b)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*(-3*a^2*c^2+7*a*b*c+2*b^2)*(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d
^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 489, 596, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,(-3a^2c^2+7abc+2b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{x(-3a^2c^2+7abc+2b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15a^2d^2}-\frac{c^{3/2}(b-3ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15ad^{5/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{x(b-3ac)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15ad^2}+\frac{x^3(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*((2*b^2 + 7*a*b*c - 3*a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
])/(a^2*d^2) + ((b - 3*a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)])/(15*a*d^2) + (x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/
(5*d) + (Sqrt[c]*(2*b^2 + 7*a*b*c - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c
+ d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*a^2*d^(5
/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(b -
3*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(15*a*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c
)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^4\sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+a\left(c+dx^2\right)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{x^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int}{5d\sqrt{b+a(}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{x^3(c+dx^2)\sqrt{b+ac+a}}{5d\sqrt{b+a(c}}$$

$$= \frac{(b-3ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15ad^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{x^3(c+dx^2)\sqrt{b+ac+a}}{5d\sqrt{b+a(c}}$$

$$= -\frac{\left(2b^2+7abc-3a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-3ac)x(c+dx^2)}{15ad^2}$$

$$= -\frac{\left(2b^2+7abc-3a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15a^2d^2\sqrt{b+a\left(c+dx^2\right)}} + \frac{(b-3ac)x(c+dx^2)}{15ad^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.58, size = 293, normalized size = 0.80

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x(c+dx^2)\left(b^2-2ab(c-2dx^2)-3a^2(c^2-d^2x^4)\right)+ic(2b^2+7abc-3a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-ibc(b+9ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{15ad^2\sqrt{\frac{ad}{b+ac}}\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b^2 - 2*a*b*(c - 2*d*x^2) - 3*a^2*(c^2 - d^2*x^4)) + I*c*(2*b^2 + 7*a*b*c
 - 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellip
ticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(b + 9*a*c)*S
qrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(15*a*d^2*Sqrt[(a*d)/(b + a*c)]*(b
 + a*(c + d*x^2)))
```

Maple [A]
time = 0.08, size = 662, normalized size = 1.80

method	result
default	$\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^2d^3x^7+3\sqrt{-\frac{ad}{ac+b}}\,a^2c\,d^2x^5+4\sqrt{-\frac{ad}{ac+b}}\,ab\,d^2x^5-3\sqrt{-\frac{ad}{ac+b}}\,a^2c^2d\,x^3+3\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\text{Ell}\right.}{}$
risch	$-\frac{x\left(-3ad\,x^2+3ac-b\right)\left(d\,x^2+c\right)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{15d^2a}+\frac{\left(-\frac{2\left(3a^2c^2d-7abcd-2b^2d\right)\left(c^2a+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7+3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2*x
^5+4*(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5-3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^3+
3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*
c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3+2*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3
+9*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a
*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
a*b*c^2-3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x+(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+((a
*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))
^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-2*
(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+(-a*d/(a*c+b))^(1/2)*b^2*c*x)*(d*x^2+c)*((a*
```

```
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)/(-a*d/(a*c+b))^(1/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^4*(a + b/(c + d*x^2))^(1/2), x)
```

3.326 $\int x^2 \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=282

$$\frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{\sqrt{c}\,(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right.\right.}{3ad^{3/2}\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

```
[Out] 1/3*(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d+1/3*x*(d*x^2+c)*((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/d-1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c)
)^(1/2)-1/3*(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/a/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1986, 489, 545, 429, 506, 422}

$$-\frac{c^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{\sqrt{c}\,(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3ad}+\frac{x(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*a*d) + (x*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (Sqrt[c]*(b - a*c)*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/(3*a*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]
) - (c^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{a+\frac{b}{c+dx^2}}\,dx &= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a(c+dx^2)}}\\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}dx}{\sqrt{b+a(c+dx^2)}}\\
&= \frac{x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a(c+dx^2)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \cdots}{3d\sqrt{b+a(c \cdots}}\\
&= \frac{x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a(c+dx^2)}} + \frac{\left((b-ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx \cdots}} \cdots\right.}{3\sqrt{b+ \cdots}}\\
&= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a(c+dx^2)}} + \frac{x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+ \cdots}}{3d\sqrt{b+a(c+dx^2)}}\\
&= \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3ad\sqrt{b+a(c+dx^2)}} + \frac{x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+ \cdots}}{3d\sqrt{b+a(c+dx^2)}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.39, size = 250, normalized size = 0.89

$$
\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x(c+dx^2)(b+ac+adx^2)+ic(-b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)+2ibc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{3d\sqrt{\frac{ad}{b+ac}}(b+a(c+dx^2))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*S
qrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c
)] + (2*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Elli
pticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*d*Sqrt[(a*d)/(b
 + a*c)]*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.04, size = 406, normalized size = 1.44

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^5+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^3-\sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\, \sqrt{\frac{d\, x^2+c}{c}}\, \mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right)\right.}{3d\sqrt{a\, d^2 x}}$
risch	$\frac{x\left(d\, x^2+c\right)\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}{3d}-\frac{\left(-\frac{2(acd-bd)\left(c^2 a+bc\right)\sqrt{1+\frac{ad\, x^2}{ac+b}}\, \sqrt{1+\frac{d\, x^2}{c}}\, \left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\, \sqrt{a\, d^2 x^4+2acd\, x^2+bd\, x^2+c^2 a-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d/
(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*a*c^2*x-2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c+((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + \frac{b}{d\,x^2 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^2*(a + b/(c + d*x^2))^(1/2), x)
```

3.327 $\int \sqrt{a+\frac{b}{c+dx^2}}\,dx$

Optimal. Leaf size=213

$$x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}-\frac{\sqrt{c}\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}+\frac{\sqrt{c}\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,\sqrt{\frac{c\,(b+ac+adx^2)}{(b+ac)\,(c+dx^2)}}}$$

```
[Out] x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1985, 1986, 433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] - (Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[d]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
 a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{c+dx^2}}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left((b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{b+a(c+dx^2)}} + \frac{\left(ad\sqrt{c+\right.}{}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a(c+dx^2)}} + \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\,F\left(\tan^{-1}\right.}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{b+a(c+dx^2)}} - \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}\,E\left(\tan^{-1}\right.}{\sqrt{d}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}\sqrt{b+a(c+}}$$

Mathematica [A]
time = 10.05, size = 98, normalized size = 0.46

$$\frac{\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}\,x\right)|\frac{ac}{b+ac}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin
[Sqrt[-(d/c)]*x], (a*c)/(b + a*c)])/(Sqrt[-(d/c)]*Sqrt[(b + a*c + a*d*x^2)/
(b + a*c)])
```

Maple [A]
time = 0.02, size = 199, normalized size = 0.93

method	result
default	$\frac{\left(ac\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)b\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\,(d\,x^2+c)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(d\,x^2+c)\,(ad\,x^2+ac+b)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*c*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b)*((d*x^2+c)/c)^(1/2)*((a*d*x^2+a*c+
b)/(a*c+b))^(1/2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{c + dx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+\frac{b}{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2),x)
[Out] int((a + b/(c + d*x^2))^(1/2), x)
```

3.328 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx$

Optimal. Leaf size=265

$$\frac{dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}-\frac{\sqrt{d}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+\frac{a\sqrt{c}}{}$$

```
[Out] d*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c/x-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 21, 433, 429, 506, 422}

$$\frac{a\sqrt{c}\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{dx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] (d*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/(c*x) - (Sqrt[d]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*
(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[d]*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b
+ a*c)])/((b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
```

```
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^2\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a(c+dx^2)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\ }{\sqrt{c}}}{c\sqrt{b+a(c+}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a(c+dx^2)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int}{c\sqrt{b+a(c+dx^2}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a(c+dx^2)}} + \frac{\left(ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int}{\sqrt{b+a(c+}}$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a(c+dx^2)}} +$$

$$= \frac{dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a(c+dx^2)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.40, size = 141, normalized size = 0.53

$$\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-\frac{1}{x}-\frac{dx}{c}-\frac{iad\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)}{\sqrt{\frac{ad}{b+ac}}\,(b+a(c+dx^2))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^2,x]
```

```
[Out] Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-x^(-1) - (d*x)/c - (I*a*d*Sqrt[(b +
 a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*
d)/(b + a*c)]*x], 1 + b/(a*c)])/(Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
)
```

Maple [A]
time = 0.05, size = 272, normalized size = 1.03

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^4 - adc\sqrt{\frac{a d\, x^2+ac+b}{ac+b}}\,\sqrt{\frac{d\, x^2+c}{c}}\, x \,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^2+\sqrt{-\frac{ac}{ac+\cdots}}\right.}{\sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+c^2a+bc}\,\sqrt{-\frac{ad}{ac+b}}\, xc\sqrt{(d\, x^2+c\cdots}}$
risch	$-\frac{(d\, x^2+c)\sqrt{\frac{a d\, x^2+ac+b}{d\, x^2+c}}}{cx}+\frac{ad\left(-\frac{2d(c^2a+bc)\sqrt{1+\frac{a d\, x^2}{ac+b}}\,\sqrt{1+\frac{d\, x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)-\cdots\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\, d^2x^4+2acd\, x^2+bd\, x^2+c^2a-\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*a*c*d*x^2+(-a*d/(a*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(
1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/
x/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^2,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^2, x)
```

3.329 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx$

Optimal. Leaf size=362

$$-\frac{(2b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)}-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3cx^3}+\frac{(2b+ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2(b+ac)x}+\frac{(2b\cdots}{\cdots}$$

```
[Out] -1/3*(a*c+2*b)*d^2*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)-1/3*(d*x
^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3+1/3*(a*c+2*b)*d*(d*x^2+c)*((a
*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x+1/3*(a*c+2*b)*d^(3/2)*(1/(1+d*
x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/
2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)/(c*
(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)/c^(1/2)/(c*(a*d*x^2+a*c
+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$\frac{d^{3/2}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{ad^{3/2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(ac+b)\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{d^2x(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2(ac+b)}+\frac{d(ac+2b)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x(ac+b)}-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*((2*b + a*c)*d^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c^2*(b + a*
c)) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c*x^3) + ((2*b
 + a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*(b + a*
c)*x) + ((2*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*d^(3/2)*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*Sqrt[c]*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^4\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a(c+dx^2)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{}{x^2\sqrt{}}}{3c\sqrt{b+a(c+}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a(c+dx^2)}} + \frac{(2b+ac)d(c+dx^2)\sqrt{b+ac+adx^2}}{3c^2(b+ac)x\sqrt{b+a(c+}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3cx^3\sqrt{b+a(c+dx^2)}} + \frac{(2b+ac)d(c+dx^2)\sqrt{b+ac+adx^2}}{3c^2(b+ac)x\sqrt{b+a(c+}}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+}}{3cx^3\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(2b+ac)d^2x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3c^2(b+ac)\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+}}{3cx^3\sqrt{b+a(c+dx^2)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.66, size = 314, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\,(c+dx^2)\,(b^2(c-2dx^2)+a^2c(c^2-d^2x^4)+2ab(c^2-cdx^2-d^2x^4))-iac(2b+ac)d^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)+iabcd^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)\right)}{3ac^2dx^3\,(b+a\,(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*
d)/(b + a*c)]*(c + d*x^2)*(b^2*(c - 2*d*x^2) + a^2*c*(c^2 - d^2*x^4) + 2*a*
b*(c^2 - c*d*x^2 - d^2*x^4)) - I*a*c*(2*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*
d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a
*c)]*x], 1 + b/(a*c)] + I*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a
*c)]))/(a*c^2*d*x^3*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.07, size = 571, normalized size = 1.58

method	result
risch	$-\frac{(dx^2+c)(-acdx^2-2bdx^2+c^2a+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{3c^2x^3(ac+b)}-\frac{ad^2\left(-\frac{2(acd+2bd)(c^2a+bc)\sqrt{1+\frac{adx^2}{ac+b}}\sqrt{1+\frac{dx^2}{c}}\left(\mathrm{EllipticF}(\ldots\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{ad^2x^4\ldots}}\right.}{\ldots}$ …
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,a^2cd^3x^6-2\sqrt{-\frac{ad}{ac+b}}\,abd^3x^6+\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2d^2x^3\ldots\right.}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^6-2*(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^
6+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*
c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2*x^3-(-a*d/(a*c+b))^(1/2)*a^2*c
^2*d^2*x^4-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d^2*x^3+2*((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b
)/a/c)^(1/2))*a*b*c*d^2*x^3-4*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+(-a*d/(a*c
+b))^(1/2)*a^2*c^3*d*x^2-2*(-a*d/(a*c+b))^(1/2)*b^2*d^2*x^4+(-a*d/(a*c+b))^
(1/2)*a^2*c^4-(-a*d/(a*c+b))^(1/2)*b^2*c*d*x^2+2*(-a*d/(a*c+b))^(1/2)*a*b*c
^3+(-a*d/(a*c+b))^(1/2)*b^2*c^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c
+b)/x^3/c^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+\frac{b}{d\,x^2+c}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^4,x)
[Out] int((a + b/(c + d*x^2))^(1/2)/x^4, x)
```

3.330 $\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx$

Optimal. Leaf size=466

$$\frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{15c^3(b+ac)^2}-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5cx^5}+\frac{(4b+3ac)d(c+dx^2)\sqrt{\frac{b+ac+}{c+dx}}}{15c^2(b+ac)x^3}$$

```
[Out] 1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^3*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3
/(a*c+b)^2-1/5*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/15*(3*a*
c+4*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/(a*c+b)/x^3-1/15*(
3*a^2*c^2+13*a*b*c+8*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
^3/(a*c+b)^2/x-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(5/2)/(a*c+b)^2/(c*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/15*a*(3*a*c+4*b)*d^(5/2)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(3/2)/(a*c+b)^2/(c*(a*d*x^
2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$-\frac{d^{5/2}(3a^2c^2+13abc+8b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{5/2}(ac+b)^2\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{d^3x(3a^2c^2+13abc+8b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3(ac+b)^2}-\frac{d^2(3a^2c^2+13abc+8b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^3x(ac+b)^2}+\frac{ad^{5/2}(3ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15c^{3/2}(ac+b)^2\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{d(3ac+4b)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{15c^2x^3(ac+b)}-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(15*c^3*(b + a*c)^2) - ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
)/(5*c*x^5) + ((4*b + 3*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)])/(15*c^2*(b + a*c)*x^3) - ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*d^2*(c + d*
x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(15*c^3*(b + a*c)^2*x) - ((8*b^
2 + 13*a*b*c + 3*a^2*c^2)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Ell
ipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(15*c^(5/2)*(b + a*c)^2*S
qrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*(4*b + 3*a*c)*d^
(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], b/(b + a*c)])/(15*c^(3/2)*(b + a*c)^2*Sqrt[(c*(b + a*c + a*d*x^2))/
((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{c+dx^2}}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+a(c+dx^2)}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\sqrt{b+ac+adx^2}}{x^6\sqrt{c+dx^2}}\,dx}{\sqrt{b+a(c+dx^2)}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a(c+dx^2)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{}{x^4 \cdots}}{5c\sqrt{b+a(c \cdots}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a(c+dx^2)}} + \frac{(4b+3ac)d(c+dx^2)\sqrt{b+ac+ad \cdots}}{15c^2(b+ac)x^3\sqrt{b+a(c \cdots}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a(c+dx^2)}} + \frac{(4b+3ac)d(c+dx^2)\sqrt{b+ac+ad \cdots}}{15c^2(b+ac)x^3\sqrt{b+a(c \cdots}}$$

$$= -\frac{(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5cx^5\sqrt{b+a(c+dx^2)}} + \frac{(4b+3ac)d(c+dx^2)\sqrt{b+ac+ad \cdots}}{15c^2(b+ac)x^3\sqrt{b+a(c \cdots}}$$

$$= \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+ \cdots}}{5cx^5\sqrt{b+ \cdots}}$$

$$= \frac{(8b^2+13abc+3a^2c^2)\,d^3x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{15c^3(b+ac)^2\sqrt{b+a(c+dx^2)}} - \frac{(c+dx^2)\sqrt{b+ac+ \cdots}}{5cx^5\sqrt{b+ \cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.73, size = 402, normalized size = 0.86

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}(c+dx^2)\left(b^3(3c^2-4cdx^2+8d^2x^4)+3a^3c^2(c^3+d^3x^6)+ab^2(9c^3-8c^2dx^2+17cd^2x^4+8d^3x^6)+a^2bc(9c^3-4c^2dx^2+9cd^2x^4+13d^3x^6)\right)+iac(8b^2+13abc+3a^2c^2)\,d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)\middle|1+\frac{b}{ac}\right)-2iabc(2b+3ac)d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)\middle|1+\frac{b}{ac}\right)\right)}{15c^3(b+ac)^2\sqrt{\frac{ad}{b+ac}}\,x^5\,(b+a(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^6,x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*
x^2)*(b^3*(3*c^2 - 4*c*d*x^2 + 8*d^2*x^4) + 3*a^3*c^2*(c^3 + d^3*x^6) + a*b
^2*(9*c^3 - 8*c^2*d*x^2 + 17*c*d^2*x^4 + 8*d^3*x^6) + a^2*b*c*(9*c^3 - 4*c^
2*d*x^2 + 9*c*d^2*x^4 + 13*d^3*x^6)) + I*a*c*(8*b^2 + 13*a*b*c + 3*a^2*c^2)
*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*a*b*c*(2*b + 3*a*c
)*d^3*x^5*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(c^3*(b + a*c)^2*Sqrt[(
a*d)/(b + a*c)]*x^5*(b + a*(c + d*x^2)))
```

Maple [**A**]
time = 0.07, size = 955, normalized size = 2.05

method	result
risch	$-\frac{(d x^2+c)(3a^2c^2d^2x^4+13acd^2bx^4-3a^2c^3dx^2+8b^2d^2x^4-7abc^2dx^2+3a^2c^4-4b^2cdx^2+6abc^3+3b^2c^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{15c^3x^5(ac+b)^2}+\frac{d^3a\left(-\right.}{}$
default	$-\frac{\left(3\sqrt{-\frac{ad}{ac+b}}\,a^3c^2d^4x^8+13\sqrt{-\frac{ad}{ac+b}}\,a^2bcd^4x^8-3\sqrt{\frac{adx^2+ac+b}{ac+b}}\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^3c^3d^3\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(3*(-a*d/(a*c+b))^(1/2)*a^3*c^2*d^4*x^8+13*(-a*d/(a*c+b))^(1/2)*a^2*b
*c*d^4*x^8-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^3*c^3*d^3*x^5+3*(-a*d/(a*c+b)
)^(1/2)*a^3*c^3*d^3*x^6+8*(-a*d/(a*c+b))^(1/2)*a*b^2*d^4*x^8+6*((a*d*x^2+a*
c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((
a*c+b)/a/c)^(1/2))*a^2*b*c^2*d^3*x^5-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*
b*c^2*d^3*x^5+22*(-a*d/(a*c+b))^(1/2)*a^2*b*c^2*d^3*x^6+4*((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b
)/a/c)^(1/2))*a*b^2*c*d^3*x^5-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b^2*c*d^3*
x^5+25*(-a*d/(a*c+b))^(1/2)*a*b^2*c*d^3*x^6+5*(-a*d/(a*c+b))^(1/2)*a^2*b*c^
3*d^2*x^4+8*(-a*d/(a*c+b))^(1/2)*b^3*d^3*x^6+3*(-a*d/(a*c+b))^(1/2)*a^3*c^5
*d*x^2+9*(-a*d/(a*c+b))^(1/2)*a*b^2*c^2*d^2*x^4+5*(-a*d/(a*c+b))^(1/2)*a^2*
b*c^4*d*x^2+4*(-a*d/(a*c+b))^(1/2)*b^3*c*d^2*x^4+3*(-a*d/(a*c+b))^(1/2)*a^3
*c^6+(-a*d/(a*c+b))^(1/2)*a*b^2*c^3*d*x^2+9*(-a*d/(a*c+b))^(1/2)*a^2*b*c^5-
(-a*d/(a*c+b))^(1/2)*b^3*c^2*d*x^2+9*(-a*d/(a*c+b))^(1/2)*a*b^2*c^4+3*(-a*d
```

```
/(a*c+b))^(1/2)*b^3*c^3)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*c+b)^2/x^
5/c^3/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(1/2)/x**6,x)
[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
[Out] integrate(sqrt(a + b/(d*x^2 + c))/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{d\,x^2 + c}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(1/2)/x^6,x)
```

```
[Out] int((a + b/(c + d*x^2))^(1/2)/x^6, x)
```

3.331 $\int x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=249

$$-\frac{bc^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^3}-\frac{(5b^2+60abc-24a^2c^2)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48ad^3}-\frac{(b+12ac)\,(c+dx^2)^2\sqrt{\frac{b+a}{c}}}{24d^3}$$

```
[Out] 1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/a/d^3-1/16*b*(-24*a^2*c^2
+12*a*b*c+b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d
^3-b*c^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3-1/48*(-24*a^2*c^2+60*a*b*c+5
*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^3-1/24*(12*a*c+b)*(d*
x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^3
```

Rubi [A]
time = 0.34, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1981, 1980, 474, 466, 1171, 396, 214}

$$-\frac{(-24a^2c^2+60abc+5b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3}-\frac{b(-24a^2c^2+12abc+b^2)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}-\frac{bc^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^3}+\frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{5/2}}{6ad^3}-\frac{(12ac+b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b*c^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^3) - ((5*b^2 + 60*a*b*c -
 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*a*d^3)
- ((b + 12*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*d^
3) + ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*x^2))^(5/2))/(6*a*d^3) - (b
*(b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]/Sqrt[a]])/(16*a^(3/2)*d^3)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^5\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x^2(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx^2)\right)^2}{bd^3}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\right.}{bd^3\sqrt{b+}}$$

$$=-\frac{c^2\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx^2)\right)^2}{bd^3}+\frac{\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx\right.}{6ad^3}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx^2)\right)}{24abd^3}-\frac{c^2\sqrt{a+\frac{}{c}}}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)(c+}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)(c+}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)(c+}{}$$

$$=-\frac{\left(b^2+12abc-24a^2c^2\right)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{16ad^3}-\frac{\left(b^2+12abc-24a^2c^2\right)(c+}{}$$

Mathematica [A]

time = 0.25, size = 150, normalized size = 0.60

$$\frac{\sqrt{a}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(3b^2(c+dx^2)-2ab(47c^2+16cdx^2-7d^2x^4)+8a^2(c^3+d^3x^6)\right)-3b(b^2+12abc-24a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{3/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(
47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a
*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/
(48*a^(3/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(227) = 454$.
time = 0.13, size = 1018, normalized size = 4.09

method	result
risch	$\frac{(8d^2a^2x^4-8a^2cdx^2+14abdx^2+8a^2c^2-46abc+3b^2)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48d^3a}+\left(\frac{3ba\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2\right.}{4d^2\sqrt{ad^2}}\right.$
default	$-\left(48\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}\,a^2cd^2x^4-12\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*(48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2
*c*d^2*x^4-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
*a*b*d^2*x^4-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^
2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d^2*x^2+48*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2*d*x^2+
36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d^2*x^2-16*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*d*x^2+96*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c*d*x^2-72*ln(1/2*(2*a*d^2*
x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
+b*d)/(a*d^2)^(1/2))*a^2*b*c^3*d+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3
*d^2*x^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*d*x^2+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
```

```
^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c^2*d+96*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c^2-16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))
*b^3*c*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*c)/d^3/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

Maxima [A]
time = 0.52, size = 368, normalized size = 1.48

$$-\frac{bc^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^3}-\frac{3\left(8a^2bc^2-20ab^2c+b^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\left(6a^3bc^2-12a^2b^2c-ab^3\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\left(8a^4bc^2-12a^3b^2c-a^2b^3\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^4d^3-\frac{3(adx^2+ac+b)a^3d^3}{dx^2+c}+\frac{3(adx^2+ac+b)^2a^2d^3}{(dx^2+c)^2}-\frac{(adx^2+ac+b)^3ad^3}{(dx^2+c)^3}\right)}-\frac{(24a^2c^2-12abc-b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32a^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 - 2
0*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 -
 12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b
*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*
d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a
^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1/32*(2
4*a^2*c^2 - 12*a*b*c - b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d^3)
```

Fricas [A]
time = 0.42, size = 427, normalized size = 1.71

$$\left[\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{a}\log\left(8a^2d^2x^4+8a^2c^2+8\left(2a^2c+ab\right)dx^2+8abc+b^2+4\left(2ad^2x^4+(4ac+b)dx^2+2ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(8a^3d^3x^6+14a^2bd^2x^4+8a^3c^3-94a^2bc^2+3ab^2c-\left(32a^2bc-3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192a^2d^3},-\frac{3\left(24a^2bc^2-12ab^2c-b^3\right)\sqrt{-a}\arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right)-2\left(8a^3d^3x^6+14a^2bd^2x^4+8a^3c^3-94a^2bc^2+3ab^2c-\left(32a^2bc-3ab^2\right)dx^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96a^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a
^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c
+ b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
+ 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*
c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a
^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*
a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^2*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(229) = 458.
time = 3.67, size = 527, normalized size = 2.12

$$\frac{1}{48}\sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\left(2\left(\frac{4\,ax^2\mathrm{sgn}(dx^2+c)}{d} - \frac{4\,a^3cd^6\mathrm{sgn}(dx^2+c) - 7\,a^2bd^6\mathrm{sgn}(dx^2+c)}{a^2d^8}\right)x^2 + \frac{8\,a^3c^2d^5\mathrm{sgn}(dx^2+c) - 46\,a^2bcd^5\mathrm{sgn}(dx^2+c) + 3\,ab^2d^5\mathrm{sgn}(dx^2+c)}{a^2d^8}\right) - \frac{\left(24\,a^{\frac{5}{2}}bc^2\mathrm{sgn}(dx^2+c) - 12\,a^{\frac{3}{2}}b^2c\mathrm{sgn}(dx^2+c) - \sqrt{a}\,b^3\mathrm{sgn}(dx^2+c)\right)\log\left(\left|-2\,a^{\frac{5}{2}}c^3d - 6\left(\sqrt{ad^2}\,x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)a^2c^2|d| - 6\left(\sqrt{ad^2}\,x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)^2 a^{\frac{3}{2}}cd - a^{\frac{3}{2}}bc^2d - 2\left(\sqrt{ad^2}\,x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)^3 a|d| - 2\left(\sqrt{ad^2}\,x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)abc|d| - \left(\sqrt{ad^2}\,x^2 - \sqrt{ad^2x^4 + 2\,acdx^2 + bdx^2 + ac^2 + bc}\right)^2\sqrt{a}\,bd\right|\right)}{96\,a^2d^2|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn(
d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/(a
^2*d^8))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b*c*d^5*sgn(d*x^2 + c
) + 3*a*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8)) - 1/96*(24*a^(5/2)*b*c^2*sgn(d*x
^2 + c) - 12*a^(3/2)*b^2*c*sgn(d*x^2 + c) - sqrt(a)*b^3*sgn(d*x^2 + c))*log
(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 +
 b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d
- 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d
*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a^2*d^2*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

3.332 $\int x^3 \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=172

$$\frac{bc\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d^2}+\frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8d^2}+\frac{a\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4d^2}+\frac{3b(b-4ac)\tanh}{}$$

```
[Out] 3/8*b*(-4*a*c+b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/d^2/a^(
1/2)+b*c*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/8*(-4*a*c+5*b)*(d*x^2+c)*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+1/4*a*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*
x^2+c))^(1/2)/d^2
```

Rubi [A]
time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1981, 1980, 466, 1171, 396, 214}

$$\frac{a\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2}+\frac{(5b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2}+\frac{bc\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2}+\frac{3b(b-4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d^2 + ((5*b - 4*a*c)*(c + d*x^2
)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*d^2) + (a*(c + d*x^2)^2*Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)])/(4*d^2) + (3*b*(b - 4*a*c)*ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^3\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int\frac{x(b+ac+adx)^{3/2}}{(c+dx)^{3/2}}dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx^2)\right)^2}{bd^2}+\frac{\left((b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Su}}{2bd\sqrt{b+a\left(c+\right.}}$$

$$=\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c+dx^2)\right)}{4bd^2}+\frac{c\sqrt{a+\frac{b}{c+dx^2}}\left(b+a(c\right.}{bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4bd^2}$$

$$=\frac{3(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2}+\frac{(b-4ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}\left(b+\right.}{4bd^2}$$

Mathematica [A]

time = 0.18, size = 113, normalized size = 0.66

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(13bc-2ac^2+5bdx^2+2ad^2x^4)}{8d^2}-\frac{3b(-b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a*
d^2*x^4))/(8*d^2) - (3*b*(-b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c +
 d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(154) = 308$.
time = 0.10, size = 593, normalized size = 3.45

method	result
risch	$-\frac{(-2adx^2+2ac-5b)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8d^2}+\frac{\left(-\frac{3b\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right.}{4d\sqrt{ad^2}}\right.}{}$
default	$-\frac{\left(-4\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}\,ad^2x^4+12\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+b}}{2\sqrt{ad^2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*(-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d
^2*x^4+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d^2*x^2-3*ln(1/2*(2*a*
d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2)+b*d)/(a*d^2)^(1/2))*b^2*d^2*x^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*(a*d^2)^(1/2)*b*d*x^2+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*
a*b*c^2*d+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a
*c^2-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*d-10*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b*c-16*(a*d^2)^(1/2)*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*b*c)/d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1
/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [A]
time = 0.52, size = 247, normalized size = 1.44

$$\frac{bc\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2}+\frac{3\,(4\,ac-b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,\sqrt{a}\,d^2}-\frac{(4\,abc-5\,b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc-3\,ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2-\frac{2\,(adx^2+ac+b)ad^2}{dx^2+c}+\frac{(adx^2+ac+b)^2d^2}{(dx^2+c)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(sq
rt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2 + a
*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a*d*x^
2 + a*c + b)^2*d^2/(d*x^2 + c)^2)
```

Fricas [A]
time = 0.42, size = 335, normalized size = 1.95

$$\left[\frac{3\,(4\,abc-b^2)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(2\,a^2d^2x^4+5\,abdx^2-2\,a^2c^2+13\,abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,ad^2},\ \frac{3\,(4\,abc-b^2)\sqrt{-a}\,\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\,\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)+2\,(2\,a^2d^2x^4+5\,abdx^2-2\,a^2c^2+13\,abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,ad^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
+ 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c +
 b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b
)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c)))/(a*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(156) = 312.
time = 4.03, size = 438, normalized size = 2.55

$$\frac{1}{8}\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,ax^2\mathrm{sgn}(dx^2+c)}{d}-\frac{2\,a^2cd^2\mathrm{sgn}(dx^2+c)-5\,abd^2\mathrm{sgn}(dx^2+c)}{ad^4}\right)+\frac{\left(4\,a^{\frac{3}{2}}bc\mathrm{sgn}(dx^2+c)-\sqrt{a}\,b^2\mathrm{sgn}(dx^2+c)\right)\log\left(\left|-2\,a^{\frac{5}{2}}c^3d-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^2c^2|d|-6\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}cd-a^{\frac{3}{2}}bc^2d-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3a|d|-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)abc|d|-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2\sqrt{a}\,bd\right|\right)}{16\,ad|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x^
2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4))
 + 1/16*(4*a^(3/2)*b*c*sgn(d*x^2 + c) - sqrt(a)*b^2*sgn(d*x^2 + c))*log(abs
(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b*c^2*d - 2*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3
*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a
*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a*d*abs(d))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3*(a + b/(c + d*x^2))^(3/2), x)
```

3.333 $\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx$

Optimal. Leaf size=94

$$-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

```
[Out] 1/2*(d*x^2+c)*(a+b/(d*x^2+c))^(3/2)/d+3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a
^(1/2))*a^(1/2)/d-3/2*b*(a+b/(d*x^2+c))^(1/2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1605, 248, 43, 52, 65, 214}

$$\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-3*b*Sqrt[a + b/(c + d*x^2)])/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/
2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int x\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\text{Subst}\left(\int\left(a+\frac{b}{x}\right)^{3/2}dx,x,c+dx^2\right)}{2d}$$

$$=-\frac{\text{Subst}\left(\int\frac{(a+bx)^{3/2}}{x^2}dx,x,\frac{1}{c+dx^2}\right)}{2d}$$

$$=\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3b)\text{Subst}\left(\int\frac{\sqrt{a+bx}}{x}dx,x,\frac{1}{c+dx^2}\right)}{4d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3ab)\text{Subst}\left(\int\frac{1}{x\sqrt{a+bx}}dx,\right.}{4d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}-\frac{(3a)\text{Subst}\left(\int\frac{1}{-\frac{a}{b}+\frac{x^2}{b}}dx,x,\sqrt{\right.}}{2d}$$

$$=-\frac{3b\sqrt{a+\frac{b}{c+dx^2}}}{2d}+\frac{(c+dx^2)\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{2d}+\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right.}{2d}$$

Mathematica [A]
time = 0.11, size = 87, normalized size = 0.93

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2b+a(c+dx^2)\right)+3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b
*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(78) = 156.
time = 0.12, size = 336, normalized size = 3.57

method	result

derivativedivides	$\frac{\sqrt{\frac{a(dx^2+c)+b}{dx^2+c}}\left(3\ln\left(\frac{2\sqrt{a(dx^2+c)^2+b(dx^2+c)}\sqrt{a}+2a(dx^2+c)+b}{2\sqrt{a}}\right)ab(dx^2+c)^2+6a^{\frac{3}{2}}\sqrt{a(d}\right.}{4d(dx^2+c)\sqrt{(dx^2+c)(a(dx^2+c)+}}$
risch	$\frac{(dx^2+c)a\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2d}+\frac{\left(\frac{3ba\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right)}{4\sqrt{ad^2}}-\frac{}{\sqrt{}}\right.}{}$
default	$\frac{\left(3\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\sqrt{ad^2}+bd}{2\sqrt{ad^2}}\right)abd^2x^2+2\sqrt{ad^2x^4+2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*d^2*x^2+2*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2+3*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2)*a*c-4*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b)/d*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(
1/2)
```

Maxima [A]
time = 0.51, size = 156, normalized size = 1.66

$$-\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad-\frac{(adx^2+ac+b)d}{dx^2+c}\right)}-\frac{3\sqrt{a}\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,d}-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d
/(d*x^2 + c)) - 3/4*sqrt(a)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/d - b*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/d
```

Fricas [A]
time = 0.41, size = 269, normalized size = 2.86

$$\left[\frac{3\,\sqrt{a}\,b\log\left(8\,a^{2}d^{2}x^{4}+8\,a^{2}c^{2}+8\,(2\,a^{2}c+ab)dx^{2}+8\,abc+b^{2}+4\,(2\,ad^{2}x^{4}+(4\,ac+b)dx^{2}+2\,ac^{2}+bc)\sqrt{a}\,\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}\right)+4\,(adx^{2}+ac-2\,b)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{8\,d},\;-\frac{3\,\sqrt{-a}\,b\arctan\left(\frac{(2\,adx^{2}+2\,ac+b)\sqrt{-a}\,\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{2\,(a^{2}dx^{2}+a^{2}c+ab)}\right)-2\,(adx^{2}+ac-2\,b)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{4\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 +
 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c - 2*b)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/d, -1/4*(3*sqrt(-a)*b*arctan(1/2*(2*a*d*x^2
+ 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^
2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\frac{ac+adx^{2}+b}{c+dx^{2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(78) = 156.
time = 5.26, size = 387, normalized size = 4.12

$$-\frac{\sqrt{a}\,b\log\left(\left|-2\,a^{\frac{5}{2}}c^{3}d-6\left(\sqrt{ad^{2}}\,x^{2}-\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\right)a^{2}c^{2}|d|-6\left(\sqrt{ad^{2}}\,x^{2}-\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\right)^{2}a^{\frac{3}{2}}cd-a^{\frac{3}{2}}bc^{2}d-2\left(\sqrt{ad^{2}}\,x^{2}-\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\right)^{3}a|d|-2\left(\sqrt{ad^{2}}\,x^{2}-\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\right)abc|d|-\left(\sqrt{ad^{2}}\,x^{2}-\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\right)^{2}\sqrt{a}\,bd\right|\right)\mathrm{sgn}(dx^{2}+c)}{4\,|d|}-\frac{\sqrt{a}\,b|d|\log\left(|a|\right)\mathrm{sgn}(dx^{2}+c)}{4\,d^{2}}+\frac{\sqrt{ad^{2}x^{4}+2\,acdx^{2}+bdx^{2}+ac^{2}+bc}\,a\,\mathrm{sgn}(dx^{2}+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(a)*b*log(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x
^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d
- a^(3/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d
*x^2 + a*c^2 + b*c))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a
*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))*sgn(d*x^2
```

```
+ c)/abs(d) - 1/4*sqrt(a)*b*abs(d)*log(abs(a))*sgn(d*x^2 + c)/d^2 + 1/2*sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*a*sgn(d*x^2 + c)/d
```

Mupad [B]
time = 3.74, size = 61, normalized size = 0.65

$$-\frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{a\,\left(d\,x^2+c\right)}{b}\right)}{d\,{\left(\frac{a\,\left(d\,x^2+c\right)}{b}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] -((a + b/(c + d*x^2))^(3/2)*(c + d*x^2)*hypergeom([-3/2, -1/2], 1/2, -(a*(c
 + d*x^2))/b))/(d*((a*(c + d*x^2))/b + 1)^(3/2))
```

3.334 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=126

$$\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}+a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(b+ac)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{c^{3/2}}$$

```
[Out] a^(3/2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))-(a*c+b)^(3/2)*ar
ctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/c^(3/2)+b*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c
```

Rubi [A]
time = 0.25, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 490, 536, 214}

$$a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)-\frac{(ac+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c + a^(3/2)*ArcTanh[Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)]/Sqrt[a]] - ((b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/c^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 490

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p +
 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q) + 1))), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{\frac{1}{2}(b+ac)^2d+\frac{1}{2}a^2cd^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\right.}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left((b+ac)^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+}}\right.}{2c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{\left(a^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+}\right.}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} - \frac{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx}}{\sqrt{c}\sqrt{b+a(c+dx}}\right.}{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{a+\frac{b}{c+dx^2}}}{c} + \frac{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b}{}$$

Mathematica [A]
time = 0.19, size = 139, normalized size = 1.10

$$\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c} - \frac{(-b-ac)^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-b-ac}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{b+ac}\right)}{c^{3/2}} + a^{3/2}\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x,x]

[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((-b - a*c)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/c^(3/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(108) = 216$.
time = 0.07, size = 652, normalized size = 5.17

method	result
default	$\frac{\left(\ln\left(\frac{2a\,d^2x^2+2acd+2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2c^2d^2x^2-\sqrt{a\,d^2}\,\sqrt{c^2a+bc}\,\ln\left(2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^2*d^2*x^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c*d*x^2+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*d*x^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b*c+2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)

Maxima [**A**]
time = 0.53, size = 201, normalized size = 1.60

$$-\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)+\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c}+\frac{(a^2c^2+2\,abc+b^2)\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")

[Out] -1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/(d

```
*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

Fricas [A]
time = 0.47, size = 1073, normalized size = 8.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((
8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8
*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c
^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
 c))*sqrt((a*c + b)/c))/x^4) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c
, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((
a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c
^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*
c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 +
 a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*b*sqrt((a*d*x^2 + a*c +
b)/(d*x^2 + c)))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2
*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*
c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqr
t(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x
^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c, -1/2*(
sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-(a*c + b)/
c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c
+ b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x, x)
```

3.335 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=138

$$-\frac{3bd\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2c^2}-\frac{(c+dx^2)\left(\frac{b+ac+adx^2}{c+dx^2}\right)^{3/2}}{2cx^2}+\frac{3b\sqrt{b+ac}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(3/2)/c/x^2+3/2*b*d*arctanh(c^(1
/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*(a*c+b)^(1/2)/c^(5/2)-
3/2*b*d*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1985, 1981, 1980, 294, 327, 214}

$$\frac{3bd\sqrt{ac+b}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}}-\frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2}-\frac{(c+dx^2)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]
```

```
[Out] (-3*b*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*c^2) - ((c + d*x^2)*((b +
 a*c + a*d*x^2)/(c + d*x^2))^(3/2))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*ArcTan
h[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*c^(5/2
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{(b+a(c+dx^2))^{3/2}}{x^3(c+dx^2)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{(b+ac+adx^2)^{3/2}}{x^3(c+dx^2)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{2cx^2} - \frac{\left(3bd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\mathrm{Subst}\left(\int \frac{\sqrt{b-}}{x}\right.}{4c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{\right.}{}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2}\sqrt{\right.}{}$$

$$= -\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{2cx^2} + \frac{3b\sqrt{b+ac}\,d\sqrt{c+dx^2}\sqrt{}}{2}$$

Mathematica [A]
time = 0.21, size = 124, normalized size = 0.90

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(ac(c+dx^2)+b(c+3dx^2))}{2c^2x^2} + \frac{3b\sqrt{-b-ac}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]
[Out] -1/2*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(a*c*(c + d*x^2) + b*(c + 3*d*x
^2)))/(c^2*x^2) + (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(118) = 236$.
time = 0.11, size = 820, normalized size = 5.94

method	result
risch	$-\frac{(ac+b)(dx^2+c)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2c^2x^2}+\frac{\left(\frac{3bd\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\sqrt{c^2a+bc+(2acd+bd)x^2+}}{x^2}\right.}{4c\sqrt{c^2a+bc}}\right.}{}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\left(-2\sqrt{c^2a+bc}\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,ad^3x^6-3\ln\left(\frac{2acdx^2+bdx^2+2c^2a+2\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*d^3*x^6-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*a*b*c^2*d^2*x^4-6*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*a*c*d^2*x^4-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c
)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^2*c*d
^2*x^4-2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
b*d^2*x^4-3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b*c^3*d*x^2-4*(a*c^2+b*c
)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*c^2*d*x^2-3*ln((2
*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^2*c^2*d*x^2+4*(a*c^2+b*c)^(1/2)*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*b*c*d*x^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*d*x^2-2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*b*c*d*x^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*c)/(a*c^2+b*c)^(1/2)/x^2/c^3/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

Maxima [A]
time = 0.50, size = 202, normalized size = 1.46

$$-\frac{(abc+b^2)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3+bc^2-\frac{(adx^2+ac+b)c^3}{dx^2+c}\right)}-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2}-\frac{3\,(abc+b^2)d\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b*c + b^2)*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2 -
 (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c))/c^2 - 3/4*(a*b*c + b^2)*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*
c + b)*c)))/(sqrt((a*c + b)*c)*c^2)
```

Fricas [A]
time = 0.44, size = 404, normalized size = 2.93

$$\left[\frac{3\,bdx^2\sqrt{\frac{ac+b}{c}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac^2+bc)d^2x^4+2\,ac^4+2\,bc^3+(4\,ac^3+3\,bc^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{\frac{ac+b}{c}}}{x^4}\right)-4\,((ac+3\,b)dx^2+ac^2+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,c^2x^2},-\frac{3\,bdx^2\sqrt{-\frac{ac+b}{c}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{ac+b}{c}}}{2\,(a^2c^2+(a^2c+ab)dx^2+2\,abc+b^2)}\right)+2\,((ac+3\,b)dx^2+ac^2+bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4
+ 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*
x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*
d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((
a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2
*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2
*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a
^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c
^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^3,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^3, x)
```

3.336 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=205

$$\frac{bd^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^3}+\frac{(9b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c^3x^2}-\frac{(b+ac)\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c^3x^4}-\frac{3b(5b\ldots}{}$$

```
[Out] -3/8*b*(4*a*c+5*b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a
*c+b)^(1/2))/c^(7/2)/(a*c+b)^(1/2)+b*d^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^3+1/8*(4*a*c+9*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^2-1
/4*(a*c+b)*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^4
```

Rubi [A]

time = 0.28, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1981, 1980, 466, 1171, 396, 214}

$$-\frac{3bd^2(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}}+\frac{bd^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3}+\frac{d(4ac+9b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2}-\frac{(ac+b)\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (b*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^3 + ((9*b + 4*a*c)*d*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c^3*x^2) - ((b + a*c)*(c +
d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c^3*x^4) - (3*b*(5*b + 4
*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b +
a*c]])/(8*c^(7/2)*Sqrt[b + a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{(b+a(c+dx^2))^{3/2}}{x^5(c+dx^2)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{(b+ac+adx^2)^{3/2}}{x^5(c+dx^2)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^3(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))^2}{4c(b+ac)x^4} - \frac{\left((5b+4ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}}{8c(b+ac)\sqrt{b+a\left(c+d\right.}}$$

$$= \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))^2}{4c(b+ac)x^4} + \ldots$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a\ldots}}{}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a\ldots}}{}$$

$$= \frac{3b(5b+4ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a+\frac{b}{c+dx^2}}\,(b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a\ldots}}{}$$

Mathematica [A]
time = 0.22, size = 153, normalized size = 0.75

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-2bc^2-2ac^3+5bcdx^2+15bd^2x^4+2acd^2x^4\right)}{8c^3x^4} + \frac{3b(5b+4ac)d^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{7/2}\sqrt{-b-ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 +
15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) + (3*b*(5*b + 4*a*c)*d^2*ArcTan[
(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(7/2)
*Sqrt[-b - a*c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(183) = 366$.
time = 0.15, size = 1653, normalized size = 8.06

method	result
risch	$-\frac{(d\,x^2+c)(-2acd\,x^2-7bd\,x^2+2c^2a+2bc)\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}{8c^3x^4}+\frac{\left(-\frac{3d^2b\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc}}{x^2}\right.}{4c^2\sqrt{c^2a+bc}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*c*d^4*x^8+12*ln((2*a*c*d*x^2+b*d*x^2
+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)+2*b*c)/x^2)*a^3*b*c^5*d^3*x^6+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)*(a*c^2+b*c)^(3/2)*a*b*d^4*x^8+39*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*a^2*b^2*c^4*d^3*x^6+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(
a*c^2+b*c)^(3/2)*a^2*c^2*d^3*x^6+12*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^
2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^
3*b*c^6*d^2*x^4+42*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^3*c^3*d^3*x^6+6
2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a*b*c*d
^3*x^6+39*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b^2*c^5*d^2*x^4+15*ln((2
*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^4*c^2*d^3*x^6+20*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*c^3*d^2*x^4+18*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*b^2*d^3*x^6+42*ln((2*a*
c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+
a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^3*c^4*d^2*x^4-12*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c*d^2*x^4+44*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a*b*c^2*d^2*x^4-16*(a*c^2+b*
c)^(3/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2*d^2*x^4+15*ln((2*a*c*d*x
```

```
^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)+2*b*c)/x^2)*b^4*c^3*d^2*x^4-18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*d^2*x^4+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*b^2*c*d^2*x^4-16*(a*c^2+b*c)^(3/2)*((
d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c*d^2*x^4-8*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c^2*d*x^2-14*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*c*d*x^2+4*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*c^3+4*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b*c^2)/(a*c^2+b*c)^(3/2)/x^4
/(a*c+b)/c^4/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [A]
time = 0.51, size = 313, normalized size = 1.53

$$\frac{bd^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^3}+\frac{3\,(4\,abc+5\,b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,\sqrt{(ac+b)c}\,c^3}-\frac{(4\,abc^2+9\,b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+11\,ab^2c+7\,b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2c^5+2\,abc^4+b^2c^3+\frac{(adx^2+ac+b)^2c^5}{(dx^2+c)^2}-\frac{2\,(ac^5+bc^4)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] b*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^3 + 3/16*(4*a*b*c + 5*b^2)*d^
2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*
c^3) - 1/8*((4*a*b*c^2 + 9*b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/
2) - (4*a^2*b*c^2 + 11*a*b^2*c + 7*b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2
 + c)))/(a^2*c^5 + 2*a*b*c^4 + b^2*c^3 + (a*d*x^2 + a*c + b)^2*c^5/(d*x^2 +
 c)^2 - 2*(a*c^5 + b*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Fricas [A]
time = 0.70, size = 557, normalized size = 2.72

$$\left[\frac{3\,(4\,abc+5\,b^2)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\,(2\,a^2c^5-(2\,a^2c^3+17\,abc^2+15\,b^2c)d^2x^4+4\,abc^4+2\,b^2c^3-5\,(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,(ac^5+bc^4)x^4},\ \frac{3\,(4\,abc+5\,b^2)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)-2\,(2\,a^2c^5-(2\,a^2c^3+17\,abc^2+15\,b^2c)d^2x^4+4\,abc^4+2\,b^2c^3-5\,(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,(ac^5+bc^4)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*
b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*
a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*
c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b
*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 -
b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 -
```

```
 b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2
 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2
*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5,x)
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^5,x)
[Out] int((a + b/(c + d*x^2))^(3/2)/x^5, x)
```

3.337 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=292

$$-\frac{bd^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^4}-\frac{(79b^2+108abc+24a^2c^2)\,d^2(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{48c^4(b+ac)x^2}+\frac{(11b+12ac)d(c+dx^2)^2\sqrt{\ }}{24c^4x^4}$$

```
[Out] -1/6*(d*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(5/2)/c^2/(a*c+b)/x^6+1/16*b*(
24*a^2*c^2+60*a*b*c+35*b^2)*d^3*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(a*c+b)^(1/2))/c^(9/2)/(a*c+b)^(3/2)-b*d^3*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^4-1/48*(24*a^2*c^2+108*a*b*c+79*b^2)*d^2*(d*x^2+c)*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x^2+1/24*(12*a*c+11*b)*d*(d*x^2+c)^2*((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/x^4
```

Rubi [A]
time = 0.42, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1981, 1980, 474, 466, 1171, 396, 214}

$$\frac{bd^3(24a^2c^2+60abc+35b^2)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{16c^{9/2}(ac+b)^{3/2}}-\frac{d^2(24a^2c^2+108abc+79b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)}-\frac{bd^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^4}+\frac{d(12ac+11b)(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24c^4x^4}-\frac{(c+dx^2)^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{5/2}}{6c^2x^6(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] -((b*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c^4) - ((79*b^2 + 108*a*b*c
 + 24*a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(48*c
^4*(b + a*c)*x^2) + ((11*b + 12*a*c)*d*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(24*c^4*x^4) - ((c + d*x^2)^3*((b + a*c + a*d*x^2)/(c + d*
x^2))^(5/2))/(6*c^2*(b + a*c)*x^6) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^
3*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(
16*c^(9/2)*(b + a*c)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 474

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)
/(a*b^2*e*n*(p + 1))), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^
n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0]
&& IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
```

```
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^7\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^4(c+dx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} - \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Subst}\left(\int \frac{\frac{7}{2}b(b+ac)d+}{x^3(c+dx)^{3/2}\sqrt{b+}}\right.}{6c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} + \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\text{Su}}{12c^2(b+}$$

$$= -\frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a+\frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+32ac)d^2\sqrt{a+\frac{b}{c+dx^2}}}{48c^3(b+ac)x^2}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a}}{24}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a}}{24}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a}}{24}$$

$$= -\frac{\left(105b^2+110abc+8a^2c^2\right)d^3\sqrt{a+\frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a+\frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a}}{24}$$

Mathematica [A]

time = 0.45, size = 226, normalized size = 0.77

$$\frac{-\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(8a^2c^2(c^3+d^3x^6)+2abc(8c^3-7c^2dx^2+16cd^2x^4+55d^3x^6)+b^2(8c^3-14c^2dx^2+35cd^2x^4+105d^3x^6)\right)}{(b+ac)x^6}+\frac{3b(35b^2+60abc+24a^2c^2)d^3\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}}{48c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7,x]
```

```
[Out] (-((Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(8*a^2*c^2*(c^3 + d^3*x^6
) + 2*a*b*c*(8*c^3 - 7*c^2*d*x^2 + 16*c*d^2*x^4 + 55*d^3*x^6) + b^2*(8*c^3
- 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6)))/((b + a*c)*x^6)) + (3*b*(35*
b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(
c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)^(3/2))/(48*c^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2604 vs. $2(266) = 532$.
time = 0.15, size = 2605, normalized size = 8.92

method	result
risch	$-\frac{(dx^2+c)(8a^2c^2d^2x^4+62acd^2bx^4-8a^2c^3dx^2+57b^2d^2x^4-30abc^2dx^2+8a^2c^4-22b^2cdx^2+16abc^3+8b^2c^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48c^4x^6(ac+b)}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-105*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*b^6*c^3*d^4*x^8-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(a*c^2+b*c)^(5/2)*b^3*d^4*x^8+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(3/2)*(a*c^2+b*c)^(5/2)*a^2*c^5+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^3-927*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)
*a^2*b^4*c^6*d^3*x^6+96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*
c^2+b*c)^(5/2)*a^2*c^2*d^3*x^6-495*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2
+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b
^5*c^5*d^3*x^6+48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*
c)^(5/2)*a^2*c^3*d^2*x^4-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*c^2+b*c)^(5/2)*b^3*c*d^3*x^6+96*(a*c^2+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)*b^3*c*d^3*x^6-32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3
```

```
/2)*(a*c^2+b*c)^(5/2)*a^2*c^4*d*x^2+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c*d^2*x^4-44*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*c^2*d*x^2+276*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c*d^3*x^6-564*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*a*b^2*c^2*d^3*x^6+
192*(a*c^2+b*c)^(5/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c^2*d^3*x^6+1
68*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^
2*d^2*x^4-76*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5
/2)*a*b*c^3*d*x^2-105*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^6*c^4*d^3*x^6+
174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*b^2*d
^3*x^6-276*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2
)*a^2*b*c*d^5*x^10-816*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c
^2+b*c)^(5/2)*a^2*b*c^2*d^4*x^8-540*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(5/2)*a^2*b*c^3*d^3*x^6+96*(a*c^2+b*c)^(5/2)*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^3*d^3*x^6-738*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*a*b^2*c*d^4*x^8+32*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(5/2)*a*b*c^4-72*ln((2*a*c*d*x^2+
b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)+2*b*c)/x^2)*a^5*b*c^8*d^4*x^8-96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*a^3*c^2*d^5*x^10-396*ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*a^4*b^2*c^7*d^4*x^8-72*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)
*a^5*b*c^9*d^3*x^6-861*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^3*b^3*c^6*d^4
*x^8-240*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*
a^3*c^3*d^4*x^8-174*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+
b*c)^(5/2)*a*b^2*d^5*x^10-396*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)
^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^4*b^2*
c^8*d^3*x^6-927*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*
x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b^4*c^5*d^4*x^8-86
1*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^3*b^3*c^7*d^3*x^6-495*ln((2*a*c*d*
x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)+2*b*c)/x^2)*a*b^5*c^4*d^4*x^8-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*a^3*c^4*d^3*x^6)/(a*c^2+b*c)^(5/2)/x^
6/(a*c+b)^2/c^5/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(266) = 532$.

time = 0.54, size = 534, normalized size = 1.83

$$-\frac{(24\,a^2bc^2+60\,ab^2c+35\,b^3)d^3\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{32\,(ac^5+bc^4)\sqrt{(ac+b)c}}-\frac{bd^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^4}-\frac{3\,(8\,a^2bc^4+36\,ab^2c^3+29\,b^3c^2)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-8\,(6\,a^3bc^4+30\,a^2b^2c^3+41\,ab^3c^2+17\,b^4c)d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,(8\,a^4bc^4+44\,a^3b^2c^3+83\,a^2b^3c^2+66\,ab^4c+19\,b^5)d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^4c^8+4\,a^3bc^7+6\,a^2b^2c^6+4\,ab^3c^5+b^4c^4-\frac{(ac^8+bc^7)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{3\,(a^2c^8+2\,abc^7+b^2c^6)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{3\,(a^3c^8+3\,a^2bc^7+3\,ab^2c^6+b^3c^5)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)) + sqrt((a*c + b)*c)))/((a*c^5 + b*c^4)*sqrt((a*c + b)*c)) - b*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*c^3
+ 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4
+ 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2
 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66*a*b^4*
c + 19*b^5)*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4*a^3*b*c
^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*x^2 + a*c
 + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c +
b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d
*x^2 + a*c + b)/(d*x^2 + c))
```

Fricas [**A**]
time = 0.95, size = 733, normalized size = 2.51

$$\left[\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\sqrt{ac^2+bc}\,d^3x^6\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\,(8\,a^3c^7+(8\,a^3c^4+118\,a^2bc^3+215\,ab^2c^2+105\,b^3c)d^3x^6+24\,a^2bc^6+24\,ab^2c^5+8\,b^3c^4+(32\,a^2bc^4+67\,ab^2c^3+35\,b^3c^2)d^2x^4-14\,(a^2bc^5+2\,ab^2c^4+b^3c^3)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{192\,(a^2c^7+2\,abc^6+b^2c^5)x^6},\ -\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)\sqrt{-ac^2-bc}\,d^3x^6\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(8\,a^3c^7+(8\,a^3c^4+118\,a^2bc^3+215\,ab^2c^2+105\,b^3c)d^3x^6+24\,a^2bc^6+24\,ab^2c^5+8\,b^3c^4+(32\,a^2bc^4+67\,ab^2c^3+35\,b^3c^2)d^2x^4-14\,(a^2bc^5+2\,ab^2c^4+b^3c^3)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{96\,(a^2c^7+2\,abc^6+b^2c^5)x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*lo
g(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2
 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c
^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 21
5*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4
+ (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*
b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7
+ 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*
sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)
*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*
c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a
^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5
 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2
*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**7,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^7,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^7, x)
```

3.338 $\int x^4 \left(a+\frac{b}{c+dx^2}\right)^{3/2} \, dx$

Optimal. Leaf size=405

$$\frac{(b^2-14abc+a^2c^2)\, x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5ad^2}+\frac{(7b-ac)x(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5d^2}+\frac{6ax^3(c+dx^2)\sqrt{\frac{b+ac+a\cdots}{c+dx^2}}}{5d}$$

```
[Out] 1/5*(a^2*c^2-14*a*b*c+b^2)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2+1/5*(-
a*c+7*b)*x*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2+6/5*a*x^3*(d*x^2
+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x^3*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/d-1/5*c^(3/2)*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/
c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*(
(a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+
c))^(1/2)-1/5*(a^2*c^2-14*a*b*c+b^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1985, 1986, 478, 595, 596, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,(a^2c^2-14abc+b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5ad^{5/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{x(a^2c^2-14abc+b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5ad^2}-\frac{c^{3/2}(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5d^{5/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{x(7b-ac)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d^2}+\frac{6ax^3(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5d}-\frac{x^3(ac+adx^2+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((b^2 - 14*a*b*c + a^2*c^2)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*a*d
^2) + ((7*b - a*c)*x*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*
d^2) + (6*a*x^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*d) -
(x^3*(b + a*c + a*d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c
]*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
E[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*a*d^(5/2)*Sqrt[(c*(b + a*c
+ a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (c^(3/2)*(7*b - a*c)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)]
)/(5*d^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))])])*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}dx=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^4\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x^3(b+ac+adx^2)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\sqrt{\ }}{}}{d\sqrt{b+a\left(c\right.}}$$

$$=\frac{6ax^3(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x^3(b+ac+adx^2)^{3/2}\sqrt{a+}}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3(c+dx^2)\sqrt{b+a}}{5d\sqrt{b+}}$$

$$=\frac{(7b-ac)x(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5d^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{6ax^3(c+dx^2)\sqrt{b+a}}{5d\sqrt{b+}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x(c+dx^2)}{5d^2\sqrt{}}$$

$$=\frac{\left(b^2-14abc+a^2c^2\right)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{5ad^2\sqrt{b+a\left(c+dx^2\right)}}+\frac{(7b-ac)x(c+dx^2)}{5d^2\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.59, size = 308, normalized size = 0.76

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x\left(-a^2(c-dx^2)\left(c+dx^2\right)^2+b^2(7c+2dx^2)+3ab(2c^2+3cdx^2+d^2x^4)\right)-ic(b^2-14abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)+8ibc(b-ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{5d^2\sqrt{\frac{ad}{b+ac}}\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-(a^2*(c -
 d*x^2)*(c + d*x^2)^2) + b^2*(7*c + 2*d*x^2) + 3*a*b*(2*c^2 + 3*c*d*x^2 + d
^2*x^4)) - I*c*(b^2 - 14*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c
)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/
(a*c)] + (8*I)*b*c*(b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(5*d
^2*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1097 vs. $2(441) = 882$.
time = 0.10, size = 1098, normalized size = 2.71 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*d^3*x^7+(-a
*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^2*x^5+3*(-a*d/(
a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^2*x^5-(-a*d/(a*c+b))^
(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d*x^3+5*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3+4*(-a*d/(a*c+
b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x^3+((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/
a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3-(-a*d/(a*c+b))^(1/2)*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3*x+2*(-a*d/(a*c+b))^(1/2)*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d*x^3+8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2-14*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x
^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*
(a*d*x^2+a*c+b))^(1/2)*a*b*c^2*x-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*b^2*c+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c
)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*b^2*c+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(-a*d/(a*c+b))^(1/2)*b^2*c*x+2*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a
*c+b))^(1/2)*b^2*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/(a*d^2*x^4+2*a*
c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^4*(a + b/(c + d*x^2))^(3/2), x)
```

3.339 $\int x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2} \, dx$

Optimal. Leaf size=331

$$\frac{(7b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}+\frac{4ax(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3d}-\frac{x(b+ac+adx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{d}-\frac{\sqrt{c}\,(7b$$

```
[Out] 1/3*(-a*c+7*b)*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d+4/3*a*x*(d*x^2+c)*((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d-x*(a*d*x^2+a*c+b)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/d-1/3*(-a*c+7*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+
1/3*(-a*c+3*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/
c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)/d^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 478, 542, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,(3b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{\sqrt{c}\,(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3d^{3/2}\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{4ax(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}-\frac{x(ac+adx^2+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d}+\frac{x(7b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((7*b - a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) + (4*a*x*(c + d
*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*d) - (x*(b + a*c + a*d*x^2)
*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/d - (Sqrt[c]*(7*b - a*c)*Sqrt[(b +
a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a
*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (
Sqrt[c]*(3*b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*d^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))
/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}\,dx=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{x^2\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$=-\frac{x(b+ac+adx^2)^{3/2}\sqrt{a+\frac{b}{c+dx^2}}}{d\sqrt{b+a\left(c+dx^2\right)}}+\frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int\frac{\sqrt{b+\cdots}}{\cdots}}{d\sqrt{b+a\left(c+d\cdots\right.}}$$

$$=\frac{4ax(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x(b+ac+adx^2)^{3/2}\sqrt{a+\frac{\cdots}{c+\cdots}}}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{4ax(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}-\frac{x(b+ac+adx^2)^{3/2}\sqrt{a+\frac{\cdots}{c+\cdots}}}{d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{\cdots}}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

$$=\frac{(7b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{3d\sqrt{b+a\left(c+dx^2\right)}}+\frac{4ax(c+dx^2)\sqrt{b+ac+adx^2}\sqrt{\cdots}}{3d\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.53, size = 270, normalized size = 0.82

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\,x\left(-3b^2-2ab(c+dx^2)+a^2(c+dx^2)^2\right)+iac(-7b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)+ib(-3b+5ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)\right)}{3d\sqrt{\frac{ad}{b+ac}}\,(b+a\left(c+dx^2\right))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(-3*b^2 - 2
*a*b*(c + d*x^2) + a^2*(c + d*x^2)^2) + I*a*c*(-7*b + a*c)*Sqrt[(b + a*c +
a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b +
 a*c)]*x], 1 + b/(a*c)] + I*b*(-3*b + 5*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)]))/(3*d*Sqrt[(a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(371) = 742$.
time = 0.08, size = 820, normalized size = 2.48

method	result
default	$\frac{\left(\sqrt{\left(d x^2+c\right)\left(a d x^2+a c+b\right)}\sqrt{-\frac{a d}{a c+b}}\, a^2 d^2 x^5+2\sqrt{\left(d x^2+c\right)\left(a d x^2+a c+b\right)}\sqrt{-\frac{a d}{a c+b}}\, a^2 c d x^3+\sqrt{\cdots}\right.}{\cdots}$
risch	$\frac{a x\left(d x^2+c\right)\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}{3 d}-\frac{\left(-\frac{2 a^2 c d\left(c^2 a+b c\right)\sqrt{1+\frac{a d x^2}{a c+b}}\sqrt{1+\frac{d x^2}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{a d}{a c+b}},\sqrt{-1+\frac{2 a c d+b d}{d c a}}\right.\right.}{\sqrt{-\frac{a d}{a c+b}}\sqrt{a d^2 x^4+2 a c d x^2+b d x^2+c^2 a}}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*d^2*x^5+2*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c*d*x^3+((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3-((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
E(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2-3*(a*d^2*x^4+2*a*c*d*
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d*x^3+((d*x^2+c)*(a*d
*x^2+a*c+b))^(1/2)*(-a*d/(a*c+b))^(1/2)*a^2*c^2*x-5*((d*x^2+c)*(a*d*x^2+a*c
+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+7*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+((d*x^2+c)*(a*d*x^2+a*c+b))^(
1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+3*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d
*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(
1/2),((a*c+b)/a/c)^(1/2))*b^2-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x-3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d/(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2
+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{ac + adx^2 + b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + \frac{b}{d\,x^2 + c}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^2*(a + b/(c + d*x^2))^(3/2), x)
```

3.340 $\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx$

Optimal. Leaf size=260

$$\frac{bx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}-\frac{(b-ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c}+\frac{(b-ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}}}+-$$

```
[Out] b*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c-(-a*c+b)*x*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c+(-a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+
c))^(1/2)/c^(1/2)/d^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/
(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c
)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^(1/2
)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1985, 1986, 424, 545, 429, 506, 422}

$$\frac{a\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{x(b-ac)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}+\frac{bx\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (b*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/c - ((b - a*c)*x*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)])/c + ((b - a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(Sqrt[c]*Sqrt[d]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[c]*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b +
a*c)])/(Sqrt[d]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \left(a+\frac{b}{c+dx^2}\right)^{3/2} dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{ac(b+c}{\sqrt{c+dx^2}}}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(a(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int}{\sqrt{b+a\left(c+dx\right.}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{bx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}} - \frac{(b-ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c\sqrt{b+a\left(c+dx^2\right)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.39, size = 243, normalized size = 0.93

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\sqrt{\frac{ad}{b+ac}}x(b+a(c+dx^2)) - iac(-b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right) - 2iabc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{c\sqrt{\frac{ad}{b+ac}}\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c
 + d*x^2)) - I*a*c*(-b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*
I)*a*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(c*Sqrt[(a*d)/(b + a*c)]
*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.04, size = 515, normalized size = 1.98

method	result
default	$\frac{\left(\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+c^{2}a+bc}\,\sqrt{-\frac{ad}{ac+b}}\,abd\,x^{3}+\sqrt{\left(d\,x^{2}+c\right)\left(ad\,x^{2}+ac+b\right)}\,\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*d
*x^3+((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c
^2+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c
-((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c+(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*x+(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*x)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)/(-a*d/(a*c+b))^(1/2)/c/(a*d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{c + dx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2),x)
[Out] Integral((a + b/(c + d*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] integrate((a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2),x)
[Out] int((a + b/(c + d*x^2))^(3/2), x)
```

3.341 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=312

$$\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx}+\frac{(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2}-\frac{(2b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{c^2x}-\frac{(2b+ac)\sqrt{d}\,\sqrt{\cdots}}{\cdots}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x+(a*c+2*b)*d*x*((a*d*x^2+a*c+b)/(d*x
^2+c))^(1/2)/c^2-(a*c+2*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/
x-(a*c+2*b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/c^(3/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+a*(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
b/(a*c+b))^(1/2))*d^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^(1/2)/(c*(a*d
*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$-\frac{\sqrt{d}(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{c^{3/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{a\sqrt{d}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{c}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{dx(ac+2b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2}-\frac{(ac+2b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^2x}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x) + ((2*b + a*c)*d*x*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)])/c^2 - ((2*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)])/(c^2*x) - ((2*b + a*c)*Sqrt[d]*Sqrt[(b + a*c + a*
d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(c
^(3/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (a*Sqrt[d]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]]
, b/(b + a*c)])/(Sqrt[c]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2
))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(2b+ac)d}{x^2\sqrt{c+dx^2}\sqrt{b}}}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+d}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} - \frac{(2b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+d}}}{c^2x\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(}{}$$

$$= \frac{b\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{cx\sqrt{b+a\left(c+dx^2\right)}} + \frac{(2b+ac)dx\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}}{c^2\sqrt{b+a\left(c+dx^2\right)}} - \frac{(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.49, size = 278, normalized size = 0.89

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\left(2ab(c+dx^2)^2+a^2c(c+dx^2)^2+b^2(c+2dx^2)\right)+iac(2b+ac)dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-iabcdx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{c^2\sqrt{\frac{ad}{b+ac}}x\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^2,x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(2*a*b*(c +
 d*x^2)^2 + a^2*c*(c + d*x^2)^2 + b^2*(c + 2*d*x^2)) + I*a*c*(2*b + a*c)*d*
x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSi
nh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*d*x*Sqrt[(b + a*c + a*d
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)]))/(c^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 872 vs. 2(360) = 720.
time = 0.07, size = 873, normalized size = 2.80

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^{2}+c\right)\left(ad\,x^{2}+ac+b\right)}\,a^{2}c\,d^{2}x^{4}+\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+c^{2}a+b\right.}{}$
risch	$-\frac{(ac+b)\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{c^{2}x}+\frac{d\left(-\frac{2a^{2}cd\left(c^{2}a+bc\right)\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+2a}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^2*x^4+(-a*
d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b*d^2*x^
4+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^2*x^4-((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d*x+2*(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d*x^2+((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x-2*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x+(-a*d/
(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b*c*d*x^2+
3*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x^2+(-a*d/
```

```
(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d*x^2+(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3+(-a*d/(a*c+b))
^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d*x^2+2*(-a*d/(a*c+b))^(1/2)*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a
*d*x^2+a*c+b))^(1/2)*b^2*c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/x/(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c^2/(a*d*x^2+a*c+
b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**2,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^2,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^2, x)
```

3.342 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=388

$$\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^3}-\frac{(8b+ac)d^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^3}-\frac{(4b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{3c^2x^3}+\frac{(8b+ac)d(c+$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^3-1/3*(a*c+8*b)*d^2*x*((a*d*x^2+a*c
+b)/(d*x^2+c))^(1/2)/c^3-1/3*(a*c+4*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^2/x^3+1/3*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
/c^3/x+1/3*(a*c+8*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)
/(d*x^2+c))^(1/2)/c^(5/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a
*(a*c+4*b)*d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c^(3/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$-\frac{ad^{3/2}(ac+4b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{3/2}(ac+b)\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^{3/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3c^{5/2}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{d^2x(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^3}+\frac{d(ac+8b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^3x}-\frac{(ac+4b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{3c^2x^3}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^3) - ((8*b + a*c)*d^2*x*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3) - ((4*b + a*c)*(c + d*x^2)*Sqrt[
(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^2*x^3) + ((8*b + a*c)*d*(c + d*x^2)*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(3*c^3*x) + ((8*b + a*c)*d^(3/2)*Sqr
t[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*c^(5/2)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)]) - (a*(4*b + a*c)*d^(3/2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*c^(3/2)*(b + a*c)*Sqrt[(c*(
b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^4\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(4b+ac)}{x^4\sqrt{c+dx^2}\,\sqrt{}}}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(4b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{3c^2x^3\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} -$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^3\sqrt{b+a\left(c+dx^2\right)}} - \frac{(8b+ac)d^2x\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{3c^3\sqrt{b+a\left(c+dx^2\right)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.57, size = 329, normalized size = 0.85

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\left(a^2c(c-dx^2)\left(c+dx^2\right)^2+b^2(c^2-4cdx^2-8d^2x^4)+ab(2c^3-3c^2dx^2-13cd^2x^4-8d^3x^6)\right)-iac(8b+ac)d^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)+4iabcd^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)\right)}{3c^3\sqrt{\frac{ad}{b+ac}}\,x^3\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^4,x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(a^2*c*(
c - d*x^2)*(c + d*x^2)^2 + b^2*(c^2 - 4*c*d*x^2 - 8*d^2*x^4) + a*b*(2*c^3 -
 3*c^2*d*x^2 - 13*c*d^2*x^4 - 8*d^3*x^6)) - I*a*c*(8*b + a*c)*d^2*x^3*Sqrt[
(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt
[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] + (4*I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*
d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a
*c)]*x], 1 + b/(a*c)]))/(c^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(424) = 848$.
time = 0.07, size = 1039, normalized size = 2.68 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^3*x^6+3
*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b*d
^3*x^6+5*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^3*x^6
-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c
+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d
^2*x^3+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d^2*x
^4+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/
(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c
*d^2*x^3-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*a*b*c*d^2*x^3+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*a*b*c*d^2*x^4+10*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b)
)^(1/2)*a*b*c*d^2*x^4+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*b^2*d^2*x^4-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+
b))^(1/2)*a^2*c^3*d*x^2+5*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)*b^2*d^2*x^4+3*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*
a*b*c^2*d*x^2-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^
4+4*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c*d*x^2-2*(-
a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^3-(-a*d/(a*c+b))
^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c
))^(1/2)/x^3/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))
^(1/2)/c^3/(a*d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**4,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^4,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^4, x)
```

3.343 $\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=494

$$\frac{b\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{cx^5}+\frac{(16b^2+16abc+a^2c^2)\,d^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^4(b+ac)}-\frac{(6b+ac)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{5c^2x^5}+\frac{(8b}{}$$

```
[Out] b*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c/x^5+1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^3
*x*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)-1/5*(a*c+6*b)*(d*x^2+c)*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^2/x^5+1/5*(a*c+8*b)*d*(d*x^2+c)*((a*d*x^2
+a*c+b)/(d*x^2+c))^(1/2)/c^3/x^3-1/5*(a^2*c^2+16*a*b*c+16*b^2)*d^2*(d*x^2+c
)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/c^4/(a*c+b)/x-1/5*(a^2*c^2+16*a*b*c+16*
b^2)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/c^(7/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/5*a*(a*c+8*
b)*d^(5/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/
c^(5/2)/(a*c+b)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$-\frac{d^{5/2}(a^2c^2+16abc+16b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{7/2}(ac+b)\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{d^3x(a^2c^2+16abc+16b^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4(ac+b)}-\frac{d^2(a^2c^2+16abc+16b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^4x(ac+b)}+\frac{ad^{5/2}(ac+8b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5c^{5/2}(ac+b)\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{d(ac+8b)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^3x^3}-\frac{(ac+6b)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{5c^2x^5}+\frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(c*x^5) + ((16*b^2 + 16*a*b*c + a
^2*c^2)*d^3*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^4*(b + a*c)) - ((
6*b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^2*x^5) +
 ((8*b + a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(5*c^3*x
^3) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)])/(5*c^4*(b + a*c)*x) - ((16*b^2 + 16*a*b*c + a^2*c^2)*d^(5/
2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], b/(b + a*c)])/(5*c^(7/2)*(b + a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b +
a*c)*(c + d*x^2))]) + (a*(8*b + a*c)*d^(5/2)*Sqrt[(b + a*c + a*d*x^2)/(c +
d*x^2)]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(5*c^(5/2)*(b
+ a*c)*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+a\left(c+dx^2\right)\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{\left(b+ac+adx^2\right)^{3/2}}{x^6\left(c+dx^2\right)^{3/2}}\,dx}{\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{\left(\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}\right)\int \frac{-(b+ac)(6b+ac)}{x^6\sqrt{c+dx^2}\,\sqrt{}}}{cd\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} - \frac{(6b+ac)\left(c+dx^2\right)\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+}}}{5c^2x^5\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\,\sqrt{a}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

$$= \frac{b\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}{cx^5\sqrt{b+a\left(c+dx^2\right)}} + \frac{\left(16b^2+16abc+a^2c^2\right)d^3x\sqrt{b+ac+adx^2}\,\sqrt{a}}{5c^4(b+ac)\sqrt{b+a\left(c+dx^2\right)}}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.

time = 10.76, size = 430, normalized size = 0.87

$$-\frac{\sqrt{\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}(b^3(c^3-2c^2dx^2+8cd^2x^4+16d^3x^6)+a^3c^2(c^4+c^3dx^2+cd^3x^6+d^4x^8)+a^2bc(3c^4+5c^2d^2x^4+24cd^3x^6+16d^4x^8)+ab^2(3c^4-3c^3dx^2+13c^2d^2x^4+40cd^3x^6+16d^4x^8))+iac(16b^2+16abc+a^2c^2)\,d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-iabc(8b+7ac)d^3x^5\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{5ac^4dx^5\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^6,x]
```

```
[Out] -1/5*(Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*
d)/(b + a*c)]*(b^3*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6) + a^3*c^2
*(c^4 + c^3*d*x^2 + c*d^3*x^6 + d^4*x^8) + a^2*b*c*(3*c^4 + 5*c^2*d^2*x^4 +
 24*c*d^3*x^6 + 16*d^4*x^8) + a*b^2*(3*c^4 - 3*c^3*d*x^2 + 13*c^2*d^2*x^4 +
 40*c*d^3*x^6 + 16*d^4*x^8)) + I*a*c*(16*b^2 + 16*a*b*c + a^2*c^2)*d^3*x^5*
Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*a*b*c*(8*b + 7*a*c)*d^3*x^5*Sqr
t[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sq
rt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a*c^4*d*x^5*(b + a*(c + d*x^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1665 vs. $2(526) = 1052$.

time = 0.09, size = 1666, normalized size = 3.37

method	result
risch	$-\frac{(dx^2+c)(a^2c^2d^2x^4+11acd^2bx^4-a^2c^3dx^2+11b^2d^2x^4-4abc^2dx^2+a^2c^4-3b^2cdx^2+2abc^3+b^2c^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{5c^4x^5(ac+b)}+\frac{d^3\left(-\frac{2(a^3}{}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(5*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*b^3*d^3*x^6+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^
6+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^3*c^3+11*(-a*d/(
a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c*d^4*x^8+19*(-a*d/(a
*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^2*d^3*x^6+5*(-a*d/(a
*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b^2*d^4*x^8-
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^3*d^
3*x^5+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^2*d^4*x^
8+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^3*d^3*x^6+30
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c*d^3*x^6+5*(
-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^3*d^2*x^4+13*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c^2*d^2*x^4-3*
```

```
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c^3*d*x^2-16*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b
))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^2*d
^3*x^5+8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-
a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a
*b^2*c*d^3*x^5-16*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b)
)^(1/2)*a*b^2*c*d^3*x^5+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^
2+a*c+b))^(1/2)*a^2*b*c^2*d^3*x^5+5*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a^2*b*c*d^4*x^8+5*(-a*d/(a*c+b))^(1/2)*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a^2*b*c^2*d^3*x^6+10*(-a*d/(a*c+b
))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b^2*c*d^3*x^6+11
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*d^4*x^8+(-a*d
/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^3*c^5*d*x^2+8*(-a*d/(a*
c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^3*c*d^2*x^4-2*(-a*d/(a*c+b)
)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^3*c^2*d*x^2+11*(-a*d/(a*c+b))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^3*d^3*x^6+3*(-a*d/(a*c+b))^(1/2)*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^5+3*(-a*d/(a*c+b))^(1/2)*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c^4)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/x^5/(
a*c+b)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)
/c^4/(a*d*x^2+a*c+b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x**6,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(3/2)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x^6,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x^6, x)
```

3.344 $\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=225

$$\frac{(5b^2+12abc+8a^2c^2)\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^3d^3}-\frac{(5b+8ac)\,(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{24a^2d^3}+\frac{x^2(c+dx^2)^2 \,\sqrt{\ }}{6}$$

```
[Out] -1/16*b*(8*a^2*c^2+12*a*b*c+5*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/a^(1/2))/a^(7/2)/d^3+1/16*(8*a^2*c^2+12*a*b*c+5*b^2)*(d*x^2+c)*((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/a^3/d^3-1/24*(8*a*c+5*b)*(d*x^2+c)^2*((a*d*x^2+a*c+
b)/(d*x^2+c))^(1/2)/a^2/d^3+1/6*x^2*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/a/d^2
```

Rubi [A]
time = 0.29, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1981, 1980, 424, 393, 205, 214}

$$-\frac{(8ac+5b)\,(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^2d^3}-\frac{b(8a^2c^2+12abc+5b^2)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3}+\frac{(8a^2c^2+12abc+5b^2)\,(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^3d^3}+\frac{x^2(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)])/(16*a^3*d^3) - ((5*b + 8*a*c)*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/(24*a^2*d^3) + (x^2*(c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/
(c + d*x^2)])/(6*a*d^2) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[(b
 + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(7/2)*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^2(c+dx^2)\left(b+a(c+dx^2)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\Bigg(\int \frac{\sqrt{c+dx}\,\left(-c(b+ac)-\frac{1}{2}(5b\cdots\right.}{\sqrt{b+ac+adx}}}{6ad^2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(5b+8ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)\left(b+a(c+dx^2)\right)}{6ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\Big(\big(-2ac(\cdots}{\cdots}$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= \frac{\left(5b^2+12abc+8a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^3d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(5b+8ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

Mathematica [A]

time = 0.24, size = 148, normalized size = 0.66

$$\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(15b^2+2ab(13c-5dx^2)+8a^2(c^2-cdx^2+d^2x^4))-3b(5b^2+12abc+8a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{7/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*
(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c
 + 8*a^2*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a
^(7/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(205) = 410$.
time = 0.08, size = 533, normalized size = 2.37

method	result
risch	$\frac{(8d^2a^2x^4-8a^2cd\,x^2-10abd\,x^2+8a^2c^2+26abc+15b^2)(ad\,x^2+ac+b)}{48d^3a^3\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(-\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{c^2a+bc+(2acd+bd)}\right.}{4d^2a\sqrt{a\,d^2}}\right.}{}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\;x^2ca^2d\sqrt{a\,d^2}-36\sqrt{a\,d^2x^4+2acd\,x^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)-36*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)-24*ln(1/2*(2*a
*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*b^2*c*a*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(
1/2)+36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)
-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+30*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
/(a*d^2)^(1/2)
```

Maxima [A]

time = 0.51, size = 340, normalized size = 1.51

$$-\frac{3\left(8\,a^{2}bc^{2}+12\,ab^{2}c+5\,b^{3}\right)\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{5}{2}}-8\left(6\,a^{3}bc^{2}+12\,a^{2}b^{2}c+5\,ab^{3}\right)\left(\frac{adx^{2}+ac+b}{dx^{2}+c}\right)^{\frac{3}{2}}+3\left(8\,a^{4}bc^{2}+20\,a^{3}b^{2}c+11\,a^{2}b^{3}\right)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{48\left(a^{6}d^{3}-\frac{3\,(adx^{2}+ac+b)a^{5}d^{3}}{dx^{2}+c}+\frac{3\,(adx^{2}+ac+b)^{2}a^{4}d^{3}}{(dx^{2}+c)^{2}}-\frac{(adx^{2}+ac+b)^{3}a^{3}d^{3}}{(dx^{2}+c)^{3}}\right)}+\frac{\left(8\,a^{2}c^{2}+12\,abc+5\,b^{2}\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{\sqrt{a}+\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}\right)}{32\,a^{\frac{7}{2}}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c
))^(5/2) - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/(d
*x^2 + c))^(3/2) + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*sqrt((a*d*x^
2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*x^2
+ c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^
3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*log(-(sqrt
(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c))))/(a^(7/2)*d^3)
```

Fricas [A]

time = 0.37, size = 425, normalized size = 1.89

$$\left[\frac{3\left(8\,a^{2}bc^{2}+12\,ab^{2}c+5\,b^{3}\right)\sqrt{a}\log\left(8\,a^{2}d^{2}x^{4}+8\,a^{2}c^{2}+8\left(2\,a^{2}c+ab\right)dx^{2}+8\,abc+b^{2}-4\left(2\,ad^{2}x^{4}+\left(4\,ac+b\right)dx^{2}+2\,ac^{2}+bc\right)\sqrt{a}\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}\right)+4\left(8\,a^{3}d^{3}x^{6}-10\,a^{2}bd^{2}x^{4}+8\,a^{3}c^{3}+26\,a^{2}bc^{2}+15\,ab^{2}c+\left(16\,a^{2}bc+15\,ab^{2}\right)dx^{2}\right)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{192\,a^{4}d^{3}},\ \frac{3\left(8\,a^{2}bc^{2}+12\,ab^{2}c+5\,b^{3}\right)\sqrt{-a}\arctan\left(\frac{\left(2\,adx^{2}+2\,ac+b\right)\sqrt{-a}\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{2\left(a^{2}dx^{2}+a^{2}c+ab\right)}\right)+2\left(8\,a^{3}d^{3}x^{6}-10\,a^{2}bd^{2}x^{4}+8\,a^{3}c^{3}+26\,a^{2}bc^{2}+15\,ab^{2}c+\left(16\,a^{2}bc+15\,ab^{2}\right)dx^{2}\right)\sqrt{\frac{adx^{2}+ac+b}{dx^{2}+c}}}{96\,a^{4}d^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*
a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c
 + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
 + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^
2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*
(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2
*d*x^2 + a^2*c + a*b)) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 +
26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\sqrt{\frac{ac+adx^{2}+b}{c+dx^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A]
time = 3.41, size = 226, normalized size = 1.00

$$\frac{2\,\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\,\left(2\,x^2\left(\frac{4\,x^2}{ad}-\frac{4\,a^2cd^3+5\,abd^3}{a^3d^5}\right)+\frac{8\,a^2c^2d^2+26\,abcd^2+15\,b^2d^2}{a^3d^5}\right)+\frac{3\,(8\,a^2bc^2+12\,ab^2c+5\,b^3)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{7}{2}}d^2|d|}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2
/(a*d) - (4*a^2*c*d^3 + 5*a*b*d^3)/(a^3*d^5)) + (8*a^2*c^2*d^2 + 26*a*b*c*d
^2 + 15*b^2*d^2)/(a^3*d^5)) + 3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(abs(
-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(7/2)*d^2*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(1/2), x)
```

3.345 $\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=148

$$-\frac{(3b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4ad^2}+\frac{b(3b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+a}{c+dx}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

```
[Out] 1/8*b*(4*a*c+3*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2
)/d^2-1/8*(4*a*c+3*b)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2+1
/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 393, 205, 214}

$$\frac{b(4ac+3b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}-\frac{(4ac+3b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/8*((3*b + 4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(a^2
*d^2) + ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a*d^2) + (
b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*
a^(5/2)*d^2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((3b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{b+ac+}}\right.}{8ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(3b+4ac)\left(b+a(c+dx^2)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a}\right.}{}$$

$$= -\frac{(3b+4ac)\left(b+a(c+dx^2)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a}\right.}{8}$$

$$= -\frac{(3b+4ac)\left(b+a(c+dx^2)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a}\right.}{}$$

$$= -\frac{(3b+4ac)\left(b+a(c+dx^2)\right)}{8a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4ad^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(3b+4ac)\sqrt{b+a\,(}}{8a^{5/2}d^2}$$

Mathematica [A]

time = 0.14, size = 108, normalized size = 0.73

$$\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(-3b-2ac+2adx^2)+b(3b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b - 2*a*c +
2*a*d*x^2) + b*(3*b + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/
Sqrt[a]])/(8*a^(5/2)*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(132) = 264$.
time = 0.06, size = 354, normalized size = 2.39

method	result
risch	$-\frac{(-2adx^2+2ac+3b)(adx^2+ac+b)}{8d^2a^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{\left(\frac{b\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right)c}{4da\sqrt{ad^2}}+\frac{3b^2\ln\left(\frac{acd}{\ }\right.}{\ }\right.}{\sqrt{\ }}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,(dx^2+c)\left(-4\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}\,adx^2-4\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2}}{\ }\right.\right.}{\ }$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(-4*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2-4*ln(1/2*(2*a*d^2*x^2+
2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d
)/(a*d^2)^(1/2))*a*b*c*d+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*d^2)^(1/2)*a*c-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d+6*(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)/a^2/(a*d^2)^(1/2)
```

Maxima [A]

time = 0.52, size = 223, normalized size = 1.51

$$-\frac{\left(4\,abc+3\,b^2\right)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-\left(4\,a^2bc+5\,ab^2\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2-\frac{2\left(adx^2+ac+b\right)a^3d^2}{dx^2+c}+\frac{\left(adx^2+ac+b\right)^2a^2d^2}{\left(dx^2+c\right)^2}\right)}-\frac{\left(4\,ac+3\,b\right)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{5}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*
c + 5*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2 - 2*(a*d*x^2 +
 a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c)^2
) - 1/16*(4*a*c + 3*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d^2)
```

Fricas [A]
time = 0.38, size = 333, normalized size = 2.25

$$\left[\frac{\left(4\,abc+3\,b^2\right)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\left(2\,a^2c+ab\right)dx^2+8\,abc+b^2+4\left(2\,ad^2x^4+\left(4\,ac+b\right)dx^2+2\,ac^2+bc\right)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\left(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,a^3d^2},-\frac{\left(4\,abc+3\,b^2\right)\sqrt{-a}\arctan\left(\frac{\left(2\,adx^2+2\,ac+b\right)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2dx^2+a^2c+ab\right)}\right)-2\left(2\,a^2d^2x^4-3\,abdx^2-2\,a^2c^2-3\,abc\right)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,a^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c
 + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^
2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4
- 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c
 + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a
*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2
+ a*c + b)/(d*x^2 + c)))/(a^3*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [A]
time = 4.46, size = 167, normalized size = 1.13

$$\frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\left(\frac{2\,x^2}{ad}-\frac{2\,acd+3\,bd}{a^2d^3}\right)-\frac{(4\,abc+3\,b^2)\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{5}{2}}d|d|}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d)
- (2*a*c*d + 3*b*d)/(a^2*d^3)) - (4*a*b*c + 3*b^2)*log(abs(-2*a*c*d - 2*(sq
rt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt
(a)*abs(d) - b*d))/(a^(5/2)*d*abs(d)))/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(1/2), x)
```

3.346 $\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=72

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

```
[Out] -1/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)/d+1/2*(d*x^2+c)*(a+b/
(d*x^2+c))^(1/2)/a/d
```

Rubi [A]

time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1605, 248, 44, 65, 214}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c +
d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx, x, c+dx^2\right)}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d}$$

$$= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4ad}$$

$$= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2ad}$$

$$= \frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2ad} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 1.22

$$\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2ad}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*a*d) - (b*ArcTanh[Sq
rt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(60) = 120.
time = 0.24, size = 184, normalized size = 2.56

method	result
derivativedivides	$\frac{\sqrt{\frac{a(dx^2+c)+b}{dx^2+c}}\,(dx^2+c)\left(2\sqrt{(dx^2+c)(a(dx^2+c)+b)}\,\sqrt{a}-b\ln\left(\frac{2\sqrt{(dx^2+c)(a(dx^2+}{2\sqrt{}}\right.\right.}{4d\sqrt{(dx^2+c)(a(dx^2+c)+b)}\,a^{\frac{3}{2}}}$
risch	$\frac{adx^2+ac+b}{2da\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}-\frac{\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right)b\sqrt{(dx^2+c}}{4a\sqrt{ad^2}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}(dx^2+c)}$
default	$\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,(dx^2+c)\left(-\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}+bd}{2\sqrt{ad^2}}\right)\right.}{4\sqrt{(dx^2+c)(adx^2+ac+b)}\,ad\sqrt{ad}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*c
*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*
d^2)^(1/2))*b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/d/(a*d^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.

time = 0.51, size = 129, normalized size = 1.79

$$-\frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2d-\frac{(adx^2+ac+b)ad}{dx^2+c}\right)}+\frac{b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*a
*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c
)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)
```

Fricas [A]
time = 0.36, size = 267, normalized size = 3.71

$$\left[\frac{\sqrt{a}\,b\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,a^2d},\ \frac{\sqrt{-a}\,b\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)+2\,(adx^2+ac)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,a^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8
*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 +
a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a
*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c +
 a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(60) = 120.

time = 4.82, size = 129, normalized size = 1.79

$$\frac{\frac{b\log\left(\left|-2\,acd-2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)\sqrt{a}\,|d|-bd\right|\right)}{a^{\frac{3}{2}}|d|}+\frac{2\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{ad}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2
 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(3/2)*abs(d)) + 2*sqrt
(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d))/sgn(d*x^2 + c)
```

Mupad [B]

time = 3.34, size = 111, normalized size = 1.54

$$\frac{\sqrt{\frac{a\,\left(d\,x^2+c\right)}{b}+1}\,\left(d\,x^2+c\right)\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,\left(d\,x^2+c\right)}}{2\,a\,\left(d\,x^2+c\right)}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{d\,x^2+c}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,3{}\mathrm{i}}{2\,a^{3/2}\,{\left(d\,x^2+c\right)}^{3/2}}\right)}{3\,d\,\sqrt{a+\frac{b}{d\,x^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] (((a*(c + d*x^2))/b + 1)^(1/2)*(c + d*x^2)*((b^(3/2)*asin((a^(1/2)*(c + d*x
^2)^(1/2)*1i)/b^(1/2))*3i)/(2*a^(3/2)*(c + d*x^2)^(3/2)) + (3*b^(1/2)*(b +
a*(c + d*x^2))^(1/2))/(2*a*(c + d*x^2))))/(3*d*(a + b/(c + d*x^2))^(1/2))
```

3.347 $\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{\sqrt{b+ac}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(1/2)-arctanh(c^(1/2)*
((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c^(1/2)/(a*c+b)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1985, 1981, 1980, 400, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt
[b + a*c]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \sqrt{c+dx^2}\right)}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= -\frac{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \cdots$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{\sqrt{a}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \cdots$$

Mathematica [A]
time = 0.11, size = 101, normalized size = 1.05

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{-b-ac}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a
*c]])/Sqrt[-b - a*c] + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a
]]/Sqrt[a]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(80) = 160$.
time = 0.04, size = 312, normalized size = 3.25

method	result
default	$\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+2 \sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+c^{2} a+b c}\,\sqrt{a\,d^{2}}+b d}{2 \sqrt{a\,d^{2}}}\right) a c d+\ln\left(\frac{2 a\,d^{2} x}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*a*d^2*x^2+2*a*c*
d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d
^2)^(1/2))*a*c*d+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b*d-(a*c^2+b*c)^(1/2)
*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)/(a*c+b)/(a*d^2)^(1/2)
```

Maxima [A]
time = 0.53, size = 155, normalized size = 1.61

$$\frac{c\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*c*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b)*
c) - 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) +
sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(80) = 160$.
time = 0.46, size = 972, normalized size = 10.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*
c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*
((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 +
(4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(c/(a*c + b)))/x^4) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*
a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2
*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/4*(a*sqr
t(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*
b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2
 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3
+ 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/
(a*c + b)))/x^4) - 2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)))/a, 1/4*(2*
a*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*
c)) + sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a
*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sq
rt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a, 1/2*(a*sqrt(-c/(a*c + b))*arctan(1
/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) - sqrt(-a)*arctan(1/2*(2*
a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*
x^2 + a^2*c + a*b)))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)
```

3.348 $\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=108

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{2(b+ac)x^2}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2\sqrt{c}\,(b+ac)^{3/2}}$$

```
[Out] -1/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(
a*c+b)^(3/2)/c^(1/2)-1/2*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b
)/x^2
```

Rubi [A]

time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1985, 1981, 1980, 205, 214}

$$-\frac{(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac+b)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2\sqrt{c}\,(ac+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2) -
(b*d*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]]
)/(2*Sqrt[c]*(b + a*c)^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+a(c+dx^2)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{x^3\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b+a(c+dx^2)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(bd\sqrt{b+a(c+dx^2)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ \cdots}}\right.}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+d\cdots}}}$$

$$= -\frac{b+a(c+dx^2)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(bd\sqrt{b+a(c+dx^2)}\right)\text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2}\,dx, \cdots\right.}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+\cdots}}}$$

$$= -\frac{b+a(c+dx^2)}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{bd\sqrt{b+a(c+dx^2)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx\cdots}}{\sqrt{c}\sqrt{b+a(c+dx\cdots}}\right.}{2\sqrt{c}(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [A]
time = 0.23, size = 114, normalized size = 1.06

$$\frac{1}{2}\left(-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x^2} - \frac{bd\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{\sqrt{c}(-b-ac)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (-(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x^2)) - (
b*d*ArcTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])
/(Sqrt[c]*(-b - a*c)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(92) = 184$.
time = 0.07, size = 452, normalized size = 4.19

method	result
risch	$-\frac{a d x^2+a c+b}{2(ac+b)x^2\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}} - \frac{b d \ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc+(2acd+bd)x^2+a d^2 x}}{x^2}\right.}{4(ac+b)\sqrt{c^2a+bc}\,\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)}$
default	$-\frac{\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}\,(d x^2+c)\left(-2a d^2\sqrt{a d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,x^4\sqrt{c^2a+bc}+\ln\left(\frac{2acd\,x^2+bd\,x^2+2c}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*a*b*c^2*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*a*c*d*x^2*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b
*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^2*c
*d*x^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b*d*x^2*(a*c^2+b*c
)^(1/2)+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)
)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/c/x^2/(a*c^2+b*c)^(1/2)
```

Maxima [A]
time = 0.51, size = 173, normalized size = 1.60

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2+2\,abc+b^2-\frac{(adx^2+ac+b)(ac^2+bc)}{dx^2+c}\right)}+\frac{bd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}\,(ac+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^2 + 2*a*b*c + b^2 - (
a*d*x^2 + a*c + b)*(a*c^2 + b*c)/(d*x^2 + c)) + 1/4*b*d*log((c*sqrt((a*d*x^
```

```
2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/
(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*(a*c + b))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(92) = 184$.
time = 0.42, size = 451, normalized size = 4.18

$$\left[\frac{\sqrt{ac^2+bc}\,bdx^2\log\left(\frac{(8a^2c^2+8abc+b^2)d^2x^4+8a^2c^4+16abc^3+8b^2c^2+8(2a^2c^3+3abc^2+b^2c)dx^2-4((2ac+b)d^2x^4+2ac^3+(4ac^2+3bc)dx^2+2bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8(a^2c^3+2abc^2+b^2c)x^2}, \frac{\sqrt{-ac^2-bc}\,bdx^2\arctan\left(\frac{((2ac+b)dx^2+2ac^2+2bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(a^2c^3+2abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)-2(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4(a^2c^3+2abc^2+b^2c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 +
8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^
2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*s
qrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (
a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^
3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2
*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) -
 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 +
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(92) = 184$.
time = 4.95, size = 292, normalized size = 2.70

$$\frac{\frac{bd\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{\sqrt{-ac^2-bc}\,(ac+b)}-\frac{2a^{\frac{3}{2}}c^2|d|+2\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)acd+2\sqrt{a}\,bc|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)bd}{\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)(ac+b)}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
+ a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c + b)) - (2*a^(
3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x
^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*(a*
c + b)))/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)
```

3.349 $\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=177

$$\frac{(b+4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8c(b+ac)^2x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4c(b+ac)x^4}+\frac{b(b+4ac)d^2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+}{c+d}}}{\sqrt{b+ac}}\right.}{8c^{3/2}(b+ac)^{5/2}}$$

```
[Out] 1/8*b*(4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+
b)^(1/2))/c^(3/2)/(a*c+b)^(5/2)+1/8*(4*a*c+b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)/c/(a*c+b)^2/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/c/(a*c+b)/x^4
```

Rubi [A]

time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 393, 205, 214}

$$\frac{bd^2(4ac+b)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}}+\frac{d(4ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] ((b + 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*c*(b +
 a*c)^2*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*c*(
b + a*c)*x^4) + (b*(b + 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)])/Sqrt[b + a*c]])/(8*c^(3/2)*(b + a*c)^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^5\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^5\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3\sqrt{b+ac+adx}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((b+4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{\sqrt{c}}{x^2\sqrt{b+}}\right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+}}}$$

$$= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b}\right.}{}$$

$$= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(b(b+4ac)d^2\sqrt{b}\right.}{}$$

$$= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{b(b+4ac)d^2\sqrt{b+}}{8c^{3/2}(b}$$

Mathematica [A]
time = 0.26, size = 149, normalized size = 0.84

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(2ac(c-dx^2)+b(2c+dx^2)\right)}{8c(b+ac)^2x^4} - \frac{b(b+4ac)d^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8c^{3/2}(-b-ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*a*c*(c - d*x^2)
+ b*(2*c + d*x^2)))/(c*(b + a*c)^2*x^4) - (b*(b + 4*a*c)*d^2*ArcTan[(Sqrt[c
]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*c^(3/2)*(-b -
a*c)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 922 vs. $2(157) = 314$.

time = 0.09, size = 923, normalized size = 5.21

method	result
risch	$-\frac{(a d\,x^2+ac+b)\left(-2acd\,x^2+bd\,x^2+2c^2a+2bc\right)}{8(ac+b)^2x^4c\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(\frac{d^2 b\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc+(2acd+}}{x^2}\right.}{4(ac+b)^2\sqrt{c^2a+bc}}\right.}{}$
default	$-\frac{\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(12a^2d^3\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,x^6c(c^2a+bc)^{\frac{3}{2}}-4\ln\left(\frac{2acd\,x^2+bd\,x^2+2c^2}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(12*a^2*d^3*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^6*c*(a*c^2+b*c)^(3/2)-4*ln((2*a*c*d*x^
2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)+2*b*c)/x^2)*a^3*b*c^5*d^2*x^4+2*a*d^3*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*x^6*b*(a*c^2+b*c)^(3/2)-9*ln((2*a*c*d*x^2+b*d*x^2+2*c
^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*
b*c)/x^2)*a^2*b^2*c^4*d^2*x^4+20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*a^2*c^2*d^2*x^4*(a*c^2+b*c)^(3/2)-6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2
*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x
^2)*a*b^3*c^3*d^2*x^4+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*
c*d^2*b*x^4*(a*c^2+b*c)^(3/2)-ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)
^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^4*c^2*
d^2*x^4+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*d^2*x^4*(a*c^
2+b*c)^(3/2)-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*c*d*x^2*(
a*c^2+b*c)^(3/2)-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*b*d*x^2*
(a*c^2+b*c)^(3/2)+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+
b*c)^(3/2)*a*c^2+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b
*c)^(3/2)*b*c)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/c^2/x^4/(a*c^2+b
*c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(157) = 314$.

time = 0.53, size = 359, normalized size = 2.03

$$-\frac{(4\,abc+b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^2c^3+2\,abc^2+b^2c)\sqrt{(ac+b)c}}-\frac{(4\,abc^2+b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(4\,a^2bc^2+3\,ab^2c-b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c+\frac{(a^2c^5+2\,abc^4+b^2c^3)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{2\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*a*b*c + b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 +
b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c^2 + 3*a*b^2
*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*b*c^4
 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a
*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3
+ b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))
```

Fricas [**A**]
time = 0.54, size = 593, normalized size = 3.35

$$\left[\frac{(4\,abc+b^2)\sqrt{ac^2+bc}\,d^2x^4\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)-4\,(2\,a^2c^5-(2\,a^2c^3+abc^2-b^2c)d^2x^4+4\,abc^4+2\,b^2c^3+3\,(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^4},\ -\frac{(4\,abc+b^2)\sqrt{-ac^2-bc}\,d^2x^4\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)+2\,(2\,a^2c^5-(2\,a^2c^3+abc^2-b^2c)d^2x^4+4\,abc^4+2\,b^2c^3+3\,(abc^3+b^2c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,(a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c
+ b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*
c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d
*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^
4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b
^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))
)/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c +
b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*c^3 + 2
*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 +
a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*
x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b
^2*c^3 + b^3*c^2)*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^5\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. 2(157) = 314.
time = 4.86, size = 778, normalized size = 4.40

$$-\frac{\frac{(4abcd^2+b^2d^2)\arctan\left(-\frac{\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}}{\sqrt{-ac^2-bc}}\right)}{(a^2c^3+2abc^2+b^2c)\sqrt{-ac^2-bc}}-\frac{8a^{\frac{7}{2}}c^5d|d|+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^3c^4d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}c^3d|d|+16a^{\frac{5}{2}}bc^4d|d|+28\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2bc^3d^2+16\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bc^2d|d|+8a^{\frac{3}{2}}b^2c^3d|d|+4\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3abcd^2+13\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2c^2d^2+8\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2\sqrt{a}\,b^2cd|d|+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3b^2d^2+\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)b^3cd^2}{(a^2c^3+2abc^2+b^2c)\left(ac^2-\left(\sqrt{ad^2}\,x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2+bc\right)^2}}{8\,\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*((4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2
*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3 + 2*a*b*
c^2 + b^2*c)*sqrt(-a*c^2 - b*c)) - (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2
)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2
 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c
))^2*a^(5/2)*c^3*d*abs(d) + 16*a^(5/2)*b*c^4*d*abs(d) + 28*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 1
6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*c^2*d*abs(d) + 8*a^(3/2)*b^2*c^3*d*abs(d) + 4*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 13*
(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a
*b^2*c^2*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2
 + a*c^2 + b*c))^2*sqrt(a)*b^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^
4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x^2 - sq
rt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a^2*c^3 +
 2*a*b*c^2 + b^2*c)*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^
2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2))/sgn(d*x^2 + c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)
```

3.350 $\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=443

$$-\frac{(4b+3ac)x(b+ac+adx^2)}{15a^2d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{x^3(b+ac+adx^2)}{5ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(8b^2+13abc+3a^2c^2)\,x(b+ac+adx^2)}{15a^3d^2\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}\,(8b^2+1}{15a^3d^5}$$

```
[Out] -1/15*(3*a*c+4*b)*x*(a*d*x^2+a*c+b)/a^2/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/
2)+1/5*x^3*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/15*(3*a^
2*c^2+13*a*b*c+8*b^2)*x*(a*d*x^2+a*c+b)/a^3/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/
(d*x^2+c))^(1/2)+1/15*c^(3/2)*(3*a*c+4*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(
1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*
c+b))^(1/2))/a^2/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*
d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/15*(3*a^2*c^2+13*a*b*c+8*b^2)*(a*d*
x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(5/2)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 489, 596, 545, 429, 506, 422}

$$\frac{c^{3/2}(3ac+4b)\,(ac+adx^2+b)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^2d^{5/2}\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{x(3ac+4b)\,(ac+adx^2+b)}{15a^2d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\,(3a^2c^2+13abc+8b^2)\,(ac+adx^2+b)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{15a^3d^{5/2}\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{x(3a^2c^2+13abc+8b^2)\,(ac+adx^2+b)}{15a^3d^2\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{x^3(ac+adx^2+b)}{5ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*((4*b + 3*a*c)*x*(b + a*c + a*d*x^2))/(a^2*d^2*Sqrt[(b + a*c + a*d*x^
2)/(c + d*x^2)]) + (x^3*(b + a*c + a*d*x^2))/(5*a*d*Sqrt[(b + a*c + a*d*x^2
)/(c + d*x^2)]) + ((8*b^2 + 13*a*b*c + 3*a^2*c^2)*x*(b + a*c + a*d*x^2))/(1
5*a^3*d^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(8*
b^2 + 13*a*b*c + 3*a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x
)/Sqrt[c]], b/(b + a*c)])/(15*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (
c^(3/2)*(4*b + 3*a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt
[c]], b/(b + a*c)])/(15*a^2*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a(c+dx^2)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)}\int \frac{x^4\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{5ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a(c+dx^2)}\int \frac{x^2(3c(b+ac)+(4b+3ac)\cdots}{\sqrt{c+dx^2}\sqrt{b+ac+\cdots}}}{5ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a(c\cdots}}{5ad\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a(c\cdots}}{5ad\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a(c\cdots}}{5ad\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(4b+3ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{15a^2d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{x^3\sqrt{b+ac+adx^2}\sqrt{b+a(c\cdots}}{5ad\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 297, normalized size = 0.67

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x(c+dx^2)(4b^2+ab(7c+dx^2)+3a^2(c^2-d^2x^4))+ic(8b^2+13abc+3a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\middle|1+\frac{b}{ac}\right)-2ibc(2b+3ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\middle|1+\frac{b}{ac}\right)\right)}{15a^2d^2\sqrt{\frac{ad}{b+ac}}(b+a(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] -1/15*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c +
d*x^2)*(4*b^2 + a*b*(7*c + d*x^2) + 3*a^2*(c^2 - d^2*x^4)) + I*c*(8*b^2 + 1
```

```
3*a*b*c + 3*a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - (2*I)*b*c*(2
*b + 3*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^2*d^2*Sqrt[(a*d)/(
b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.06, size = 664, normalized size = 1.50

method	result
default	$-\frac{\left(-3\sqrt{-\frac{ad}{ac+b}}\,a^{2}d^{3}x^{7}-3\sqrt{-\frac{ad}{ac+b}}\,a^{2}c\,d^{2}x^{5}+\sqrt{-\frac{ad}{ac+b}}\,ab\,d^{2}x^{5}+3\sqrt{-\frac{ad}{ac+b}}\,a^{2}c^{2}d\,x^{3}+8\sqrt{-\frac{ad}{ac+b}}\,abcd\,x^{3}-3\sqrt{ad}\cdots\right.}{}$
risch	$-\frac{x\left(-3ad\,x^{2}+3ac+4b\right)\left(ad\,x^{2}+ac+b\right)}{15d^{2}a^{2}\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}+\frac{\left(-\frac{2\left(3a^{2}c^{2}d+13abcd+8b^{2}d\right)\left(c^{2}a+bc\right)\sqrt{1+\frac{ad\,x^{2}}{ac+b}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+\cdots}}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+\cdots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(-3*(-a*d/(a*c+b))^(1/2)*a^2*d^3*x^7-3*(-a*d/(a*c+b))^(1/2)*a^2*c*d^2
*x^5+(-a*d/(a*c+b))^(1/2)*a*b*d^2*x^5+3*(-a*d/(a*c+b))^(1/2)*a^2*c^2*d*x^3+
8*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^
2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^3
+3*(-a*d/(a*c+b))^(1/2)*a^2*c^3*x+4*(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+6*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*a*b*c^2-13*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c^2+
7*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+
c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c-8*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b
))^(1/2),((a*c+b)/a/c)^(1/2))*b^2*c+4*(-a*d/(a*c+b))^(1/2)*b^2*c*x)*(d*x^2+
c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^2/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(1/2),x)
[Out] Integral(x**4/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/(c + d*x^2))^(1/2),x)
[Out] int(x^4/(a + b/(c + d*x^2))^(1/2), x)
```

3.351 $\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=354

$$\frac{x(b+ac+adx^2)}{3ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(2b+ac)x(b+ac+adx^2)}{3a^2d\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{c}\,(2b+ac)\left(b+ac+adx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\right.}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\,(b+ac+a}{(b+ac)\,(c+}}}$$

```
[Out] 1/3*x*(a*d*x^2+a*c+b)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+2*b)*x
*(a*d*x^2+a*c+b)/a^2/d/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3
/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/d^(3/2)/(d*x^2+c)/((a*d
*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/
3*(a*c+2*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/d^(3/
2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(
d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1986, 489, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,(ac+2b)\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+2b)\left(ac+adx^2+b\right)}{3a^2d\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3ad^{3/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(ac+adx^2+b\right)}{3ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*(b + a*c + a*d*x^2))/(3*a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((2
*b + a*c)*x*(b + a*c + a*d*x^2))/(3*a^2*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)]) + (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^2*d^(3/2)*(c + d*x^2)*Sqrt[(b +
 a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d
*x^2))]) - (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], b/(b + a*c)])/(3*a*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)+(2b+ac)dx^2}{\sqrt{c+dx^2}\sqrt{b+ac+a}}}{3ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((2b+ac)\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{}{\sqrt{c+dx^2}}}{3a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right.}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right.}}{3a^2d\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.04, size = 253, normalized size = 0.71

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x(c+dx^2)\left(b+ac+adx^2\right)+ic(2b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-ibc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{3ad\sqrt{\frac{ad}{b+ac}}\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(b + a*c + a*d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*
Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*
```

```
c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a*d*Sqrt[(a*d)/(b
+ a*c)]*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.05, size = 409, normalized size = 1.16

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\, a\, d^2 x^5+2\sqrt{-\frac{ad}{ac+b}}\, acd\, x^3+\sqrt{-\frac{ad}{ac+b}}\, bd\, x^3-\sqrt{\frac{ad\, x^2+ac+b}{ac+b}}\sqrt{\frac{d\, x^2+c}{c}}\, \mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right.\right.}{3d\sqrt{a\, d^2}}$
risch	$\frac{x\left(ad\, x^2+ac+b\right)}{3ad\sqrt{\frac{ad\, x^2+ac+b}{d\, x^2+c}}}-\frac{\left(-\frac{2(acd+2bd)\left(c^2 a+bc\right)\sqrt{1+\frac{ad\, x^2}{ac+b}}\sqrt{1+\frac{d\, x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{-1+\frac{2acd+bd}{dca}}\right)-\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a\, d^2 x^4+2acd\, x^2+bd\, x^2+c^2 a+bc}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*a*d^2*x^5+2*(-a*d/(a*c+b))^(1/2)*a*c*d*x^3+(-a*d/
(a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*
EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*c^2+(-a*d/(a*c+b))^
(1/2)*a*c^2*x+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF
(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*c-2*((a*d*x^2+a*c+b)/(a*c+b)
)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^
(1/2))*b*c+(-a*d/(a*c+b))^(1/2)*b*c*x)*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/d/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1
/2)/a/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^2/(a + b/(c + d*x^2))^(1/2), x)
```

3.352 $\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=286

$$\frac{x(b+ac+adx^2)}{a\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}\left(b+ac+adx^2\right)E\left(\tan^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{\frac{c\left(b+ac+adx^2\right)}{(b+ac)\left(c+dx^2\right)}}}+\frac{c^{3/2}(b+ac}{(b+ac)\sqrt{d}\left(c+d\right.}$$

```
[Out] x*(a*d*x^2+a*c+b)/a/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*
x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*
d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a/(d*x^2+c)/d^(1/2)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1985, 1986, 433, 429, 506, 422}

$$\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(ac+adx^2+b\right)}{a\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (x*(b + a*c + a*d*x^2))/(a*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) - (Sqrt[c]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(a*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(b + a*c +
a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*Sq
rt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c +
a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\left(c\sqrt{b+a(c+dx^2)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a(c+dx^2)}\right)\int \cdots}{\sqrt{c+dx^2} \cdots}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{a(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{c^{3/2}\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}\,F\left(\cdots\right.}{(b+ac)\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \cdots}$$

$$= \frac{x\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{a(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}\,E\left(\cdots\right.}{a\sqrt{d}\,(c+dx^2)\sqrt{\frac{c(b+ac+adx^2)}{(b+ac)(c+dx^2)}} \cdots}$$

Mathematica [A]
time = 8.83, size = 107, normalized size = 0.37

$$\frac{\sqrt{\frac{b+ac+adx^2}{b+ac}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)}{\sqrt{-\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\sqrt{1+\frac{dx^2}{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*EllipticE[ArcSin[Sqrt[-((a*d)/(b + a*c
))]*x], 1 + b/(a*c)])/(Sqrt[-((a*d)/(b + a*c))]*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]*Sqrt[1 + (d*x^2)/c])
```

Maple [A]
time = 0.02, size = 164, normalized size = 0.57

method	result	size
default	$\frac{\operatorname{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}, \sqrt{\frac{ac+b}{ac}}\right)\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,c\left(d\,x^{2}+c\right)\sqrt{\frac{ad\,x^{2}+ac+b}{d\,x^{2}+c}}}{\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+c^{2}a+bc}\,\sqrt{-\frac{ad}{ac+b}}\,\sqrt{\left(d\,x^{2}+c\right)\left(ad\,x^{2}+ac+b\right)}}$	164

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)/c)^(1/2)*(
(a*d*x^2+a*c+b)/(a*c+b))^(1/2)*c*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{c + dx^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/(c + d*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a + b/(d*x^2 + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/(c + d*x^2))^(1/2),x)
```

```
[Out] int(1/(a + b/(c + d*x^2))^(1/2), x)
```

3.353 $\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx$

Optimal. Leaf size=343

$$-\frac{b+ac+adx^2}{(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{dx(b+ac+adx^2)}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}\,\sqrt{d}\,(b+ac+adx^2)\,E\Big(\tan^{-1}\Big(}{(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,\sqrt{}}$$

```
[Out] (-a*d*x^2-a*c-b)/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+d*x*(a*d*x^2+a
*c+b)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(a*d*x^2+a*c+b)*(
1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2
/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*
c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/(a*c+b)/(d*x^2+c)/((a*
d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 21, 433, 429, 506, 422}

$$\frac{\sqrt{c}\,\sqrt{d}\,(ac+adx^2+b)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{\sqrt{c}\,\sqrt{d}\,(ac+adx^2+b)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{ac+adx^2+b}{x(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{dx(ac+adx^2+b)}{(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -((b + a*c + a*d*x^2)/((b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]))
+ (d*x*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*
x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) +
(Sqrt[c]*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
 b/(b + a*c)])/((b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]
*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+a(c+dx^2)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)}\int \frac{\sqrt{c+dx^2}}{x^2\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)}\int \frac{(b+ac)d+ad^2x^2}{\sqrt{c+dx^2}\sqrt{b+ac+}}}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a(c+dx^2)}\right)\int \frac{\sqrt{b+ac+adx^2}}{\sqrt{c+dx^2}}}{(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a(c+dx^2)}\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{b+}}}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)x\sqrt{a+\frac{b}{c+dx^2}}} + \frac{dx\sqrt{b+ac+adx^2}\sqrt{b+a(c+dx^2)}}{(b+ac)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}} -$$

Mathematica [A]

time = 9.16, size = 151, normalized size = 0.44

$$-\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{(b+ac)x}+\frac{d\sqrt{\frac{c+dx^2}{c}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}E\left(\sin^{-1}\left(\sqrt{-\frac{d}{c}}x\right)|\frac{ac}{b+ac}\right)}{(b+ac)\sqrt{-\frac{d}{c}}\sqrt{\frac{b+ac+adx^2}{b+ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -(((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/((b + a*c)*x)) + (d*S
qrt[(c + d*x^2)/c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*EllipticE[ArcSin[S
qrt[-(d/c)]*x], (a*c)/(b + a*c)])/((b + a*c)*Sqrt[-(d/c)]*Sqrt[(b + a*c + a
*d*x^2)/(b + a*c)])
```

Maple [A]
time = 0.05, size = 345, normalized size = 1.01

method	result
default	$-\frac{\left(\sqrt{-\frac{ad}{ac+b}}\,a\,d^2x^4-adc\sqrt{\frac{ad\,x^2+ac+b}{ac+b}}\sqrt{\frac{d\,x^2+c}{c}}\,x\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)+2\sqrt{-\frac{ad}{ac+b}}\,acd\,x^2-\sqrt{\frac{ad\,x}{}}\right.}{\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+}}$
risch	$-\frac{ad\,x^2+ac+b}{(ac+b)x\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{d\left(-\frac{2ad\left(c^2a+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-a*d/(a*c+b))^(1/2)*a*d^2*x^4-a*d*c*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*
x^2+c)/c)^(1/2)*x*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))+2*(
-a*d/(a*c+b))^(1/2)*a*c*d*x^2-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*b*d*x+(-a*d/(a
*c+b))^(1/2)*b*d*x^2+(-a*d/(a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*b*c)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)/((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(1/2)), x)
```

3.354 $\int \frac{1}{x^4 \sqrt{a+\frac{b}{c+dx^2}}} \, dx$

Optimal. Leaf size=435

$$\frac{-b-ac-adx^2}{3(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(b-ac)d(b+ac+adx^2)}{3c(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(b-ac)d^2x(b+ac+adx^2)}{3c(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{}{3\cdots}$$

```
[Out] 1/3*(-a*d*x^2-a*c-b)/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*
c+b)*d*(a*d*x^2+a*c+b)/c/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*
(-a*c+b)*d^2*x*(a*d*x^2+a*c+b)/c/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^
2+c))^(1/2)-1/3*(-a*c+b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d
*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/
2))/(a*c+b)^2/(d*x^2+c)/c^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x
^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-1/3*a*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^
2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2)
,(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))
^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 431, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 486, 597, 545, 429, 506, 422}

$$-\frac{a\sqrt{c}\,d^{3/2}(ac+adx^2+b)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}-\frac{d^{3/2}(b-ac)(ac+adx^2+b)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3\sqrt{c}(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c(ac+adx^2+b)}{(ac+b)(c+dx^2)}}}+\frac{d^2x(b-ac)(ac+adx^2+b)}{3c(ac+b)^2(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{d(b-ac)(ac+adx^2+b)}{3cx(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{ac+adx^2+b}{3x^3(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] -1/3*(b + a*c + a*d*x^2)/((b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]) - ((b - a*c)*d*(b + a*c + a*d*x^2))/(3*c*(b + a*c)^2*x*Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]) + ((b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*c*(b + a*
c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((b - a*c)*d^(3/2
)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/
(3*Sqrt[c]*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sq
rt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) - (a*Sqrt[c]*d^(3/2)*(
b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*
(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b +
a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+\frac{b}{c+dx^2}}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+a\left(c+dx^2\right)}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}}{x^4\sqrt{b+ac+adx^2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{(b-ac)d-ad^2x^2}{x^2\sqrt{c+dx^2}\sqrt{b+ac}}}{3(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx\right.}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx\right.}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx\right.}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3(b+ac)x^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-ac)d\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx\right.}}{3c(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

<u>**Mathematica**</u> **[C]** Result contains complex when optimal does not.
time = 10.63, size = 314, normalized size = 0.72

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(-\sqrt{\frac{ad}{b+ac}}\left(c+dx^2\right)\left(b^2\left(c+dx^2\right)+a^2c\left(c^2-d^2x^4\right)+ab\left(2c^2+cdx^2+d^2x^4\right)\right)+iac(-b+ac)d^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\middle|1+\frac{b}{ac}\right)+2iabcd^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)\middle|1+\frac{b}{ac}\right)\right)}{3c(b+ac)^2\sqrt{\frac{ad}{b+ac}}x^3\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b/(c + d*x^2)]),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b^2*(c + d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + c*d*x^2 + d^2*x^4)
```

```
)) + I*a*c*(-b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] + (2*
I)*a*b*c*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*El
lipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*c*(b + a*c)^2
*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.05, size = 593, normalized size = 1.36

method	result
risch	$-\frac{(a d x^2+ac+b)(-acd x^2+bd x^2+c^2 a+bc)}{3(ac+b)^2 x^3 c\sqrt{\frac{a d x^2+ac+b}{d x^2+c}}} - \frac{d^2 a\left(-\frac{2(acd-bd)(c^2a+bc)\sqrt{1+\frac{a d x^2}{ac+b}}\sqrt{1+\frac{d x^2}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a d^2 x^4+2acd x^2}}\right.}{}$
default	$-\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\, a^2 c d^3 x^6+\sqrt{-\frac{ad}{ac+b}}\, a b d^3 x^6+\sqrt{\frac{a d x^2+ac+b}{ac+b}}\sqrt{\frac{d x^2+c}{c}}\,\text{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{\frac{ac+b}{ac}}\right)a^2c^2d^2x^3-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-(-a*d/(a*c+b))^(1/2)*a^2*c*d^3*x^6+(-a*d/(a*c+b))^(1/2)*a*b*d^3*x^6+
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*a^2*c^2*d^2*x^3-(-a*d/(a*c+b))^(1/2)*a^2*c^2
*d^2*x^4+2*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*
(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*a*b*c*d^2*x^3-((a*d*x^2+a*c+b)/(a
*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/
a/c)^(1/2))*a*b*c*d^2*x^3+2*(-a*d/(a*c+b))^(1/2)*a*b*c*d^2*x^4+(-a*d/(a*c+b
))^(1/2)*a^2*c^3*d*x^2+(-a*d/(a*c+b))^(1/2)*b^2*d^2*x^4+3*(-a*d/(a*c+b))^(1
/2)*a*b*c^2*d*x^2+(-a*d/(a*c+b))^(1/2)*a^2*c^4+2*(-a*d/(a*c+b))^(1/2)*b^2*c
*d*x^2+2*(-a*d/(a*c+b))^(1/2)*a*b*c^3+(-a*d/(a*c+b))^(1/2)*b^2*c^2)*(d*x^2+
c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/c/x^3/(a*c+b)^2/((d*x^2+c)*(a*d*x^2+a*c+b))^
(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a + b/(d*x^2 + c))*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a+\frac{b}{d\,x^2+c}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(1/2)), x)
```

3.355 $\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=310

$$-\frac{(b+ac)^2(c+dx^2)^3}{ab^2d^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(35b^2+60abc+24a^2c^2)(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{16a^4d^3}-\frac{(35b^2+60abc+24a^2c^2)(c}{24a}$$

```
[Out] -1/16*b*(24*a^2*c^2+60*a*b*c+35*b^2)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1
/2)/a^(1/2))/a^(9/2)/d^3-(a*c+b)^2*(d*x^2+c)^3/a/b^2/d^3/((a*d*x^2+a*c+b)/(
d*x^2+c))^(1/2)+1/16*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)*((a*d*x^2+a*c+b
)/(d*x^2+c))^(1/2)/a^4/d^3-1/24*(24*a^2*c^2+60*a*b*c+35*b^2)*(d*x^2+c)^2*((
a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/b/d^3+1/6*(6*a^2*c^2+12*a*b*c+7*b^2)*(d
*x^2+c)^3*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/b^2/d^3
```

Rubi [A]

time = 0.34, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1985, 1981, 1980, 473, 393, 205, 214}

$$\frac{(6a^2c^2+12abc+7b^2)(c+dx^2)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3}-\frac{b(24a^2c^2+60abc+35b^2)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3}+\frac{(24a^2c^2+60abc+35b^2)(c+dx^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^4d^3}-\frac{(24a^2c^2+60abc+35b^2)(c+dx^2)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^3bd^3}-\frac{(ac+b)^2(c+dx^2)^3}{ab^2d^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -(((b + a*c)^2*(c + d*x^2)^3)/(a*b^2*d^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^
2)])) + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x
^2)/(c + d*x^2)])/(16*a^4*d^3) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x
^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(24*a^3*b*d^3) + ((7*b^2 + 12*
a*b*c + 6*a^2*c^2)*(c + d*x^2)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(6*
a^2*b^2*d^3) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*ArcTanh[Sqrt[(b + a*c +
a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(16*a^(9/2)*d^3)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{x^5}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^5\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(\frac{1}{2}(b+ac)(5b+4ac)d-\frac{1}{2}abd^2x\right)}{\sqrt{b+ac+adx}}\,d\right.}{a^2bd^3\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c+dx^2\right)^2\left(b+a(c+dx^2)\right)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(\left(35b^2+60abc+24a^2c^2\right)\sqrt{b\right.}{12a^2}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(35b^2+60abc+24a^2c^2\right)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(c+}{6a}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(35b^2+60abc}{24}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(35b^2+60abc}{24}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(35b^2+60abc}{24}$$

$$= \frac{(b+ac)^2\left(c+dx^2\right)^2}{a^2bd^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(35b^2+60abc+24a^2c^2\right)\left(b+a(c+dx^2)\right)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(35b^2+60abc}{24}$$

Mathematica [A]
time = 0.35, size = 183, normalized size = 0.59

$$\frac{\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(105b^3+5ab^2(43c+7dx^2)+2a^2b(59c^2+16cdx^2-7d^2x^4)+8a^3(c^3+d^3x^6))}{b+a(c+dx^2)}-3b(35b^2+60abc+24a^2c^2)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{48a^{9/2}d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3 + 5*a*
b^2*(43*c + 7*d*x^2) + 2*a^2*b*(59*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^3*(c
^3 + d^3*x^6)))/(b + a*(c + d*x^2)) - 3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*
ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(48*a^(9/2)*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(288) = 576$.
time = 0.12, size = 1240, normalized size = 4.00

method	result
risch	$\frac{(8d^2a^2x^4-8a^2cd\,x^2-22abd\,x^2+8a^2c^2+62abc+57b^2)(ad\,x^2+ac+b)}{48d^3a^4\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(-\frac{3b\ln\left(\frac{acd+\frac{1}{2}bd+a\,d^2x^2}{\sqrt{a\,d^2}}+\sqrt{c^2a+bc+(2acd+bd}\right.}{4a^2d^2\sqrt{a\,d^2}}\right.}{}$
default	$\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-48\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{a\,d^2}\,a^3c\,d^2x^4-60\sqrt{a\,d^2x^4+2acd\,x^2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^4/d^3*(-48*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^3*c*d^2*x^4-60*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*d^2*x^4-72*ln(1/
2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a
*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*b*c^2*d^2*x^2-48*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^3*c^2*d*x^2-180*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+
b*d)/(a*d^2)^(1/2))*a^2*b^2*c*d^2*x^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c
^2+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*d*x^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2
))*a^3*b*c^3*d-105*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^3*d^2*x^2+54*(a
```

```
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^2*d*x^2-252
*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b^2*c^2*d+16*(a*d^2*x^4+2*a*c*d*x
^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a^2*c+108*(a*d^2*x^4+2*a*c*d*x^2+
b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*b*c^2-285*ln(1/2*(2*a*d^2*x^2+2*
a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/
(a*d^2)^(1/2))*a*b^3*c*d+96*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)
*a^2*b*c^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)
*a*b+222*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b^
2*c-105*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+
b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^4*d+192*(a*d^2)^(1/2)*((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2*c+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*(a*d^2)^(1/2)*b^3+96*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*b^3)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [A]
time = 0.52, size = 389, normalized size = 1.25

$$\frac{48\,a^5bc^2+96\,a^4b^2c+48\,a^3b^3-\frac{3\,(24\,a^2bc^2+60\,ab^2c+35\,b^3)(adx^2+ac+b)^3}{(dx^2+c)^3}+\frac{8\,(24\,a^3bc^2+60\,a^2b^2c+35\,ab^3)(adx^2+ac+b)^2}{(dx^2+c)^2}-\frac{3\,(56\,a^4bc^2+132\,a^3b^2c+77\,a^2b^3)(adx^2+ac+b)}{dx^2+c}}{48\left(a^7d^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-3\,a^6d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+3\,a^5d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-a^4d^3\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{7}{2}}\right)}+\frac{(24\,a^2c^2+60\,abc+35\,b^2)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32\,a^{\frac{9}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^2
*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a^2
*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2 +
132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x^2 +
c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d^3*((a
*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c + 35*b^
2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)
```

Fricas [A]
time = 0.48, size = 675, normalized size = 2.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*
c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2
```

```
 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d
*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8
*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 +
(18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*
c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c
 + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b*
c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c +
 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt
((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4
*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*
b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 15
0*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(288) = 576.
time = 4.07, size = 597, normalized size = 1.93

$$\frac{1}{48}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\left(2x^2\left(\frac{4x^2}{a^2d\,\mathrm{sgn}\left(dx^2+c\right)}-\frac{4a^{11}cd^6\mathrm{sgn}(dx^2+c)+11a^{10}bd^6\mathrm{sgn}(dx^2+c)}{a^{13}d^8}\right)+\frac{8a^{11}c^2d^5\mathrm{sgn}(dx^2+c)+62a^{10}bcd^5\mathrm{sgn}(dx^2+c)+57a^9b^2d^5\mathrm{sgn}(dx^2+c)}{a^{13}d^8}\right)+\frac{\left(24a^{\frac{5}{2}}bc^2+60a^{\frac{3}{2}}b^2c+35\sqrt{a}b^3\right)\log\left(\left|-2a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}cd-5a^{\frac{5}{2}}bc^2d-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3a^2|d|-10\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2bc|d|-5\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bd-4a^{\frac{3}{2}}b^2cd-4\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2|d|-\sqrt{a}b^3d\right|\right)}{96a^5d^2|d|\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a
^2*d*sgn(d*x^2 + c)) - (4*a^11*c*d^6*sgn(d*x^2 + c) + 11*a^10*b*d^6*sgn(d*x
^2 + c))/(a^13*d^8)) + (8*a^11*c^2*d^5*sgn(d*x^2 + c) + 62*a^10*b*c*d^5*sgn
(d*x^2 + c) + 57*a^9*b^2*d^5*sgn(d*x^2 + c))/(a^13*d^8)) + 1/96*(24*a^(5/2)
*b*c^2 + 60*a^(3/2)*b^2*c + 35*sqrt(a)*b^3)*log(abs(-2*a^(7/2)*c^3*d - 6*(s
qrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3
*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 +
 a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqr
t(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt
(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*
c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*
```

```
c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a
*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3
*d))/(a^5*d^2*abs(d)*sgn(d*x^2 + c))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a+\frac{b}{d x^2+c}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5/(a + b/(c + d*x^2))^(3/2), x)
```

3.356 $\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}} dx$

Optimal. Leaf size=187

$$-\frac{b(b+ac)}{a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(7b+4ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8a^3d^2}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4a^2d^2}+\frac{3b(5b+4ac}{}$$

```
[Out] 3/8*b*(4*a*c+5*b)*arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(7/2
)/d^2-b*(a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*(4*a*c+7*b)*(
d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^3/d^2+1/4*(d*x^2+c)^2*((a*d*x^
2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d^2
```

Rubi [A]
time = 0.26, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 467, 464, 214}

$$\frac{3b(4ac+5b)\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}-\frac{(4ac+7b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2}-\frac{b(ac+b)}{a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((b*(b + a*c))/(a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((7*b +
4*a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*a^3*d^2) + ((c
 + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*a^2*d^2) + (3*b*(5*b
+ 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2)
*d^2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^3\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\mathrm{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(b+ac+adx)^{3/2}}\,dx, x, x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((5b+4ac)\sqrt{b+a\left(c+dx^2\right)}\right)\mathrm{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{b+ac+adx}}\,dx\right.}{2abd\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a(c+dx^2)\right)}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(3(5b+4ac)\sqrt{b+\right.}{8}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a(c+dx^2)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a(c\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a(c+dx^2)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a(c\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a(c+dx^2)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a(c\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{(b+ac)\left(c+dx^2\right)^2}{abd^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\left(b+a(c+dx^2)\right)}{8a^3d^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(5b+4ac)\left(c+dx^2\right)\left(b+a(c\right.}{4a^2bd^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [A]

time = 0.24, size = 144, normalized size = 0.77

$$-\frac{\frac{\sqrt{a}\,(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(15b^2+ab(17c+5dx^2)+2a^2(c^2-d^2x^4))}{b+a(c+dx^2)}+3b(5b+4ac)\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (-((Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + a*b
*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4)))/(b + a*(c + d*x^2))) + 3*b*(5*b
 + 4*a*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2
)*d^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. $2(169) = 338$.
time = 0.10, size = 783, normalized size = 4.19

method	result
risch	$-\frac{(-2adx^2+2ac+7b)(adx^2+ac+b)}{8d^2a^3\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}+\frac{\left(\frac{3b\ln\left(\frac{acd+\frac{1}{2}bd+ad^2x^2}{\sqrt{ad^2}}+\sqrt{c^2a+bc+(2acd+bd)x^2+ad^2x^4}\right)c}{4a^2d\sqrt{ad^2}}+\frac{15b^2\ln\left(\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\,(dx^2+c)\left(-4\sqrt{ad^2x^4+2acdx^2+bdx^2+c^2a+bc}\,\sqrt{ad^2}\,a^2d^2x^4-12\ln\left(\frac{2ad^2x^2+2acd+2\sqrt{\cdots}}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^2*(-4*(a*d^2*x^4+2*
a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*d^2*x^4-12*ln(1/2*(2*a
*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^
(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c*d^2*x^2-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^
(1/2))*a*b^2*d^2*x^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2
*b*a*d*(a*d^2)^(1/2)-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d+4*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c^2-27*ln
(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)
*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*a*d+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)-15*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(
```

```
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(
1/2))*b^3*d+16*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c+14*(a*
d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2)+16*(a*d^2)^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [A]
time = 0.54, size = 262, normalized size = 1.40

$$-\frac{8\,a^3bc+8\,a^2b^2+\frac{3\,(adx^2+ac+b)^2(4\,abc+5\,b^2)}{(dx^2+c)^2}-\frac{5\,(4\,a^2bc+5\,ab^2)(adx^2+ac+b)}{dx^2+c}}{8\left(a^5d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-2\,a^4d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+a^3d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)}-\frac{3\,(4\,ac+5\,b)b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\,a^{\frac{7}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2)/(d*
x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^5*
d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 2*a^4*d^2*((a*d*x^2 + a*c + b)/
(d*x^2 + c))^(3/2) + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2)) - 3/1
6*(4*a*c + 5*b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(s
qrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(7/2)*d^2)
```

Fricas [A]
time = 0.48, size = 541, normalized size = 2.89

$$\left[\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{a}\,\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2+4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(2\,a^3d^3x^6+(2\,a^3c-5\,a^2b)d^2x^4-2\,a^3c^3-17\,a^2bc^2-15\,ab^2c-(2\,a^3c^2+22\,a^2bc+15\,ab^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,(a^5d^3x^2+(a^5c+a^4b)d^2)},-\frac{3\,(4\,a^2bc^2+9\,ab^2c+(4\,a^2bc+5\,ab^2)dx^2+5\,b^3)\sqrt{-a}\,\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)-2\,(2\,a^3d^3x^6+(2\,a^3c-5\,a^2b)d^2x^4-2\,a^3c^3-17\,a^2bc^2-15\,ab^2c-(2\,a^3c^2+22\,a^2bc+15\,ab^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,(a^5d^3x^2+(a^5c+a^4b)d^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sq
rt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b
^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x
^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a
*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c +
 a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x
^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^3*d^3*x^6 +
 (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a
^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. 2(169) = 338.
time = 4.99, size = 510, normalized size = 2.73

$$\frac{1}{8}\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\left(\frac{2x^2}{a^2d\operatorname{sgn}(dx^2+c)}-\frac{2a^6cd^2+7a^5bd^2}{a^8d^4\operatorname{sgn}(dx^2+c)}\right)-\frac{\left(4a^{\frac{3}{2}}bc+5\sqrt{a}b^2\right)\log\left(\left|-2a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}cd-5a^{\frac{5}{2}}bc^2d-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^3a^2|d|-10\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)a^2bc|d|-5\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bd-4a^{\frac{3}{2}}b^2cd-4\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2acdx^2+bdx^2+ac^2+bc}\right)ab^2|d|-\sqrt{a}b^3d\right|\right)}{16a^4d|d|\operatorname{sgn}(dx^2+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn
(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16
*(4*a^(3/2)*b*c + 5*sqrt(a)*b^2)*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*
x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d)
 - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*
c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4
 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2
 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) -
5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))
^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 +
2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^4*d
*abs(d)*sgn(d*x^2 + c))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^3/(a + b/(c + d*x^2))^(3/2), x)
```

3.357 $\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

```
[Out] -3/2*b*arctanh((a+b/(d*x^2+c))^(1/2)/a^(1/2))/a^(5/2)/d+3/2*b/a^2/d/(a+b/(d
*x^2+c))^(1/2)+1/2*(d*x^2+c)/a/d/(a+b/(d*x^2+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1605, 248, 44, 53, 65, 214}

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}+\frac{3b}{2a^2d\sqrt{a+\frac{b}{c+dx^2}}}+\frac{c+dx^2}{2ad\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (3*b)/(2*a^2*d*Sqrt[a + b/(c + d*x^2)]) + (c + d*x^2)/(2*a*d*Sqrt[a + b/(c
+ d*x^2)]) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx, x, c+dx^2\right)}{2d} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{c+dx^2}\right)}{2d} \\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{2ad} \\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{c+dx^2}\right)}{4a^2d} \\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{c+dx^2}}\right)}{2a^2d} \\
&= -\frac{c+dx^2}{ad\sqrt{a+\frac{b}{c+dx^2}}} + \frac{3(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2a^2d} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.17, size = 114, normalized size = 1.14

$$
\frac{(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\,(3b+ac+adx^2)}{2a^2d\,(b+ac+adx^2)} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b + a*c + a*d*x^2))/(
2*a^2*d*(b + a*c + a*d*x^2)) - (3*b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d
*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 477 vs. 2(84) = 168.
time = 0.27, size = 478, normalized size = 4.78

method	result
risch	$\frac{a d x^2+a c+b}{2 d a^2 \sqrt{\frac{a d x^2+a c+b}{d x^2+c}}}+\frac{\left(-\frac{3 b \ln \left(\frac{a c d+\frac{1}{2} b d+a d^2 x^2}{\sqrt{a d^2}}+\sqrt{c^2 a+b c+(2 a c d+b d) x^2+a d^2 x^4}\right)}{4 a^2 \sqrt{a d^2}}+\frac{}{a^2 \sqrt{a}}\right.}{}$
derivativedivides	$-\frac{\sqrt{\frac{a(d x^2+c)+b}{d x^2+c}}\,(d x^2+c)\left(-6 \sqrt{(d x^2+c)(a(d x^2+c)+b)}\, a^{\frac{5}{2}} (d x^2+c)^2+3 \ln \left(\frac{2 \sqrt{(d x^2+c)(}}{}\right.\right.}{}$
default	$\frac{\sqrt{\frac{a d x^2+a c+b}{d x^2+c}}\,(d x^2+c)\left(-3 \ln \left(\frac{2 a d^2 x^2+2 a c d+2 \sqrt{a d^2 x^4+2 a c d x^2+b d x^2+c^2 a+b c}\, \sqrt{a d^2}+b d}{2 \sqrt{a d^2}}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^2/d*(-3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+
b*d)/(a*d^2)^(1/2))*a*b*d^2*x^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)
^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b*c*d
-3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^
(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+4*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+
a*c+b))^(1/2)*b+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(
1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [A]
time = 0.53, size = 161, normalized size = 1.61

$$\frac{2\,ab-\frac{3\,(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)}+\frac{3\,b\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4\,a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a*
c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/4*
b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(84) = 168$.
time = 0.40, size = 395, normalized size = 3.95

$$\left[\frac{3\,(abdx^2+abc+b^2)\sqrt{a}\log\left(8\,a^2d^2x^4+8\,a^2c^2+8\,(2\,a^2c+ab)dx^2+8\,abc+b^2-4\,(2\,ad^2x^4+(4\,ac+b)dx^2+2\,ac^2+bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)+4\,(a^2d^2x^4+a^2c^2+(2\,a^2c+3\,ab)dx^2+3\,abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,(a^4d^2x^2+(a^4c+a^3b)d)}, \frac{3\,(abdx^2+abc+b^2)\sqrt{-a}\arctan\left(\frac{(2\,adx^2+2\,ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2dx^2+a^2c+ab)}\right)+2\,(a^2d^2x^4+a^2c^2+(2\,a^2c+3\,ab)dx^2+3\,abc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,(a^4d^2x^2+(a^4c+a^3b)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8
*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2
 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d
^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*
b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x
^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^
2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2
+ c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(84) = 168$.
time = 4.84, size = 449, normalized size = 4.49

$$\frac{b\log\left(\left|-2\,a^{\frac{7}{2}}c^3d-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^3c^2|d|-6\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{5}{2}}cd-5\,a^{\frac{5}{2}}bc^2d-2\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^3a^2|d|-10\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)a^2bc|d|-5\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)^2a^{\frac{3}{2}}bd-4\,a^{\frac{3}{2}}b^2cd-4\left(\sqrt{ad^2}x^2-\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}\right)ab^2|d|-\sqrt{a}b^3d\right|\right)}{4\,a^{\frac{5}{2}}|d|\mathrm{sgn}\left(dx^2+c\right)}+\frac{\sqrt{ad^2x^4+2\,acdx^2+bdx^2+ac^2+bc}}{2\,a^2d\mathrm{sgn}\left(dx^2+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

[Out] 1/4*b*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^(5/2)*abs(d)*sgn(d*x^2 + c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a^2*d*sgn(d*x^2 + c))

Mupad [B]
time = 3.93, size = 61, normalized size = 0.61

$$\frac{\left(\frac{a\,\left(d\,x^2+c\right)}{b}+1\right)^{3/2}\,\left(d\,x^2+c\right)\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a\,\left(d\,x^2+c\right)}{b}\right)}{5\,d\,{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/(c + d*x^2))^(3/2),x)

[Out] (((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*(c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))

3.358 $\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{(b+ac)^{3/2}}$$

```
[Out] arctanh(((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^(1/2))/a^(3/2)-c^(3/2)*arctanh(
c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(3/2)-b/a/
(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 491, 536, 214}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{(ac+b)^{3/2}}-\frac{b}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ArcTanh[Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*
Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{\frac{1}{2}ac^2d+\frac{1}{2}(b+ac)d^2x}{x\sqrt{c+dx}\sqrt{b+ac+adx}}\right.}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c^2\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac-}}\right.}{2(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx,x,\sqrt{c+dx}\right.}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^{3/2}\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a\left(c+dx^2\right)}}\right.}{(b+ac)^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b}{a(b+ac)\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a\left(c+dx^2\right)}}\right)}{a^{3/2}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c^3}{}$$

Mathematica [A]

time = 0.42, size = 140, normalized size = 1.04

$$-\frac{b}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{(-b-ac)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - (c^(3/2)*ArcTan[
(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(-b - a*c)
^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(116) = 232$.
time = 0.07, size = 1014, normalized size = 7.57

method	result
default	$\frac{\sqrt{\frac{a d\,x^{2}+a c+b}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(\ln\left(\frac{2 a\,d^{2} x^{2}+2 a c d+2\sqrt{a\,d^{2} x^{4}+2 a c d\,x^{2}+b d\,x^{2}+c^{2} a+b c}\,\sqrt{a\,d^{2}}+b d}{2\sqrt{a\,d^{2}}}\right) a^{3} c^{2} d^{2} x^{2}+2\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a*(ln(1/2*(2*a*d^2*x^2+2*a*
c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a
*d^2)^(1/2))*a^3*c^2*d^2*x^2+2*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c
*d^2*x^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*
(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^
2)*a^2*c*d*x^2+ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^3*c^3*d+ln(1/2*(2*a*d
^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2*x^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^
2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2)
)*a^2*b*c^2*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2
*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*
c)/x^2)*a^2*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*a*d-(a*d^2)^(1
/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b*c+ln(1/2*(2*
```

```
a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)
^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b)
)^(1/2)*a*b*c-2*(a*d^2)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2)/((d*x^
2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^2/(a*d^2)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [**A**]
time = 0.53, size = 201, normalized size = 1.50

$$\frac{c^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}\,(ac+b)}-\frac{b}{(a^2c+ab)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}-\frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(
c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/(sqrt((a*c +
b)*c)*(a*c + b)) - b/((a^2*c + a*b)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
- 1/2*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqr
t((a*d*x^2 + a*c + b)/(d*x^2 + c))))/a^(3/2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(116) = 232.
time = 0.51, size = 1477, normalized size = 11.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2
*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4
 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2
*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a
^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4
+ 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x
^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*
d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c
+ a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2
+ 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt(
(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2
+ a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2
```

```
*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*
c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 +
 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c +
 b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*
x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*
b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arct
an(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^
2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c +
 a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a
^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*
a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2
+ a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*
b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*
c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2
 + a*c + b)/(d*x^2 + c))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^
2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2
*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 +
 a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x*(a + b/(c + d*x^2))^(3/2)), x)
```

3.359 $\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{3bd}{2(b+ac)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{c+dx^2}{2(b+ac)x^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{b+ac}}\right)}{2(b+ac)^{5/2}}$$

```
[Out] -3/2*b*d*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))*c
^(1/2)/(a*c+b)^(5/2)+3/2*b*d/(a*c+b)^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/
2*(-d*x^2-c)/(a*c+b)/x^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 296, 331, 214}

$$\frac{3bd}{2(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c+dx^2}{2x^2(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3b\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2(ac+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (c + d*x^2)
/(2*(b + a*c)*x^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (3*b*Sqrt[c]*d*A
rcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(2*(
b + a*c)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
```

```
b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
x]
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^3\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bd\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}}\,d\right.}{4(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx\right.\right.}}{4(b}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a\left(c+dx\right.\right.}}{2(}$$

$$= \frac{3bd}{2(b+ac)^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}\,d\sqrt{b+a\left(c+d\right.}}{2(b+ac)^{5/}}$$

Mathematica [A]
time = 0.25, size = 148, normalized size = 1.01

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\left(c-2dx^2\right)+ac\left(c+dx^2\right)\right)}{2(b+ac)^2x^2\left(b+a\left(c+dx^2\right)\right)} + \frac{3b\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{2(-b-ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/2*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*(c - 2*d*x^2) +
a*c*(c + d*x^2)))/((b + a*c)^2*x^2*(b + a*(c + d*x^2))) + (3*b*Sqrt[c]*d*Ar
cTan[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(2*(-
b - a*c)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(129) = 258$.

time = 0.11, size = 1088, normalized size = 7.45

method	result
risch	$-\frac{c(ad\,x^2+ac+b)}{2(ac+b)^2x^2\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(-\frac{3bdc\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc+(2acd+bd)\,x^2+a\,}}{x^2}\right.}{4(ac+b)^2\sqrt{c^2a+bc}}\right.}{}$
default	$-\frac{\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}\,(d\,x^2+c)\left(-2\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{c^2a+bc}\,a^2d^3x^6+3\ln\left(\frac{2acd\,x^2+bd\,x^2+2c}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*(a*d^2*x^4+2*a*c*d*x^2
+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*a^2*d^3*x^6+3*ln((2*a*c*d*x^2+b
*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c
)^(1/2)+2*b*c)/x^2)*a^2*b*c^2*d^2*x^4-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*a^2*c*d^2*x^4+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^
2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b
*c)/x^2)*a*b^2*c*d^2*x^4-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*
(a*c^2+b*c)^(1/2)*a*b*d^2*x^4+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*
c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b*
c^3*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/
2)*a^2*c^2*d*x^2+6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d
^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^2*c^2*d*x^2+2*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*a*d*x^2-6*
(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*a*b*c*d*x
^2-4*(a*c^2+b*c)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x^2+3*ln((
2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^3*c*d*x^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*b^2*d*x^2-4*(a*c^2+b*c)^(1/2)*((d*x^2
+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d*x^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(3/2)*(a*c^2+b*c)^(1/2)*a*c+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(3/2)*(a*c^2+b*c)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^3/x
^2/(a*c^2+b*c)^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [A]
time = 0.52, size = 247, normalized size = 1.69

$$\frac{3\,bcd\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{4\,(a^2c^2+2\,abc+b^2)\sqrt{(ac+b)c}}+\frac{\frac{3\,(adx^2+ac+b)bcd}{dx^2+c}-2\,(abc+b^2)d}{2\left((a^2c^3+2\,abc^2+b^2c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}-(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*b*c*d*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))
/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^2 +
 2*a*b*c + b^2)*sqrt((a*c + b)*c)) + 1/2*(3*(a*d*x^2 + a*c + b)*b*c*d/(d*x^
2 + c) - 2*(a*b*c + b^2)*d)/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*((a*d*x^2 + a*c
+ b)/(d*x^2 + c))^(3/2) - (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a
*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Fricas [A]
time = 0.48, size = 599, normalized size = 4.10

$$\left[\frac{3\,(abd^2x^4+(abc+b^2)dx^2)\sqrt{\frac{c}{ac+b}}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2-4\,((2\,a^2c^2+3\,abc+b^2)d^2x^4+2\,a^2c^4+4\,abc^3+2\,b^2c^2+(4\,a^2c^3+7\,abc^2+3\,b^2c)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{\frac{c}{ac+b}}}{x^4}\right)-4\,((ac-2\,b)d^2x^4+ac^3+(2\,ac^2-bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\,((a^3c^2+2\,a^2bc+ab^2)dx^4+(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)x^2)},\ \frac{3\,(abd^2x^4+(abc+b^2)dx^2)\sqrt{-\frac{c}{ac+b}}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\sqrt{-\frac{c}{ac+b}}}{2\,(acdx^2+ac^2+bc)}\right)-2\,((ac-2\,b)d^2x^4+ac^3+(2\,ac^2-bc)dx^2+bc^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4\,((a^3c^2+2\,a^2bc+ab^2)dx^4+(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c
^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2
*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 +
2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2
)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*((a*c -
 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c
+ b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*
b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*s
qrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b))/(a*c*d*x^2 + a*c^2 + b*c))
 - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*
d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*
c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)
[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)
```

3.360 $\int \frac{1}{x^5\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=212

$$-\frac{abd^2}{(b+ac)^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(3b-4ac)d(c+dx^2)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{8(b+ac)^3x^2}-\frac{(c+dx^2)^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{4(b+ac)^2x^4}-\frac{3b(b$$

[Out] -3/8*b*(-4*a*c+b)*d^2*arctanh(c^(1/2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^(1/2))/(a*c+b)^(7/2)/c^(1/2)-a*b*d^2/(a*c+b)^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/8*(-4*a*c+3*b)*d*(d*x^2+c)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^3/x^2-1/4*(d*x^2+c)^2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*c+b)^2/x^4

Rubi [A]
time = 0.31, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1985, 1981, 1980, 467, 464, 214}

$$-\frac{abd^2}{(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{3bd^2(b-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8\sqrt{c}\,(ac+b)^{7/2}}-\frac{d(3b-4ac)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4x^4(ac+b)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]

[Out] -((a*b*d^2)/((b + a*c)^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((3*b - 4*a*c)*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(8*(b + a*c)^3*x^2) - ((c + d*x^2)^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(4*(b + a*c)^2*x^4) - (3*b*(b - 4*a*c)*d^2*ArcTanh[(Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[b + a*c]])/(8*Sqrt[c]*(b + a*c)^(7/2))

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rule 1981

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*((
a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^5\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(b+ac+adx)^{3/2}}\,dx,x,x^2\right)}{2\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left((b-4ac)d\sqrt{b+a\left(c+dx^2\right)}\right)\text{Subst}\left(\int \frac{(c+
}{x^2(b+ac
}\right.}{8c(b+ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+d
}}}$$

$$= -\frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d^2\right.}{}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(b-4ac)d\left(c+dx^2\right)}{8c(b+ac)^2x^2\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left(c+dx^2\right)^2}{4c(b+ac)x^4\sqrt{a+}}$$

Mathematica [A]
time = 0.35, size = 192, normalized size = 0.91

$$-\frac{\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b^2(2c+5dx^2)+2a^2c(c^2-d^2x^4)+ab(4c^2+5cdx^2+13d^2x^4)\right)}{8(b+ac)^3x^4\left(b+a\left(c+dx^2\right)\right)} - \frac{3b(b-4ac)d^2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}{\sqrt{-b-ac}}\right)}{8\sqrt{c}\,(-b-ac)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/8*((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b^2*(2*c + 5*d*x^2
) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)))/((b +
a*c)^3*x^4*(b + a*(c + d*x^2))) - (3*b*(b - 4*a*c)*d^2*ArcTan[(Sqrt[c]*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)])/Sqrt[-b - a*c]])/(8*Sqrt[c]*(-b - a*c)^(
7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1946 vs. $2(190) = 380$.
time = 0.14, size = 1947, normalized size = 9.18

method	result
risch	$-\frac{(a d\,x^2+ac+b)\left(-2acd\,x^2+5bd\,x^2+2c^2a+2bc\right)}{8(ac+b)^3 x^4\sqrt{\frac{a d\,x^2+ac+b}{d\,x^2+c}}}+\frac{\left(\frac{3d^2 b\ln\left(\frac{2c^2a+2bc+(2acd+bd)x^2+2\sqrt{c^2a+bc}\,\sqrt{c^2a+bc+(2acd+}}{x^2}\right.}{4(ac+b)^3\sqrt{c^2a+bc}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-8*(a*d^2*x^4+2*a*c*d*x^
2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c^2*d*x^2-2*(a*d^2*x^4+2*a
*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*c*d*x^2+12*(a*d^2*x
^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^3*c*d^4*x^8+6*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b^2*d*x^2+
8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*c^2
+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c^
3+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*b^2*c
+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*b*
c*d^3*x^6+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3
/2)*a^2*b*c^2*d^2*x^4+3*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)
*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*b^5*c^2*d^2*x^
4-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*b^3*d
^2*x^4+16*(a*c^2+b*c)^(3/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*b*c^2*d^2
*x^4-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a
*b^2*c*d^2*x^4+16*(a*c^2+b*c)^(3/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b^2
*c*d^2*x^4-33*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^3*b^2*c^5*d^2*x^4+3*ln
((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*
d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^4*c^2*d^3*x^6+20*(a*d^2*x^4+2*a*c*d*
```

```
x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a^3*c^3*d^2*x^4-12*(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*a*b^2*d^3*x^6-27*l
n((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b^3*c^4*d^2*x^4-12*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a^2*c*d^2*x^4-3*ln((2*a*c
*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a
*c^2+b*c)^(1/2)+2*b*c)/x^2)*a*b^4*c^3*d^2*x^4+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*
x^2+a*c^2+b*c)^(3/2)*(a*c^2+b*c)^(3/2)*a*b*d^2*x^4-12*ln((2*a*c*d*x^2+b*d*x
^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1
/2)+2*b*c)/x^2)*a^4*b*c^5*d^3*x^6-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*
c)^(1/2)*(a*c^2+b*c)^(3/2)*a^2*b*d^4*x^8-21*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a
+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)
/x^2)*a^3*b^2*c^4*d^3*x^6+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2
)*(a*c^2+b*c)^(3/2)*a^3*c^2*d^3*x^6-12*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a
*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)
*a^4*b*c^6*d^2*x^4-6*ln((2*a*c*d*x^2+b*d*x^2+2*c^2*a+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)+2*b*c)/x^2)*a^2*b^3*c^3*d^3*x
^6)/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^4/x^4/(a*c^2+b*c)^(3/2)/(a*
d*x^2+a*c+b)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(192) = 384$.

time = 0.55, size = 450, normalized size = 2.12

$$-\frac{3\,(4\,abc-b^2)d^2\log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}-\sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}}+\sqrt{(ac+b)c}}\right)}{16\,(a^3c^3+3\,a^2bc^2+3\,ab^2c+b^3)\sqrt{(ac+b)c}}-\frac{8\,(a^3bc^2+2\,a^2b^2c+ab^3)d^2+\frac{3\,(4\,abc^2-b^2c)(adx^2+ac+b)^2d^2}{(dx^2+c)^2}-\frac{5\,(4\,a^2bc^2+3\,ab^2c-b^3)(adx^2+ac+b)d^2}{dx^2+c}}{8\left((a^3c^5+3\,a^2bc^4+3\,ab^2c^3+b^3c^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}-2\,(a^4c^5+4\,a^3bc^4+6\,a^2b^2c^3+4\,ab^3c^2+b^4c)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}+(a^5c^5+5\,a^4bc^4+10\,a^3b^2c^3+10\,a^2b^3c^2+5\,ab^4c+b^5)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -3/16*(4*a*b*c - b^2)*d^2*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sq
rt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*
c)))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*sqrt((a*c + b)*c)) - 1/8*(8
*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2 + a
*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x^2 +
a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*
((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*
b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + (a
^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c + b^5)*s
qrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(192) = 384$.

time = 0.77, size = 961, normalized size = 4.53

$$\left[\frac{3\,((4\,a^2bc-ab^2)d^3x^6+(4\,a^2bc^2+3\,ab^2c-b^3)d^2x^4)\sqrt{ac^2+bc}\log\left(\frac{(8\,a^2c^2+8\,abc+b^2)d^2x^4+8\,a^2c^4+16\,abc^3+8\,b^2c^2+8\,(2\,a^2c^3+3\,abc^2+b^2c)dx^2+4\,((2\,ac+b)d^2x^4+2\,ac^3+(4\,ac^2+3\,bc)dx^2+2\,bc^2)\sqrt{ac^2+bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{x^4}\right)+4\,((2\,a^3c^3-11\,a^2bc^2-13\,ab^2c)d^3x^6-2\,a^3c^6-6\,a^2bc^5-6\,ab^2c^4+(2\,a^3c^4-16\,a^2bc^3-23\,ab^2c^2-5\,b^3c)d^2x^4-2\,b^3c^3-(2\,a^3c^5+11\,a^2bc^4+16\,ab^2c^3+7\,b^3c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{32\,((a^5c^5+4\,a^4bc^4+6\,a^3b^2c^3+4\,a^2b^3c^2+ab^4c)dx^6+(a^5c^6+5\,a^4bc^5+10\,a^3b^2c^4+10\,a^2b^3c^3+5\,ab^4c^2+b^5c)x^4)},-\frac{3\,((4\,a^2bc-ab^2)d^3x^6+(4\,a^2bc^2+3\,ab^2c-b^3)d^2x^4)\sqrt{-ac^2-bc}\arctan\left(\frac{((2\,ac+b)dx^2+2\,ac^2+2\,bc)\sqrt{-ac^2-bc}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\,(a^2c^3+2\,abc^2+(a^2c^2+abc)dx^2+b^2c)}\right)-2\,((2\,a^3c^3-11\,a^2bc^2-13\,ab^2c)d^3x^6-2\,a^3c^6-6\,a^2bc^5-6\,ab^2c^4+(2\,a^3c^4-16\,a^2bc^3-23\,ab^2c^2-5\,b^3c)d^2x^4-2\,b^3c^3-(2\,a^3c^5+11\,a^2bc^4+16\,ab^2c^3+7\,b^3c^2)dx^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{16\,((a^5c^5+4\,a^4bc^4+6\,a^3b^2c^3+4\,a^2b^3c^2+ab^4c)dx^6+(a^5c^6+5\,a^4bc^5+10\,a^3b^2c^4+10\,a^2b^3c^3+5\,ab^4c^2+b^5c)x^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2
*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^
4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((
2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^
2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a
^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2
*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*
a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a
*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c
^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*
c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4
*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*
c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)
/(d*x^2 + c))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*
((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5
- 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4
 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)
*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2
*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2
*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] undef
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(a + \frac{b}{d x^2+c}\right)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)
[Out] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)
```

3.361 $\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=482

$$-\frac{x^3(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{5a^3d^2\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{6x^3(b+ac+adx^2)}{5a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(16b^2+16abc+a^2c^2)\,x(b+ac}{5a^4d^2\,(c+dx^2)\,\sqrt{\frac{b+ac+}{c+dx}}}$$

```
[Out] -x^3*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/5*(a*c+8*b)*x*(a*d*x
^2+a*c+b)/a^3/d^2/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+6/5*x^3*(a*d*x^2+a*c+b)
/a^2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*(a^2*c^2+16*a*b*c+16*b^2)*x*(a
*d*x^2+a*c+b)/a^4/d^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/5*c^(3/
2)*(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^3/d^(5/2)/(d*x
^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c
))^(1/2)-1/5*(a^2*c^2+16*a*b*c+16*b^2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b
))^(1/2))*c^(1/2)/a^4/d^(5/2)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(
c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1985, 1986, 478, 595, 596, 545, 429, 506, 422}

$$\frac{c^{3/2}(ac+8b)\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^3d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{x(ac+8b)\left(ac+adx^2+b\right)}{5a^3d^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{6x^3\left(ac+adx^2+b\right)}{5a^2d\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{\sqrt{c}\left(a^2c^2+16abc+16b^2\right)\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{5a^4d^{5/2}\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x\left(a^2c^2+16abc+16b^2\right)\left(ac+adx^2+b\right)}{5a^4d^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{x^3\left(c+dx^2\right)}{ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x^3*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) - ((8*b +
a*c)*x*(b + a*c + a*d*x^2))/(5*a^3*d^2*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]) + (6*x^3*(b + a*c + a*d*x^2))/(5*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x
^2)]) + ((16*b^2 + 16*a*b*c + a^2*c^2)*x*(b + a*c + a*d*x^2))/(5*a^4*d^2*(c
 + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(16*b^2 + 16*a*
b*c + a^2*c^2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(5*a^4*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (c^(3/2)*(8*b +
 a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c
)])/(5*a^3*d^(5/2)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(
c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 595

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*((c + d*x^n)^q/(b*g*(m + n*(p + q + 1) + 1))), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
```

```
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 596

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(b*d*(m + n*(p + q + 1) +
 1))), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^4}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^4\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\sqrt{c+dx^2}\,(3c+6dx^2)}{\sqrt{b+ac+adx^2}}\,d}{ad\sqrt{c+dx^2}\,\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} + \frac{6x^3\sqrt{b+ac+adx^2}\,\sqrt{b+a\left(c+dx^2\right)}}{5a^2d\sqrt{a+\frac{b}{c+dx^2}}} +$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\,\sqrt{b+a\left(c+dx\right.}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\,\sqrt{b+a\left(c+dx\right.}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\,\sqrt{b+a\left(c+dx\right.}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x^3(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\,\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b+ac)x\sqrt{b+ac+adx^2}\,\sqrt{b+a\left(c+dx\right.}}{5a^3d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.58, size = 296, normalized size = 0.61

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\,x(c+dx^2)\left(8b^2+ab(9c+2dx^2)+a^2(c^2-d^2x^4)\right)+ic(16b^2+16abc+a^2c^2)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)-ibc(8b+7ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)\right)}{5a^3d^2\sqrt{\frac{ad}{b+ac}}\,(b+a(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -1/5*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d
*x^2)*(8*b^2 + a*b*(9*c + 2*d*x^2) + a^2*(c^2 - d^2*x^4)) + I*c*(16*b^2 + 1
6*a*b*c + a^2*c^2)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*
EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)] - I*b*c*(8*b + 7
*a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^3*d^2*Sqrt[(a*d)/(b + a*
c)]*(b + a*(c + d*x^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(518) = 1036$.
time = 0.11, size = 1159, normalized size = 2.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*d^3*x^7-(
-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^2*x^5+2*(-a*d
/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^2*x^5+(-a*d/(a*c+b)
)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d*x^3+5*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c*d*x^3+6*(-a*d/(a*
c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c*d*x^3-((a*d*x^2+a*c+b)/
(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b
)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3+5*(a*d^2*x^4+2*a*c*
d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*d*x^3+(-a*d/(a*c+b)
)^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^3*x+3*(-a*d/(a*c+b))^(1/2)*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*d*x^3+7*((a*d*x^2+a*c+b)/(a*c+b))^(1/
2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2)
)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2-16*((a*d*x^2+a*c+b)/(a*c+b))^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2
))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*a*b*c^2*x+4*(-a*d/(a*c+b))^(1/2)*(
(d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^2*x+8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2
)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))
*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c-16*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*
((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c+5*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b^2*c*x+3*(-a*d/(a*c+b))^(1/2)*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/d^2/a^
3/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d
*x^2+a*c+b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**4/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^4/(a + b/(c + d*x^2))^(3/2), x)
```

3.362 $\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=409

$$-\frac{x(c+dx^2)}{ad\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{4x(b+ac+adx^2)}{3a^2d\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(8b+ac)x(b+ac+adx^2)}{3a^3d\,(c+dx^2)\,\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{c}\,(8b+ac)\,(b+ac+a}{3a^3d^{3/2}\,(c+dx^2)\,\sqrt{\frac{b+a}{c}}}$$

```
[Out] -x*(d*x^2+c)/a/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+4/3*x*(a*d*x^2+a*c+b)/a^
2/d/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(a*c+8*b)*x*(a*d*x^2+a*c+b)/a^3/d
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*c^(3/2)*(a*c+4*b)*(a*d*x^2
+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a^2/(a*c+b)/d^(3/2)/(d*x^2+c)/((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*
(a*c+8*b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE
(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^3/d^(3/2)
/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*
x^2+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 478, 542, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,(ac+8b)\,(ac+adx^2+b)\,E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^3d^{3/2}\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}-\frac{x(ac+8b)\,(ac+adx^2+b)}{3a^3d\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{c^{3/2}(ac+4b)\,(ac+adx^2+b)\,F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3a^2d^{3/2}(ac+b)\,(c+dx^2)\,\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\,\sqrt{\frac{c\,(ac+adx^2+b)}{(ac+b)\,(c+dx^2)}}}+\frac{4x(ac+adx^2+b)}{3a^2d\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{x(c+dx^2)}{ad\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] -((x*(c + d*x^2))/(a*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + (4*x*(b +
a*c + a*d*x^2))/(3*a^2*d*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - ((8*b + a
*c)*x*(b + a*c + a*d*x^2))/(3*a^3*d*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c
 + d*x^2)]) + (Sqrt[c]*(8*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], b/(b + a*c)])/(3*a^3*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c +
 a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))
]) - (c^(3/2)*(4*b + a*c)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/
Sqrt[c]], b/(b + a*c)])/(3*a^2*(b + a*c)*d^(3/2)*(c + d*x^2)*Sqrt[(b + a*c
+ a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2)
)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*n*(p + 1))), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{x^2\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\sqrt{c+dx^2}\,(c+4dx^2)}{\sqrt{b+ac+adx^2}}\,dx}{ad\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{x(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b}}{}$$

$$= -\frac{x(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\left((8b\right.}{}$$

$$= -\frac{x(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b}{}$$

$$= -\frac{x(c+dx^2)\sqrt{b+a\left(c+dx^2\right)}}{ad\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{4x\sqrt{b+ac+adx^2}\sqrt{b+a\left(c+dx^2\right)}}{3a^2d\sqrt{a+\frac{b}{c+dx^2}}} - \frac{(8b}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.39, size = 255, normalized size = 0.62

$$\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}x(c+dx^2)(4b+ac+adx^2)+ic(8b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-4ibc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{3a^2d\sqrt{\frac{ad}{b+ac}}\,(b+a(c+dx^2))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b/(c + d*x^2))^(3/2),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*x*(c + d*x^2)
*(4*b + a*c + a*d*x^2) + I*c*(8*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(
a*c)] - (4*I)*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(3*a^2*d*Sqrt[(
a*d)/(b + a*c)]*(b + a*(c + d*x^2)))
```

Maple [A]
time = 0.08, size = 667, normalized size = 1.63

method	result
default	$\frac{\left(\sqrt{-\frac{ad}{ac+b}}\sqrt{(d\,x^2+c)\,(ad\,x^2+ac+b)}\,a\,d^2x^5+2\sqrt{-\frac{ad}{ac+b}}\sqrt{(d\,x^2+c)\,(ad\,x^2+ac+b)}\,acd\,x^3+3\sqrt{}\right.}{}$
risch	$\frac{x(ad\,x^2+ac+b)}{3a^2d\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}-\frac{\left(-\frac{2d(ac+5b)(c^2a+bc)\sqrt{1+\frac{ad\,x^2}{ac+b}}\sqrt{1+\frac{d\,x^2}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+bd}{dca}}\right)-\right.}{\sqrt{-\frac{ad}{ac+b}}\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*d^2*x^5+2*(-a
*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c*d*x^3+3*(a*d^2*x^4+
2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*x^3+(-a*d/(a*
c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*d*x^3-((a*d*x^2+a*c+b)/(a*c
+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/
c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2+(-a*d/(a*c+b))^(1/2)*((d*
x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2*x+4*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*
```

```
x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c-8*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2
+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+
c)*(a*d*x^2+a*c+b))^(1/2)*b*c+3*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(
1/2)*(-a*d/(a*c+b))^(1/2)*b*c*x+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*
c+b))^(1/2)*b*c*x)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/a^2/d/(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/(a*d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**2/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a + b/(d*x^2 + c))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a+\frac{b}{d\,x^2+c}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b/(c + d*x^2))^(3/2),x)
[Out] int(x^2/(a + b/(c + d*x^2))^(3/2), x)
```

3.363 $\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=356

$$-\frac{bx}{a(b+ac)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(2b+ac)x(b+ac+adx^2)}{a^2(b+ac)\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{\sqrt{c}\,(2b+ac)\left(b+ac+adx^2\right)E\Big(}{a^2(b+ac)\sqrt{d}\,\left(c+dx^2\right)\sqrt{\frac{b+ac+a}{c+dx}}}$$

```
[Out] -b*x/a/(a*c+b)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(a*c+2*b)*x*(a*d*x^2+a*c+b
)/a^2/(a*c+b)/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+
a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))/a/(a*c+b)/(d*x^2+c)/d^(1/2)/((a*d*x^2+
a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)-(a*c+2*
b)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)/a^2/(a*c+b)/(d*x^2
+c)/d^(1/2)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d
*x^2+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {1985, 1986, 424, 545, 429, 506, 422}

$$-\frac{\sqrt{c}\,(ac+2b)\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a^2\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{x(ac+2b)\left(ac+adx^2+b\right)}{a^2(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{c^{3/2}\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a\sqrt{d}\,(ac+b)\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{bx}{a(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((b*x)/(a*(b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((2*b + a*c)
*x*(b + a*c + a*d*x^2))/(a^2*(b + a*c)*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)
/(c + d*x^2)]) - (Sqrt[c]*(2*b + a*c)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a^2*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt
[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(
c + d*x^2))]) + (c^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/S
qrt[c]], b/(b + a*c)])/(a*(b + a*c)*Sqrt[d]*(c + d*x^2)*Sqrt[(b + a*c + a*d
*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(b+ac)d+(2b+a}{\sqrt{c+dx^2}\sqrt{b+a}}}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a+\frac{}{c+}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a\left(c+dx^2\right)}\right)\int \frac{}{\sqrt{c+dx^2}\sqrt{b}}}{a\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+d}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx}}}$$

$$= -\frac{bx\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(2b+ac)x\sqrt{b+ac+adx^2}\sqrt{b+a(c}}{a^2(b+ac)\left(c+dx^2\right)\sqrt{a+\frac{b}{c+dx}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.34, size = 241, normalized size = 0.68

$$-\frac{\sqrt{\frac{ad}{b+ac}}\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(b\sqrt{\frac{ad}{b+ac}}x(c+dx^2)+ic(2b+ac)\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)-ibc\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{a^2d\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(-3/2),x]
```

```
[Out] -((Sqrt[(a*d)/(b + a*c)]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*Sqrt[(a*d
)/(b + a*c)]*x*(c + d*x^2) + I*c*(2*b + a*c)*Sqrt[(b + a*c + a*d*x^2)/(b +
a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 +
 b/(a*c)] - I*b*c*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*E
llipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/(a^2*d*(b + a*(
c + d*x^2))))
```

Maple [A]
time = 0.04, size = 466, normalized size = 1.31

method	result
default	$-\frac{\left(\sqrt{a\,d^{2}x^{4}+2acd\,x^{2}+bd\,x^{2}+c^{2}a+bc}\,\sqrt{-\frac{ad}{ac+b}}\,bd\,x^{3}-\sqrt{\frac{ad\,x^{2}+ac+b}{ac+b}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\mathrm{EllipticE}\left(x\sqrt{-\frac{ad}{ac+b}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d*
x^3-((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(
a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2+
((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+
b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c-2*((a*
d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^
(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c+(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*x)/a*((a*d*x
^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/
(-a*d/(a*c+b))^(1/2)/(a*c+b)/(a*d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral((a + b/(c + d*x**2))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/(d*x^2 + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(1/(a + b/(c + d*x^2))^(3/2), x)
```

3.364 $\int \frac{1}{x^2\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=410

$$-\frac{b}{a(b+ac)x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(b-ac)\left(b+ac+adx^2\right)}{a(b+ac)^2x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(b-ac)dx(b+ac+adx^2)}{a(b+ac)^2\left(c+dx^2\right)\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{\sqrt{}}{a(b}$$

```
[Out] -b/a/(a*c+b)/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)*(a*d*x^2+a*c+b)/a
/(a*c+b)^2/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-(-a*c+b)*d*x*(a*d*x^2+a*c+b)
/a/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+c^(3/2)*(a*d*x^2+a
*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*d^(1/2)/(a*c+b)^2/(d*x^2+c)/((a*d*x^2+a
*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)+(-a*c+b)
*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1/2)*d^(1/2)/a/(a*c+b)^2/
(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x
^2+c))^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$,
Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\frac{c^{3/2}\sqrt{d}\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{\sqrt{c}\sqrt{d}(b-ac)\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{a(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{(b-ac)\left(ac+adx^2+b\right)}{ax(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{dx(b-ac)\left(ac+adx^2+b\right)}{a(ac+b)^2\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{b}{ax(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((b - a*c)*(b
+ a*c + a*d*x^2))/(a*(b + a*c)^2*x*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) -
 ((b - a*c)*d*x*(b + a*c + a*d*x^2))/(a*(b + a*c)^2*(c + d*x^2)*Sqrt[(b + a
*c + a*d*x^2)/(c + d*x^2)]) + (Sqrt[c]*(b - a*c)*Sqrt[d]*(b + a*c + a*d*x^2
)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/(a*(b + a*c)^2*(c +
d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/(
(b + a*c)*(c + d*x^2))]) + (c^(3/2)*Sqrt[d]*(b + a*c + a*d*x^2)*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], b/(b + a*c)])/((b + a*c)^2*(c + d*x^2)*Sqrt[(b
+ a*c + a*d*x^2)/(c + d*x^2)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c +
d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
```

```
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+\frac{b}{c+dx^2}\right)^{3/2}} \, dx = \frac{\sqrt{b+a\left(c+dx^2\right)} \int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+a\left(c+dx^2\right)\right)^{3/2}} dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)} \int \frac{\left(c+dx^2\right)^{3/2}}{x^2\left(b+ac+adx^2\right)^{3/2}} dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)} \int \frac{c(b-ac)}{x^2\sqrt{c+dx^2}}}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(b-ac)\sqrt{b+ac+adx^2}\sqrt{b+a}}{a(b+ac)^2x\sqrt{a+\frac{b}{c+dx^2}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.45, size = 268, normalized size = 0.65

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\left(c+dx^2\right)\left(b\left(c-dx^2\right)+ac\left(c+dx^2\right)\right)+ic(-b+ac)dx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)+2ibcdx\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}x\right)|1+\frac{b}{ac}\right)\right)}{(b+ac)^2\sqrt{\frac{ad}{b+ac}}x\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -((Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x^2)
*(b*(c - d*x^2) + a*c*(c + d*x^2)) + I*c*(-b + a*c)*d*x*Sqrt[(b + a*c + a*d
```

```
*x^2)/(b + a*c)]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*
c)]*x], 1 + b/(a*c)] + (2*I)*b*c*d*x*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sq
rt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)
]))/((b + a*c)^2*Sqrt[(a*d)/(b + a*c)]*x*(b + a*(c + d*x^2))))
```

Maple [A]
time = 0.07, size = 686, normalized size = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{ad}{ac+b}}\,\sqrt{(d\,x^2+c)\,(ad\,x^2+ac+b)}\,acd^2x^4+\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a+bc}\,\sqrt{-\frac{ad}{ac+b}}\right.}{}$
risch	$-\frac{c(ad\,x^2+ac+b)}{(ac+b)^2x\sqrt{\frac{ad\,x^2+ac+b}{d\,x^2+c}}}+\frac{d\left(-\frac{2a^2cd\left(c^2a+bc\right)\sqrt{1+\frac{ad\,x^2}{ac+b}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{ad}{ac+b}},\sqrt{-1+\frac{2acd+ba}{dca}}\right.\right.}{\sqrt{-\frac{ad}{ac+b}}\,\sqrt{a\,d^2x^4+2acd\,x^2+bd\,x^2+c^2a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c*d^2*x^4+(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*d^2*x^4+a*
c^2*d*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d
/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*x*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)-2*
(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^2*d*x^2+2*((a*d*
x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*d/(a*c+b))^(1
/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c*d*x-((a*d*x^
2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a*c+b))^(1/2
),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c*d*x+(a*d^2*x^4
+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(-a*d/(a*c+b))^(1/2)*b*c*d*x^2-(-a*d/
(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b*c*d*x^2-(-a*d/(a*c+b))^(
1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*c^3-(-a*d/(a*c+b))^(1/2)*((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)*b*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2*x^
4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/x/(a*c+b)^2/(a*
d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b/(c + d*x^2))^(3/2)), x)
```

3.365 $\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=490

$$-\frac{b}{a(b+ac)x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(3b-ac)\left(b+ac+adx^2\right)}{3a(b+ac)^2x^3\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}-\frac{(7b-ac)d(b+ac+adx^2)}{3(b+ac)^3x\sqrt{\frac{b+ac+adx^2}{c+dx^2}}}+\frac{(7b}{3(b+ac}$$

```
[Out] -b/a/(a*c+b)/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*(a*d*x^2+
a*c+b)/a/(a*c+b)^2/x^3/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)-1/3*(-a*c+7*b)*d*(
a*d*x^2+a*c+b)/(a*c+b)^3/x/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+7*b)
*d^2*x*(a*d*x^2+a*c+b)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2
)-1/3*(-a*c+7*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(b/(a*c+b))^(1/2))*c^(1
/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(c*(a*d*x^2+a*c+b
)/(a*c+b)/(d*x^2+c))^(1/2)+1/3*(-a*c+3*b)*d^(3/2)*(a*d*x^2+a*c+b)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(b/(a*c+b))^(1/2))*c^(1/2)/(a*c+b)^3/(d*x^2+c)/((a*d*x^2+a*c+b)/(d*x^2+c)
)^(1/2)/(c*(a*d*x^2+a*c+b)/(a*c+b)/(d*x^2+c))^(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 490, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1985, 1986, 479, 597, 545, 429, 506, 422}

$$\frac{\sqrt{c}\,d^{3/2}(3b-ac)\left(ac+adx^2+b\right)F\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}-\frac{\sqrt{c}\,d^{3/2}(7b-ac)\left(ac+adx^2+b\right)E\left(\text{ArcTan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)|\frac{b}{b+ac}\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}\sqrt{\frac{c\left(ac+adx^2+b\right)}{(ac+b)\left(c+dx^2\right)}}}+\frac{d^2x(7b-ac)\left(ac+adx^2+b\right)}{3(ac+b)^3\left(c+dx^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{d(7b-ac)\left(ac+adx^2+b\right)}{3x(ac+b)^3\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}+\frac{(3b-ac)\left(ac+adx^2+b\right)}{3ax^3(ac+b)^2\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}-\frac{b}{ax^3(ac+b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -(b/(a*(b + a*c)*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])) + ((3*b - a*c)
*(b + a*c + a*d*x^2))/(3*a*(b + a*c)^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(c + d*
x^2)]) - ((7*b - a*c)*d*(b + a*c + a*d*x^2))/(3*(b + a*c)^3*x*Sqrt[(b + a*c
 + a*d*x^2)/(c + d*x^2)]) + ((7*b - a*c)*d^2*x*(b + a*c + a*d*x^2))/(3*(b +
 a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]) - (Sqrt[c]*(7*b
- a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b
/(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2
)]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))]) + (Sqrt[c]*(3*b -
 a*c)*d^(3/2)*(b + a*c + a*d*x^2)*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], b/
(b + a*c)])/(3*(b + a*c)^3*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)
]*Sqrt[(c*(b + a*c + a*d*x^2))/((b + a*c)*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(-(c*b - a*d))*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)
*((c + d*x^n)^(q - 1)/(a*b*e*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[
(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*
b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n
, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q,
x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
```

```
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1985

```
Int[(u_.)*((a_) + (b_.)/((c_) + (d_.)*(x_)^(n_)))^(p_), x_Symbol] :> Int[u*
((b + a*c + a*d*x^n)/(c + d*x^n))^p, x] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rule 1986

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Dist[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))], Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*r
), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+\frac{b}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+a\left(c+dx^2\right)\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{\left(c+dx^2\right)^{3/2}}{x^4\left(b+ac+adx^2\right)^{3/2}}\,dx}{\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} - \frac{\sqrt{b+a\left(c+dx^2\right)}\int \frac{c(3b-ac)\cdots}{x^4\sqrt{c+dx^2}\cdots}}{a(b+ac)d\sqrt{c+dx^2}\sqrt{a\cdots}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+\cdots}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+d\cdots}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+\cdots}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+d\cdots}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+\cdots}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+d\cdots}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+\cdots}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+d\cdots}}}$$

$$= -\frac{b\sqrt{b+a\left(c+dx^2\right)}}{a(b+ac)x^3\sqrt{b+ac+adx^2}\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(3b-ac)\sqrt{b+ac+adx^2}\sqrt{b+\cdots}}{3a(b+ac)^2x^3\sqrt{a+\frac{b}{c+d\cdots}}}$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 10.58, size = 319, normalized size = 0.65

$$-\frac{\sqrt{\frac{b+ac+adx^2}{c+dx^2}}\left(\sqrt{\frac{ad}{b+ac}}\left(c+dx^2\right)\left(b^2(c+4dx^2)+a^2c(c^2-d^2x^4)+ab(2c^2+4cdx^2+7d^2x^4)\right)-iac(-7b+ac)d^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)+ib(3b-5ac)d^2x^3\sqrt{\frac{b+ac+adx^2}{b+ac}}\sqrt{1+\frac{dx^2}{c}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{ad}{b+ac}}\,x\right)|1+\frac{b}{ac}\right)\right)}{3(b+ac)^3\sqrt{\frac{ad}{b+ac}}\,x^3\left(b+a\left(c+dx^2\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] -1/3*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(Sqrt[(a*d)/(b + a*c)]*(c + d*x
^2)*(b^2*(c + 4*d*x^2) + a^2*c*(c^2 - d^2*x^4) + a*b*(2*c^2 + 4*c*d*x^2 + 7
*d^2*x^4)) - I*a*c*(-7*b + a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]
*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a
*c)] + I*b*(3*b - 5*a*c)*d^2*x^3*Sqrt[(b + a*c + a*d*x^2)/(b + a*c)]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[(a*d)/(b + a*c)]*x], 1 + b/(a*c)]))/
((b + a*c)^3*Sqrt[(a*d)/(b + a*c)]*x^3*(b + a*(c + d*x^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(526) = 1052$.
time = 0.08, size = 1080, normalized size = 2.20 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c*d^3*x^6
+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*a*b
*d^3*x^6+4*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*d^3*x
^6+((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-a*d/(a
*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2
*d^2*x^3-(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^2*d^2
*x^4+5*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-a*
d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b
*c*d^2*x^3-7*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)*a*b*c*d^2*x^3+3*(-a*d/(a*c+b))^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^
2+b*c)^(1/2)*a*b*c*d^2*x^4+8*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b
))^(1/2)*a*b*c*d^2*x^4-3*((a*d*x^2+a*c+b)/(a*c+b))^(1/2)*((d*x^2+c)/c)^(1/2
)*EllipticF(x*(-a*d/(a*c+b))^(1/2),((a*c+b)/a/c)^(1/2))*((d*x^2+c)*(a*d*x^2
+a*c+b))^(1/2)*b^2*d^2*x^3+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))
^(1/2)*a^2*c^3*d*x^2+4*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/
2)*b^2*d^2*x^4+6*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b
*c^2*d*x^2+(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a^2*c^4+5
*(-a*d/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c*d*x^2+2*(-a*d
/(a*c+b))^(1/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*a*b*c^3+(-a*d/(a*c+b))^(1
/2)*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*b^2*c^2)*((a*d*x^2+a*c+b)/(d*x^2+c))^
(1/2)/(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)/(-a*d/(a*c+b))^(1/2)/
x^3/(a*c+b)^3/(a*d*x^2+a*c+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a + b/(d*x^2 + c))^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(a+\frac{b}{d\,x^2+c}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b/(c + d*x^2))^(3/2)), x)
```

3.366 $\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=75

$$-\frac{3\sqrt{ax^{23}}\,\sqrt{1+x^5}}{20x^9}+\frac{\sqrt{ax^{23}}\,\sqrt{1+x^5}}{10x^4}+\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}$$

```
[Out] 3/20*arcsinh(x^(5/2))*(a*x^23)^(1/2)/x^(23/2)-3/20*(a*x^23)^(1/2)*(x^5+1)^(
1/2)/x^9+1/10*(a*x^23)^(1/2)*(x^5+1)^(1/2)/x^4
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 327, 335, 281, 221}

$$\frac{3\sqrt{ax^{23}}\,\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}-\frac{3\sqrt{x^5+1}\,\sqrt{ax^{23}}}{20x^9}+\frac{\sqrt{x^5+1}\,\sqrt{ax^{23}}}{10x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10
*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{23}}\int \frac{x^{23/2}}{\sqrt{1+x^5}}\,dx}{x^{23/2}}\\
&= \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} - \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{8x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{4x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{\left(3\sqrt{ax^{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{20x^{23/2}}\\
&= -\frac{3\sqrt{ax^{23}}\sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}}\sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}}\sinh^{-1}\left(x^{5/2}\right)}{20x^{23/2}}
\end{aligned}$$

Mathematica [A]
time = 0.94, size = 59, normalized size = 0.79

$$\frac{\sqrt{ax^{23}}\left(x^{5/2}\sqrt{1+x^5}\,(-3+2x^5)+3\tanh^{-1}\left(\frac{x^{5/2}}{\sqrt{1+x^5}}\right)\right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcTanh[x^(5/2)/Sqrt[
1 + x^5]]))/(20*x^(23/2))
```

Maple [A]
time = 0.23, size = 48, normalized size = 0.64

method	result	size
meijerg	$\frac{\sqrt{a\,x^{23}}\,\left(-\frac{\sqrt{\pi}\,x^{\frac{5}{2}}\left(-10x^{5}+15\right)\sqrt{x^{5}+1}}{20}+\frac{3\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{4}\right)}{5x^{\frac{23}{2}}\sqrt{\pi}}$	48
risch	$\frac{(2x^{5}-3)\sqrt{x^{5}+1}\,\sqrt{a\,x^{23}}}{20x^{9}}+\frac{3\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{23}}\,\sqrt{ax\,(x^{5}+1)}}{20\sqrt{a}\,x^{12}\sqrt{x^{5}+1}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*x^23)^(1/2)/x^(23/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(5/2)*(-10*x^5+15)*(
x^5+1)^(1/2)+3/4*Pi^(1/2)*arcsinh(x^(5/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Fricas [A]
time = 0.42, size = 169, normalized size = 2.25

$$\left[\frac{3\,\sqrt{a}\,x^{9}\log\left(-\frac{8\,ax^{19}+8\,ax^{14}+ax^{9}+4\,\sqrt{ax^{23}}\,(2\,x^{5}+1)\sqrt{x^{5}+1}\,\sqrt{a}}{x^{9}}\right)+4\,\sqrt{ax^{23}}\,(2\,x^{5}-3)\sqrt{x^{5}+1}}{80\,x^{9}},-\frac{3\,\sqrt{-a}\,x^{9}\arctan\left(\frac{\sqrt{ax^{23}}\,(2\,x^{5}+1)\sqrt{x^{5}+1}\,\sqrt{-a}}{2\,(ax^{19}+ax^{14})}\right)-2\,\sqrt{ax^{23}}\,(2\,x^{5}-3)\sqrt{x^{5}+1}}{40\,x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23)*(2*
x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5
+ 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x
^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 +
 1))/x^9]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^{23}}}{\sqrt{(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^{23}}}{\sqrt{x^5+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.367 $\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

```
[Out] -1/5*arcsinh(x^(5/2))*(a*x^13)^(1/2)/x^(13/2)+1/5*(a*x^13)^(1/2)*(x^5+1)^(1
/2)/x^4
```

Rubi [A]

time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {15, 327, 335, 281, 221}

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4}-\frac{\sqrt{ax^{13}}\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x
^(13/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^{13}}\int \frac{x^{13/2}}{\sqrt{1+x^5}}\,dx}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{2x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{13/2}}\\
&= \frac{\sqrt{ax^{13}}\sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.15, size = 52, normalized size = 1.04

$$\frac{\sqrt{ax^{13}}\left(x^{5/2}\sqrt{1+x^5} - \tanh^{-1}\left(\frac{x^{5/2}}{\sqrt{1+x^5}}\right)\right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5],x]
```

```
[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - ArcTanh[x^(5/2)/Sqrt[1 + x^5]]))/(5*
x^(13/2))
```

Maple [**A**]
time = 0.22, size = 40, normalized size = 0.80

method	result	size
meijerg	$\frac{\sqrt{a\,x^{13}}\,\left(\sqrt{\pi}\,x^{\frac{5}{2}}\sqrt{x^5+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\right)}{5x^{\frac{13}{2}}\sqrt{\pi}}$	40
risch	$\frac{\sqrt{a\,x^{13}}\,\sqrt{x^5+1}}{5x^4}-\frac{\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^{13}}\,\sqrt{ax\,(x^5+1)}}{5\sqrt{a}\,x^7\sqrt{x^5+1}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(a*x^13)^(1/2)/x^(13/2)/Pi^(1/2)*(Pi^(1/2)*x^(5/2)*(x^5+1)^(1/2)-Pi^(1/
2)*arcsinh(x^(5/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^13)/sqrt(x^5 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.
time = 0.43, size = 153, normalized size = 3.06

$$\left[\frac{\sqrt{a}\,x^4\log\left(-\frac{8\,ax^{14}+8\,ax^9+ax^4-4\,\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+1}\,\sqrt{a}}{x^4}\right)+4\,\sqrt{ax^{13}}\,\sqrt{x^5+1}}{20\,x^4},\ \frac{\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{ax^{13}}\,(2\,x^5+1)\sqrt{x^5+1}\,\sqrt{-a}}{2\,(ax^{14}+ax^9)}\right)+2\,\sqrt{ax^{13}}\,\sqrt{x^5+1}}{10\,x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5
 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10
*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(
a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^{13}}}{\sqrt{(x+1)\,(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [A]
time = 4.53, size = 68, normalized size = 1.36

$$\frac{a^{\frac{11}{2}} \log \left(-\sqrt{ax}\, a^{\frac{5}{2}} x^2 + \sqrt{a^6 x^5 + a^6}\right)}{5\, |a|^5} + \frac{\sqrt{a^6 x^5 + a^6}\, \sqrt{ax}\, x^2}{5\, a^2 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*a^(11/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))/abs(a)^5 + 1
/5*sqrt(a^6*x^5 + a^6)*sqrt(a*x)*x^2/(a^2*abs(a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\, x^{13}}}{\sqrt{x^5 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.368 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

```
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Rubi [A]

time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 335, 281, 221}

$$\frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^5}}\,dx}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}\\
&= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^{5/2}\right)}{5x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\,\sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.13, size = 34, normalized size = 1.42

$$\frac{2\sqrt{ax^3}\,\tanh^{-1}\left(\frac{x^{5/2}}{\sqrt{1+x^5}}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5],x]
[Out] (2*Sqrt[a*x^3]*ArcTanh[x^(5/2)/Sqrt[1 + x^5]])/(5*x^(3/2))
```

Maple [**A**]
time = 0.20, size = 17, normalized size = 0.71

method	result	size
meijerg	$\frac{2\,\text{arcsinh}\left(x^{\frac{5}{2}}\right)\sqrt{a\,x^3}}{5x^{\frac{3}{2}}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(16) = 32$.
time = 0.41, size = 98, normalized size = 4.08

$$\left[\frac{1}{10}\sqrt{a}\log\left(-8\,ax^{10}-8\,ax^{5}-4\,(2\,x^{6}+x)\sqrt{x^{5}+1}\sqrt{ax^{3}}\sqrt{a}-a\right),-\frac{1}{5}\sqrt{-a}\arctan\left(\frac{(2\,x^{5}+1)\sqrt{x^{5}+1}\sqrt{ax^{3}}\sqrt{-a}}{2\,(ax^{9}+ax^{4})}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*
x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(
a*x^3)*sqrt(-a)/(a*x^9 + a*x^4))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^{3}}}{\sqrt{(x+1)\,(x^{4}-x^{3}+x^{2}-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(16) = 32$.
time = 3.67, size = 58, normalized size = 2.42

$$-\frac{2\,a^{\frac{3}{2}}\log\left(-\sqrt{ax}\,a^{\frac{5}{2}}x^{2}+\sqrt{a^{6}x^{5}+a^{6}}\right)\operatorname{sgn}(x)}{5\,|a|}+\frac{2\,a^{\frac{3}{2}}\log\left(a^{2}|a|\right)\operatorname{sgn}(x)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(a
) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{a\,x^{3}}}{\sqrt{x^{5}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)
```

3.369 $\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{5}\sqrt{\frac{a}{x^7}}\,x\sqrt{1+x^5}$$

```
[Out] -2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 270}

$$-\frac{2}{5}x\sqrt{x^5+1}\,\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}}\,dx &= \left(\sqrt{\frac{a}{x^7}}\,x^{7/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{5}\sqrt{\frac{a}{x^7}}\,x\sqrt{1+x^5}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 23, normalized size = 1.00

$$-\frac{2}{5}\sqrt{\frac{a}{x^7}}\,x\sqrt{1+x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5
```

Maple [A]
time = 0.20, size = 18, normalized size = 0.78

method	result	size
meijerg	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
risch	$-\frac{2x\sqrt{\frac{a}{x^7}}\,\sqrt{x^5+1}}{5}$	18
gosper	$-\frac{2x(1+x)(x^4-x^3+x^2-x+1)\sqrt{\frac{a}{x^7}}}{5\sqrt{x^5+1}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*x*(a/x^7)^(1/2)*(x^5+1)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(17) = 34.
time = 0.64, size = 41, normalized size = 1.78

$$-\frac{2\left(\sqrt{a}\,x^6+\sqrt{a}\,x\right)}{5\sqrt{x^4-x^3+x^2-x+1}\,\sqrt{x+1}\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*(sqrt(a)*x^6 + sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x
^(7/2))
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.74

$$-\frac{2}{5}\sqrt{x^5+1}\,x\sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(x^5 + 1)*x*sqrt(a/x^7)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [A]
time = 3.86, size = 28, normalized size = 1.22

$$-\frac{2\,a^4\left(\frac{\sqrt{a+\frac{a}{x^5}}}{a^3}-\frac{1}{a^{\frac{5}{2}}}\right)}{5\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*a^4*(sqrt(a + a/x^5)/a^3 - 1/a^(5/2))/abs(a)
```

Mupad [B]
time = 2.69, size = 17, normalized size = 0.74

$$-\frac{2\,x\,\sqrt{x^5+1}\,\sqrt{\frac{a}{x^7}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)
```

```
[Out] -(2*x*(x^5 + 1)^(1/2)*(a/x^7)^(1/2))/5
```

3.370 $\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx$

Optimal. Leaf size=49

$$-\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}\,x^6\sqrt{1+x^5}$$

```
[Out] -2/15*x*(a/x^17)^(1/2)*(x^5+1)^(1/2)+4/15*x^6*(a/x^17)^(1/2)*(x^5+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 277, 270}

$$\frac{4}{15}x^6\sqrt{x^5+1}\,\sqrt{\frac{a}{x^{17}}}-\frac{2}{15}x\sqrt{x^5+1}\,\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/1
5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}}\,dx &= \left(\sqrt{\frac{a}{x^{17}}}\,x^{17/2}\right)\int \frac{1}{x^{17/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5}-\frac{1}{3}\left(2\sqrt{\frac{a}{x^{17}}}\,x^{17/2}\right)\int \frac{1}{x^{7/2}\sqrt{1+x^5}}\,dx\\ &= -\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5}+\frac{4}{15}\sqrt{\frac{a}{x^{17}}}\,x^6\sqrt{1+x^5}\end{aligned}$$

Mathematica [A]
time = 0.83, size = 30, normalized size = 0.61

$$\frac{2}{15}\sqrt{\frac{a}{x^{17}}}\,x\sqrt{1+x^5}\left(-1+2x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5],x]
```

```
[Out] (2*Sqrt[a/x^17]*x*Sqrt[1 + x^5]*(-1 + 2*x^5))/15
```

Maple [A]
time = 0.20, size = 25, normalized size = 0.51

method	result	size
meijerg	$-\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(-2x^5+1\right)\sqrt{x^5+1}}{15}$	25
risch	$\frac{2\sqrt{\frac{a}{x^{17}}}\,x\left(2x^{10}+x^5-1\right)}{15\sqrt{x^5+1}}$	28
gosper	$\frac{2x(1+x)\left(x^4-x^3+x^2-x+1\right)\left(2x^5-1\right)\sqrt{\frac{a}{x^{17}}}}{15\sqrt{x^5+1}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15*(a/x^17)^(1/2)*x*(-2*x^5+1)*(x^5+1)^(1/2)
```

Maxima [A]
time = 0.67, size = 50, normalized size = 1.02

$$\frac{2\left(2\sqrt{a}\,x^{11}+\sqrt{a}\,x^6-\sqrt{a}\,x\right)}{15\sqrt{x^4-x^3+x^2-x+1}\,\sqrt{x+1}\,x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x +
 1)*sqrt(x + 1)*x^(17/2))
```

Fricas [A]
time = 0.33, size = 25, normalized size = 0.51

$$\frac{2}{15}\left(2x^{6}-x\right)\sqrt{x^{5}+1}\,\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)\left(x^{4}-x^{3}+x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 2.67, size = 29, normalized size = 0.59

$$\frac{\sqrt{\frac{a}{x^{17}}}\,\left(\frac{4\,x^{11}}{15}+\frac{2\,x^{6}}{15}-\frac{2\,x}{15}\right)}{\sqrt{x^{5}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)
[Out] ((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)
```

3.371 $\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {15, 304, 209, 212}

$$\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\text{ArcTan}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] -1/2*(Sqrt[a*x^6]*ArcTan[x])/x^3 + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^6}}{x(1-x^4)}\,dx &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3} - \frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\ &= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\,(2\tan^{-1}(x)+\log(1-x)-\log(1+x))}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

Maple [A]
time = 0.20, size = 28, normalized size = 0.76

method	result	size
default	$-\frac{\sqrt{a\,x^6}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^3}$	28
meijerg	$-\frac{\sqrt{a\,x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	44
risch	$-\frac{i\sqrt{a\,x^6}\,\ln(x+i)}{4x^3}+\frac{i\sqrt{a\,x^6}\,\ln(x-i)}{4x^3}+\frac{\sqrt{a\,x^6}\,\ln(1+x)}{4x^3}-\frac{\sqrt{a\,x^6}\,\ln(-1+x)}{4x^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\,\arctan(x)+\frac{1}{4}\sqrt{a}\,\log(x+1)-\frac{1}{4}\sqrt{a}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A]
time = 6.82, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x)-\log(|x+1|)\operatorname{sgn}(x)+\log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$-\int\frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)
```

```
[Out] -int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)
```

3.372 $\int \frac{\sqrt{ax^6}}{x-x^5} \, dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

```
[Out] -1/2*arctan(x)*(a*x^6)^(1/2)/x^3+1/2*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 1598, 304, 209, 212}

$$\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\text{ArcTan}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] -1/2*(Sqrt[a*x^6]*ArcTan[x])/x^3 + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^6}}{x-x^5}\,dx &= \frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\ &= \frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}-\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\ &= -\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6}\left(2\tan^{-1}(x)+\log(1-x)-\log(1+x)\right)}{4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^6]/(x - x^5),x]
[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3
```

Maple [A]
time = 0.20, size = 28, normalized size = 0.76

method	result	size
default	$-\frac{\sqrt{a\,x^6}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^3}$	28
meijerg	$-\frac{\sqrt{a\,x^6}\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$	44
risch	$-\frac{i\sqrt{a\,x^6}\,\ln(x+i)}{4x^3}+\frac{i\sqrt{a\,x^6}\,\ln(x-i)}{4x^3}+\frac{\sqrt{a\,x^6}\,\ln(1+x)}{4x^3}-\frac{\sqrt{a\,x^6}\,\ln(-1+x)}{4x^3}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3
```

Maxima [A]
time = 0.86, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\,\arctan(x)+\frac{1}{4}\sqrt{a}\,\log(x+1)-\frac{1}{4}\sqrt{a}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6}\,\left(2\arctan(x)-\log\left(\frac{x+1}{x-1}\right)\right)}{4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{ax^6}}{x^5-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(1/2)/(-x**5+x),x)
[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A]
time = 4.34, size = 29, normalized size = 0.78

$$-\frac{1}{4}\left(2\arctan(x)\operatorname{sgn}(x)-\log(|x+1|)\operatorname{sgn}(x)+\log(|x-1|)\operatorname{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))
*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x - x^5),x)
[Out] int((a*x^6)^(1/2)/(x - x^5), x)
```

3.373 $\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx$

Optimal. Leaf size=71

$$-\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] -a*(a*x^6)^(1/2)/x^2-1/5*a*x^2*(a*x^6)^(1/2)+1/2*a*arctan(x)*(a*x^6)^(1/2)/
x^3+1/2*a*arctanh(x)*(a*x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {15, 308, 218, 212, 209}

$$\frac{a\sqrt{ax^6}\,\text{ArcTan}(x)}{2x^3}+\frac{a\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}-\frac{1}{5}ax^2\sqrt{ax^6}-\frac{a\sqrt{ax^6}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/
(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax^6)^{3/2}}{x(1-x^4)}\,dx &= \frac{\left(a\sqrt{ax^6}\right)\int \frac{x^8}{1-x^4}\,dx}{x^3}\\
&= \frac{\left(a\sqrt{ax^6}\right)\int \left(-1-x^4+\frac{1}{1-x^4}\right)\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^4}\,dx}{x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\left(a\sqrt{ax^6}\right)\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= -\frac{a\sqrt{ax^6}}{x^2}-\frac{1}{5}ax^2\sqrt{ax^6}+\frac{a\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}+\frac{a\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.62

$$
-\frac{a\sqrt{ax^6}\,\left(20x+4x^5-10\tan^{-1}(x)+5\log(1-x)-5\log(1+x)\right)}{20x^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]
```

```
[Out] -1/20*(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*Log[1
+ x]))/x^3
```

Maple [A]
time = 0.21, size = 38, normalized size = 0.54

method	result

default	$-\frac{(a\,x^{6})^{\frac{3}{2}}\left(4x^{5}+5\ln(-1+x)-5\ln(1+x)-10\arctan(x)+20x\right)}{20x^{9}}$
meijerg	$-\frac{(a\,x^{6})^{\frac{3}{2}}(-1)^{\frac{3}{4}}\left(-\frac{4x(-1)^{\frac{1}{4}}(9x^{4}+45)}{45}-\frac{x(-1)^{\frac{1}{4}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{(x^{4})^{\frac{1}{4}}}\right)}{4x^{9}}$
risch	$-\frac{a\,x^{2}\sqrt{a\,x^{6}}}{5}-\frac{a\sqrt{a\,x^{6}}}{x^{2}}-\frac{a\sqrt{a\,x^{6}}\,\ln(-1+x)}{4x^{3}}-\frac{ia\sqrt{a\,x^{6}}\,\ln(x-i)}{4x^{3}}+\frac{ia\sqrt{a\,x^{6}}\,\ln(x+i)}{4x^{3}}+\frac{a\sqrt{a\,x^{6}}\,\ln(1+x)}{4x^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(3/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(a*x^6)^(3/2)*(4*x^5+5*ln(-1+x)-5*ln(1+x)-10*arctan(x)+20*x)/x^9
```

Maxima [A]
time = 0.60, size = 40, normalized size = 0.56

$$-\frac{1}{5}\,a^{\frac{3}{2}}x^{5}-a^{\frac{3}{2}}x+\frac{1}{2}\,a^{\frac{3}{2}}\arctan\left(x\right)+\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x+1\right)-\frac{1}{4}\,a^{\frac{3}{2}}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*a^(3/2)*x^5 - a^(3/2)*x + 1/2*a^(3/2)*arctan(x) + 1/4*a^(3/2)*log(x +
1) - 1/4*a^(3/2)*log(x - 1)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 0.58

$$-\frac{\sqrt{ax^{6}}\left(4\,ax^{5}+20\,ax-10\,a\arctan\left(x\right)-5\,a\log\left(\frac{x+1}{x-1}\right)\right)}{20\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x -
 1)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{(ax^{6})^{\frac{3}{2}}}{x^{5}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**6)**(3/2)/x/(-x**4+1),x)
```

[Out] -Integral((a*x**6)**(3/2)/(x**5 - x), x)

Giac [A]
time = 4.09, size = 42, normalized size = 0.59

$$-\frac{1}{20}\left(4\,x^5\mathrm{sgn}(x)+20\,x\mathrm{sgn}(x)-10\,\arctan\left(x\right)\mathrm{sgn}(x)-5\,\log\left(|x+1|\right)\mathrm{sgn}(x)+5\,\log\left(|x-1|\right)\mathrm{sgn}(x)\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(4*x^5*sgn(x) + 20*x*sgn(x) - 10*arctan(x)*sgn(x) - 5*log(abs(x + 1))
*sgn(x) + 5*log(abs(x - 1))*sgn(x))*a^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{\left(a\,x^6\right)^{3/2}}{x\,\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(a*x^6)^(3/2)/(x*(x^4 - 1)),x)
```

```
[Out] -int((a*x^6)^(3/2)/(x*(x^4 - 1)), x)
```

3.374 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x(1-x^4)}\right)\,dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\,\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}=0.152$, Rules used = {218, 212, 209, 15, 304}

$$\frac{\sqrt{ax^6}\,\text{ArcTan}(x)}{2x^3}-\frac{\sqrt{ax^6}\,\tanh^{-1}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x\,(1-x^4)}\right)dx&=\int\frac{1}{1-x^4}\,dx-\int\frac{\sqrt{ax^6}}{x\,(1-x^4)}\,dx\\&=\frac{1}{2}\int\frac{1}{1-x^2}\,dx+\frac{1}{2}\int\frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int\frac{x^2}{1-x^4}\,dx}{x^3}\\&=\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int\frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int\frac{1}{1+x^2}\,dx}{2x^3}\\&=\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 42, normalized size = 0.86

$$\frac{\left(x^3+\sqrt{ax^6}\right)\tan^{-1}(x)+\left(x^3-\sqrt{ax^6}\right)\tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)),x]
[Out] ((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Maple [A]
time = 0.23, size = 37, normalized size = 0.76

method	result
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}+\frac{\sqrt{a\,x^6}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^3}$
meijerg	$-\frac{x\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)-2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{1}{4}}}+\frac{\sqrt{a\,x^6}\,\left(\ln\left(1-(x^4)^{\frac{1}{4}}\right)-\ln\left(1+(x^4)^{\frac{1}{4}}\right)+2\arctan\left((x^4)^{\frac{1}{4}}\right)\right)}{4(x^4)^{\frac{3}{4}}}$

risch	$\frac{i\ln(x+i)x^3-i\ln(x-i)x^3-\ln(-1+x)x^3+\ln(1+x)x^3+i\sqrt{a\,x^6}\,\ln(x+i)-i\sqrt{a\,x^6}\,\ln(x-i)+\sqrt{a\,x^6}\,\ln(-1+x)-\sqrt{a\,x^6}\,\ln(1+}{4x^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x
))/x^3
```

Maxima [A]
time = 0.66, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan\left(x\right)-\frac{1}{4}\sqrt{a}\log\left(x+1\right)+\frac{1}{4}\sqrt{a}\log\left(x-1\right)+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(x+1\right)-\frac{1}{4}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.
time = 0.39, size = 256, normalized size = 5.22

$$\left[\frac{x^3\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{ax^6}\right)\sqrt{-\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{x^4+x^2}\right)+x^3\log\left(x+1\right)-x^3\log\left(x-1\right)-\sqrt{ax^6}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)}{4\,x^3},\ \frac{2\,x^3\sqrt{\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}\arctan\left(-\frac{\left(x^3-\sqrt{ax^6}\right)\sqrt{\frac{(a+1)x^3+2\sqrt{ax^6}}{x^3}}}{(a-1)x^2}\right)+x^3\log\left(x+1\right)-x^3\log\left(x-1\right)-\sqrt{ax^6}\left(\log\left(x+1\right)-\log\left(x-1\right)\right)}{4\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x^5-x}\,dx-\int\left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)
```

```
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A]
time = 3.35, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(|x+1|\right)\operatorname{sgn}(x)+\log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^6}}{x\,\left(x^4-1\right)}-\frac{1}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)
```

```
[Out] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)
```

3.375 $\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right) dx$

Optimal. Leaf size=49

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(a*x^6)^(1/2)/x^3-1/2*arctanh(x)
*(a*x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {218, 212, 209, 15, 1598, 304}

$$\frac{\sqrt{ax^6}\,\text{ArcTan}(x)}{2x^3}-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]
*ArcTanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{1-x^4}-\frac{\sqrt{ax^6}}{x-x^5}\right)dx &= \int \frac{1}{1-x^4}\,dx-\int \frac{\sqrt{ax^6}}{x-x^5}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{\sqrt{ax^6}\int \frac{x^3}{x-x^5}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int \frac{x^2}{1-x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{ax^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{ax^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{ax^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 0.86

$$\frac{\left(x^3+\sqrt{ax^6}\right)\tan^{-1}(x)+\left(x^3-\sqrt{ax^6}\right)\tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5),x]
[Out] ((x^3 + Sqrt[a*x^6])*ArcTan[x] + (x^3 - Sqrt[a*x^6])*ArcTanh[x])/(2*x^3)
```

Maple [A]
time = 0.22, size = 37, normalized size = 0.76

method	result
default	$\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}+\frac{\sqrt{a\,x^{6}}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^{3}}$
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{a\,x^{6}}\,\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$
risch	$\frac{i\ln(x+i)x^{3}-i\ln(x-i)x^{3}-\ln(-1+x)x^{3}+\ln(1+x)x^{3}+i\sqrt{a\,x^{6}}\,\ln(x+i)-i\sqrt{a\,x^{6}}\,\ln(x-i)+\sqrt{a\,x^{6}}\,\ln(-1+x)-\sqrt{a\,x^{6}}\,\ln(1+}{4x^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/4*(a*x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x
))/x^3
```

Maxima [A]
time = 0.66, size = 42, normalized size = 0.86

$$\frac{1}{2}\sqrt{a}\arctan(x)-\frac{1}{4}\sqrt{a}\log(x+1)+\frac{1}{4}\sqrt{a}\log(x-1)+\frac{1}{2}\arctan(x)+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(37) = 74.
time = 0.34, size = 256, normalized size = 5.22

$$\left[\frac{x^{3}\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}\log\left(\frac{(a-1)x^{4}-(a-1)x^{2}-2\left(x^{3}-\sqrt{ax^{6}}\right)\sqrt{-\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}}{x^{4}+x^{2}}\right)+x^{3}\log(x+1)-x^{3}\log(x-1)-\sqrt{ax^{6}}\left(\log(x+1)-\log(x-1)\right)}{4\,x^{3}}, \frac{2\,x^{3}\sqrt{\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}\arctan\left(-\frac{\left(x^{3}-\sqrt{ax^{6}}\right)\sqrt{\frac{(a+1)x^{3}+2\sqrt{ax^{6}}}{x^{3}}}}{(a-1)x^{2}}\right)+x^{3}\log(x+1)-x^{3}\log(x-1)-\sqrt{ax^{6}}\left(\log(x+1)-\log(x-1)\right)}{4\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="fricas")
```

```
[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*log(((a - 1)*x^4 - (a -
1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3))/(x
^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - lo
g(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)*arctan(-
(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6))/x^3)/((a - 1)*x^2))
```

```
+ x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/
x^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x}\,dx-\int \left(-\frac{\sqrt{ax^6}}{x^5-x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)
[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)
```

Giac [A]
time = 3.57, size = 48, normalized size = 0.98

$$\frac{1}{4}\left(2\arctan\left(x\right)\operatorname{sgn}(x)-\log\left(|x+1|\right)\operatorname{sgn}(x)+\log\left(|x-1|\right)\operatorname{sgn}(x)\right)\sqrt{a}+\frac{1}{2}\arctan\left(x\right)+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")
[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{1}{x^4-1}-\frac{\sqrt{a\,x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)
[Out] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)
```

3.376 $\int \frac{\sqrt{ax^3}}{x-x^3}\,dx$

Optimal. Leaf size=44

$$-\frac{\sqrt{ax^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] -arctan(x^(1/2))*(a*x^3)^(1/2)/x^(3/2)+arctanh(x^(1/2))*(a*x^3)^(1/2)/x^(3/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {15, 1598, 335, 304, 209, 212}

$$\frac{\sqrt{ax^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\,\text{ArcTan}\left(\sqrt{x}\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x
^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^3}}{x-x^3}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{x-x^3}\,dx}{x^{3/2}}\\ &= \frac{\sqrt{ax^3}\int \frac{\sqrt{x}}{1-x^2}\,dx}{x^{3/2}}\\ &= \frac{\left(2\sqrt{ax^3}\right)\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\ &= \frac{\sqrt{ax^3}\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}-\frac{\sqrt{ax^3}\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)}{x^{3/2}}\\ &= -\frac{\sqrt{ax^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\frac{\sqrt{ax^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.68

$$\frac{\sqrt{ax^3}\left(-\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\right)}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/(x - x^3),x]
```

```
[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)
```

Maple [A]
time = 0.20, size = 44, normalized size = 1.00

method	result	size
default	$-\frac{\sqrt{a\,x^{3}}\,\sqrt{a}\,\left(\arctan\left(\frac{\sqrt{a x}}{\sqrt{a}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{a x}}{\sqrt{a}}\right)\right)}{x\sqrt{a x}}$	44
meijerg	$-\frac{\sqrt{a\,x^{3}}\,\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] -(a*x^3)^(1/2)*a^(1/2)*(arctan((a*x)^(1/2)/a^(1/2))-arctanh((a*x)^(1/2)/a^(
1/2)))/x/(a*x)^(1/2)
```

Maxima [A]
time = 0.63, size = 32, normalized size = 0.73

$$-\sqrt{a}\arctan\left(\sqrt{x}\right)+\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}+1\right)-\frac{1}{2}\sqrt{a}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(s
qrt(x) - 1)
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.
time = 0.34, size = 127, normalized size = 2.89

$$\left[-\sqrt{a}\arctan\left(\frac{\sqrt{ax^{3}}}{\sqrt{a}\,x}\right)+\frac{1}{2}\sqrt{a}\log\left(\frac{ax^{2}+ax+2\sqrt{ax^{3}}\sqrt{a}}{x^{2}-x}\right), -\sqrt{-a}\arctan\left(\frac{\sqrt{ax^{3}}\sqrt{-a}}{ax}\right)+\frac{1}{2}\sqrt{-a}\log\left(\frac{ax^{2}-ax-2\sqrt{ax^{3}}\sqrt{-a}}{x^{2}+x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="fricas")
```

```
[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x +
2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a
*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{ax^{3}}}{x^{3}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)
```

Giac [A]
time = 3.74, size = 43, normalized size = 0.98

$$-\frac{\left(\frac{a^2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)\right)\operatorname{sgn}(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")
```

```
[Out] -(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(a
)))*sgn(x)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^3}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x - x^3),x)
```

```
[Out] int((a*x^3)^(1/2)/(x - x^3), x)
```

3.377 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=44

$$\frac{\sqrt{ax^4}\,\sqrt{1+x^2}}{2x}-\frac{\sqrt{ax^4}\,\sinh^{-1}(x)}{2x^2}$$

```
[Out] -1/2*arcsinh(x)*(a*x^4)^(1/2)/x^2+1/2*(a*x^4)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 327, 221}

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^4}}{2x}-\frac{\sqrt{ax^4}\,\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}}\,dx = \frac{\sqrt{ax^4}\int \frac{x^2}{\sqrt{1+x^2}}\,dx}{x^2}$$
$$= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\int \frac{1}{\sqrt{1+x^2}}\,dx}{2x^2}$$
$$= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.95

$$\frac{\sqrt{ax^4}\left(x\sqrt{1+x^2}-\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]
[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcTanh[x/Sqrt[1 + x^2]]))/(2*x^2)
```

Maple [A]
time = 0.20, size = 27, normalized size = 0.61

method	result	size
default	$\frac{\sqrt{a\,x^4}\,\left(x\sqrt{x^2+1}-\operatorname{arcsinh}(x)\right)}{2x^2}$	27
meijerg	$\frac{\sqrt{a\,x^4}\,\left(\sqrt{\pi}\,x\sqrt{x^2+1}-\sqrt{\pi}\,\operatorname{arcsinh}(x)\right)}{2x^2\sqrt{\pi}}$	36
risch	$\frac{\sqrt{a\,x^4}\,\sqrt{x^2+1}}{2x}-\frac{\ln\left(x\sqrt{a}+\sqrt{a\,x^2+a}\right)\sqrt{a\,x^4}\,\sqrt{(x^2+1)\,a}}{2\sqrt{a}\,x^2\sqrt{x^2+1}}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 42, normalized size = 0.95

$$\frac{\sqrt{ax^4}\,\sqrt{x^2+1}\,x+\sqrt{ax^4}\,\log\left(-x+\sqrt{x^2+1}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

Giac [A]
time = 3.62, size = 27, normalized size = 0.61

$$\frac{1}{2}\left(\sqrt{x^2+1}\,x+\log\left(-x+\sqrt{x^2+1}\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{a\,x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.378 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{ax^3}\sqrt{1+x^2}}{3x}-\frac{\sqrt{ax^3}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}$$

```
[Out] 2/3*(a*x^3)^(1/2)*(x^2+1)^(1/2)/x-1/3*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2
)/cos(2*arctan(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a*x
^3)^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/x^(3/2)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {15, 327, 335, 226}

$$\frac{2\sqrt{x^2+1}\sqrt{ax^3}}{3x}-\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\sqrt{ax^3}\,F\left(2\text{ArcTan}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{3x^{3/2}\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*Sqrt[1 + x^2])/(3*x) - (Sqrt[a*x^3]*(1 + x)*Sqrt[(1 + x^2)/(
1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/(3*x^(3/2)*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^2}}\,dx}{x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\,\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\int \frac{1}{\sqrt{x}\,\sqrt{1+x^2}}\,dx}{3x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\,\sqrt{1+x^2}}{3x} - \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)}{3x^{3/2}}\\
&= \frac{2\sqrt{ax^3}\,\sqrt{1+x^2}}{3x} - \frac{\sqrt{ax^3}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)\middle|\frac{1}{2}\right)}{3x^{3/2}\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 43, normalized size = 0.52

$$\frac{2\sqrt{ax^3}\left(\sqrt{1+x^2} - {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x^3]*(Sqrt[1 + x^2] - Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]))/(3
*x)
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 76, normalized size = 0.92

method	result
meijerg	$\frac{2\sqrt{a\,x^{3}}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-x^{2}\right)}{5}$
default	$-\frac{\sqrt{a\,x^{3}}\left(i\sqrt{-i\left(x+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-x+i\right)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i\left(x+i\right)},\frac{\sqrt{2}}{2}\right)-2x^{3}-2x\right)}{3x^{2}\sqrt{x^{2}+1}}$
risch	$\frac{2\sqrt{a\,x^{3}}\,\sqrt{x^{2}+1}}{3x}-\frac{i\sqrt{-i\left(x+i\right)}\,\sqrt{2}\,\sqrt{i\left(x-i\right)}\,\sqrt{ix}\,\mathrm{EllipticF}\left(\sqrt{-i\left(x+i\right)},\frac{\sqrt{2}}{2}\right)\sqrt{a\,x^{3}}\,\sqrt{ax\,(}}{3\sqrt{a\,x^{3}+ax}\,x^{2}\sqrt{x^{2}+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(a*x^3)^(1/2)/x^2/(x^2+1)^(1/2)*(I*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I)
)^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))-2*x^3-2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 31, normalized size = 0.37

$$-\frac{2\left(\sqrt{a}\,x\mathrm{weierstrassPInverse}(-4,0,x)-\sqrt{ax^{3}}\,\sqrt{x^{2}+1}\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(a)*x*weierstrassPInverse(-4, 0, x) - sqrt(a*x^3)*sqrt(x^2 + 1))/
x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax^{3}}}{\sqrt{x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^3}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a*x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.379 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{ax^2}\,\sqrt{1+x^2}}{x}$$

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 267}

$$\frac{\sqrt{x^2+1}\,\sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}}\,dx &= \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}\\ &= \frac{\sqrt{ax^2}\,\sqrt{1+x^2}}{x}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{ax^2}\,\sqrt{1+x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x
```

Maple [A]
time = 0.19, size = 19, normalized size = 0.86

method	result	size
gosper	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
default	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
risch	$\frac{\sqrt{a\,x^{2}}\,\sqrt{x^{2}+1}}{x}$	19
meijerg	$\frac{\sqrt{a\,x^{2}}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{2}+1}\right)}{2x\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Maxima [A]
time = 0.49, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\,x^2+\sqrt{a}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)
```

Fricas [A]
time = 0.38, size = 18, normalized size = 0.82

$$\frac{\sqrt{ax^2}\,\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x
```

Sympy [A]
time = 0.15, size = 17, normalized size = 0.77

$$\frac{\sqrt{ax^2}\,\sqrt{x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a*x**2)*sqrt(x**2 + 1)/x
```

Giac [A]
time = 3.33, size = 19, normalized size = 0.86

$$\left(\sqrt{x^2+1}\,\mathrm{sgn}(x) - \mathrm{sgn}(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)
```

Mupad [B]
time = 2.67, size = 19, normalized size = 0.86

$$\frac{\sqrt{a}\,\sqrt{x^2+1}\,\sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] (a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x
```

3.380 $\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=131

$$\frac{2\sqrt{ax}\,\sqrt{1+x^2}}{1+x}-\frac{2\sqrt{a}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}+\frac{\sqrt{a}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\right.\right.}{\sqrt{1+x^2}}$$

```
[Out] 2*(a*x)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)
))^2)^(1/2)/cos(2*arctan((a*x)^(1/2)/a^(1/2)))*EllipticE(sin(2*arctan((a*x)
^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
+(1+x)*(cos(2*arctan((a*x)^(1/2)/a^(1/2)))^2)^(1/2)/cos(2*arctan((a*x)^(1/2
)/a^(1/2)))*EllipticF(sin(2*arctan((a*x)^(1/2)/a^(1/2))),1/2*2^(1/2))*a^(1/
2)*((x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {335, 311, 226, 1210}

$$\frac{\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2\sqrt{a}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{2\sqrt{x^2+1}\,\sqrt{ax}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*Sqrt[a*x]*Sqrt[1 + x^2])/(1 + x) - (2*Sqrt[a]*(1 + x)*Sqrt[(1 + x^2)/(1
+ x)^2]*EllipticE[2*ArcTan[Sqrt[a*x]/Sqrt[a]], 1/2])/Sqrt[1 + x^2] + (Sqrt[
a]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[a*x]/Sqrt[a]],
 1/2])/Sqrt[1 + x^2]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax}}{\sqrt{1+x^2}}\,dx &= \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)}{a}\\ &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right) - 2\text{Subst}\left(\int \frac{1-\frac{x^2}{a}}{\sqrt{1+\frac{x^4}{a^2}}}\,dx, x, \sqrt{ax}\right)\\ &= \frac{2\sqrt{ax}\,\sqrt{1+x^2}}{1+x} - \frac{2\sqrt{a}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}} + \frac{\sqrt{a}\,(1+x)\sqrt{\frac{1}{(1}}}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 27, normalized size = 0.21

$$\frac{2}{3}x\sqrt{ax}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^2],x]
```

```
[Out] (2*x*Sqrt[a*x]*Hypergeometric2F1[1/2, 3/4, 7/4, -x^2])/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 81, normalized size = 0.62

method	result
meijerg	$\frac{2\sqrt{ax}\, x\, \mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-x^{2}\right)}{3}$
default	$\frac{\sqrt{ax}\,\sqrt{-i\left(x+i\right)}\,\sqrt{2}\,\sqrt{-i\left(-x+i\right)}\,\sqrt{ix}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{-i\left(x+i\right)},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{-i\left(x+i\right.}\right.\right.}{\sqrt{x^{2}+1}\, x}$
elliptic	$\frac{i\sqrt{ax}\,\sqrt{ax\left(x^{2}+1\right)}\,\sqrt{-i\left(x+i\right)}\,\sqrt{2}\,\sqrt{i\left(x-i\right)}\,\sqrt{ix}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(x+i\right)},\frac{\sqrt{2}}{2}\right)+i\,\mathrm{Elliptic}\right.}{\sqrt{x^{2}+1}\, x\sqrt{a\,x^{3}+ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^
(1/2)*(2*EllipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-EllipticF((-I*(x+I))^(1/2)
,1/2*2^(1/2)))/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 12, normalized size = 0.09

$$-2\,\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 36, normalized size = 0.27

$$\frac{\sqrt{a}\,x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|x^{2}e^{i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(a)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(I*pi))
/(2*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a*x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.381 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{\frac{a}{x}}\,\sqrt{x}\,(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}$$

```
[Out] (1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*EllipticF(sin
(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x)^(1/2)*x^(1/2)*((x^2+1)/(1+x)^2)^(1/2
)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 335, 226}

$$\frac{\sqrt{x}\,(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,\sqrt{\frac{a}{x}}\,F\left(2\text{ArcTan}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[a/x]*Sqrt[x]*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqr
t[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^2}}\,dx\\
&= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{\frac{a}{x}}\sqrt{x}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,F\left(2\tan^{-1}\left(\sqrt{x}\right)\mid\frac{1}{2}\right)}{\sqrt{1+x^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 27, normalized size = 0.50

$$2\sqrt{\frac{a}{x}}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^2],x]
```

```
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -x^2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 62, normalized size = 1.15

method	result	size
meijerg	$2\sqrt{\frac{a}{x}}\,x\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-x^2\right)$	22
default	$\frac{i\sqrt{\frac{a}{x}}\sqrt{-i(x+i)}\sqrt{2}\sqrt{-i(-x+i)}\sqrt{ix}\,\text{EllipticF}\left(\sqrt{-i(x+i)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x^2+1}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] I*(a/x)^(1/2)/(x^2+1)^(1/2)*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x
)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 9, normalized size = 0.17

$$2\sqrt{a}\ \text{weierstrassPInverse}(-4,0,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a)*weierstrassPInverse(-4, 0, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(sqrt(a/x)/sqrt(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x)/sqrt(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.382 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=22

$$-\sqrt{\frac{a}{x^2}}\, x \tanh^{-1}\left(\sqrt{1+x^2}\right)$$

```
[Out] -x*arctanh((x^2+1)^(1/2))*(a/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 272, 65, 213}

$$x\left(-\sqrt{\frac{a}{x^2}}\right)\tanh^{-1}\left(\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,x^2\right)\\ &= \left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+x^2}\right)\\ &= -\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$-\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])
```

Maple [A]
time = 0.20, size = 19, normalized size = 0.86

method	result	size
default	$-\sqrt{\frac{a}{x^2}}\,x\,\text{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\,x\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)+(-2\ln(2)+2\ln(x))\sqrt{\pi}\right)}{2\sqrt{\pi}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^2 + 1), x)
```

Fricas [**A**] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(18) = 36$.
time = 0.33, size = 76, normalized size = 3.45

$$\left[x\sqrt{\frac{a}{x^2}}\log\left(\frac{\sqrt{x^2+1}-1}{x}\right), 2\sqrt{-a}\arctan\left(-\frac{\sqrt{-a}\,x^2\sqrt{\frac{a}{x^2}}-\sqrt{x^2+1}\,\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2
*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt(x**2 + 1), x)
```

Giac [**A**]
time = 3.67, size = 30, normalized size = 1.36

$$-\frac{1}{2}\sqrt{a}\left(\log\left(\sqrt{x^2+1}+1\right)-\log\left(\sqrt{x^2+1}-1\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((a/x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.383 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=159

$$-2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2}+\frac{2\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^2}}{1+x}-\frac{2\sqrt{\frac{a}{x^3}}\,x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{1+x^2}}+\frac{\sqrt{\frac{a}{x^3}}\,x^{3/2}\cdots}{\cdots}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^2+1)^(1/2)+2*x^2*(a/x^3)^(1/2)*(x^2+1)^(1/2)/(1+x)-2*
x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arctan(x^(1/2)))*Ellip
ticE(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((x^2+1)/(1+x)^2)^(1
/2)/(x^2+1)^(1/2)+x^(3/2)*(1+x)*(cos(2*arctan(x^(1/2)))^2)^(1/2)/cos(2*arct
an(x^(1/2)))*EllipticF(sin(2*arctan(x^(1/2))),1/2*2^(1/2))*(a/x^3)^(1/2)*((
x^2+1)/(1+x)^2)^(1/2)/(x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {15, 331, 335, 311, 226, 1210}

$$\frac{(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,F\left(2\text{ArcTan}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}-\frac{2(x+1)\sqrt{\frac{x^2+1}{(x+1)^2}}\,x^{3/2}\sqrt{\frac{a}{x^3}}\,E\left(2\text{ArcTan}\left(\sqrt{x}\right)|\frac{1}{2}\right)}{\sqrt{x^2+1}}+\frac{2\sqrt{x^2+1}\,x^2\sqrt{\frac{a}{x^3}}}{x+1}-2\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^2] + (2*Sqrt[a/x^3]*x^2*Sqrt[1 + x^2])/(1 + x)
- (2*Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x^2)/(1 + x)^2]*EllipticE[2*ArcT
an[Sqrt[x]], 1/2])/Sqrt[1 + x^2] + (Sqrt[a/x^3]*x^(3/2)*(1 + x)*Sqrt[(1 + x
^2)/(1 + x)^2]*EllipticF[2*ArcTan[Sqrt[x]], 1/2])/Sqrt[1 + x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 311

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^2}}\,dx\\
&= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2}+\left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{\sqrt{x}}{\sqrt{1+x^2}}\,dx\\
&= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)\\
&= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2}+\left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^4}}\,dx,x,\sqrt{x}\right)-\left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\text{S}\\
&= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^2}+\frac{2\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^2}}{1+x}-\frac{2\sqrt{\frac{a}{x^3}}\,x^{3/2}(1+x)\sqrt{\frac{1+x^2}{(1+x)^2}}\,E\left(2\tan^{-1}\left(\sqrt{\right.\right.}}{\sqrt{1+x^2}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 27, normalized size = 0.17

$$-2\sqrt{\frac{a}{x^3}}\,x\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^2],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/4, 1/2, 3/4, -x^2]
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 116, normalized size = 0.73

method	result
meijerg	$-2\sqrt{\frac{a}{x^3}}\,x\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],-x^2\right)$
default	$\frac{\sqrt{\frac{a}{x^3}}\,x\left(2\sqrt{-i\,(x+i)}\,\sqrt{2}\,\sqrt{-i\,(-x+i)}\,\sqrt{ix}\,\mathrm{EllipticE}\left(\sqrt{-i\,(x+i)},\frac{\sqrt{2}}{2}\right)-\sqrt{-i\,(x+i)}\,\sqrt{2}\right.}{\sqrt{x^2+1}}$
risch	$-2x\sqrt{\frac{a}{x^3}}\,\sqrt{x^2+1}+\frac{i\sqrt{-i\,(x+i)}\,\sqrt{2}\,\sqrt{i\,(x-i)}\,\sqrt{ix}\,\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\,(x+i)},\frac{\sqrt{2}}{2}\right)+i\,\mathrm{Ellip}\right.}{\sqrt{a\,x^3+ax}\,\sqrt{x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a/x^3)^(1/2)*x*(2*(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))^(1/2)*(I*x)^(1/2)*E
llipticE((-I*(x+I))^(1/2),1/2*2^(1/2))-(-I*(x+I))^(1/2)*2^(1/2)*(-I*(-x+I))
^(1/2)*(I*x)^(1/2)*EllipticF((-I*(x+I))^(1/2),1/2*2^(1/2))-2*x^2-2)/(x^2+1)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 30, normalized size = 0.19

$$-2\sqrt{x^2+1}\,x\sqrt{\frac{a}{x^3}}-2\sqrt{a}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x^2 + 1)*x*sqrt(a/x^3) - 2*sqrt(a)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, x))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**3)/sqrt(x**2 + 1), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^3)/sqrt(x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((a/x^3)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.384 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$-\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^2}$$

```
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 270}

$$x\sqrt{x^2+1}\left(-\sqrt{\frac{a}{x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}}\,dx &= \left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1}{x^2\sqrt{1+x^2}}\,dx\\ &= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^2}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$-\sqrt{\frac{a}{x^4}}\, x\sqrt{1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2],x]
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])
```

Maple [A]
time = 0.24, size = 18, normalized size = 0.86

method	result	size
gosper	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
default	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
meijerg	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^2+1}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 23, normalized size = 1.10

$$-\frac{\sqrt{a}\, x^2+\sqrt{a}}{\sqrt{x^2+1}\, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)
```

Fricas [A]
time = 0.33, size = 30, normalized size = 1.43

$$-x^2\sqrt{\frac{a}{x^4}} - \sqrt{x^2+1}\, x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

Giac [A]
time = 3.55, size = 22, normalized size = 1.05

$$\frac{2\sqrt{a}}{\left(x-\sqrt{x^2+1}\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)
```

Mupad [B]
time = 2.87, size = 18, normalized size = 0.86

$$-\sqrt{a}\,x\,\sqrt{x^2+1}\,\sqrt{\frac{1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] -a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)
```

3.385 $\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{ax^4}\,\sqrt{1+x^3}}{3x^2}$$

[Out] 2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 267}

$$\frac{2\sqrt{x^3+1}\,\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3],x]

[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}} \, dx &= \frac{\sqrt{ax^4}\,\int \frac{x^2}{\sqrt{1+x^3}} \, dx}{x^2} \\
&= \frac{2\sqrt{ax^4}\,\sqrt{1+x^3}}{3x^2}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\sqrt{ax^4}\,\sqrt{1+x^3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3],x]
[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)
```

Maple [A]
time = 0.20, size = 20, normalized size = 0.80

method	result	size
default	$\frac{2\sqrt{a\,x^{4}}\,\sqrt{x^{3}+1}}{3x^{2}}$	20
risch	$\frac{2\sqrt{a\,x^{4}}\,\sqrt{x^{3}+1}}{3x^{2}}$	20
gosper	$\frac{2(1+x)(x^{2}-x+1)\sqrt{a\,x^{4}}}{3x^{2}\sqrt{x^{3}+1}}$	31
meijerg	$\frac{\sqrt{a\,x^{4}}\,\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{x^{3}+1}\right)}{3x^{2}\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(a*x^4)^(1/2)*(x^3+1)^(1/2)/x^2
```

Maxima [A]
time = 0.52, size = 28, normalized size = 1.12

$$\frac{2\left(\sqrt{a}\,x^{3}+\sqrt{a}\right)}{3\sqrt{x^{2}-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{2\sqrt{ax^{4}}\,\sqrt{x^{3}+1}}{3\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^4}}{\sqrt{(x+1)(x^2-x+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 3.27, size = 12, normalized size = 0.48

$$\frac{2}{3}\sqrt{x^3+1}\,\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)
```

Mupad [B]
time = 2.91, size = 20, normalized size = 0.80

$$\frac{2\sqrt{a}\,\sqrt{x^3+1}\,\sqrt{x^4}}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] (2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)
```

3.386 $\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=292

$$\frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\,\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)}-\frac{\sqrt[4]{3}\,\sqrt{ax^3}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\right.\right.}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] (1+3^(1/2))*(a*x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+x*(1+3^(1/2)))-3^(1/4)*(1+x)*(
(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^
(1/2)))*EllipticE((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/x/(x^
3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)-1/6*(1+x)*((1+x*(1-3^(1/2)))
^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF
((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1-3^(1/2))*(a*x^3)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/x/
(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{ax^3}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{ax^3}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{x^3+1}}+\frac{\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,\sqrt{ax^3}}{x\left(\left(1+\sqrt{3}\right)x+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] ((1 + Sqrt[3])*Sqrt[a*x^3]*Sqrt[1 + x^3])/(x*(1 + (1 + Sqrt[3])*x)) - (3^(1
/4)*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellipti
cE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(
x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])
*Sqrt[a*x^3]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(2*3
^(1/4)*x*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
```

```
&& !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^3}}\,dx = \frac{\sqrt{ax^3}\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx}{x^{3/2}}$$

$$= \frac{\left(2\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}}$$

$$= -\frac{\sqrt{ax^3}\,\text{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\left(\left(-1+\sqrt{3}\right)\sqrt{ax^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\right.}{x^{3/2}}$$

$$= \frac{\left(1+\sqrt{3}\right)\sqrt{ax^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} - \frac{\sqrt[4]{3}\sqrt{ax^3}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-1}\left(\frac{1+(}{1+(}\right.\right.}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 29, normalized size = 0.10

$$\frac{2}{5}x\sqrt{ax^3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^3],x]
```

```
[Out] (2*x*Sqrt[a*x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

Maple [C] Result contains complex when optimal does not.
time = 0.43, size = 1521, normalized size = 5.21

method	result	size
meijerg	$\frac{2\sqrt{a\,x^3}\,x\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{5}{6}\right],\left[\frac{11}{6}\right],-x^3\right)}{5}$	22
default	Expression too large to display	1521

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(a*x^3)^(1/2)/x*(x^3+1)^(1/2)*a*(I*3^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2)
)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2
```

```
*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(
1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/
2))*x^2+2*I*3^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)
+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))
^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2)
)*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+I*3^(1/2)*((I*3^(1/2
)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^
(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3
)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))
/(I*3^(1/2)+3))^(1/2))-2*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^
(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(
1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^
(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+3*((I*3^(1/2)
+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(
1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)
*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/
(I*3^(1/2)+3))^(1/2))*x^2-I*3^(1/2)*x^3-4*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1
+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1
)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x)
)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*
x+6*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^
(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE
(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/
(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+I*x^2*3^(1/2)-2*((I*3^(1/2)+3)*x/(1+
I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*
3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3
^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2
)+3))^(1/2))+3*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-
1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2
)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+
I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-I*3^(1/2)*x-3*x^3+3*x^2-3*x
)/(x*(x^3+1)*a)^(1/2)/(I*3^(1/2)+3)/(-a*x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2
)-2*x+1))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3)/sqrt(x^3 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x^3}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a*x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.387 $\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=260

$$\frac{2\sqrt{ax^2}\,\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{ax^2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}+x}{1+\sqrt{3}+x}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}+\cdots$$

```
[Out] 2*(a*x^2)^(1/2)*(x^3+1)^(1/2)/x/(1+x+3^(1/2))+2/3*(1+x)*EllipticF((1+x-3^(1
/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2)*(a*x^2)^(1/2)*((x^2-x+1)/(1+x+3^(
1/2))^2)^(1/2)*3^(3/4)/x/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)-3^(1/4
)*(1+x)*EllipticE((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*(a*x^2)^(1/2)*
(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)/x/(x^3+1)^(1/2)
/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 309, 224, 1891}

$$\frac{2\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{ax^2}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{ax^2}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{x\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}+\frac{2\sqrt{x^3+1}\,\sqrt{ax^2}}{x\left(x+\sqrt{3}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x^2]*Sqrt[1 + x^3])/(x*(1 + Sqrt[3] + x)) - (3^(1/4)*Sqrt[2 - Sqr
t[3]]*Sqrt[a*x^2]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticE
[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(x*Sqrt[(1 +
 x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3]) + (2*Sqrt[2]*Sqrt[a*x^2]*(1 + x)*Sq
rt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1
 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[(1 + x)/(1 + Sqrt[3] + x
)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2}}{\sqrt{1+x^3}}\,dx &= \frac{\sqrt{ax^2}\int \frac{x}{\sqrt{1+x^3}}\,dx}{x} \\
&= \frac{\sqrt{ax^2}\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx}{x} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\sqrt{ax^2}\right)\int \frac{1}{\sqrt{1+x^3}}\,dx}{x} \\
&= \frac{2\sqrt{ax^2}\sqrt{1+x^3}}{x\left(1+\sqrt{3}+x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{ax^2}\,(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}}{1+\sqrt{3}}\right.\right.}{x\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\sqrt{1+x^3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.02, size = 29, normalized size = 0.11

$$\frac{1}{2}x\sqrt{ax^2}\ {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^3],x]
[Out] (x*Sqrt[a*x^2]*Hypergeometric2F1[1/2, 2/3, 5/3, -x^3])/2
```

Maple [A]
time = 0.28, size = 270, normalized size = 1.04

method	result
meijerg	$\frac{\sqrt{a\,x^2}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{2}{3}\right],\left[\frac{5}{3}\right],-x^3\right)}{2}$
default	$\sqrt{a\,x^2}\left(-3+i\sqrt{3}\right)\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\left(i\,\mathrm{EllipticE}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*x^2)^(1/2)*(-3+I*3^(1/2))*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2
)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*(I*E
llipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1
/2))*3^(1/2)-I*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(
I*3^(1/2)+3))^(1/2))*3^(1/2)+3*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-
(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1
/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2)))/x/(x^3+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 19, normalized size = 0.07

$$-\frac{2\sqrt{ax^2}\ \mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(a*x^2)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a*x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^2)/sqrt(x^3 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x^2}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a*x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.388 $\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{a}\ \sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

```
[Out] 2/3*arcsinh((a*x)^(3/2)/a^(3/2))*a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {335, 281, 221}

$$\frac{2}{3}\sqrt{a}\ \sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}}\,dx = \frac{2\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}}\,dx, x, \sqrt{ax}\right)}{a}$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}}\,dx, x, (ax)^{3/2}\right)}{3a}$$
$$= \frac{2}{3}\sqrt{a}\,\sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Mathematica [A]
time = 0.09, size = 32, normalized size = 1.39

$$\frac{2\sqrt{ax}\,\tanh^{-1}\left(\frac{x^{3/2}}{\sqrt{1+x^3}}\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3],x]
```

```
[Out] (2*Sqrt[a*x]*ArcTanh[x^(3/2)/Sqrt[1 + x^3]])/(3*Sqrt[x])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.34, size = 321, normalized size = 13.96

method	result
meijerg	$\frac{2\sqrt{ax}\,\operatorname{arcsinh}\left(x^{\frac{3}{2}}\right)}{3\sqrt{x}}$
elliptic	$-\frac{2\sqrt{ax}\,\sqrt{x\left(x^3+1\right)a}\,\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^2\sqrt{-\frac{x-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(1+x)}}\,\sqrt{-\frac{x-\frac{1}{2}-\frac{i\sqrt{3}}{2}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}}}{\sqrt{x^3+1}\,x\left(\frac{3}{2}+\right.}$

default	$-\frac{4\sqrt{ax}\,\sqrt{x^3+1}\,a\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(1+i\sqrt{3}\right)(1+x)}}\,(1+x)^2\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)(1+x)}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{\left(1+i\sqrt{3}\right)(1+x)}}\,\left(\mathrm{EllipticF}\right.}{\sqrt{x\left(x^3+1\right)a}\,\left(i\sqrt{3}+3\right)\sqrt{-ax}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(1+I*3^(1/2))*((I*3^(1/2)+3)*x/(1+I*3^(1/2))
/(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^
(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*(EllipticF(((I*3^(1/2)+3)*x/(1+I*3^
(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)
+3))^(1/2))-EllipticPi(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),(1+I*3^(
1/2))/(I*3^(1/2)+3),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)
+3))^(1/2)))/(x*(x^3+1)*a)^(1/2)/(I*3^(1/2)+3)/(-a*x*(1+x)*(I*3^(1/2)+2*x-1
)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(15) = 30$.
time = 0.38, size = 85, normalized size = 3.70

$$\left[\frac{1}{6}\sqrt{a}\,\log\left(-8\,ax^6-8\,ax^3-4\left(2\,x^4+x\right)\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{a}-a\right),-\frac{1}{3}\sqrt{-a}\,\arctan\left(\frac{2\sqrt{x^3+1}\,\sqrt{ax}\,\sqrt{-a}\,x}{2\,ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)
*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2
*a*x^3 + a))]
```

Sympy [A]
time = 0.45, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a}\,\operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] 2*sqrt(a)*asinh(x**(3/2))/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(15) = 30.
time = 4.06, size = 35, normalized size = 1.52

$$-\frac{2\,a^{\frac{5}{2}}\log\left(-\sqrt{ax}\,a^{\frac{3}{2}}x+\sqrt{a^4x^3+a^4}\right)}{3\,|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.389 $\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=116

$$\frac{\sqrt{\frac{a}{x}}\, x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\, F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] 1/3*x*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/(1+x*(1-3^(1/2)
))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x)^(1/2)*((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 335, 231}

$$\frac{x(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{\frac{a}{x}}\, F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] (Sqrt[a/x]*x*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[
ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(
1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{1+x^3}}\,dx\\
&= \left(2\sqrt{\frac{a}{x}}\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{\frac{a}{x}}\,x(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,F\left(\cos^{-1}\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 27, normalized size = 0.23

$$2\sqrt{\frac{a}{x}}\,x\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x]/Sqrt[1 + x^3],x]
```

```
[Out] 2*Sqrt[a/x]*x*Hypergeometric2F1[1/6, 1/2, 7/6, -x^3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.37, size = 232, normalized size = 2.00

method	result
meijerg	$2\sqrt{\frac{a}{x}}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2}\right],\left[\frac{7}{6}\right],-x^{3}\right)$
default	$\frac{4\sqrt{\frac{a}{x}}\, x\sqrt{x^{3}+1}\,\left(1+i\sqrt{3}\right)\sqrt{\frac{\left(i\sqrt{3}+3\right)x}{\left(1+i\sqrt{3}\right)\left(1+x\right)}}\,\left(1+x\right)^{2}\sqrt{\frac{i\sqrt{3}+2x-1}{\left(-1+i\sqrt{3}\right)\left(1+x\right)}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{\left(1+i\sqrt{3}\right)\left(1+x\right)}}\,\mathrm{EllipticF}\left(\sqrt{\right.}\right.}{\sqrt{x\left(x^{3}+1\right)}\,\left(i\sqrt{3}+3\right)\sqrt{-x\left(1+x\right)\left(i\sqrt{3}+2x-1\right)\left(i\sqrt{3}-2\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*(a/x)^(1/2)*x*(x^3+1)^(1/2)*(1+I*3^(1/2))*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/
(1+x))^(1/2)*(1+x)^2*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(
1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1
/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3
))^(1/2))/(x*(x^3+1))^(1/2)/(I*3^(1/2)+3)/(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^
(1/2)-2*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 11, normalized size = 0.09

$$-2\sqrt{a}\,\mathrm{weierstrassPInverse}\left(0,-4,\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a)*weierstrassPInverse(0, -4, 1/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x}}}{\sqrt{(x+1)\left(x^{2}-x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x)/sqrt(x^3 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{a}{x}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.390 $\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} \, dx$

Optimal. Leaf size=24

$$-\frac{2}{3}\sqrt{\frac{a}{x^2}}\, x \tanh^{-1}\left(\sqrt{1+x^3}\right)$$

```
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 272, 65, 213}

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}}\tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
```

```
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}}\,dx &= \left(\sqrt{\frac{a}{x^2}}\,x\right)\int \frac{1}{x\sqrt{1+x^3}}\,dx \\
&= \frac{1}{3}\left(\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^3\right) \\
&= \frac{1}{3}\left(2\sqrt{\frac{a}{x^2}}\,x\right)\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^3}\right) \\
&= -\frac{2}{3}\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^3}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2}{3}\sqrt{\frac{a}{x^2}}\,x\tanh^{-1}\left(\sqrt{1+x^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3],x]
[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3
```

Maple [A]
time = 0.20, size = 19, normalized size = 0.79

method	result	size
default	$-\frac{2x\,\mathrm{arctanh}\left(\sqrt{x^3+1}\right)\sqrt{\frac{a}{x^2}}}{3}$	19
meijerg	$\frac{\sqrt{\frac{a}{x^2}}\,x\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^3+1}}{2}\right)+(-2\ln(2)+3\ln(x))\sqrt{\pi}\right)}{3\sqrt{\pi}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)
```

Fricas [A] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.
time = 0.33, size = 68, normalized size = 2.83

$$\left[\frac{1}{3}\,x\sqrt{\frac{a}{x^2}}\,\log\left(\frac{x^3-2\,\sqrt{x^3+1}+2}{x^3}\right),\ \frac{2}{3}\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{x^3+1}\,\sqrt{-a}\,x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arcta
n(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [A]
time = 3.45, size = 31, normalized size = 1.29

$$-\frac{1}{3}\,\sqrt{a}\,\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.391 $\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=312

$$-2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3}+\frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x}-\frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}\,x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\cos^{-}\right.}{\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\sqrt{}}$$

```
[Out] -2*x*(a/x^3)^(1/2)*(x^3+1)^(1/2)+2*x^2*(1+3^(1/2))*(a/x^3)^(1/2)*(x^3+1)^(1
/2)/(1+x*(1+3^(1/2)))-2*3^(1/4)*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1
/2)))^2)^(1/2)/(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticE((1-(1+x*(1-3^(
1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a/x^3)^(1/2)*
((x^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3^(1/2
)))^2)^(1/2)-1/3*x^2*(1+x)*((1+x*(1-3^(1/2)))^2/(1+x*(1+3^(1/2)))^2)^(1/2)/
(1+x*(1-3^(1/2)))*(1+x*(1+3^(1/2)))*EllipticF((1-(1+x*(1-3^(1/2)))^2/(1+x*(
1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(a/x^3)^(1/2)*((x
^2-x+1)/(1+x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/(x*(1+x)/(1+x*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {15, 331, 335, 314, 231, 1895}

$$-\frac{\left(1-\sqrt{3}\right)(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,x^2\sqrt{\frac{a}{x^3}}\,F\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-\frac{2\sqrt[4]{3}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\,x^2\sqrt{\frac{a}{x^3}}\,E\left(\text{ArcCos}\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}-2\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^3}}+\frac{2\left(1+\sqrt{3}\right)\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^3}}}{\left(1+\sqrt{3}\right)x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
```

```
[Out] -2*Sqrt[a/x^3]*x*Sqrt[1 + x^3] + (2*(1 + Sqrt[3])*Sqrt[a/x^3]*x^2*Sqrt[1 +
x^3])/(1 + (1 + Sqrt[3])*x) - (2*3^(1/4)*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 -
x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3]
)*x)^2]*Sqrt[1 + x^3]) - ((1 - Sqrt[3])*Sqrt[a/x^3]*x^2*(1 + x)*Sqrt[(1 - x
 + x^2)/(1 + (1 + Sqrt[3])*x)^2]*EllipticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1
+ (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4])/(3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 +
Sqrt[3])*x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{1}{x^{3/2}\sqrt{1+x^3}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3} + \left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\int \frac{x^{3/2}}{\sqrt{1+x^3}}\,dx$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3} + \left(4\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3} - \left(2\sqrt{\frac{a}{x^3}}\,x^{3/2}\right)\mathrm{Subst}\left(\int \frac{-1+\sqrt{3}-2x^4}{\sqrt{1+x^6}}\,dx, x, \sqrt{x}\right) + \Big(2\Big(-1 \cdots$$
$$= -2\sqrt{\frac{a}{x^3}}\,x\sqrt{1+x^3} + \frac{2\left(1+\sqrt{3}\right)\sqrt{\frac{a}{x^3}}\,x^2\sqrt{1+x^3}}{1+\left(1+\sqrt{3}\right)x} - \frac{2\sqrt[4]{3}\sqrt{\frac{a}{x^3}}\,x^2(1+x)\sqrt{\frac{1-\cdots}{\left(1+\left(1\cdots\right.\right.}}}{\sqrt{\frac{\cdots}{\left(1\cdots\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.01, size = 27, normalized size = 0.09

$$-2\sqrt{\frac{a}{x^3}}\,x\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^3]/Sqrt[1 + x^3],x]
[Out] -2*Sqrt[a/x^3]*x*Hypergeometric2F1[-1/6, 1/2, 5/6, -x^3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.39, size = 1784, normalized size = 5.72

method	result
meijerg	$-2\sqrt{\frac{a}{x^3}}\,x\,\mathrm{hypergeom}\left(\left[-\frac{1}{6},\frac{1}{2}\right],\left[\frac{5}{6}\right],-x^3\right)$

risch	$-2x\sqrt{\frac{a}{x^{3}}}\,\sqrt{x^{3}+1}+\frac{2\left(x\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(1+x)}}\,(1+x)^{2}\sqrt{-\frac{}{\left(\frac{1}{2}\right.}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a/x^3)^(1/2)*x*(-4*I*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1
+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1
)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x)
)^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*
3^(1/2)*x-I*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)*3^(1/2)+4*
(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2
*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(
1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*
(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+2*I*(x*(x^3+1))^(1/2
)*3^(1/2)*x^3-6*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/
2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3
^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),
((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x^2+8*(x
*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x
-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/
2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1
+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-12*(x*(x^3+1))^(1/2)*((I
*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/
(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^
(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3
^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*I*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I
*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3
^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^
(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)
+3))^(1/2))*3^(1/2)*x^2-2*I*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2)
)/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2
*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(
1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/
```

```
2))*3^(1/2)+4*(x*(x^3+1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)
*((I*3^(1/2)+2*x-1)/(-1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(
1/2))/(1+x))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((
-3+I*3^(1/2))*(1+I*3^(1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-6*(x*(x^3+
1))^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)+2*x-1)/(-
1+I*3^(1/2))/(1+x))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(1+x))^(1/2)*Ell
ipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(1+x))^(1/2),((-3+I*3^(1/2))*(1+I*3^(
1/2))/(-1+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+2*I*(x*(x^3+1))^(1/2)*3^(1/2)*x-
2*I*(x*(x^3+1))^(1/2)*3^(1/2)*x^2+6*(x*(x^3+1))^(1/2)*x^3-3*(-x*(1+x)*(I*3^
(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)*x^3-6*(x*(x^3+1))^(1/2)*x^2-I*(-x*(1+
x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)*3^(1/2)*x^3+6*(x*(x^3+1))^(1/
2)*x-3*(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2))/(x^3+1)^(1/2)/
(I*3^(1/2)+3)/(-x*(1+x)*(I*3^(1/2)+2*x-1)*(I*3^(1/2)-2*x+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 14, normalized size = 0.04

$$2\sqrt{a}\,\text{weierstrassZeta}\left(0, -4, \text{weierstrassPInverse}\left(0, -4, \frac{1}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(a)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, 1/x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{(x+1)(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**3)**(1/2)/(x**3+1)**(1/2),x)
[Out] Integral(sqrt(a/x**3)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^3)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a/x^3)/sqrt(x^3 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a}{x^3}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2),x)
[Out] int((a/x^3)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.392 $\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx$

Optimal. Leaf size=281

$$-\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3}+\frac{\sqrt{\frac{a}{x^4}}\,x^2\sqrt{1+x^3}}{1+\sqrt{3}+x}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{\frac{a}{x^4}}\,x^2(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\sqrt{3}+x\right)^2}}\,E\left(\sin^{-1}\left(\frac{1-\sqrt{3}\ldots}{1+\sqrt{3}\ldots}\right.\right.}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right)^2}}\,\sqrt{1+x^3}}$$

```
[Out] -x*(a/x^4)^(1/2)*(x^3+1)^(1/2)+x^2*(a/x^4)^(1/2)*(x^3+1)^(1/2)/(1+x+3^(1/2)
)+1/3*x^2*(1+x)*EllipticF((1+x-3^(1/2))/(1+x+3^(1/2)),I*3^(1/2)+2*I)*2^(1/2
)*(a/x^4)^(1/2)*((x^2-x+1)/(1+x+3^(1/2))^2)^(1/2)*3^(3/4)/(x^3+1)^(1/2)/((1
+x)/(1+x+3^(1/2))^2)^(1/2)-1/2*3^(1/4)*x^2*(1+x)*EllipticE((1+x-3^(1/2))/(1
+x+3^(1/2)),I*3^(1/2)+2*I)*(a/x^4)^(1/2)*(1/2*6^(1/2)-1/2*2^(1/2))*((x^2-x+
1)/(1+x+3^(1/2))^2)^(1/2)/(x^3+1)^(1/2)/((1+x)/(1+x+3^(1/2))^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 331, 309, 224, 1891}

$$\frac{\sqrt{2}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,x^2\sqrt{\frac{a}{x^4}}\,F\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,(x+1)\sqrt{\frac{x^2-x+1}{\left(x+\sqrt{3}+1\right)^2}}\,x^2\sqrt{\frac{a}{x^4}}\,E\left(\text{ArcSin}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{\left(x+\sqrt{3}+1\right)^2}}\,\sqrt{x^3+1}}-\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^4}}+\frac{\sqrt{x^3+1}\,x^2\sqrt{\frac{a}{x^4}}}{x+\sqrt{3}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
```

```
[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^3]) + (Sqrt[a/x^4]*x^2*Sqrt[1 + x^3])/(1 + Sqrt[
3] + x) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x +
x^2)/(1 + Sqrt[3] + x)^2]*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] +
 x)], -7 - 4*Sqrt[3]])/(2*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
+ (Sqrt[2]*Sqrt[a/x^4]*x^2*(1 + x)*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]*
EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4*Sqrt[3]])/(3^
(1/4)*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]*Sqrt[1 + x^3])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 309

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 - Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1891

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 - Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 - Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 + Sqrt[3])*s + r*x))), x] - S
imp[3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[s*((s + r*x)/((1 + Sqrt
[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])
*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^3}}\,dx = \left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1}{x^2\sqrt{1+x^3}}\,dx$$
$$= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3} + \frac{1}{2}\left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{x}{\sqrt{1+x^3}}\,dx$$
$$= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3} + \frac{1}{2}\left(\sqrt{\frac{a}{x^4}}\,x^2\right)\int \frac{1-\sqrt{3}+x}{\sqrt{1+x^3}}\,dx + \left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,\sqrt{\frac{a}{x^4}}\,x^2\right)\int$$
$$= -\sqrt{\frac{a}{x^4}}\,x\sqrt{1+x^3} + \frac{\sqrt{\frac{a}{x^4}}\,x^2\sqrt{1+x^3}}{1+\sqrt{3}+x} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt{\frac{a}{x^4}}\,x^2(1+x)\sqrt{\frac{1-x+x}{\left(1+\sqrt{3}+\right.}}}{2\sqrt{\frac{1+x}{\left(1+\sqrt{3}+x\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 27, normalized size = 0.10

$$-\sqrt{\frac{a}{x^4}}\,x\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^3],x]
[Out] -(Sqrt[a/x^4]*x*Hypergeometric2F1[-1/3, 1/2, 2/3, -x^3])
```

Maple [A]
time = 0.32, size = 353, normalized size = 1.26

method	result
meijerg	$-\sqrt{\frac{a}{x^4}}\,x\,\mathrm{hypergeom}\left(\left[-\frac{1}{3},\frac{1}{2}\right],\left[\frac{2}{3}\right],-x^3\right)$
risch	$-x\sqrt{\frac{a}{x^4}}\,\sqrt{x^3+1} - \frac{i\sqrt{3}\,\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{\frac{1+x}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\,\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}\,\left(\left(\frac{3}{2}\right.\right.}{}$

default	$\frac{\sqrt{\frac{a}{x^4}}\, x\left(i\sqrt{3}\,\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\,\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{2(1+x)}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^3+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a/x^4)^(1/2)*x*(I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-
2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*Ellipt
icF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*
x-6*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)
*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)
))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x+3*(-2*(1+x)/(-3+I*3^(1/2)
))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3
^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(
I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/(x^3+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 37, normalized size = 0.13

$$-x^2\sqrt{\frac{a}{x^4}}\,\mathrm{weierstrassZeta}(0,-4,\mathrm{weierstrassPInverse}(0,-4,x))-\sqrt{x^3+1}\,x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -x^2*sqrt(a/x^4)*weierstrassZeta(0, -4, weierstrassPInverse(0, -4, x)) - sq
rt(x^3 + 1)*x*sqrt(a/x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a}{x^4}}}{\sqrt{(x+1)\,(x^2-x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a/x^4)/sqrt(x^3 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2),x)
```

```
[Out] int((a/x^4)^(1/2)/(x^3 + 1)^(1/2), x)
```

3.393 $\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=37

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}$$

```
[Out] x*hypergeom([1/2, 1+1/n],[2+1/n],-x^n)*(a*x^(2*n))^(1/2)/(1+n)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {15, 371}

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n}\sqrt{ax^{2n}}\right)\int \frac{x^n}{\sqrt{1+x^n}}\,dx\\ &= \frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.00

$$\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n],x]
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n)
```

Maple [A]
time = 0.28, size = 36, normalized size = 0.97

method	result	size
meijerg	$\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{1}{n}\right],\left[2+\frac{1}{n}\right],-x^{n}\right)\sqrt{a\,x^{2n}}}{1+n}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*hypergeom([1/2,1+1/n],[2+1/n],-x^n)*(a*x^(2*n))^(1/2)/(1+n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**(2*n))/sqrt(x**n + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{a\,x^{2\,n}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2),x)
[Out] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2), x)
```

3.394 $\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=48

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 371}

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2
 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^n}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/2}\sqrt{ax^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx\\ &= \frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.83

$$\frac{2x\sqrt{ax^n}\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{2+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^n]/Sqrt[1 + x^n],x]
[Out] (2*x*Sqrt[a*x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/
(2 + n)
```

Maple [A]
time = 0.22, size = 35, normalized size = 0.73

method	result	size
meijerg	$\frac{2x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}+\frac{1}{n}\right],\left[\frac{3}{2}+\frac{1}{n}\right],-x^n\right)\sqrt{a\,x^n}}{2+n}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*hypergeom([1/2,1/2+1/n],[3/2+1/n],-x^n)*(a*x^n)^(1/2)/(2+n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**n)**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**n)/sqrt(x**n + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^n)^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^n)/sqrt(x^n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^n}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^n)^(1/2)/(x^n + 1)^(1/2),x)
```

```
[Out] int((a*x^n)^(1/2)/(x^n + 1)^(1/2), x)
```

3.395 $\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx$

Optimal. Leaf size=52

$$\frac{4x\sqrt{ax^{n/2}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{4+n}$$

```
[Out] 4*x*hypergeom([1/2, 1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {15, 371}

$$\frac{4x\sqrt{ax^{n/2}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, (1 + 4/n)/4, (5 + 4/n)/4, -x^n]
)/(4 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^{n/2}}}{\sqrt{1+x^n}}\,dx &= \left(x^{-n/4}\sqrt{ax^{n/2}}\right)\int \frac{x^{n/4}}{\sqrt{1+x^n}}\,dx\\ &= \frac{4x\sqrt{ax^{n/2}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}\left(1+\frac{4}{n}\right);\frac{1}{4}\left(5+\frac{4}{n}\right);-x^n\right)}{4+n}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 44, normalized size = 0.85

$$\frac{4x\sqrt{ax^{n/2}}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}+\frac{1}{n};\frac{5}{4}+\frac{1}{n};-x^n\right)}{4+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(n/2)]/Sqrt[1 + x^n],x]
```

```
[Out] (4*x*Sqrt[a*x^(n/2)]*Hypergeometric2F1[1/2, 1/4 + n^(-1), 5/4 + n^(-1), -x^
n])/(4 + n)
```

Maple [A]
time = 0.23, size = 37, normalized size = 0.71

method	result	size
meijerg	$\frac{4x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{4}+\frac{1}{n}\right],\left[\frac{5}{4}+\frac{1}{n}\right],-x^{n}\right)\sqrt{a\,x^{\frac{n}{2}}}}{4+n}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*x*hypergeom([1/2,1/4+1/n],[5/4+1/n],-x^n)*(a*x^(1/2*n))^(1/2)/(4+n)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^{\frac{n}{2}}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(1/2*n))**(1/2)/(1+x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**(n/2))/sqrt(x**n + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(1/2*n))^(1/2)/(1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(1/2*n))/sqrt(x^n + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^{n/2}}}{\sqrt{x^n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2),x)
```

```
[Out] int((a*x^(n/2))^(1/2)/(x^n + 1)^(1/2), x)
```

3.396 $\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx$

Optimal. Leaf size=34

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\sqrt{1+x^n}}{2+n}$$

```
[Out] 2*x^(1-n)*(a*x^(2*n))^(1/2)*(1+x^n)^(1/2)/(2+n)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {15, 371, 251}

$$\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{n+2}+\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1
 + x^n]),x]
```

```
[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1
 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(
-1), -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int\left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}+\frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}}\right)dx=\frac{2\int\frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx}{2+n}+\int\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}}\,dx$$

$$=\left(x^{-n}\sqrt{ax^{2n}}\right)\int\frac{x^n}{\sqrt{1+x^n}}\,dx+\frac{\left(2x^{-n}\sqrt{ax^{2n}}\right)\int\frac{1}{\sqrt{1+x^n}}\,dx}{2+n}$$

$$=\frac{x\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},1+\frac{1}{n};2+\frac{1}{n};-x^n\right)}{1+n}+\frac{2x^{1-n}\sqrt{ax^{2n}}\,{}_2F_1\left(\frac{1}{2},\frac{1}{n};1+\frac{1}{n};-x^n\right)}{2+n}$$

Mathematica [A]
time = 0.05, size = 33, normalized size = 0.97

$$\frac{2ax^{1+n}\sqrt{1+x^n}}{(2+n)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*
Sqrt[1 + x^n]),x]
```

```
[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])
```

Maple [A]
time = 0.25, size = 30, normalized size = 0.88

method	result	size
risch	$\frac{2x\sqrt{1+x^n}\,\sqrt{a\,x^{2n}}\,x^{-n}}{2+n}$	30
meijerg	$\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1+\frac{1}{n}\right],\left[2+\frac{1}{n}\right],-x^n\right)\sqrt{a\,x^{2n}}}{1+n}+\frac{2\sqrt{a\,x^{2n}}\,x^{1-n}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{n}\right],\left[1+\frac{1}{n}\right],-x^n\right)}{2+n}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)
^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(1+x^n)^(1/2)/(2+n)*(a*(x^n)^2)^(1/2)/(x^n)
```

Maxima [A]
time = 0.33, size = 18, normalized size = 0.53

$$\frac{2\sqrt{a}\,\sqrt{x^n+1}\,x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx + \int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}}\,dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(
x**n)/(1+x**n)**(1/2),x)
```

```
[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n
))/sqrt(x**n + 1), x) + Integral(2*sqrt(a*x**(2*n))/(x**n*sqrt(x**n + 1)),
x))/(n + 2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(
1+x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x
^n + 1)*x^n), x)
```

Mupad [B]
time = 2.89, size = 43, normalized size = 1.26

$$\frac{\sqrt{a\,x^{2\,n}}\,\left(\frac{2\,x}{n+2}+\frac{2\,x^{n+1}}{n+2}\right)}{x^n\,\sqrt{x^n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n + 1
)^(1/2)*(n + 2)),x)
[Out] ((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)^
(1/2))
```

3.397 $\int \frac{\sqrt{ax}}{\sqrt{d+ex}\,\sqrt{e+fx}}\,dx$

Optimal. Leaf size=114

$$\frac{2\sqrt{-e^2+df}\,\sqrt{ax}\,\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\,\sqrt{-\frac{ex}{d}}\,\sqrt{e+fx}}$$

```
[Out] 2*EllipticE(f^(1/2)*(e*x+d)^(1/2)/(d*f-e^2)^(1/2),(1-e^2/d/f)^(1/2))*(d*f-e
^2)^(1/2)*(a*x)^(1/2)*(e*(f*x+e)/(-d*f+e^2))^(1/2)/e/f^(1/2)/(-e*x/d)^(1/2)
/(f*x+e)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {115, 114}

$$\frac{2\sqrt{ax}\,\sqrt{df-e^2}\,\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{f}\,\sqrt{d+ex}}{\sqrt{df-e^2}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\,\sqrt{-\frac{ex}{d}}\,\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
```

```
[Out] (2*Sqrt[-e^2 + d*f]*Sqrt[a*x]*Sqrt[(e*(e + f*x))/(e^2 - d*f)]*EllipticE[Arc
Sin[(Sqrt[f]*Sqrt[d + e*x])/Sqrt[-e^2 + d*f]], 1 - e^2/(d*f)])/(e*Sqrt[f]*S
qrt[-((e*x)/d)]*Sqrt[e + f*x])
```

Rule 114

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Simp[(2/b)*Rt[-(b*e - a*f)/d, 2]*EllipticE[ArcSin[Sqrt[a
+ b*x]/Rt[-(b*c - a*d)/d, 2]], f*((b*c - a*d)/(d*(b*e - a*f)))], x] /; Free
Q[{a, b, c, d, e, f}, x] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0]
&&  !LtQ[-(b*c - a*d)/d, 0] &&  !(SimplerQ[c + d*x, a + b*x] && GtQ[-d/(b*c
 - a*d), 0] && GtQ[d/(d*e - c*f), 0] &&  !LtQ[(b*c - a*d)/b, 0])
```

Rule 115

```
Int[Sqrt[(e_.) + (f_.)*(x_)]/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_
)]), x_Symbol] :> Dist[Sqrt[e + f*x]*(Sqrt[b*((c + d*x)/(b*c - a*d))]/(Sqrt
```

```
[c + d*x]*Sqrt[b*((e + f*x)/(b*e - a*f))])), Int[Sqrt[b*(e/(b*e - a*f)) + b
*f*(x/(b*e - a*f))]/(Sqrt[a + b*x]*Sqrt[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a
*d))]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !(GtQ[b/(b*c - a*d), 0]
&& GtQ[b/(b*e - a*f), 0]) &&  !LtQ[-(b*c - a*d)/d, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax}}{\sqrt{d+ex}\sqrt{e+fx}}\,dx = \frac{\left(\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\right)\int \frac{\sqrt{-\frac{ex}{d}}}{\sqrt{d+ex}\sqrt{\frac{e^2}{e^2-df}+\frac{efx}{e^2-df}}}\,dx}{\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$
$$= \frac{2\sqrt{-e^2+df}\sqrt{ax}\sqrt{\frac{e(e+fx)}{e^2-df}}\,E\left(\sin^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{-e^2+df}}\right)|1-\frac{e^2}{df}\right)}{e\sqrt{f}\sqrt{-\frac{ex}{d}}\sqrt{e+fx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.91, size = 106, normalized size = 0.93

$$-\frac{2ie\sqrt{ax}\sqrt{1+\frac{fx}{e}}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{ex}{d}}\right)|\frac{df}{e^2}\right)\right)}{f\sqrt{\frac{ex}{d+ex}}\sqrt{d+ex}\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x]/(Sqrt[d + e*x]*Sqrt[e + f*x]),x]
[Out] ((-2*I)*e*Sqrt[a*x]*Sqrt[1 + (f*x)/e]*(EllipticE[I*ArcSinh[Sqrt[(e*x)/d]],
(d*f)/e^2] - EllipticF[I*ArcSinh[Sqrt[(e*x)/d]], (d*f)/e^2]))/(f*Sqrt[(e*x)
/(d + e*x)]*Sqrt[d + e*x]*Sqrt[e + f*x])
```

Maple [A]
time = 0.23, size = 191, normalized size = 1.68

method	result
default	$-\frac{2\left(d\,\mathrm{EllipticF}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)f-\mathrm{EllipticE}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)df+\mathrm{EllipticE}\left(\sqrt{\frac{fx+e}{e}},\sqrt{-\frac{e^2}{df-e^2}}\right)e^2}{f^2x(ef\,x^2+dfx+e^2x+de)}$

elliptic	$\frac{2\sqrt{ax}\,\sqrt{(ex+d)\,(fx+e)\,ax}\,e\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}}\,\sqrt{\frac{x+\frac{d}{e}}{-\frac{e}{f}+\frac{d}{e}}}\,\sqrt{-\frac{fx}{e}}\left(\left(-\frac{e}{f}+\frac{d}{e}\right)\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{e}{f}\right)f}{e}},\sqrt{-\frac{e}{f\left(-\frac{e}{f}\right.}}\right.\right.}{\sqrt{ex+d}\,\sqrt{fx+e}\,xf\sqrt{aef\,x^3+adf\,x^2+a\,e^2x^2+adex}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d*EllipticF(((f*x+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*f-EllipticE(((f*x
+e)/e)^(1/2),(-e^2/(d*f-e^2))^(1/2))*d*f+EllipticE(((f*x+e)/e)^(1/2),(-e^2/
(d*f-e^2))^(1/2))*e^2)*(-f*x/e)^(1/2)*((e*x+d)*f/(d*f-e^2))^(1/2)*((f*x+e)/
e)^(1/2)*(a*x)^(1/2)*(e*x+d)^(1/2)*(f*x+e)^(1/2)/f^2/x/(e*f*x^2+d*f*x+e^2*x
+d*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(f*x + e)*sqrt(x*e + d)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 250, normalized size = 2.19

$$-\frac{2\left(\sqrt{af}\,(df+e^2)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(\frac{4\,(d^2f^2-dfe^2+e^4)e^{(-2)}}{3\,f^2},-\frac{4\,(2\,d^3f^3-3\,d^2f^2e^2-3\,dfe^4+2\,e^6)e^{(-3)}}{27\,f^3},\frac{(3\,fxe+df+e^2)e^{(-1)}}{3\,f}\right)+3\,\sqrt{af}\,fe^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(\frac{4\,(d^2f^2-dfe^2+e^4)e^{(-2)}}{3\,f^2},-\frac{4\,(2\,d^3f^3-3\,d^2f^2e^2-3\,dfe^4+2\,e^6)e^{(-3)}}{27\,f^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(d^2f^2-dfe^2+e^4)e^{(-2)}}{3\,f^2},-\frac{4\,(2\,d^3f^3-3\,d^2f^2e^2-3\,dfe^4+2\,e^6)e^{(-3)}}{27\,f^3},\frac{(3\,fxe+df+e^2)e^{(-1)}}{3\,f}\right)\right)\right)e^{(-2)}}{3\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(sqrt(a*f)*(d*f + e^2)*e^(1/2)*weierstrassPInverse(4/3*(d^2*f^2 - d*f*
e^2 + e^4)*e^(-2)/f^2, -4/27*(2*d^3*f^3 - 3*d^2*f^2*e^2 - 3*d*f*e^4 + 2*e^6
)*e^(-3)/f^3, 1/3*(3*f*x*e + d*f + e^2)*e^(-1)/f) + 3*sqrt(a*f)*f*e^(3/2)*w
eierstrassZeta(4/3*(d^2*f^2 - d*f*e^2 + e^4)*e^(-2)/f^2, -4/27*(2*d^3*f^3 -
 3*d^2*f^2*e^2 - 3*d*f*e^4 + 2*e^6)*e^(-3)/f^3, weierstrassPInverse(4/3*(d^
2*f^2 - d*f*e^2 + e^4)*e^(-2)/f^2, -4/27*(2*d^3*f^3 - 3*d^2*f^2*e^2 - 3*d*f
*e^4 + 2*e^6)*e^(-3)/f^3, 1/3*(3*f*x*e + d*f + e^2)*e^(-1)/f)))*e^(-2)/f^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax}}{\sqrt{d+ex}\,\sqrt{e+fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)**(1/2)/(e*x+d)**(1/2)/(f*x+e)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x)/(sqrt(d + e*x)*sqrt(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x)^(1/2)/(e*x+d)^(1/2)/(f*x+e)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x)/(sqrt(f*x + e)*sqrt(x*e + d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x}}{\sqrt{e+f\,x}\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)),x)
```

```
[Out] int((a*x)^(1/2)/((e + f*x)^(1/2)*(d + e*x)^(1/2)), x)
```

3.398 $\int (ax^m)^r \, dx$

Optimal. Leaf size=16

$$\frac{x(ax^m)^r}{1+mr}$$

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (ax^m)^r \, dx &= \left(x^{-mr}(ax^m)^r\right) \int x^{mr} \, dx \\ &= \frac{x(ax^m)^r}{1+mr}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(ax^m)^r}{1+mr}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r,x]
```

```
[Out] (x*(a*x^m)^r)/(1 + m*r)
```

Maple [A]
time = 0.03, size = 17, normalized size = 1.06

method	result	size
gosper	$\frac{x(a\,x^{m})^{r}}{mr+1}$	17
norman	$\frac{x\,\mathrm{e}^{r\ln\left(a\,\mathrm{e}^{m\ln(x)}\right)}}{mr+1}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x,method=_RETURNVERBOSE)
```

```
[Out] x*(a*x^m)^r/(m*r+1)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 1.06

$$\frac{a^{r}x(x^{m})^{r}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="maxima")
```

```
[Out] a^r*x*(x^m)^r/(m*r + 1)
```

Fricas [A]
time = 0.34, size = 20, normalized size = 1.25

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x(ax^{m})^{r}}{mr+1} & \text{for}\: m \neq -\frac{1}{r} \\ \int \left(ax^{-\frac{1}{r}}\right)^{r}\, dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r,x)
```

```
[Out] Piecewise((x*(a*x**m)**r/(m*r + 1), Ne(m, -1/r)), (Integral((a/x**(1/r))**r
, x), True))
```

Giac [A]
time = 3.70, size = 20, normalized size = 1.25

$$\frac{xe^{(mr\log(x)+r\log(a))}}{mr+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)
```

Mupad [B]
time = 3.40, size = 16, normalized size = 1.00

$$\frac{x\,{\left(a\,x^m\right)}^r}{m\,r+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r,x)
```

```
[Out] (x*(a*x^m)^r)/(m*r + 1)
```

3.399 $\int (ax^m)^r (bx^n)^s \, dx$

Optimal. Leaf size=26

$$\frac{x(ax^m)^r (bx^n)^s}{1+mr+ns}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$\frac{x(ax^m)^r (bx^n)^s}{mr+ns+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s \, dx &= \left(x^{-mr}(ax^m)^r\right) \int x^{mr}(bx^n)^s \, dx \\
&= \left(x^{-mr-ns}(ax^m)^r (bx^n)^s\right) \int x^{mr+ns} \, dx \\
&= \frac{x(ax^m)^r (bx^n)^s}{1+mr+ns}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x(ax^m)^r (bx^n)^s}{1+mr+ns}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)
```

Maple [A]
time = 0.31, size = 27, normalized size = 1.04

method	result
gosper	$\frac{x(a\,x^{m})^{r}(b\,x^{n})^{s}}{mr+ns+1}$
risch	$\frac{x\,\mathrm{e}^{-\frac{i\pi\,\mathrm{csgn}(ia\,x^{m})^{3}r}{2}+\frac{i\pi\,\mathrm{csgn}(ia\,x^{m})^{2}\mathrm{csgn}(ia)r}{2}+\frac{i\pi\,\mathrm{csgn}(ia\,x^{m})^{2}\mathrm{csgn}(ix^{m})r}{2}-\frac{i\pi\,\mathrm{csgn}(ia\,x^{m})\mathrm{csgn}(ia)\mathrm{csgn}(ix^{m})r}{2}+r\ln(a)+\ln(x^{m})r-\frac{i\pi\,\mathrm{csgn}(ib\,x}{2}}{mr+ns+1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x,method=_RETURNVERBOSE)
```

```
[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.23

$$\frac{a^{r}b^{s}xe^{(r\log(x^{m})+s\log(x^{n}))}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")
```

```
[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)
```

Fricas [A]
time = 0.33, size = 32, normalized size = 1.23

$$\frac{xe^{(mr\log(x)+ns\log(x)+r\log(a)+s\log(b))}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases}\frac{x(ax^{m})^{r}(bx^{n})^{s}}{mr+ns+1} & \text{for}\ m\neq\frac{-ns-1}{r}\\ \int(bx^{n})^{s}\left(ax^{-\frac{1}{r}}x^{-\frac{ns}{r}}\right)^{r}\,dx & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s,x)
```

```
[Out] Piecewise((x*(a*x**m)**r*(b*x**n)**s/(m*r + n*s + 1), Ne(m, (-n*s - 1)/r)),
 (Integral((b*x**n)**s*(a/(x**(1/r)*x**(n*s/r)))**r, x), True))
```

Giac [A]
time = 4.37, size = 32, normalized size = 1.23

$$\frac{xe^{(mr\log(x)+ns\log(x)+r\log(a)+s\log(b))}}{mr+ns+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)
```

Mupad [B]
time = 2.92, size = 26, normalized size = 1.00

$$\frac{x\,{\left(a\,x^m\right)}^r\,{\left(b\,x^n\right)}^s}{m\,r+n\,s+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s,x)
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)
```

3.400 $\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx$

Optimal. Leaf size=36

$$\frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{1+mr+ns+pt}$$

```
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 30}

$$\frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{mr+ns+pt+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (ax^m)^r (bx^n)^s (cx^p)^t \, dx &= \left(x^{-mr}(ax^m)^r\right) \int x^{mr}(bx^n)^s (cx^p)^t \, dx \\
&= \left(x^{-mr-ns}(ax^m)^r (bx^n)^s\right) \int x^{mr+ns}(cx^p)^t \, dx \\
&= \left(x^{-mr-ns-pt}(ax^m)^r (bx^n)^s (cx^p)^t\right) \int x^{mr+ns+pt} \, dx \\
&= \frac{x(ax^m)^r (bx^n)^s (cx^p)^t}{1+mr+ns+pt}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 1.00

$$\frac{x(ax^m)^r\,(bx^n)^s\,(cx^p)^t}{1+mr+ns+pt}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)
```

Maple [A]
time = 24.71, size = 37, normalized size = 1.03

method	result
gosper	$\frac{x(a\,x^m)^r(b\,x^n)^s(c\,x^p)^t}{mr+ns+pt+1}$
risch	$x\,\mathrm{e}^{-\frac{i\pi\mathrm{csgn}(ia\,x^m)^3 r}{2}-\frac{i\mathrm{csgn}(ic\,x^p)^3\pi t}{2}+\frac{i\mathrm{csgn}(ic\,x^p)^2\mathrm{csgn}(ix^p)\pi t}{2}-\frac{i\pi\,\mathrm{csgn}(ia\,x^m)\mathrm{csgn}(ia)\mathrm{csgn}(ix^m)r}{2}+r\ln(a)+\ln(x^m)r+\frac{i\pi\mathrm{csgn}(ib\,x^n)^2\mathrm{csgn}(\ldots}{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x,method=_RETURNVERBOSE)
[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)
```

Maxima [A]
time = 0.30, size = 44, normalized size = 1.22

$$\frac{a^r b^s c^t x e^{(r\log(x^m)+s\log(x^n)+t\log(x^p))}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")
[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1
)
```

Fricas [A]
time = 0.37, size = 44, normalized size = 1.22

$$\frac{xe^{(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c))}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")
```

[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**A**]
time = 4.48, size = 44, normalized size = 1.22

$$\frac{xe^{(mr\log(x)+ns\log(x)+pt\log(x)+r\log(a)+s\log(b)+t\log(c))}}{mr+ns+pt+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")
```

```
[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c))
/(m*r + n*s + p*t + 1)
```

Mupad [**B**]
time = 3.10, size = 36, normalized size = 1.00

$$\frac{x\,{\left(a\,x^m\right)}^r\,{\left(b\,x^n\right)}^s\,{\left(c\,x^p\right)}^t}{m\,r+n\,s+p\,t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)
```

```
[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)
```

3.401 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)^{5/2}}{5b^3(a-c)}-\frac{2(c+bx)^{7/2}}{7b^3(a-c)}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(a-c)-4/5*a*(b*x+a)^(5/2)/b^3/(a-c)+2/7*(b*x+a)^(
7/2)/b^3/(a-c)-2/3*c^2*(b*x+c)^(3/2)/b^3/(a-c)+4/5*c*(b*x+c)^(5/2)/b^3/(a-c
)-2/7*(b*x+c)^(7/2)/b^3/(a-c)
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2129, 45}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}-\frac{2(bx+c)^{7/2}}{7b^3(a-c)}+\frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3
*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*
b^3*(a - c))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{b\int x^2\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x^2\sqrt{c+bx}\,dx}{-ab+bc}$$
$$= -\frac{b\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{-ab+bc}+\frac{b\int\left(\frac{c^2\sqrt{c+bx}}{b^2}-\frac{2c(c+bx)^{3/2}}{b^2}\right.}{-ab+bc}$$
$$= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(a-c)}+\frac{2(a+bx)^{7/2}}{7b^3(a-c)}-\frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)}+\frac{4c(c+bx)}{5b^3(a-c}$$

Mathematica [A]
time = 0.48, size = 140, normalized size = 0.95

$$\frac{2\left(8a^3\sqrt{a+bx}-4a^2bx\sqrt{a+bx}+3ab^2x^2\sqrt{a+bx}-8c^3\sqrt{c+bx}+4bc^2x\sqrt{c+bx}-3b^2cx^2\sqrt{c+bx}+15b^3x^3\left(\sqrt{a+bx}-\sqrt{c+bx}\right)\right)}{105b^3(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (2*(8*a^3*Sqrt[a + b*x] - 4*a^2*b*x*Sqrt[a + b*x] + 3*a*b^2*x^2*Sqrt[a + b*
x] - 8*c^3*Sqrt[c + b*x] + 4*b*c^2*x*Sqrt[c + b*x] - 3*b^2*c*x^2*Sqrt[c + b
*x] + 15*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x])))/(105*b^3*(a - c))
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{(a-c)b^3}-\frac{2\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{c^2(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)b^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-2
/(a-c)/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A]
time = 0.36, size = 94, normalized size = 0.64

$$\frac{2\left((15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}-(15\,b^3x^3+3\,b^2cx^2-4\,bc^2x+8\,c^3)\sqrt{bx+c}\right)}{105\,(ab^3-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a) - (15*b
^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*sqrt(b*x + c))/(a*b^3 - b^3*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 390 vs. 2(123) = 246.
time = 3.67, size = 390, normalized size = 2.65

$$-\frac{2}{105}\left(\left(\left(3\,(bx+a)\left(\frac{5\,(a^2b^9-2\,ab^9c+b^9c^2)(bx+a)}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}-\frac{15\,a^3b^9-31\,a^2b^9c+17\,ab^9c^2-b^9c^3}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)+\frac{45\,a^4b^9-96\,a^3b^9c+53\,a^2b^9c^2+2\,ab^9c^3-4\,b^9c^4}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)(bx+a)-\frac{15\,a^5b^9-33\,a^4b^9c+17\,a^3b^9c^2-3\,a^2b^9c^3+12\,ab^9c^4-8\,b^9c^5}{a^3b^{12}-3\,a^2b^{12}c+3\,ab^{12}c^2-b^{12}c^3}\right)\sqrt{bx+c}+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,(ab^3-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^12
 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c + 1
7*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))
 + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^4)/(
a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^5*b^9
 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b^9*c^5
)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*sqrt(b*x + c) + 2/10
5*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/(a*b
^3 - b^3*c)
```

Mupad [B]
time = 2.95, size = 179, normalized size = 1.22

$$\frac{2\,x^3\,\sqrt{a+b\,x}}{7\,(a-c)}-\frac{2\,x^3\,\sqrt{c+b\,x}}{7\,(a-c)}+\frac{16\,a^3\,\sqrt{a+b\,x}}{105\,b^3\,(a-c)}-\frac{16\,c^3\,\sqrt{c+b\,x}}{105\,b^3\,(a-c)}+\frac{2\,a\,x^2\,\sqrt{a+b\,x}}{35\,b\,(a-c)}-\frac{8\,a^2\,x\,\sqrt{a+b\,x}}{105\,b^2\,(a-c)}-\frac{2\,c\,x^2\,\sqrt{c+b\,x}}{35\,b\,(a-c)}+\frac{8\,c^2\,x\,\sqrt{c+b\,x}}{105\,b^2\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(105
*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c)) + (8
*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))
```

3.402 $\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=95

$$-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(a-c)+2/5*(b*x+a)^(5/2)/b^2/(a-c)+2/3*c*(b*x+c)^(3
/2)/b^2/(a-c)-2/5*(b*x+c)^(5/2)/b^2/(a-c)
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2129, 45}

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}-\frac{2(bx+c)^{5/2}}{5b^2(a-c)}+\frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)
) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a -
 c))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx = -\frac{b\int x\sqrt{a+bx}\,dx}{-ab+bc}+\frac{b\int x\sqrt{c+bx}\,dx}{-ab+bc}$$
$$= -\frac{b\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{-ab+bc}+\frac{b\int\left(-\frac{c\sqrt{c+bx}}{b}+\frac{(c+bx)^{3/2}}{b}\right)dx}{-ab+bc}$$
$$= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)}+\frac{2(a+bx)^{5/2}}{5b^2(a-c)}+\frac{2c(c+bx)^{3/2}}{3b^2(a-c)}-\frac{2(c+bx)^{5/2}}{5b^2(a-c)}$$

Mathematica [A]
time = 0.36, size = 101, normalized size = 1.06

$$-\frac{2\sqrt{a+bx}\,(2a^2+ac-3c^2-a(c+bx)+6c(c+bx)-3(c+bx)^2)}{15b^2(a-c)}+\frac{2\left(5c(c+bx)^{3/2}-3(c+bx)^{5/2}\right)}{15b^2(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(c
 + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2))
)/(15*b^2*(a - c))
```

Maple [A]
time = 0.03, size = 66, normalized size = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(a-c)b^2}-\frac{2\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{(bx+c)^{\frac{3}{2}}c}{3}\right)}{(a-c)b^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(a-c)/b^2*(1/5*(b*x+c
)^(5/2)-1/3*(b*x+c)^(3/2)*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A]
time = 0.32, size = 70, normalized size = 0.74

$$\frac{2\left((3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}-(3\,b^2x^2+bcx-2\,c^2)\sqrt{bx+c}\right)}{15\,(ab^2-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^
2)*sqrt(b*x + c))/(a*b^2 - b^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{a+bx}+\sqrt{bx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(79) = 158.
time = 4.34, size = 206, normalized size = 2.17

$$-\frac{2\left(\left((bx+a)\left(\frac{3\,(ab^2-b^2c)\,(bx+a)}{a^2b^3-2\,ab^3c+b^3c^2}-\frac{6\,a^2b^2-7\,ab^2c+b^2c^2}{a^2b^3-2\,ab^3c+b^3c^2}\right)+\frac{3\,a^3b^2-4\,a^2b^2c-ab^2c^2+2\,b^2c^3}{a^2b^3-2\,ab^3c+b^3c^2}\right)\sqrt{bx+c}-\frac{3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a}{ab-bc}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)) +
(3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c + b^
3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b - b*
c))/b
```

Mupad [B]
time = 2.70, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(a-c\right)}-\frac{2\,x^2\,\sqrt{c+b\,x}}{5\,\left(a-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{4\,c^2\,\sqrt{c+b\,x}}{15\,b^2\,\left(a-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(a-c\right)}-\frac{2\,c\,x\,\sqrt{c+b\,x}}{15\,b\,\left(a-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b^2
*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1/2)
)/(15*b*(a - c))
```

3.403 $\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {6821}

$$\frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}+\sqrt{c+bx}}\,dx &= \frac{\int \left(\sqrt{a+bx}-\sqrt{c+bx}\right)\,dx}{a-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(a-c)}-\frac{2(c+bx)^{3/2}}{3b(a-c)}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 35, normalized size = 0.74

$$\frac{2\left((a+bx)^{3/2}-(c+bx)^{3/2}\right)}{3b(a-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1),x]
```

```
[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(a-c)}-\frac{2(bx+c)^{\frac{3}{2}}}{3b(a-c)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)
```

Fricas [A]
time = 0.32, size = 29, normalized size = 0.62

$$\frac{2\left((bx+a)^{\frac{3}{2}}-(bx+c)^{\frac{3}{2}}\right)}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(32) = 64$.
time = 0.48, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}}+\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for}\ b\neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a
+ b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) +
 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(
b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

Giac [A]
time = 4.39, size = 75, normalized size = 1.60

$$-\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c}-\frac{ab-bc}{ab^2-b^2c}\right)+\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2*
c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)
```

Mupad [B]
time = 2.71, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,(a-c)}-\frac{2\,x\,\sqrt{c+b\,x}}{3\,(a-c)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,(a-c)}-\frac{2\,c\,\sqrt{c+b\,x}}{3\,b\,(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))
```

3.404 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)+2*arctanh((b*x+c)^(1/2)/c^(
1/2))*c^(1/2)/(a-c)+2*(b*x+a)^(1/2)/(a-c)-2*(b*x+c)^(1/2)/(a-c)
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2129, 52, 65, 214}

$$\frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{bx+c}}{a-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]
)/(a - c)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx &= -\frac{b\int \frac{\sqrt{a+bx}}{x}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x}\,dx}{-ab+bc}\\ &= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{a-c}-\frac{c\int \frac{1}{x\sqrt{c+bx}}\,dx}{a-c}\\ &= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}+\frac{(2a)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(a-c)}-\frac{(2\\ &= \frac{2\sqrt{a+bx}}{a-c}-\frac{2\sqrt{c+bx}}{a-c}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c}+\frac{2\sqrt{c}\tanh^{-1}}{a-}\end{aligned}$$

Mathematica [A]
time = 0.50, size = 160, normalized size = 1.65

$$\frac{2\left(\sqrt{a+bx}-\sqrt{c+bx}-\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\tan^{-1}\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[(Sq
rt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]] - Sqrt[-(Sqrt[a]
 + Sqrt[c])^2]*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt
[c])^2]]))/(a - c)
```

Maple [A]
time = 0.01, size = 73, normalized size = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a-c}-\frac{2\sqrt{bx+c}-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*
(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A]
time = 0.37, size = 318, normalized size = 3.28

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},\ -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{bx+c}}{a-c},\ \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}-2\sqrt{bx+c}}{a-c},\ \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{bx+a}-\sqrt{bx+c}\right)}{a-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x
- 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a
 - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2
*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a -
c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqr
t(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c),
2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c
)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1016 vs. $2(81) = 162$.
time = 4.70, size = 1016, normalized size = 10.47

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,(a-c)} - \frac{2\left(a^4c - a^3c^2 - a^2c^3 + ac^4 + 2\left(ac^2 + \sqrt{ac}\,c^2\right)(a-c)^2\mathrm{sgn}(-a+c) - 2\left(ac^2 + \sqrt{ac}\,ac\right)(a-c)^2 + \left(a^2c^2 - 2ac^3 + c^4 - \left(a^2c - 2ac^2 + c^3\right)\sqrt{ac}\right)|-a+c|\,\mathrm{sgn}(-a+c) - \left(a^3c - 2a^2c^2 + ac^3 + \left(a^2c - 2ac^2 + c^3\right)\sqrt{ac}\right)|-a+c| - \left(a^4c - a^3c^2 - a^2c^3 + ac^4 + \left(a^3c - a^2c^2 - ac^3 + c^4\right)\sqrt{ac}\right)\mathrm{sgn}(-a+c) + \left(a^4 - a^3c - a^2c^2 + ac^3\right)\sqrt{ac}\right)\arctan\left(-\frac{\sqrt{bx+a} - \sqrt{bx+c}}{\sqrt{-\frac{a^2 - c^2 + \sqrt{(a^2-c^2)^2 - (a^3 - 3a^2c + 3ac^2 - c^3)(a-c)}}{a-c}}}\right)}{\left(\sqrt{-a}\,a^4 - a^4\sqrt{-c} - 4\sqrt{-a}\,a^3c + 4a^3\sqrt{-c}\,c + 6\sqrt{-a}\,a^2c^2 - 6a^2\sqrt{-c}\,c^2 - 4\sqrt{-a}\,ac^3 + 4a\sqrt{-c}\,c^3 + \sqrt{-a}\,c^4 - \sqrt{-c}\,c^4\right)|-a+c|} + \frac{2\left(a^4c - a^3c^2 - a^2c^3 + ac^4 - 2\left(ac^2 + \sqrt{ac}\,c^2\right)(a-c)^2\mathrm{sgn}(-a+c) - 2\left(ac^2 - \sqrt{ac}\,ac\right)(a-c)^2 + \left(a^2c^2 - 2ac^3 + c^4 - \left(a^2c - 2ac^2 + c^3\right)\sqrt{ac}\right)|-a+c|\,\mathrm{sgn}(-a+c) + \left(a^3c - 2a^2c^2 + ac^3 + \left(a^2c - 2ac^2 + c^3\right)\sqrt{ac}\right)|-a+c| + \left(a^4c - a^3c^2 - a^2c^3 + ac^4 - \left(a^3c - a^2c^2 - ac^3 + c^4\right)\sqrt{ac}\right)\mathrm{sgn}(-a+c) + \left(a^4 - a^3c - a^2c^2 + ac^3\right)\sqrt{ac}\right)\arctan\left(-\frac{\sqrt{bx+a} - \sqrt{bx+c}}{\sqrt{-\frac{a^2 - c^2 - \sqrt{(a^2-c^2)^2 - (a^3 - 3a^2c + 3ac^2 - c^3)(a-c)}}{a-c}}}\right)}{\left(\sqrt{-a}\,a^4 - a^4\sqrt{-c} - 4\sqrt{-a}\,a^3c + 4a^3\sqrt{-c}\,c + 6\sqrt{-a}\,a^2c^2 - 6a^2\sqrt{-c}\,c^2 - 4\sqrt{-a}\,ac^3 + 4a\sqrt{-c}\,c^3 + \sqrt{-a}\,c^4 - \sqrt{-c}\,c^4\right)|-a+c|} + \frac{2\sqrt{bx+a}}{a-c} - \frac{2\sqrt{bx+c}}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) - 2*(a^4*c - a^3*c^2
- a^2*c^3 + a*c^4 + 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) - 2*(a*
c^2 + sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^
2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) - (a^3*c - 2*a^2*c^2 + a*c^3 +
(a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) - (a^4*c - a^3*c^2 - a^2*c^3
 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (a^4 -
a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))
/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a
 - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c + 4*a^3*
sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c^3 + 4
*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*(a^4*c - a^
3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*sgn(-a + c) -
 2*(a*c^2 - sqrt(a*c)*a*c)*(a - c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c -
2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c)*sgn(-a + c) + (a^3*c - 2*a^2*c^2 + a*
c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*abs(-a + c) + (a^4*c - a^3*c^2 - a
^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*sgn(-a + c) + (
a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*arctan(-(sqrt(b*x + a) - sqrt(b*x
 + c))/sqrt(-(a^2 - c^2 - sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c
^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4 - a^4*sqrt(-c) - 4*sqrt(-a)*a^3*c +
4*a^3*sqrt(-c)*c + 6*sqrt(-a)*a^2*c^2 - 6*a^2*sqrt(-c)*c^2 - 4*sqrt(-a)*a*c
^3 + 4*a*sqrt(-c)*c^3 + sqrt(-a)*c^4 - sqrt(-c)*c^4)*abs(-a + c)) + 2*sqrt(
b*x + a)/(a - c) - 2*sqrt(b*x + c)/(a - c)
```

Mupad [B]
time = 18.08, size = 2500, normalized size = 25.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)
```

```
[Out] (atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/
```

2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^
(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 +
a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c -
2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^
(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2
)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/
2) - 2*a^4*c^2*(a + b*x)^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5
/2)*c^(7/2)*(a + b*x)^(1/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)
c^(3/2)(a + b*x)^(1/2) + 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^
(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*
c^2)^(1/2)*2i - 4*a^(3/2)*c - 8*a*c^(3/2) + atan((a^2*c^(5/2)*(a*c^3 + a^3*
c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
- a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3
*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2
)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a
^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/
2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c +
b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)
^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4*c^(5/2)
+ 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 - 2*a^2*c^
4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)^(1/2) -
2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1/2) + 2*
a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2) + 2*a^2
c^4(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2
)))*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + 4*a^(3/2)*c^(1/2
)*(c + b*x)^(1/2) - 3*a*c^(3/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) - 3*a^(3/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^
(1/2) - c^(1/2))) + 8*a*c*(c + b*x)^(1/2) - c^(1/2)*atan((a^2*c^(5/2)*(a*c^
3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(
1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c
^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2
*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2
)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i
+ a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a
^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/
2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(
c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2) - 4*a^4
*c^(5/2) + 2*a^(5/2)*c^4 + 2*a^3*c^(7/2) - 4*a^(7/2)*c^3 + 2*a^(9/2)*c^2 -
2*a^2*c^4*(a + b*x)^(1/2) + 4*a^3*c^3*(a + b*x)^(1/2) - 2*a^4*c^2*(a + b*x)
^(1/2) - 2*a^(3/2)*c^(9/2)*(a + b*x)^(1/2) + 2*a^(5/2)*c^(7/2)*(a + b*x)^(1
/2) + 2*a^(7/2)*c^(5/2)*(a + b*x)^(1/2) - 2*a^(9/2)*c^(3/2)*(a + b*x)^(1/2)

+ 2*a^2*c^4*(c + b*x)^(1/2) - 4*a^3*c^3*(c + b*x)^(1/2) + 2*a^4*c^2*(c + b*x)^(1/2)))*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) + a^(3/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + a^(3/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + b*x)^(1/2) + 2*a*c*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(c + b*x)^(1/2))/(a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*(a^(1/2) + c^(1/2))^2*((a + b*x)^(1/2) + (c + b*x)^(1/2) - a^(1/2) - c^(1/2))) - (c^2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) - 4*c^(3/2)*(c + b*x)^(1/2) + 4*c^2 + atan((a^2*c^(5/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^3*c^(3/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(7/2)*c*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^2*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a*c^3*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^3*c*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(3/2)*c^(5/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i + a^(5/2)*c^(3/2)*(a + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^2*c^2*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*4i - a^(3/2)*c^(5/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i - a^(5/2)*c^(3/2)*(c + b*x)^(1/2)*(a*c^3 + a^3*c - 2*a^2*c^2)^(1/2)*2i)/(2*a^5*c^(3/2)...

3.405 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)/a^(1/2)+b*arctanh((b*x+c)^(1/2)/c^(
1/2))/(a-c)/c^(1/2)-(b*x+a)^(1/2)/(a-c)/x+(b*x+c)^(1/2)/(a-c)/x
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2129, 43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x(a-c)}+\frac{\sqrt{bx+c}}{x(a-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}\,dx = -\frac{b\int \frac{\sqrt{a+bx}}{x^2}\,dx}{-ab+bc}+\frac{b\int \frac{\sqrt{c+bx}}{x^2}\,dx}{-ab+bc}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(a-c)}-\frac{b\int \frac{1}{x\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a-c}-\frac{\text{Subst}}{}$$

$$= -\frac{\sqrt{a+bx}}{(a-c)x}+\frac{\sqrt{c+bx}}{(a-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+b}}{\sqrt{c}}\right.}{(a-c)\sqrt{c}}$$

Mathematica [A]
time = 0.81, size = 187, normalized size = 1.82

$$\frac{\sqrt{a}\sqrt{c}\left(-\sqrt{a+bx}+\sqrt{c+bx}\right)+b\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\,x\tan^{-1}\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)-b\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}\,x\tan^{-1}\left(\frac{\sqrt{a+bx}-\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)}{\sqrt{a}\,(a-c)\sqrt{c}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]
```

```
[Out] (Sqrt[a]*Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[c + b*x]) + b*Sqrt[-(Sqrt[a] - Sqrt
[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^
2]] - b*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*x*ArcTan[(Sqrt[a + b*x] - Sqrt[c + b*x
])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/(Sqrt[a]*(a - c)*Sqrt[c]*x)
```

Maple [A]
time = 0.01, size = 88, normalized size = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{a-c}-\frac{2b\left(-\frac{\sqrt{bx+c}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)}{a-c}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(a-c)*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)
```

Fricas [A]
time = 0.36, size = 399, normalized size = 3.87

$$\left[-\frac{\sqrt{a}\,bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+ab\sqrt{c}\,x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{bx+a}\,ac-2\sqrt{bx+c}\,ac}{2(a^2c-ac^2)x},-\frac{2ab\sqrt{-c}\,x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+\sqrt{a}\,bcx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,ac-2\sqrt{bx+c}\,ac}{2(a^2c-ac^2)x},\frac{2\sqrt{-a}\,bcx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-ab\sqrt{c}\,x\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2\sqrt{bx+a}\,ac+2\sqrt{bx+c}\,ac}{2(a^2c-ac^2)x},\frac{\sqrt{-a}\,bcx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-ab\sqrt{-c}\,x\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)-\sqrt{bx+a}\,ac+\sqrt{bx+c}\,ac}{(a^2c-ac^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqr
t(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c -
 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sq
rt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a)
+ 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),
 1/2*(2*sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - a*b*sqrt(c)*x*log
((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a)*a*c + 2*sqrt(b*
x + c)*a*c)/((a^2*c - a*c^2)*x), (sqrt(-a)*b*c*x*arctan(sqrt(b*x + a)*sqrt(
-a)/a) - a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) - sqrt(b*x + a)*a*
c + sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(87) = 174$.
time = 9.23, size = 1190, normalized size = 11.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(a - c)) + (2*(a*c^2 + sqrt(a*c)
*c^2)*(a - c)^2*b*sgn(2*a - 2*c) + 2*(a*c^2 + sqrt(a*c)*a*c)*(a - c)^2*b +
(a^2*c^2 - 2*a*c^3 + c^4 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c)*
sgn(2*a - 2*c) + (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*sqrt(
a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2
 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c) - (a^4*c - a^3*c^2 - a^2*c^3 + a
*c^4 + (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a)
 - sqrt(b*x + c))/sqrt(-(a^2 - c^2 + sqrt((a^2 - c^2)^2 - (a^3 - 3*a^2*c +
3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-a)*a^4*c - a^4*sqrt(-c)*c - 4*sq
rt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqrt(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c
^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 + sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs
(a - c)) - (2*(a*c^2 + sqrt(a*c)*c^2)*(a - c)^2*b*sgn(2*a - 2*c) - 2*(a*c^2
 + sqrt(a*c)*a*c)*(a - c)^2*b + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2
 + c^3)*sqrt(a*c))*b*abs(a - c)*sgn(2*a - 2*c) - (a^3*c - 2*a^2*c^2 + a*c^3
 + (a^2*c - 2*a*c^2 + c^3)*sqrt(a*c))*b*abs(a - c) - (a^4*c - a^3*c^2 - a^2
*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*sqrt(a*c))*b*sgn(2*a - 2*c)
+ (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - (a^4 - a^3*c - a^2*c^2 + a*c^3)*sqrt
(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^2 - c^2 - sqrt((
a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a - c)))/(a - c)))/((sqrt(-
a)*a^4*c - a^4*sqrt(-c)*c - 4*sqrt(-a)*a^3*c^2 + 4*a^3*sqrt(-c)*c^2 + 6*sqr
t(-a)*a^2*c^3 - 6*a^2*sqrt(-c)*c^3 - 4*sqrt(-a)*a*c^4 + 4*a*sqrt(-c)*c^4 +
sqrt(-a)*c^5 - sqrt(-c)*c^5)*abs(a - c)) - 2*(b*(sqrt(b*x + a) - sqrt(b*x +
 c))^3 - a*b*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c*(sqrt(b*x + a) - sqrt(b*
x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) - sqrt(b*
x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^2)*(a -
 c)) - sqrt(b*x + a)/((a - c)*x)
```

Mupad [B]
time = 18.88, size = 2642, normalized size = 25.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)

[Out] (b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))/(((a^(3/2)*b^2*c^(7/2))/2 - a^(5/2)*b^2*c^(5/2) + (a^(7/2)*b^2*c^(3/2))/2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(a^(3/2)*b^2*c^(7/2) - 2*a^(5/2)*b^2*c^(5/2) + a^(7/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*

```
c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a
^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*
a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(2*a^(3/2)*b*c^5 - 2*a^5*
b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/
2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)
*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5
- 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^6 - 12*a^3*c
^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(
a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2
))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^
3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a
^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))))*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3
/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))
^(1/2)*1i)/(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)) - ((
a^(1/2)*b)/(4*(a*c - a^2)) - (b*c^(1/2))/(4*(a*c - c^2)) - (((a^(1/2)*((a^2
*b)/4 - (b*c^2)/4 + (a*b*c)/4))/(a^3*c - a^2*c^2) - (c^(1/2)*((b*c^2)/4 - (
a^2*b)/4 + (a*b*c)/4))/(a*c^3 - a^2*c^2))*((a + b*x)^(1/2) - a^(1/2))^2)/((
c + b*x)^(1/2) - c^(1/2))^2 + (((a^(1/2)*((a*b)/4 - (3*b*c)/4))/(a*c^2 - a^
2*c) - (c^(1/2)*((3*a*b)/4 - (b*c)/4))/(a*c^2 - a^2*c))*((a + b*x)^(1/2) -
a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b
*x)^(1/2) - c^(1/2)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(
1/2))^3 - ((a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*
x)^(1/2) - c^(1/2))^2)) - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)))*(b/(2*a^(1/2)*c) - (b*(a^(1/2) + c^(1/2)))/(2*c*(a - c))) - (b*
((a + b*x)^(1/2) - a^(1/2)))/(4*a^(1/2)*c^(1/2)*(a^(1/2) - c^(1/2))*((c + b
*x)^(1/2) - c^(1/2)))
```

3.406 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=228

$$\frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}+\frac{5(a+$$

```
[Out] 1/3*(a+c)*x^3/(a-c)^2+1/2*b*x^4/(a-c)^2+5/12*(a+c)*(b*x+a)^(3/2)*(b*x+c)^(3
/2)/b^3/(a-c)^2-1/2*x*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2/(a-c)^2-1/32*(4*a*c-5
*(a+c)^2)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^3+1/16*(4*a*c-5*(a+c)^2)*(
b*x+a)^(3/2)*(b*x+c)^(1/2)/b^3/(a-c)^2-1/32*(4*a*c-5*(a+c)^2)*(b*x+a)^(1/2)
*(b*x+c)^(1/2)/b^3/(a-c)
```

Rubi [A]
time = 0.26, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6821, 92, 81, 52, 65, 223, 212}

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2}-\frac{\left(4ac-5(a+c)^2\right)\sqrt{a+bx}\,\sqrt{bx+c}}{32b^3(a-c)}-\frac{\left(4ac-5(a+c)^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3}-\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{x^3(a+c)}{3(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2
)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a
 + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2
)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))
/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c +
b*x]])/(32*b^3)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 92

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[b*(a + b*x)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(
d*f*(n + p + 3))), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x^2+2bx^3-2x^2\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{2\int x^2\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}-\frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2}-\frac{\int \sqrt{a+bx}\,\sqrt{c+bx}\,(}{2b^2(a}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2}-\frac{x(a+bx)^{3/2}(c}{2b^2(a-c}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(4ac-5(a+c)^2\right)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2}+\frac{5(a+c}{}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{(4ac-5}{}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{(4ac-5}{}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{(4ac-5}{}$$

$$= \frac{(a+c)x^3}{3(a-c)^2}+\frac{bx^4}{2(a-c)^2}+\frac{\left(5a^2+6ac+5c^2\right)\sqrt{a+bx}\,\sqrt{c+bx}}{32b^3(a-c)}+\frac{(4ac-5}{}$$

Mathematica [A]
time = 0.52, size = 187, normalized size = 0.82

$$\frac{-\frac{\sqrt{a+bx}\,\sqrt{c+bx}\,\left(15a^3+15c^3-10bc^2x+8b^2cx^2+48b^3x^3-a^2(7c+10bx)+a\left(-7c^2+4bcx+8b^2x^2\right)\right)}{(a-c)^2}+\frac{16\left(-c^4+2b^3cx^3+3b^4x^4+2a\left(c^3+b^3x^3\right)\right)}{(a-c)^2}+3\left(5a^2+6ac+5c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{a+bx}}\right)}{96b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (-((Sqrt[a + b*x]*Sqrt[c + b*x]*(15*a^3 + 15*c^3 - 10*b*c^2*x + 8*b^2*c*x^2
 + 48*b^3*x^3 - a^2*(7*c + 10*b*x) + a*(-7*c^2 + 4*b*c*x + 8*b^2*x^2)))/(a
- c)^2) + (16*(-c^4 + 2*b^3*c*x^3 + 3*b^4*x^4 + 2*a*(c^3 + b^3*x^3)))/(a -
c)^2 + 3*(5*a^2 + 6*a*c + 5*c^2)*ArcTanh[Sqrt[c + b*x]/Sqrt[a + b*x]])/(96*
b^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 604, normalized size = 2.65

method	result
default	$\frac{x^3 a}{3(a-c)^2}+\frac{x^3 c}{3(a-c)^2}+\frac{b\,x^4}{2(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(96\,\mathrm{csgn}(b)x^3b^3\sqrt{b^2x^2+abx+bcx+ac}+16\,\mathrm{csgn}(b)x^2a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/(a-c)^2*a+1/3*x^3/(a-c)^2*c+1/2*b*x^4/(a-c)^2-1/192/(a-c)^2*(b*x+a)
^(1/2)*(b*x+c)^(1/2)*(96*csgn(b)*x^3*b^3*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16
*csgn(b)*x^2*a*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+16*csgn(b)*x^2*b^2*c*(b^
2*x^2+a*b*x+b*c*x+a*c)^(1/2)-20*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a
^2*b+8*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b*c-20*csgn(b)*(b^2*x^2+
a*b*x+b*c*x+a*c)^(1/2)*x*b*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a
^3-14*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2*c-14*csgn(b)*(b^2*x^2+a*b
*x+b*c*x+a*c)^(1/2)*a*c^2+30*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*c^3-15
*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^4+
12*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*a^
3*c+6*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))
*a^2*c^2+12*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*cs
gn(b))*a*c^3-15*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c
)*csgn(b))*c^4)*csgn(b)/b^3/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A]
time = 0.33, size = 196, normalized size = 0.86

$$\frac{96\,b^4x^4+64\,(ab^3+b^3c)x^3-2\,(48\,b^3x^3+15\,a^3-7\,a^2c-7\,ac^2+15\,c^3+8\,(ab^2+b^2c)x^2-2\,(5\,a^2b-2\,abc+5\,bc^2)x)\sqrt{bx+a}\,\sqrt{bx+c}-3\,(5\,a^4-4\,a^3c-2\,a^2c^2-4\,ac^3+5\,c^4)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)}{192\,(a^2b^3-2\,ab^3c+b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2
*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*
c^2)*x)*sqrt(b*x + a)*sqrt(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*
c^3 + 5*c^4)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^3
- 2*a*b^3*c + b^3*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(194) = 388.
time = 3.95, size = 797, normalized size = 3.50

$$-\frac{1}{96}\left(2\left(4\,(bx+a)\left(\frac{6\,(a^5b^9-5\,a^4b^9c+10\,a^3b^9c^2-10\,a^2b^9c^3+5\,ab^9c^4-b^9c^5)(bx+a)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}-\frac{17\,a^6b^9-86\,a^5b^9c+175\,a^4b^9c^2-180\,a^3b^9c^3+95\,a^2b^9c^4-22\,ab^9c^5+b^9c^6}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)+\frac{59\,a^7b^9-301\,a^6b^9c+615\,a^5b^9c^2-625\,a^4b^9c^3+305\,a^3b^9c^4-39\,a^2b^9c^5-19\,ab^9c^6+5\,b^9c^7}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)(bx+a)-\frac{3\,(5\,a^8b^9-24\,a^7b^9c+44\,a^6b^9c^2-40\,a^5b^9c^3+30\,a^4b^9c^4-40\,a^3b^9c^5+44\,a^2b^9c^6-24\,ab^9c^7+5\,b^9c^8)}{a^7b^{12}-7\,a^6b^{12}c+21\,a^5b^{12}c^2-35\,a^4b^{12}c^3+35\,a^3b^{12}c^4-21\,a^2b^{12}c^5+7\,ab^{12}c^6-b^{12}c^7}\right)\sqrt{bx+a}\,\sqrt{bx+c}+\frac{3\,(bx+a)^4-10\,(bx+a)^3a+12\,(bx+a)^2a^2-6\,(bx+a)a^3+2\,(bx+a)^3c-6\,(bx+a)^2ac+6\,(bx+a)a^2c}{6\,(a^2b^3-2\,ab^3c+b^3c^2)}-\frac{(5\,a^2+6\,ac+5\,c^2)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*b
^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5
*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*
c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^3*b^
9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^12*c +
 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*
a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*c^2 - 6
25*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 + 5*b^9*c^
7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^
12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*(b*x + a) - 3*(5*a^8*b
^9 - 24*a^7*b^9*c + 44*a^6*b^9*c^2 - 40*a^5*b^9*c^3 + 30*a^4*b^9*c^4 - 40*a
^3*b^9*c^5 + 44*a^2*b^9*c^6 - 24*a*b^9*c^7 + 5*b^9*c^8)/(a^7*b^12 - 7*a^6*b
^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c
^5 + 7*a*b^12*c^6 - b^12*c^7))*sqrt(b*x + a)*sqrt(b*x + c) + 1/6*(3*(b*x +
a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a
)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2*a*b^3*c + b^3*c
^2) - 1/32*(5*a^2 + 6*a*c + 5*c^2)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))
/b^3
```

Mupad [B]
time = 81.17, size = 1358, normalized size = 5.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (x^3*(a + c))/(3*(a - c)^2) - ((((a + b*x)^(1/2) - a^(1/2))^15*((3*a*c)/8 +
 (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))^15) + (((a + b*
x)^(1/2) - a^(1/2))^3*((23*a*c^3)/12 + (23*a^3*c)/12 - (115*a^4)/48 - (115*
c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^3 + b^3*c
^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^13*((23*a*c^3)/12 + (23*a^3
*c)/12 - (115*a^4)/48 - (115*c^4)/48 + (349*a^2*c^2)/8))/(((c + b*x)^(1/2)
 - c^(1/2))^13*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2
))^5*((3917*a*c^3)/12 + (3917*a^3*c)/12 + (383*a^4)/48 + (383*c^4)/48 + (72
79*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^3 + b^3*c^2 - 2*a*b^3
*c)) + (((a + b*x)^(1/2) - a^(1/2))^11*((3917*a*c^3)/12 + (3917*a^3*c)/12 +
 (383*a^4)/48 + (383*c^4)/48 + (7279*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/
2))^11*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))^7*((
17567*a*c^3)/12 + (17567*a^3*c)/12 + (2789*a^4)/48 + (2789*c^4)/48 + (28213
*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c
)) + (((a + b*x)^(1/2) - a^(1/2))^9*((17567*a*c^3)/12 + (17567*a^3*c)/12 +
(2789*a^4)/48 + (2789*c^4)/48 + (28213*a^2*c^2)/8))/(((c + b*x)^(1/2) - c^(
1/2))^9*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) + (((a + b*x)^(1/2) - a^(1/2))*((3
*a*c)/8 + (5*a^2)/16 + (5*c^2)/16))/(b^3*((c + b*x)^(1/2) - c^(1/2))) - (a^
(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^4)/(((c +
 b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2
)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^12)/(((c + b*x)^(1/2)
 - c^(1/2))^12*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*
x)^(1/2) - a^(1/2))^6*((5120*a*c^2)/3 + (5120*a^2*c)/3 + 256*a^3 + 256*c^3)
)/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2
)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^10*((5120*a*c^2)/3 + (5120*a^2*c)/3 +
 256*a^3 + 256*c^3))/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^3 + b^3*c^2 - 2
*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*((10112*a*c^2)/
3 + (10112*a^2*c)/3 + 512*a^3 + 512*c^3))/(((c + b*x)^(1/2) - c^(1/2))^8*(a
^2*b^3 + b^3*c^2 - 2*a*b^3*c)))/((28*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b
*x)^(1/2) - c^(1/2))^4 - (8*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2)
 - c^(1/2))^2 - (56*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/
2))^6 + (70*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 -
(56*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + (28*((
a + b*x)^(1/2) - a^(1/2))^12)/((c + b*x)^(1/2) - c^(1/2))^12 - (8*((a + b*x
)^(1/2) - a^(1/2))^14)/((c + b*x)^(1/2) - c^(1/2))^14 + ((a + b*x)^(1/2) -
a^(1/2))^16/((c + b*x)^(1/2) - c^(1/2))^16 + 1) + (b*x^4)/(2*(a - c)^2) + (
atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(6*a*c + 5*a
^2 + 5*c^2))/(16*b^3)
```

3.407 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=165

$$\frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}-\frac{(a}{}$$

```
[Out] 1/2*(a+c)*x^2/(a-c)^2+2/3*b*x^3/(a-c)^2-2/3*(b*x+a)^(3/2)*(b*x+c)^(3/2)/b^2
/(a-c)^2-1/4*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b^2+1/2*(a+c)*(b*x+
a)^(3/2)*(b*x+c)^(1/2)/b^2/(a-c)^2-1/4*(a+c)*(b*x+a)^(1/2)*(b*x+c)^(1/2)/b^
2/(a-c)
```

Rubi [A]

time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6821, 81, 52, 65, 223, 212}

$$-\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{bx+c}}{4b^2(a-c)}-\frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2}+\frac{2bx^3}{3(a-c)^2}+\frac{x^2(a+c)}{2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b
*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x]
)/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2)
 - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)x+2bx^2-2x\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2}+\frac{(a+c)\int\sqrt{a+bx}\,\sqrt{c}}{b(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}+\frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2}-\frac{2(a+bx)^{3/2}(c+b}{3b^2(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}}{2b^2(a-c)^2}$$

$$= \frac{(a+c)x^2}{2(a-c)^2}+\frac{2bx^3}{3(a-c)^2}-\frac{(a+c)\sqrt{a+bx}\,\sqrt{c+bx}}{4b^2(a-c)}+\frac{(a+c)(a+bx)^{3/2}}{2b^2(a-c)^2}$$

Mathematica [A]
time = 0.35, size = 131, normalized size = 0.79

$$\frac{2(c+bx)\left(-3ac+c^2+3abx-bcx+4b^2x^2\right)+\sqrt{a+bx}\,\sqrt{c+bx}\left(3a^2+3c^2-2bcx-8b^2x^2-2a(c+bx)\right)-3(a-c)^2(a+c)\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{a+bx}}\right)}{12b^2(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]
```

```
[Out] (2*(c + b*x)*(-3*a*c + c^2 + 3*a*b*x - b*c*x + 4*b^2*x^2) + Sqrt[a + b*x]*S
qrt[c + b*x]*(3*a^2 + 3*c^2 - 2*b*c*x - 8*b^2*x^2 - 2*a*(c + b*x)) - 3*(a -
 c)^2*(a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[a + b*x]])/(12*b^2*(a - c)^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 431, normalized size = 2.61

method	result
default	$\frac{x^2 a}{2(a-c)^2}+\frac{x^2 c}{2(a-c)^2}+\frac{2b\,x^3}{3(a-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(16\,\mathrm{csgn}(b)x^2b^2\sqrt{b^2x^2+abx+bcx+ac}+4\,\mathrm{csgn}(b)\sqrt{b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^
(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*c
sgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*x*a*b+4*csgn(b)*(b^2*x^2+a*b*x+b*c*x
+a*c)^(1/2)*x*b*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a^2+4*csgn(b)*(
b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*a*c-6*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2
)*c^2+3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b
))*a^3-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(
b))*a^2*c-3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*cs
gn(b))*a*c^2+3*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)
*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)
```

Fricas [A]
time = 0.34, size = 149, normalized size = 0.90

$$\frac{16\,b^3x^3+12\,(ab^2+b^2c)x^2-2\,(8\,b^2x^2-3\,a^2+2\,ac-3\,c^2+2\,(ab+bc)x)\sqrt{bx+a}\,\sqrt{bx+c}+3\,(a^3-a^2c-ac^2+c^3)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)}{24\,(a^2b^2-2\,ab^2c+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c -
3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c
^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c))/(a^2*b^2 - 2
*a*b^2*c + b^2*c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(137) = 274.
time = 3.56, size = 445, normalized size = 2.70

$$-\frac{\left(2\,(bx+a)\left(\frac{4\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)(bx+a)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}-\frac{7\,a^4b^2-22\,a^3b^2c+24\,a^2b^2c^2-10\,ab^2c^3+b^2c^4}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)+\frac{3\,(a^5b^2-3\,a^4b^2c+2\,a^3b^2c^2+2\,a^2b^2c^3-3\,ab^2c^4+b^2c^5)}{a^5b^3-5\,a^4b^3c+10\,a^3b^3c^2-10\,a^2b^3c^3+5\,ab^3c^4-b^3c^5}\right)\sqrt{bx+a}\,\sqrt{bx+c}-\frac{3\,(a+c)\log\left(\left|-\sqrt{bx+a}+\sqrt{bx+c}\right|\right)}{b}-\frac{2\left(4\,(bx+a)^3-9\,(bx+a)^2a+6\,(bx+a)a^2+3\,(bx+a)^2c-6\,(bx+a)ac\right)}{a^2b-2\,abc+bc^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x
 + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^
4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3 +
b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3
*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^2*c^3
 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*
b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a + c)*l
og(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b*x + a)^
2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b - 2*a*b*c
 + b*c^2))/b
```

Mupad [B]
time = 37.52, size = 1012, normalized size = 6.13

$$\frac{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+bx}-\sqrt{c}\right)}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^{11}\left(\frac{a}{2}+\frac{c}{2}\right)}{b^2\left(\sqrt{c+bx}-\sqrt{c}\right)^{11}}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3\left(\frac{17a^3}{6}+\frac{101a^2c}{2}+\frac{101ac^2}{2}+\frac{17c^3}{6}\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^3\left(a^2b^2-2ab^2c+b^2c^2\right)}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^9\left(\frac{17a^3}{6}+\frac{101a^2c}{2}+\frac{101ac^2}{2}+\frac{17c^3}{6}\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^9\left(a^2b^2-2ab^2c+b^2c^2\right)}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^5\left(19a^3+269a^2c+269ac^2+19c^3\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^5\left(a^2b^2-2ab^2c+b^2c^2\right)}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^7\left(19a^3+269a^2c+269ac^2+19c^3\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^7\left(a^2b^2-2ab^2c+b^2c^2\right)}+\frac{16a^{3/2}c^{3/2}\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{c+bx}-\sqrt{c}\right)^2\left(a^2b^2-2ab^2c+b^2c^2\right)}+\frac{16a^{3/2}c^{3/2}\left(\sqrt{a+bx}-\sqrt{a}\right)^{10}}{\left(\sqrt{c+bx}-\sqrt{c}\right)^{10}\left(a^2b^2-2ab^2c+b^2c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^4\left(64a^2+192ac+64c^2\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^4\left(a^2b^2-2ab^2c+b^2c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^8\left(64a^2+192ac+64c^2\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^8\left(a^2b^2-2ab^2c+b^2c^2\right)}+\frac{\sqrt{a}\,\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)^6\left(128a^2+\frac{1312ac}{3}+128c^2\right)}{\left(\sqrt{c+bx}-\sqrt{c}\right)^6\left(a^2b^2-2ab^2c+b^2c^2\right)}}{\frac{15\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{c+bx}-\sqrt{c}\right)^4}-\frac{6\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{c+bx}-\sqrt{c}\right)^2}-\frac{20\left(\sqrt{a+bx}-\sqrt{a}\right)^6}{\left(\sqrt{c+bx}-\sqrt{c}\right)^6}+\frac{15\left(\sqrt{a+bx}-\sqrt{a}\right)^8}{\left(\sqrt{c+bx}-\sqrt{c}\right)^8}-\frac{6\left(\sqrt{a+bx}-\sqrt{a}\right)^{10}}{\left(\sqrt{c+bx}-\sqrt{c}\right)^{10}}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^{12}}{\left(\sqrt{c+bx}-\sqrt{c}\right)^{12}}+1}-\frac{\mathrm{atanh}\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{c+bx}-\sqrt{c}}\right)(a+c)}{2b^2}+\frac{x^2\,(a+c)}{2\,(a-c)^2}+\frac{2\,b\,x^3}{3\,(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))
) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^
(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)/2
+ (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b^2*c
^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (101*a^2
```

```
*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2*b^2 +
 b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 + 269*a^
2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b^2*c^2 -
 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2*c + 19*a
^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c
)) + (16*a^(3/2)*c^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) -
 c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*((a + b*
x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^2 + b^2*c^2
- 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^4*(192*a*c + 6
4*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b^2*c^2 - 2*a*b^
2*c)) + (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*(192*a*c + 64*a^2 +
64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) +
(a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c)/3 + 128*a^2 + 12
8*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)))/((
15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) - c^(1/2))^4 - (6*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 - (20*((a + b*x)^(1/
2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/2))^6 + (15*((a + b*x)^(1/2) - a^(
1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 - (6*((a + b*x)^(1/2) - a^(1/2))^10)
/((c + b*x)^(1/2) - c^(1/2))^10 + ((a + b*x)^(1/2) - a^(1/2))^12/((c + b*x)
^(1/2) - c^(1/2))^12 + 1) - (atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(
1/2) - c^(1/2)))*(a + c))/(2*b^2) + (x^2*(a + c))/(2*(a - c)^2) + (2*b*x^3)
/(3*(a - c)^2)
```

3.408 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=63

$$\frac{(a-c)^2}{8b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{2b}$$

```
[Out] 1/2*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/b+1/8*(a-c)^2/b/((b*x+a)^(1/2)+(b*
x+c)^(1/2))^4
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {6821, 52, 65, 223, 212}

$$\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{bx+c}}{2b(a-c)}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}+\frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(
2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt
[a + b*x]/Sqrt[c + b*x]]/(2*b)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(a\left(1+\frac{c}{a}\right)+2bx-2\sqrt{a+bx}\,\sqrt{c+bx}\right)dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{2\int \sqrt{a+bx}\,\sqrt{c+bx}\,dx}{(a-c)^2}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}}\,dx}{2(a-c)}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{1}{4}\int$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Sub}}{}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\text{Sub}}{}$$

$$= \frac{(a+c)x}{(a-c)^2}+\frac{bx^2}{(a-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{c+bx}}{2b(a-c)}-\frac{(a+bx)^{3/2}\sqrt{c+bx}}{b(a-c)^2}+\frac{\tan}{}$$

Mathematica [A]

time = 0.21, size = 81, normalized size = 1.29

$$\frac{\frac{2(a+bx)(c+bx)}{(a-c)^2} - \frac{\sqrt{a+bx}\,\sqrt{c+bx}\,(a+c+2bx)}{(a-c)^2} + \tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{a+bx}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2),x]
```

```
[Out] ((2*(a + b*x)*(c + b*x))/(a - c)^2 - (Sqrt[a + b*x]*Sqrt[c + b*x]*(a + c + 2*b*x))/(a - c)^2 + ArcTanh[Sqrt[c + b*x]/Sqrt[a + b*x]])/(2*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(51) = 102$.
time = 0.02, size = 377, normalized size = 5.98

method	result
default	$\frac{xa}{(a-c)^2} + \frac{xc}{(a-c)^2} + \frac{b\,x^2}{(a-c)^2} - \frac{\sqrt{bx+a}\,(bx+c)^{\frac{3}{2}}}{(a-c)^2 b} - \frac{\sqrt{bx+c}\,\sqrt{bx+a}\,a}{2(a-c)^2 b} + \frac{\sqrt{bx+c}\,\sqrt{bx+a}\,c}{2(a-c)^2 b} + \frac{\sqrt{(bx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(a-c)^2*a+x/(a-c)^2*c+b*x^2/(a-c)^2-1/(a-c)^2/b*(b*x+a)^(1/2)*(b*x+c)^(3/2)-1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*a+1/2/(a-c)^2/b*(b*x+c)^(1/2)*(b*x+a)^(1/2)*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)*a^2-1/2/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)*a*c+1/4/(a-c)^2*((b*x+a)*(b*x+c))^(1/2)/(b*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*b*c+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*c)*x+a*c)^(1/2))/(b^2)^(1/2)*c^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(51) = 102$.

time = 0.35, size = 103, normalized size = 1.63

$$\frac{4\,b^2x^2 - 2\,(2\,bx + a + c)\sqrt{bx + a}\,\sqrt{bx + c} + 4\,(ab + bc)x - (a^2 - 2\,ac + c^2)\log\left(-2\,bx + 2\,\sqrt{bx + a}\,\sqrt{bx + c} - a - c\right)}{4\,(a^2b - 2\,abc + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*x^2 - 2*(2*b*x + a + c)*sqrt(b*x + a)*sqrt(b*x + c) + 4*(a*b + b
*c)*x - (a^2 - 2*a*c + c^2)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a
- c))/(a^2*b - 2*a*b*c + b*c^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(48) = 96$.

time = 0.51, size = 388, normalized size = 6.16

$$\begin{cases} \frac{2a\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{4bx\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{2c\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{c}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} + \frac{4\sqrt{a+bx}\,\sqrt{bx+c}\log\left(\sqrt{a+bx}+\sqrt{bx+c}\right)}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\,\sqrt{bx+c}} & \text{for}\ b \neq 0 \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c
 + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqr
t(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b
 + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b
**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) +
sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c))
 + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*sqrt(
a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x
 + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(
c))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(51) = 102$.

time = 3.76, size = 189, normalized size = 3.00

$$-\frac{1}{2}\,\sqrt{bx + a}\,\sqrt{bx + c}\left(\frac{2\,(ab - bc)(bx + a)}{a^3b^2 - 3\,a^2b^2c + 3\,ab^2c^2 - b^2c^3} - \frac{a^2b - 2\,abc + bc^2}{a^3b^2 - 3\,a^2b^2c + 3\,ab^2c^2 - b^2c^3}\right) + \frac{(bx + a)^2 - (bx + a)a + (bx + a)c}{a^2b - 2\,abc + bc^2} - \frac{\log\left(\left|-\sqrt{bx + a} + \sqrt{bx + c}\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + a)*sqrt(b*x + c)*(2*(a*b - b*c)*(b*x + a)/(a^3*b^2 - 3*a^2*
b^2*c + 3*a*b^2*c^2 - b^2*c^3) - (a^2*b - 2*a*b*c + b*c^2)/(a^3*b^2 - 3*a^2
```

```
*b^2*c + 3*a*b^2*c^2 - b^2*c^3)) + ((b*x + a)^2 - (b*x + a)*a + (b*x + a)*c
)/(a^2*b - 2*a*b*c + b*c^2) - 1/2*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/
b
```

Mupad [B]
time = 0.24, size = 110, normalized size = 1.75

$$\frac{b\,x^2}{{\left(a-c\right)}^2}+\frac{x\,\left(a+c\right)}{{\left(a-c\right)}^2}+\frac{\ln\left(a+c+2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}+2\,b\,x\right)\,{\left(a\,b-b\,c\right)}^2}{4\,b^3\,{\left(a-c\right)}^2}-\frac{2\,\sqrt{a+b\,x}\,\sqrt{c+b\,x}\,\left(\frac{x}{2}+\frac{a\,b+b\,c}{4\,b^2}\right)}{{\left(a-c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)
```

```
[Out] (b*x^2)/(a - c)^2 + (x*(a + c))/(a - c)^2 + (log(a + c + 2*(a + b*x)^(1/2)*
(c + b*x)^(1/2) + 2*b*x)*(a*b - b*c)^2)/(4*b^3*(a - c)^2) - (2*(a + b*x)^(1
/2)*(c + b*x)^(1/2)*(x/2 + (a*b + b*c)/(4*b^2)))/(a - c)^2
```

3.409 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=133

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{(a+c}{(a}$$

```
[Out] 2*b*x/(a-c)^2-2*(a+c)*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+(a+c)*ln
(x)/(a-c)^2+4*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))*a^(1/2)*
c^(1/2)/(a-c)^2-2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2
```

Rubi [A]
time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6821, 103, 163, 65, 223, 212, 95, 214}

$$\frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*
ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTan
h[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Lo
g[x])/(a - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(2b+\frac{a\left(1+\frac{c}{a}\right)}{x}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{x}\right)dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x}\,dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}+\frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\sqrt{c-}}}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(2ac)\int \frac{1}{x\sqrt{a+bx}}}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{(a+c)\log(x)}{(a-c)^2}-\frac{(4ac)\text{Subst}\left(\int \frac{1}{-a+}\right.}{(a}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}+\frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2}-\frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2}-\frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{4\sqrt{}}{}$$

Mathematica [A]
time = 0.27, size = 137, normalized size = 1.03

$$\frac{\log\left(\sqrt{a}\sqrt{c}+bx-\sqrt{a+bx}\sqrt{c+bx}\right)}{\left(\sqrt{a}+\sqrt{c}\right)^2}+\frac{2\left(c+bx-\sqrt{a+bx}\sqrt{c+bx}\right)+\left(\sqrt{a}+\sqrt{c}\right)^2\log\left(\sqrt{a}\sqrt{c}-bx+\sqrt{a+bx}\sqrt{c+bx}\right)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] Log[Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c]
)^2 + (2*(c + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[a] + Sqrt[c])^2*Lo
g[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]])/(a - c)^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 258, normalized size = 1.94

method	result

default	$\frac{a\ln(x)}{(a-c)^2}+\frac{c\ln(x)}{(a-c)^2}+\frac{2bx}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(2\,\mathrm{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+ac}+2ac}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(
b*x+c)^(1/2)*(2*csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+
a*c)^(1/2)+2*a*c)/x)*a*c-2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1
/2)-ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*csgn(b))*(
a*c)^(1/2)*a-ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*b*x+a+c)*c
sgn(b))*(a*c)^(1/2)*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A]
time = 0.35, size = 290, normalized size = 2.18

$$\left[\frac{2\,bx+(a+c)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+(a+c)\log\left(x\right)+2\,\sqrt{ac}\,\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}\,(a+c)\right)\sqrt{bx+a}\,\sqrt{bx+c}+(a^2b+2\,abc+bc^2)x+2\,(2\,ac+(ab+bc)x)\sqrt{ac}}{x}\right)-2\,\sqrt{bx+a}\,\sqrt{bx+c}}{a^2-2\,ac+c^2},\ \frac{2\,bx+(a+c)\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+(a+c)\log\left(x\right)-4\,\sqrt{-ac}\,\arctan\left(-\frac{\sqrt{-ac}\,bx-\sqrt{-ac}\,\sqrt{bx+a}\,\sqrt{bx+c}}{ac}\right)-2\,\sqrt{bx+a}\,\sqrt{bx+c}}{a^2-2\,ac+c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a
+ c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c)*(a
+ c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c
+ (a*b + b*c)*x)*sqrt(a*c))/x) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*
c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a -
 c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sq
rt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2
*a*c + c^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Giac [A]
time = 4.35, size = 194, normalized size = 1.46

$$\frac{4\,ac\arctan\left(\frac{\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2-a-c}{2\sqrt{-ac}}\right)}{\left(a^2-2\,ac+c^2\right)\sqrt{-ac}}-\frac{2\left(a^2-2\,ac+c^2\right)\sqrt{bx+a}\,\sqrt{bx+c}}{a^4-4\,a^3c+6\,a^2c^2-4\,ac^3+c^4}+\frac{(a+c)\log\left(\left(\sqrt{bx+a}-\sqrt{bx+c}\right)^2\right)}{a^2-2\,ac+c^2}+\frac{(a+c)\log\left(|bx|\right)}{a^2-2\,ac+c^2}+\frac{2\,(bx+a)}{a^2-2\,ac+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((
a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b
*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(b*x
 + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(a^2
- 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)
```

Mupad [B]
time = 11.14, size = 524, normalized size = 3.94

$$\frac{2\,b\,x}{{(a-c)}^2}-\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}+1\right)\,\left(\frac{4\,c}{{(a-c)}^2}+\frac{2}{a-c}\right)-\frac{\frac{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^3\,(4\,a+4\,c)}{{\left(\sqrt{c+b\,x}-\sqrt{c}\right)}^3\,(a^2-2\,a\,c+c^2)}+\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)\,(4\,a+4\,c)}{\left(\sqrt{c+b\,x}-\sqrt{c}\right)\,(a^2-2\,a\,c+c^2)}-\frac{16\,\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{c+b\,x}-\sqrt{c}\right)}^2\,(a^2-2\,a\,c+c^2)}}{\frac{{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^4}{{\left(\sqrt{c+b\,x}-\sqrt{c}\right)}^4}-\frac{2\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{c+b\,x}-\sqrt{c}\right)}^2}+1}+\frac{2\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}-1\right)\,(a+c)}{{(a-c)}^2}+\frac{\ln\left(x\right)\,(a+c)}{a^2-2\,a\,c+c^2}+\frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{c+b\,x}-\sqrt{c}}\right)}{{(a-c)}^2}-\frac{2\,\sqrt{a}\,\sqrt{c}\,\ln\left(\frac{a\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}}-\sqrt{a}\,\sqrt{c}+\frac{c\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{c+b\,x}-\sqrt{c}}-\frac{\sqrt{a}\,\sqrt{c}\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{c+b\,x}-\sqrt{c}\right)}^2}\right)}{a^2-2\,a\,c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - ((((a + b*x)^(1/2) - a^(1/2))^3*(
4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a + b*
x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*a*c
+ c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1
/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2)))/(((a + b*x)^(1/2) - a^(1/2))^4/((c +
 b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/
2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2)
- c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c + c^2) +
 (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1
/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) - a^(1/2)))/(
(c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1/2
)))/((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/
2))^2)/((c + b*x)^(1/2) - c^(1/2))^2))/(a^2 - 2*a*c + c^2)
```

3.410 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx$

Optimal. Leaf size=141

$$-\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{\sqrt{a}\,(a-c)^2\sqrt{c}}+\frac{2b\log(x)}{(a-c)^2}$$

```
[Out] (-a-c)/(a-c)^2/x-4*b*arctanh((b*x+a)^(1/2)/(b*x+c)^(1/2))/(a-c)^2+2*b*ln(x)
/(a-c)^2+2*b*(a+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/a^(1/2)/(b*x+c)^(1/2))/(a-
c)^2/a^(1/2)/c^(1/2)+2*(b*x+a)^(1/2)*(b*x+c)^(1/2)/(a-c)^2/x
```

Rubi [A]
time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6821, 99, 163, 65, 223, 212, 95, 214}

$$\frac{2\sqrt{a+bx}\,\sqrt{bx+c}}{x(a-c)^2}+\frac{2b\log(x)}{(a-c)^2}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{bx+c}}\right)}{\sqrt{a}\,\sqrt{c}\,(a-c)^2}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2}-\frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) -
(4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (2*b*(a + c)*ArcTanh
[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[
c]) + (2*b*Log[x])/(a - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^2}\,dx = \frac{\int \left(\frac{a\left(1+\frac{c}{a}\right)}{x^2}+\frac{2b}{x}-\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}\right)dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{c+bx}}{x^2}dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{2\int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\,\sqrt{c+b}}}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{(2b^2)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c}}}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b\log(x)}{(a-c)^2}-\frac{(4b)\text{Subst}\left(\int \frac{}{\sqrt{-a+}}\right.}{(}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}+\frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{a}\,\sqrt{c+bx}}\right)}{\sqrt{a}\,(a-c)^2\sqrt{c}}$$

$$= -\frac{a+c}{(a-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{c+bx}}{(a-c)^2x}-\frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2}+\frac{2b(a+}{}$$

Mathematica [A]
time = 0.52, size = 128, normalized size = 0.91

$$\frac{\frac{2b(a+c)\tanh^{-1}\left(\frac{-bx+\sqrt{a+bx}\,\sqrt{c+bx}}{\sqrt{a}\,\sqrt{c}}\right)}{\sqrt{a}\,\sqrt{c}}-\frac{a+c-2bx-2\sqrt{a+bx}\,\sqrt{c+bx}-2bx\log\left(bx\left(a+c+2bx-2\sqrt{a+bx}\,\sqrt{c+bx}\right)\right)}{x}}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] ((2*b*(a + c)*ArcTanh[(-(b*x) + Sqrt[a + b*x]*Sqrt[c + b*x])/(Sqrt[a]*Sqrt[
c])])/(Sqrt[a]*Sqrt[c]) - (a + c - 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x] -
2*b*x*Log[b*x*(a + c + 2*b*x - 2*Sqrt[a + b*x]*Sqrt[c + b*x])])/x)/(a - c)^
2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 274, normalized size = 1.94

method	result
default	$-\frac{a}{x(a-c)^2}-\frac{c}{x(a-c)^2}+\frac{2b\ln(x)}{(a-c)^2}+\frac{\sqrt{bx+a}\,\sqrt{bx+c}\,\left(\operatorname{csgn}(b)\ln\left(\frac{abx+bcx+2\sqrt{ac}\,\sqrt{b^2x^2+abx+bcx+a}}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x
+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^
(1/2)+2*a*c)/x)*x*a*b+csgn(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+
b*c*x+a*c)^(1/2)+2*a*c)/x)*x*b*c-2*ln(1/2*(2*csgn(b)*(b^2*x^2+a*b*x+b*c*x+a
*c)^(1/2)+2*b*x+a+c)*csgn(b))*x*b*(a*c)^(1/2)+2*csgn(b)*(b^2*x^2+a*b*x+b*c*
x+a*c)^(1/2)*(a*c)^(1/2))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)
```

Fricas [A]
time = 0.38, size = 367, normalized size = 2.60

$$\left[\frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx+(ab+bc)\sqrt{ac}\,x\log\left(\frac{2\,a^2c+2\,ac^2+2\left(2\,ac+\sqrt{ac}\,(a+c)\right)\sqrt{bx+a}\,\sqrt{bx+c}+(a^2b+2\,abc+bc^2)x+2\,(2\,ac+(ab+bc)x)\sqrt{ac}}{x}\right)+2\,\sqrt{bx+a}\,\sqrt{bx+c}\,ac-a^2c-ac^2}{(a^3c-2\,a^2c^2+ac^3)x},\ \frac{2\,abcx\log\left(-2\,bx+2\,\sqrt{bx+a}\,\sqrt{bx+c}-a-c\right)+2\,abcx\log\left(x\right)+2\,abcx-2\,(ab+bc)\sqrt{-ac}\,x\arctan\left(-\frac{\sqrt{-ac}\,bx-\sqrt{-ac}\,\sqrt{bx+a}\,\sqrt{bx+c}}{ac}\right)+2\,\sqrt{bx+a}\,\sqrt{bx+c}\,ac-a^2c-ac^2}{(a^3c-2\,a^2c^2+ac^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*x*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x
*log(x) + 2*a*b*c*x + (a*b + b*c)*sqrt(a*c)*x*log((2*a^2*c + 2*a*c^2 + 2*(2
*a*c + sqrt(a*c)*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c +
b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c))/x) + 2*sqrt(b*x + a)*sqrt(b
*x + c)*a*c - a^2*c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x), (2*a*b*c*x*lo
g(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + 2*a*b*c*x*log(x) + 2*a*
b*c*x - 2*(a*b + b*c)*sqrt(-a*c)*x*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqr
t(b*x + a)*sqrt(b*x + c))/(a*c)) + 2*sqrt(b*x + a)*sqrt(b*x + c)*a*c - a^2*
c - a*c^2)/((a^3*c - 2*a^2*c^2 + a*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\sqrt{a+bx} + \sqrt{bx+c}\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(121) = 242.
time = 5.04, size = 311, normalized size = 2.21

$$\frac{2\,b\log\left(\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2\right)}{a^2 - 2\,ac + c^2} + \frac{2\,b\log\left(|bx|\right)}{a^2 - 2\,ac + c^2} + \frac{2\,(ab+bc)\arctan\left(\frac{\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2 - a - c}{2\,\sqrt{-ac}}\right)}{(a^2 - 2\,ac + c^2)\sqrt{-ac}} - \frac{4\,\left(ab\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2 + bc\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2 - a^2b + 2\,abc - bc^2\right)}{\left(\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^4 - 2\,a\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2 - 2\,c\left(\sqrt{bx+a} - \sqrt{bx+c}\right)^2 + a^2 - 2\,ac + c^2\right)(a^2 - 2\,ac + c^2)} - \frac{2\,(bx+a)b - ab + bc}{(a^2 - 2\,ac + c^2)bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*b*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + 2*b*log(ab
s(b*x))/(a^2 - 2*a*c + c^2) + 2*(a*b + b*c)*arctan(1/2*((sqrt(b*x + a) - sq
rt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 4*(a
*b*(sqrt(b*x + a) - sqrt(b*x + c))^2 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^
2 - a^2*b + 2*a*b*c - b*c^2)/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqr
t(b*x + a) - sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2
 - 2*a*c + c^2)*(a^2 - 2*a*c + c^2)) - (2*(b*x + a)*b - a*b + b*c)/((a^2 -
2*a*c + c^2)*b*x)
```

Mupad [B]
time = 28.82, size = 2500, normalized size = 17.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)
```

```
[Out] (2*b*log(x))/(a^2 - 2*a*c + c^2) - ((((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b)
/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^3*
c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2 + (
b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^3 +
a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2
)) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((a + c)
*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) - c^(1/2)
```

)^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7 + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13/2) + 231*a^(29/2)*b*c^(11/2) - 43*a^(31/2)*b*c^(9/2) + 4*a^(33/2)*b*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (4*(a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2) + 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(23/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (2*((a + b*x)^(1/2) - a^(1/2))*(73*a^4*b^2*c^14 - 570*a^5*b^2*c^13 + 2053*a^6*b^2*c^12 - 4568*a^7*b^2*c^11 + 7090*a^8*b^2*c^10 - 8156*a^9*b^2*c^9 + 7090*a^10*b^2*c^8 - 4568*a^11*b^2*c^7 + 2053*a^12*b^2*c^6 - 570*a^13*b^2*c^5 + 73*a^14*b^2*c^4))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7))))/(a - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^3*c^(25/2) - 427*a^(9/2)*b^3*c^(23/2) + 1256*a^(11/2)*b^3*c^(21/2) - 1856*a^(13/2)*b^3*c^(19/2) + 962*a^(15/2)*b^3*c^(17/2) + 962*a^(17/2)*b^3*c^(15/2) - 1856*a^(19/2)*b^3*c^(13/2) + 1256*a^(21/2)*b^3*c^(11/2) - 427*a^(23/2)*b^3*c^(9/2) + 65*a^(25/2)*b^3*c^(7/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))*4i)/(a - c

)^2 + (b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3
*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 +
4*a^12*b^3*c^4))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a
^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(
a^(7/2)*b^2*c^(29/2) + 12*a^(9/2)*b^2*c^(27/2) - 100*a^(11/2)*b^2*c^(25/2)
+ 285*a^(13/2)*b^2*c^(23/2) - 390*a^(15/2)*b^2*c^(21/2) + 192*a^(17/2)*b^2*
c^(19/2) + 192*a^(19/2)*b^2*c^(17/2) - 390*a^(21/2)*b^2*c^(15/2) + 285*a^(2
3/2)*b^2*c^(13/2) - 100*a^(25/2)*b^2*c^(11/2) + 12*a^(27/2)*b^2*c^(9/2) + a
^(29/2)*b^2*c^(7/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 +
 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*
((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^
9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^
7 + 16*a^14*b*c^6 - 4*a^15*b*c^5))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 5
6*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15
*c^7) - (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)...

3.411 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=375

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}+\frac{2(a+$$

```
[Out] -8/3*a^3*(b*x+a)^(3/2)/b^3/(a-c)^3+2/3*a^2*(a+3*c)*(b*x+a)^(3/2)/b^3/(a-c)^
3+24/5*a^2*(b*x+a)^(5/2)/b^3/(a-c)^3-4/5*a*(a+3*c)*(b*x+a)^(5/2)/b^3/(a-c)^
3-24/7*a*(b*x+a)^(7/2)/b^3/(a-c)^3+2/7*(a+3*c)*(b*x+a)^(7/2)/b^3/(a-c)^3+8/
9*(b*x+a)^(9/2)/b^3/(a-c)^3+8/3*c^3*(b*x+c)^(3/2)/b^3/(a-c)^3-2/3*c^2*(3*a+
c)*(b*x+c)^(3/2)/b^3/(a-c)^3-24/5*c^2*(b*x+c)^(5/2)/b^3/(a-c)^3+4/5*c*(3*a+
c)*(b*x+c)^(5/2)/b^3/(a-c)^3+24/7*c*(b*x+c)^(7/2)/b^3/(a-c)^3-2/7*(3*a+c)*(
b*x+c)^(7/2)/b^3/(a-c)^3-8/9*(b*x+c)^(9/2)/b^3/(a-c)^3
```

Rubi [A]
time = 0.28, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6821, 45}

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3}+\frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3}+\frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3}-\frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3}-\frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}+\frac{8(a+bx)^{9/2}}{9b^3(a-c)^3}+\frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3}-\frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3}-\frac{8(bx+c)^{9/2}}{9b^3(a-c)^3}+\frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3}-\frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3}+\frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (-8*a^3*(a + b*x)^(3/2))/(3*b^3*(a - c)^3) + (2*a^2*(a + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(a - c)^3) + (24*a^2*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (4*a*(
a + 3*c)*(a + b*x)^(5/2))/(5*b^3*(a - c)^3) - (24*a*(a + b*x)^(7/2))/(7*b^3
*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(7/2))/(7*b^3*(a - c)^3) + (8*(a + b*x
)^(9/2))/(9*b^3*(a - c)^3) + (8*c^3*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (2
*c^2*(3*a + c)*(c + b*x)^(3/2))/(3*b^3*(a - c)^3) - (24*c^2*(c + b*x)^(5/2)
)/(5*b^3*(a - c)^3) + (4*c*(3*a + c)*(c + b*x)^(5/2))/(5*b^3*(a - c)^3) + (
24*c*(c + b*x)^(7/2))/(7*b^3*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(7/2))/(7*
b^3*(a - c)^3) - (8*(c + b*x)^(9/2))/(9*b^3*(a - c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x^2\sqrt{a+bx}+4bx^3\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x^2\sqrt{c+bx}-4bx\right.}{(a-c)^3}$$

$$= \frac{(4b)\int x^3\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^3\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3}$$

$$= \frac{(4b)\int \left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx}{(a-c)^3} - \frac{(4b)\int}{}$$

$$= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{4a(a+3c}{5b^3(}$$

Mathematica [A]
time = 0.78, size = 138, normalized size = 0.37

$$\frac{2\left((a+bx)^{3/2}\left(-40a^3+12a^2(6c+5bx)-3abx(36c+25bx)+5b^2x^2(27c+28bx)\right)+(c+bx)^{3/2}\left(-9a\left(8c^2-12bcx+15b^2x^2\right)+5\left(8c^3-12bc^2x+15b^2cx^2-28b^3x^3\right)\right)\right)}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*((a + b*x)^(3/2)*(-40*a^3 + 12*a^2*(6*c + 5*b*x) - 3*a*b*x*(36*c + 25*b*
x) + 5*b^2*x^2*(27*c + 28*b*x)) + (c + b*x)^(3/2)*(-9*a*(8*c^2 - 12*b*c*x +
 15*b^2*x^2) + 5*(8*c^3 - 12*b*c^2*x + 15*b^2*c*x^2 - 28*b^3*x^3))))/(315*b
^3*(a - c)^3)
```

Maple [A]
time = 0.05, size = 294, normalized size = 0.78

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}+\frac{a^2(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}+\frac{a^2(bx+a)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}-\frac{6a\left(\frac{(bx+c)^{\frac{7}{2}}}{7}-\frac{2(bx+c)^{\frac{5}{2}}c}{5}+\frac{c^2(bx+c)^{\frac{3}{2}}}{3}\right)}{(a-c)^3b^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)^3*a/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2
))+6/(a-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(
3/2))-6/(a-c)^3*a/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c
)^(3/2))-2/(a-c)^3*c/b^3*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*
x+c)^(3/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b
*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(
b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A]
time = 0.33, size = 208, normalized size = 0.55

$$\frac{2\left((140\,b^4x^4-40\,a^4+72\,a^3c+5\,(13\,ab^3+27\,b^3c)x^3-3\,(5\,a^2b^2-9\,ab^2c)x^2+4\,(5\,a^3b-9\,a^2bc)x)\sqrt{bx+a}-(140\,b^4x^4+72\,ac^3-40\,c^4+5\,(27\,ab^3+13\,b^3c)x^3+3\,(9\,ab^2c-5\,b^2c^2)x^2-4\,(9\,abc^2-5\,bc^3)x)\sqrt{bx+c}\right)}{315\,(a^3b^3-3\,a^2b^3c+3\,ab^3c^2-b^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(
5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (14
0*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c
- 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a
^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. 2(319) = 638.
time = 5.58, size = 1447, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/315*(((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*a
^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*a^
2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 - 778*a
^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c^4 - 12
348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2*b^12*c^
8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^
15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b
^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b
^15*c^10 - 12*a*b^15*c^11 + b^15*c^12)) + 3*(145*a^11*b^12 - 1361*a^10*b^12
*c + 5719*a^9*b^12*c^2 - 14151*a^8*b^12*c^3 + 22794*a^7*b^12*c^4 - 24906*a^
6*b^12*c^5 + 18606*a^5*b^12*c^6 - 9294*a^4*b^12*c^7 + 2901*a^3*b^12*c^8 - 4
69*a^2*b^12*c^9 + 11*a*b^12*c^10 + 5*b^12*c^11)/(a^12*b^15 - 12*a^11*b^15*c
 + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^
5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c
^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))*(b*x + a) - (155*a^12*
b^12 - 1536*a^11*b^12*c + 6855*a^10*b^12*c^2 - 18170*a^9*b^12*c^3 + 31770*a
^8*b^12*c^4 - 38520*a^7*b^12*c^5 + 33222*a^6*b^12*c^6 - 20700*a^5*b^12*c^7
 + 9495*a^4*b^12*c^8 - 3320*a^3*b^12*c^9 + 915*a^2*b^12*c^10 - 186*a*b^12*c^
11 + 20*b^12*c^12)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9
*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^
5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*
b^15*c^11 + b^15*c^12))*(b*x + a) + (5*a^13*b^12 - 83*a^12*b^12*c + 543*a^1
1*b^12*c^2 - 1925*a^10*b^12*c^3 + 4070*a^9*b^12*c^4 - 4950*a^8*b^12*c^5 + 2
046*a^7*b^12*c^6 + 3894*a^6*b^12*c^7 - 8415*a^5*b^12*c^8 + 8305*a^4*b^12*c^
9 - 5005*a^3*b^12*c^10 + 1887*a^2*b^12*c^11 - 412*a*b^12*c^12 + 40*b^12*c^1
3)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*
a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495
*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15
*c^12))*sqrt(b*x + c) + 2/315*(140*(b*x + a)^(9/2) - 495*(b*x + a)^(7/2)*a
 + 630*(b*x + a)^(5/2)*a^2 - 315*(b*x + a)^(3/2)*a^3 + 135*(b*x + a)^(7/2)*c
 - 378*(b*x + a)^(5/2)*a*c + 315*(b*x + a)^(3/2)*a^2*c)/(a^3*b^3 - 3*a^2*b^
3*c + 3*a*b^3*c^2 - b^3*c^3)
```

Mupad [B]
time = 3.30, size = 529, normalized size = 1.41

$$\frac{x^3\left(\frac{64bc}{9(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)\sqrt{c+bx}}{7b}-\frac{x^3\left(\frac{64ab}{9(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)\sqrt{a+bx}}{7b}-\frac{8c^2\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{6c\left(\frac{64bc}{9(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{7b}\right)\sqrt{c+bx}}{15b^3}-\frac{x^2\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{6c\left(\frac{64bc}{9(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{7b}\right)\sqrt{c+bx}}{5b}+\frac{8a^2\left(\frac{2(a^2+3ca)}{(a-c)^3}+\frac{6a\left(\frac{64ab}{9(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{7b}\right)\sqrt{a+bx}}{15b^3}+\frac{x^2\left(\frac{2(a^2+3ca)}{(a-c)^3}+\frac{6a\left(\frac{64ab}{9(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{7b}\right)\sqrt{a+bx}}{5b}+\frac{8bx^4\sqrt{a+bx}}{9(a-c)^3}-\frac{8bx^4\sqrt{c+bx}}{9(a-c)^3}+\frac{4cx\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{6c\left(\frac{64bc}{9(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{7b}\right)\sqrt{c+bx}}{15b^2}-\frac{4ax\left(\frac{2(a^2+3ca)}{(a-c)^3}+\frac{6a\left(\frac{64ab}{9(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{7b}\right)\sqrt{a+bx}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(
a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b^3)
- (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a*c + a
^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3
))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a - c)^3 + (
6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x
)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x^4*(c + b*
x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c
)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15
*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3)
- (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^2)
```

3.412 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}-\frac{8c^2(c+bx)^{3/2}}{3b^2(a-c)^3}$$

```
[Out] 8/3*a^2*(b*x+a)^(3/2)/b^2/(a-c)^3-2/3*a*(a+3*c)*(b*x+a)^(3/2)/b^2/(a-c)^3-1
6/5*a*(b*x+a)^(5/2)/b^2/(a-c)^3+2/5*(a+3*c)*(b*x+a)^(5/2)/b^2/(a-c)^3+8/7*(
b*x+a)^(7/2)/b^2/(a-c)^3-8/3*c^2*(b*x+c)^(3/2)/b^2/(a-c)^3+2/3*c*(3*a+c)*(b
*x+c)^(3/2)/b^2/(a-c)^3+16/5*c*(b*x+c)^(5/2)/b^2/(a-c)^3-2/5*(3*a+c)*(b*x+c
)^(5/2)/b^2/(a-c)^3-8/7*(b*x+c)^(7/2)/b^2/(a-c)^3
```

Rubi [A]

time = 0.17, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6821, 45}

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3}+\frac{8(a+bx)^{7/2}}{7b^2(a-c)^3}+\frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3}-\frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3}-\frac{8(bx+c)^{7/2}}{7b^2(a-c)^3}+\frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3}-\frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3}+\frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))
/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c
)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3
) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3
/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a
 + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a -
c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)x\sqrt{a+bx}+4bx^2\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)x\sqrt{c+bx}-4bx\right.}{(a-c)^3}$$

$$= \frac{(4b)\int x^2\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b)\int x^2\sqrt{c+bx}\,dx}{(a-c)^3} - \frac{(3a+c)\int x\sqrt{c+bx}\,dx}{(a-c)^3}$$

$$= \frac{(4b)\int \left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx}{(a-c)^3} - \frac{(4b)\int \left(\frac{c^2\sqrt{c+bx}}{b^2}\right.}{}$$

$$= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a}{5b^2(a-}$$

Mathematica [A]
time = 0.61, size = 93, normalized size = 0.36

$$\frac{2\left((c+bx)^{3/2}\left(-6c^2+9bcx-20b^2x^2+7a(2c-3bx)\right)+(a+bx)^{3/2}\left(6a^2-a(14c+9bx)+bx(21c+20bx)\right)\right)}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*((c + b*x)^(3/2)*(-6*c^2 + 9*b*c*x - 20*b^2*x^2 + 7*a*(2*c - 3*b*x)) + (
a + b*x)^(3/2)*(6*a^2 - a*(14*c + 9*b*x) + b*x*(21*c + 20*b*x))))/(35*b^2*(
a - c)^3)
```

Maple [A]
time = 0.05, size = 222, normalized size = 0.85

method	result
default	$\frac{2a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^3b^2}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(a-c)^3b^2}-\frac{6a\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{(bx+c)^{\frac{3}{2}}c}{3}\right)}{(a-c)^3b^2}-\frac{2c\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{(bx+c)^{\frac{3}{2}}c}{3}\right)}{(a-c)^3b^2}+\frac{8(bx+a)}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+6/(a-c)^3*c/b^2*(1/
5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-6/(a-c)^3*a/b^2*(1/5*(b*x+c)^(5/2)-1/3
*(b*x+c)^(3/2)*c)-2/(a-c)^3*c/b^2*(1/5*(b*x+c)^(5/2)-1/3*(b*x+c)^(3/2)*c)+8
/(a-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-
8/(a-c)^3/b^2*(1/7*(b*x+c)^(7/2)-2/5*(b*x+c)^(5/2)*c+1/3*c^2*(b*x+c)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)
```

Fricas [A]
time = 0.34, size = 159, normalized size = 0.61

$$\frac{2\left((20\,b^3x^3+6\,a^3-14\,a^2c+(11\,ab^2+21\,b^2c)x^2-(3\,a^2b-7\,abc)x)\sqrt{bx+a}-(20\,b^3x^3-14\,ac^2+6\,c^3+(21\,ab^2+11\,b^2c)x^2+(7\,abc-3\,bc^2)x)\sqrt{bx+c}\right)}{35\,(a^3b^2-3\,a^2b^2c+3\,ab^2c^2-b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b
 - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 +
 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2
*c + 3*a*b^2*c^2 - b^2*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 942 vs. $2(240) = 480$.
time = 0.90, size = 942, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 14
0*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x
) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b*
*2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10
5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
 + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
```

```
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b*
*2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. 2(221) = 442.
time = 4.56, size = 866, normalized size = 3.32

$$-\frac{2\left(\left(\left(\left(bx+a\right)\left(\frac{20\left(a^6b^3-6a^5b^3c+15a^4b^3c^2-20a^3b^3c^3+15a^2b^3c^4-6ab^3c^5+b^3c^6\right)(bx+a)}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}-\frac{39a^7b^3-245a^6b^3c+651a^5b^3c^2-945a^4b^3c^3+805a^3b^3c^4-399a^2b^3c^5+105ab^3c^6-11b^3c^7}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)+\frac{3\left(6a^8b^3-41a^7b^3c+119a^6b^3c^2-189a^5b^3c^3+175a^4b^3c^4-91a^3b^3c^5+21a^2b^3c^6+ab^3c^7-b^3c^8\right)}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)(bx+a)+\frac{a^9b^3-2a^8b^3c-20a^7b^3c^2+112a^6b^3c^3-266a^5b^3c^4+364a^4b^3c^5-308a^3b^3c^6+160a^2b^3c^7-47ab^3c^8+6b^3c^9}{a^9b^4-9a^8b^4c+36a^7b^4c^2-84a^6b^4c^3+126a^5b^4c^4-126a^4b^4c^5+84a^3b^4c^6-36a^2b^4c^7+9ab^4c^8-b^4c^9}\right)\sqrt{bx+c}-\frac{20(bx+a)^{\frac{7}{2}}-49(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2+21(bx+a)^{\frac{5}{2}}c-35(bx+a)^{\frac{3}{2}}ac}{a^3b-3a^2bc+3abc^2-bc^3}\right)}{35b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3
*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8*b
^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
+ 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^3 -
245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 - 399*a
^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^
4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6
 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7*b^3*c +
 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*c^5 + 21*
a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2
- 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*
a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*a^8*b^3*c -
20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b^3*c^5 - 308*
a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^9*b^4 - 9*a^8*
b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5
 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*sqrt(b*x + c)
- (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 21*
(b*x + a)^(5/2)*c - 35*(b*x + a)^(3/2)*a*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2
- b*c^3))/b
```

Mupad [B]
time = 3.21, size = 385, normalized size = 1.48

$$\frac{x^2\left(\frac{48bc}{7(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)\sqrt{c+bx}}{5b}-\frac{x^2\left(\frac{48ab}{7(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)\sqrt{a+bx}}{5b}-\frac{2a\left(\frac{2a(a+3c)}{(a-c)^3}+\frac{4a\left(\frac{48ab}{7(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{5b}\right)\sqrt{a+bx}}{3b^2}+\frac{8bx^3\sqrt{a+bx}}{7(a-c)^3}+\frac{2c\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{4c\left(\frac{48bc}{7(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{5b}\right)\sqrt{c+bx}}{3b^2}-\frac{8bx^3\sqrt{c+bx}}{7(a-c)^3}+\frac{x\left(\frac{2a(a+3c)}{(a-c)^3}+\frac{4a\left(\frac{48ab}{7(a-c)^3}-\frac{2b(5a+3c)}{(a-c)^3}\right)}{5b}\right)\sqrt{a+bx}}{3b}-\frac{x\left(\frac{2c(3a+c)}{(a-c)^3}+\frac{4c\left(\frac{48bc}{7(a-c)^3}-\frac{2b(3a+5c)}{(a-c)^3}\right)}{5b}\right)\sqrt{c+bx}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)

[Out] (x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a
- c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (
8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)^3 +
(4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*
x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2*a*(a +
 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)
^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3 + (4*c*(
(48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/
2))/(3*b)
```

3.413 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{(a-c)^2}{10b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^5}-\frac{1}{2b\left(\sqrt{a+bx}+\sqrt{c+bx}\right)}$$

```
[Out] 1/10*(a-c)^2/b/((b*x+a)^(1/2)+(b*x+c)^(1/2))^5-1/2/b/((b*x+a)^(1/2)+(b*x+c)
^(1/2))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$,
Rules used = {6821, 45}

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3}+\frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8a(a+bx)^{3/2}}{3b(a-c)^3}-\frac{8(bx+c)^{5/2}}{5b(a-c)^3}+\frac{8c(bx+c)^{3/2}}{3b(a-c)^3}-\frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b
*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(
3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*
x)^(5/2))/(5*b*(a - c)^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}+4bx\sqrt{a+bx}-3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}-4bx\sqrt{c+\right.}{(a-c)^3}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b)\int x\sqrt{a+bx}\,dx}{(a-c)^3} - \frac{(4b}{}$$

$$= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b)\int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+}{}\right.}{(a-c)^3}$$

$$= -\frac{8a(a+bx)^{3/2}}{3b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} + \frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{8c(c+bx)^{3/2}}{3b(a-c)^3} -$$

Mathematica [A]
time = 0.49, size = 55, normalized size = 0.86

$$\frac{2\left((-5a+c-4bx)(c+bx)^{3/2}+(a+bx)^{3/2}(-a+5c+4bx)\right)}{5b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3),x]
```

```
[Out] (2*((-5*a + c - 4*b*x)*(c + b*x)^(3/2) + (a + b*x)^(3/2)*(-a + 5*c + 4*b*x)
))/(5*b*(a - c)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(52) = 104$.
time = 0.03, size = 146, normalized size = 2.28

method	result	size
default	$\frac{2a(bx+a)^{\frac{3}{2}}}{3b(a-c)^3}+\frac{2c(bx+a)^{\frac{3}{2}}}{(a-c)^3 b}-\frac{2a(bx+c)^{\frac{3}{2}}}{(a-c)^3 b}-\frac{2c(bx+c)^{\frac{3}{2}}}{3b(a-c)^3}+\frac{\frac{8(bx+a)^{\frac{5}{2}}}{5}-\frac{8(bx+a)^{\frac{3}{2}}a}{3}}{b(a-c)^3}-\frac{8\left(\frac{(bx+c)^{\frac{5}{2}}}{5}-\frac{(bx+c)^{\frac{3}{2}}c}{3}\right)}{(a-c)^3 b}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^(3/2)/b-2/(a-c)^3*a*(b*x+
c)^(3/2)/b-2/3*c*(b*x+c)^(3/2)/b/(a-c)^3+8/(a-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3
*(b*x+a)^(3/2)*a)-8/(a-c)^3/b*(1/5*(b*x+c)^(5/2)-1/3*(b*x+c)^(3/2)*c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(52) = 104$.
time = 0.34, size = 106, normalized size = 1.66

$$\frac{2\left((4b^2x^2 - a^2 + 5ac + (3ab + 5bc)x)\sqrt{bx+a} - (4b^2x^2 + 5ac - c^2 + (5ab + 3bc)x)\sqrt{bx+c}\right)}{5(a^3b - 3a^2bc + 3abc^2 - bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x
^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3
*a*b*c^2 - b*c^3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(48) = 96$.
time = 0.77, size = 384, normalized size = 6.00

$$\begin{cases} -\frac{2a}{5ab\sqrt{a+bx} + 15ab\sqrt{bx+c} + 20b^2x\sqrt{a+bx} + 20b^2x\sqrt{bx+c} + 15bc\sqrt{a+bx} + 5bc\sqrt{bx+c}} - \frac{4bx}{5ab\sqrt{a+bx} + 15ab\sqrt{bx+c} + 20b^2x\sqrt{a+bx} + 20b^2x\sqrt{bx+c} + 15bc\sqrt{a+bx} + 5bc\sqrt{bx+c}} - \frac{2c}{5ab\sqrt{a+bx} + 15ab\sqrt{bx+c} + 20b^2x\sqrt{a+bx} + 20b^2x\sqrt{bx+c} + 15bc\sqrt{a+bx} + 5bc\sqrt{bx+c}} - \frac{6\sqrt{a+bx}\sqrt{bx+c}}{5ab\sqrt{a+bx} + 15ab\sqrt{bx+c} + 20b^2x\sqrt{a+bx} + 20b^2x\sqrt{bx+c} + 15bc\sqrt{a+bx} + 5bc\sqrt{bx+c}} & \text{for } b \neq 0 \\ \frac{x}{\left(\sqrt{a}+\sqrt{c}\right)^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqr
t(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*
x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(
a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x
+ c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*
x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a
+ b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(52) = 104$.
time = 5.24, size = 427, normalized size = 6.67

$$-\frac{2}{5}\left((bx+a)\left(\frac{4(a^3b^2 - 3a^2b^2c + 3ab^2c^2 - b^2c^3)(bx+a)}{a^6b^3 - 6a^5b^3c + 15a^4b^3c^2 - 20a^3b^3c^3 + 15a^2b^3c^4 - 6ab^3c^5 + b^3c^6} - \frac{3(a^4b^2 - 4a^3b^2c + 6a^2b^2c^2 - 4ab^2c^3 + b^2c^4)}{a^6b^3 - 6a^5b^3c + 15a^4b^3c^2 - 20a^3b^3c^3 + 15a^2b^3c^4 - 6ab^3c^5 + b^3c^6}\right) - \frac{a^5b^2 - 5a^4b^2c + 10a^3b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5}{a^6b^3 - 6a^5b^3c + 15a^4b^3c^2 - 20a^3b^3c^3 + 15a^2b^3c^4 - 6ab^3c^5 + b^3c^6}\right)\sqrt{bx+c} + \frac{2\left(4(bx+a)^{\frac{5}{2}} - 5(bx+a)^{\frac{3}{2}}a + 5(bx+a)^{\frac{3}{2}}c\right)}{5(a^3b - 3a^2bc + 3abc^2 - bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a
)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4
 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4*a*
b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3
 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c + 10*a
^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5*b^3*c
 + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6
))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5*(b*x +
a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

Mupad [B]
time = 2.99, size = 252, normalized size = 3.94

$$\frac{\left(\frac{2\,\left(a^2+3\,c\,a\right)}{{\left(a-c\right)}^3}+\frac{2\,a\,\left(\frac{32\,a\,b}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(5\,a+3\,c\right)}{{\left(a-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{a+b\,x}}{b}-\frac{\left(\frac{2\,c\,\left(3\,a+c\right)}{{\left(a-c\right)}^3}+\frac{2\,c\,\left(\frac{32\,b\,c}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(3\,a+5\,c\right)}{{\left(a-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{c+b\,x}}{b}+\frac{8\,b\,x^2\,\sqrt{a+b\,x}}{5\,{\left(a-c\right)}^3}-\frac{8\,b\,x^2\,\sqrt{c+b\,x}}{5\,{\left(a-c\right)}^3}-\frac{x\,\left(\frac{32\,a\,b}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(5\,a+3\,c\right)}{{\left(a-c\right)}^3}\right)\,\sqrt{a+b\,x}}{3\,b}+\frac{x\,\left(\frac{32\,b\,c}{5\,{\left(a-c\right)}^3}-\frac{2\,b\,\left(3\,a+5\,c\right)}{{\left(a-c\right)}^3}\right)\,\sqrt{c+b\,x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)
```

```
[Out] (((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a +
3*c))/(a - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((2*c*(3*a + c))/(a - c)^3 +
 (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c + b
*x)^(1/2))/b + (8*b*x^2*(a + b*x)^(1/2))/(5*(a - c)^3) - (8*b*x^2*(c + b*x)
^(1/2))/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a -
 c)^3)*(a + b*x)^(1/2))/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*
c))/(a - c)^3)*(c + b*x)^(1/2))/(3*b)
```

3.414 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=157

$$\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+$$

```
[Out] 8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3-2*(a+3*c)*arctanh((b*x+
a)^(1/2)/a^(1/2))*a^(1/2)/(a-c)^3+2*(3*a+c)*arctanh((b*x+c)^(1/2)/c^(1/2))*
c^(1/2)/(a-c)^3+2*(a+3*c)*(b*x+a)^(1/2)/(a-c)^3-2*(3*a+c)*(b*x+c)^(1/2)/(a-
c)^3
```

Rubi [A]
time = 0.16, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6821, 52, 65, 214}

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3}+\frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}-\frac{8(bx+c)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3}-\frac{2\sqrt{a}\,(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{2\sqrt{c}\,(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) -
 (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3)
- (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[
c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(4b\sqrt{a+bx}+\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x}-4b\sqrt{c+bx}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x}\right)}{(a-c)^3}$$

$$= \frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}+\frac{(a+3c)\int \frac{\sqrt{a}}{}}{(a-c)}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}$$

$$= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3}+\frac{8(a+bx)^{3/2}}{3(a-c)^3}-\frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3}-\frac{8(c+bx)^{3/2}}{3(a-c)^3}$$

Mathematica [A]
time = 0.96, size = 244, normalized size = 1.55

$$\frac{2}{3}\left(-\frac{\sqrt{c+bx}\,(9a+7c+4bx)}{(a-c)^3}+\frac{\sqrt{a+bx}\,(7a+9c+4bx)}{(a-c)^3}-\frac{3\left(\sqrt{a}-\sqrt{c}\right)\tan^{-1}\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}}\right)}{\sqrt{-\left(\sqrt{a}-\sqrt{c}\right)^2}\left(\sqrt{a}+\sqrt{c}\right)^3}-\frac{3\left(\sqrt{a}+\sqrt{c}\right)\tan^{-1}\left(\frac{-\sqrt{a+bx}+\sqrt{c+bx}}{\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)}{\left(\sqrt{a}-\sqrt{c}\right)^3\sqrt{-\left(\sqrt{a}+\sqrt{c}\right)^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]

[Out] (2*(-((Sqrt[c + b*x]*(9*a + 7*c + 4*b*x))/(a - c)^3) + (Sqrt[a + b*x]*(7*a + 9*c + 4*b*x))/(a - c)^3 - (3*(Sqrt[a] - Sqrt[c])*ArcTan[(-Sqrt[a + b*x] + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] - Sqrt[c])^2]])/(Sqrt[-(Sqrt[a] - Sqrt[c])^2]*(Sqrt[a] + Sqrt[c])^3) - (3*(Sqrt[a] + Sqrt[c])*ArcTan[(-Sqrt[a + b*x] + Sqrt[c + b*x])/Sqrt[-(Sqrt[a] + Sqrt[c])^2]])/((Sqrt[a] - Sqrt[c])^3*Sqrt[-(Sqrt[a] + Sqrt[c])^2])))/3

Maple [A]
time = 0.01, size = 181, normalized size = 1.15

method	result
default	$\frac{a\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(a-c)^3}+\frac{8(bx+a)^{\frac{3}{2}}}{3(a-c)^3}-\frac{8(bx+c)^{\frac{3}{2}}}{3(a-c)^3}+\frac{3c\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.\right.}{(a-c)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)

[Out] 1/(a-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/3*(b*x+a)^(3/2)/(a-c)^3-8/3*(b*x+c)^(3/2)/(a-c)^3+3/(a-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(a-c)^3*a*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))-1/(a-c)^3*c*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x+c)^(1/2)/c^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)

Fricas [A]
time = 0.35, size = 516, normalized size = 3.29

$$\left[-\frac{3(a+3c)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3(3a+c)\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)-2(4bx+7a+9c)\sqrt{bx+a}+2(4bx+9a+7c)\sqrt{bx+c}}{3(a^3-3a^2c+3ac^2-c^3)},-\frac{6(3a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+3(a+3c)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2(4bx+7a+9c)\sqrt{bx+a}+2(4bx+9a+7c)\sqrt{bx+c}}{3(a^3-3a^2c+3ac^2-c^3)},\frac{6\sqrt{-a}(a+3c)\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3(3a+c)\sqrt{c}\log\left(\frac{bx-2\sqrt{bx+c}\sqrt{c}+2c}{x}\right)+2(4bx+7a+9c)\sqrt{bx+a}-2(4bx+9a+7c)\sqrt{bx+c}}{3(a^3-3a^2c+3ac^2-c^3)},\frac{2\left(3\sqrt{-a}(a+3c)\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3(3a+c)\sqrt{-c}\arctan\left(\frac{\sqrt{bx+c}\sqrt{-c}}{c}\right)+(4bx+7a+9c)\sqrt{bx+a}-(4bx+9a+7c)\sqrt{bx+c}\right)}{3(a^3-3a^2c+3ac^2-c^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")

[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x

```
+ 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c))/(a^3 - 3*
a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*sqr
t(-c)/c) + 3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
 - 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) + 2*(4*b*x + 9*a + 7*c)*sqrt(b*x + c
))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*sqrt(-a)*(a + 3*c)*arctan(sqrt(b
*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c
) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*sqrt(b*x + a) - 2*(4*b*x + 9*a + 7*c)*s
qrt(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*sqrt(-a)*(a + 3*c)*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(3*a + c)*sqrt(-c)*arctan(sqrt(b*x + c)*
sqrt(-c)/c) + (4*b*x + 7*a + 9*c)*sqrt(b*x + a) - (4*b*x + 9*a + 7*c)*sqrt(
b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2652 vs. 2(133) = 266.
time = 5.52, size = 2652, normalized size = 16.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a^
5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*a^
3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3*c^
3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*arctan(sqrt(b*x + a)/sqr
t(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) + 2/3*(4*(b*x + a)^(3/2)*
a^6 + 3*sqrt(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*sqrt(b*x + a)*a^6*
c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*sqrt(b*x + a)*a^5*c^2 - 80*(b*x + a)^(3/
2)*a^3*c^3 + 75*sqrt(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4 - 135*sq
rt(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*sqrt(b*x + a)*a^2*c^5
+ 4*(b*x + a)^(3/2)*c^6 - 51*sqrt(b*x + a)*a*c^6 + 9*sqrt(b*x + a)*c^7)/(a^
9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^5 + 84*a^3*
c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) - 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 -
6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - 2*(3*
```

a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3
)^2*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 - (3*a^2*c +
a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c
^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*
c^3 - 6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^
3)*sgn(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3
- 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*
c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) + (3*a^9*
c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c
^7 - 2*a^2*c^8 + a*c^9 - (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 2
0*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*
a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^
5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqr
t(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^
4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 +
5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2
- c^3)))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*
c + 28*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^
5*sqrt(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^
3*c^5 + 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8
*sqrt(-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3
+ 3*a^2*c - 3*a*c^2 + c^3)) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*
c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + 2*(3*a^2*c^
2 + a*c^3 + (3*a*c^2 + c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*sg
n(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2)
*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c^3 + 1
4*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 -
6*a^2*c^4 - a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn
(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 - 6*a^
3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2*c^4 -
a*c^5 + c^6)*sqrt(a*c))*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (3*a^9*c - 14
*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2
*a^2*c^8 + a*c^9 + (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 20*a^4*
c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*sqrt(a*c))*sgn(a^3 - 3*a^2*c
+ 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^5*c^4
+ 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*sqrt(a*c))*arctan(-(sqrt(b*x
+ a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2*
a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c
^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)
))/((sqrt(-a)*a^8 - a^8*sqrt(-c) - 8*sqrt(-a)*a^7*c + 8*a^7*sqrt(-c)*c + 28
*sqrt(-a)*a^6*c^2 - 28*a^6*sqrt(-c)*c^2 - 56*sqrt(-a)*a^5*c^3 + 56*a^5*sqrt
(-c)*c^3 + 70*sqrt(-a)*a^4*c^4 - 70*a^4*sqrt(-c)*c^4 - 56*sqrt(-a)*a^3*c^5
+ 56*a^3*sqrt(-c)*c^5 + 28*sqrt(-a)*a^2*c^6 - 28*a^2*sqrt(-c)*c^6 - 8*sqrt(
-a)*a*c^7 + 8*a*sqrt(-c)*c^7 + sqrt(-a)*c^8 - sqrt(-c)*c^8)*abs(-a^3 + 3*a^
2*c - 3*a*c^2 + c^3))

Mupad [B]
time = 27.72, size = 2500, normalized size = 15.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] ((((a^(1/2)*(16*a + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(16*a
 + 16*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a + b*x)^(1/2) - a^(1/2)))/((c
 + b*x)^(1/2) - c^(1/2)) + (((a^(1/2)*(12*a + 20*c))/(3*a*c^2 - 3*a^2*c + a
^3 - c^3) + (c^(1/2)*(20*a + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3))*((a +
b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + (a^(1/2)*((28*a)/3
 + 12*c))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) + (c^(1/2)*(12*a + (28*c)/3))/(3*
a*c^2 - 3*a^2*c + a^3 - c^3))/((3*((a + b*x)^(1/2) - a^(1/2)))/((c + b*x)^(
1/2) - c^(1/2)) + (3*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1
/2))^2 + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 + 1) +
 (log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a*(a^(1/2)
+ 3*c^(1/2)) + c*(3*a^(1/2) + c^(1/2))))/(3*a*c^2 - 3*a^2*c + a^3 - c^3) +
(atan(((((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/
2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5
- 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(
9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3
 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^(11/
2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(13/2
)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*
a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c^9 +
 a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4 + 22
*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(
3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2))
)^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3
/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3
+ 5*a^5*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2
) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(
11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 +
15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3
- 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*
1i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) - (((
a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/
2)*c^(1/2)))^(1/2)*((((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^
(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(
13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 -
 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c
^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4
```

+ 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2) + 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*1i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2))/((2*(a^(1/2)*c^(9/2) - 4*a^(3/2)*c^(7/2) + 6*a^(5/2)*c^(5/2) - 4*a^(7/2)*c^(3/2) + a^(9/2)*c^(1/2)))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) + (((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2) - 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*a^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c...

3.415 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx$

Optimal. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{3b(a+$$

```
[Out] -3*b*(3*a+c)*arctanh((b*x+a)^(1/2)/a^(1/2))/(a-c)^3/a^(1/2)-3*b*(a+3*c)*arc
tanh((b*x+c)^(1/2)/c^(1/2))/(-a+c)^3/c^(1/2)+8*b*(b*x+a)^(1/2)/(a-c)^3-(a+3
*c)*(b*x+a)^(1/2)/(a-c)^3/x-8*b*(b*x+c)^(1/2)/(a-c)^3+(3*a+c)*(b*x+c)^(1/2)
/(a-c)^3/x
```

Rubi [A]
time = 0.19, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6821, 43, 65, 214, 52}

$$\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{8b\sqrt{bx+c}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3}+\frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3}-\frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(a-c)^3}-\frac{8\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3}+\frac{b(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}\,(a-c)^3}+\frac{8b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (
8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8
*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh
[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c
+ b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(
(a - c)^3*Sqrt[c])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{c+bx}\right)^3}\,dx = \frac{\int \left(\frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x^2}+\frac{4b\sqrt{a+bx}}{x}-\frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x^2}-\frac{4b\sqrt{c+bx}}{x}\right.}{(a-c)^3}$$

$$=\frac{(4b)\int \frac{\sqrt{a+bx}}{x}\,dx}{(a-c)^3}-\frac{(4b)\int \frac{\sqrt{c+bx}}{x}\,dx}{(a-c)^3}-\frac{(3a+c)\int \frac{\sqrt{c+bx}}{x^2}\,dx}{(a-c)^3}$$

$$=\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}$$

$$=\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}$$

$$=\frac{8b\sqrt{a+bx}}{(a-c)^3}-\frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x}-\frac{8b\sqrt{c+bx}}{(a-c)^3}+\frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x}$$

Mathematica [A]

time = 10.28, size = 112, normalized size = 0.69

$$\frac{-\frac{(a+3c-8bx)\sqrt{a+bx}}{x}+\frac{(3a+c-8bx)\sqrt{c+bx}}{x}-\frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{3b(a+3c)\tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{\sqrt{c}}}{(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]
```

```
[Out] (-(((a + 3*c - 8*b*x)*Sqrt[a + b*x])/x) + ((3*a + c - 8*b*x)*Sqrt[c + b*x])
/x - (3*b*(3*a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a] + (3*b*(a + 3*c
)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/Sqrt[c])/(a - c)^3
```

Maple [A]
time = 0.02, size = 252, normalized size = 1.56

method	result
default	$\frac{2ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}+\frac{6cb\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(a-c)^3}-\frac{6ab\left(-\frac{\sqrt{bx+c}}{2xb}-\right.}{(a-c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+6/(a-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2))-6/(a-c)^3*a*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arctanh((b*
x+c)^(1/2)/c^(1/2)))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^(1/2)/x/b-1/2/c^(1/2)*arct
anh((b*x+c)^(1/2)/c^(1/2)))+4/(a-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh(
(b*x+a)^(1/2)/a^(1/2)))-4/(a-c)^3*b*(2*(b*x+c)^(1/2)-2*c^(1/2)*arctanh((b*x
+c)^(1/2)/c^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)
```

Fricas [A]
time = 0.38, size = 675, normalized size = 4.17

$$\left[-\frac{3\,(3\,abc+bc^{2})\sqrt{a}\,x\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+3\,(a^{2}b+3\,abc)\sqrt{c}\,x\log\left(\frac{bx-2\,\sqrt{bx+c}\,\sqrt{c}+2\,c}{x}\right)-2\,(8\,abcx-a^{2}c-3\,ac^{2})\sqrt{bx+a}+2\,(8\,abcx-3\,a^{2}c-ac^{2})\sqrt{bx+c}}{2\,(a^{4}c-3\,a^{3}c^{2}+3\,a^{2}c^{3}-ac^{4})x},\ -\frac{6\,(a^{2}b+3\,abc)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{bx+c}\,\sqrt{-c}}{c}\right)+3\,(3\,abc+bc^{2})\sqrt{a}\,x\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\,(8\,abcx-a^{2}c-3\,ac^{2})\sqrt{bx+a}+2\,(8\,abcx-3\,a^{2}c-ac^{2})\sqrt{bx+c}}{2\,(a^{4}c-3\,a^{3}c^{2}+3\,a^{2}c^{3}-ac^{4})x},\ \frac{6\,(3\,abc+bc^{2})\sqrt{-a}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-3\,(a^{2}b+3\,abc)\sqrt{c}\,x\log\left(\frac{bx-2\,\sqrt{bx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,(8\,abcx-a^{2}c-3\,ac^{2})\sqrt{bx+a}-2\,(8\,abcx-3\,a^{2}c-ac^{2})\sqrt{bx+c}}{2\,(a^{4}c-3\,a^{3}c^{2}+3\,a^{2}c^{3}-ac^{4})x},\ \frac{3\,(3\,abc+bc^{2})\sqrt{-a}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-3\,(a^{2}b+3\,abc)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{bx+c}\,\sqrt{-c}}{c}\right)+(8\,abcx-a^{2}c-3\,ac^{2})\sqrt{bx+a}-(8\,abcx-3\,a^{2}c-ac^{2})\sqrt{bx+c}}{(a^{4}c-3\,a^{3}c^{2}+3\,a^{2}c^{3}-ac^{4})x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2
*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) +
 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3
*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x),
 -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*
(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*
c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*
a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a
*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x
 - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*
x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*s
qrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x
*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x
+ a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3
*a^2*c^3 - a*c^4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(\sqrt{a+bx}+\sqrt{bx+c}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2594 vs. $2(142) = 284$.
time = 24.44, size = 2594, normalized size = 16.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

[Out] 8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a)) /((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 - (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7 - 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + 2*(a^2*c^2 + 3*a*c^3 + (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b - (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 - (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)

```
*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56
*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)
*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7
- 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a
^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 2*(3*a*b*(sqrt(b*x + a) - sqrt(b*x + c))^3
 + b*c*(sqrt(b*x + a) - sqrt(b*x + c))^3 - 3*a^2*b*(sqrt(b*x + a) - sqrt(b*
x + c)) + 2*a*b*c*(sqrt(b*x + a) - sqrt(b*x + c)) + b*c^2*(sqrt(b*x + a) -
sqrt(b*x + c)))/(((sqrt(b*x + a) - sqrt(b*x + c))^4 - 2*a*(sqrt(b*x + a) -
sqrt(b*x + c))^2 - 2*c*(sqrt(b*x + a) - sqrt(b*x + c))^2 + a^2 - 2*a*c + c^
2)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)) - (sqrt(b*x + a)*a*b + 3*sqrt(b*x + a)*
b*c)/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*b*x)
```

Mupad [B]
time = 33.22, size = 2500, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)
```

```
[Out] (b*atan(((b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c
^(3/2) + a^(3/2)*c^(1/2))))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a
^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3
*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*
a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*
(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 -
48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4
))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^
6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^
(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(1
1/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9
 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)
- (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*
a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2)
)/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6
*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3
/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*(a*c^(7/2)
+ a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2*a^(5/2)*c^2
))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^3 - a^7*c^2)
))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) +
 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a
^6*c^3 - a^7*c^2)) + (b*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c
 + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2))))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*
b*c^6 - 24*a^5*b*c^(9/2) + 24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/
2)*b*c^4 - 3*a^2*b*c^(15/2) + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a
```

```
^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2)
 - a^(1/2))*(6*a^(3/2)*b*c^8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7
/2)*b*c^6 - 48*a^5*b*c^(9/2) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^
(11/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5
*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) + (3*b*((a^(5/2)*c^(
19/2) - 5*a^(7/2)*c^(17/2) + 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a
^(13/2)*c^(11/2) + 9*a^(15/2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/
2))/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4
 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^
4*c^8 - 164*a^5*c^7 + 200*a^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 +
 4*a^10*c^2))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*
c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) -
2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)
*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a^2*c^(5/2) + 2
*a^(5/2)*c^2))/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a^5*c^4 + 5*a^6*c^
3 - a^7*c^2)))*(a*c^(7/2) + a^(7/2)*c - 3*a^3*c^(3/2) - 3*a^(3/2)*c^3 + 2*a
^2*c^(5/2) + 2*a^(5/2)*c^2)*3i)/(2*(a^2*c^7 - 5*a^3*c^6 + 10*a^4*c^5 - 10*a
^5*c^4 + 5*a^6*c^3 - a^7*c^2)))/(((9*a^(3/2)*b^2*c^(11/2))/2 - 18*a^(5/2)*b
^2*c^(9/2) + 27*a^(7/2)*b^2*c^(7/2) - 18*a^(9/2)*b^2*c^(5/2) + (9*a^(11/2)*
b^2*c^(3/2))/2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5
 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(1/2) - a^(1/2))*(72*a^3*b^2*c^4 - 72
*a^2*b^2*c^5 + 72*a^4*b^2*c^3 - 72*a^5*b^2*c^2 + 27*a^(3/2)*b^2*c^(11/2) +
36*a^(5/2)*b^2*c^(9/2) - 126*a^(7/2)*b^2*c^(7/2) + 36*a^(9/2)*b^2*c^(5/2) +
 27*a^(11/2)*b^2*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^
8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a
^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2
)*c^(1/2)))^(1/2)*((9*a^6*b*c^(7/2) - 9*a^(7/2)*b*c^6 - 24*a^5*b*c^(9/2) +
24*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4 - 3*a^2*b*c^(15/2)
 + 3*a^(15/2)*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^
7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^(1/2) - a^(1/2))*(6*a^(3/2)*b*c^
8 - 6*a^8*b*c^(3/2) + 36*a^6*b*c^(7/2) - 36*a^(7/2)*b*c^6 - 48*a^5*b*c^(9/2
) + 48*a^(9/2)*b*c^5 + 18*a^4*b*c^(11/2) - 18*a^(11/2)*b*c^4))/(2*((c + b*x
)^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*
c^5 - 6*a^8*c^4 + a^9*c^3)) - (3*b*((a^(5/2)*c^(19/2) - 5*a^(7/2)*c^(17/2)
+ 9*a^(9/2)*c^(15/2) - 5*a^(11/2)*c^(13/2) - 5*a^(13/2)*c^(11/2) + 9*a^(15/
2)*c^(9/2) - 5*a^(17/2)*c^(7/2) + a^(19/2)*c^(5/2))/(a^3*c^9 - 6*a^4*c^8 +
15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) - (((a + b*x)^(
1/2) - a^(1/2))*(4*a^2*c^10 - 28*a^3*c^9 + 88*a^4*c^8 - 164*a^5*c^7 + 200*a
^6*c^6 - 164*a^7*c^5 + 88*a^8*c^4 - 28*a^9*c^3 + 4*a^10*c^2))/(2*((c + b*x)
^(1/2) - c^(1/2))*(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c
^5 - 6*a^8*c^4 + a^9*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2
*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2...
```

3.416 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$-\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2131, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx &= -\int \sqrt{x}\,dx+\int \sqrt{1+x}\,dx\\ &= -\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 1.00

$$-\frac{2x^{3/2}}{3}+\frac{2}{3}(1+x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
default	$-\frac{2x^{\frac{3}{2}}}{3}+\frac{2(1+x)^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{\frac{4\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2\sqrt{\pi}\,x^{\frac{3}{2}}\left(2+\frac{2}{x}\right)\sqrt{1+\frac{1}{x}}}{3}}{2\sqrt{\pi}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Fricas [A]
time = 0.32, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.
time = 0.20, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\,\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x+1}}+\frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Giac [A]
time = 4.66, size = 13, normalized size = 0.62

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}-\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Mupad [B]
time = 2.97, size = 21, normalized size = 1.00

$$\frac{2\,x\,\sqrt{x+1}}{3}+\frac{2\,\sqrt{x+1}}{3}-\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*x*(x + 1)^(1/2))/3 + (2*(x + 1)^(1/2))/3 - (2*x^(3/2))/3
```

3.417 $\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

```
[Out] -2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2131, 30, 32}

$$\frac{2x^{3/2}}{3}-\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{x}}\,dx &= -\int \sqrt{-1+x}\,dx+\int \sqrt{x}\,dx\\ &= -\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 1.00

$$-\frac{2}{3}(-1+x)^{3/2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1),x]
[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.67

method	result	size
default	$-\frac{2(-1+x)^{\frac{3}{2}}}{3}+\frac{2x^{\frac{3}{2}}}{3}$	14
meijerg	$-\frac{i\left(\frac{4i\sqrt{\pi}\,x^{\frac{3}{2}}}{3}-\frac{2i\sqrt{\pi}\,x^{\frac{3}{2}}\left(2-\frac{2}{x}\right)\sqrt{1-\frac{1}{x}}}{3}\right)}{2\sqrt{\pi}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^(1/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)
```

Fricas [A]
time = 0.32, size = 13, normalized size = 0.62

$$-\frac{2}{3}\,(x-1)^{\frac{3}{2}}+\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.
time = 0.18, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x}+3\sqrt{x-1}}+\frac{4x}{3\sqrt{x}+3\sqrt{x-1}}-\frac{2}{3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

Giac [A]
time = 4.35, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Mupad [B]
time = 2.94, size = 21, normalized size = 1.00

$$\frac{2\sqrt{x-1}}{3}-\frac{2x\sqrt{x-1}}{3}+\frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + x^(1/2)),x)
```

```
[Out] (2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3
```

3.418 $\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

```
[Out] -1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {6821}

$$\frac{1}{3}(x+1)^{3/2}-\frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3
```

Rule 6821

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x}+\sqrt{1+x}}\,dx &= -\left(\frac{1}{2}\int \left(\sqrt{-1+x}-\sqrt{1+x}\right)\,dx\right)\\ &= -\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 23, normalized size = 1.00

$$-\frac{1}{3}(-1+x)^{3/2}+\frac{1}{3}(1+x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1),x]
```

[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3

Maple [A]
time = 0.01, size = 16, normalized size = 0.70

method	result	size
default	$-\frac{(-1+x)^{\frac{3}{2}}}{3}+\frac{(1+x)^{\frac{3}{2}}}{3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-1+x)^(3/2)+1/3*(1+x)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A]
time = 0.33, size = 15, normalized size = 0.65

$$\frac{1}{3}(x+1)^{\frac{3}{2}}-\frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.
time = 0.19, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1}+3\sqrt{x+1}}+\frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x -
 1) + 3*sqrt(x + 1))
```

Giac [A]
time = 6.28, size = 15, normalized size = 0.65

$$\frac{1}{3}\,(x+1)^{\frac{3}{2}}-\frac{1}{3}\,(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)
```

Mupad [B]
time = 2.84, size = 15, normalized size = 0.65

$$\frac{(x+1)^{3/2}}{3}-\frac{(x-1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
[Out] (x + 1)^(3/2)/3 - (x - 1)^(3/2)/3
```

3.419 $\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal. Leaf size=38

$$\frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}$$

```
[Out] 1/2*x^4-2/3*(-x^2+1)^(3/2)+2/5*(-x^2+1)^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6874, 272, 45}

$$\frac{x^4}{2}+\frac{2}{5}\left(1-x^2\right)^{5/2}-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx &= \int \left(2x^3+2x^3\sqrt{1-x^2}\right) dx\\ &= \frac{x^4}{2}+2\int x^3\sqrt{1-x^2}\,dx\\ &= \frac{x^4}{2}+\text{Subst}\left(\int \sqrt{1-x}\,x\,dx,x,x^2\right)\\ &= \frac{x^4}{2}+\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right) dx,x,x^2\right)\\ &= \frac{x^4}{2}-\frac{2}{3}\left(1-x^2\right)^{3/2}+\frac{2}{5}\left(1-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 44, normalized size = 1.16

$$\frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
[Out] ((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))/30
```

Maple [A]
time = 0.22, size = 33, normalized size = 0.87

method	result	size
default	$\frac{x^4}{2}+\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(x^2-1\right)\left(3x^2+2\right)}{15}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*x^4+2/15*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A]
time = 0.49, size = 31, normalized size = 0.82

$$\frac{1}{2}x^4-\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)

Fricas [A]
time = 0.35, size = 32, normalized size = 0.84

$$\frac{1}{2}x^4 + \frac{2}{15}\left(3x^4 - x^2 - 2\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")

[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.
time = 3.35, size = 77, normalized size = 2.03

$$\frac{1}{2}x^4 + \frac{1}{60}\left(\left(2\left(3\left(4x - 17\right)(x+1) + 133\right)(x+1) - 295\right)(x+1) + 195\right)\sqrt{x+1}\,\sqrt{-x+1} + \frac{1}{12}\left(\left(2\left(3x - 10\right)(x+1) + 43\right)(x+1) - 39\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")

[Out] 1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 19
5)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 3
9)*sqrt(x + 1)*sqrt(-x + 1)

Mupad [B]
time = 3.02, size = 45, normalized size = 1.18

$$\frac{x^4}{2} - \frac{\sqrt{1-x}\,\left(-\frac{2x^5}{5} - \frac{2x^4}{5} + \frac{2x^3}{15} + \frac{2x^2}{15} + \frac{4x}{15} + \frac{4}{15}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)

[Out] x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (2
*x^5)/5 + 4/15))/(x + 1)^(1/2)

3.420 $\int x^2(\sqrt{1-x}+\sqrt{1+x})^2\,dx$

Optimal. Leaf size=48

$$\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)$$

```
[Out] 2/3*x^3+1/4*arcsin(x)-1/4*x*(-x^2+1)^(1/2)+1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {6874, 285, 327, 222}

$$\frac{\text{ArcSin}(x)}{4}+\frac{2x^3}{3}-\frac{1}{4}\sqrt{1-x^2}\,x+\frac{1}{2}\sqrt{1-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx&=\int\left(2x^2+2x^2\sqrt{1-x^2}\right)\,dx\\&=\frac{2x^3}{3}+2\int x^2\sqrt{1-x^2}\,dx\\&=\frac{2x^3}{3}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\&=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{2x^3}{3}-\frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^3\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 1.23

$$\frac{1}{12}\left(8-3x\sqrt{1-x^2}+x^3\left(8+6\sqrt{1-x^2}\right)+6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] (8 - 3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 6*ArcTan[Sqrt[1 + x]/S
qrt[1 - x]])/12
```

Maple [A]
time = 0.22, size = 59, normalized size = 1.23

method	result	size
default	$\frac{2x^3}{3}+\frac{\sqrt{1-x}\,\sqrt{1+x}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^3+1/4*(1-x)^(1/2)*(1+x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+
arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.71

$$\frac{2}{3}x^3-\frac{1}{2}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{1}{4}\sqrt{-x^2+1}\,x+\frac{1}{4}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 1/2*(-x^2 + 1)^(3/2)*x + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

Fricas [A]
time = 0.34, size = 51, normalized size = 1.06

$$\frac{2}{3}x^3 + \frac{1}{4}\left(2x^3 - x\right)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{2}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1
)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 159.23, size = 270, normalized size = 5.62

$$-\frac{x^4}{4} + \frac{x^3}{3} + \frac{(x+1)^4}{4} - \frac{2(x+1)^3}{3} + \frac{(x+1)^2}{2} + 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right) - 8\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right) + 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{3} - \frac{\sqrt{1-x}\sqrt{x+1}\left(-5x-2(x+1)^3+6(x+1)^2-4\right)}{16} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] -x**4/4 + x**3/3 + (x + 1)**4/4 - 2*(x + 1)**3/3 + (x + 1)**2/2 + 4*Piecewi
se((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x +
1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) - 8*Piecewise((x*sqrt(1 - x)*sqr
t(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/
2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) + 4*Piecewise((x*sq
rt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/3 - sqrt(1 - x)*sqr
t(x + 1)*(-5*x - 2*(x + 1)**3 + 6*(x + 1)**2 - 4)/16 + 5*asin(sqrt(2)*sqrt(
x + 1)/2)/8, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.
time = 4.09, size = 76, normalized size = 1.58

$$\frac{2}{3}x^3 + \frac{1}{12}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(
-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 14.09, size = 563, normalized size = 11.73

$$\frac{\frac{4\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}-\frac{28\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{28\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{4\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}}{\frac{4\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{6\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{4\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+1}-\operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{3\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1}+\frac{23\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}-\frac{333\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{671\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}-\frac{671\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}+\frac{333\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}-\frac{23\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-\frac{3\left(\sqrt{1-x}-1\right)^{15}}{\left(\sqrt{x+1}-1\right)^{15}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{1-x}-1\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}+1}+\frac{2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] ((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/(
(x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (
4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/
((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (
4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x
+ 1)^(1/2) - 1)^8 + 1) - atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((
3*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) + (23*((1 - x)^(1/2) - 1)^3)/((x
 + 1)^(1/2) - 1)^3 - (333*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 + (6
71*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7 - (671*((1 - x)^(1/2) - 1)^
9)/((x + 1)^(1/2) - 1)^9 + (333*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)
^11 - (23*((1 - x)^(1/2) - 1)^13)/((x + 1)^(1/2) - 1)^13 - (3*((1 - x)^(1/2
) - 1)^15)/((x + 1)^(1/2) - 1)^15)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2
) - 1)^2 + (28*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^
(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(
1/2) - 1)^8 + (56*((1 - x)^(1/2) - 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1
- x)^(1/2) - 1)^12)/((x + 1)^(1/2) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x
 + 1)^(1/2) - 1)^14 + ((1 - x)^(1/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) +
(2*x^3)/3
```

3.421 $\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx$

Optimal. Leaf size=19

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] x^2-2/3*(-x^2+1)^(3/2)
```

Rubi [A]
time = 0.04, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6874, 267}

$$x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] x^2 - (2*(1 - x^2)^(3/2))/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx &= \int \left(2x+2x\sqrt{1-x^2}\right)\,dx\\ &= x^2+2\int x\sqrt{1-x^2}\,dx\\ &= x^2-\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 25, normalized size = 1.32

$$\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] ((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))/3
```

Maple [A]
time = 0.21, size = 24, normalized size = 1.26

method	result	size
default	$x^{2}+\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(x^{2}-1\right)}{3}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^2+2/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)
```

Maxima [A]
time = 0.48, size = 15, normalized size = 0.79

$$x^{2}-\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] x^2 - 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.36, size = 23, normalized size = 1.21

$$x^{2}+\frac{2}{3}\left(x^{2}-1\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] x^2 + 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A]
time = 53.90, size = 144, normalized size = 7.58

$$-\frac{x^{3}}{3}-x+\frac{\left(x+1\right)^{3}}{3}-4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)+4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] -x**3/3 - x + (x + 1)**3/3 - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asi
n(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(
2)))) + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**
(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x
+ 1) > -sqrt(2)))) - 1
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(15) = 30$.
time = 4.01, size = 51, normalized size = 2.68

$$(x+1)^2 + \frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}\,(x-2)\sqrt{-x+1} - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x +
 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2
```

Mupad [B]
time = 2.98, size = 33, normalized size = 1.74

$$x^2 - \frac{\sqrt{1-x}\,\left(-\frac{2x^3}{3} - \frac{2x^2}{3} + \frac{2x}{3} + \frac{2}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^2 - ((1 - x)^(1/2)*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x + 1)^(1/2)
```

3.422 $\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx$

Optimal. Leaf size=19

$$2x+x\sqrt{1-x^2}+\sin^{-1}(x)$$

```
[Out] 2*x+arcsin(x)+x*(-x^2+1)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6874, 201, 222}

$$\text{ArcSin}(x)+\sqrt{1-x^2}\,x+2x$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(\sqrt{1-x}+\sqrt{1+x}\right)^2 \, dx &= \int \left(2+2\sqrt{1-x^2}\right) \, dx\\
&= 2x+2\int \sqrt{1-x^2} \, dx\\
&= 2x+x\sqrt{1-x^2}+\int \frac{1}{\sqrt{1-x^2}} \, dx\\
&= 2x+x\sqrt{1-x^2}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 37, normalized size = 1.95

$$2+x\left(2+\sqrt{1-x^2}\right)+2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]
```

```
[Out] 2 + x*(2 + Sqrt[1 - x^2]) + 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.
time = 0.21, size = 58, normalized size = 3.05

method	result	size
default	$2x+\sqrt{1-x}\,(1+x)^{\frac{3}{2}}-\sqrt{1-x}\,\sqrt{1+x}+\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*x+(1-x)^(1/2)*(1+x)^(3/2)-(1-x)^(1/2)*(1+x)^(1/2)+((1-x)*(1+x))^(1/2)/(1+
x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.89

$$\sqrt{-x^2+1}\,x+2\,x+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.
time = 0.40, size = 40, normalized size = 2.11

$$\sqrt{x+1}\,x\sqrt{-x+1}+2\,x-2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/
x)
```

Sympy [A]
time = 16.25, size = 61, normalized size = 3.21

$$2x + 4\left(\begin{cases} \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] 2*x + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2
)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) + 2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.
time = 4.82, size = 48, normalized size = 2.53

$$\sqrt{x+1}\,(x-2)\sqrt{-x+1} + 2\,x + 2\,\sqrt{x+1}\,\sqrt{-x+1} + 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1)) + 2
```

Mupad [B]
time = 7.72, size = 206, normalized size = 10.84

$$2\,x - 4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{4\,\left(\sqrt{1-x}-1\right)}{\sqrt{x+1}-1} - \frac{28\,{\left(\sqrt{1-x}-1\right)}^3}{{\left(\sqrt{x+1}-1\right)}^3} + \frac{28\,{\left(\sqrt{1-x}-1\right)}^5}{{\left(\sqrt{x+1}-1\right)}^5} - \frac{4\,{\left(\sqrt{1-x}-1\right)}^7}{{\left(\sqrt{x+1}-1\right)}^7}}{\frac{4\,{\left(\sqrt{1-x}-1\right)}^2}{{\left(\sqrt{x+1}-1\right)}^2} + \frac{6\,{\left(\sqrt{1-x}-1\right)}^4}{{\left(\sqrt{x+1}-1\right)}^4} + \frac{4\,{\left(\sqrt{1-x}-1\right)}^6}{{\left(\sqrt{x+1}-1\right)}^6} + \frac{{\left(\sqrt{1-x}-1\right)}^8}{{\left(\sqrt{x+1}-1\right)}^8} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 2*x - 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

3.423 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x} dx$

Optimal. Leaf size=32

$$2\sqrt{1-x^2} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\log(x)$$

```
[Out] -2*arctanh((-x^2+1)^(1/2))+2*ln(x)+2*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6874, 272, 52, 65, 212}

$$2\sqrt{1-x^2} - 2\tanh^{-1}\left(\sqrt{1-x^2}\right) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx &= \int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx \\
&= 2\log(x)+2\int \frac{\sqrt{1-x^2}}{x}\,dx \\
&= 2\log(x)+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)+\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right) \\
&= 2\sqrt{1-x^2}+2\log(x)-2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right) \\
&= 2\sqrt{1-x^2}-2\tanh^{-1}\left(\sqrt{1-x^2}\right)+2\log(x)
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 81 vs. $2(32) = 64$.
time = 0.18, size = 81, normalized size = 2.53

$$
\frac{4\left(-1+\sqrt{1-x}\right)^2\left(-1+\sqrt{1+x}\right)^2}{\left(-2+\sqrt{1-x}+\sqrt{1+x}\right)^2}-8\tanh^{-1}\left(\frac{-2-x+2\sqrt{1+x}}{-2+2\sqrt{1-x}+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]
```

```
[Out] (4*(-1 + Sqrt[1 - x])^2*(-1 + Sqrt[1 + x])^2)/(-2 + Sqrt[1 - x] + Sqrt[1 +
x])^2 - 8*ArcTanh[(-2 - x + 2*Sqrt[1 + x])/(-2 + 2*Sqrt[1 - x] + x)]
```

Maple [A]
time = 0.21, size = 51, normalized size = 1.59

method	result	size
default	$2\ln\left(x\right)+\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(x)+2*(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/
(-x^2+1)^(1/2)))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 1.28

$$2\sqrt{-x^{2}+1}+2\log\left(x\right)-2\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 0.32, size = 41, normalized size = 1.28

$$2\sqrt{x+1}\,\sqrt{-x+1}+2\log\left(x\right)+2\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1
)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x, x)
```

Giac [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [B]

time = 4.10, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+2\ln\left(x\right)+\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(((1 - x)^(1/
2) - 1)/((x + 1)^(1/2) - 1)) + 2*log(x) + (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.424 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2} \, dx$

Optimal. Leaf size=26

$$-\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)$$

[Out] -2/x-2*arcsin(x)-2*(-x^2+1)^(1/2)/x

Rubi [A]
time = 0.06, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {6874, 283, 222}

$$-2\text{ArcSin}(x)-\frac{2\sqrt{1-x^2}}{x}-\frac{2}{x}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]

[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx &= \int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx \\
&= -\frac{2}{x}+2\int \frac{\sqrt{1-x^2}}{x^2}\,dx \\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2}{x}-\frac{2\sqrt{1-x^2}}{x}-2\sin^{-1}(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.10, size = 39, normalized size = 1.50

$$
-\frac{2\left(1+\sqrt{1-x^2}+2x\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)\right)}{x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]
```

```
[Out] (-2*(1 + Sqrt[1 - x^2] + 2*x*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]))/x
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.22, size = 50, normalized size = 1.92

method	result	size
default	$-\frac{2}{x}+\frac{2\left(-\arcsin(x)x-\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{1+x}}{x\sqrt{-x^2+1}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/x/(-x^2+1)^(1/
2)
```

Maxima [**A**]
time = 0.49, size = 24, normalized size = 0.92

$$
-\frac{2\sqrt{-x^2+1}}{x}-\frac{2}{x}-2\,\arcsin\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.69

$$\frac{2\left(2x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
- 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.
time = 4.47, size = 149, normalized size = 5.73

$$-2\pi-\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}-\frac{2}{x}-4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sq
rt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
- sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-
x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B]
time = 3.79, size = 120, normalized size = 4.62

$$8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}-\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^2
)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)
 - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) - 2/x
```

3.425 $\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] -1/x^2+arctanh((-x^2+1)^(1/2))-(-x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6874, 272, 43, 65, 212}

$$-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
```

```
[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx &= \int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\ &= -\frac{1}{x^2}+2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\ &= -\frac{1}{x^2}+\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\ &= -\frac{1}{x^2}-\frac{\sqrt{1-x^2}}{x^2}+\tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 43, normalized size = 1.26

$$-\frac{1+\sqrt{1-x^2}+2ix^2\tan^{-1}\left(x+i\sqrt{1-x^2}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]
[Out] -((1 + Sqrt[1 - x^2] + (2*I)*x^2*ArcTan[x + I*Sqrt[1 - x^2]])/x^2)
```

Maple [A]
time = 0.21, size = 58, normalized size = 1.71

method	result	size
default	$-\frac{1}{x^{2}}+\frac{\sqrt{1-x}\,\sqrt{1+x}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right) x^{2}-\sqrt{-x^{2}+1}\right)}{x^{2} \sqrt{-x^{2}+1}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x^2+(1-x)^(1/2)*(1+x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2
))/x^2/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.59

$$-\sqrt{-x^{2}+1}-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}-\frac{1}{x^{2}}+\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1) - (-x^2 + 1)^(3/2)/x^2 - 1/x^2 + log(2*sqrt(-x^2 + 1)/abs(x
) + 2/abs(x))
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.29

$$-\frac{x^{2}\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\,\sqrt{-x+1}+1}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)
/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [B]
time = 4.88, size = 189, normalized size = 5.56

$$\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)+\frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}-\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}-\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - log(((1 - x)^(1/2) - 1)^2/((
x + 1)^(1/2) - 1)^2 - 1) + ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 - (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) - 1/x^2
```

3.426 $\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3}+\frac{4a(a+cx)^{5/2}}{5(b-c)c^3}-\frac{2(a+cx)^{7/2}}{7(b-c)c^3}$$

```
[Out] 2/3*a^2*(b*x+a)^(3/2)/b^3/(b-c)-4/5*a*(b*x+a)^(5/2)/b^3/(b-c)+2/7*(b*x+a)^(
7/2)/b^3/(b-c)-2/3*a^2*(c*x+a)^(3/2)/(b-c)/c^3+4/5*a*(c*x+a)^(5/2)/(b-c)/c^
3-2/7*(c*x+a)^(7/2)/(b-c)/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2128, 45}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)}-\frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)}+\frac{2(a+bx)^{7/2}}{7b^3(b-c)}-\frac{4a(a+bx)^{5/2}}{5b^3(b-c)}-\frac{2(a+cx)^{7/2}}{7c^3(b-c)}+\frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b -
 c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b
- c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*
(b - c)*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\int x^2\sqrt{a+bx}\,dx}{b-c} - \frac{\int x^2\sqrt{a+cx}\,dx}{b-c}$$
$$= \frac{\int \left(\frac{a^2\sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2}\right)dx}{b-c} - \frac{\int \left(\frac{a^2\sqrt{a+cx}}{c^2} - \frac{2a(a+cx)^{3/2}}{c^2}\right.}{b-c}$$
$$= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx}{5(b-c)}$$

Mathematica [A]
time = 1.09, size = 155, normalized size = 1.05

$$\frac{30b^3c^3x^3\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+6ab^2c^2x^2\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)+8a^2bcx\left(-c^2\sqrt{a+bx}+b^2\sqrt{a+cx}\right)+16a^3\left(c^3\sqrt{a+bx}-b^3\sqrt{a+cx}\right)}{105b^3(b-c)c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (30*b^3*c^3*x^3*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 6*a*b^2*c^2*x^2*(c*Sqrt[a
 + b*x] - b*Sqrt[a + c*x]) + 8*a^2*b*c*x*(-(c^2*Sqrt[a + b*x]) + b^2*Sqrt[a
 + c*x]) + 16*a^3*(c^3*Sqrt[a + b*x] - b^3*Sqrt[a + c*x]))/(105*b^3*(b - c)
*c^3)
```

Maple [A]
time = 0.04, size = 90, normalized size = 0.61

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{(b-c)b^3} - \frac{2\left(\frac{(cx+a)^{\frac{7}{2}}}{7}-\frac{2a(cx+a)^{\frac{5}{2}}}{5}+\frac{a^2(cx+a)^{\frac{3}{2}}}{3}\right)}{(b-c)c^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)/b^3*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-2
/(b-c)/c^3*(1/7*(c*x+a)^(7/2)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A]
time = 0.34, size = 122, normalized size = 0.83

$$\frac{2\left(\left(15\,b^{3}c^{3}x^{3}+3\,ab^{2}c^{3}x^{2}-4\,a^{2}bc^{3}x+8\,a^{3}c^{3}\right)\sqrt{bx+a}-\left(15\,b^{3}c^{3}x^{3}+3\,ab^{3}c^{2}x^{2}-4\,a^{2}b^{3}cx+8\,a^{3}b^{3}\right)\sqrt{cx+a}\right)}{105\,\left(b^{4}c^{3}-b^{3}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*sqrt(
b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*s
qrt(c*x + a))/(b^4*c^3 - b^3*c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(123) = 246.
time = 3.96, size = 451, normalized size = 3.07

$$-\frac{2}{105}\,\sqrt{ab^{2}+(bx+a)bc-abc}\left(\left(3\,(bx+a)\left(\frac{5\,(b^{17}c^{5}|b|-2\,b^{16}c^{6}|b|+b^{15}c^{7}|b|)(bx+a)}{b^{23}c^{5}-3\,b^{22}c^{6}+3\,b^{21}c^{7}-b^{20}c^{8}}+\frac{ab^{18}c^{4}|b|-17\,ab^{17}c^{5}|b|+31\,ab^{16}c^{6}|b|-15\,ab^{15}c^{7}|b|}{b^{23}c^{5}-3\,b^{22}c^{6}+3\,b^{21}c^{7}-b^{20}c^{8}}\right)-\frac{4\,a^{2}b^{19}c^{3}|b|-2\,a^{2}b^{18}c^{4}|b|-53\,a^{2}b^{17}c^{5}|b|+96\,a^{2}b^{16}c^{6}|b|-45\,a^{2}b^{15}c^{7}|b|}{b^{23}c^{5}-3\,b^{22}c^{6}+3\,b^{21}c^{7}-b^{20}c^{8}}\right)(bx+a)+\frac{8\,a^{3}b^{20}c^{2}|b|-12\,a^{3}b^{19}c^{3}|b|+3\,a^{3}b^{18}c^{4}|b|-17\,a^{3}b^{17}c^{5}|b|+33\,a^{3}b^{16}c^{6}|b|-15\,a^{3}b^{15}c^{7}|b|}{b^{23}c^{5}-3\,b^{22}c^{6}+3\,b^{21}c^{7}-b^{20}c^{8}}\right)+\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}\right)}{105\,(b^{4}-b^{3}c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/105*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*abs(b
) - 2*b^16*c^6*abs(b) + b^15*c^7*abs(b))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6 +
 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*abs(b) - 17*a*b^17*c^5*abs(b) + 31*a*
b^16*c^6*abs(b) - 15*a*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7
 - b^20*c^8)) - (4*a^2*b^19*c^3*abs(b) - 2*a^2*b^18*c^4*abs(b) - 53*a^2*b^1
7*c^5*abs(b) + 96*a^2*b^16*c^6*abs(b) - 45*a^2*b^15*c^7*abs(b))/(b^23*c^5 -
 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*abs(b) -
12*a^3*b^19*c^3*abs(b) + 3*a^3*b^18*c^4*abs(b) - 17*a^3*b^17*c^5*abs(b) + 3
3*a^3*b^16*c^6*abs(b) - 15*a^3*b^15*c^7*abs(b))/(b^23*c^5 - 3*b^22*c^6 + 3*
```

```
b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2)/(b^4 - b^3*c)
```

Mupad [B]
time = 2.92, size = 179, normalized size = 1.22

$$\frac{2x^3\sqrt{a+bx}}{7(b-c)}-\frac{2x^3\sqrt{a+cx}}{7(b-c)}+\frac{16a^3\sqrt{a+bx}}{105b^3(b-c)}-\frac{16a^3\sqrt{a+cx}}{105c^3(b-c)}+\frac{2ax^2\sqrt{a+bx}}{35b(b-c)}-\frac{8a^2x\sqrt{a+bx}}{105b^2(b-c)}-\frac{2ax^2\sqrt{a+cx}}{35c(b-c)}+\frac{8a^2x\sqrt{a+cx}}{105c^2(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^3*(a + b*x)^(1/2))/(7*(b - c)) - (2*x^3*(a + c*x)^(1/2))/(7*(b - c)) +
 (16*a^3*(a + b*x)^(1/2))/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^(1/2))/(105
*c^3*(b - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(b - c)) - (8*a^2*x*(a + b*
x)^(1/2))/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^(1/2))/(35*c*(b - c)) + (8
*a^2*x*(a + c*x)^(1/2))/(105*c^2*(b - c))
```

3.427 $\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=95

$$-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}+\frac{2(a+bx)^{5/2}}{5b^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3(b-c)c^2}-\frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

```
[Out] -2/3*a*(b*x+a)^(3/2)/b^2/(b-c)+2/5*(b*x+a)^(5/2)/b^2/(b-c)+2/3*a*(c*x+a)^(3
/2)/(b-c)/c^2-2/5*(c*x+a)^(5/2)/(b-c)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2128, 45}

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)}-\frac{2a(a+bx)^{3/2}}{3b^2(b-c)}-\frac{2(a+cx)^{5/2}}{5c^2(b-c)}+\frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)
) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*
c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx = \frac{\int x\sqrt{a+bx}\,dx}{b-c} - \frac{\int x\sqrt{a+cx}\,dx}{b-c}$$
$$= \frac{\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{b-c} - \frac{\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)dx}{b-c}$$
$$= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)} + \frac{2(a+bx)^{5/2}}{5b^2(b-c)} + \frac{2a(a+cx)^{3/2}}{3(b-c)c^2} - \frac{2(a+cx)^{5/2}}{5(b-c)c^2}$$

Mathematica [A]
time = 0.74, size = 113, normalized size = 1.19

$$\frac{6b^2c^2x^2\left(\sqrt{a+bx}-\sqrt{a+cx}\right)+2abcx\left(c\sqrt{a+bx}-b\sqrt{a+cx}\right)+a^2\left(-4c^2\sqrt{a+bx}+4b^2\sqrt{a+cx}\right)}{15b^2(b-c)c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (6*b^2*c^2*x^2*(Sqrt[a + b*x] - Sqrt[a + c*x]) + 2*a*b*c*x*(c*Sqrt[a + b*x]
 - b*Sqrt[a + c*x]) + a^2*(-4*c^2*Sqrt[a + b*x] + 4*b^2*Sqrt[a + c*x]))/(15
*b^2*(b - c)*c^2)
```

Maple [A]
time = 0.03, size = 66, normalized size = 0.69

method	result	size
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)b^2}-\frac{2\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)c^2}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-2/(b-c)/c^2*(1/5*(c*x+a
)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [**A**]
time = 0.33, size = 92, normalized size = 0.97

$$\frac{2\left(\left(3\,b^2c^2x^2+abc^2x-2\,a^2c^2\right)\sqrt{bx+a}-\left(3\,b^2c^2x^2+ab^2cx-2\,a^2b^2\right)\sqrt{cx+a}\right)}{15\,(b^3c^2-b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*sqrt(b*x + a) - (3*b^2*c^2*x^
2 + a*b^2*c*x - 2*a^2*b^2)*sqrt(c*x + a))/(b^3*c^2 - b^2*c^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(79) = 158.
time = 4.90, size = 255, normalized size = 2.68

$$-\frac{2}{15}\sqrt{ab^2+(bx+a)bc-abc}\left((bx+a)\left(\frac{3\,(b^9c^3|b|-b^8c^4|b|)(bx+a)}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}+\frac{ab^{10}c^2|b|-7\,ab^9c^3|b|+6\,ab^8c^4|b|}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right)-\frac{2\,a^2b^{11}c|b|-a^2b^{10}c^2|b|-4\,a^2b^9c^3|b|+3\,a^2b^8c^4|b|}{b^{14}c^3-2\,b^{13}c^4+b^{12}c^5}\right)+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,(b^3-b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/15*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) - b
^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2*a
bs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b
^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*abs(
b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) + 2/15*(3*(b
*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)
```

Mupad [**B**]
time = 2.86, size = 129, normalized size = 1.36

$$\frac{2\,x^2\,\sqrt{a+b\,x}}{5\,\left(b-c\right)}-\frac{2\,x^2\,\sqrt{a+c\,x}}{5\,\left(b-c\right)}-\frac{4\,a^2\,\sqrt{a+b\,x}}{15\,b^2\,\left(b-c\right)}+\frac{4\,a^2\,\sqrt{a+c\,x}}{15\,c^2\,\left(b-c\right)}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{15\,b\,\left(b-c\right)}-\frac{2\,a\,x\,\sqrt{a+c\,x}}{15\,c\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c)) -
 (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c^2
*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1/2)
)/(15*c*(b - c))
```

3.428 $\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}$$

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2128, 32}

$$\frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \sqrt{a+bx}\,dx}{b-c}-\frac{\int \sqrt{a+cx}\,dx}{b-c}\\ &= \frac{2(a+bx)^{3/2}}{3b(b-c)}-\frac{2(a+cx)^{3/2}}{3(b-c)c}\end{aligned}$$

Mathematica [A]
time = 0.49, size = 71, normalized size = 1.51

$$\frac{2ac\sqrt{a+bx}+2bcx\sqrt{a+bx}-2ab\sqrt{a+cx}-2bcx\sqrt{a+cx}}{3b^2c-3bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]
```

```
[Out] (2*a*c*Sqrt[a + b*x] + 2*b*c*x*Sqrt[a + b*x] - 2*a*b*Sqrt[a + c*x] - 2*b*c*
x*Sqrt[a + c*x])/(3*b^2*c - 3*b*c^2)
```

Maple [A]
time = 0.01, size = 40, normalized size = 0.85

method	result	size
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3b(b-c)}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)c}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b/(b-c)-2/3*(c*x+a)^(3/2)/(b-c)/c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A]
time = 0.33, size = 50, normalized size = 1.06

$$\frac{2\left((bcx+ac)\sqrt{bx+a}-(bcx+ab)\sqrt{cx+a}\right)}{3\left(b^2c-bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*((b*c*x + a*c)*sqrt(b*x + a) - (b*c*x + a*b)*sqrt(c*x + a))/(b^2*c - b*
c^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(39) = 78.
time = 5.18, size = 107, normalized size = 2.28

$$-\frac{2\left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2}+\frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2}\right)\sqrt{ab^2+(bx+a)bc-abc}-\frac{(bx+a)^{\frac{3}{2}}}{b-c}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*a
bs(b))/(b^5*c - b^4*c^2))*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)^(
3/2)/(b - c))/b
```

Mupad [B]
time = 2.91, size = 79, normalized size = 1.68

$$\frac{2\,x\,\sqrt{a+b\,x}}{3\,\left(b-c\right)}-\frac{2\,x\,\sqrt{a+c\,x}}{3\,\left(b-c\right)}+\frac{2\,a\,\sqrt{a+b\,x}}{3\,b\,\left(b-c\right)}-\frac{2\,a\,\sqrt{a+c\,x}}{3\,c\,\left(b-c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] (2*x*(a + b*x)^(1/2))/(3*(b - c)) - (2*x*(a + c*x)^(1/2))/(3*(b - c)) + (2*
a*(a + b*x)^(1/2))/(3*b*(b - c)) - (2*a*(a + c*x)^(1/2))/(3*c*(b - c))
```

3.429 $\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)+2*arctanh((c*x+a)^(1/2)/a^(
1/2))*a^(1/2)/(b-c)+2*(b*x+a)^(1/2)/(b-c)-2*(c*x+a)^(1/2)/(b-c)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6822, 52, 65, 214}

$$\frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
```

```
[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]
)/(b - c)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx &= \frac{\int \left(\frac{\sqrt{a+bx}}{x}-\frac{\sqrt{a+cx}}{x}\right)dx}{b-c} \\
&= \frac{\int \frac{\sqrt{a+bx}}{x}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x}\,dx}{b-c} \\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{a\int \frac{1}{x\sqrt{a+bx}}\,dx}{b-c}-\frac{a\int \frac{1}{x\sqrt{a+cx}}\,dx}{b-c} \\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}+\frac{(2a)\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b(b-c)}-\frac{(2a)\mathrm{Su}\cdots}{\cdots} \\
&= \frac{2\sqrt{a+bx}}{b-c}-\frac{2\sqrt{a+cx}}{b-c}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c}+\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right)}{b-c}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 263 vs. 2(97) = 194.
time = 1.40, size = 263, normalized size = 2.71

$$
2\left(\frac{\sqrt{a+bx}}{b-c}+\frac{\sqrt{a+cx}}{-b+c}+\frac{\sqrt{a}\left(\sqrt{b}-\sqrt{\frac{b}{c}}\sqrt{c}\right)\tan^{-1}\left(\frac{\sqrt{c}\left(-\sqrt{a+bx}+\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{\sqrt{a}\sqrt{-\left(\sqrt{b}-\sqrt{c}\right)^2}}\right)}{\sqrt{b}\sqrt{-\left(\sqrt{b}-\sqrt{c}\right)^2}\left(\sqrt{b}+\sqrt{c}\right)}+\frac{\sqrt{a}\left(\sqrt{b}+\sqrt{\frac{b}{c}}\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{c}\left(-\sqrt{a+bx}+\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)}{\sqrt{a}\left(\sqrt{b}+\sqrt{c}\right)}\right)}{\sqrt{b}\,(b-c)}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1),x]
[Out] 2*(Sqrt[a + b*x]/(b - c) + Sqrt[a + c*x]/(-b + c) + (Sqrt[a]*(Sqrt[b] - Sqr
t[b/c]*Sqrt[c])*ArcTan[(Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[b/c]*Sqrt[a + c*x]))
```

```
/(Sqrt[a]*Sqrt[-(Sqrt[b] - Sqrt[c])^2])])/(Sqrt[b]*Sqrt[-(Sqrt[b] - Sqrt[c]
)^2]*(Sqrt[b] + Sqrt[c])) + (Sqrt[a]*(Sqrt[b] + Sqrt[b/c]*Sqrt[c])*ArcTanh[
(Sqrt[c]*(-Sqrt[a + b*x] + Sqrt[b/c]*Sqrt[a + c*x]))/(Sqrt[a]*(Sqrt[b] + Sq
rt[c]))])/(Sqrt[b]*(b - c)))
```

Maple [A]
time = 0.01, size = 73, normalized size = 0.75

method	result	size
default	$\frac{2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b-c}-\frac{2\sqrt{cx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{b-c}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/(b-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-1/(b-c)*
(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)
```

Fricas [A]
time = 0.35, size = 158, normalized size = 1.63

$$\left[-\frac{\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}+2\sqrt{cx+a}}{b-c},\ \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-a}\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}-\sqrt{cx+a}\right)}{b-c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x
- 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b
 - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}+\sqrt{a+cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(81) = 162$.
time = 4.71, size = 1093, normalized size = 11.27

$$-\frac{2\sqrt{ab^2+(bx+a)bc-abc}\,|b|}{b^3-b^2c}+\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,(b-c)}+\frac{2\sqrt{bx+a}}{b-c}-\frac{2\left(2\left(ab^3c-ab^2c^2\right)\left(ab^2-abc\right)^2\sqrt{-a}\,|b|\,\mathrm{sgn}(b-c)+2\left(ab^3-ab^2c\right)\left(ab^2-abc\right)^2\sqrt{-abc}\,|b|+\left(a^2b^5-3a^2b^4c+3a^2b^3c^2-a^2b^2c^3\right)\sqrt{-abc}\,|ab^2-abc|\,|b|\,\mathrm{sgn}(b-c)+\left(a^2b^6-3a^2b^5c+3a^2b^4c^2-a^2b^3c^3\right)\sqrt{-a}\,|ab^2-abc|\,|b|+\left(a^3b^7c-2a^3b^6c^2+2a^3b^4c^4-a^3b^3c^5\right)\sqrt{-a}\,|b|\,\mathrm{sgn}(b-c)+\left(a^3b^7-2a^3b^6c+2a^3b^4c^3-a^3b^3c^4\right)\sqrt{-abc}\,|b|\right)\arctan\left(-\frac{\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}}{\sqrt{-\frac{ab^3-abc^2+\sqrt{\left(ab^3-abc^2\right)^2-\left(a^2b^5-3a^2b^4c+3a^2b^3c^2-a^2b^2c^3\right)(b-c)}}{b-c}}}\right)}{\left(b^8-5b^7c+10b^6c^2-10b^5c^3+5b^4c^4-b^3c^5\right)a^2|ab^2-abc|}+\frac{2\left(2\left(ab^3c-ab^2c^2\right)\left(ab^2-abc\right)^2\sqrt{-a}\,|b|\,\mathrm{sgn}(b-c)+2\left(ab^3-ab^2c\right)\left(ab^2-abc\right)^2\sqrt{-abc}\,|b|+\left(a^2b^5-3a^2b^4c+3a^2b^3c^2-a^2b^2c^3\right)\sqrt{-abc}\,|ab^2-abc|\,|b|\,\mathrm{sgn}(b-c)+\left(a^2b^6-3a^2b^5c+3a^2b^4c^2-a^2b^3c^3\right)\sqrt{-a}\,|ab^2-abc|\,|b|+\left(a^3b^7c-2a^3b^6c^2+2a^3b^4c^4-a^3b^3c^5\right)\sqrt{-a}\,|b|\,\mathrm{sgn}(b-c)+\left(a^3b^7-2a^3b^6c+2a^3b^4c^3-a^3b^3c^4\right)\sqrt{-abc}\,|b|\right)\arctan\left(-\frac{\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}}{\sqrt{-\frac{ab^3-abc^2-\sqrt{\left(ab^3-abc^2\right)^2-\left(a^2b^5-3a^2b^4c+3a^2b^3c^2-a^2b^2c^3\right)(b-c)}}{b-c}}}\right)}{\left(b^8-5b^7c+10b^6c^2-10b^5c^3+5b^4c^4-b^3c^5\right)a^2|ab^2-abc|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*abs(b)/(b^3 - b^2*c) + 2*a*arctan(sq
rt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) + 2*sqrt(b*x + a)/(b - c) - 2*(2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(b - c) + 2*(a*b^
3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5 - 3*a^2*b^4*c
 + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c)*abs(a*b^2 - a*b*c)*abs(b)*sgn(
b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs
(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b
^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3
 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqr
t(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*
b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/
(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*
a^2*abs(a*b^2 - a*b*c)) + 2*(2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt
(-a)*abs(b)*sgn(b - c) + 2*(a*b^3 - a*b^2*c)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)
*abs(b) + (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*sqrt(-a*b*c
)*abs(a*b^2 - a*b*c)*abs(b)*sgn(b - c) + (a^2*b^6 - 3*a^2*b^5*c + 3*a^2*b^4
*c^2 - a^2*b^3*c^3)*sqrt(-a)*abs(a*b^2 - a*b*c)*abs(b) + (a^3*b^7*c - 2*a^3
*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(b)*sgn(b - c) + (a^3*b
^7 - 2*a^3*b^6*c + 2*a^3*b^4*c^3 - a^3*b^3*c^4)*sqrt(-a*b*c)*abs(b))*arctan
(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*
b^3 - a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b
^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10
*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^2*abs(a*b^2 - a*b*c))
```

Mupad [B]
time = 4.33, size = 213, normalized size = 2.20

$$-\frac{2\sqrt{a}\,c\left(\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)-2\sqrt{a}\,b\left(\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{2\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+4\right)}{(b-c)\left(b-\frac{c\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)
```

```
[Out] -(2*a^(1/2)*c*((2*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))
+ (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^
(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2) - 2*a^(1/2)*b*(log(((a +
 b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))) - (2*((a + b*x)^(1/2) -
 a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 4))/((b - c)*(b - (c*((a + b*x)^(1
/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2))
```

3.430 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

```
[Out] -b*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)/a^(1/2)+c*arctanh((c*x+a)^(1/2)/a^(
1/2))/(b-c)/a^(1/2)-(b*x+a)^(1/2)/(b-c)/x+(c*x+a)^(1/2)/(b-c)/x
```

Rubi [A]

time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2128, 43, 65, 214}

$$-\frac{\sqrt{a+bx}}{x(b-c)}+\frac{\sqrt{a+cx}}{x(b-c)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(
Sqrt[a]*(b - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx &= \frac{\int \frac{\sqrt{a+bx}}{x^2}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^2}\,dx}{b-c}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}+\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2(b-c)}-\frac{c\int \frac{1}{x\sqrt{a+cx}}\,dx}{2(b-c)}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b-c}-\frac{\text{Subst}\Big(}{}\\
&= -\frac{\sqrt{a+bx}}{(b-c)x}+\frac{\sqrt{a+cx}}{(b-c)x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)}+\frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right.}{\sqrt{a}\,(b-c)}
\end{aligned}
$$

Mathematica [A]
time = 10.15, size = 81, normalized size = 0.79

$$
\frac{-\sqrt{a+bx}+\sqrt{a+cx}-\frac{bx\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}}}{bx-cx}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (-Sqrt[a + b*x] + Sqrt[a + c*x] - (b*x*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt
[a] + (c*x*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/Sqrt[a])/(b*x - c*x)
```

Maple [A]
time = 0.01, size = 88, normalized size = 0.85

method	result	size
default	$\frac{2b\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}-\frac{2c\left(-\frac{\sqrt{cx+a}}{2xc}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{b-c}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
))-2/(b-c)*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A]
time = 0.38, size = 182, normalized size = 1.77

$$\left[-\frac{\sqrt{a}\,bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\sqrt{a}\,cx\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a-2\sqrt{cx+a}\,a}{2(ab-ac)x},\ \frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{-a}\,cx\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a+\sqrt{cx+a}\,a}{(ab-ac)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c
*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqr
t(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/
a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt
(c*x + a)*a)/((a*b - a*c)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(87) = 174$.
time = 7.40, size = 1402, normalized size = 13.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b*
x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)*s
qrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sqrt(
b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/((a^2
*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2
+ (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) + (2*(a*b
^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(
a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3
*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c
)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2
*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^
3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7
*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arct
an(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(
a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2
*b^3*c^2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 -
10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c)) - (2*(a*b^3*c^2
- a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c
 - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^
4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c)*abs(b
)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^
4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a
^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*
a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arctan(-(sq
rt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 -
 a*b*c^2 - sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^
2 - a^2*b^2*c^3)*(b - c)))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*
c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c))
```

Mupad [B]
time = 10.93, size = 1637, normalized size = 15.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] (2*a*b - 2*a*c + a*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - 2*a^(1/2)*b*(a + c*x)^(1/2) + 2*a^(1/2)*c*(a + b*x)^(1/2) + a*b*at
an((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1
i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a +
 b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(
1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*2i - a*c*atan((b^3*(a + b*x)^(1/2)*1i
- b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2
)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)
^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x
)^(1/2)))*2i + a*b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/
2))) + b*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b
*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i
)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2
)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(
1/2)*2i - c*atan((b^3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a
+ b*x)^(1/2)*1i - a^(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)
*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(
1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*(a + c*x
)^(1/2)*2i + b*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))
*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^(1/2) - a^(1/2))/((a +
c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) - a^(1/2)*b*atan((b^
3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^
(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^
(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b
*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i - a^(1/2)*b*atan((b^3*(a
+ b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)
*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2)
 - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2
+ b*c^2*(a + c*x)^(1/2)))*(a + c*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x
)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*
1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^
3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c
^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a
+ c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a
 + c*x)^(1/2)))*(a + c*x)^(1/2)*2i - a^(1/2)*b*log(((a + b*x)^(1/2) - a^(1/
```

```
2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2) - a^(1/2)*b*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + c*x)^(1/2) - a^(1/2)*c*
log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2
) - a^(1/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*
(a + c*x)^(1/2))/(2*a^(1/2)*(b - c)*((a + b*x)^(1/2) - a^(1/2))*((a + c*x)^
(1/2) - a^(1/2)))
```

3.431 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}$$

```
[Out] 1/4*b^2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/4*c^2*arctanh((c*x+a
)^(1/2)/a^(1/2))/a^(3/2)/(b-c)-1/2*(b*x+a)^(1/2)/(b-c)/x^2-1/4*b*(b*x+a)^(1
/2)/a/(b-c)/x+1/2*(c*x+a)^(1/2)/(b-c)/x^2+1/4*c*(c*x+a)^(1/2)/a/(b-c)/x
```

Rubi [A]

time = 0.08, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2128, 43, 44, 65, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)}-\frac{\sqrt{a+bx}}{2x^2(b-c)}+\frac{\sqrt{a+cx}}{2x^2(b-c)}-\frac{b\sqrt{a+bx}}{4ax(b-c)}+\frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -1/2*Sqrt[a + b*x]/((b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt
[a + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/
Sqrt[a]])/(4*a^(3/2)*(b - c))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx=\frac{\int \frac{\sqrt{a+bx}}{x^3}\,dx}{b-c}-\frac{\int \frac{\sqrt{a+cx}}{x^3}\,dx}{b-c}$$

$$=-\frac{\sqrt{a+bx}}{2(b-c)x^2}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{b\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4(b-c)}-\frac{c\int \frac{1}{x^2\sqrt{a+cx}}\,dx}{4(b-c)}$$

$$=-\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}}{8a(b-c)}$$

$$=-\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}-\frac{b\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+}\right.}{4a}$$

$$=-\frac{\sqrt{a+bx}}{2(b-c)x^2}-\frac{b\sqrt{a+bx}}{4a(b-c)x}+\frac{\sqrt{a+cx}}{2(b-c)x^2}+\frac{c\sqrt{a+cx}}{4a(b-c)x}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{4a^{3/2}(b-}$$

Mathematica [A]
time = 10.14, size = 123, normalized size = 0.72

$$\frac{\sqrt{a}\left(-2a\sqrt{a+bx}-bx\sqrt{a+bx}+2a\sqrt{a+cx}+cx\sqrt{a+cx}\right)+b^2x^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-c^2x^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] (Sqrt[a]*(-2*a*Sqrt[a + b*x] - b*x*Sqrt[a + b*x] + 2*a*Sqrt[a + c*x] + c*x*
Sqrt[a + c*x]) + b^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] - c^2*x^2*ArcTanh[S
qrt[a + c*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)*x^2)
```

Maple [A]
time = 0.01, size = 120, normalized size = 0.70

method	result	siz
default	$\frac{2b^{2}\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^{2}b^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}-\frac{2c^{2}\left(\frac{-\frac{(cx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{cx+a}}{8}}{x^{2}c^{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{b-c}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8/a^(3/2)*a
rctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+
a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)
```

Fricas [A]
time = 0.35, size = 243, normalized size = 1.42

$$\left[-\frac{\sqrt{a}\,b^{2}x^{2}\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+\sqrt{a}\,c^{2}x^{2}\log\left(\frac{cx+2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right)+2\,(abx+2\,a^{2})\sqrt{bx+a}-2\,(acx+2\,a^{2})\sqrt{cx+a}}{8\,(a^{2}b-a^{2}c)x^{2}},\ -\frac{\sqrt{-a}\,b^{2}x^{2}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\sqrt{-a}\,c^{2}x^{2}\arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right)+(abx+2\,a^{2})\sqrt{bx+a}-(acx+2\,a^{2})\sqrt{cx+a}}{4\,(a^{2}b-a^{2}c)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(
a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)
*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2), -1
/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*ar
ctan(sqrt(c*x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a) - (a*c*x + 2
*a^2)*sqrt(c*x + a))/((a^2*b - a^2*c)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. $2(139) = 278$.
time = 15.75, size = 1895, normalized size = 11.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/((a*b - a*c)*sqrt(-a)) - 1/2*((sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^6*c^2*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*
b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c -
a*b*c))*a^3*b^4*c^4*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*
b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^4*c^2*abs(b) - 10*(sqrt(b*c)*sqrt(b*x
 + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^3*c^3*abs(b) + 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^2*c^4*ab
s(b) + 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*
a*b^2*c^2*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^5*a*b*c^3*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
 a)*b*c - a*b*c))^7*c^2*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (
sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(a*b -
a*c)) - 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((a*b - a*c)*b^2*x^
2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*s
gn(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a
*b*c)*abs(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*
sqrt(-a*b*c)*abs(a^2*b^2 - a^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^
2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2
*b*c)*abs(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*
sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b
^4*c^5 - a^5*b^3*c^6)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a)
```

```
 - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 + sqrt((
a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c
^3)*(a*b - a*c)))/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 +
 5*b^4*c^4 - b^3*c^5)*a^5*abs(a^2*b^2 - a^2*b*c)) + 1/4*(2*(a*b^3*c^3 - a*b
^2*c^4)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a)*abs(b)*sgn(8*a*b - 8*a*c) + 2*(a*b^3
*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^3*b^5*c^2
- 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*sqrt(-a*b*c)*abs(a^2*b^2 - a
^2*b*c)*abs(b)*sgn(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^
4*c^4 - a^3*b^3*c^5)*sqrt(-a)*abs(a^2*b^2 - a^2*b*c)*abs(b) + (a^5*b^7*c^3
- 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*sqrt(-a)*abs(b)*sgn(8*a*b -
8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*sqrt(-
a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))/sqrt(-(a^2*b^3 - a^2*b*c^2 - sqrt((a^2*b^3 - a^2*b*c^2)^2 - (a^
3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c)))/(a*b - a*c
)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*ab
s(a^2*b^2 - a^2*b*c))
```

Mupad [B]
time = 11.85, size = 1610, normalized size = 9.42

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)
```

```
[Out] ((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1/
2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(
1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a^(1
/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2/8)*
((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^3))/(
((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c)*((a
+ b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((a + b*x
)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2)) - (((c*(b + c))/(4
*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b*x)^(1/2)
 - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2) - a^(1/2))
/((a + c*x)^(1/2) - a^(1/2)))*(a^(3/2)*b^2 + a^(3/2)*c^2))/(8*a^3*b - 8*a^3
*c) + (atan((((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 -
 a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*
a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^
(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
+ ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*
b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c)*((16*a^3*b^4 +
16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b
*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3
```

```
 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*(
(a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)
^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2))))*
1i)/(8*a^3))/(((b + c)*(((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3
- a^6*b*c^2)) - (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128
*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a
^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2))
 + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6
*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (b*c^4 - b^5)/(32*(a^6*c^3
 - a^6*b*c^2)) + ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b
*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) -
 (((a + b*x)^(1/2) - a^(1/2))*(64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 12
8*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a
^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 -
a^6*b*c^2)*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) + (((a + b*x)^(1/2) - a^(
1/2))*(b*c^4 - b^4*c + b^2*c^3 - b^3*c^2))/(16*(a^6*c^3 - a^6*b*c^2)*((a +
c*x)^(1/2) - a^(1/2)))))*(a^(3/2)*b + a^(3/2)*c)*1i)/(4*a^3) + (c^2*((a + b
*x)^(1/2) - a^(1/2))^2)/(16*a^(3/2)*(b - c)*((a + c*x)^(1/2) - a^(1/2))^2)
```

3.432 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}-$$

```
[Out] a*x^2/(b-c)^2+1/3*(b+c)*x^3/(b-c)^2-2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/b/(b-c)
^2/c-1/4*a^3*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/b^(
5/2)/c^(5/2)+1/2*a*(b+c)*(b*x+a)^(3/2)*(c*x+a)^(1/2)/b^2/(b-c)^2/c+1/4*a^2*
(b+c)*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b^2/(b-c)/c^2
```

Rubi [A]
time = 0.25, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {6822, 81, 52, 65, 223, 212}

$$-\frac{a^3(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2c^2(b-c)}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2}+\frac{ax^2}{(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}+\frac{x^3(b+c)}{3(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x
]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a +
c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c
)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x
])])/(4*b^(5/2)*c^(5/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2ax+b\left(1+\frac{c}{b}\right)x^2-2x\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2\int x\sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}-\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c}+\frac{(a(b+c))\int\sqrt{a+bx}}{b(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c}-\frac{2(a+bx)^{3/2}(a+}{3b(b-c)^2c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^3}{2b^2(b-c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^3}{2b^2(b-c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^3}{2b^2(b-c}$$

$$= \frac{ax^2}{(b-c)^2}+\frac{(b+c)x^3}{3(b-c)^2}+\frac{a^2(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4b^2(b-c)c^2}+\frac{a(b+c)(a+bx)^3}{2b^2(b-c}$$

Mathematica [A]
time = 1.18, size = 213, normalized size = 1.09

$$\frac{\frac{b\left(4a^3b^2(b-2c)+a^2c(3b^2-2bc+3c^2)\sqrt{a+bx}\,\sqrt{a+cx}+4b^2c^3x^2\left(bx+cx-2\sqrt{a+bx}\,\sqrt{a+cx}\right)-2abc^2x\left(-6bcx+b\sqrt{a+bx}\,\sqrt{a+cx}+c\sqrt{a+bx}\,\sqrt{a+cx}\right)\right)}{(b-c)^2}+3a^3\sqrt{\frac{b}{c}}\,c(b+c)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{c}}\,\sqrt{a+cx}\right)}{12b^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b*(4*a^3*b^2*(b - 2*c) + a^2*c*(3*b^2 - 2*b*c + 3*c^2)*Sqrt[a + b*x]*Sqrt
[a + c*x] + 4*b^2*c^3*x^2*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - 2*a
*b*c^2*x*(-6*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a
 + c*x])))/(b - c)^2 + 3*a^3*Sqrt[b/c]*c*(b + c)*Log[Sqrt[a + b*x] - Sqrt[b
/c]*Sqrt[a + c*x]])/(12*b^3*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(161) = 322$.

time = 0.02, size = 517, normalized size = 2.65

method	result
default	$\frac{x^{3} b}{3(b-c)^{2}}+\frac{x^{3} c}{3(b-c)^{2}}+\frac{a\,x^{2}}{(b-c)^{2}}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(16x^{2}b^{2}c^{2}\sqrt{bc}\,\sqrt{bc\,x^{2}+abx+acx+a^{2}}+3\ln\left(\frac{2bcx+2\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2
)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)
+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(
b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*
c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^2*c-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b*c^2+3*ln(1/2*(2*b
*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^
3*c^3+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b^2*c+4*(b*c)^(1/2)
*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*a*b*c^2-6*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c
*x+a^2)^(1/2)*a^2*b^2+4*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*b*c
-6*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*a^2*c^2)/(b*c*x^2+a*b*x+a*c*
x+a^2)^(1/2)/b^2/c^2/(b*c)^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A]

time = 0.36, size = 479, normalized size = 2.46

$$\left[\frac{24\,ab^{3}c^{3}x^{2}+8\,(b^{4}c^{3}+b^{3}c^{4})x^{3}+3\,(a^{3}b^{3}-a^{3}b^{2}c-a^{3}bc^{2}+a^{3}c^{3})\sqrt{bc}\,\log\left(ab^{2}+2\,abc+ac^{2}+2\left(2\,bc-\sqrt{bc}\,(b+c)\right)\sqrt{bx+a}\,\sqrt{cx+a}+2\,(b^{2}c+bc^{2})x-2\,(2\,bcx+ab+ac)\sqrt{bc}\right)-2\,(8\,b^{3}c^{3}x^{2}-3\,a^{2}b^{3}c+2\,a^{2}b^{2}c^{2}-3\,a^{2}bc^{3}+2\,(ab^{3}c^{2}+ab^{2}c^{3})x)\sqrt{bx+a}\,\sqrt{cx+a}}{24\,(b^{5}c^{3}-2\,b^{4}c^{4}+b^{3}c^{5})},\ \frac{12\,ab^{3}c^{3}x^{2}+4\,(b^{4}c^{3}+b^{3}c^{4})x^{3}+3\,(a^{3}b^{3}-a^{3}b^{2}c-a^{3}bc^{2}+a^{3}c^{3})\sqrt{-bc}\,\arctan\left(\frac{\sqrt{-bc}\,\sqrt{bx+a}\,\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)-(8\,b^{3}c^{3}x^{2}-3\,a^{2}b^{3}c+2\,a^{2}b^{2}c^{2}-3\,a^{2}bc^{3}+2\,(ab^{3}c^{2}+ab^{2}c^{3})x)\sqrt{bx+a}\,\sqrt{cx+a}}{12\,(b^{5}c^{3}-2\,b^{4}c^{4}+b^{3}c^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*
c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c -
```

```
 sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(
2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^
2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x +
 a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 +
 b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*ar
ctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*
b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*
b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(161) = 322.
time = 4.67, size = 511, normalized size = 2.62

$$-\frac{1}{12}\sqrt{ab^2+(bx+a)bc-abc}\left(2(bx+a)\left(\frac{4(b^{11}c^4|b|-3b^{10}c^5|b|+3b^9c^6|b|-b^8c^7|b|)(bx+a)}{b^{17}c^4-5b^{16}c^5+10b^{15}c^6-10b^{14}c^7+5b^{13}c^8-b^{12}c^9}+\frac{ab^{12}c^3|b|-10ab^{11}c^4|b|+24ab^{10}c^5|b|-22ab^9c^6|b|+7ab^8c^7|b|}{b^{17}c^4-5b^{16}c^5+10b^{15}c^6-10b^{14}c^7+5b^{13}c^8-b^{12}c^9}\right)-\frac{3(a^2b^{13}c^2|b|-3a^2b^{12}c^3|b|+2a^2b^{11}c^4|b|+2a^2b^{10}c^5|b|-3a^2b^9c^6|b|+a^2b^8c^7|b|)}{b^{17}c^4-5b^{16}c^5+10b^{15}c^6-10b^{14}c^7+5b^{13}c^8-b^{12}c^9}\right)\sqrt{bx+a}+\frac{(bx+a)^3b-3(bx+a)a^2b+(bx+a)^3c-3(bx+a)^2ac+3(bx+a)a^2c}{3(b^5-2b^4c+b^3c^2)}+\frac{(a^3b|b|+a^3c|b|)\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{4\sqrt{bc}\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b)
- 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*c^
4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a*b^
12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*c^6*
abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14
*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*c^3*ab
s(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6*abs(b)
 + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 +
 5*b^13*c^8 - b^12*c^9))*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b*x + a)*a
^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^5 - 2*b^4*
c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b*c)*sqrt(b*
x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c^2)
```

Mupad [**B**]
time = 18.15, size = 1107, normalized size = 5.68

$$\frac{\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^6\left(128a^3b^3c+\frac{1312a^3b^2c^2}{3}+128a^3bc^3\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^6}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^7\left(19a^3b^3c+269a^3b^2c^2+269a^3bc^3+19a^3c^4\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^7}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^5\left(19a^3b^4+269a^3b^3c+269a^3b^2c^2+19a^3bc^3\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^5}+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^4\left(64a^3b^4+192a^3b^3c+64a^3b^2c^2\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^4}+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^8\left(64a^3b^2c^2+192a^3bc^3+64a^3c^4\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^8}+\frac{16a^3b^4{\left(\sqrt{a+bx}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^2}+\frac{16a^3c^4{\left(\sqrt{a+bx}-\sqrt{a}\right)}^{10}}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^{10}}+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^{11}\left(a^3b^3c^3-a^3b^2c^4-a^3bc^5+a^3c^6\right)}{2b^2{\left(\sqrt{a+cx}-\sqrt{a}\right)}^{11}}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^3\left(17a^3b^5+303a^3b^4c+303a^3b^3c^2+17a^3b^2c^3\right)}{6c{\left(\sqrt{a+cx}-\sqrt{a}\right)}^3}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^9\left(17a^3b^3c^2+303a^3b^2c^3+303a^3bc^4+17a^3c^5\right)}{6b{\left(\sqrt{a+cx}-\sqrt{a}\right)}^9}+\frac{\left(a^3b+a^3c\right)\left(\sqrt{a+bx}-\sqrt{a}\right)\left(b^5-2b^4c+b^3c^2\right)}{2c^2\left(\sqrt{a+cx}-\sqrt{a}\right)}}{b^8-2b^7c+b^6c^2+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^{12}\left(b^2c^6-2bc^7+c^8\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^{12}}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^2\left(6b^7c-12b^6c^2+6b^5c^3\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^2}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^{10}\left(6b^3c^5-12b^2c^6+6bc^7\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^{10}}+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^4\left(15b^6c^2-30b^5c^3+15b^4c^4\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^4}+\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^8\left(15b^4c^4-30b^3c^5+15b^2c^6\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^8}-\frac{{\left(\sqrt{a+bx}-\sqrt{a}\right)}^6\left(20b^5c^3-40b^4c^4+20b^3c^5\right)}{{\left(\sqrt{a+cx}-\sqrt{a}\right)}^6}}+\frac{x^3(b+c)}{3{(b-c)}^2}+\frac{ax^2}{{(b-c)}^2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{a+cx}-\sqrt{a}\right)}\right)(b+c)}{2b^{5/2}c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] ((((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3*
b^2*c^2)/3))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1/2) - a^(1/2))^7
*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^
(1/2) - a^(1/2))^7 - (((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^3*b*
c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^5 + (((
a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a^3*c^4
+ 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 + (16*a^3*
b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (16*a^3*
c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^10 + (((a +
 b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a^3*b^3*c^3)
)/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*(
17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2))/(6*c*((a +
c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17*a^3*c^5 + 303
*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c*x)^(1/2) - a^(
1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^5 - 2*b^4*c + b^
3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2))))/(b^8 - 2*b^7*c + b^6*c^2 + (((
a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/((a + c*x)^(1/2) -
a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c + 6*b^5*c^3 - 12*b^6*
c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^10*(6*b*
c^7 - 12*b^2*c^6 + 6*b^3*c^5))/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)
^(1/2) - a^(1/2))^4*(15*b^4*c^4 - 30*b^5*c^3 + 15*b^6*c^2))/((a + c*x)^(1/2
) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(15*b^2*c^6 - 30*b^3*c^5 +
15*b^4*c^4))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^(1/2))^6
*(20*b^3*c^5 - 40*b^4*c^4 + 20*b^5*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) + (
x^3*(b + c))/(3*(b - c)^2) + (a*x^2)/(b - c)^2 - (a^3*atanh((c^(1/2)*((a +
b*x)^(1/2) - a^(1/2)))/(b^(1/2)*((a + c*x)^(1/2) - a^(1/2))))*(b + c))/(2*b
^(5/2)*c^(5/2))
```

3.433 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

```
[Out] 2*a*x/(b-c)^2+1/2*(b+c)*x^2/(b-c)^2+1/2*a^2*arctanh(c^(1/2)*(b*x+a)^(1/2)/b
^(1/2)/(c*x+a)^(1/2))/b^(3/2)/c^(3/2)-(b*x+a)^(3/2)*(c*x+a)^(1/2)/b/(b-c)^2
-1/2*a*(b*x+a)^(1/2)*(c*x+a)^(1/2)/b/(b-c)/c
```

Rubi [A]
time = 0.17, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6822, 52, 65, 223, 212}

$$\frac{a^2\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}+\frac{2ax}{(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2bc(b-c)}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+\frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a +
 c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a
^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(
3/2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(2a+b\left(1+\frac{c}{b}\right)x-2\sqrt{a+bx}\,\sqrt{a+cx}\right)dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{2\int \sqrt{a+bx}\,\sqrt{a+cx}\,dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}-\frac{a\int \frac{\sqrt{a+bx}}{\sqrt{a+cx}}\,dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+$$

$$= \frac{2ax}{(b-c)^2}+\frac{(b+c)x^2}{2(b-c)^2}-\frac{a\sqrt{a+bx}\,\sqrt{a+cx}}{2b(b-c)c}-\frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2}+$$

Mathematica [A]
time = 0.67, size = 161, normalized size = 1.13

$$\frac{\frac{b\left(-a^2b(b-3c)+bc^2x\left(bx+cx-2\sqrt{a+bx}\,\sqrt{a+cx}\right)-ac\left(-4bcx+b\sqrt{a+bx}\,\sqrt{a+cx}+c\sqrt{a+bx}\,\sqrt{a+cx}\right)\right)}{(b-c)^2}-a^2\sqrt{\frac{b}{c}}\,c\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{c}}\,\sqrt{a+cx}\right)}{2b^2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b*(-(a^2*b*(b - 3*c)) + b*c^2*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x
]) - a*c*(-4*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a
 + c*x])))/(b - c)^2 - a^2*Sqrt[b/c]*c*Log[Sqrt[a + b*x] - Sqrt[b/c]*Sqrt[a
 + c*x]])/(2*b^2*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(116) = 232$.
time = 0.01, size = 385, normalized size = 2.71

method	result
default	$\frac{x^2 b}{2(b-c)^2}+\frac{x^2 c}{2(b-c)^2}+\frac{2ax}{(b-c)^2}-\frac{\sqrt{bx+a}\,(cx+a)^{\frac{3}{2}}}{(b-c)^2 c}+\frac{\sqrt{cx+a}\,\sqrt{bx+a}\,a}{2(b-c)^2 c}-\frac{\sqrt{cx+a}\,\sqrt{bx+a}\,a}{2(b-c)^2 b}+\frac{\sqrt{(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*x+a)^(1/2)
*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a-1/2/(b-c)^2/b*(c
*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(
1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c
)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2*b-1/2/(b-c)^2*((b*x+a)*(c*x+a))^(1/2)/(c*x+
a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b
+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((b*x+a)*(c*x+a))^(1/2)
/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^
2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A]
time = 0.37, size = 372, normalized size = 2.62

$$\left[\frac{8\,ab^2c^2x+2\,(b^3c^2+b^2c^3)x^2+(a^2b^2-2\,a^2bc+a^2c^2)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc+\sqrt{bc}\,(b+c)\right)\sqrt{bx+a}\,\sqrt{cx+a}+2\,(b^2c+bc^2)x+2\,(2\,bcx+ab+ac)\sqrt{bc}\right)-2\,(2\,b^2c^2x+ab^2c+abc^2)\sqrt{bx+a}\,\sqrt{cx+a}}{4\,(b^4c^2-2\,b^3c^3+b^2c^4)},\ \frac{4\,ab^2c^2x+(b^3c^2+b^2c^3)x^2-(a^2b^2-2\,a^2bc+a^2c^2)\sqrt{-bc}\,\arctan\left(\frac{\sqrt{-bc}\,\sqrt{bx+a}\,\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)-(2\,b^2c^2x+ab^2c+abc^2)\sqrt{bx+a}\,\sqrt{cx+a}}{2\,(b^4c^2-2\,b^3c^3+b^2c^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^
2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c)*(b + c)
)*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*
c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x
 + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*
c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c)*sq
rt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c
 + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(116) = 232.
time = 4.74, size = 272, normalized size = 1.92

$$-\frac{1}{2}\,\sqrt{ab^2+(bx+a)bc-abc}\,\sqrt{bx+a}\left(\frac{2\,(b^4c^2|b|-b^3c^3|b|)(bx+a)}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}+\frac{ab^5c|b|-2\,ab^4c^2|b|+ab^3c^3|b|}{b^9c^2-3\,b^8c^3+3\,b^7c^4-b^6c^5}\right)-\frac{a^2|b|\log\left(\left|-\sqrt{bc}\,\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{2\,\sqrt{bc}\,b^2c}+\frac{(bx+a)^2b+2\,(bx+a)ab+(bx+a)^2c-2\,(bx+a)ac}{2\,(b^4-2\,b^3c+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b) -
 b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) + (a
*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*c^3
 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a)
+ sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x + a)^
2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3*c + b
^2*c^2)
```

Mupad [B]
time = 0.25, size = 129, normalized size = 0.91

$$\frac{2\,a\,x}{{\left(b-c\right)}^2}+\frac{x^2\,\left(b+c\right)}{2\,{\left(b-c\right)}^2}-\frac{2\,\left(\frac{x}{2}+\frac{a\,b+a\,c}{4\,b\,c}\right)\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}}{{\left(b-c\right)}^2}+\frac{\ln\left(a\,b+a\,c+2\,b\,c\,x+2\,\sqrt{b}\,\sqrt{c}\,\sqrt{a+b\,x}\,\sqrt{a+c\,x}\right)\,{\left(a\,b-a\,c\right)}^2}{4\,b^{3/2}\,c^{3/2}\,{\left(b-c\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*x)/(b - c)^2 + (x^2*(b + c))/(2*(b - c)^2) - (2*(x/2 + (a*b + a*c)/(4*
b*c))*(a + b*x)^(1/2)*(a + c*x)^(1/2))/(b - c)^2 + (log(a*b + a*c + 2*b*c*x
 + 2*b^(1/2)*c^(1/2)*(a + b*x)^(1/2)*(a + c*x)^(1/2))*(a*b - a*c)^2)/(4*b^(
3/2)*c^(3/2)*(b - c)^2)
```

3.434 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=135

$$\frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{\sqrt{b}\,(b-c)^2\sqrt{c}}+\frac{2a\log(x)}{(b-c)^2}$$

```
[Out] (b+c)*x/(b-c)^2+4*a*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+2*a*ln(x)/
(b-c)^2-2*a*(b+c)*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))/(b-c
)^2/b^(1/2)/c^(1/2)-2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2
```

Rubi [A]

time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6822, 103, 163, 65, 223, 212, 95, 214}

$$-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{\sqrt{b}\,\sqrt{c}\,(b-c)^2}+\frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanh[(Sqrt[
c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (
2*a*Log[x])/(b - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(b\left(1+\frac{c}{b}\right)+\frac{2a}{x}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x}\right)dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}+\frac{2\int \frac{-a^2-\frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\,\sqrt{a+cx}}\,dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(2a^2)\int \frac{1}{x\sqrt{a+bx}\,\sqrt{a+cx}}}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-\frac{(4a^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+ax^2}\,dx,\right.}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{2a\log(x)}{(b-c)^2}-$$

$$= \frac{(b+c)x}{(b-c)^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2}+\frac{4a\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{2a(b+c)\,\mathrm{ta}}{\sqrt{}}$$

Mathematica [A]
time = 0.91, size = 232, normalized size = 1.72

$$\frac{4a\sqrt{b}\sqrt{\frac{b}{c}}\,c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\left(-bx+\sqrt{\frac{b}{c}}\sqrt{a+bx}\,\sqrt{a+cx}\right)}{a\sqrt{b}}\right)+2ac\left(b\left(-2+\sqrt{\frac{b}{c}}\right)+\sqrt{\frac{b}{c}}\,c\right)\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)+b\left(a(b+c)+c\left(bx+cx-2\sqrt{a+bx}\,\sqrt{a+cx}\right)+2ac\log\left(bcx\left(a(b+c)+2bcx-2\sqrt{\frac{b}{c}}\,c\sqrt{a+bx}\,\sqrt{a+cx}\right)\right)\right)}{b(b-c)^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (4*a*Sqrt[b]*Sqrt[b/c]*c^(3/2)*ArcTanh[(Sqrt[c]*(-(b*x) + Sqrt[b/c]*Sqrt[a
+ b*x]*Sqrt[a + c*x]))/(a*Sqrt[b])] + 2*a*c*(b*(-2 + Sqrt[b/c]) + Sqrt[b/c]
*c)*Log[Sqrt[a + b*x] - Sqrt[b/c]*Sqrt[a + c*x]] + b*(a*(b + c) + c*(b*x +
c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) + 2*a*c*Log[b*c*x*(a*(b + c) + 2*b*c*x
 - 2*Sqrt[b/c]*c*Sqrt[a + b*x]*Sqrt[a + c*x])]))/(b*(b - c)^2*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 266, normalized size = 1.97

method	result
default	$\frac{xb}{(b-c)^2}+\frac{xc}{(b-c)^2}+\frac{2a\ln(x)}{(b-c)^2}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+ac}{2\sqrt{bc}}\right)\mathrm{csgn}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(
1/2)*(ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c
)/(b*c)^(1/2))*csgn(a)*a*b+ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2
)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*csgn(a)*a*c+2*(b*c)^(1/2)*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*csgn(a)-2*(b*c)^(1/2)*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^
(1/2)*csgn(a)+b*x+c*x+2*a)/x)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b
*c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)
```

Fricas [A]
time = 0.38, size = 346, normalized size = 2.56

$$\left[\frac{2\,abc\log\left(x\right)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,\sqrt{cx+a}\,bc+(ab+ac)\sqrt{bc}\,\log\left(ab^2+2\,abc+ac^2+2\left(2\,bc-\sqrt{bc}\,(b+c)\right)\sqrt{bx+a}\,\sqrt{cx+a}+2\,(b^2c+bc^2)x-2\,(2\,bcx+ab+ac)\sqrt{bc}\right)+(b^2c+bc^2)x}{b^3c-2\,b^2c^2+bc^3},\ \frac{2\,abc\log\left(x\right)-2\,abc\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)-2\,\sqrt{bx+a}\,\sqrt{cx+a}\,bc+2\,(ab+ac)\sqrt{-bc}\,\arctan\left(\frac{\sqrt{-bc}\,\sqrt{bx+a}\,\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)+(b^2c+bc^2)x}{b^3c-2\,b^2c^2+bc^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a)
+ 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a
*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c
*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c
 + b*c^2)*x)/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((
b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c
*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqr
t(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x)/(b^3*c - 2*b^2*c^2
 + b*c^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(115) = 230.
time = 5.13, size = 306, normalized size = 2.27

$$\frac{\frac{\sqrt{bc}\,a(b+c)|b|\log\left(\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3c-2\,b^2c^2+bc^3}-\frac{4\,\sqrt{bc}\,a|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\,\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\,\sqrt{-bc}\,ab}\right)}{(b^2-2\,bc+c^2)\sqrt{-bc}}+\frac{2\,ab\log(|bx|)}{b^2-2\,bc+c^2}-\frac{2\,\sqrt{ab^2+(bx+a)bc-abc}\,(b^2|b|-2\,bc|b|+c^2|b|)\sqrt{bx+a}}{b^5-4\,b^4c+6\,b^3c^2-4\,b^2c^3+bc^4}+\frac{(bx+a)b+(bx+a)c}{b^2-2\,bc+c^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (sqrt(b*c)*a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*
x + a)*b*c - a*b*c))^2)/(b^3*c - 2*b^2*c^2 + b*c^3) - 4*sqrt(b*c)*a*abs(b)*
arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)) + 2*
a*b*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c +
6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c +
 c^2))/b
```

Mupad [B]
time = 19.76, size = 2500, normalized size = 18.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2) -
 a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b*x
)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(1/2
) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2 - (
((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c*x)^(
1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b^2*c^2
))/((a + c*x)^(1/2) - a^(1/2))^4) - (2*a*log((((a + b*x)^(1/2) - (a + c*x)^
```

```
(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/(
(a + c*x)^(1/2) - a^(1/2))))/(b^2 - 2*b*c + c^2) + (2*a*log(((a + b*x)^(1/2
) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))/(b - c)^2 + (x*(b + c))/(b - c)^
2 + (a*atan(((a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3
*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^
6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4
*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 +
 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^
2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2
*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5
 + 4*a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a +
b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10
- 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^
2*b^9*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(a*b^4*c^12 + 7
*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 +
7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (
2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^1
1 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b
^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^
14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4
*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a
 + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6
*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11
*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3
*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b
^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2
*c^2) - (a*(b*c)^(1/2)*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3
*b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2
*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*
c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/(
((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) +
(2*a*(b*c)^(1/2)*(b + c)*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*
c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*
a^2*b^10*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^
(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*
a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9
*c^5 + 16*a^2*b^10*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*
c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c)*((2*((a + b*x)^(1/2) - a
^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 6
48*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5
))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)
) - (4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8
*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3
```

```
+ c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c)*((4*(4*b^5*c^13 - b^4*c^14 -
5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12
*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*
x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12
 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7
- 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c -
4*b*c^3 + c^4 + 6*b^2*c^2))))/(b*c^3 + b^3*c - 2*b^2*c^2)))/(b*c^3 + b^3*c
- 2*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2*c^2)
)/((4*((a + b*x)^(1/2) - a^(1/2))*(128*a^4*b^3*c^7 + 256*a^4*b^4*c^6 + 128*
a^4*b^5*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 +
 6*b^2*c^2)) - (8*(16*a^4*b^3*c^7 + 56*a^4*b^4*c^6 + 56*a^4*b^5*c^5 + 16*a^
4*b^6*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*
(b + c)*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 +
 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*
b^8*c^4))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4...
```

3.435 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=138

$$-\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b-}{(}$$

```
[Out] -2*a/(b-c)^2/x+2*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/(b-c)^2+(b+c)*l
n(x)/(b-c)^2-4*arctanh(c^(1/2)*(b*x+a)^(1/2)/b^(1/2)/(c*x+a)^(1/2))*b^(1/2)
*c^(1/2)/(b-c)^2+2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/(b-c)^2/x
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6822, 99, 163, 65, 223, 212, 95, 214}

$$-\frac{2a}{x(b-c)^2}+\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x(b-c)^2}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\sqrt{b}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(
b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b
+ c)*Log[x])/(b - c)^2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^p/(b*(
m + 1))), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^2}+\frac{b\left(1+\frac{c}{b}\right)}{x}-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^2}\,dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{2\int \frac{\frac{1}{2}a(b+c)+b\cdots}{x\sqrt{a+bx}\sqrt{a\cdots}}}{(b-c)^2\cdots}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(2bc)\int \frac{1\cdots}{\sqrt{a+bx}\cdots}}{(b-c\cdots}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{(b+c)\log(x)}{(b-c)^2}-\frac{(4c)\text{Subst}\left(\int \frac{\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}+\frac{(b\cdots}{\cdots}$$

$$= -\frac{2a}{(b-c)^2x}+\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x}+\frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}-\frac{4\cdots}{\cdots}$$

Mathematica [A]
time = 1.20, size = 215, normalized size = 1.56

$$\frac{2\sqrt{\frac{b}{c}}\sqrt{c}\,(b+c)x\tanh^{-1}\left(\frac{\sqrt{c}\left(-bx+\sqrt{\frac{b}{c}}\sqrt{a+bx}\sqrt{a+cx}\right)}{a\sqrt{b}}\right)+\sqrt{b}\left(-2a+2\sqrt{a+bx}\sqrt{a+cx}-2\left(b+c-2\sqrt{\frac{b}{c}}c\right)x\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{c}}\sqrt{a+cx}\right)+(b+c)x\log\left(bcx\left(a(b+c)+2bcx-2\sqrt{\frac{b}{c}}c\sqrt{a+bx}\sqrt{a+cx}\right)\right)\right)}{\sqrt{b}\,(b-c)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2),x]
```

```
[Out] (2*Sqrt[b/c]*Sqrt[c]*(b + c)*x*ArcTanh[(Sqrt[c]*(-(b*x) + Sqrt[b/c]*Sqrt[a
+ b*x]*Sqrt[a + c*x]))/(a*Sqrt[b])] + Sqrt[b]*(-2*a + 2*Sqrt[a + b*x]*Sqrt[
a + c*x] - 2*(b + c - 2*Sqrt[b/c]*c)*x*Log[Sqrt[a + b*x] - Sqrt[b/c]*Sqrt[a
 + c*x]] + (b + c)*x*Log[b*c*x*(a*(b + c) + 2*b*c*x - 2*Sqrt[b/c]*c*Sqrt[a
+ b*x]*Sqrt[a + c*x])]))/(Sqrt[b]*(b - c)^2*x)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.01, size = 272, normalized size = 1.97

method	result
default	$\frac{b\ln(x)}{(b-c)^2}+\frac{c\ln(x)}{(b-c)^2}-\frac{2a}{(b-c)^2x}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(2\,\mathrm{csgn}(a)\ln\left(\frac{2bcx+2\sqrt{bc\,x^2+abx+acx+a^2}\,\sqrt{bc}+ab+a}{2\sqrt{bc}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)

[Out] 1/(b-c)^2*b*ln(x)+1/(b-c)^2*c*ln(x)-2*a/(b-c)^2/x-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(1/2)*(2*csgn(a)*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*x*b*c-ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*x*b*(b*c)^(1/2)-ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*x*c*(b*c)^(1/2)-2*(b*c)^(1/2)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x/(b*c)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-2), x)

Fricas [A]
time = 0.35, size = 317, normalized size = 2.30

$$\left[\frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+4\sqrt{bc}\,x\log\left(ab^2+2abc+ac^2+2\left(2bc-\sqrt{bc}(b+c)\right)\sqrt{bx+a}\sqrt{cx+a}+2(b^2c+bc^2)x-2(2bcx+ab+ac)\sqrt{bc}\right)+(b+c)x+4\sqrt{bx+a}\sqrt{cx+a}-4a}{2(b^2-2bc+c^2)x},\ \frac{2(b+c)x\log(x)-2(b+c)x\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\sqrt{cx+a}+2a}{x}\right)+8\sqrt{-bc}\,x\arctan\left(\frac{\sqrt{-bc}\sqrt{bx+a}\sqrt{cx+a}-\sqrt{-bc}\,a}{bcx}\right)+(b+c)x+4\sqrt{bx+a}\sqrt{cx+a}-4a}{2(b^2-2bc+c^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")

[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 438 vs. 2(118) = 236.
time = 5.17, size = 438, normalized size = 3.17

$$\frac{2\sqrt{bc}\,|b|\log\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2\right)}{b^3-2b^2c+bc^2}-\frac{2\sqrt{bc}\,(b+c)|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{(b^2-2bc+c^2)\sqrt{-bc}\,b}+\frac{(b+c)\log(|bx|)}{b^2-2bc+c^2}-\frac{4\left(\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a(b+c)|b|-(b^3-2b^2c+bc^2)\sqrt{bc}\,a^2|b|\right)}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+(b^4-2b^3c+b^2c^2)a^2\right)(b^2-2bc+c^2)}-\frac{(bx+a)b+ab+(bx+a)c-ac}{(b^2-2bc+c^2)bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*
c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) - 2*sqrt(b*c)*(b + c)*abs(b)*arctan(
1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
- a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b + c)
*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a)
- sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2*b^2*c
+ b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b
^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)*(b^2 - 2*
b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*b*c + c^2)*
b*x)
```

Mupad [B]
time = 17.44, size = 2500, normalized size = 18.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)
```

```
[Out] (atan((((b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^
10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 -
 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36
*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3
 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^1
```

2 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - c)^2 - ((b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - c)^2)/((4*(b*c)^(1/2)*((4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*b^5*c^11 - 42*b^6*c^10 + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^10*c^6 + b^11*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^12 - 36*b^7*c^10 + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^11*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^13 - b^4*c^14 - 5*b^6*c^12 + b^7*c^11 + b^8*c^10 + b^9*c^9 + b^10*c^8 - 5*b^11*c^7 + 4*b^12*c^6 - b^13*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^15 - 31*b^4*c^14 + 120*b^5*c^13 - 300*b^6*c^12 + 516*

b^7*c^11 - 618*b^8*c^10 + 516*b^9*c^9 - 300*b^10*c^8 + 120*b^11*c^7 - 31*b^12*c^6 + 4*b^13*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(4*b^3*c^14 - 27*b^4*c^13 + 99*b^5*c^12 - 175*b^6*c^11 + 99*b^7*c^10 + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^10*c^7 - 27*b^11*c^6 + 4*b^12*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (2*((a + b*x)^(1/2) - a^(1/2))*(73*b^4*c^12 - 278*b^5*c^11 + 503*b^6*c^10 - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^10*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(65*b^4*c^11 - 167*b^5*c^10 + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))))/(b - c)^2 - (8*(14*b^5*c^9 + 42*b^6*c^8 + 42*b^7*c^7 + 14*b^8*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(4*b^5*c^10 + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^(1/2)*((4*(b^4*c^12 + 16*...

3.436 $\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}$$

```
[Out] -a/(b-c)^2/x^2+(-b-c)/(b-c)^2/x-1/2*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a+
(c*x+a)^(3/2)*(b*x+a)^(1/2)/a/(b-c)^2/x^2+1/2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a
/(b-c)/x
```

Rubi [A]
time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6822, 96, 95, 214}

$$\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{ax^2(b-c)^2}-\frac{a}{x^2(b-c)^2}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2ax(b-c)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a}-\frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x]
)/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - Arc
Tanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^3}+\frac{b\left(1+\frac{c}{b}\right)}{x^2}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^3}\right)dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^3}\,dx}{(b-c)^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}-\frac{\int \frac{\sqrt{a+cx}}{x^2\sqrt{a+bx}}\,dx}{2(b-c)}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}$$

$$= -\frac{a}{(b-c)^2x^2}-\frac{b+c}{(b-c)^2x}+\frac{\sqrt{a+bx}\,\sqrt{a+cx}}{2a(b-c)x}+\frac{\sqrt{a+bx}\,(a+cx)^{3/2}}{a(b-c)^2x^2}$$

Mathematica [A]
time = 1.85, size = 148, normalized size = 1.20

$$\frac{\frac{-2a^2+(b+c)x\sqrt{a+bx}\,\sqrt{a+cx}+2a\left(-bx-cx+\sqrt{a+bx}\,\sqrt{a+cx}\right)}{(b-c)^2x^2}-\frac{\sqrt{\frac{b}{c}}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\left(-bx+\sqrt{\frac{b}{c}}\,\sqrt{a+bx}\,\sqrt{a+cx}\right)}{a\sqrt{b}}\right)}{\sqrt{b}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] ((-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt
[a + b*x]*Sqrt[a + c*x]))/((b - c)^2*x^2) - (Sqrt[b/c]*Sqrt[c]*ArcTanh[(Sqr
t[c]*(-(b*x) + Sqrt[b/c]*Sqrt[a + b*x]*Sqrt[a + c*x]))/(a*Sqrt[b])])/Sqrt[b
])/(2*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 313, normalized size = 2.54

method	result
default	$-\frac{b}{x(b-c)^2}-\frac{c}{x(b-c)^2}-\frac{a}{(b-c)^2x^2}+\frac{\sqrt{bx+a}\,\sqrt{cx+a}\left(-\ln\left(\frac{a\left(2\sqrt{bc\,x^2+abx+acx+a^2}\,\mathrm{csgn}(a)+bx+cx+2}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a*(-ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)
*x^2*b^2+2*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*
x^2*b*c-ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*x^2
*c^2+2*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b+2*csgn(a)*(b*c*x^2+a*b*x
+a*c*x+a^2)^(1/2)*x*c+4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*csgn(a)/
(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A]
time = 0.36, size = 126, normalized size = 1.02

$$\frac{4\,(b^2-2\,bc+c^2)x^2\log\left(-\frac{(b+c)x-2\,\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)+(b^2+2\,bc+c^2)x^2+8\,((b+c)x+2\,a)\sqrt{bx+a}\,\sqrt{cx+a}-16\,a^2-16\,(ab+ac)x}{16\,(ab^2-2\,abc+ac^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b^2 - 2*b*c + c^2)*x^2*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x
+ a) + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*sqrt(b*x + a
)*sqrt(c*x + a) - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(107) = 214.
time = 7.31, size = 532, normalized size = 4.33

$$\frac{\sqrt{bc}\,|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{2\sqrt{-bc}\,ab}-\frac{(b^2+6bc+c^2)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6|b|-(3b^4+5b^3c+5b^2c^2+3bc^3)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4a|b|+(3b^6-4b^5c+2b^4c^2-4b^3c^3+3b^2c^4)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a^2|b|-(b^8-3b^7c+2b^6c^2+2b^5c^3-3b^4c^4+b^3c^5)\sqrt{bc}\,a^3|b|}{\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+(b^4-2b^3c+b^2c^2)a^2\right)^2(b^2-2bc+c^2)}-\frac{(bx+a)b^2+(bx+a)bc-abc}{(b^2-2bc+c^2)b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) -
 sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-b*c)*a*b)
 - ((b^2 + 6*b*c + c^2)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*c^3)*s
qrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*
a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*sqrt(b*c)*
(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a^2*abs(b
) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)*sqrt(b*c)
*a^3*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c + c^2)) - ((
b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^2*x^2)
```

Mupad [B]
time = 12.32, size = 787, normalized size = 6.40

$$\frac{\ln\left(\frac{\left(\sqrt{a+bx}-\sqrt{a+cx}\right)\left(b-\frac{c\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}\right)}{\sqrt{a+cx}-\sqrt{a}}\right)}{4a}-\frac{\frac{b^4}{2}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4\left(-\frac{b^4}{2}+4b^3c+\frac{3b^2c^2}{2}+4bc^3-\frac{c^4}{2}\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4}-\frac{(2b^4+2cb^3)\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}-\frac{(b^2c^2+bc^3)\left(\sqrt{a+bx}-\sqrt{a}\right)^5}{\left(\sqrt{a+cx}-\sqrt{a}\right)^5}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^2\left(\frac{5b^4}{2}+6b^3c+\frac{5b^2c^2}{2}\right)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3(b^4+6b^3c+6b^2c^2+bc^3)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^3}}{\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4(8ab^4+16ab^3c-48ab^2c^2+16abc^3+8ac^4)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^3(16ab^4-16ab^3c-16ab^2c^2+16abc^3)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^3}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^5(16ab^3c-16ab^2c^2-16abc^3+16ac^4)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^5}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^2(8ab^4-16ab^3c+8ab^2c^2)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}+\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^6(8ab^2c^2-16abc^3+8ac^4)}{\left(\sqrt{a+cx}-\sqrt{a}\right)^6}}-\frac{\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)}{4a}-\frac{a+x(b+c)}{x^2(b^2-2bc+c^2)}-\frac{c^2\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{16a(b-c)^2\left(\sqrt{a+cx}-\sqrt{a}\right)^2}+\frac{c(b+c)\left(\sqrt{a+bx}-\sqrt{a}\right)}{8a(b-c)^2\left(\sqrt{a+cx}-\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)
))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^4/
2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 + (3*
b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b*x)^
(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a + b*
x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) -
a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2
))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^2*c^2))/
((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*a*b^4 + 8
*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2
))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 + 16*a*b*c^3
 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/2) - a^(1/2)
)^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) -
 a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b^2*c^2 - 16*a*
b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^6*(8*a
*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) - log(((a
+ b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (a + x*(b + c)
)/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a*(b
- c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a + b*x)^(1/2) - a^(1/
2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

3.437 $\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2} dx$

Optimal. Leaf size=174

$$-\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)}{3a^2(b-c)^2x^3}$$

```
[Out] -2/3*a/(b-c)^2/x^3+1/2*(-b-c)/(b-c)^2/x^2+2/3*(b*x+a)^(3/2)*(c*x+a)^(3/2)/a
^2/(b-c)^2/x^3+1/4*(b+c)*arctanh((b*x+a)^(1/2)/(c*x+a)^(1/2))/a^2-1/2*(b+c)
*(c*x+a)^(3/2)*(b*x+a)^(1/2)/a^2/(b-c)^2/x^2-1/4*(b+c)*(b*x+a)^(1/2)*(c*x+a
)^(1/2)/a^2/(b-c)/x
```

Rubi [A]
time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6822, 98, 96, 95, 214}

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2}-\frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2}-\frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)}+\frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2}-\frac{2a}{3x^3(b-c)^2}-\frac{b+c}{2x^2(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*
x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2
))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b -
c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
```

```
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx = \frac{\int \left(\frac{2a}{x^4}+\frac{b\left(1+\frac{c}{b}\right)}{x^3}-\frac{2\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}\right)dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{2\int \frac{\sqrt{a+bx}\,\sqrt{a+cx}}{x^4}dx}{(b-c)^2}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}+\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2x^3}+\frac{(b+c)\int \sqrt{}}{}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,(a+cx)^{3/2}}{2a^2(b-c)^2x^2}+\frac{2(a+b}{3a}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{}}{2a}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{}}{2a}$$

$$= -\frac{2a}{3(b-c)^2x^3}-\frac{b+c}{2(b-c)^2x^2}-\frac{(b+c)\sqrt{a+bx}\,\sqrt{a+cx}}{4a^2(b-c)x}-\frac{(b+c)\sqrt{}}{2a}$$

Mathematica [A]
time = 10.27, size = 164, normalized size = 0.94

$$\frac{\frac{2\left(-8a^3+2a(b+c)x\sqrt{a+bx}\,\sqrt{a+cx}+(-3b^2+2bc-3c^2)x^2\sqrt{a+bx}\,\sqrt{a+cx}+a^2\left(-6bx-6cx+8\sqrt{a+bx}\,\sqrt{a+cx}\right)\right)}{(b-c)^2x^3}-3(b+c)\log(x)+3(b+c)\log\left(2a+bx+cx+2\sqrt{a+bx}\,\sqrt{a+cx}\right)}{24a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]
```

```
[Out] ((2*(-8*a^3 + 2*a*(b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + (-3*b^2 + 2*b*c -
 3*c^2)*x^2*Sqrt[a + b*x]*Sqrt[a + c*x] + a^2*(-6*b*x - 6*c*x + 8*Sqrt[a +
b*x]*Sqrt[a + c*x])))/((b - c)^2*x^3) - 3*(b + c)*Log[x] + 3*(b + c)*Log[2*
a + b*x + c*x + 2*Sqrt[a + b*x]*Sqrt[a + c*x]])/(24*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 457, normalized size = 2.63

method	result

default	$-\frac{b}{2x^2(b-c)^2}-\frac{c}{2x^2(b-c)^2}-\frac{2a}{3(b-c)^2x^3}-\frac{\sqrt{bx+a}\,\sqrt{cx+a}\,\left(-3\ln\left(\frac{a\left(2\sqrt{bc\,x^2+abx+acx+a^2}\,\operatorname{csgn}(a)+bx\right.}{x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a
)+b*x+c*x+2*a)/x)*x^3*b^3+3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)
+b*x+c*x+2*a)/x)*x^3*b^2*c+3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a
)+b*x+c*x+2*a)/x)*x^3*b*c^2-3*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(
a)+b*x+c*x+2*a)/x)*x^3*c^3+6*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b^
2-4*csgn(a)*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x^2*b*c+6*csgn(a)*(b*c*x^2+a*b*
x+a*c*x+a^2)^(1/2)*x^2*c^2-4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*b-
4*csgn(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a))*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)
```

Fricas [A]
time = 0.33, size = 182, normalized size = 1.05

$$-\frac{12\,(b^3-b^2c-bc^2+c^3)x^3\log\left(-\frac{(b+c)x-2\sqrt{bx+a}\,\sqrt{cx+a}+2\,a}{x}\right)+(5\,b^3+3\,b^2c+3\,bc^2+5\,c^3)x^3+64\,a^3+8\,((3\,b^2-2\,bc+3\,c^2)x^2-8\,a^2-2\,(ab+ac)x)\sqrt{bx+a}\,\sqrt{cx+a}+48\,(a^2b+a^2c)x}{96\,(a^2b^2-2\,a^2bc+a^2c^2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-((b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
 + 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(146) = 292.
time = 6.36, size = 802, normalized size = 4.61

$$-\frac{\sqrt{bc}\,(b+c)|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{-bc}\,ab}\right)}{4\sqrt{-bc}\,a^2b}+\frac{3\left(b^3-b^2c-bc^2+c^3\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^{10}|b|-3\left(5b^5+22b^3c^2+5bc^4\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^8a|b|+2\left(15b^7-b^6c+18b^5c^2+18b^4c^3-b^3c^4+15b^2c^5\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^6a^2|b|-6\left(5b^9-6b^8c-5b^7c^2+12b^6c^3-5b^5c^4-6b^4c^5+5b^3c^6\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4a^3|b|+3\left(5b^{11}-17b^{10}c+21b^9c^2-9b^8c^3-9b^7c^4+21b^6c^5-17b^5c^6+5b^4c^7\right)\sqrt{bc}\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a^4|b|-\left(3b^{13}-20b^{12}c+60b^{11}c^2-108b^{10}c^3+130b^9c^4-108b^8c^5+60b^7c^6-20b^6c^7+3b^5c^8\right)\sqrt{bc}\,a^5|b|}{6\left(\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^4-2\left(b^2+bc\right)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+\left(b^4-2b^3c+b^2c^2\right)a^2\right)^3\left(b^2-2bc+c^2\right)a}-\frac{3(bx+a)b^3+ab^3+3(bx+a)b^2c-3ab^2c}{6\left(b^2-2bc+c^2\right)b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/(sqrt(-
b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 + 22*b
^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4*c^3 -
 b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2 + 12*b^
6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x +
a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^11 - 17*b^1
0*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c^6 + 5*b^4*
c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^10*c^3 + 130*b
^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*sqrt(b*c)*a^5*a
bs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4
- 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c + c^2)*a) - 1/6*(3
*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/((b^2 - 2*b*c + c^2
)*b^3*x^3)
```

Mupad [**B**]
time = 18.74, size = 1290, normalized size = 7.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)
```

```
[Out] (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(8*a
^2) - ((((a + b*x)^(1/2) - a^(1/2))^7*(3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2*c
^4 + 3*b^3*c^3 - 15*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2))^7 - (((a + b*x)^(
```

1/2) - a^(1/2))^5*(26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*b^4*
c^2))/((a + c*x)^(1/2) - a^(1/2))^5 - b^6/3 + ((b^5*c + b^6)*((a + b*x)^(1/
2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (((a + b*x)^(1/2) - a^(1/2))^8
*(c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2))/((a + c*x)^(1/2) - a^(1
/2))^8 + (((a + b*x)^(1/2) - a^(1/2))^6*(6*b*c^5 + 6*b^5*c - (5*b^6)/3 - (5
*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2))/((a + c*x)^(1/2) - a^(1/2)
)^6 - (((17*b^6)/3 + (17*b^3*c^3)/3)*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c
*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(b^6 - 4*b^5*c + b^
4*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(18*
b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2))/((a + c*x)^(1/2) - a^(
1/2))^4)/((((a + b*x)^(1/2) - a^(1/2))^5*(96*a^2*b^5 + 96*a^2*b*c^4 + 96*a^
2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^5
- (((a + b*x)^(1/2) - a^(1/2))^8*(96*a^2*c^5 - 96*a^2*b*c^4 - 96*a^2*b^2*c^
3 + 96*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^
(1/2))^6*(32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a^2*b^4*c - 256*a^2
*b^2*c^3 - 256*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^6 - (((a + b*x)^(1
/2) - a^(1/2))^4*(96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 96*a^2*b^3*c
^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^7*(96*a^2
*c^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c - 384*a^2*b^2*c^3 + 96*a^2*b^3*c^2))/((a
 + c*x)^(1/2) - a^(1/2))^7 + (((a + b*x)^(1/2) - a^(1/2))^3*(32*a^2*b^5 - 6
4*a^2*b^4*c + 32*a^2*b^3*c^2))/((a + c*x)^(1/2) - a^(1/2))^3 + (((a + b*x)^
(1/2) - a^(1/2))^9*(32*a^2*c^5 - 64*a^2*b*c^4 + 32*a^2*b^2*c^3))/((a + c*x)
^(1/2) - a^(1/2))^9) - (((c*(8*b*c + 3*b^2 + 3*c^2))/(16*a^2*(b - c)^2) - (
c*(17*b*c + 4*b^2 + 4*c^2))/(32*a^2*(b - c)^2))*((a + b*x)^(1/2) - a^(1/2))
)/((a + c*x)^(1/2) - a^(1/2)) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(
b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x
)^(1/2) - a^(1/2)))*(b + c))/(8*a^2) - ((2*a)/3 + x*(b/2 + c/2))/(x^3*(b^2
- 2*b*c + c^2)) + (c^3*((a + b*x)^(1/2) - a^(1/2))^3)/(96*a^2*(b - c)^2*((a
 + c*x)^(1/2) - a^(1/2))^3)

3.438 $\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=277

$$-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}+\frac{8a}{3}$$

```
[Out] -8/3*a^2*(b*x+a)^(3/2)/b^2/(b-c)^3+2/3*a^2*(b+3*c)*(b*x+a)^(3/2)/b^3/(b-c)^
3+8/5*a*(b*x+a)^(5/2)/b^2/(b-c)^3-4/5*a*(b+3*c)*(b*x+a)^(5/2)/b^3/(b-c)^3+2
/7*(b+3*c)*(b*x+a)^(7/2)/b^3/(b-c)^3+8/3*a^2*(c*x+a)^(3/2)/(b-c)^3/c^2-2/3*
a^2*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c^3-8/5*a*(c*x+a)^(5/2)/(b-c)^3/c^2+4/5*a
*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^3-2/7*(3*b+c)*(c*x+a)^(7/2)/(b-c)^3/c^3
```

Rubi [A]

time = 0.23, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6822, 45}

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3}-\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3}+\frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3}-\frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3}+\frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3}+\frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3}-\frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/
2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b +
 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7
*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b +
 c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3
*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a
 + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4ax\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x^2\sqrt{a+bx}-4ax\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x^2\right.}{(b-c)^3}$$

$$= \frac{(4a)\int x\sqrt{a+bx}\,dx}{(b-c)^3} - \frac{(4a)\int x\sqrt{a+cx}\,dx}{(b-c)^3} - \frac{(3b+c)\int x^2\sqrt{a+cx}\,dx}{(b-c)^3}$$

$$= \frac{(4a)\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx}{(b-c)^3} - \frac{(4a)\int \left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right.}{(b-c)^3}$$

$$= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(}{5b^3(b-}$$

Mathematica [A]
time = 10.27, size = 114, normalized size = 0.41

$$-\frac{2\left(b^3(a+cx)^{3/2}\left(8a^2(b-2c)-12a(b-2c)cx+5c^2(3b+c)x^2\right)+c^3(a+bx)^{3/2}\left(8a^2(2b-c)+12ab(-2b+c)x-5b^2(b+3c)x^2\right)\right)}{35b^3(b-c)^3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*(b^3*(a + c*x)^(3/2)*(8*a^2*(b - 2*c) - 12*a*(b - 2*c)*c*x + 5*c^2*(3*b
 + c)*x^2) + c^3*(a + b*x)^(3/2)*(8*a^2*(2*b - c) + 12*a*b*(-2*b + c)*x - 5
*b^2*(b + 3*c)*x^2)))/(35*b^3*(b - c)^3*c^3)
```

Maple [A]
time = 0.06, size = 246, normalized size = 0.89

method	result
default	$\frac{\frac{2(bx+a)^{\frac{7}{2}}}{7}-\frac{4a(bx+a)^{\frac{5}{2}}}{5}+\frac{2a^2(bx+a)^{\frac{3}{2}}}{3}}{(b-c)^3b^2}+\frac{8a\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3b^2}-\frac{8a\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3c^2}+\frac{6c\left(\frac{(bx+a)^{\frac{7}{2}}}{7}-\frac{2a(bx+a)^{\frac{5}{2}}}{5}+\frac{a^2}{}\right.}{(b-c)^3b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3/b^2*(1/7*(b*x+a)^(7/2)-2/5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))
+8/(b-c)^3*a/b^2*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)-8/(b-c)^3*a/c^2*(1
/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)+6/(b-c)^3*c/b^3*(1/7*(b*x+a)^(7/2)-2/
5*a*(b*x+a)^(5/2)+1/3*a^2*(b*x+a)^(3/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/2)
```

```
-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7/2
)-2/5*a*(c*x+a)^(5/2)+1/3*a^2*(c*x+a)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A]
time = 0.34, size = 225, normalized size = 0.81

$$-\frac{2\left((16a^3bc^3-8a^3c^4-5(b^4c^3+3b^3c^4)x^3-(29ab^3c^3+3ab^2c^4)x^2-4(2a^2b^2c^3-a^2bc^4)x)\sqrt{bx+a}+(8a^3b^4-16a^3b^3c+5(3b^4c^3+b^3c^4)x^3+(3ab^4c^2+29ab^3c^3)x^2-4(a^2b^4c-2a^2b^3c^2)x)\sqrt{cx+a}\right)}{35(b^6c^3-3b^5c^4+3b^4c^5-b^3c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*
c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (
8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*
a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 -
 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 932 vs. 2(237) = 474.
time = 4.24, size = 932, normalized size = 3.36

$$-\frac{2}{35}\sqrt{ab^2+(bx+a)bc-abc}\left(\left((bx+a)\left(\frac{5(3b^{22}c^5|b|-17b^{21}c^6|b|+39b^{20}c^7|b|-45b^{19}c^8|b|+25b^{18}c^9|b|-3b^{17}c^{10}|b|-3b^{16}c^{11}|b|+b^{15}c^{12}|b|)(bx+a)}{b^{29}c^5-9b^{28}c^6+36b^{27}c^7-84b^{26}c^8+126b^{25}c^9-126b^{24}c^{10}+84b^{23}c^{11}-36b^{22}c^{12}+9b^{21}c^{13}-b^{20}c^{14}}+\frac{3ab^{23}c^4|b|-34ab^{22}c^5|b|+126ab^{21}c^6|b|-210ab^{20}c^7|b|+140ab^{19}c^8|b|+42ab^{18}c^9|b|-126ab^{17}c^{10}|b|+74ab^{16}c^{11}|b|-15ab^{15}c^{12}|b|}{b^{29}c^5-9b^{28}c^6+36b^{27}c^7-84b^{26}c^8+126b^{25}c^9-126b^{24}c^{10}+84b^{23}c^{11}-36b^{22}c^{12}+9b^{21}c^{13}-b^{20}c^{14}}\right)-\frac{4a^2b^{24}c^3|b|-26a^2b^{23}c^4|b|+85a^2b^{22}c^5|b|-203a^2b^{21}c^6|b|+385a^2b^{20}c^7|b|-539a^2b^{19}c^8|b|+511a^2b^{18}c^9|b|-305a^2b^{17}c^{10}|b|+103a^2b^{16}c^{11}|b|-15a^2b^{15}c^{12}|b|}{b^{29}c^5-9b^{28}c^6+36b^{27}c^7-84b^{26}c^8+126b^{25}c^9-126b^{24}c^{10}+84b^{23}c^{11}-36b^{22}c^{12}+9b^{21}c^{13}-b^{20}c^{14}}\right)(bx+a)+\frac{8a^3b^{25}c^2|b|-60a^3b^{24}c^3|b|+187a^3b^{23}c^4|b|-296a^3b^{22}c^5|b|+196a^3b^{21}c^6|b|+112a^3b^{20}c^7|b|-350a^3b^{19}c^8|b|+328a^3b^{18}c^9|b|-164a^3b^{17}c^{10}|b|+44a^3b^{16}c^{11}|b|-5a^3b^{15}c^{12}|b|}{b^{29}c^5-9b^{28}c^6+36b^{27}c^7-84b^{26}c^8+126b^{25}c^9-126b^{24}c^{10}+84b^{23}c^{11}-36b^{22}c^{12}+9b^{21}c^{13}-b^{20}c^{14}}\right)+\frac{2\left(5(bx+a)^{\frac{7}{2}}b+14(bx+a)^{\frac{5}{2}}ab-35(bx+a)^{\frac{3}{2}}a^2b+15(bx+a)^{\frac{7}{2}}c-42(bx+a)^{\frac{5}{2}}ac+35(bx+a)^{\frac{3}{2}}a^2c\right)}{35(b^6-3b^5c+3b^4c^2-b^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b)
 - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18*c
^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*(b*
x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 -
126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14) + (3
*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 210*a*b
^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*a*b^17*
c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*c^5 - 9*
b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^
23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*c^3*abs(b)
 - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^21*c^6*abs(b
) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2*b^18*c^9*ab
s(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 15*a^2*b^15*c^
12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^
9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14))
*(b*x + a) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b) + 187*a^3*b^23
*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*abs(b) + 112*a^3*b
^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c^9*abs(b) - 164*a^
3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^15*c^12*abs(b))/(b^2
9*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^
10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + 2/35*(5*(b*x
 + a)^(7/2)*b + 14*(b*x + a)^(5/2)*a*b - 35*(b*x + a)^(3/2)*a^2*b + 15*(b*x
 + a)^(7/2)*c - 42*(b*x + a)^(5/2)*a*c + 35*(b*x + a)^(3/2)*a^2*c)/(b^6 - 3
*b^5*c + 3*b^4*c^2 - b^3*c^3)
```

Mupad [B]

time = 3.34, size = 429, normalized size = 1.55

$$\frac{x^2\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)\,\sqrt{a+c\,x}}{5\,c}-\frac{2\,a\,\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\,\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\,\sqrt{a+b\,x}}{3\,b^2}+\frac{x\,\left(\frac{8\,a^2}{(b-c)^3}-\frac{4\,a\,\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)}{5\,b}\right)\,\sqrt{a+b\,x}}{3\,b}+\frac{2\,a\,\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\,\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\,\sqrt{a+c\,x}}{3\,c^2}+\frac{x^2\,\left(\frac{2\,a\,(5\,b+3\,c)}{(b-c)^3}-\frac{12\,a\,\left(b^2+3\,c\,b\right)}{7\,b\,(b-c)^3}\right)\,\sqrt{a+b\,x}}{5\,b}-\frac{x\,\left(\frac{8\,a^2}{(b-c)^3}+\frac{4\,a\,\left(\frac{12\,a\,(3\,b+c)}{7\,(b-c)^3}-\frac{2\,a\,(3\,b+5\,c)}{(b-c)^3}\right)}{5\,c}\right)\,\sqrt{a+c\,x}}{3\,c}-\frac{2\,x^3\,(3\,b+c)\,\sqrt{a+c\,x}}{7\,(b-c)^3}+\frac{2\,x^3\,\left(b^2+3\,c\,b\right)\,\sqrt{a+b\,x}}{7\,b\,(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c*
x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)
^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b^2)
 + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c
 + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^2)/(b
 - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^
3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c)^3 - (
12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((8*a^2)/(
b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)
^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(1/2))/(7*(
b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3)
```

3.439 $\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=163

$$\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}+\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3(b-c)^3c}+\frac{2a(3b+c)(a+cx)^{3/2}}{3(b-c)^3c^2}-\frac{2(3b+\ldots}{5\ldots}$$

```
[Out] 8/3*a*(b*x+a)^(3/2)/b/(b-c)^3-2/3*a*(b+3*c)*(b*x+a)^(3/2)/b^2/(b-c)^3+2/5*(
b+3*c)*(b*x+a)^(5/2)/b^2/(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+2/3*a*(3*b+c
)*(c*x+a)^(3/2)/(b-c)^3/c^2-2/5*(3*b+c)*(c*x+a)^(5/2)/(b-c)^3/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6822, 45}

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3}-\frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3}-\frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3}+\frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3}+\frac{8a(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*
b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a
+ c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^
3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(4a\sqrt{a+bx}+b\left(1+\frac{3c}{b}\right)x\sqrt{a+bx}-4a\sqrt{a+cx}-3b\left(1+\frac{c}{3b}\right)x\sqrt{a-\right.}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} - \frac{(3b+c)\int x\sqrt{a+cx}\,dx}{(b-c)^3} + \frac{(b+3c)\int x}{(b-}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} - \frac{(3b+c)\int\left(-\frac{a\sqrt{a+cx}}{c}+\frac{(a+cx)^{3/2}}{c}\right)d}{(b-c)^3}$$

$$= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+c}{3(b-c}$$

Mathematica [A]
time = 1.06, size = 197, normalized size = 1.21

$$\frac{2\sqrt{a-\frac{ab}{c}+\frac{b(a+cx)}{c}}\left(a^2b^3-4a^2b^2c+5a^2bc^2-2a^2c^3-2ab^3(a+cx)+ab^2c(a+cx)+abc^2(a+cx)+b^3(a+cx)^2+3b^2c(a+cx)^2\right)}{5b^2(b-c)^3c^2}+\frac{2\left(5ab(a+cx)^{3/2}-5ac(a+cx)^{3/2}-3b(a+cx)^{5/2}-c(a+cx)^{5/2}\right)}{5(b-c)^3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^2
 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x) +
b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*b*(
a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c*x)^
(5/2)))/(5*(b - c)^3*c^2)
```

Maple [A]
time = 0.03, size = 172, normalized size = 1.06

method	result
default	$\frac{\frac{2(bx+a)^{\frac{5}{2}}}{5}-\frac{2(bx+a)^{\frac{3}{2}}a}{3}}{(b-c)^3b}+\frac{8a(bx+a)^{\frac{3}{2}}}{3b(b-c)^3}-\frac{8a(cx+a)^{\frac{3}{2}}}{3(b-c)^3c}+\frac{6c\left(\frac{(bx+a)^{\frac{5}{2}}}{5}-\frac{(bx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3b^2}-\frac{6b\left(\frac{(cx+a)^{\frac{5}{2}}}{5}-\frac{(cx+a)^{\frac{3}{2}}a}{3}\right)}{(b-c)^3c^2}-\frac{2\left(\frac{(cx+a)^{\frac{5}{2}}}{5}\right.}{(b-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3/b*(1/5*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a)+8/3*a*(b*x+a)^(3/2)/b/(
b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(1/5*(b*x+a)^(5/2)-1/3
*(b*x+a)^(3/2)*a)-6/(b-c)^3*b/c^2*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)-2
/(b-c)^3/c*(1/5*(c*x+a)^(5/2)-1/3*(c*x+a)^(3/2)*a)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A]
time = 0.34, size = 167, normalized size = 1.02

$$\frac{2\left((6a^2bc^2 - 2a^2c^3 + (b^3c^2 + 3b^2c^3)x^2 + (7ab^2c^2 + abc^3)x)\sqrt{bx+a} + (2a^2b^3 - 6a^2b^2c - (3b^3c^2 + b^2c^3)x^2 - (ab^3c + 7ab^2c^2)x)\sqrt{cx+a}\right)}{5(b^5c^2 - 3b^4c^3 + 3b^3c^4 - b^2c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 +
a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3
)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*
b^3*c^4 - b^2*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\sqrt{a+bx} + \sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(139) = 278.
time = 6.79, size = 480, normalized size = 2.94

$$-\frac{2}{5}\sqrt{ab^2 + (bx+a)bc - abc}\left((bx+a)\left(\frac{(3b^{12}c^3|b| - 8b^{11}c^4|b| + 6b^{10}c^5|b| - b^8c^7|b|)(bx+a)}{b^{18}c^3 - 6b^{17}c^4 + 15b^{16}c^5 - 20b^{15}c^6 + 15b^{14}c^7 - 6b^{13}c^8 + b^{12}c^9} + \frac{ab^{13}c^2|b| - 2ab^{12}c^3|b| - 2ab^{11}c^4|b| + 8ab^{10}c^5|b| - 7ab^9c^6|b| + 2ab^8c^7|b|}{b^{18}c^3 - 6b^{17}c^4 + 15b^{16}c^5 - 20b^{15}c^6 + 15b^{14}c^7 - 6b^{13}c^8 + b^{12}c^9}\right) - \frac{2a^2b^{14}c|b| - 11a^2b^{13}c^2|b| + 25a^2b^{12}c^3|b| - 30a^2b^{11}c^4|b| + 20a^2b^{10}c^5|b| - 7a^2b^9c^6|b| + a^2b^8c^7|b|}{b^{18}c^3 - 6b^{17}c^4 + 15b^{16}c^5 - 20b^{15}c^6 + 15b^{14}c^7 - 6b^{13}c^8 + b^{12}c^9}\right) + \frac{2\left((bx+a)^{\frac{5}{2}}b + 5(bx+a)^{\frac{3}{2}}ab + 3(bx+a)^{\frac{5}{2}}c - 5(bx+a)^{\frac{3}{2}}ac\right)}{5(b^5 - 3b^4c + 3b^3c^2 - b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) - 8
*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^3
```

```
- 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*
c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) + 8*a
*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3 - 6*b
^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9))
 - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*abs(b) -
 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*abs(b) + a
^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*
b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(b*x + a)^(
3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3*b^4*c + 3*
b^3*c^2 - b^2*c^3)
```

Mupad [B]
time = 3.36, size = 268, normalized size = 1.64

$$\frac{\left(\frac{8\,a^2}{{\left(b-c\right)}^3}+\frac{2\,a\,\left(\frac{8\,a\,\left(b+3\,c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(5\,b+3\,c\right)}{{\left(b-c\right)}^3}\right)}{3\,b}\right)\,\sqrt{a+b\,x}}{b}-\frac{\left(\frac{8\,a^2}{{\left(b-c\right)}^3}+\frac{2\,a\,\left(\frac{8\,a\,\left(3\,b+c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(3\,b+5\,c\right)}{{\left(b-c\right)}^3}\right)}{3\,c}\right)\,\sqrt{a+c\,x}}{c}-\frac{x\,\left(\frac{8\,a\,\left(b+3\,c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(5\,b+3\,c\right)}{{\left(b-c\right)}^3}\right)\,\sqrt{a+b\,x}}{3\,b}+\frac{x\,\left(\frac{8\,a\,\left(3\,b+c\right)}{5\,{\left(b-c\right)}^3}-\frac{2\,a\,\left(3\,b+5\,c\right)}{{\left(b-c\right)}^3}\right)\,\sqrt{a+c\,x}}{3\,c}+\frac{2\,x^2\,\left(b+3\,c\right)\,\sqrt{a+b\,x}}{5\,{\left(b-c\right)}^3}-\frac{2\,x^2\,\left(3\,b+c\right)\,\sqrt{a+c\,x}}{5\,{\left(b-c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c
))/(b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8*a
*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(3*c))*(a + c*x)^
(1/2))/c - (x*((8*a*(b + 3*c))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c)^3)
*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c))/(5*(b - c)^3) - (2*a*(3*b + 5
*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(1/2))/
(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

3.440 $\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=155

$$\frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}}{}$$

```
[Out] 2/3*(b+3*c)*(b*x+a)^(3/2)/b/(b-c)^3-2/3*(3*b+c)*(c*x+a)^(3/2)/(b-c)^3/c-8*a
^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3+8*a^(3/2)*arctanh((c*x+a)^(1/
2)/a^(1/2))/(b-c)^3+8*a*(b*x+a)^(1/2)/(b-c)^3-8*a*(c*x+a)^(1/2)/(b-c)^3
```

Rubi [A]

time = 0.14, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6822, 52, 65, 214}

$$-\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\sqrt{a+bx}}{(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^
3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b -
c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)
*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}+\frac{4a\sqrt{a+bx}}{x}-3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}-\frac{4a\sqrt{a+c}}{x}\right.}{(b-c)^3}$$

$$= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c}+\frac{(4a)\int\frac{\sqrt{a+bx}}{x}\,dx}{(b-c)^3}-\frac{(4a}{}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/}}{3(b-c)^3c}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/}}{3(b-c)^3c}$$

$$= \frac{8a\sqrt{a+bx}}{(b-c)^3}+\frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3}-\frac{8a\sqrt{a+cx}}{(b-c)^3}-\frac{2(3b+c)(a+cx)^{3/}}{3(b-c)^3c}$$

Mathematica [A]
time = 10.22, size = 127, normalized size = 0.82

$$-\frac{2\left(b\sqrt{a+cx}\,(a(3b+13c)+c(3b+c)x)-c\sqrt{a+bx}\,(a(13b+3c)+b(b+3c)x)+12a^{3/2}bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-12a^{3/2}bc\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)\right)}{3b(b-c)^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-2*(b*Sqrt[a + c*x]*(a*(3*b + 13*c) + c*(3*b + c)*x) - c*Sqrt[a + b*x]*(a*
(13*b + 3*c) + b*(b + 3*c)*x) + 12*a^(3/2)*b*c*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]] - 12*a^(3/2)*b*c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(3*b*(b - c)^3*c)
```

Maple [A]
time = 0.01, size = 148, normalized size = 0.95

method	result
default	$\frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^{3}}+\frac{2c(bx+a)^{\frac{3}{2}}}{(b-c)^{3}b}-\frac{2b(cx+a)^{\frac{3}{2}}}{(b-c)^{3}c}-\frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^{3}}+\frac{4a\left(2\sqrt{bx+a}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^{3}}-\frac{4a\left(2\sqrt{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(
3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4*a/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc
tanh((b*x+a)^(1/2)/a^(1/2)))-4*a/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh
((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A]
time = 0.36, size = 321, normalized size = 2.07

$$\left[-\frac{2\left(6\,a^{\frac{3}{2}}bc\log\left(\frac{bx+2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+6\,a^{\frac{3}{2}}bc\log\left(\frac{cx-2\,\sqrt{cx+a}\,\sqrt{a}+2\,a}{x}\right)-(13\,abc+3\,ac^{2}+(b^{2}c+3\,bc^{2})x)\sqrt{bx+a}+(3\,ab^{2}+13\,abc+(3\,b^{2}c+bc^{2})x)\sqrt{cx+a}\right)}{3\,(b^{4}c-3\,b^{3}c^{2}+3\,b^{2}c^{3}-bc^{4})},\ \frac{2\left(12\,\sqrt{-a}\,abc\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-12\,\sqrt{-a}\,abc\arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right)+(13\,abc+3\,ac^{2}+(b^{2}c+3\,bc^{2})x)\sqrt{bx+a}-(3\,ab^{2}+13\,abc+(3\,b^{2}c+bc^{2})x)\sqrt{cx+a}\right)}{3\,(b^{4}c-3\,b^{3}c^{2}+3\,b^{2}c^{3}-bc^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/
2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 +
 (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^
2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(
-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(
c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x
+ a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3
*b^3*c^2 + 3*b^2*c^3 - b*c^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. $2(131) = 262$.
time = 5.70, size = 2374, normalized size = 15.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(b
) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^12*c - 6*b^11*c^2 + 15*
b^10*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs(b)
 + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^4*c^
4*abs(b) + a*b^3*c^5*abs(b))/(b^12*c - 6*b^11*c^2 + 15*b^10*c^3 - 20*b^9*c^
4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*arctan(sqrt(b*x + a)/sqrt(-a
))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) + 2/3*((b*x + a)^(3/2)*b^9 +
12*sqrt(b*x + a)*a*b^9 - 3*(b*x + a)^(3/2)*b^8*c - 72*sqrt(b*x + a)*a*b^8*c
 - 3*(b*x + a)^(3/2)*b^7*c^2 + 180*sqrt(b*x + a)*a*b^7*c^2 + 25*(b*x + a)^(
3/2)*b^6*c^3 - 240*sqrt(b*x + a)*a*b^6*c^3 - 45*(b*x + a)^(3/2)*b^5*c^4 + 1
80*sqrt(b*x + a)*a*b^5*c^4 + 39*(b*x + a)^(3/2)*b^4*c^5 - 72*sqrt(b*x + a)*
a*b^4*c^5 - 17*(b*x + a)^(3/2)*b^3*c^6 + 12*sqrt(b*x + a)*a*b^3*c^6 + 3*(b*
x + a)^(3/2)*b^2*c^7)/(b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8
*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9) - 8*(2*
(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a
)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^
2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2
*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqr
t(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 -
3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2
*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3
*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^
3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*
b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6
*a^3*b^10*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5
*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)
*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^
4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c
^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^
3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b
*c^2 - c^3)))/((b^12 - 9*b^11*c + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 -
```

```
126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 +
 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) + 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^
2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*
b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b
^2*c)*sqrt(-a*b*c)*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^
2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*
c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2
*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*
b^3*c^5)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) +
(a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6
*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a)*abs(b)*sgn(b^
3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^11 - 6*a^3*b^10*c + 14*a^3*b^9*c^2 -
14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*
c^8)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - s
qrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c
+ 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b
^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^12 - 9*b^11*c
 + 36*b^10*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b
^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*
c^3))
```

Mupad [**B**]
time = 7.02, size = 762, normalized size = 4.92

$$\frac{4\,a^{3/2}\,b^4-\frac{4\,a^{3/2}\,c^4\left(\frac{4\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{15\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}+\frac{24\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{6\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^6}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^6}\right)}{3}-\frac{4\,a^{3/2}\,b^2\,c^2\left(\frac{24\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{12\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}+\frac{12\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}-\frac{15\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}+\frac{18\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-3\right)}{3}+\frac{4\,a^{3/2}\,b\,c^3\left(\frac{6\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{12\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}+\frac{66\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}-\frac{24\left(\sqrt{a+b\,x}-\sqrt{a}\right)^5}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^5}+\frac{18\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^4}\right)}{3}+\frac{4\,a^{3/2}\,b^3\,c\left(6\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)-\frac{24\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a+c\,x}-\sqrt{a}}+\frac{6\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{4\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)^3}+26\right)}{3}}{c\,{\left(b-c\right)}^3\,{\left(b-\frac{c\,{\left(\sqrt{a+b\,x}-\sqrt{a}\right)}^2}{{\left(\sqrt{a+c\,x}-\sqrt{a}\right)}^2}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*
x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2)
 - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/
2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a
 + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(3/2)*
b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (12
*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*((a + b
*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2
) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)^(1/2) -
a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^2)/((a +
c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b*c^3*((6*((a + b*x)^(1/2) - a
^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2) - a^(1/2))^
3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1/2))^4)/((a +
 c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1
/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a
```

```
^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4))/3 +
 (4*a^(3/2)*b^3*c*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(
1/2))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (6*
((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (4*((a + b*x
)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + 26))/3)/(c*(b - c)^3*
(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)^3)
```

3.441 $\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3} \, dx$

Optimal. Leaf size=157

$$\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-\frac{6\sqrt{a}\,(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+6$$

```
[Out] -6*(b+c)*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+6*(b+c)*arctanh((c*
x+a)^(1/2)/a^(1/2))*a^(1/2)/(b-c)^3+2*(b+3*c)*(b*x+a)^(1/2)/(b-c)^3-4*a*(b*
x+a)^(1/2)/(b-c)^3/x-2*(3*b+c)*(c*x+a)^(1/2)/(b-c)^3+4*a*(c*x+a)^(1/2)/(b-c
)^3/x
```

Rubi [A]
time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6822, 43, 65, 214, 52}

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3}+\frac{4a\sqrt{a+cx}}{x(b-c)^3}+\frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}-\frac{2\sqrt{a}\,(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}-\frac{4\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{4\sqrt{a}\,c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{2\sqrt{a}\,(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) -
 (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x)
- (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b +
3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[
a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(b - c)^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x}-\frac{4a\sqrt{a+cx}}{x^2}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x}\right) dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^2}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x}\,dx}{(b-c)^3}+$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}+$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3}-\frac{4a\sqrt{a+bx}}{(b-c)^3x}-\frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3}+\frac{4a\sqrt{a+cx}}{(b-c)^3x}-$$

Mathematica [A]

time = 10.15, size = 143, normalized size = 0.91

$$\left(\frac{2(b+3c)}{(b-c)^3}-\frac{4a}{(b-c)^3x}\right)\sqrt{a+bx}+\left(-\frac{2(3b+c)}{(b-c)^3}+\frac{4a}{(b-c)^3x}\right)\sqrt{a+cx}-\frac{6\sqrt{a}\,(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3}+\frac{6\sqrt{a}\,(b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] ((2*(b + 3*c))/(b - c)^3 - (4*a)/((b - c)^3*x))*Sqrt[a + b*x] + ((-2*(3*b +
 c))/(b - c)^3 + (4*a)/((b - c)^3*x))*Sqrt[a + c*x] - (6*Sqrt[a]*(b + c)*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (6*Sqrt[a]*(b + c)*ArcTanh[Sqrt[a
 + c*x]/Sqrt[a]])/(b - c)^3
```

Maple [A]
time = 0.01, size = 237, normalized size = 1.51

method	result
default	$\frac{b\left(2\sqrt{bx+a}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{(b-c)^3}+\frac{8ab\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}-\frac{8ac\left(-\frac{\sqrt{cx+a}}{2xc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] b/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8*a/(b
-c)^3*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
-8*a/(b-c)^3*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^
(1/2)))-c/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+3*c/(b-c)^3*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3*
b/(b-c)^3*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

Fricas [A]
time = 0.36, size = 260, normalized size = 1.66

$$\left[-\frac{3\sqrt{a}\,(b+c)x\log\left(\frac{bx+2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+3\sqrt{a}\,(b+c)x\log\left(\frac{cx-2\sqrt{cx+a}\,\sqrt{a}+2a}{x}\right)-2\left((b+3c)x-2a\right)\sqrt{bx+a}+2\left((3b+c)x-2a\right)\sqrt{cx+a}}{(b^3-3b^2c+3bc^2-c^3)x},\ \frac{2\left(3\sqrt{-a}\,(b+c)x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-3\sqrt{-a}\,(b+c)x\arctan\left(\frac{\sqrt{cx+a}\,\sqrt{-a}}{a}\right)+((b+3c)x-2a)\sqrt{bx+a}-((3b+c)x-2a)\sqrt{cx+a}\right)}{(b^3-3b^2c+3bc^2-c^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")

[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqrt(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*x - 2*a)*sqrt(b*x + a) + 2*((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x), 2*(3*sqrt(-a)*(b + c)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*(b + c)*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) + ((b + 3*c)*x - 2*a)*sqrt(b*x + a) - ((3*b + c)*x - 2*a)*sqrt(c*x + a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2318 vs. 2(137) = 274.
time = 23.33, size = 2318, normalized size = 14.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out] -2*(sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b^3*c + 3*b^2*c^2 - b*c^3) + 2*sqrt(b*x + a)*a*b/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x) - 3*(a*b^2 + a*b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(-a)) - (sqrt(b*x + a)*b^2 + 3*sqrt(b*x + a)*b*c)/(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^3*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^2*b^2*c^2*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a*b*c*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(

```
a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6
*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4
 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 -
c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c
^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*
abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 1
4*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(
b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^
10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 -
a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a
*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b
*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^
2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b
^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 -
9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6
 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c
^2 + a*b*c^3)) - 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^
2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^
4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*sqrt(-a*b*c)*a
bs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*
c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*
c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5
*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(-a
*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11
*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5
*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 -
c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*
b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))
*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sq
rt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c +
2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2
*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(
b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^
3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c
^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)))/b
```

Mupad [B]

time = 7.49, size = 559, normalized size = 3.56

$$\frac{2\sqrt{a}\,b^2\left(\sqrt{a+cx}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}-\frac{2\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}+1\right)-2\sqrt{a}\,c^2\left(\sqrt{a+cx}-\sqrt{a}\right)\left(\frac{2\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}-\frac{\left(\sqrt{a+bx}-\sqrt{a}\right)^4}{\left(\sqrt{a+cx}-\sqrt{a}\right)^4}+\frac{3\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{\left(\sqrt{a+cx}-\sqrt{a}\right)^3}\right)+2\sqrt{a}\,b\,c\left(\sqrt{a+cx}-\sqrt{a}\right)\left(\frac{8\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}-\frac{14\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}+\frac{3\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)\left(\sqrt{a+bx}-\sqrt{a}\right)}{\sqrt{a+cx}-\sqrt{a}}-\frac{3\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)\left(\sqrt{a+bx}-\sqrt{a}\right)^3}{\left(\sqrt{a+cx}-\sqrt{a}\right)^3}\right)}{(b-c)^3\left(\sqrt{a+bx}-\sqrt{a}\right)\left(b-\frac{c\left(\sqrt{a+bx}-\sqrt{a}\right)^2}{\left(\sqrt{a+cx}-\sqrt{a}\right)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))
/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)
```

```
^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
 a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1) -
2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/2))^2
)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^
(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3) + 2
*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/(
(a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^
(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) -
a^(1/2)))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (3*log
(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2)
- a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3))/((b - c)^3*((a + b*x)^(1/2) -
 a^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2)
)^2))
```

3.442 $\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$

Optimal. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{(2b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{(3b+2c)\sqrt{a+cx}}{(b-c)^3x}-\frac{3bc\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{3bc\tanh}{\sqrt{}}$$

```
[Out] -3*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)+3*b*c*arctanh((c*x+a)
^(1/2)/a^(1/2))/(b-c)^3/a^(1/2)-2*a*(b*x+a)^(1/2)/(b-c)^3/x^2-(2*b+3*c)*(b*
x+a)^(1/2)/(b-c)^3/x+2*a*(c*x+a)^(1/2)/(b-c)^3/x^2+(3*b+2*c)*(c*x+a)^(1/2)/
(b-c)^3/x
```

Rubi [A]
time = 0.13, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6822, 43, 44, 65, 214}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{c^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3}+\frac{2a\sqrt{a+cx}}{x^2(b-c)^3}-\frac{b\sqrt{a+bx}}{x(b-c)^3}-\frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3}+\frac{c\sqrt{a+cx}}{x(b-c)^3}+\frac{(3b+c)\sqrt{a+cx}}{x(b-c)^3}-\frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}+\frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((
b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2)
 + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x
) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a
+ c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/S
qrt[a]])/(Sqrt[a]*(b - c)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
```

```
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 6822

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3}+\frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2}-\frac{4a\sqrt{a+cx}}{x^3}-\frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2}\right) dx}{(b-c)^3}$$

$$= \frac{(4a)\int \frac{\sqrt{a+bx}}{x^3}\,dx}{(b-c)^3}-\frac{(4a)\int \frac{\sqrt{a+cx}}{x^3}\,dx}{(b-c)^3}-\frac{(3b+c)\int \frac{\sqrt{a+cx}}{x^2}\,dx}{(b-c)^3}+$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x}+$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+}}{(b-c}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+}}{(b-c}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2}-\frac{b\sqrt{a+bx}}{(b-c)^3x}-\frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x}+\frac{2a\sqrt{a+cx}}{(b-c)^3x^2}+\frac{c\sqrt{a+}}{(b-c}$$

Mathematica [A]
time = 10.16, size = 146, normalized size = 0.89

$$\frac{\sqrt{a}\left(-2a\sqrt{a+bx}-2bx\sqrt{a+bx}-3cx\sqrt{a+bx}+2a\sqrt{a+cx}+3bx\sqrt{a+cx}+2cx\sqrt{a+cx}\right)-3bcx^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+3bcx^2\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}\,(b-c)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3),x]
```

```
[Out] (Sqrt[a]*(-2*a*Sqrt[a + b*x] - 2*b*x*Sqrt[a + b*x] - 3*c*x*Sqrt[a + b*x] +
2*a*Sqrt[a + c*x] + 3*b*x*Sqrt[a + c*x] + 2*c*x*Sqrt[a + c*x]) - 3*b*c*x^2*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*b*c*x^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/
(Sqrt[a]*(b - c)^3*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(144) = 288.
time = 0.02, size = 300, normalized size = 1.83

method	result
default	$\frac{2b^2\left(-\frac{\sqrt{bx+a}}{2xb}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}\right)}{(b-c)^3}+\frac{8a\,b^2\left(\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{x^2b^2}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}\right)}{(b-c)^3}-\frac{8a\,c^2\left(-\frac{(cx}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^
(1/2))+8/(b-c)^3*a*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/
8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8/a*(c*x+a)^
(3/2)-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2))
)+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/
a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((c*x+a)^
(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/x/c-1/2/a^(1/2)*arctanh((
c*x+a)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
```

Fricas [A]
time = 0.36, size = 297, normalized size = 1.81

$$\left[-\frac{3\sqrt{a}bcx^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+3\sqrt{a}bcx^2\log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a}+2a}{x}\right)+2(2a^2+(2ab+3ac)x)\sqrt{bx+a}-2(2a^2+(3ab+2ac)x)\sqrt{cx+a}}{2(ab^3-3ab^2c+3abc^2-ac^3)x^2}, \frac{3\sqrt{-a}bcx^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-3\sqrt{-a}bcx^2\arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right)-(2a^2+(2ab+3ac)x)\sqrt{bx+a}+(2a^2+(3ab+2ac)x)\sqrt{cx+a}}{(ab^3-3ab^2c+3abc^2-ac^3)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*s
qrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2
*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a
))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arcta
n(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(
-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*
c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\sqrt{a+bx}+\sqrt{a+cx}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2766 vs. 2(144) = 288.
time = 55.11, size = 2766, normalized size = 16.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 3*b*c*arctan(sqrt(b*x + a)/sqrt(-a))/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*sqrt(
-a)) - 2*(3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))
*a^3*b^7*c*abs(b) - 7*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c
 - a*b*c))*a^3*b^6*c^2*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (
b*x + a)*b*c - a*b*c))*a^3*b^5*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sq
rt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^4*c^4*abs(b) - 2*(sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a^3*b^3*c^5*abs(b) - 3*(sqrt
(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^5*c*abs(
b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^
```

2*b^3*c^3*abs(b) + 6*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*a^2*b^2*c^4*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^3*c*abs(b) - 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b^2*c^2*abs(b) - 6*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^5*a*b*c^3*abs(b) + 3*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*b*c*abs(b) + 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^7*c^2*abs(b)) /((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)^2*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) - 3*(2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^7*c - 5*a^2*b^6*c^2 + 10*a^2*b^5*c^3 - 10*a^2*b^4*c^4 + 5*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^2*b^8*c - 5*a^2*b^7*c^2 + 10*a^2*b^6*c^3 - 10*a^2*b^5*c^4 + 5*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b) + (a^3*b^11*c^2 - 6*a^3*b^10*c^3 + 14*a^3*b^9*c^4 - 14*a^3*b^8*c^5 + 14*a^3*b^6*c^7 - 14*a^3*b^5*c^8 + 6*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^3*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)) + 3*(2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*sqrt(-a*b*c)*abs(b) + (a^2*b^7*c - 5*a^2*b^6*c^2 + 10*a^2*b^5*c^3 - 10*a^2*b^4*c^4 + 5*a^2*b^3*c^5 - a^2*b^2*c^6)*sqrt(-a*b*c)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^2*b^8*c - 5*a^2*b^7*c^2 + 10*a^2*b^6*c^3 - 10*a^2*b^5*c^4 + 5*a^2*b^4*c^5 - a^2*b^3*c^6)*sqrt(-a)*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)*abs(b) + (a^3*b^11*c^2 - 6*a^3*b^10*c^3 + 14*a^3*b^9*c^4 - 14*a^3*b^8*c^5 + 14*a^3*b^6*c^7 - 14*a^3*b^5*c^8 + 6*a^3*b^4*c^9 - a^3*b^3*c^10)*sqrt(-a)*abs(b)*sgn(2*b^3 - 6*b^2*c + 6*b*c^2 - 2*c^3) + (a^3*b^11*c - 6*a^3*b^10*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - sqrt((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^

```
5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*
c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c
^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b
^3*c^8 - b^2*c^9)*a^3*abs(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)) - (2*
(b*x + a)^(3/2)*b^2 + 3*(b*x + a)^(3/2)*b*c - 3*sqrt(b*x + a)*a*b*c)/((b^3
- 3*b^2*c + 3*b*c^2 - c^3)*b^2*x^2)
```

Mupad [B]
time = 5.74, size = 287, normalized size = 1.75

$$\frac{c^2\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{4\,\sqrt{a}\,\left(b-c\right)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}-\frac{\left(\frac{\sqrt{a}\,b^2}{4\,\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}-\frac{\sqrt{a}\,\left(b^2+c\,b\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\left(\sqrt{a+c\,x}-\sqrt{a}\right)\left(a\,b^3-3\,a\,b^2\,c+3\,a\,b\,c^2-a\,c^3\right)}\right)\left(\sqrt{a+c\,x}-\sqrt{a}\right)^2}{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}+\frac{3\,b\,c\,\ln\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{a+c\,x}-\sqrt{a}}\right)}{\sqrt{a}\,\left(b^3-3\,b^2\,c+3\,b\,c^2-c^3\right)}-\frac{c\,\left(b+c\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{a}\,\left(b-c\right)^3\left(\sqrt{a+c\,x}-\sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2) -
 a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c))
- (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2)))/(((a + c*x)^(1/2) - a^(
1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1/2))
^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1/2))/
((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3)) - (
c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2)))
```

3.443 $\int \sqrt{1-x} \left(\sqrt{1-x}+\sqrt{1+x}\right) \, dx$

Optimal. Leaf size=31

$$x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] x-1/2*x^2+1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6820, 201, 222}

$$\frac{\text{ArcSin}(x)}{2}-\frac{x^2}{2}+\frac{1}{2}\sqrt{1-x^2}\,x+x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= \int \left(1-x+\sqrt{1-x^2}\right)dx\\ &= x-\frac{x^2}{2}+\int \sqrt{1-x^2}\,dx\\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 1.45

$$x-\frac{x^2}{2}+\frac{1}{2}x\sqrt{1-x^2}-\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(23) = 46$.
time = 0.24, size = 63, normalized size = 2.03

method	result	size
default	$x-\frac{x^2}{2}+\frac{\sqrt{1-x}\,(1+x)^{\frac{3}{2}}}{2}-\frac{\sqrt{1-x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{2\sqrt{1+x}\,\sqrt{1-x}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/2*x^2+1/2*(1-x)^(1/2)*(1+x)^(3/2)-1/2*(1-x)^(1/2)*(1+x)^(1/2)+1/2*((1-x
)*(1+x))^(1/2)/(1+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.49, size = 23, normalized size = 0.74

$$-\frac{1}{2}x^2+\frac{1}{2}\sqrt{-x^2+1}\,x+x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(-x^2 + 1)*x + x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.42

$$-\frac{1}{2}\,x^2+\frac{1}{2}\,\sqrt{x+1}\,x\sqrt{-x+1}\,+x-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*sqrt(x + 1)*x*sqrt(-x + 1) + x - arctan((sqrt(x + 1)*sqrt(-x
 + 1) - 1)/x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.
time = 1.19, size = 65, normalized size = 2.10

$$-\frac{(1-x)^2}{2}-2\left(\begin{cases}-\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{1-x}}{2}\right)}{2} & \text{for } \sqrt{1-x}\,>-\sqrt{2}\,\wedge\sqrt{1-x}\,<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(1 - x)**2/2 - 2*Piecewise((-x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sq
rt(1 - x)/2)/2, (sqrt(1 - x) < sqrt(2)) & (sqrt(1 - x) > -sqrt(2))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(23) = 46$.
time = 2.73, size = 54, normalized size = 1.74

$$-\frac{1}{2}\,(x-1)^2+\frac{1}{2}\,(x+2)\sqrt{x+1}\,\sqrt{-x+1}\,-\sqrt{x+1}\,\sqrt{-x+1}\,-\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(x - 1)^2 + 1/2*(x + 2)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*sqrt(-x
 + 1) - arcsin(1/2*sqrt(2)*sqrt(-x + 1))
```

Mupad [B]
time = 8.12, size = 209, normalized size = 6.74

$$x-2\,\mathrm{atan}\left(\frac{\sqrt{1-x}\,-1}{\sqrt{x+1}\,-1}\right)-\frac{\frac{2\left(\sqrt{1-x}\,-1\right)}{\sqrt{x+1}\,-1}-\frac{14\left(\sqrt{1-x}\,-1\right)^3}{\left(\sqrt{x+1}\,-1\right)^3}+\frac{14\left(\sqrt{1-x}\,-1\right)^5}{\left(\sqrt{x+1}\,-1\right)^5}-\frac{2\left(\sqrt{1-x}\,-1\right)^7}{\left(\sqrt{x+1}\,-1\right)^7}}{\frac{4\left(\sqrt{1-x}\,-1\right)^2}{\left(\sqrt{x+1}\,-1\right)^2}+\frac{6\left(\sqrt{1-x}\,-1\right)^4}{\left(\sqrt{x+1}\,-1\right)^4}+\frac{4\left(\sqrt{1-x}\,-1\right)^6}{\left(\sqrt{x+1}\,-1\right)^6}+\frac{\left(\sqrt{1-x}\,-1\right)^8}{\left(\sqrt{x+1}\,-1\right)^8}+1}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2),x)
```

```
[Out] x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) -
1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3
+ (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) - 1)
^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2
 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)
^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1
) - x^2/2
```

3.444 $\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)$

Optimal. Leaf size=38

$$-\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}$$

```
[Out] -1/2*x^4+2/3*(-x^2+1)^(3/2)-2/5*(-x^2+1)^(5/2)
```

Rubi [A]
time = 0.24, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6820, 6874, 272, 45}

$$-\frac{x^4}{2}-\frac{2}{5}\left(1-x^2\right)^{5/2}+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -1/2*x^4 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^3\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x^3\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx \\ &= -\int \left(2x^3+2x^3\sqrt{1-x^2}\right)dx \\ &= -\frac{x^4}{2}-2\int x^3\sqrt{1-x^2}\,dx \\ &= -\frac{x^4}{2}-\text{Subst}\left(\int \sqrt{1-x}\,x\,dx,x,x^2\right) \\ &= -\frac{x^4}{2}-\text{Subst}\left(\int \left(\sqrt{1-x}-(1-x)^{3/2}\right)dx,x,x \\ &= -\frac{x^4}{2}+\frac{2}{3}\left(1-x^2\right)^{3/2}-\frac{2}{5}\left(1-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.16

$$-\frac{1}{30}\left(-1+x^2\right)\left(15+8\sqrt{1-x^2}+3x^2\left(5+4\sqrt{1-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -1/30*((-1 + x^2)*(15 + 8*Sqrt[1 - x^2] + 3*x^2*(5 + 4*Sqrt[1 - x^2])))
```

Maple [A]
time = 0.26, size = 33, normalized size = 0.87

method	result	size
default	$-\frac{x^4}{2}-\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(x^2-1\right)\left(3x^2+2\right)}{15}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETU
RNVERBOSE)
[Out] -1/2*x^4-2/15*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)*(3*x^2+2)
```

Maxima [A]
time = 0.48, size = 31, normalized size = 0.82

$$-\frac{1}{2}x^4+\frac{2}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2+\frac{4}{15}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)
```

Fricas [**A**]
time = 0.33, size = 32, normalized size = 0.84

$$-\frac{1}{2}\,x^4-\frac{2}{15}\,\left(3\,x^4-x^2-2\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.
time = 3.18, size = 77, normalized size = 2.03

$$-\frac{1}{2}\,x^4-\frac{1}{60}\,\left(\left(2\,\left(3\,\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{12}\,\left(\left(2\,\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 1
95)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) -
39)*sqrt(x + 1)*sqrt(-x + 1)
```

Mupad [**B**]
time = 3.06, size = 42, normalized size = 1.11

$$\sqrt{1-x}\,\left(\frac{4\,\sqrt{x+1}}{15}+\frac{2\,x^2\,\sqrt{x+1}}{15}-\frac{2\,x^4\,\sqrt{x+1}}{5}\right)-\frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] (1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x
+ 1)^(1/2))/5) - x^4/2
```

3.445 $\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)$

Optimal. Leaf size=48

$$-\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] -2/3*x^3-1/4*arcsin(x)+1/4*x*(-x^2+1)^(1/2)-1/2*x^3*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}=0.119$, Rules used = {6820, 6874, 285, 327, 222}

$$-\frac{\text{ArcSin}(x)}{4}-\frac{2x^3}{3}+\frac{1}{4}\sqrt{1-x^2}\,x-\frac{1}{2}\sqrt{1-x^2}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-2*x^3)/3 + (x*Sqrt[1 - x^2])/4 - (x^3*Sqrt[1 - x^2])/2 - ArcSin[x]/4
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x^2\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)\,dx &= -\int x^2\left(\sqrt{1-x}+\sqrt{1+x}\right)^2\,dx\\ &= -\int\left(2x^2+2x^2\sqrt{1-x^2}\right)dx\\ &= -\frac{2x^3}{3}-2\int x^2\sqrt{1-x^2}\,dx\\ &= -\frac{2x^3}{3}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\int\frac{}{\sqrt{}}\\ &= -\frac{2x^3}{3}+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{2}x^3\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 1.25

$$\frac{1}{12}\left(-8+3x\sqrt{1-x^2}-x^3\left(8+6\sqrt{1-x^2}\right)-6\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] (-8 + 3*x*Sqrt[1 - x^2] - x^3*(8 + 6*Sqrt[1 - x^2]) - 6*ArcTan[Sqrt[1 + x]/
Sqrt[1 - x]])/12
```

Maple [A]
time = 0.26, size = 59, normalized size = 1.23

method	result	size
default	$-\frac{2x^3}{3}-\frac{\sqrt{1-x}\,\sqrt{1+x}\,\left(2x^3\sqrt{-x^2+1}-x\sqrt{-x^2+1}+\arcsin(x)\right)}{4\sqrt{-x^2+1}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3*x^3-1/4*(1-x)^(1/2)*(1+x)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)
+arcsin(x))/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.71

$$-\frac{2}{3}\,x^3+\frac{1}{2}\,{\left(-x^2+1\right)}^{\frac{3}{2}}x-\frac{1}{4}\,\sqrt{-x^2+1}\,x-\frac{1}{4}\,\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="maxima")
```

```
[Out] -2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Fricas [A]
time = 0.40, size = 51, normalized size = 1.06

$$-\frac{2}{3}\,x^3-\frac{1}{4}\,\left(2\,x^3-x\right)\sqrt{x+1}\,\sqrt{-x+1}\,+\frac{1}{2}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="fricas")
```

```
[Out] -2/3*x^3 - 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arctan((sqrt(x +
1)*sqrt(-x + 1) - 1)/x)
```

Sympy [A]
time = 143.80, size = 270, normalized size = 5.62

$$\frac{x^4}{4}-\frac{x^3}{3}-\frac{(x+1)^4}{4}+\frac{2(x+1)^3}{3}-\frac{(x+1)^2}{2}-4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)+8\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)-4\left(\begin{cases}\frac{x\sqrt{1-x}\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{3}-\frac{\sqrt{1-x}\sqrt{x+1}\left(-5x-2(x+1)^3+6(x+1)^2-4\right)}{16}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**4/4 - x**3/3 - (x + 1)**4/4 + 2*(x + 1)**3/3 - (x + 1)**2/2 - 4*Piecewis
e((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1
) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) + 8*Piecewise((x*sqrt(1 - x)*sqrt
(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2
, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) - 4*Piecewise((x*sqr
t(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/3 - sqrt(1 - x)*sqrt
(x + 1)*(-5*x - 2*(x + 1)**3 + 6*(x + 1)**2 - 4)/16 + 5*asin(sqrt(2)*sqrt(x
 + 1)/2)/8, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(36) = 72$.
time = 3.93, size = 76, normalized size = 1.58

$$-\frac{2}{3}\,x^3-\frac{1}{12}\,\left(\left(2\,\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\,\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algor
ithm="giac")
```

```
[Out] -2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt
(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arcsi
n(1/2*sqrt(2)*sqrt(x + 1))
```

Mupad [B]
time = 10.39, size = 381, normalized size = 7.94

$$\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{35\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{273\left(\sqrt{1-x}-1\right)^5}{\left(\sqrt{x+1}-1\right)^5}-\frac{715\left(\sqrt{1-x}-1\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{715\left(\sqrt{1-x}-1\right)^9}{\left(\sqrt{x+1}-1\right)^9}-\frac{273\left(\sqrt{1-x}-1\right)^{11}}{\left(\sqrt{x+1}-1\right)^{11}}+\frac{35\left(\sqrt{1-x}-1\right)^{13}}{\left(\sqrt{x+1}-1\right)^{13}}-\frac{\left(\sqrt{1-x}-1\right)^{15}}{\left(\sqrt{x+1}-1\right)^{15}}}{\frac{8\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{28\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{56\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{1-x}-1\right)^8}{\left(\sqrt{x+1}-1\right)^8}+\frac{56\left(\sqrt{1-x}-1\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{1-x}-1\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{8\left(\sqrt{1-x}-1\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{1-x}-1\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}+1}-\frac{2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - (((1 - x)^(1/2) - 1)/((x +
1)^(1/2) - 1) - (35*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (273*((1
 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (715*((1 - x)^(1/2) - 1)^7)/((x
 + 1)^(1/2) - 1)^7 + (715*((1 - x)^(1/2) - 1)^9)/((x + 1)^(1/2) - 1)^9 - (2
73*((1 - x)^(1/2) - 1)^11)/((x + 1)^(1/2) - 1)^11 + (35*((1 - x)^(1/2) - 1)
^13)/((x + 1)^(1/2) - 1)^13 - ((1 - x)^(1/2) - 1)^15/((x + 1)^(1/2) - 1)^15
)/((8*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (28*((1 - x)^(1/2) - 1
)^4)/((x + 1)^(1/2) - 1)^4 + (56*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)
^6 + (70*((1 - x)^(1/2) - 1)^8)/((x + 1)^(1/2) - 1)^8 + (56*((1 - x)^(1/2)
- 1)^10)/((x + 1)^(1/2) - 1)^10 + (28*((1 - x)^(1/2) - 1)^12)/((x + 1)^(1/2
) - 1)^12 + (8*((1 - x)^(1/2) - 1)^14)/((x + 1)^(1/2) - 1)^14 + ((1 - x)^(1
/2) - 1)^16/((x + 1)^(1/2) - 1)^16 + 1) - (2*x^3)/3
```

3.446 $\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$

Optimal. Leaf size=21

$$-x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] -x^2+2/3*(-x^2+1)^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {6820, 6874, 267}

$$\frac{2}{3}\left(1-x^2\right)^{3/2}-x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -x^2 + (2*(1 - x^2)^(3/2))/3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int x\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int x\left(\sqrt{1-x}+\sqrt{1+x}\right)^2 dx\\ &= -\int\left(2x+2x\sqrt{1-x^2}\right)dx\\ &= -x^2-2\int x\sqrt{1-x^2}\,dx\\ &= -x^2+\frac{2}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.19

$$-\frac{1}{3}(-1+x)(1+x)\left(3+2\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] -1/3*((-1 + x)*(1 + x)*(3 + 2*Sqrt[1 - x^2]))
```

Maple [A]
time = 0.25, size = 26, normalized size = 1.24

method	result	size
default	$-x^{2}-\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(x^{2}-1\right)}{3}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURN
VERBOSE)
```

```
[Out] -x^2-2/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.81

$$-x^{2}+\frac{2}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] -x^2 + 2/3*(-x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.35, size = 25, normalized size = 1.19

$$-x^{2}-\frac{2}{3}\left(x^{2}-1\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -x^2 - 2/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A]
time = 54.64, size = 144, normalized size = 6.86

$$\frac{x^3}{3}+x-\frac{(x+1)^3}{3}+4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)-4\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin
(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2
)))) - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(
3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x +
 1) > -sqrt(2)))) + 1
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 3.04, size = 54, normalized size = 2.57

$$-(x+1)^2-\frac{1}{3}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\,(x-2)\sqrt{-x+1}+2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] -(x + 1)^2 - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x
+ 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2
```

Mupad [B]
time = 3.06, size = 25, normalized size = 1.19

$$-x^2-\frac{2\left(x^2-1\right)\sqrt{1-x}\,\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] - x^2 - (2*(x^2 - 1)*(1 - x)^(1/2)*(x + 1)^(1/2))/3
```

3.447 $\int \left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right) dx$

Optimal. Leaf size=22

$$-2x - x\sqrt{1-x^2} - \sin^{-1}(x)$$

[Out] -2*x-arcsin(x)-x*(-x^2+1)^(1/2)

Rubi [A]

time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6820, 6874, 201, 222}

$$-\text{ArcSin}(x) - \sqrt{1-x^2}\,x - 2x$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)dx &= -\int\left(\sqrt{1-x}+\sqrt{1+x}\right)^2dx\\ &= -\int\left(2+2\sqrt{1-x^2}\right)dx\\ &= -2x-2\int\sqrt{1-x^2}\,dx\\ &= -2x-x\sqrt{1-x^2}-\int\frac{1}{\sqrt{1-x^2}}dx\\ &= -2x-x\sqrt{1-x^2}-\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.73

$$-2-x\left(2+\sqrt{1-x^2}\right)-2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] -2 - x*(2 + Sqrt[1 - x^2]) - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.
time = 0.25, size = 59, normalized size = 2.68

method	result	size
default	$-2x-\sqrt{1-x}\,(1+x)^{\frac{3}{2}}+\sqrt{1-x}\,\sqrt{1+x}-\frac{\sqrt{(1-x)(1+x)}\,\arcsin(x)}{\sqrt{1+x}\,\sqrt{1-x}}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVE
RBOSE)
[Out] -2*x-(1-x)^(1/2)*(1+x)^(3/2)+(1-x)^(1/2)*(1+x)^(1/2)-((1-x)*(1+x))^(1/2)/(1
+x)^(1/2)/(1-x)^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 20, normalized size = 0.91

$$-\sqrt{-x^2+1}\,x-2\,x-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(20) = 40$.
time = 0.37, size = 41, normalized size = 1.86

$$-\sqrt{x+1}\,x\sqrt{-x+1} - 2\,x + 2\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

Sympy [A]
time = 21.12, size = 63, normalized size = 2.86

$$-2x - 4\left(\begin{cases} \frac{x\sqrt{1-x}\,\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1} > -\sqrt{2} \wedge \sqrt{x+1} < \sqrt{2} \end{cases}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -2*x - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/
2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) - 2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.
time = 3.38, size = 49, normalized size = 2.23

$$-\sqrt{x+1}\,(x-2)\sqrt{-x+1} - 2\,x - 2\,\sqrt{x+1}\,\sqrt{-x+1} - 2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] -sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*ar
csin(1/2*sqrt(2)*sqrt(x + 1)) - 2
```

Mupad [B]
time = 3.71, size = 205, normalized size = 9.32

$$4\,\mathrm{atan}\left(\frac{\sqrt{1-x}\ -1}{\sqrt{x+1}\ -1}\right)-2\,x+\frac{\frac{4\left(\sqrt{1-x}\,-1\right)}{\sqrt{x+1}\,-1}-\frac{28\left(\sqrt{1-x}\,-1\right)^3}{\left(\sqrt{x+1}\,-1\right)^3}+\frac{28\left(\sqrt{1-x}\,-1\right)^5}{\left(\sqrt{x+1}\,-1\right)^5}-\frac{4\left(\sqrt{1-x}\,-1\right)^7}{\left(\sqrt{x+1}\,-1\right)^7}}{\frac{4\left(\sqrt{1-x}\,-1\right)^2}{\left(\sqrt{x+1}\,-1\right)^2}+\frac{6\left(\sqrt{1-x}\,-1\right)^4}{\left(\sqrt{x+1}\,-1\right)^4}+\frac{4\left(\sqrt{1-x}\,-1\right)^6}{\left(\sqrt{x+1}\,-1\right)^6}+\frac{\left(\sqrt{1-x}\,-1\right)^8}{\left(\sqrt{x+1}\,-1\right)^8}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2)
- 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^
3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) -
1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)
^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) -
1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 +
 1)
```

3.448 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

```
[Out] 2*arctanh((-x^2+1)^(1/2))-2*ln(x)-2*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6820, 6874, 272, 52, 65, 212}

$$-2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x}\,dx \\
&= -\int \left(\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}\right)dx \\
&= -2\log(x)-2\int \frac{\sqrt{1-x^2}}{x}\,dx \\
&= -2\log(x)-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx,x,x^2\right) \\
&= -2\sqrt{1-x^2}-2\log(x)-\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x \right. \\
&= -2\sqrt{1-x^2}-2\log(x)+2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x, \right. \\
&= -2\sqrt{1-x^2}+2\tanh^{-1}\left(\sqrt{1-x^2}\right)-2\log(x)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(32) = 64.
time = 0.02, size = 81, normalized size = 2.53

$$
-\frac{4\left(-1+\sqrt{1-x}\right)^2\left(-1+\sqrt{1+x}\right)^2}{\left(-2+\sqrt{1-x}+\sqrt{1+x}\right)^2}+8\tanh^{-1}\left(\frac{-2-x+2\sqrt{1+x}}{-2+2\sqrt{1-x}+x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]
```

```
[Out] (-4*(-1 + Sqrt[1 - x])^2*(-1 + Sqrt[1 + x])^2)/(-2 + Sqrt[1 - x] + Sqrt[1 +
 x])^2 + 8*ArcTanh[(-2 - x + 2*Sqrt[1 + x])/(-2 + 2*Sqrt[1 - x] + x)]
```

Maple [A]
time = 0.26, size = 51, normalized size = 1.59

method	result	size
default	$-2\ln\left(x\right)-\frac{2\sqrt{1-x}\,\sqrt{1+x}\,\left(\sqrt{-x^{2}+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right)\right)}{\sqrt{-x^{2}+1}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x,method=_RETURN
VERBOSE)
```

```
[Out] -2*ln(x)-2*(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1
/(-x^2+1)^(1/2)))
```

Maxima [A]
time = 0.49, size = 41, normalized size = 1.28

$$-2\sqrt{-x^{2}+1}-2\log\left(x\right)+2\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.28

$$-2\sqrt{x+1}\,\sqrt{-x+1}-2\log\left(x\right)-2\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="fricas")
```

```
[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) -
1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x}\,dx - \int \frac{2\sqrt{1-x}\,\sqrt{x+1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)
```

```
[Out] -Integral(2/x, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorit
hm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [B]
time = 4.13, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-2\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-2\ln\left(x\right)-\frac{16\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2\left(\frac{2\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}+\frac{\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x,x)
```

```
[Out] 2*log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*log(((1 - x)^(1/2) - 1)^
2/((x + 1)^(1/2) - 1)^2 - 1) - 2*log(x) - (16*((1 - x)^(1/2) - 1)^2)/(((x +
 1)^(1/2) - 1)^2*((2*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + ((1 - x
)^(1/2) - 1)^4/((x + 1)^(1/2) - 1)^4 + 1))
```

3.449 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\sin^{-1}(x)$$

[Out] 2/x+2*arcsin(x)+2*(-x^2+1)^(1/2)/x

Rubi [A]
time = 0.14, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6820, 6874, 283, 222}

$$2\text{ArcSin}(x)+\frac{2\sqrt{1-x^2}}{x}+\frac{2}{x}$$

Antiderivative was successfully verified.

[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]

[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^2}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^2}\,dx \\
&= -\int \left(\frac{2}{x^2}+\frac{2\sqrt{1-x^2}}{x^2}\right)dx \\
&= \frac{2}{x}-2\int \frac{\sqrt{1-x^2}}{x^2}\,dx \\
&= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{2}{x}+\frac{2\sqrt{1-x^2}}{x}+2\sin^{-1}(x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 39, normalized size = 1.50

$$\frac{2\left(1+\sqrt{1-x^2}+2x\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]
[Out] (2*(1 + Sqrt[1 - x^2] + 2*x*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]))/x
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.26, size = 50, normalized size = 1.92

method	result	size
default	$\frac{2}{x}-\frac{2\left(-\arcsin(x)x-\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{1+x}}{x\sqrt{-x^2+1}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x,method=_RETU
RNVERBOSE)
[Out] 2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/x/(-x^2+1)^(1/2
)
```

Maxima [A]
time = 0.48, size = 24, normalized size = 0.92

$$\frac{2\sqrt{-x^2+1}}{x}+\frac{2}{x}+2\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="maxima")
```

```
[Out] 2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)
```

Fricas [A]
time = 0.39, size = 44, normalized size = 1.69

$$-\frac{2\left(2\,x\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\,\sqrt{-x+1}-1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="fricas")
```

```
[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1)
 - 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{2}{x^2}\,dx-\int\frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**2,x)
```

```
[Out] -Integral(2/x**2, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(24) = 48.
time = 4.20, size = 149, normalized size = 5.73

$$2\,\pi+\frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2-4}+\frac{2}{x}+4\arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algor
ithm="giac")
```

```
[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) -
 sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x
 + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

Mupad [B]
time = 3.79, size = 118, normalized size = 4.54

$$\frac{\frac{5\left(\sqrt{1-x}-1\right)^2}{2\left(\sqrt{x+1}-1\right)^2}-\frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}-\frac{\left(\sqrt{1-x}-1\right)^3}{\left(\sqrt{x+1}-1\right)^3}}-8\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)+\frac{\sqrt{1-x}-1}{2\left(\sqrt{x+1}-1\right)}+\frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)
```

```
[Out] ((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2)
 - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) -
8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + ((1 - x)^(1/2) - 1)/(2*((
x + 1)^(1/2) - 1)) + 2/x
```

3.450 $\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3} dx$

Optimal. Leaf size=33

$$\frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

```
[Out] 1/x^2-arctanh((-x^2+1)^(1/2))+(-x^2+1)^(1/2)/x^2
```

Rubi [A]
time = 0.15, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6820, 6874, 272, 43, 65, 212}

$$\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-\sqrt{1-x}-\sqrt{1+x}\right)\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x^3}\,dx &= -\int \frac{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}{x^3}\,dx\\
&= -\int \left(\frac{2}{x^3}+\frac{2\sqrt{1-x^2}}{x^3}\right)dx\\
&= \frac{1}{x^2}-2\int \frac{\sqrt{1-x^2}}{x^3}\,dx\\
&= \frac{1}{x^2}-\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{1}{x^2}+\frac{\sqrt{1-x^2}}{x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.01, size = 42, normalized size = 1.27

$$\frac{1+\sqrt{1-x^2}+2ix^2\tan^{-1}\left(x+i\sqrt{1-x^2}\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]
```

```
[Out] (1 + Sqrt[1 - x^2] + (2*I)*x^2*ArcTan[x + I*Sqrt[1 - x^2]])/x^2
```

Maple [A]
time = 0.26, size = 57, normalized size = 1.73

method	result	size
default	$\frac{1}{x^{2}}-\frac{\sqrt{1-x}\,\sqrt{1+x}\,\left(\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^{2}+1}}\right) x^{2}-\sqrt{-x^{2}+1}\right)}{x^{2} \sqrt{-x^{2}+1}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/x^2-(1-x)^(1/2)*(1+x)^(1/2)*(arctanh(1/(-x^2+1)^(1/2))*x^2-(-x^2+1)^(1/2)
)/x^2/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 51, normalized size = 1.55

$$\sqrt{-x^{2}+1}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}}+\frac{1}{x^{2}}-\log\left(\frac{2\sqrt{-x^{2}+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="maxima")
```

```
[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x)
 + 2/abs(x))
```

Fricas [A]
time = 0.34, size = 43, normalized size = 1.30

$$\frac{x^{2}\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+\sqrt{x+1}\,\sqrt{-x+1}+1}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="fricas")
```

```
[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/
x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x^3}\,dx - \int \frac{2\sqrt{1-x}\,\sqrt{x+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)
```

```
[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [B]
time = 4.78, size = 186, normalized size = 5.64

$$\ln\left(\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{\left(\sqrt{1-x}-1\right)^2}{16\left(\sqrt{x+1}-1\right)^2}+\frac{\frac{\left(\sqrt{1-x}-1\right)^2}{8\left(\sqrt{x+1}-1\right)^2}+\frac{15\left(\sqrt{1-x}-1\right)^4}{16\left(\sqrt{x+1}-1\right)^4}-\frac{1}{16}}{\frac{\left(\sqrt{1-x}-1\right)^2}{\left(\sqrt{x+1}-1\right)^2}-\frac{2\left(\sqrt{1-x}-1\right)^4}{\left(\sqrt{x+1}-1\right)^4}+\frac{\left(\sqrt{1-x}-1\right)^6}{\left(\sqrt{x+1}-1\right)^6}}+\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^3,x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) - ((1 - x)^(1/2) - 1)^2/(16*((x + 1)^(1/2) - 1)^2)
 + (((1 - x)^(1/2) - 1)^2/(8*((x + 1)^(1/2) - 1)^2) + (15*((1 - x)^(1/2) -
1)^4)/(16*((x + 1)^(1/2) - 1)^4) - 1/16)/(((1 - x)^(1/2) - 1)^2/((x + 1)^(1
/2) - 1)^2 - (2*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + ((1 - x)^(1/
2) - 1)^6/((x + 1)^(1/2) - 1)^6) + 1/x^2
```

3.451 $\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=28

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

```
[Out] -arctanh((-x^2+1)^(1/2))+ln(x)+(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}=0.180$, Rules used = {2128, 6820, 14, 272, 52, 65, 212}

$$\sqrt{1-x^2}-\tanh^{-1}\left(\sqrt{1-x^2}\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2128

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{\sqrt{1-x}+\sqrt{1+x}}{-\sqrt{1-x}+\sqrt{1+x}}\,dx = \frac{1}{2}\int \frac{\sqrt{1-x}\left(\sqrt{1-x}+\sqrt{1+x}\right)}{x}\,dx + \frac{1}{2}\int \frac{\sqrt{1+x}\left(\sqrt{1-x}+\sqrt{1+x}\right.}{x}$$
$$= \frac{1}{2}\int \frac{1-x+\sqrt{1-x^2}}{x}\,dx + \frac{1}{2}\int \frac{1+x+\sqrt{1-x^2}}{x}\,dx$$
$$= \frac{1}{2}\int \left(-1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx + \frac{1}{2}\int \left(1+\frac{1}{x}+\frac{\sqrt{1-x^2}}{x}\right)dx$$
$$= \log(x) + 2\left(\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x}\,dx\right)$$
$$= \log(x) + 2\left(\frac{1}{4}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, x^2\right)\right)$$
$$= \log(x) + 2\left(\frac{\sqrt{1-x^2}}{2} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, x^2\right)\right)$$
$$= \log(x) + 2\left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)\right)$$
$$= 2\left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2}\tanh^{-1}\left(\sqrt{1-x^2}\right)\right) + \log(x)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(28) = 56.
time = 0.18, size = 81, normalized size = 2.89

$$\frac{2\left(-1+\sqrt{1-x}\right)^2\left(-1+\sqrt{1+x}\right)^2}{\left(-2+\sqrt{1-x}+\sqrt{1+x}\right)^2} - 4\tanh^{-1}\left(\frac{-2-x+2\sqrt{1+x}}{-2+2\sqrt{1-x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
[Out] (2*(-1 + Sqrt[1 - x])^2*(-1 + Sqrt[1 + x])^2)/(-2 + Sqrt[1 - x] + Sqrt[1 +
x])^2 - 4*ArcTanh[(-2 - x + 2*Sqrt[1 + x])/(-2 + 2*Sqrt[1 - x] + x)]
```

Maple [**A**]
time = 0.22, size = 48, normalized size = 1.71

method	result	size

default	$\ln(x)+\frac{\sqrt{1-x}\,\sqrt{1+x}\,\left(\sqrt{-x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)}{\sqrt{-x^2+1}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x,method=_RETURNVE
RBOSE)
```

```
[Out] ln(x)+(1-x)^(1/2)*(1+x)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^
2+1)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)
```

Fricas [A]
time = 0.39, size = 36, normalized size = 1.29

$$\sqrt{x+1}\,\sqrt{-x+1}+\log(x)+\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}}\,dx-\int\frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1
)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm
="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [B]
time = 4.49, size = 93, normalized size = 3.32

$$\ln\left(\frac{\left(\sqrt{1-x}\ -1\right)^2}{\left(\sqrt{x+1}\ -1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}\ -1}{\sqrt{x+1}\ -1}\right)+\ln\left(x\right)-\frac{8\left(x-2\sqrt{x+1}\ +2\right)\ \left(x+2\sqrt{1-x}\ -2\right)}{\left(2\sqrt{x+1}\ +2\sqrt{1-x}\ -4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) -
 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(1
- x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

3.452 $\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{-1+x}\,x\sqrt{1+x}\,+\frac{1}{2}\cosh^{-1}(x)$$

```
[Out] 1/2*x^2+1/2*arccosh(x)-1/2*x*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.114$, Rules used = {2129, 6874, 38, 54}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{x-1}\,\sqrt{x+1}\,x+\frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[x*
(a + b*x)^m*((c + d*x)^m/(2*m + 1)), x] + Dist[2*a*c*(m/(2*m + 1)), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2129

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[-d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b
/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{-\sqrt{-1+x}+\sqrt{1+x}}{\sqrt{-1+x}+\sqrt{1+x}}\,dx &= -\left(\frac{1}{2}\int \sqrt{-1+x}\left(-\sqrt{-1+x}+\sqrt{1+x}\right)dx\right)+\frac{1}{2}\int \sqrt{1+x}\left(-\sqrt{-1\right.\\ &= \frac{1}{2}\int\left(1+x-\sqrt{-1+x}\sqrt{1+x}\right)dx-\frac{1}{2}\int\left(1-x+\sqrt{-1+x}\sqrt{1+x}\right.\\ &= \frac{x^2}{2}-2\left(\frac{1}{2}\int\sqrt{-1+x}\sqrt{1+x}\,dx\right)\\ &= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\int\frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx\right)\\ &= \frac{x^2}{2}-2\left(\frac{1}{4}\sqrt{-1+x}\,x\sqrt{1+x}-\frac{1}{4}\cosh^{-1}(x)\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 42, normalized size = 1.27

$$\frac{1}{2}\left(-1+x^2-\sqrt{-1+x}\,x\sqrt{1+x}+2\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]
```

```
[Out] (-1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + 2*ArcTanh[1/Sqrt[(-1 + x)/(1 + x)]
])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.
time = 0.23, size = 62, normalized size = 1.88

method	result	size
default	$-\frac{\sqrt{-1+x}\,(1+x)^{\frac{3}{2}}}{2}+\frac{\sqrt{-1+x}\,\sqrt{1+x}}{2}+\frac{\sqrt{(1+x)(-1+x)}\,\ln\left(x+\sqrt{x^2-1}\right)}{2\sqrt{1+x}\,\sqrt{-1+x}}+\frac{x^2}{2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/2*(-1+x)^(1/2)*(1+x)^(3/2)+1/2*(-1+x)^(1/2)*(1+x)^(1/2)+1/2*((1+x)*(-1+x
))^(1/2)/(1+x)^(1/2)/(-1+x)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="maxima")
```

```
[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 1.12

$$-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x+\frac{1}{2}\,x^2-\frac{1}{2}\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1)
- x)
```

Sympy [A]
time = 16.55, size = 224, normalized size = 6.79

$$-\frac{(x-1)^{\frac{5}{2}}}{4\sqrt{x+1}}-\frac{3(x-1)^{\frac{3}{2}}}{4\sqrt{x+1}}-\frac{\sqrt{x-1}}{2\sqrt{x+1}}+\frac{(x-1)^2}{4}+2\left(\begin{cases}\frac{(x+1)^2}{8}+\frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}}+\frac{3(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}-\frac{\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } |x+1|>2\\ \frac{(x+1)^2}{8}-\frac{i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}}-\frac{3i(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}+\frac{i\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}\right)+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -(x - 1)**(5/2)/(4*sqrt(x + 1)) - 3*(x - 1)**(3/2)/(4*sqrt(x + 1)) - sqrt(x
 - 1)/(2*sqrt(x + 1)) + (x - 1)**2/4 + 2*Piecewise(((x + 1)**2/8 + acosh(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(5/2)/(8*sqrt(x - 1)) + 3*(x + 1)**(3/2)/
(8*sqrt(x - 1)) - sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1) > 2), ((x + 1)**2
/8 - I*asin(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(5/2)/(8*sqrt(1 - x)) - 3
*I*(x + 1)**(3/2)/(8*sqrt(1 - x)) + I*sqrt(x + 1)/(4*sqrt(1 - x)), True)) +
 asinh(sqrt(2)*sqrt(x - 1)/2)/2
```

Giac [A]
time = 3.72, size = 41, normalized size = 1.24

$$\frac{1}{2}\,(x+1)^2-\frac{1}{2}\sqrt{x+1}\sqrt{x-1}\,x-x-\log\left(\sqrt{x+1}-\sqrt{x-1}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorit
hm="giac")
```

```
[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt(
x - 1)) - 1
```

Mupad [B]
time = 10.85, size = 200, normalized size = 6.06

$$\operatorname{acosh}(x)-2\operatorname{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)+\frac{\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{14\left(\sqrt{x-1}-\mathrm{i}\right)^5}{\left(\sqrt{x+1}-1\right)^5}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)^7}{\left(\sqrt{x+1}-1\right)^7}+\frac{2\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}{1+\frac{6\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)
```

```
[Out] acosh(x) - 2*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)) + ((14*((x - 1
)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (14*((x - 1)^(1/2) - 1i)^5)/((x +
1)^(1/2) - 1)^5 + (2*((x - 1)^(1/2) - 1i)^7)/((x + 1)^(1/2) - 1)^7 + (2*((x
 - 1)^(1/2) - 1i))/((x + 1)^(1/2) - 1))/((6*((x - 1)^(1/2) - 1i)^4)/((x + 1
)^(1/2) - 1)^4 - (4*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (4*((x
- 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + ((x - 1)^(1/2) - 1i)^8/((x + 1)
^(1/2) - 1)^8 + 1) + x^2/2
```

3.453 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=121

$$\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(2,1+n;2+n;\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)}{2d^2e(1+n)}$$

```
[Out] 1/2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*a*f^2*hypergeom([2, 1
+n],[2+n],(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))/d)*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)
)^(1+n)/d^2/e/(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2142, 961, 66}

$$\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{d+ex+f\sqrt{\frac{e^2x^2}{f^2}+a}}{d}\right)}{2d^2e(n+1)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (a*f^2*(d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n,
(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d])/(2*d^2*e*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
```

```
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\text{Subst}\left(\int \frac{x^n(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(x^n+\frac{af^2x^n}{(d-x)^2}\right)dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{(af^2)\,\text{Subst}\left(\int \frac{x^n}{(d-x)^2}\,dx, x, d+ex+\right.}{2e}$$

$$= \frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\ {}_2}{2d^2}$$

Mathematica [A]
time = 0.45, size = 86, normalized size = 0.71

$$\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{1+n}\left(d^2+af^2\,{}_2F_1\left(2,1+n;2+n;\frac{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}{d}\right)\right)}{2d^2e(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(1 + n)*(d^2 + a*f^2*Hypergeometric2
F1[2, 1 + n, 2 + n, (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])/d]))/(2*d^2*e*(1
+ n))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.454 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=175

$$-\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}{4e}+\frac{\left(d+ex+f\sqrt{\right.}}{8e}$$

```
[Out] 3/2*a*d^2*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^3*f^2/e/(e*x+f*(a+e
^2*x^2/f^2)^(1/2))+a*d*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/4*a*f^2*(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))^2/e+1/8*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^4/e
```

Rubi [A]

time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2142, 907}

$$-\frac{ad^3f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{3ad^2f^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{4e}+\frac{adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] -1/2*(a*d^3*f^2)/(e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*
Sqrt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^
2)/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log
[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
```

`eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]`

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3 dx = \frac{\text{Subst}\left(\int \frac{x^3(d^2+af^2-2dx+x^2)}{(d-x)^2} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(2adf^2+\frac{ad^3f^2}{(d-x)^2}-\frac{3ad^2f^2}{d-x}+af^2x+x^3\right) dx, x, d+ex+f\sqrt{a\right.}}{2e}$$

$$= -\frac{ad^3f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{adf^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e} + \frac{af^2\left(d-\right.}{}$$

Mathematica [**A**]
time = 0.78, size = 159, normalized size = 0.91

$$\frac{1}{2}\left(x\left(2d^3+6adf^2+3d^2ex+3aef^2x+4de^2x^2+2e^3x^3\right)+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\left(2af^3(2d+ex)+efx(3d^2+4dex+2e^2x^2)\right)}{e}-\frac{3ad^2f\log\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (x*(2*d^3 + 6*a*d*f^2 + 3*d^2*e*x + 3*a*e*f^2*x + 4*d*e^2*x^2 + 2*e^3*x^3)
+ (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3*(2*d + e*x) + e*f*x*(3*d^2 + 4*d*e*x +
2*e^2*x^2)))/e - (3*a*d^2*f*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2
]])/Sqrt[e^2/f^2])/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(157) = 314$.
time = 0.21, size = 341, normalized size = 1.95

method	result

default	$f^{3}\left(\frac{x\left(a+\frac{e^{2}x^{2}}{f^{2}}\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}}{2}+\frac{a\ln\left(\frac{e^{2}x}{f^{2}\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}}\right)}{4}\right)+3f^{2}\left(\frac{e^{3}x^{4}}{4f^{2}}+\frac{d\,e^{2}x^{3}}{3f^{2}}+\frac{ae\,x^{2}}{2}+ad$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/4*x*(a+1/f^2*e^2*x^2)^(3/2)+3/4*a*(1/2*x*(a+1/f^2*e^2*x^2)^(1/2)+1/2
*a*ln(1/f^2*e^2*x/(1/f^2*e^2)^(1/2)+(a+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1
/2)))+3*f^2*(1/4/f^2*e^3*x^4+1/3*d/f^2*e^2*x^3+1/2*a*e*x^2+a*d*x)+3*f*(e^2*
(1/4*x*(a+1/f^2*e^2*x^2)^(3/2)*f^2/e^2-1/4*a*f^2/e^2*(1/2*x*(a+1/f^2*e^2*x^
2)^(1/2)+1/2*a*ln(1/f^2*e^2*x/(1/f^2*e^2)^(1/2)+(a+1/f^2*e^2*x^2)^(1/2))/(1
/f^2*e^2)^(1/2)))+2/3*d/e*f^2*((e^2*x^2+a*f^2)/f^2)^(3/2)+d^2*(1/2*x*(a+1/f
^2*e^2*x^2)^(1/2)+1/2*a*ln(1/f^2*e^2*x/(1/f^2*e^2)^(1/2)+(a+1/f^2*e^2*x^2)^
(1/2))/(1/f^2*e^2)^(1/2)))+1/4*(e*x+d)^4/e
```

Maxima [A]
time = 0.28, size = 252, normalized size = 1.44

$$\frac{3}{4}\left(a+\frac{x^{2}e^{2}}{f^{2}}\right)^{2}f^{4}e^{(-1)}+\frac{1}{4}x^{4}e^{3}+\frac{1}{8}\left(3a^{2}f\operatorname{arsinh}\left(\frac{xe}{\sqrt{a}f}\right)e^{(-1)}+2\left(a+\frac{x^{2}e^{2}}{f^{2}}\right)^{\frac{3}{2}}x+3\sqrt{a+\frac{x^{2}e^{2}}{f^{2}}}ax\right)f^{3}+d^{3}x+\frac{3}{2}\left(x^{2}e+\left(af\operatorname{arsinh}\left(\frac{xe}{\sqrt{a}f}\right)e^{(-1)}+\sqrt{a+\frac{x^{2}e^{2}}{f^{2}}}x\right)f\right)d^{2}-\frac{3}{8}\left(a^{2}f^{3}\operatorname{arsinh}\left(\frac{xe}{\sqrt{a}f}\right)e^{(-3)}-2\left(a+\frac{x^{2}e^{2}}{f^{2}}\right)^{\frac{3}{2}}f^{2}xe^{(-2)}+\sqrt{a+\frac{x^{2}e^{2}}{f^{2}}}af^{2}xe^{(-2)}\right)fe^{2}+\left(2\left(a+\frac{x^{2}e^{2}}{f^{2}}\right)^{\frac{3}{2}}f^{3}e^{(-1)}+x^{3}e^{2}+\left(3ax+\frac{x^{3}e^{2}}{f^{2}}\right)f^{2}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*(a + x^2*e^2/f^2)^2*f^4*e^(-1) + 1/4*x^4*e^3 + 1/8*(3*a^2*f*arcsinh(x*e
/(sqrt(a)*f))*e^(-1) + 2*(a + x^2*e^2/f^2)^(3/2)*x + 3*sqrt(a + x^2*e^2/f^2
)*a*x)*f^3 + d^3*x + 3/2*(x^2*e + (a*f*arcsinh(x*e/(sqrt(a)*f))*e^(-1) + sq
rt(a + x^2*e^2/f^2)*x)*f)*d^2 - 3/8*(a^2*f^3*arcsinh(x*e/(sqrt(a)*f))*e^(-3
) - 2*(a + x^2*e^2/f^2)^(3/2)*f^2*x*e^(-2) + sqrt(a + x^2*e^2/f^2)*a*f^2*x*
e^(-2))*f*e^2 + (2*(a + x^2*e^2/f^2)^(3/2)*f^3*e^(-1) + x^3*e^2 + (3*a*x +
x^3*e^2/f^2)*f^2)*d
```

Fricas [A]
time = 0.36, size = 150, normalized size = 0.86

$$-\frac{1}{2}\left(3\,ad^{2}f^{2}\log\left(-xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}\right)-2\,x^{4}e^{4}-4\,dx^{3}e^{3}-3\left(af^{2}+d^{2}\right)x^{2}e^{2}-2\left(3\,adf^{2}+d^{3}\right)xe-\left(4\,adf^{3}+2\,fx^{3}e^{3}+4\,dfx^{2}e^{2}+\left(2\,af^{3}+3\,d^{2}f\right)xe\right)\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a*d^2*f^2*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2)) - 2*x^4*e^4 - 4
*d*x^3*e^3 - 3*(a*f^2 + d^2)*x^2*e^2 - 2*(3*a*d*f^2 + d^3)*x*e - (4*a*d*f^3
 + 2*f*x^3*e^3 + 4*d*f*x^2*e^2 + (2*a*f^3 + 3*d^2*f)*x*e)*sqrt((a*f^2 + x^2
*e^2)/f^2))*e^(-1)
```

Sympy [A]
time = 5.47, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}} f^{3} x \sqrt{1 + \frac{e^{2} x^{2}}{a f^{2}}}}{2} + \frac{a^{\frac{3}{2}} f^{3} x}{2 \sqrt{1 + \frac{e^{2} x^{2}}{a f^{2}}}} + \frac{3 \sqrt{a} d^{2} f x \sqrt{1 + \frac{e^{2} x^{2}}{a f^{2}}}}{2} + \frac{3 \sqrt{a} e^{2} f x^{3}}{2 \sqrt{1 + \frac{e^{2} x^{2}}{a f^{2}}}} + \frac{3 a d^{2} f^{2} \operatorname{asinh}{\left(\frac{e x}{\sqrt{a} f} \right)}}{2 e} + 3 a d f^{2} x + \frac{3 a e f^{2} x^{2}}{2} + d^{3} x + \frac{3 d^{2} e x^{2}}{2} + 2 d e^{2} x^{3} + 6 d e f \left(\begin{cases} \frac{\sqrt{a} x^{2}}{2} & \text{for}\: e^{2} = 0 \\ \frac{f^{2} \left(a + \frac{e^{2} x^{2}}{f^{2}}\right)^{\frac{3}{2}}}{3 e^{2}} & \text{otherwise} \end{cases}\right) + e^{3} x^{4} + \frac{e^{4} x^{5}}{\sqrt{a} f \sqrt{1 + \frac{e^{2} x^{2}}{a f^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/(2*sqrt(1
+ e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e**2*x**2/(a*f**2))/2
+ 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*a*d**2*f**2*as
inh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*
d**2*e*x**2/2 + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2,
 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4
*x**5/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))
```

Giac [A]
time = 3.64, size = 163, normalized size = 0.93

$$-\frac{3}{2} a d^{2} f |f| e^{(-1)} \log\left(\left|-x e + \sqrt{a f^{2} + x^{2} e^{2}}\right|\right) + \frac{3}{2} a f^{2} x^{2} e + 3 a d f^{2} x + x^{4} e^{3} + 2 d x^{3} e^{2} + \frac{3}{2} d^{2} x^{2} e + d^{3} x + \frac{1}{2} \left(4 a d f |f| e^{(-1)} + \left(2 \left(\frac{x |f| e^{2}}{f} + \frac{2 d |f| e}{f}\right) x + \frac{(2 a f^{4} |f| e^{4} + 3 d^{2} f^{2} |f| e^{4}) e^{(-4)}}{f^{3}}\right) x\right) \sqrt{a f^{2} + x^{2} e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + 3/2*a*f
^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 1/
2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f + 2*d*abs(f)*e/f)*x + (2*a*f^
4*abs(f)*e^4 + 3*d^2*f^2*abs(f)*e^4)*e^(-4)/f^3)*x)*sqrt(a*f^2 + x^2*e^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.455 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=136

$$-\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}{6e}+\frac{adf^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e}$$

```
[Out] a*d*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*d^2*f^2/e/(e*x+f*(a+e^2*x^2
/f^2)^(1/2))+1/2*a*f^2*(e*x+f*(a+e^2*x^2/f^2)^(1/2))/e+1/6*(d+e*x+f*(a+e^2*
x^2/f^2)^(1/2))^3/e
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2142, 907}

$$-\frac{ad^2f^2}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}+\frac{adf^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] -1/2*(a*d^2*f^2)/(e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sq
rt[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*
e) + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
```

```
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2 dx = \frac{\text{Subst}\left(\int \frac{x^2(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(af^2+\frac{ad^2f^2}{(d-x)^2}-\frac{2adf^2}{d-x}+x^2\right) dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{ad^2f^2}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} + \frac{\left(d+ex\right.}{}$$

Mathematica [A]
time = 0.60, size = 118, normalized size = 0.87

$$d^2x+af^2x+dex^2+\frac{2e^2x^3}{3}+\frac{\sqrt{a+\frac{e^2x^2}{f^2}}\,(2af^3+efx(3d+2ex))}{3e}-\frac{adf\log\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]
```

```
[Out] d^2*x + a*f^2*x + d*e*x^2 + (2*e^2*x^3)/3 + (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f
^3 + e*f*x*(3*d + 2*e*x)))/(3*e) - (a*d*f*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a +
 (e^2*x^2)/f^2]])/Sqrt[e^2/f^2]
```

Maple [A]
time = 0.21, size = 124, normalized size = 0.91

method	result

default	$\frac{e^{2} x^{3}}{3}+a\,f^{2} x+2 f\left(\frac{f^{2} \left(\frac{e^{2} x^{2}+a\,f^{2}}{f^{2}}\right)^{\frac{3}{2}}}{3 e}+d\left(\frac{x\sqrt{a+\frac{e^{2} x^{2}}{f^{2}}}}{2}+\frac{a\ln\left(\frac{e^{2} x}{f^{2} \sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2} x^{2}}{f^{2}}}\right)}{2 \sqrt{\frac{e^{2}}{f^{2}}}}\right)\right)+\frac{(e x+d)^{3}}{3 e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e^2*x^3+a*f^2*x+2*f*(1/3/e*f^2*((e^2*x^2+a*f^2)/f^2)^(3/2)+d*(1/2*x*(a+
1/f^2*e^2*x^2)^(1/2)+1/2*a*ln(1/f^2*e^2*x/(1/f^2*e^2)^(1/2)+(a+1/f^2*e^2*x^
2)^(1/2))/(1/f^2*e^2)^(1/2)))+1/3*(e*x+d)^3/e
```

Maxima [A]
time = 0.27, size = 95, normalized size = 0.70

$$\frac{2}{3}\left(a+\frac{x^{2} e^{2}}{f^{2}}\right)^{\frac{3}{2}} f^{3} e^{(-1)}+\frac{1}{3} x^{3} e^{2}+\frac{1}{3}\left(3 a x+\frac{x^{3} e^{2}}{f^{2}}\right) f^{2}+d^{2} x+\left(x^{2} e+\left(a f \operatorname{arsinh}\left(\frac{x e}{\sqrt{a} f}\right) e^{(-1)}+\sqrt{a+\frac{x^{2} e^{2}}{f^{2}}}\, x\right) f\right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(a + x^2*e^2/f^2)^(3/2)*f^3*e^(-1) + 1/3*x^3*e^2 + 1/3*(3*a*x + x^3*e^2
/f^2)*f^2 + d^2*x + (x^2*e + (a*f*arcsinh(x*e/(sqrt(a)*f))*e^(-1) + sqrt(a
+ x^2*e^2/f^2)*x)*f)*d
```

Fricas [A]
time = 0.34, size = 110, normalized size = 0.81

$$-\frac{1}{3}\left(3 a d f^{2} \log\left(-x e+f\sqrt{\frac{a f^{2}+x^{2} e^{2}}{f^{2}}}\right)-2 x^{3} e^{3}-3 d x^{2} e^{2}-3\left(a f^{2}+d^{2}\right) x e-\left(2 a f^{3}+2 f x^{2} e^{2}+3 d f x e\right)\sqrt{\frac{a f^{2}+x^{2} e^{2}}{f^{2}}}\right) e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*d*f^2*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2)) - 2*x^3*e^3 - 3*d
*x^2*e^2 - 3*(a*f^2 + d^2)*x*e - (2*a*f^3 + 2*f*x^2*e^2 + 3*d*f*x*e)*sqrt((
a*f^2 + x^2*e^2)/f^2))*e^(-1)
```

Sympy [A]
time = 2.16, size = 116, normalized size = 0.85

$$\sqrt{a}\, d f x\sqrt{1+\frac{e^{2} x^{2}}{a f^{2}}}+\frac{a d f^{2} \operatorname{asinh}\left(\frac{e x}{\sqrt{a}\, f}\right)}{e}+a f^{2} x+d^{2} x+d e x^{2}+\frac{2 e^{2} x^{3}}{3}+2 e f\left(\begin{cases}\frac{\sqrt{a}\, x^{2}}{2} & \text{for}\ e^{2}=0\\ \frac{f^{2}\left(a+\frac{e^{2} x^{2}}{f^{2}}\right)^{\frac{3}{2}}}{3 e^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(sqrt(a)*f)
)/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*e*f*Piecewise((sqrt(
a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True))
```

Giac [A]
time = 3.37, size = 103, normalized size = 0.76

$$-adf|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)+af^2x+\frac{2}{3}x^3e^2+dx^2e+d^2x+\frac{1}{3}\left(2af|f|e^{(-1)}+\left(\frac{2x|f|e}{f}+\frac{3d|f|}{f}\right)x\right)\sqrt{af^2+x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -a*d*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^2*x + 2/3
*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) + (2*x*abs(f)*e/f + 3
*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)
```

Mupad [B]
time = 4.66, size = 210, normalized size = 1.54

$$\begin{cases} x\left(d+\sqrt{a}\,f\right)^2 & \text{if } e=0\\ x\left(d^2+a\,f^2\right)+\frac{2\,e^2\,x^3}{3}+d\,e\,x^2+\frac{2\,a\,f^3\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{e}-\frac{2\,f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\,\left(2\,a\,f^2-e^2\,x^2\right)}{3\,e}+d\,f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}+\frac{2\,a\,d\,f\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}}-\frac{a\,d\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}} & \text{if } e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^3
)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^2)
/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2) +
(2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^(1/2
) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^
2/f^2)^(3/2)))
```

3.456 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\,dx$

Optimal. Leaf size=68

$$dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}$$

```
[Out] d*x+1/2*e*x^2+1/2*a*f^2*arctanh(e*x/f/(a+e^2*x^2/f^2)^(1/2))/e+1/2*f*x*(a+e
^2*x^2/f^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {201, 223, 212}

$$\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f
*Sqrt[a + (e^2*x^2)/f^2])])/(2*e)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)dx &= dx+\frac{ex^2}{2}+f\int\sqrt{a+\frac{e^2x^2}{f^2}}\,dx\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\int\frac{1}{\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{1}{2}(af)\text{Subst}\left(\int\frac{1}{1-\frac{e^2x^2}{f^2}}\,dx,x,\frac{x}{\sqrt{a+\dots}}\right)\\ &= dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}+\frac{af^2\tanh^{-1}\left(\frac{ex}{f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{2e}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 83, normalized size = 1.22

$$dx+\frac{ex^2}{2}+\frac{1}{2}fx\sqrt{a+\frac{e^2x^2}{f^2}}-\frac{af\log\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2\sqrt{\frac{e^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]
[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 - (a*f*Log[-(Sqrt[e^2/f^2
]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*Sqrt[e^2/f^2])
```

Maple [A]
time = 0.24, size = 75, normalized size = 1.10

method	result	size
default	$dx+\frac{e\,x^{2}}{2}+\frac{fx\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}}{2}+\frac{fa\ln\left(\frac{\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}\right)}{2\sqrt{\frac{e^{2}}{f^{2}}}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x+1/2*e*x^2+1/2*f*x*(a+1/f^2*e^2*x^2)^(1/2)+1/2*f*a*ln(1/f^2*e^2*x/(1/f^2
*e^2)^(1/2)+(a+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 46, normalized size = 0.68

$$\frac{1}{2}\,x^{2}e+\frac{1}{2}\left(af\,\mathrm{arsinh}\left(\frac{xe}{\sqrt{a}\,f}\right)e^{(-1)}+\sqrt{a+\frac{x^{2}e^{2}}{f^{2}}}\,x\right)f+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*e + 1/2*(a*f*arcsinh(x*e/(sqrt(a)*f))*e^(-1) + sqrt(a + x^2*e^2/f^2
)*x)*f + d*x
```

Fricas [A]
time = 0.35, size = 74, normalized size = 1.09

$$-\frac{1}{2}\left(af^{2}\log\left(-xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}\right)-fx\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}\,e-x^{2}e^{2}-2\,dxe\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*f^2*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2)) - f*x*sqrt((a*f^2 + x
^2*e^2)/f^2)*e - x^2*e^2 - 2*d*x*e)*e^(-1)
```

Sympy [A]
time = 1.17, size = 54, normalized size = 0.79

$$dx+\frac{ex^{2}}{2}+f\left(\frac{\sqrt{a}\,x\sqrt{1+\frac{e^{2}x^{2}}{af^{2}}}}{2}+\frac{af\,\mathrm{asinh}\left(\frac{ex}{\sqrt{a}\,f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a*f*asinh(e*
x/(sqrt(a)*f))/(2*e))
```

Giac [A]
time = 3.31, size = 65, normalized size = 0.96

$$\frac{1}{2}\,x^2e+dx-\frac{\left(af^2e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)-\sqrt{af^2+x^2e^2}\,x\right)|f|}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))
- sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f
```

Mupad [B]
time = 3.89, size = 136, normalized size = 2.00

$$\begin{cases} x\left(d+\sqrt{a}\,f\right) & \text{if}\ \ e=0\\ d\,x+\frac{e\,x^2}{2}+\frac{f\,x\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}{2}+\frac{a\,e^2\,\ln\left(x\,\sqrt{\frac{e^2}{f^2}}+\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}}-\frac{a\,e^2\,\ln\left(2\,x\,\sqrt{\frac{e^2}{f^2}}+2\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}{2\,f\,{\left(\frac{e^2}{f^2}\right)}^{3/2}} & \text{if}\ \ e\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] piecewise(e == 0, x*(d + a^(1/2)*f), e ~= 0, d*x + (e*x^2)/2 + (f*x*(a + (e
^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^
(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*
x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))
```

3.457 $\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=117

$$-\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e}+\frac{\left(1+\frac{af^2}{d^2}\right)\log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

```
[Out] -1/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^2/e+1/2*(1+a*f^2/d^2)*ln(d+e*x
+f*(a+e^2*x^2/f^2)^(1/2))/e-1/2*a*f^2/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2142, 907}

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e}+\frac{\left(\frac{af^2}{d^2}+1\right)\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e}-\frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -1/2*(a*f^2)/(d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*S
qrt[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqr
t[a + (e^2*x^2)/f^2]])/(2*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
```

```
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2}+\frac{af^2}{d^2(d-x)}+\frac{d^2+af^2}{d^2x}\right)\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1+\frac{af^2}{d^2}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(117) = 234.
time = 0.90, size = 337, normalized size = 2.88

$$\frac{2de^2x-2def\sqrt{a+\frac{e^2x^2}{f^2}}-\left(af^2\left(e-\sqrt{\frac{e^2}{f^2}}\,f\right)+d^2\left(e+\sqrt{\frac{e^2}{f^2}}\,f\right)\right)\log\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)+\sqrt{\frac{e^2}{f^2}}\,f(d^2+af^2)\log\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)+e(d^2+af^2)\log\left(de\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)-\sqrt{\frac{e^2}{f^2}}\,f(d^2+af^2)\log\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)+e(d^2+af^2)\log\left(d^2e\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}\,x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{4d^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (2*d*e^2*x - 2*d*e*f*Sqrt[a + (e^2*x^2)/f^2] - (a*f^2*(e - Sqrt[e^2/f^2]*f)
 + d^2*(e + Sqrt[e^2/f^2]*f))*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f
^2]] + Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[a*f + d*(-(Sqrt[e^2/f^2]*x) + Sqrt
[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d*e*(a*f + d*(-(Sqrt[e^2/f^2]*x
) + Sqrt[a + (e^2*x^2)/f^2]))] - Sqrt[e^2/f^2]*f*(d^2 + a*f^2)*Log[d + f*(-
(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2])] + e*(d^2 + a*f^2)*Log[d^2*e*(
d + f*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]))])/(4*d^2*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. 2(105) = 210.
time = 0.03, size = 1325, normalized size = 11.32

method	result
default	$-\frac{f\sqrt{\frac{4e^2\left(x+\frac{-a f^2+d^2}{2de}\right)^2}{f^2}+\frac{4e(a f^2-d^2)\left(x+\frac{-a f^2+d^2}{2de}\right)}{d f^2}+\frac{a^2f^4+2a d^2f^2+d^4}{d^2f^2}}}{4de}-\frac{f\ln\left(\frac{\frac{e\left(a f^2-d^2\right)}{2d f^2}+\frac{e^2\left(x+\frac{-a f^2+d^2}{2de}\right)}{f^2}}{\sqrt{\frac{e^2}{f^2}}}+\sqrt{\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*f/d/e*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1
/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)-1/4*f/d^2*ln(
(1/2*e*(a*f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e))/(1/f^2*e^2)^(1
/2)+(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^
2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)
*a+1/4/f*ln((1/2*e*(a*f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e))/(1
/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2*(
x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(1/f^
2*e^2)^(1/2)+1/4*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/
2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)
/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f
^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*
d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*
f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f
^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d
^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/
f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1
/2*(-a*f^2+d^2)/d/e))*a+1/4/f*d/e/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)
*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f
^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/
2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f
^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2*ln(a*f^2-
2*d*e*x-d^2)/e+1/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4/e*ln(-a*f
^2+2*d*e*x+d^2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*e + sqrt(a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 0.35, size = 187, normalized size = 1.60

$$\frac{\left(2\,dxe-2\,df\sqrt{\frac{af^2+x^2e^2}{f^2}}+(af^2+d^2)\log\left(af^2-dxe+df\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)+(af^2+d^2)\log\left(-af^2+2\,dxe+d^2\right)-(af^2+d^2)\log\left(-xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}-d\right)+(af^2-d^2)\log\left(-xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\right)e^{(-1)}}{4\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e - 2*d*f*sqrt((a*f^2 + x^2*e^2)/f^2) + (a*f^2 + d^2)*log(a*f^2
- d*x*e + d*f*sqrt((a*f^2 + x^2*e^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d
*x*e + d^2) - (a*f^2 + d^2)*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) - d) +
 (a*f^2 - d^2)*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2)))*e^(-1)/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(102) = 204.
time = 5.20, size = 219, normalized size = 1.87

$$\frac{(af^2+d^2)e^{(-1)}\log\left(|-af^2+2\,dxe+d^2|\right)}{4\,d^2}-\frac{\sqrt{af^2+x^2e^2}\,|f|e^{(-1)}}{2\,df}+\frac{x}{2\,d}+\frac{(af^2|f|+d^2|f|)e^{(-1)}\log\left(\left|af^2-\left(xe-\sqrt{af^2+x^2e^2}\right)d\right|\right)}{4\,d^2f}-\frac{(af^2|f|+d^2|f|)e^{(-1)}\log\left(\left|-xe-d+\sqrt{af^2+x^2e^2}\right|\right)}{4\,d^2f}+\frac{(af^2|f|-d^2|f|)e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)}{4\,d^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*(a*f^2 + d^2)*e^(-1)*log(abs(-a*f^2 + 2*d*x*e + d^2))/d^2 - 1/2*sqrt(a*
f^2 + x^2*e^2)*abs(f)*e^(-1)/(d*f) + 1/2*x/d + 1/4*(a*f^2*abs(f) + d^2*abs(
f))*e^(-1)*log(abs(a*f^2 - (x*e - sqrt(a*f^2 + x^2*e^2))*d))/(d^2*f) - 1/4*
(a*f^2*abs(f) + d^2*abs(f))*e^(-1)*log(abs(-x*e - d + sqrt(a*f^2 + x^2*e^2)
))/(d^2*f) + 1/4*(a*f^2*abs(f) - d^2*abs(f))*e^(-1)*log(abs(-x*e + sqrt(a*f
^2 + x^2*e^2)))/(d^2*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

3.458 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=151

$$-\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{af^2\log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{d^3e}+\frac{af^2\log\left(d\right.}{}$$

```
[Out] -a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^3/e+a*f^2*ln(d+e*x+f*(a+e^2*x^2/f^
2)^(1/2))/d^3/e-1/2*a*f^2/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/2*(-1-a*f^2
/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2142, 907}

$$-\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e}+\frac{af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}-\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] -1/2*(a*f^2)/(d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/
(2*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (
e^2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/
(d^3*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
```

```
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2}+\frac{2af^2}{d^3(d-x)}+\frac{d^2+af^2}{d^2x^2}+\frac{2af^2}{d^3x}\right)dx, x, d+ex+f\sqrt{a+\frac{e}{}}\right.}{2e}$$

$$= -\frac{af^2}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2\,\mathrm{l}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 378 vs. 2(151) = 302.
time = 1.45, size = 378, normalized size = 2.50

$$\frac{\frac{2dx(d^4+a^2f^4+d^3ex-adef^2x)}{(d^2-af^2)(d^2-af^2+2dex)}+\frac{2d(af^3-defx)\sqrt{a+\frac{e^2x^2}{f^2}}}{e(d^2-af^2+2dex)}+\frac{af^2\left(-e+\sqrt{\frac{e^2}{f^2}}f\right)\log\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e^2}+\frac{af\log\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\sqrt{\frac{e^2}{f^2}}}+\frac{af^2\log\left(d^2e\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{e}-\frac{af\log\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\sqrt{\frac{e^2}{f^2}}}+\frac{af^2\log\left(d^3e\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{e}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] ((2*d*x*(d^4 + a^2*f^4 + d^3*e*x - a*d*e*f^2*x))/((d^2 - a*f^2)*(d^2 - a*f^
2 + 2*d*e*x)) + (2*d*(a*f^3 - d*e*f*x)*Sqrt[a + (e^2*x^2)/f^2])/(e*(d^2 - a
*f^2 + 2*d*e*x)) + (a*f^2*(-e + Sqrt[e^2/f^2]*f)*Log[-(Sqrt[e^2/f^2]*x) + S
qrt[a + (e^2*x^2)/f^2]])/e^2 + (a*f*Log[a*f + d*(-(Sqrt[e^2/f^2]*x) + Sqrt[
a + (e^2*x^2)/f^2])])/Sqrt[e^2/f^2] + (a*f^2*Log[d^2*e*(a*f + d*(-(Sqrt[e^2
/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]))])/e - (a*f*Log[d + f*(-(Sqrt[e^2/f^2]*
x) + Sqrt[a + (e^2*x^2)/f^2])])/Sqrt[e^2/f^2] + (a*f^2*Log[d^3*e*(d + f*(-(
Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]))])/e)/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4160 vs. 2(140) = 280.
time = 0.04, size = 4161, normalized size = 27.56

method	result	size
default	Expression too large to display	4161

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/e*f^7/d^4/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)
^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2
*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2
*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+
(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^4+1/2
*a*f^2/(a*f^2-2*d*e*x-d^2)/d/e-1/4/e/d^3/(-a*f^2+2*d*e*x+d^2)*a^2*f^4-f^3/d
/(a^2*f^4+2*a*d^2*f^2+d^4)*(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d
^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(
1/2)*x*a+1/4/e*f^3/d^4/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a
^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e
)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d
^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*
f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2/e*f/d^2/((a^2*f^
4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^
2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2
*(-a*f^2+d^2)/d/e))*a+1/4/e*d/(-a*f^2+2*d*e*x+d^2)+1/2/d^2*f^5/e/(a^2*f^4+2
*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+
2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*(
(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e
)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))*a^3+1/2/e/d^3*ln(-a*f^2+2*d*e*x
+d^2)*a*f^2+1/2/d^2*x+1/2*d/(a*f^2-2*d*e*x-d^2)/e+1/e^2*f^5/d/(a^2*f^4+2*a*
d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2*d/e)*(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^
2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(3/2)*a-1/2*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*
f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-
d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)
^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(
-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d
^2)/d/e))*a-1/4*f/d^3*ln((1/2*e*(a*f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+
d^2)/d/e))/(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2
-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)
^(1/2))/(1/f^2*e^2)^(1/2)*a-1/4/e*f^5/d^2/(a^2*f^4+2*a*d^2*f^2+d^4)*(4/f^2*
e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/
e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*a^2-1/4*f^5/d^3/(a^2*f^4+2*a*d^
```

```
2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e))/
(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2
*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(1/
f^2*e^2)^(1/2)*a^3+d*f^3/e^2/(a^2*f^4+2*a*d^2*f^2+d^4)/(x-1/2/e/d*a*f^2+1/2
*d/e)*(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*
f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(3/2)-3/4/d*f^3/(a^2*f
^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^
2)/d/e))/(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d
^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(
1/2))/(1/f^2*e^2)^(1/2)*a^2-3/4*d*f/(a^2*f^4+2*a*d^2*f^2+d^4)*ln((1/2*e*(a*
f^2-d^2)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e))/(1/f^2*e^2)^(1/2)+(1/f^2
*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e
)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)*a-1/4*d^4
/f/e/(a^2*f^4+2*a*d^2*f^2+d^4)/((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln
((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+
d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(
-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+
2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2*(-a*f^2+d^2)/d/e))-1/4/e*f/d^2*(4/f
^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)
/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)+1/4/f/d*ln((1/2*e*(a*f^2-d^2
)/d/f^2+1/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e))/(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x
+1/2*(-a*f^2+d^2)/d/e)^2+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(
a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(1/f^2*e^2)^(1/2)+1/4/e/f/((a^2*f^
4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2
+e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4
)/d^2/f^2)^(1/2)*(4/f^2*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^
2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2))/(x+1/2
*(-a*f^2+d^2)/d/e))+1/4*d^2*f/e/(a^2*f^4+2*a*d^2*f^2+d^4)*(4/f^2*e^2*(x+1/2
*(-a*f^2+d^2)/d/e)^2+4*e*(a*f^2-d^2)/d/f^2*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^
4+2*a*d^2*f^2+d^4)/d^2/f^2)^(1/2)-1/4*d^3/f/(a^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(135) = 270$.
time = 0.38, size = 290, normalized size = 1.92

$$-\frac{a^2f^4 + ad^2f^2 - 2\,d^2x^2e^2 - 2\,d^3xe + (a^2f^4 - 2\,adf^2xe - ad^2f^2)\log\left(-af^2xe + adf^2 + 2\,dx^2e^2 + (af^3 - 2\,dfxe)\sqrt{\frac{af^2 + x^2e^2}{f^2}}\right) + (a^2f^4 - 2\,adf^2xe - ad^2f^2)\log\left(-af^2 + 2\,dxe + d^2\right) - (a^2f^4 - 2\,adf^2xe - ad^2f^2)\log\left(-xe + f\sqrt{\frac{af^2 + x^2e^2}{f^2}} - d\right) - 2\,(adf^3 - d^2fxe)\sqrt{\frac{af^2 + x^2e^2}{f^2}}}{2\,(2\,d^4xe^2 - (ad^3f^2 - d^5)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*f^4 + a*d^2*f^2 - 2*d^2*x^2*e^2 - 2*d^3*x*e + (a^2*f^4 - 2*a*d*f^
2*x*e - a*d^2*f^2)*log(-a*f^2*x*e + a*d*f^2 + 2*d*x^2*e^2 + (a*f^3 - 2*d*f*
x*e)*sqrt((a*f^2 + x^2*e^2)/f^2)) + (a^2*f^4 - 2*a*d*f^2*x*e - a*d^2*f^2)*l
og(-a*f^2 + 2*d*x*e + d^2) - (a^2*f^4 - 2*a*d*f^2*x*e - a*d^2*f^2)*log(-x*e
 + f*sqrt((a*f^2 + x^2*e^2)/f^2) - d) - 2*(a*d*f^3 - d^2*f*x*e)*sqrt((a*f^2
 + x^2*e^2)/f^2))/(2*d^4*x*e^2 - (a*d^3*f^2 - d^5)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. 2(135) = 270.
time = 4.25, size = 398, normalized size = 2.64

$$\frac{af|f|e^{(-1)}\log\left(\left|af^2-\left(xe-\sqrt{af^2+x^2e^2}\right)d\right|\right)}{2\,d^3}+\frac{af^2e^{(-1)}\log\left(|-af^2+2\,dxe+d^2|\right)}{2\,d^3}-\frac{af|f|e^{(-1)}\log\left(\left|-xe-d+\sqrt{af^2+x^2e^2}\right|\right)}{2\,d^3}+\frac{af|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)}{2\,d^3}-\frac{\sqrt{af^2+x^2e^2}\,|f|e^{(-1)}}{2\,d^2f}+\frac{x}{2\,d^2}+\frac{(a^2f^4+2\,ad^2f^2+d^4)e^{(-1)}}{4\,(af^2-2\,dxe-d^2)d^3}+\frac{\left(\left(xe-\sqrt{af^2+x^2e^2}\right)a^2f^4|f|+2\,a^2df^4|f|+2\,ad^3f^2|f|-\left(xe-\sqrt{af^2+x^2e^2}\right)d^4|f|\right)e^{(-1)}}{4\left(\left(xe-\sqrt{af^2+x^2e^2}\right)af^2+adf^2-\left(xe-\sqrt{af^2+x^2e^2}\right)^2d-\left(xe-\sqrt{af^2+x^2e^2}\right)d^2\right)d^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*a*f*abs(f)*e^(-1)*log(abs(a*f^2 - (x*e - sqrt(a*f^2 + x^2*e^2))*d))/d^3
 + 1/2*a*f^2*e^(-1)*log(abs(-a*f^2 + 2*d*x*e + d^2))/d^3 - 1/2*a*f*abs(f)*e
^(-1)*log(abs(-x*e - d + sqrt(a*f^2 + x^2*e^2)))/d^3 + 1/2*a*f*abs(f)*e^(-1
)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))/d^3 - 1/2*sqrt(a*f^2 + x^2*e^2)*ab
s(f)*e^(-1)/(d^2*f) + 1/2*x/d^2 + 1/4*(a^2*f^4 + 2*a*d^2*f^2 + d^4)*e^(-1)/
((a*f^2 - 2*d*x*e - d^2)*d^3) + 1/4*((x*e - sqrt(a*f^2 + x^2*e^2))*a^2*f^4*
abs(f) + 2*a^2*d*f^4*abs(f) + 2*a*d^3*f^2*abs(f) - (x*e - sqrt(a*f^2 + x^2*
e^2))*d^4*abs(f))*e^(-1)/(((x*e - sqrt(a*f^2 + x^2*e^2))*a*f^2 + a*d*f^2 -
(x*e - sqrt(a*f^2 + x^2*e^2))^2*d - (x*e - sqrt(a*f^2 + x^2*e^2))*d^2)*d^3*
f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.459 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=193

$$-\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2}-\frac{af^2}{d^3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}-\frac{3af^2\log}{}$$

```
[Out] -3/2*a*f^2*ln(e*x+f*(a+e^2*x^2/f^2)^(1/2))/d^4/e+3/2*a*f^2*ln(d+e*x+f*(a+e^
2*x^2/f^2)^(1/2))/d^4/e-1/2*a*f^2/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))+1/4*(
-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2-a*f^2/d^3/e/(d+e*x+f*(a+e
^2*x^2/f^2)^(1/2))
```

Rubi [A]

time = 0.09, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2142, 907}

$$-\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e}+\frac{3af^2\log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}-\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}-\frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -1/2*(a*f^2)/(d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/
(4*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f
*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])
/(2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2}+\frac{3af^2}{d^4(d-x)}+\frac{d^2+af^2}{d^2x^3}+\frac{2af^2}{d^3x^2}+\frac{3af^2}{d^4x}\right)dx, x, d+ex+f\right.}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{1+\frac{af^2}{d^2}}{4e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} - \frac{}{d^3e}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(193) = 386.
time = 1.91, size = 464, normalized size = 2.40

$$\frac{-\frac{2d\sqrt{a+\frac{e^2x^2}{f^2}}\left(3a^2f^5+d^2efx(3d+4ex)-adf^3(5d+9ex)\right)}{e(d^2-af^2+2dex)^2}+\frac{2dx\left(2d^8+3a^4f^8+5d^7ex-3ad^5ef^2x+15a^2d^3ef^4x-9a^3def^6x+ad^4f^2(3af^2-8e^2x^2)+a^2d^2f^4(-9af^2+4e^2x^2)+d^6(af^2+4e^2x^2)\right)}{(d^2-af^2)^2(d^2-af^2+2dex)^2}-\frac{3af^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)\log\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{e^2}+\frac{3af\log\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\sqrt{\frac{e^2}{f^2}}}+\frac{3af^2\log\left(d^3e\left(af+d\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{e}+\frac{3af^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)\log\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{e^2}}{4d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] ((-2*d*Sqrt[a + (e^2*x^2)/f^2]*(3*a^2*f^5 + d^2*e*f*x*(3*d + 4*e*x) - a*d*f
^3*(5*d + 9*e*x)))/(e*(d^2 - a*f^2 + 2*d*e*x)^2) + (2*d*x*(2*d^8 + 3*a^4*f^
8 + 5*d^7*e*x - 3*a*d^5*e*f^2*x + 15*a^2*d^3*e*f^4*x - 9*a^3*d*e*f^6*x + a*
d^4*f^2*(3*a*f^2 - 8*e^2*x^2) + a^2*d^2*f^4*(-9*a*f^2 + 4*e^2*x^2) + d^6*(a
*f^2 + 4*e^2*x^2)))/((d^2 - a*f^2)^2*(d^2 - a*f^2 + 2*d*e*x)^2) - (3*a*f^2*
(e - Sqrt[e^2/f^2]*f)*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/e^
2 + (3*a*f*Log[a*f + d*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2])])/Sqr
t[e^2/f^2] + (3*a*f^2*Log[d^3*e*(a*f + d*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^
```

```
2*x^2)/f^2]))])/e + (3*a*f^2*(e - Sqrt[e^2/f^2]*f)*Log[d + f*(-(Sqrt[e^2/f^
2]*x) + Sqrt[a + (e^2*x^2)/f^2])])/e^2)/(4*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9870 vs. $2(176) = 352$.
time = 0.06, size = 9871, normalized size = 51.15

method	result	size
default	Expression too large to display	9871

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(170) = 340$.
time = 0.41, size = 521, normalized size = 2.70

$$\frac{5\,a^3f^6-6\,a^2d^2f^4-3\,ad^4f^2+8\,d^3x^3e^3+2\,(ad^2f^2+5\,d^4)x^2e^2-2\,(7\,a^2df^4+ad^3f^2-2\,d^5)xe+3\,(a^3f^6-2\,a^2d^2f^4+4\,ad^2f^2x^2e^2+ad^4f^2-4\,(a^2df^4-ad^3f^2)xe)\log\left(-af^2xe+adf^2+2\,dx^2e^2+(af^3-2\,dfxe)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)+3\,(a^3f^6-2\,a^2d^2f^4+4\,ad^2f^2x^2e^2+ad^4f^2-4\,(a^2df^4-ad^3f^2)xe)\log\left(-af^2+2\,dxe+d^2\right)-3\,(a^3f^6-2\,a^2d^2f^4+4\,ad^2f^2x^2e^2+ad^4f^2-4\,(a^2df^4-ad^3f^2)xe)\log\left(-xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}-d\right)-2\,(3\,a^2df^5-5\,ad^3f^3+4\,d^3fx^2e^2-3\,(3\,ad^2f^3-d^4f)xe)\sqrt{\frac{af^2+x^2e^2}{f^2}}}{4\,(4\,d^6x^2e^3-4\,(ad^5f^2-d^7)xe^2+(a^2d^4f^4-2\,ad^6f^2+d^8)e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 8*d^3*x^3*e^3 + 2*(a*d^2*f^2
 + 5*d^4)*x^2*e^2 - 2*(7*a^2*d*f^4 + a*d^3*f^2 - 2*d^5)*x*e + 3*(a^3*f^6 -
2*a^2*d^2*f^4 + 4*a*d^2*f^2*x^2*e^2 + a*d^4*f^2 - 4*(a^2*d*f^4 - a*d^3*f^2)
*x*e)*log(-a*f^2*x*e + a*d*f^2 + 2*d*x^2*e^2 + (a*f^3 - 2*d*f*x*e)*sqrt((a*
f^2 + x^2*e^2)/f^2)) + 3*(a^3*f^6 - 2*a^2*d^2*f^4 + 4*a*d^2*f^2*x^2*e^2 + a
*d^4*f^2 - 4*(a^2*d*f^4 - a*d^3*f^2)*x*e)*log(-a*f^2 + 2*d*x*e + d^2) - 3*(
a^3*f^6 - 2*a^2*d^2*f^4 + 4*a*d^2*f^2*x^2*e^2 + a*d^4*f^2 - 4*(a^2*d*f^4 -
a*d^3*f^2)*x*e)*log(-x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) - d) - 2*(3*a^2*d*
f^5 - 5*a*d^3*f^3 + 4*d^3*f*x^2*e^2 - 3*(3*a*d^2*f^3 - d^4*f)*x*e)*sqrt((a*
f^2 + x^2*e^2)/f^2))/(4*d^6*x^2*e^3 - 4*(a*d^5*f^2 - d^7)*x*e^2 + (a^2*d^4*
f^4 - 2*a*d^6*f^2 + d^8)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(170) = 340.
time = 4.98, size = 679, normalized size = 3.52

$$\frac{3\,af|f|e^{(-1)}\log\left(\left|af^2-\left(xe-\sqrt{af^2+x^2e^2}\right)d\right|\right)}{4\,d^4}+\frac{3\,af^2e^{(-1)}\log\left(\left|-af^2+2\,dxe+d^2\right|\right)}{4\,d^4}-\frac{3\,af|f|e^{(-1)}\log\left(\left|-xe-d+\sqrt{af^2+x^2e^2}\right|\right)}{4\,d^4}+\frac{3\,af|f|e^{(-1)}\log\left(\left|-xe+\sqrt{af^2+x^2e^2}\right|\right)}{4\,d^4}-\frac{\sqrt{af^2+x^2e^2}\,|f|e^{(-1)}}{2\,d^3f}+\frac{x}{2\,d^3}+\frac{\left(5\,a^3f^6-3\,a^2d^2f^4-9\,ad^4f^2-d^6-12\,(a^2df^4e+ad^3f^2e)x\right)e^{(-1)}}{8\,(af^2-2\,dxe-d^2)^2d^4}+\frac{\left(5\left(xe-\sqrt{af^2+x^2e^2}\right)^2a^3f^6|f|+14\left(xe-\sqrt{af^2+x^2e^2}\right)a^3df^6|f|+10\,a^3d^2f^6|f|-6\left(xe-\sqrt{af^2+x^2e^2}\right)^3a^2df^4|f|-19\left(xe-\sqrt{af^2+x^2e^2}\right)^2a^2d^2f^4|f|-14\left(xe-\sqrt{af^2+x^2e^2}\right)a^2d^3f^4|f|+2\,a^2d^4f^4|f|+2\left(xe-\sqrt{af^2+x^2e^2}\right)^3ad^3f^2|f|+\left(xe-\sqrt{af^2+x^2e^2}\right)^2ad^4f^2|f|-4\left(xe-\sqrt{af^2+x^2e^2}\right)ad^5f^2|f|+\left(xe-\sqrt{af^2+x^2e^2}\right)^2d^6|f|\right)e^{(-1)}}{8\left(\left(xe-\sqrt{af^2+x^2e^2}\right)af^2+adf^2-\left(xe-\sqrt{af^2+x^2e^2}\right)^2d-\left(xe-\sqrt{af^2+x^2e^2}\right)d^2\right)^2d^4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 3/4*a*f*abs(f)*e^(-1)*log(abs(a*f^2 - (x*e - sqrt(a*f^2 + x^2*e^2))*d))/d^4
 + 3/4*a*f^2*e^(-1)*log(abs(-a*f^2 + 2*d*x*e + d^2))/d^4 - 3/4*a*f*abs(f)*e
^(-1)*log(abs(-x*e - d + sqrt(a*f^2 + x^2*e^2)))/d^4 + 3/4*a*f*abs(f)*e^(-1
)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2)))/d^4 - 1/2*sqrt(a*f^2 + x^2*e^2)*ab
s(f)*e^(-1)/(d^3*f) + 1/2*x/d^3 + 1/8*(5*a^3*f^6 - 3*a^2*d^2*f^4 - 9*a*d^4*
f^2 - d^6 - 12*(a^2*d*f^4*e + a*d^3*f^2*e)*x)*e^(-1)/((a*f^2 - 2*d*x*e - d^
2)^2*d^4) + 1/8*(5*(x*e - sqrt(a*f^2 + x^2*e^2))^2*a^3*f^6*abs(f) + 14*(x*e
 - sqrt(a*f^2 + x^2*e^2))*a^3*d*f^6*abs(f) + 10*a^3*d^2*f^6*abs(f) - 6*(x*e
 - sqrt(a*f^2 + x^2*e^2))^3*a^2*d*f^4*abs(f) - 19*(x*e - sqrt(a*f^2 + x^2*e
^2))^2*a^2*d^2*f^4*abs(f) - 14*(x*e - sqrt(a*f^2 + x^2*e^2))*a^2*d^3*f^4*ab
s(f) + 2*a^2*d^4*f^4*abs(f) + 2*(x*e - sqrt(a*f^2 + x^2*e^2))^3*a*d^3*f^2*a
bs(f) + (x*e - sqrt(a*f^2 + x^2*e^2))^2*a*d^4*f^2*abs(f) - 4*(x*e - sqrt(a*
f^2 + x^2*e^2))*a*d^5*f^2*abs(f) + (x*e - sqrt(a*f^2 + x^2*e^2))^2*d^6*abs(
f))*e^(-1)/(((x*e - sqrt(a*f^2 + x^2*e^2))*a*f^2 + a*d*f^2 - (x*e - sqrt(a*
f^2 + x^2*e^2))^2*d - (x*e - sqrt(a*f^2 + x^2*e^2))*d^2)^2*d^4*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.460 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=225

$$\frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}+$$

```
[Out] -5/2*a*d^(3/2)*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e
+1/3*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e+1/7*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(7/2)/e+2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a
*d^2*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/
2))
```

Rubi [A]
time = 0.13, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1271, 1824, 212}

$$-\frac{5ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}-\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}+\frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}+\frac{2adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])
) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x +
f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d +
e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = \frac{\text{Subst}\left(\int \frac{x^{5/2}(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-ad^2f^2-2adf^2x^2-2af^2x^4}{d-x^2}\right.}{}$$

$$= -\frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(4adf^2+2af^2x^2+\right.\right.}{}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}$$

$$= \frac{2adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{ad^2f^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}$$

Mathematica [A]

time = 1.47, size = 212, normalized size = 0.94

$$\frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(20a^2f^4+6(d+2ex)^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-3d^2+4ex\left(19ex+13f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+4d\left(38ex+29f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}-105ad^{3/2}f^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{42e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(20*a^2*f^4 + 6*(d + 2*e*x)^3*(
e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(-3*d^2 + 4*e*x*(19*e*x + 13*f*Sqr
t[a + (e^2*x^2)/f^2]) + 4*d*(38*e*x + 29*f*Sqrt[a + (e^2*x^2)/f^2]))))/(e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]) - 105*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*
Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(42*e)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(5/2), x)
```

Fricas [A]
time = 0.42, size = 408, normalized size = 1.81

$$\left[\frac{1}{84}\left(105ad^{\frac{3}{2}}f^2\log\left(af^2-2dxe+2df\sqrt{\frac{af^2+x^2e^2}{f^2}}+2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)+2\left(116adf^2+24x^3e^3+36dx^2e^2+6d^3+(32af^2+39d^2)xe+(20af^3+24fx^2e^2+36dfxe-3d^2f)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)},\ \frac{1}{42}\left(105a\sqrt{-d}\,df^2\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)+\left(116adf^2+24x^3e^3+36dx^2e^2+6d^3+(32af^2+39d^2)xe+(20af^3+24fx^2e^2+36dfxe-3d^2f)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*x*e + 2*d*f*sqrt((a*f^2 + x^2*e^2)
/f^2) + 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e +
f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(116*a*d*f^2 + 24*x^3*e^3 + 36*d*x^
2*e^2 + 6*d^3 + (32*a*f^2 + 39*d^2)*x*e + (20*a*f^3 + 24*f*x^2*e^2 + 36*d*f
*x*e - 3*d^2*f)*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 + x^2
*e^2)/f^2) + d))*e^(-1), 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(x*e + f*sqr
t((a*f^2 + x^2*e^2)/f^2) + d)*sqrt(-d)/d) + (116*a*d*f^2 + 24*x^3*e^3 + 36*
d*x^2*e^2 + 6*d^3 + (32*a*f^2 + 39*d^2)*x*e + (20*a*f^3 + 24*f*x^2*e^2 + 36
*d*f*x*e - 3*d^2*f)*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 +
 x^2*e^2)/f^2) + d))*e^(-1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.461 $\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=183

$$\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e}-\frac{3a\sqrt{d}\,f^2\,\mathrm{t}}{}$$

```
[Out] -3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))*d^(1/2)/e
+1/5*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2)/e+a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)
^(1/2))^(1/2)/e-1/2*a*d*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e/(e*x+f*
(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1271, 1824, 212}

$$\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}+\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}-\frac{adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{3a\sqrt{d}\,f^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x
 + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d
 + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[
Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = \frac{\text{Subst}\left(\int \frac{x^{3/2}(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{adf^2+2af^2x^2-2dx^4+2x^6}{d-x^2}\,dx,\right.}{2e}$$

$$= -\frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \left(-2af^2-2x^4+\frac{3adf^2}{d-x^2}\right)\right.}{}$$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + $$

$$= \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{adf^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + $$

Mathematica [A]

time = 1.28, size = 170, normalized size = 0.93

$$\frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(2(d+2ex)^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(-d+16ex+12f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}-15a\sqrt{d}\,f^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{10e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(2*(d + 2*e*x)^2*(e*x + f*Sqrt[
a + (e^2*x^2)/f^2]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + (e^2*x^2)/f^2])))/
(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - 15*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x +
 f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(10*e)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(3/2), x)
```

Fricas [A]
time = 0.41, size = 335, normalized size = 1.83

$$\left[\frac{1}{20}\left(15\,a\sqrt{d}\,f^2\log\left(af^2-2\,dxe+2\,df\sqrt{\frac{af^2+x^2e^2}{f^2}}+2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)+2\left(12\,af^2+4\,x^2e^2+9\,dxe+2\,d^2+(4\,fxe-df)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)},\ \frac{1}{10}\left(15\,a\sqrt{-d}\,f^2\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)+\left(12\,af^2+4\,x^2e^2+9\,dxe+2\,d^2+(4\,fxe-df)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*x*e + 2*d*f*sqrt((a*f^2 + x^2*e^2)/
f^2) + 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f
*sqrt((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(12*a*f^2 + 4*x^2*e^2 + 9*d*x*e + 2*
d^2 + (4*f*x*e - d*f)*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2
 + x^2*e^2)/f^2) + d))*e^(-1), 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(x*e + f*
sqrt((a*f^2 + x^2*e^2)/f^2) + d)*sqrt(-d)/d) + (12*a*f^2 + 4*x^2*e^2 + 9*d*
x*e + 2*d^2 + (4*f*x*e - d*f)*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqr
t((a*f^2 + x^2*e^2)/f^2) + d))*e^(-1)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.462 $\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=147

$$-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

```
[Out] -1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/e/d^(1/2)
+1/3*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/
f^2)^(1/2))^(1/2)/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1271, 1167, 212}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}-\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2\sqrt{d}\,e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] -1/2*(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*(e*x + f*Sqrt[a +
 (e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*
f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*
e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx=\frac{\text{Subst}\left(\int \frac{\sqrt{x}\,(d^2+af^2-2dx+x^2)}{(d-x)^2}\,dx,x,d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$=\frac{\text{Subst}\left(\int \frac{x^2(d^2+af^2-2dx^2+x^4)}{(d-x^2)^2}\,dx,x,\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\text{Subst}\left(\int \frac{-af^2+2dx^2-2x^4}{d-x^2}\,dx,x,\sqrt{d+\right.}{2e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\text{Subst}\left(\int \left(2x^2-\frac{af^2}{d-x^2}\right)\,dx,x,\sqrt{d-\right.}{2e}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{(a\ldots}{}$$

$$=-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{af\ldots}{}$$

Mathematica [A]

time = 0.89, size = 142, normalized size = 0.97

$$\frac{\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2(d+2ex)\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}-\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{\sqrt{d}}}{6e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
[Out] ((Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*(d + 2*e*x)*(e*x
+ f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - (3*a*f^2
*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/Sqrt[d])/(6*e)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
[Out] int((d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x*e + sqrt(a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 0.40, size = 299, normalized size = 2.03

$$\left[\frac{\left(3\,a\sqrt{d}\,f^2\log\left(af^2-2\,dxe+2\,df\sqrt{\frac{af^2+x^2e^2}{f^2}}+2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)+2\left(5\,dxe-df\sqrt{\frac{af^2+x^2e^2}{f^2}}+2\,d^2\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)}}{12\,d}, \frac{\left(3\,a\sqrt{-d}\,f^2\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)+\left(5\,dxe-df\sqrt{\frac{af^2+x^2e^2}{f^2}}+2\,d^2\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)e^{(-1)}}{6\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*a*sqrt(d)*f^2*log(a*f^2 - 2*d*x*e + 2*d*f*sqrt((a*f^2 + x^2*e^2)/f^2) + 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(5*d*x*e - d*f*sqrt((a*f^2 + x^2*e^2)/f^2) + 2*d^2)*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))*e^(-1)/d, 1/6*(3*a*sqrt(-d)*f^2*arctan(sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)*sqrt(-d)/d) + (5*d*x*e - d*f*sqrt((a*f^2 + x^2*e^2)/f^2) + 2*d^2)*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))*e^(-1)/d]

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)

[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x*e + sqrt(a + x^2*e^2/f^2)*f + d), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)

[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

3.463 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=147

$$\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e}-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

```
[Out] 1/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(3/2)/e+
(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^
(1/2))^(1/2)/d/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1171, 396, 212}

$$\frac{af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}+\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[
a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*Arc
Tanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2\sqrt{x}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2-af^2+2dx^2}{d-x^2}\,dx, x, \sqrt{d+ \cdots}\right.}{2de}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{(af^2)\,\text{Su}\cdots}{\cdots}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{af^2\tanh\cdots}{\cdots}$$

<u>Mathematica</u> [A]
time = 0.87, size = 141, normalized size = 0.96

$$\frac{\frac{\sqrt{d}\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\left(-af^2+2d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} + af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]],x]
```

```
[Out] ((Sqrt[d]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]*(-(a*f^2) + 2*d*(e*x +
f*Sqrt[a + (e^2*x^2)/f^2])))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*ArcT
anh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+\frac{e^{2}x^{2}}{f^{2}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x*e + sqrt(a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 0.42, size = 296, normalized size = 2.01

$$\left[\frac{\left(a\sqrt{d}\,f^{2}\log\left(af^{2}-2\,dxe+2\,df\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}-2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}\right)\sqrt{xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+d}\right)+2\left(dxe-df\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+2\,d^{2}\right)\sqrt{xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+d}\right)e^{(-1)}}{4\,d^{2}},-\frac{\left(a\sqrt{-d}\,f^{2}\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+d}\,\sqrt{-d}}{d}\right)-\left(dxe-df\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+2\,d^{2}\right)\sqrt{xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}}{f^{2}}}+d}\right)e^{(-1)}}{2\,d^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*x*e + 2*d*f*sqrt((a*f^2 + x^2*e^2)/f^2)
 - 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqr
t((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(d*x*e - d*f*sqrt((a*f^2 + x^2*e^2)/f^2)
 + 2*d^2)*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))*e^(-1)/d^2, -1/2*(
a*sqrt(-d)*f^2*arctan(sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)*sqrt(-d
```

```
)/d) - (d*x*e - d*f*sqrt((a*f^2 + x^2*e^2)/f^2) + 2*d^2)*sqrt(x*e + f*sqrt(
(a*f^2 + x^2*e^2)/f^2) + d))*e^(-1)/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x*e + sqrt(a + x^2*e^2/f^2)*f + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d + e\,x + f\,\sqrt{a + \frac{e^2\,x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.464 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{1+\frac{af^2}{d^2}}{e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

```
[Out] 3/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(5/2)/e+
(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(
a+e^2*x^2/f^2)^(1/2))^(1/2)/d^2/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1273, 464, 212}

$$\frac{3af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2
*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2
*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/S
qrt[d]])/(2*d^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{3/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^2(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d(d^2+af^2)+(2d^2-af^2)x^2}{x^2(d-x^2)}\right.}{2}$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} +$$

$$= -\frac{d^2+af^2}{d^2e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^2e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} +$$

Mathematica [A]

time = 1.41, size = 169, normalized size = 1.07

$$\frac{-\frac{\sqrt{d}\left(2d^2\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(d+3ex+3f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}+3af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-((Sqrt[d]*(2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(d + 3*e*x + 3
*f*Sqrt[a + (e^2*x^2)/f^2])))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e
*x + f*Sqrt[a + (e^2*x^2)/f^2]])) + 3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a
 + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-3/2), x)
```

Fricas [A]
time = 0.43, size = 490, normalized size = 3.10

$$\left[-\frac{3\,(a^2f^4-2\,adf^2xe-ad^2f^2)\sqrt{d}\,\log\left(af^2-2\,dxe+2\,df\sqrt{\frac{af^2+x^2e^2}{f^2}}-2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)+2\left(2\,ad^2f^2-2\,d^2x^2e^2+2\,d^4+(3\,adf^2+d^3)xe-(3\,adf^3-2\,d^2fxe+d^3f)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}}{4\,(2\,d^4xe^2-(ad^3f^2-d^5)e)},\ \frac{3\,(a^2f^4-2\,adf^2xe-ad^2f^2)\sqrt{-d}\,\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)-\left(2\,ad^2f^2-2\,d^2x^2e^2+2\,d^4+(3\,adf^2+d^3)xe-(3\,adf^3-2\,d^2fxe+d^3f)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}}{2\,(2\,d^4xe^2-(ad^3f^2-d^5)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a^2*f^4 - 2*a*d*f^2*x*e - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*x*e
+ 2*d*f*sqrt((a*f^2 + x^2*e^2)/f^2) - 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^
2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(2*a*
d^2*f^2 - 2*d^2*x^2*e^2 + 2*d^4 + (3*a*d*f^2 + d^3)*x*e - (3*a*d*f^3 - 2*d^
2*f*x*e + d^3*f)*sqrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 + x^
2*e^2)/f^2) + d))/(2*d^4*x*e^2 - (a*d^3*f^2 - d^5)*e), 1/2*(3*(a^2*f^4 - 2*
a*d*f^2*x*e - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2
)/f^2) + d)*sqrt(-d)/d) - (2*a*d^2*f^2 - 2*d^2*x^2*e^2 + 2*d^4 + (3*a*d*f^2
 + d^3)*x*e - (3*a*d*f^3 - 2*d^2*f*x*e + d^3*f)*sqrt((a*f^2 + x^2*e^2)/f^2)
)*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))/(2*d^4*x*e^2 - (a*d^3*f^2
- d^5)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.465 $\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=199

$$-\frac{1+\frac{af^2}{d^2}}{3e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}-\frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}-\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}+\frac{5af^2\,\mathrm{ta}}{}$$

```
[Out] 5/2*a*f^2*arctanh((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)/d^(1/2))/d^(7/2)/e+
1/3*(-1-a*f^2/d^2)/e/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2)-2*a*f^2/d^3/e/(d
+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2)-1/2*a*f^2*(d+e*x+f*(a+e^2*x^2/f^2)^(1/2
))^(1/2)/d^3/e/(e*x+f*(a+e^2*x^2/f^2)^(1/2))
```

Rubi [A]

time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2142, 911, 1273, 1275, 212}

$$\frac{5af^2\tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}-\frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] -1/3*(1 + (a*f^2)/d^2)/(e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2
*a*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d +
 e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2
])) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/
(2*d^(7/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}}\,dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4(d-x^2)^2}\,dx, x, \sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2(d^2+af^2)-2d(d^2-af^2)x^2+\cdots}{x^4(d-x^2)}\right.}{\cdots}$$

$$= -\frac{af^2\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{2d^3e\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2(d^3+adf^2)}{x^4} + \frac{4af^2}{x^2} + \frac{5af\cdots}{d-x\cdots}\right.\right.}{\cdots}$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\cdots$$

$$= -\frac{d^2+af^2}{3d^2e\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}\cdots$$

Mathematica [A]

time = 1.43, size = 209, normalized size = 1.05

$$\frac{-\frac{\sqrt{d}\left(15a^2f^4+2d^3\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+af^2\left(3d^2+20d\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)+30ex\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\right)\right)}{\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}}+15af^2\tanh^{-1}\left(\frac{\sqrt{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{6d^{7/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (-((Sqrt[d]*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(
3*d^2 + 20*d*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 30*e*x*(e*x + f*Sqrt[a + (
e^2*x^2)/f^2]))))/((e*x + f*Sqrt[a + (e^2*x^2)/f^2])*(d + e*x + f*Sqrt[a +
(e^2*x^2)/f^2])^(3/2))) + 15*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x
^2)/f^2]]/Sqrt[d]])/(6*d^(7/2)*e)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(167) = 334.
time = 0.45, size = 784, normalized size = 3.94

$$\left[\frac{15\,(a^3f^6-2\,a^2d^2f^4+4\,ad^2f^2x^2e^2+ad^4f^2-4\,(a^2df^4-ad^3f^2)xe)\sqrt{d}\,\log\left(af^2-2\,dxe+2\,df\sqrt{\frac{af^2+x^2e^2}{f^2}}-2\left(\sqrt{d}\,xe-\sqrt{d}\,f\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\right)+2\left(10\,a^2d^2f^4-16\,ad^4f^2+12\,d^3x^3e^3-2\,d^6-8\,(5\,ad^2f^2-d^4)x^2e^2+(15\,a^2df^4-46\,ad^3f^2-d^5)xe-(15\,a^2df^5-22\,ad^3f^3+12\,d^3fx^2e^2-d^5f-8\,(5\,ad^2f^3-d^4f)xe)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}}{12\,(4\,d^6x^2e^3-4\,(ad^5f^2-d^7)xe^2+(a^2d^4f^4-2\,ad^6f^2+d^8)e)},\ -\frac{15\,(a^3f^6-2\,a^2d^2f^4+4\,ad^2f^2x^2e^2+ad^4f^2-4\,(a^2df^4-ad^3f^2)xe)\sqrt{-d}\,\arctan\left(\frac{\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}\,\sqrt{-d}}{d}\right)-\left(10\,a^2d^2f^4-16\,ad^4f^2+12\,d^3x^3e^3-2\,d^6-8\,(5\,ad^2f^2-d^4)x^2e^2+(15\,a^2df^4-46\,ad^3f^2-d^5)xe-(15\,a^2df^5-22\,ad^3f^3+12\,d^3fx^2e^2-d^5f-8\,(5\,ad^2f^3-d^4f)xe)\sqrt{\frac{af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{af^2+x^2e^2}{f^2}}+d}}{6\,(4\,d^6x^2e^3-4\,(ad^5f^2-d^7)xe^2+(a^2d^4f^4-2\,ad^6f^2+d^8)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(a^3*f^6 - 2*a^2*d^2*f^4 + 4*a*d^2*f^2*x^2*e^2 + a*d^4*f^2 - 4*(a
^2*d*f^4 - a*d^3*f^2)*x*e)*sqrt(d)*log(a*f^2 - 2*d*x*e + 2*d*f*sqrt((a*f^2
+ x^2*e^2)/f^2) - 2*(sqrt(d)*x*e - sqrt(d)*f*sqrt((a*f^2 + x^2*e^2)/f^2))*s
qrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)) + 2*(10*a^2*d^2*f^4 - 16*a*d^
4*f^2 + 12*d^3*x^3*e^3 - 2*d^6 - 8*(5*a*d^2*f^2 - d^4)*x^2*e^2 + (15*a^2*d*
f^4 - 46*a*d^3*f^2 - d^5)*x*e - (15*a^2*d*f^5 - 22*a*d^3*f^3 + 12*d^3*f*x^2
*e^2 - d^5*f - 8*(5*a*d^2*f^3 - d^4*f)*x*e)*sqrt((a*f^2 + x^2*e^2)/f^2))*sq
rt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))/(4*d^6*x^2*e^3 - 4*(a*d^5*f^2
- d^7)*x*e^2 + (a^2*d^4*f^4 - 2*a*d^6*f^2 + d^8)*e), -1/6*(15*(a^3*f^6 - 2*
a^2*d^2*f^4 + 4*a*d^2*f^2*x^2*e^2 + a*d^4*f^2 - 4*(a^2*d*f^4 - a*d^3*f^2)*x
*e)*sqrt(-d)*arctan(sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d)*sqrt(-d)/
d) - (10*a^2*d^2*f^4 - 16*a*d^4*f^2 + 12*d^3*x^3*e^3 - 2*d^6 - 8*(5*a*d^2*f
^2 - d^4)*x^2*e^2 + (15*a^2*d*f^4 - 46*a*d^3*f^2 - d^5)*x*e - (15*a^2*d*f^5
 - 22*a*d^3*f^3 + 12*d^3*f*x^2*e^2 - d^5*f - 8*(5*a*d^2*f^3 - d^4*f)*x*e)*s
qrt((a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((a*f^2 + x^2*e^2)/f^2) + d))/
(4*d^6*x^2*e^3 - 4*(a*d^5*f^2 - d^7)*x*e^2 + (a^2*d^4*f^4 - 2*a*d^6*f^2 + d
^8)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2)*f + d)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.466 $\int \sqrt{x-\sqrt{-4+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

```
[Out] 1/3*(x-(x^2-4)^(1/2))^(3/2)+4/(x-(x^2-4)^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2142, 14}

$$\frac{1}{3}\left(x-\sqrt{x^2-4}\right)^{3/2}+\frac{4}{\sqrt{x-\sqrt{x^2-4}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]
```

```
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x-\sqrt{-4+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{-4+x^2}{x^{3/2}}\,dx,x,x-\sqrt{-4+x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{4}{x^{3/2}}+\sqrt{x}\right)\,dx,x,x-\sqrt{-4+x^2}\right)\\ &= \frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 41, normalized size = 1.00

$$\frac{4}{\sqrt{x-\sqrt{-4+x^2}}}+\frac{1}{3}\left(x-\sqrt{-4+x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]
[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2-4)^(1/2))^(1/2),x)
[Out] int((x-(x^2-4)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Fricas [A]
time = 0.38, size = 26, normalized size = 0.63

$$\frac{2}{3}\left(2\,x+\sqrt{x^2-4}\right)\sqrt{x-\sqrt{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x-\sqrt{x^2-4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2-4)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x-\sqrt{x^2-4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^2 - 4)^(1/2))^(1/2),x)
```

```
[Out] int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

3.467 $\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=69

$$-\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

```
[Out] 1/3*(a*x+b*(c+a^2*x^2/b^2)^(1/2))^(3/2)/a-b^2*c/a/(a*x+b*(c+a^2*x^2/b^2)^(1
/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2142, 14}

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2}+c}+ax\right)^{3/2}}{3a}-\frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2}+c}+ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx = \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}}\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}}+\sqrt{x}\right)\,dx, x, ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)}{2a}$$

$$= -\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

Mathematica [**A**]
time = 0.53, size = 69, normalized size = 1.00

$$-\frac{b^2c}{a\sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}}+\frac{\left(ax+b\sqrt{c+\frac{a^2x^2}{b^2}}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]
```

```
[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a
^2*x^2)/b^2])^(3/2)/(3*a)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ax+b\sqrt{c+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((a*x+b*(c+a^2/b^2*x^2)^(1/2))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

Fricas [A]
time = 0.38, size = 59, normalized size = 0.86

$$\frac{2\left(2\,ax-b\sqrt{\frac{a^2x^2+b^2c}{b^2}}\right)\sqrt{ax+b\sqrt{\frac{a^2x^2+b^2c}{b^2}}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^
2*c)/b^2))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+b\sqrt{\frac{a^2x^2}{b^2}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c)*b), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a\,x+b\,\sqrt{c+\frac{a^2\,x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)
[Out] int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)
```

3.468 $\int \sqrt{1+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

```
[Out] -2/3*x^3/(1+(-x^2+1)^(1/2))^(3/2)+2*x/(1+(-x^2+1)^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2154}

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}}-\frac{2x^3}{3\left(\sqrt{1-x^2}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1-x^2}}\,dx=-\frac{2x^3}{3\left(1+\sqrt{1-x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1-x^2}}}$$

Mathematica [A]
time = 0.06, size = 35, normalized size = 0.78

$$\frac{2x\left(2+\sqrt{1-x^2}\right)}{3\sqrt{1+\sqrt{1-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]],x]
```

```
[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.03, size = 60, normalized size = 1.33

method	result	size
meijerg	$\frac{i\left(\frac{32i\sqrt{\pi}\,\sqrt{2}\,x^{3}\cos\left(\frac{3\arcsin(x)}{2}\right)}{3}-\frac{8i\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^{4}+\frac{2}{3}x^{2}+\frac{2}{3}\right)\sin\left(\frac{3\arcsin(x)}{2}\right)}{\sqrt{-x^{2}+1}}\right)}{8\sqrt{\pi}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(-x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*I/Pi^(1/2)*(32/3*I*Pi^(1/2)*2^(1/2)*x^3*cos(3/2*arcsin(x))-8*I*Pi^(1/2)
*2^(1/2)*(-4/3*x^4+2/3*x^2+2/3)*sin(3/2*arcsin(x))/(-x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Fricas [A]
time = 0.38, size = 34, normalized size = 0.76

$$\frac{2\left(x^{2}-\sqrt{-x^{2}+1}+1\right)\sqrt{\sqrt{-x^{2}+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 - sqrt(-x^2 + 1) + 1)*sqrt(sqrt(-x^2 + 1) + 1)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.59, size = 415, normalized size = 9.22

$$\begin{cases} -\frac{\sqrt{2}\,ix^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}-1}\,\sqrt{i\sqrt{x^{2}-1}+1}-12i\pi\sqrt{i\sqrt{x^{2}-1}+1}}-\frac{3\sqrt{2}\,x\sqrt{x^{2}-1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}-1}\,\sqrt{i\sqrt{x^{2}-1}+1}-12i\pi\sqrt{i\sqrt{x^{2}-1}+1}}+\frac{3\sqrt{2}\,ix\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}-1}\,\sqrt{i\sqrt{x^{2}-1}+1}-12i\pi\sqrt{i\sqrt{x^{2}-1}+1}} & \text{for}\ \left|x^{2}\right|>1 \\ \frac{\sqrt{2}\,x^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^{2}}\,\sqrt{\sqrt{1-x^{2}}+1}+12\pi\sqrt{\sqrt{1-x^{2}}+1}}-\frac{3\sqrt{2}\,x\sqrt{1-x^{2}}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^{2}}\,\sqrt{\sqrt{1-x^{2}}+1}+12\pi\sqrt{\sqrt{1-x^{2}}+1}}-\frac{3\sqrt{2}\,x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^{2}}\,\sqrt{\sqrt{1-x^{2}}+1}+12\pi\sqrt{\sqrt{1-x^{2}}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x**2+1)**(1/2))**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqr
t(I*sqrt(x**2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) - 3*sqrt(2)*x
*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x*
*2 - 1) + 1) - 12*I*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*x*gamma(-1
/4)*gamma(1/4)/(12*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*I*pi*s
qrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamma
(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 -
x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt
(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*s
qrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) +
 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(-x^2 + 1) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(((1 - x^2)^(1/2) + 1)^(1/2), x)
```

3.469 $\int \sqrt{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=41

$$\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

```
[Out] 2/3*x^3/(1+(x^2+1)^(1/2))^(3/2)+2*x/(1+(x^2+1)^(1/2))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2154}

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}}+\frac{2x^3}{3\left(\sqrt{x^2+1}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+x^2}}\,dx=\frac{2x^3}{3\left(1+\sqrt{1+x^2}\right)^{3/2}}+\frac{2x}{\sqrt{1+\sqrt{1+x^2}}}$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 0.76

$$\frac{2x\left(2+\sqrt{1+x^2}\right)}{3\sqrt{1+\sqrt{1+x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]],x]
```

```
[Out] (2*x*(2 + Sqrt[1 + x^2]))/(3*Sqrt[1 + Sqrt[1 + x^2]])
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.02, size = 55, normalized size = 1.34

method	result	size
meijerg	$-\frac{-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\cosh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{3}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{3}x^{4}-\frac{2}{3}x^{2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}(x)}{2}\right)}{\sqrt{x^{2}+1}}}{8\sqrt{\pi}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x^2+1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2
^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x))/(x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Fricas [A]
time = 0.38, size = 28, normalized size = 0.68

$$\frac{2\left(x^{2}+\sqrt{x^{2}+1}-1\right)\sqrt{\sqrt{x^{2}+1}+1}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.
time = 0.58, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}\,x^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+1}\,\sqrt{\sqrt{x^{2}+1}+1}+12\pi\sqrt{\sqrt{x^{2}+1}+1}}-\frac{3\sqrt{2}\,x\sqrt{x^{2}+1}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+1}\,\sqrt{\sqrt{x^{2}+1}+1}+12\pi\sqrt{\sqrt{x^{2}+1}+1}}-\frac{3\sqrt{2}\,x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+1}\,\sqrt{\sqrt{x^{2}+1}+1}+12\pi\sqrt{\sqrt{x^{2}+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x**2+1)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 +
 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gam
ma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*
sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(
x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(x^2+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)
```

3.470 $\int \sqrt{5+\sqrt{25+x^2}}\, dx$

Optimal. Leaf size=41

$$\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

```
[Out] 2/3*x^3/(5+(x^2+25)^(1/2))^(3/2)+10*x/(5+(x^2+25)^(1/2))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2154}

$$\frac{10x}{\sqrt{\sqrt{x^2+25}+5}}+\frac{2x^3}{3\left(\sqrt{x^2+25}+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{5+\sqrt{25+x^2}}\, dx=\frac{2x^3}{3\left(5+\sqrt{25+x^2}\right)^{3/2}}+\frac{10x}{\sqrt{5+\sqrt{25+x^2}}}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 0.76

$$\frac{2x\left(10+\sqrt{25+x^2}\right)}{3\sqrt{5+\sqrt{25+x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]],x]
```

```
[Out] (2*x*(10 + Sqrt[25 + x^2]))/(3*Sqrt[5 + Sqrt[25 + x^2]])
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.03, size = 64, normalized size = 1.56

method	result	size
meijerg	$-\frac{5\sqrt{5}\left(-\frac{32\sqrt{\pi}\,\sqrt{2}\,x^{3}\cosh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375}-\frac{8\sqrt{\pi}\,\sqrt{2}\,\left(-\frac{4}{1875}x^{4}-\frac{2}{75}x^{2}+\frac{2}{3}\right)\sinh\left(\frac{3\,\mathrm{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^{2}}{25}+1}}\right)}{8\sqrt{\pi}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+(x^2+25)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)
)-8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4-2/75*x^2+2/3)*sinh(3/2*arcsinh(1/5*x))/(1
/25*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Fricas [A]
time = 0.36, size = 30, normalized size = 0.73

$$\frac{2\left(x^{2}+5\sqrt{x^{2}+25}-25\right)\sqrt{\sqrt{x^{2}+25}+5}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(36) = 72.
time = 0.63, size = 197, normalized size = 4.80

$$-\frac{\sqrt{2}\,x^{3}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+25}\,\sqrt{\sqrt{x^{2}+25}+5}+60\pi\sqrt{\sqrt{x^{2}+25}+5}}-\frac{15\sqrt{2}\,x\sqrt{x^{2}+25}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+25}\,\sqrt{\sqrt{x^{2}+25}+5}+60\pi\sqrt{\sqrt{x^{2}+25}+5}}-\frac{75\sqrt{2}\,x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^{2}+25}\,\sqrt{\sqrt{x^{2}+25}+5}+60\pi\sqrt{\sqrt{x^{2}+25}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2
+ 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25
)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) +
 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12
*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25)
+ 5))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2+25}+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 25)^(1/2) + 5)^(1/2),x)
```

```
[Out] int(((x^2 + 25)^(1/2) + 5)^(1/2), x)
```

3.471 $\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$

Optimal. Leaf size=66

$$\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

```
[Out] 2/3*b^2*c*x^3/(a+b*(a^2/b^2+c*x^2)^(1/2))^(3/2)+2*a*x/(a+b*(a^2/b^2+c*x^2)^
(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2154}

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2}+cx^2}+a}}+\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2}+cx^2}+a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
```

```
[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*
Sqrt[a^2/b^2 + c*x^2]]
```

Rule 2154

```
Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[2*b^2*d
*(x^3/(3*(a + b*Sqrt[c + d*x^2])^(3/2))), x] + Simp[2*a*(x/Sqrt[a + b*Sqrt[
c + d*x^2]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx=\frac{2b^2cx^3}{3\left(a+b\sqrt{\frac{a^2}{b^2}+cx^2}\right)^{3/2}}+\frac{2ax}{\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}}$$

Mathematica [A]
time = 3.96, size = 72, normalized size = 1.09

$$\frac{2\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\left(-a^2+b^2cx^2+ab\sqrt{\frac{a^2}{b^2}+cx^2}\right)}{3b^2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]
[Out] (2*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]]*(-a^2 + b^2*c*x^2 + a*b*Sqrt[a^2/b^2 +
 c*x^2]))/(3*b^2*c*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sqrt{\frac{a^2}{b^2}+c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Fricas [A]
time = 0.41, size = 70, normalized size = 1.06

$$\frac{2\left(b^2cx^2+ab\sqrt{\frac{b^2cx^2+a^2}{b^2}}-a^2\right)\sqrt{b\sqrt{\frac{b^2cx^2+a^2}{b^2}}+a}}{3\,b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*
x^2 + a^2)/b^2) + a)/(b^2*c*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sqrt{\frac{a^2}{b^2}+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a+b\,\sqrt{c\,x^2+\frac{a^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)
[Out] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)
```

3.472 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=166

$$\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}+\frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(2,1+n;2+\right.}{2e\left(2de-bf^2\right)^2(1+n)}$$

```
[Out] 1/2*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)+1/2*f^2*(-b^2*f^2+4*a
*e^2)*hypergeom([2, 1+n],[2+n],2*e*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*
f^2+2*d*e))*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1+n)/e/(-b*f^2+2*d*e)^2/(1
+n)
```

Rubi [A]
time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2141, 961, 66}

$$\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(2,n+1;n+2;\frac{2e\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+bx+a}\right)}{2de-bf^2}\right)}{2e(n+1)\left(2de-bf^2\right)^2}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n)) + (f^2*(4
*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]))/(2*d*e - b*f^2)])/(2*e*(2*d*e - b*f^2)^2*(1 + n))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
GtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] && EqQ[2*c*d - b*e, 0]
))
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx = 2\text{Subst}\left(\int \frac{x^n(d^2e-(bd-ae)f^2-(2de-bf^2)x+ex^2)}{(-2de+bf^2+2ex)^2} dx, x, d\right.$$

$$= 2\text{Subst}\left(\int \left(\frac{x^n}{4e}+\frac{(4ae^2f^2-b^2f^4)x^n}{4e(2de-bf^2-2ex)^2}\right) dx, x, d+ex+f\sqrt{a}\right.$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{(4ae^2f^2-b^2f^4)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)} + \frac{f^2(4ae^2-b^2f^2)\left(d+ex\right.}{}$$

Mathematica [A]
time = 1.12, size = 134, normalized size = 0.81

$$\frac{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^{1+n}\left((-2de+bf^2)^2+(4ae^2f^2-b^2f^4)\,{}_2F_1\left(2,1+n;2+n;\frac{2e\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{2de-bf^2}\right)\right)}{2e\left(-2de+bf^2\right)^2(1+n)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(1 + n)*((-2*d*e + b*f^2)^2 + (4*a*e^2*f^2 - b^2*f^4)*Hypergeometric2F1[2, 1 + n, 2 + n, (2*e*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))/(2*d*e - b*f^2)]))/(2*e*(-2*d*e + b*f^2)^2*(1 + n))

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^n,x)

[Out] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^n,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^n, x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] integral((x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d)^n, x)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**n,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n,x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^n, x)
```

3.473 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx$

Optimal. Leaf size=303

$$\frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}{16e^3}$$

```
[Out] 3/32*f^2*(-b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^
2+e^2*x)/f^2)^(1/2)))/e^5+1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*(e*x+f*
(a+b*x+e^2*x^2/f^2)^(1/2))/e^4+1/16*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+
e^2*x^2/f^2)^(1/2))^2/e^3+1/8*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^4/e-1/32*
f^2*(-b*f^2+2*d*e)^3*(-b^2*f^2+4*a*e^2)/e^5/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A]

time = 0.27, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$-\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^3}{32e^5\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)^2\log\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}{32e^5}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(2de-bf^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^4}+\frac{f^2\left(4ae^2-b^2f^2\right)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}{16e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^4}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*
x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e
) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x +
 f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2
 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(
32*e^5)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3 dx = 2\text{Subst}\left(\int \frac{x^3(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d\right.$$

$$= 2\text{Subst}\left(\int \left(\frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)}{16e^4}+\frac{f^2(4ae^2-b^2f^2)\,x}{16e^3}+\right.\right.$$

$$= \frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^4}+\frac{f^2(4}{}$$

Mathematica [A]
time = 5.83, size = 490, normalized size = 1.62

$$\frac{1}{64}\left(32x(2d^3+3d^2ex+ex(3af^2+2x(bf^2+e^2x))+d(6af^2+x(3bf^2+4e^2x)))+\frac{4\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\,(3b^3f^7-2b^2ef^5(6d+ex)+4be^2f^3(3d^2-2af^2+2dex+2e^2x^2)+8e^3f(2af^2(2d+ex)+ex(3d^2+4dex+2e^2x^2)))}{e^4}-\frac{3\left(-b^2f^4\left(e+\sqrt{\frac{e^2}{f^2}}f\right)(-2de+bf^2)^2+4ae^2f^2\left(4d^2e^3-4bdef^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)+b^2f^4\left(e+\sqrt{\frac{e^2}{f^2}}f\right)\right)\right)\log\left(bf+2e\sqrt{\frac{e^2}{f^2}}x-2e\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{e^6}-\frac{48ad^2f\log\left(e\left(-bf+2e\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)\right)}{\sqrt{\frac{e^2}{f^2}}}+\frac{3\left(e-\sqrt{\frac{e^2}{f^2}}f\right)(4ae^2-b^2f^2)\,(-2def+bf^3)^2\log\left(bf+2e\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{e^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (32*x*(2*d^3 + 3*d^2*e*x + e*x*(3*a*f^2 + 2*x*(b*f^2 + e^2*x)) + d*(6*a*f^2
 + x*(3*b*f^2 + 4*e^2*x))) + (4*Sqrt[a + x*(b + (e^2*x)/f^2)]*(3*b^3*f^7 -
2*b^2*e*f^5*(6*d + e*x) + 4*b*e^2*f^3*(3*d^2 - 2*a*f^2 + 2*d*e*x + 2*e^2*x^
2) + 8*e^3*f*(2*a*f^2*(2*d + e*x) + e*x*(3*d^2 + 4*d*e*x + 2*e^2*x^2))))/e^
4 - (3*(-(b^2*f^4*(e + Sqrt[e^2/f^2]*f)*(-2*d*e + b*f^2)^2) + 4*a*e^2*f^2*(
4*d^2*e^3 - 4*b*d*e*f^2*(e + Sqrt[e^2/f^2]*f) + b^2*f^4*(e + Sqrt[e^2/f^2]*
f)))*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + x*(b + (e^2*x)/f^2)]])/e^
6 - (48*a*d^2*f*Log[e*(-(b*f) + 2*e*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (
e^2*x)/f^2)]))])/Sqrt[e^2/f^2] + (3*(e - Sqrt[e^2/f^2]*f)*(4*a*e^2 - b^2*f^
2)*(-2*d*e*f + b*f^3)^2*Log[b*f + 2*e*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b +
 (e^2*x)/f^2)])])/e^6)/64
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(283) = 566$.
time = 0.36, size = 809, normalized size = 2.67

method	result
default	$f^3 \left(\frac{\left(b+\frac{2e^2x}{f^2}\right) f^2 \left(a+bx+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{8e^2} + \frac{3\left(\frac{4e^2a}{f^2}-b^2\right) f^2 \left(\frac{\left(b+\frac{2e^2x}{f^2}\right) f^2 \sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2} + \frac{\left(\frac{4e^2a}{f^2}-b^2\right) f^2 \ln\left(\frac{\frac{b}{2}+\frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}} + \sqrt{a+b\right.}{8e^2\sqrt{\frac{e^2}{f^2}}} \right.}{16e^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*(1/8*(b+2/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(3/2)+3/16*(4/f^2*e^
2*a-b^2)*f^2/e^2*(1/4*(b+2/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)+1
/8*(4/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*
x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)))+3*f^2*(1/4/f^2*e^3*x^4+1/3*(d/f
^2*e^2+e*b)*x^3+1/2*(a*e+b*d)*x^2+a*d*x)+3*f*(e^2*(1/4*x*(a+b*x+1/f^2*e^2*x
^2)^(3/2)*f^2/e^2-5/8*b*f^2/e^2*(1/3*(a+b*x+1/f^2*e^2*x^2)^(3/2)*f^2/e^2-1/
2*b*f^2/e^2*(1/4*(b+2/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)+1/8*(4
/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*x+1/f
^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)))-1/4*a*f^2/e^2*(1/4*(b+2/f^2*e^2*x)*f
^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)+1/8*(4/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+
1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/
```

```
2)))+2*d*e*(1/3*(a+b*x+1/f^2*e^2*x^2)^(3/2)*f^2/e^2-1/2*b*f^2/e^2*(1/4*(b+2
/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)+1/8*(4/f^2*e^2*a-b^2)*f^2/e
^2*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*x+1/f^2*e^2*x^2)^(1/2))/(1
/f^2*e^2)^(1/2)))+d^2*(1/4*(b+2/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1
/2)+1/8*(4/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+
(a+b*x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)))+1/4*(e*x+d)^4/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*%e^2*a>0)', see `assume?`
 for mo
```

Fricas [A]
time = 0.35, size = 326, normalized size = 1.08

$$\frac{1}{32}\left(32\,x^4e^8+64\,dx^3e^7+16\,\left(2\,bf^2x^3+3\,\left(af^2+d^2\right)x^2\right)e^6+16\,\left(3\,bdf^2x^2+2\,\left(3\,adf^2+d^3\right)x\right)e^5+3\,\left(b^4f^8-4\,b^3df^6e+16\,abdf^4e^3-16\,ad^2f^2e^4-4\,\left(ab^2f^6-b^2d^2f^4\right)e^2\right)\log\left(-bf^2+2\,f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\,e-2\,xe^2\right)+2\,\left(3\,b^3f^7e-12\,b^2df^5e^2+16\,fx^3e^7+32\,dfx^2e^6+8\,\left(bf^3x^2+\left(2\,af^3+3\,d^2f\right)x\right)e^5+8\,\left(bdf^3x+4\,adf^3\right)e^4-2\,\left(b^2f^5x+4\,abf^5-6\,bd^2f^3\right)e^3\right)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)e^{(-5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^4*e^8 + 64*d*x^3*e^7 + 16*(2*b*f^2*x^3 + 3*(a*f^2 + d^2)*x^2)*e^
6 + 16*(3*b*d*f^2*x^2 + 2*(3*a*d*f^2 + d^3)*x)*e^5 + 3*(b^4*f^8 - 4*b^3*d*f
^6*e + 16*a*b*d*f^4*e^3 - 16*a*d^2*f^2*e^4 - 4*(a*b^2*f^6 - b^2*d^2*f^4)*e^
2)*log(-b*f^2 + 2*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - 2*x*e^2) + 2*
(3*b^3*f^7*e - 12*b^2*d*f^5*e^2 + 16*f*x^3*e^7 + 32*d*f*x^2*e^6 + 8*(b*f^3*
x^2 + (2*a*f^3 + 3*d^2*f)*x)*e^5 + 8*(b*d*f^3*x + 4*a*d*f^3)*e^4 - 2*(b^2*f
^5*x + 4*a*b*f^5 - 6*b*d^2*f^3)*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))
*e^(-5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)
```

Giac [A]
time = 3.78, size = 373, normalized size = 1.23

$$bf^2x^3e + \frac{3}{2}\,bdf^2x^2 + \frac{3}{2}\,af^2x^2e + 3\,adf^2x + x^4e^3 + 2\,dx^3e^2 + \frac{3}{2}\,d^2x^2e + d^3x + \frac{3}{32}\,\left(b^4f^7|f| - 4\,b^3df^5|f|e - 4\,ab^2f^5|f|e^2 + 4\,b^2d^2f^3|f|e^2 + 16\,abdf^3|f|e^3 - 16\,ad^2f|f|e^4\right)e^{(-5)}\log\left(\left|-bf^2 - 2\,\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + \frac{1}{16}\,\sqrt{bf^2x + af^2 + x^2e^2}\left(2\,\left(4\,\left(\frac{2\,x|f|e^2}{f} + \frac{\left(bf^4|f|e^6 + 4\,df^2|f|e^7\right)e^{(-6)}}{f^3}\right)x - \frac{\left(b^2f^6|f|e^4 - 4\,bdf^4|f|e^5 - 8\,af^4|f|e^6 - 12\,d^2f^2|f|e^6\right)e^{(-6)}}{f^3}\right)x + \frac{\left(3\,b^3f^8|f|e^2 - 12\,b^2df^6|f|e^3 - 8\,abf^6|f|e^4 + 12\,bd^2f^4|f|e^4 + 32\,adf^4|f|e^5\right)e^{(-6)}}{f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2
*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*f^5*abs
(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^3*abs(f)*e^2 + 16*a*b*d*f^3*ab
s(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e^(-5)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f
^2*x + a*f^2 + x^2*e^2))*e)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(
2*x*abs(f)*e^2/f + (b*f^4*abs(f)*e^6 + 4*d*f^2*abs(f)*e^7)*e^(-6)/f^3)*x -
(b^2*f^6*abs(f)*e^4 - 4*b*d*f^4*abs(f)*e^5 - 8*a*f^4*abs(f)*e^6 - 12*d^2*f^
2*abs(f)*e^6)*e^(-6)/f^3)*x + (3*b^3*f^8*abs(f)*e^2 - 12*b^2*d*f^6*abs(f)*e
^3 - 8*a*b*f^6*abs(f)*e^4 + 12*b*d^2*f^4*abs(f)*e^4 + 32*a*d*f^4*abs(f)*e^5
)*e^(-6)/f^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.474 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx$

Optimal. Leaf size=237

$$\frac{f^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}{6e}-\frac{f^2(2de-bf^2)^2}{16e^4\left(bf^2+2e\left(ex+f\right.\right.}$$

```
[Out] 1/8*f^2*(-b*f^2+2*d*e)*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))/e^4+1/8*f^2*(-b^2*f^2+4*a*e^2)*(e*x+f*(a+b*x+e^2*x^2/f^2
)^(1/2))/e^3+1/6*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3/e-1/16*f^2*(-b*f^2+2
*d*e)^2*(-b^2*f^2+4*a*e^2)/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1
/2)))
```

Rubi [A]

time = 0.17, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)^2}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{8e^4}+\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+ex\right)}{8e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^3}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) +
 (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)
^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 +
e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2 dx = 2\text{Subst}\left(\int \frac{x^2(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d+\right.$$

$$= 2\text{Subst}\left(\int \left(\frac{4ae^2f^2-b^2f^4}{16e^3}+\frac{x^2}{4e}+\frac{(4ae^2-b^2f^2)\,(2def-bf^3)^2}{16e^3\,(2de-bf^2-2ex)^2}\right.\right.$$

$$= \frac{f^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{8e^3}+\frac{\left(d+ex+f\sqrt{a+\right.}{6e}$$

Mathematica [A]
time = 10.30, size = 213, normalized size = 0.90

$$\frac{6ef^2(4ae^2-b^2f^2)\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+8e^3\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^3+\frac{3(-4ae^2+b^2f^2)(-2def+bf^3)^2}{bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}+6f^2(-2de+bf^2)(-4ae^2+b^2f^2)\log\left(-bf^2-2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{48e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]
```

```
[Out] (6*e*f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 8*e^
3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^3 + (3*(-4*a*e^2 + b^2*f^2)*(
-2*d*e*f + b*f^3)^2)/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))
+ 6*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)])])/(48*e^4)
```

Maple [A]
time = 0.30, size = 300, normalized size = 1.27

method	result

default	$\frac{b\,x^{2} f^{2}}{2}+\frac{e^{2} x^{3}}{3}+a\,f^{2} x+2 f \left(e \left(\frac{\left(a+bx+\frac{e^{2} x^{2}}{f^{2}}\right)^{\frac{3}{2}} f^{2}}{3 e^{2}}-\frac{b\,f^{2} \left(\frac{\left(b+\frac{2 e^{2} x}{f^{2}}\right) f^{2} \sqrt{a+bx+\frac{e^{2} x^{2}}{f^{2}}}}{4 e^{2}}+\frac{\left(\frac{4 e^{2} a}{f^{2}}-b^{2}\right) f^{2} \ln \left(\frac{\frac{b}{2}+\cdots}{\sqrt{\cdots}}\right.}{\cdots}\right.}{2 e^{2}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*x^2*f^2+1/3*e^2*x^3+a*f^2*x+2*f*(e*(1/3*(a+b*x+1/f^2*e^2*x^2)^(3/2)*f
^2/e^2-1/2*b*f^2/e^2*(1/4*(b+2/f^2*e^2*x)*f^2/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/
2)+1/8*(4/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(
a+b*x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)))+d*(1/4*(b+2/f^2*e^2*x)*f^2/
e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)+1/8*(4/f^2*e^2*a-b^2)*f^2/e^2*ln((1/2*b+1/f
^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2))
)+1/3*(e*x+d)^3/e
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*%e^2*a>0)', see `assume?`
 for mo
```

Fricas [A]
time = 0.35, size = 204, normalized size = 0.86

$$\frac{1}{24} \left(16 x^{3} e^{6}+24 d x^{2} e^{5}+12 \left(b f^{2} x^{2}+2 \left(a f^{2}+d^{2}\right) x\right) e^{4}-3 \left(b^{3} f^{6}-2 b^{2} d f^{4} e-4 a b f^{4} e^{2}+8 a d f^{2} e^{3}\right) \log \left(-b f^{2}+2 f \sqrt{\frac{b f^{2} x+a f^{2}+x^{2} e^{2}}{f^{2}}}\, e-2 x e^{2}\right)-2 \left(3 b^{2} f^{5} e-6 b d f^{3} e^{2}-8 f x^{2} e^{5}-12 d f x e^{4}-2 \left(b f^{3} x+4 a f^{3}\right) e^{3}\right) \sqrt{\frac{b f^{2} x+a f^{2}+x^{2} e^{2}}{f^{2}}}\right) e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(16*x^3*e^6 + 24*d*x^2*e^5 + 12*(b*f^2*x^2 + 2*(a*f^2 + d^2)*x)*e^4 -
3*(b^3*f^6 - 2*b^2*d*f^4*e - 4*a*b*f^4*e^2 + 8*a*d*f^2*e^3)*log(-b*f^2 + 2*
f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - 2*x*e^2) - 2*(3*b^2*f^5*e - 6*b
*d*f^3*e^2 - 8*f*x^2*e^5 - 12*d*f*x*e^4 - 2*(b*f^3*x + 4*a*f^3)*e^3)*sqrt((
b*f^2*x + a*f^2 + x^2*e^2)/f^2))*e^(-4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)
```

Giac [A]
time = 3.65, size = 224, normalized size = 0.95

$$\frac{1}{2}bf^2x^2 + af^2x + \frac{2}{3}x^3e^2 + dx^2e + d^2x - \frac{1}{8}\left(b^3f^5|f| - 2b^2df^3|f|e - 4abf^3|f|e^2 + 8adf|f|e^3\right)e^{(-4)}\log\left(\left|-bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + \frac{1}{12}\sqrt{bf^2x + af^2 + x^2e^2}\left(2\left(\frac{4x|f|e}{f} + \frac{(bf^3|f|e^3 + 6df|f|e^4)e^{(-4)}}{f^2}\right)x - \frac{(3b^2f^5|f|e - 6bdf^3|f|e^2 - 8af^3|f|e^3)e^{(-4)}}{f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^3*f^5*abs(
f) - 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*e^(-
4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/12*sq
rt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*
f*abs(f)*e^4)*e^(-4)/f^2)*x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 -
8*a*f^3*abs(f)*e^3)*e^(-4)/f^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.475 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx$

Optimal. Leaf size=118

$$dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}$$

```
[Out] d*x+1/2*e*x^2+1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(1/2*(b*f^2+2*e^2*x)/e/f/(a
+b*x+e^2*x^2/f^2)^(1/2))/e^3+1/4*f*(b*f^2+2*e^2*x)*(a+b*x+e^2*x^2/f^2)^(1/2
)/e^2
```

Rubi [A]
time = 0.05, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {626, 635, 212}

$$\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{bf^2+2e^2x}{2ef\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\right)}{8e^3}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+dx+\frac{ex^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^
2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])])/(8*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right) dx = dx+\frac{ex^2}{2}+f\int \sqrt{a+bx+\frac{e^2x^2}{f^2}}\, dx$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{8}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right.$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{1}{4}\left(f\left(4a-\frac{b^2f^2}{e^2}\right)\right.$$

$$= dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{4e^2}+\frac{f^2(4ae^2-b^2f^2)\tan}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 478 vs. 2(118) = 236.
time = 2.26, size = 478, normalized size = 4.05

$$dx+\frac{ex^2}{2}+\frac{f(bf^2+2e^2x)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{4e^2}+\frac{f(4ae^2f-b^2f^3)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\tanh^{-1}\left(\frac{-2e\sqrt{\frac{e^2}{f^2}}x+2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{bf}\right)}{8e^3\sqrt{a+bx+\frac{e^2x^2}{f^2}}}+\frac{b^2\sqrt{\frac{e^2}{f^2}}f^5\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\log\left(bf+2e\sqrt{\frac{e^2}{f^2}}x-2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{16e^4\sqrt{a+bx+\frac{e^2x^2}{f^2}}}-\frac{\sqrt{\frac{e^2}{f^2}}f(4ae^2f^2-b^2f^4)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\log\left(bf-2e\sqrt{\frac{e^2}{f^2}}x+2e\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{16e^4\sqrt{a+bx+\frac{e^2x^2}{f^2}}}-\frac{af\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\log\left(-bef-2e^2\sqrt{\frac{e^2}{f^2}}x+2e^2\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}{4\sqrt{\frac{e^2}{f^2}}\sqrt{a+bx+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2],x]
```

```
[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + x*(b + (e^2*x)/f^2)])/(4*e^
2) + (f*(4*a*e^2*f - b^2*f^3)*Sqrt[a + x*(b + (e^2*x)/f^2)]*ArcTanh[(-2*e*S
```

```
qrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2])/(b*f)])/(8*e^3*Sqrt[a +
 b*x + (e^2*x^2)/f^2]) + (b^2*Sqrt[e^2/f^2]*f^5*Sqrt[a + x*(b + (e^2*x)/f^2
)]*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*
e^4*Sqrt[a + b*x + (e^2*x^2)/f^2]) - (Sqrt[e^2/f^2]*f*(4*a*e^2*f^2 - b^2*f^
4)*Sqrt[a + x*(b + (e^2*x)/f^2)]*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a
 + b*x + (e^2*x^2)/f^2]])/(16*e^4*Sqrt[a + b*x + (e^2*x^2)/f^2]) - (a*f*Sqr
t[a + x*(b + (e^2*x)/f^2)]*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqr
t[a + b*x + (e^2*x^2)/f^2]])/(4*Sqrt[e^2/f^2]*Sqrt[a + b*x + (e^2*x^2)/f^2]
)
```

Maple [A]
time = 0.31, size = 173, normalized size = 1.47

method	result
default	$dx+\frac{e\,x^{2}}{2}+\frac{f^{3}\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\,b}{4e^{2}}+\frac{f\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\,x}{2}+\frac{f\ln\left(\frac{\frac{b}{2}+\frac{e^{2}x}{f^{2}}}{\sqrt{\frac{e^{2}}{f^{2}}}}+\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)a}{2\sqrt{\frac{e^{2}}{f^{2}}}}-\frac{f^{3}\ln\Big(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*x+1/2*e*x^2+1/4*f^3/e^2*(a+b*x+1/f^2*e^2*x^2)^(1/2)*b+1/2*f*(a+b*x+1/f^2*
e^2*x^2)^(1/2)*x+1/2*f*ln((1/2*b+1/f^2*e^2*x)/(1/f^2*e^2)^(1/2)+(a+b*x+1/f^
2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)*a-1/8*f^3/e^2*ln((1/2*b+1/f^2*e^2*x)/(1
/f^2*e^2)^(1/2)+(a+b*x+1/f^2*e^2*x^2)^(1/2))/(1/f^2*e^2)^(1/2)*b^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*%e^2*a>0)', see `assume?`
 for mo
```

Fricas [A]
time = 0.38, size = 117, normalized size = 0.99

$$\frac{1}{8}\left(4\,x^{2}e^{4}+8\,dxe^{3}+\left(b^{2}f^{4}-4\,af^{2}e^{2}\right)\log\left(-bf^{2}+2\,f\sqrt{\frac{bf^{2}x+af^{2}+x^{2}e^{2}}{f^{2}}}\,e-2\,xe^{2}\right)+2\left(bf^{3}e+2\,fxe^{3}\right)\sqrt{\frac{bf^{2}x+af^{2}+x^{2}e^{2}}{f^{2}}}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*x^2*e^4 + 8*d*x*e^3 + (b^2*f^4 - 4*a*f^2*e^2)*log(-b*f^2 + 2*f*sqrt(
(b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - 2*x*e^2) + 2*(b*f^3*e + 2*f*x*e^3)*sqr
t((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*e^(-3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

Giac [A]
time = 2.91, size = 111, normalized size = 0.94

$$\frac{1}{2}\,x^2e + dx + \frac{\left((b^2f^4 - 4\,af^2e^2)e^{(-3)}\log\left(\left|-bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right)e\right|\right) + 2\,\sqrt{bf^2x + af^2 + x^2e^2}\,\left(bf^2e^{(-2)} + 2\,x\right)\right)|f|}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*e + d*x + 1/8*((b^2*f^4 - 4*a*f^2*e^2)*e^(-3)*log(abs(-b*f^2 - 2*(x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 2*sqrt(b*f^2*x + a*f^2 + x^2*e^
2)*(b*f^2*e^(-2) + 2*x))*abs(f)/f
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)
```

```
[Out] int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)
```

3.476 $\displaystyle \int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=215

$$-\frac{f^2(4ae^2-b^2f^2)}{2e\,(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{2(d^2e-bdf^2+aef^2)\log\left(d+ex+f\sqrt{a+\cdots}\right.}{(2de-bf^2)^2}$$

```
[Out] 2*(a*e*f^2-b*d*f^2+d^2*e)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d
*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2
)^(1/2)))/e/(-b*f^2+2*d*e)^2-1/2*f^2*(-b^2*f^2+4*a*e^2)/e/(-b*f^2+2*d*e)/(b
*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.15, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$-\frac{f^2(4ae^2-b^2f^2)}{2e\,(2de-bf^2)\left(2e\left(f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{2e\,(2de-bf^2)^2}+\frac{2(aef^2-bdf^2+d^2e)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] -1/2*(f^2*(4*a*e^2 - b^2*f^2))/(e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d +
e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 -
 b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*
e*(2*d*e - b*f^2)^2)
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)x+ex^2}{x(-2de+bf^2+2ex)^2}\,dx, x, d+ex+f\right.$$

$$= 2\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2x}+\frac{4ae^2f^2-b^2f^4}{2(2de-bf^2)(2de-bf^2-2ex)^2}+\right.\right.$$

$$= -\frac{f^2(4ae^2-b^2f^2)}{2e(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}+\frac{2(d^2e}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 570 vs. 2(215) = 430.
time = 1.80, size = 570, normalized size = 2.65

$$\frac{ex}{2de-bf^2}+\frac{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{-2de+bf^2}-\frac{\log\left(bf+2e\sqrt{\frac{e^2}{f^2}}x-2e\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{4e}-\frac{bdef\log\left(bdf-2aef+2de\sqrt{\frac{e^2}{f^2}}x-b\sqrt{\frac{e^2}{f^2}}f^2x+(-2de+bf^2)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{\sqrt{\frac{e^2}{f^2}}(-2de+bf^2)^2}+\frac{\left(-bdf^2+d^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)+af^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)\right)\log\left(bdf-2aef+\sqrt{\frac{e^2}{f^2}}(2de-bf^2)x+(-2de+bf^2)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{(-2de+bf^2)^2}-\frac{\sqrt{\frac{e^2}{f^2}}f\log\left(e\left(-bf+2e\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)\right)}{4e^2}-\frac{f^2\left(-e+\sqrt{\frac{e^2}{f^2}}f\right)(-4ae^2+b^2f^2)\log\left(bf+2e\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{4e^2(-2de+bf^2)^2}+\frac{\left(e-\sqrt{\frac{e^2}{f^2}}f\right)(d^2e-bdf^2+aef^2)\log\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{e(-2de+bf^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1),x]
```

```
[Out] (e*x)/(2*d*e - b*f^2) + (f*Sqrt[a + x*(b + (e^2*x)/f^2)])/(-2*d*e + b*f^2)
- Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + x*(b + (e^2*x)/f^2)]]/(4*e)
- (b*d*e*f*Log[b*d*f - 2*a*e*f + 2*d*e*Sqrt[e^2/f^2]*x - b*Sqrt[e^2/f^2]*f^
2*x + (-2*d*e + b*f^2)*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(Sqrt[e^2/f^2]*(-2*d
*e + b*f^2)^2) + ((-(b*d*f^2) + d^2*(e + Sqrt[e^2/f^2]*f) + a*f^2*(e + Sqrt
[e^2/f^2]*f))*Log[b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*(2*d*e - b*f^2)*x + (-2*d
*e + b*f^2)*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(-2*d*e + b*f^2)^2 - (Sqrt[e^2/
f^2]*f*Log[e*(-(b*f) + 2*e*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (e^2*x)/f^
2)]))])/(4*e^2) - (f^2*(-e + Sqrt[e^2/f^2]*f)*(-4*a*e^2 + b^2*f^2)*Log[b*f
+ 2*e*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (e^2*x)/f^2)])])/(4*e^2*(-2*d*e
 + b*f^2)^2) + ((e - Sqrt[e^2/f^2]*f)*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + f
```

```
*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (e^2*x)/f^2)])])/(e*(-2*d*e + b*f^2)
^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4917 vs. $2(205) = 410$.

time = 0.04, size = 4918, normalized size = 22.87

method	result	size
default	Expression too large to display	4918

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] f/(b*f^2-2*d*e)*(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(-b^2*f^4+2*a*e^
2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)
)+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*
e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)+1/2*f^3/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+
2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+1/f^2*e^2*(x+(a*f^2-d^
2)/(b*f^2-2*d*e)))/(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e
))^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f
^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f
^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*b^2
-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^
2/(b*f^2-2*d*e)+1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(1/f^2*e^2)^(1/2)+
(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^
2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2
*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-
2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*a*e^2-f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f
^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+1/f^2*e^2*(x+(a*f^2
-d^2)/(b*f^2-2*d*e)))/(1/f^2*e^2)^(1/2)+(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*
d*e))^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(
a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^
2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*
b*d*e+1/f/(b*f^2-2*d*e)^2*ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*
e^2)/f^2/(b*f^2-2*d*e)+1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))/(1/f^2*e^2)
^(1/2)+(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(-b^2*f^4+2*a*e^2*f^2+2*b
*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^
2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/
(b*f^2-2*d*e)^2)^(1/2))/(1/f^2*e^2)^(1/2)*d^2*e^2-f^3/(b*f^2-2*d*e)^3/((a^2
*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*
d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^
2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*
((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e
^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(
```

```
-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2
)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b
*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e
)))*a^2*e^2+2*f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2
*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2
*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f
^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-
2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*
f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(1/f^
2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d
^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*
d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e
)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a*b*d*e-f^3/(b*f^2-2*d*e)^3/((a^
2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/
f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a
*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f
^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2
*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*
e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(1/f^2*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-
(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^
2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*
b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*
e)))*b^2*d^2-2*f/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*
e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^
2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2
*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f
^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^(1/2)*(1/f^2
*e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^
2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d
*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)
^2)^(1/2))/(x+(a*f^2-d^2)/(b*f^2-2*d*e)))*a*d^2*e^2+2*f/(b*f^2-2*d*e)^3/((a
^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)
/f^2/(b*f^2-2*d*e)^2)^(1/2)*ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*
a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 5.67, size = 362, normalized size = 1.68

$$-\frac{2 \, b f^{2} x e^{2} - 4 \, d x e^{3} + 2 \, {\left(b d f^{2} e - {\left(a f^{2} + d^{2}\right)} e^{2}\right)} \log\left(-b d f^{2} - 2 \, d x e^{2} + {\left(b f^{2} x + 2 \, a f^{2}\right)} e - {\left(b f^{3} - 2 \, d f e\right)} \sqrt{\frac{b f^{2} x + a f^{2} + x^{2} e^{2}}{f^{2}}}\right) + 2 \, {\left(b d f^{2} e - {\left(a f^{2} + d^{2}\right)} e^{2}\right)} \log\left(-b f^{2} x - a f^{2} + 2 \, d x e + d^{2}\right) + {\left(b^{2} f^{4} - 2 \, b d f^{2} e - 2 \, {\left(a f^{2} - d^{2}\right)} e^{2}\right)} \log\left(-b f^{2} + 2 \, f \sqrt{\frac{b f^{2} x + a f^{2} + x^{2} e^{2}}{f^{2}}} e - 2 \, x e^{2}\right) - 2 \, {\left(b d f^{2} e - {\left(a f^{2} + d^{2}\right)} e^{2}\right)} \log\left(-x e + f \sqrt{\frac{b f^{2} x + a f^{2} + x^{2} e^{2}}{f^{2}}} - d\right) - 2 \, {\left(b f^{3} e - 2 \, d f e^{2}\right)} \sqrt{\frac{b f^{2} x + a f^{2} + x^{2} e^{2}}{f^{2}}}}{2 \, {\left(b^{2} f^{4} e - 4 \, b d f^{2} e^{2} + 4 \, d^{2} e^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*f^2*x*e^2 - 4*d*x*e^3 + 2*(b*d*f^2*e - (a*f^2 + d^2)*e^2)*log(-b*
d*f^2 - 2*d*x*e^2 + (b*f^2*x + 2*a*f^2)*e - (b*f^3 - 2*d*f*e)*sqrt((b*f^2*x
 + a*f^2 + x^2*e^2)/f^2)) + 2*(b*d*f^2*e - (a*f^2 + d^2)*e^2)*log(-b*f^2*x
- a*f^2 + 2*d*x*e + d^2) + (b^2*f^4 - 2*b*d*f^2*e - 2*(a*f^2 - d^2)*e^2)*lo
g(-b*f^2 + 2*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - 2*x*e^2) - 2*(b*d*
f^2*e - (a*f^2 + d^2)*e^2)*log(-x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^
2) - d) - 2*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))/(b
^2*f^4*e - 4*b*d*f^2*e^2 + 4*d^2*e^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + e x + f \sqrt{a + b x + \frac{e^{2} x^{2}}{f^{2}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(204) = 408.
time = 3.88, size = 523, normalized size = 2.43

$$-\frac{x e}{b f^{2} - 2 \, d e} - \frac{{\left(b d f^{2} {\left| f \right|} - a f^{2} {\left| f \right|} e - d^{2} {\left| f \right|} e\right)} \log\left({\left| -{\left(x e - \sqrt{b f^{2} x + a f^{2} + x^{2} e^{2}}\right)} b f^{2} + b d f^{2} - 2 \, a f^{2} e + 2 \, {\left(x e - \sqrt{b f^{2} x + a f^{2} + x^{2} e^{2}}\right)} d e \right|}\right)}{b^{2} f^{5} - 4 \, b d f^{3} e + 4 \, d^{2} f e^{2}} - \frac{{\left(b d f^{2} - a f^{2} e - d^{2} e\right)} \log\left({\left| -b f^{2} x - a f^{2} + 2 \, d x e + d^{2} \right|}\right)}{b^{2} f^{4} - 4 \, b d f^{2} e + 4 \, d^{2} e^{2}} - \frac{{\left(b^{2} f^{4} {\left| f \right|} - 2 \, b d f^{2} {\left| f \right|} e - 2 \, a f^{2} {\left| f \right|} e^{2} + 2 \, d^{2} {\left| f \right|} e^{2}\right)} \log\left({\left| b f^{2} + 2 \, {\left(x e - \sqrt{b f^{2} x + a f^{2} + x^{2} e^{2}}\right)} e \right|}\right)}{2 \, {\left(b^{2} f^{5} e - 4 \, b d f^{3} e^{2} + 4 \, d^{2} f e^{3}\right)}} + \frac{{\left(b d f^{2} {\left| f \right|} - a f^{2} {\left| f \right|} e - d^{2} {\left| f \right|} e\right)} \log\left({\left| x e + d - \sqrt{b f^{2} x + a f^{2} + x^{2} e^{2}} \right|}\right)}{b^{2} f^{5} - 4 \, b d f^{3} e + 4 \, d^{2} f e^{2}} + \frac{{\left(b^{2} f^{5} {\left| f \right|} - 4 \, b d f^{3} {\left| f \right|} e + 4 \, d^{2} f {\left| f \right|} e^{2}\right)} \sqrt{b f^{2} x + a f^{2} + x^{2} e^{2}}}{b^{3} f^{8} - 6 \, b^{2} d f^{6} e + 12 \, b d^{2} f^{4} e^{2} - 8 \, d^{3} f^{2} e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")
```

```
[Out] -x*e/(b*f^2 - 2*d*e) - (b*d*f^2*abs(f) - a*f^2*abs(f)*e - d^2*abs(f)*e)*log
(abs(-(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b*f^2 + b*d*f^2 - 2*a*f^2*e +
 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*d*e))/(b^2*f^5 - 4*b*d*f^3*e + 4
*d^2*f*e^2) - (b*d*f^2 - a*f^2*e - d^2*e)*log(abs(-b*f^2*x - a*f^2 + 2*d*x*
e + d^2))/(b^2*f^4 - 4*b*d*f^2*e + 4*d^2*e^2) - 1/2*(b^2*f^4*abs(f) - 2*b*d
*f^2*abs(f)*e - 2*a*f^2*abs(f)*e^2 + 2*d^2*abs(f)*e^2)*log(abs(b*f^2 + 2*(x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e))/(b^2*f^5*e - 4*b*d*f^3*e^2 + 4*d^
```

```
2*f*e^3) + (b*d*f^2*abs(f) - a*f^2*abs(f)*e - d^2*abs(f)*e)*log(abs(x*e + d
 - sqrt(b*f^2*x + a*f^2 + x^2*e^2)))/(b^2*f^5 - 4*b*d*f^3*e + 4*d^2*f*e^2)
+ (b^2*f^5*abs(f) - 4*b*d*f^3*abs(f)*e + 4*d^2*f*abs(f)*e^2)*sqrt(b*f^2*x +
 a*f^2 + x^2*e^2)/(b^3*f^8 - 6*b^2*d*f^6*e + 12*b*d^2*f^4*e^2 - 8*d^3*f^2*e
^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)
```

3.477 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$

Optimal. Leaf size=266

$$-\frac{2(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

```
[Out] 2*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*d*
e)^3-2*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(
1/2)))/(-b*f^2+2*d*e)^3-2*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f
*(a+b*x+e^2*x^2/f^2)^(1/2))-f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^2/(b*f^2+
2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]

time = 0.16, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$\frac{2f^2(4ae^2-b^2f^2)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^3}-\frac{f^2(4ae^2-b^2f^2)}{(2de-bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{2f^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{(2de-bf^2)^3}-\frac{2(aef^2-bdf^2+d^2e)}{(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b
*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2
 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^
2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 -
(2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^
2*x))/f^2])])/(2*d*e - b*f^2)^3
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^2\,(-2de+bf^2+2ex)^2}\,dx, x, d+ex\right.$$

$$= 2\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\,x^2}+\frac{4ae^2f^2-b^2f^4}{(2de-bf^2)^3\,x}+\frac{4ae^3}{(2de-bf^2)^2}\right.\right.$$

$$= -\frac{2(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)} - \frac{}{(2de-bf^2)^2\,(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 551 vs. 2(266) = 532.
time = 3.77, size = 551, normalized size = 2.07

$$\frac{2(bf^3(-d+ex)+2ef(af^2-dex))\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}{(-2de+bf^2)^2\,(d^2+2dex-f^2(a+bx))}-\frac{2(4a^2e^3f^2x+bdx(2d^2e^2-2bdef^2+b^2f^4+2de^3x-be^2f^2x)+a(4d^3e^2+2be^3f^2x^2+d^2(-4bef^2+4e^3x)+d(b^2f^4-6be^2f^2x-4e^4x^2)))}{(bd-2ae)\,(-2de+bf^2)^2\,(-d^2-2dex+f^2(a+bx))}+\frac{f^2\left(e+\sqrt{\frac{e^2}{f^2}}f\right)(4ae^2-b^2f^2)\log\left(bdf-2aef+\sqrt{\frac{e^2}{f^2}}(2de-bf^2)\,x+(-2de+bf^2)\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}{e\,(2de-bf^2)^3}-\frac{f^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)(4ae^2-b^2f^2)\log\left(bf+2e\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{e\,(2de-bf^2)^3}+\frac{f^2\left(e-\sqrt{\frac{e^2}{f^2}}f\right)(4ae^2-b^2f^2)\log\left(d+f\left(-\sqrt{\frac{e^2}{f^2}}x+\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{e\,(2de-bf^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2),x]
```

```
[Out] (2*(b*f^3*(-d + e*x) + 2*e*f*(a*f^2 - d*e*x))*Sqrt[a + x*(b + (e^2*x)/f^2)]
)/((-2*d*e + b*f^2)^2*(d^2 + 2*d*e*x - f^2*(a + b*x))) - (2*(4*a^2*e^3*f^2*
x + b*d*x*(2*d^2*e^2 - 2*b*d*e*f^2 + b^2*f^4 + 2*d*e^3*x - b*e^2*f^2*x) + a
*(4*d^3*e^2 + 2*b*e^3*f^2*x^2 + d^2*(-4*b*e*f^2 + 4*e^3*x) + d*(b^2*f^4 - 6
*b*e^2*f^2*x - 4*e^4*x^2))))/((b*d - 2*a*e)*(-2*d*e + b*f^2)^2*(-d^2 - 2*d*
e*x + f^2*(a + b*x))) + (f^2*(e + Sqrt[e^2/f^2]*f)*(4*a*e^2 - b^2*f^2)*Log[
b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*(2*d*e - b*f^2)*x + (-2*d*e + b*f^2)*Sqrt[a
 + x*(b + (e^2*x)/f^2)]])/(e*(2*d*e - b*f^2)^3) - (f^2*(e - Sqrt[e^2/f^2]*f
)*(4*a*e^2 - b^2*f^2)*Log[b*f + 2*e*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (
e^2*x)/f^2)])])/(e*(2*d*e - b*f^2)^3) + (f^2*(e - Sqrt[e^2/f^2]*f)*(4*a*e^2
```

```
 - b^2*f^2)*Log[d + f*(-(Sqrt[e^2/f^2]*x) + Sqrt[a + x*(b + (e^2*x)/f^2)])]
)/(e*(2*d*e - b*f^2)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58068 vs. $2(258) = 516$.
time = 0.10, size = 58069, normalized size = 218.30

method	result	size
default	Expression too large to display	58069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(256) = 512$.
time = 3.04, size = 803, normalized size = 3.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*f^6*x + a*b^2*f^6 + 3*b^2*d^2*f^4 - 16*d^2*x^2*e^4 + 16*(b*d*f^2*
x^2 - d^3*x)*e^3 - 4*(b^2*f^4*x^2 - 2*a^2*f^4 - 5*b*d^2*f^2*x - 2*a*d^2*f^2
)*e^2 - 2*(4*b^2*d*f^4*x + 7*a*b*d*f^4 + b*d^3*f^2)*e - 2*(b^3*f^6*x + a*b^
2*f^6 - 2*b^2*d*f^4*x*e - b^2*d^2*f^4 + 8*a*d*f^2*x*e^3 - 4*(a*b*f^4*x + a^
2*f^4 - a*d^2*f^2)*e^2)*log(b^2*d*f^4 + 8*d*x^2*e^4 - 4*(b*f^2*x^2 + a*f^2*
x)*e^3 + 4*(2*b*d*f^2*x + a*d*f^2)*e^2 - (3*b^2*f^4*x + 4*a*b*f^4)*e + (b^2
*f^5 - 4*b*d*f^3*e - 8*d*f*x*e^3 + 4*(b*f^3*x + a*f^3)*e^2)*sqrt((b*f^2*x +
 a*f^2 + x^2*e^2)/f^2)) - 2*(b^3*f^6*x + a*b^2*f^6 - 2*b^2*d*f^4*x*e - b^2*
d^2*f^4 + 8*a*d*f^2*x*e^3 - 4*(a*b*f^4*x + a^2*f^4 - a*d^2*f^2)*e^2)*log(-b
*f^2*x - a*f^2 + 2*d*x*e + d^2) + 2*(b^3*f^6*x + a*b^2*f^6 - 2*b^2*d*f^4*x*
e - b^2*d^2*f^4 + 8*a*d*f^2*x*e^3 - 4*(a*b*f^4*x + a^2*f^4 - a*d^2*f^2)*e^2
)*log(-x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) - d) - 4*(b^2*d*f^5 -
```

```
4*d^2*f*x*e^3 + 4*(b*d*f^3*x + a*d*f^3)*e^2 - (b^2*f^5*x + 2*a*b*f^5 + 2*b*
d^2*f^3)*e)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))/(b^4*f^8*x + a*b^3*f^8 -
 b^3*d^2*f^6 + 16*d^4*x*e^4 - 8*(4*b*d^3*f^2*x + a*d^3*f^2 - d^5)*e^3 + 12*
(2*b^2*d^2*f^4*x + a*b*d^2*f^4 - b*d^4*f^2)*e^2 - 2*(4*b^3*d*f^6*x + 3*a*b^
2*d*f^6 - 3*b^2*d^3*f^4)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. 2(256) = 512.
time = 4.27, size = 1243, normalized size = 4.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*x*e^2/(b^2*f^4 - 4*b*d*f^2*e + 4*d^2*e^2) + (b^3*f^5*abs(f) - 2*b^2*d*f^3
*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*log(abs((x*e - sqrt(
b*f^2*x + a*f^2 + x^2*e^2))*b*f^2 - b*d*f^2 + 2*a*f^2*e - 2*(x*e - sqrt(b*f
^2*x + a*f^2 + x^2*e^2))*d*e))/(b^4*f^8 - 8*b^3*d*f^6*e + 24*b^2*d^2*f^4*e^
2 - 32*b*d^3*f^2*e^3 + 16*d^4*e^4) + (b^2*f^4 - 4*a*f^2*e^2)*log(abs(b*f^2*
x + a*f^2 - 2*d*x*e - d^2))/(b^3*f^6 - 6*b^2*d*f^4*e + 12*b*d^2*f^2*e^2 - 8
*d^3*e^3) + (b^2*f^3*abs(f)*e - 4*a*f*abs(f)*e^3)*log(abs(b*f^2 + 2*(x*e -
sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e))/(b^3*f^6*e - 6*b^2*d*f^4*e^2 + 12*b*d^
2*f^2*e^3 - 8*d^3*e^4) - (b^2*f^3*abs(f) - 4*a*f*abs(f)*e^2)*log(abs(-x*e -
 d + sqrt(b*f^2*x + a*f^2 + x^2*e^2)))/(b^3*f^6 - 6*b^2*d*f^4*e + 12*b*d^2*
f^2*e^2 - 8*d^3*e^3) - 2*(b^3*f^6*abs(f)*e - 6*b^2*d*f^4*abs(f)*e^2 + 12*b*
d^2*f^2*abs(f)*e^3 - 8*d^3*abs(f)*e^4)*sqrt(b*f^2*x + a*f^2 + x^2*e^2)/(b^5
*f^11 - 10*b^4*d*f^9*e + 40*b^3*d^2*f^7*e^2 - 80*b^2*d^3*f^5*e^3 + 80*b*d^4
*f^3*e^4 - 32*d^5*f*e^5) - 2*((x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b^3*d
*f^6*abs(f) - (x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*b^2*f^6*abs(f)*e +
a*b^2*d*f^6*abs(f)*e - a^2*b*f^6*abs(f)*e^2 - 3*(x*e - sqrt(b*f^2*x + a*f^2
 + x^2*e^2))*b^2*d^2*f^4*abs(f)*e + b^2*d^3*f^4*abs(f)*e - 6*a*b*d^2*f^4*ab
s(f)*e^2 + 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^2*f^4*abs(f)*e^3 + 4
```

```
*a^2*d*f^4*abs(f)*e^3 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b*d^3*f^2
*abs(f)*e^2 - b*d^4*f^2*abs(f)*e^2 + 4*a*d^3*f^2*abs(f)*e^3 - 2*(x*e - sqrt
(b*f^2*x + a*f^2 + x^2*e^2))*d^4*abs(f)*e^3)/((b^3*f^7 - 6*b^2*d*f^5*e + 12
*b*d^2*f^3*e^2 - 8*d^3*f*e^3)*((x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*b*
f^2 - b*d^2*f^2 + 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*f^2*e + 2*a*d
*f^2*e - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*d*e - 2*(x*e - sqrt(b*
f^2*x + a*f^2 + x^2*e^2))*d^2*e)) - 2*(b^2*d^2*f^4 - 2*a*b*d*f^4*e + a^2*f^
4*e^2 - 2*b*d^3*f^2*e + 2*a*d^2*f^2*e^2 + d^4*e^2)/((b*f^2*x + a*f^2 - 2*d*
x*e - d^2)*(b*f^2 - 2*d*e)^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)
```

3.478 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$

Optimal. Leaf size=330

$$-\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2}-\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)}-\frac{\quad}{(2de-bf^2}$$

```
[Out] 6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))/(-b*f^2+2*
d*e)^4-6*e*f^2*(-b^2*f^2+4*a*e^2)*ln(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^
2)^(1/2)))/(-b*f^2+2*d*e)^4+(-a*e*f^2+b*d*f^2-d^2*e)/(-b*f^2+2*d*e)^2/(d+e*
x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2-2*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/
(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))-2*e*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*
e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.21, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2141, 907}

$$-\frac{2f^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}+\frac{6ef^2(4ae^2-b^2f^2)\log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)}{(2de-bf^2)^4}-\frac{2ef^2(4ae^2-b^2f^2)}{(2de-bf^2)^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{6ef^2(4ae^2-b^2f^2)\log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}{(2de-bf^2)^4}-\frac{aef^2-bdf^2+d^2e}{(2de-bf^2)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x
+ (e^2*x^2)/f^2])^2)) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d +
 e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((
2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])))
 + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^
2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x
+ f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(2*d*e - b*f^2)^4
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
```

)

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^3\,(-2de+bf^2+2ex)^2}\,dx, x, d+e$$

$$= 2\text{Subst}\left(\int \left(\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\,x^3}+\frac{4ae^2f^2-b^2f^4}{(2de-bf^2)^3\,x^2}+\frac{3(4ae^3f^2-}{(2de-b}\right.\right.$$

$$= -\frac{d^2e-bdf^2+aef^2}{(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2} - \frac{}{(2de-bf^2)^3}$$

Mathematica [A]
time = 10.55, size = 300, normalized size = 0.91

$$-\frac{\frac{(-2de+bf^2)^2(d^2e-bdf^2+aef^2)}{\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^2}+\frac{2f^2(-2de+bf^2)(-4ae^2+b^2f^2)}{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}+\frac{2ef^2(2de-bf^2)(4ae^2-b^2f^2)}{bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)}-6ef^2(4ae^2-b^2f^2)\log\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+6ef^2(4ae^2-b^2f^2)\log\left(-bf^2-2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{(-2de+bf^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3),x]
```

```
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x
*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d +
 e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2
 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^
2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + 6*e*
f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)])])/(-2*d*e + b*f^2)^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295125 vs. $2(320) = 640$.
time = 0.14, size = 295126, normalized size = 894.32

method	result	size
default	Expression too large to display	295126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1942 vs. $2(319) = 638$.
time = 26.98, size = 1942, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] (3*b^4*d*f^8*x + 3*a*b^3*d*f^8 - b^3*d^3*f^6 + 32*d^3*x^3*e^6 - 8*(6*b*d^2*
f^2*x^3 - (a*d^2*f^2 + 5*d^4)*x^2)*e^5 + 8*(3*b^2*d*f^4*x^3 - (a*b*d*f^4 +
7*b*d^3*f^2)*x^2 - (7*a^2*d*f^4 + a*d^3*f^2 - 2*d^5)*x)*e^4 - 2*(2*b^3*f^6*
x^3 - 10*a^3*f^6 + 12*a^2*d^2*f^4 + 6*a*d^4*f^2 - (a*b^2*f^6 + 11*b^2*d^2*f
^4)*x^2 - 2*(7*a^2*b*f^6 + 11*a*b*d^2*f^4 - 6*b*d^4*f^2)*x)*e^3 - 2*(5*a^2*
b*d*f^6 - 16*a*b*d^3*f^4 - b*d^5*f^2 + 5*(a*b^2*d*f^6 - b^2*d^3*f^4)*x)*e^2
 - (b^4*f^8*x^2 + 4*a^2*b^2*f^8 + 4*a*b^2*d^2*f^6 + 4*b^2*d^4*f^4 + (5*a*b^
3*f^8 + 7*b^3*d^2*f^6)*x)*e + 3*(16*a*d^2*f^2*x^2*e^5 - 16*(a*b*d*f^4*x^2 +
 (a^2*d*f^4 - a*d^3*f^2)*x)*e^4 + 4*(a^3*f^6 - 2*a^2*d^2*f^4 + a*d^4*f^2 +
(a*b^2*f^6 - b^2*d^2*f^4)*x^2 + 2*(a^2*b*f^6 - a*b*d^2*f^4)*x)*e^3 + 4*(b^3
*d*f^6*x^2 + (a*b^2*d*f^6 - b^2*d^3*f^4)*x)*e^2 - (b^4*f^8*x^2 + a^2*b^2*f^
8 - 2*a*b^2*d^2*f^6 + b^2*d^4*f^4 + 2*(a*b^3*f^8 - b^3*d^2*f^6)*x)*e)*log(b
^2*d*f^4 + 8*d*x^2*e^4 - 4*(b*f^2*x^2 + a*f^2*x)*e^3 + 4*(2*b*d*f^2*x + a*d
*f^2)*e^2 - (3*b^2*f^4*x + 4*a*b*f^4)*e + (b^2*f^5 - 4*b*d*f^3*e - 8*d*f*x*
```

```
e^3 + 4*(b*f^3*x + a*f^3)*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) + 3*(
16*a*d^2*f^2*x^2*e^5 - 16*(a*b*d*f^4*x^2 + (a^2*d*f^4 - a*d^3*f^2)*x)*e^4 +
 4*(a^3*f^6 - 2*a^2*d^2*f^4 + a*d^4*f^2 + (a*b^2*f^6 - b^2*d^2*f^4)*x^2 + 2
*(a^2*b*f^6 - a*b*d^2*f^4)*x)*e^3 + 4*(b^3*d*f^6*x^2 + (a*b^2*d*f^6 - b^2*d
^3*f^4)*x)*e^2 - (b^4*f^8*x^2 + a^2*b^2*f^8 - 2*a*b^2*d^2*f^6 + b^2*d^4*f^4
 + 2*(a*b^3*f^8 - b^3*d^2*f^6)*x)*e)*log(-b*f^2*x - a*f^2 + 2*d*x*e + d^2)
- 3*(16*a*d^2*f^2*x^2*e^5 - 16*(a*b*d*f^4*x^2 + (a^2*d*f^4 - a*d^3*f^2)*x)*
e^4 + 4*(a^3*f^6 - 2*a^2*d^2*f^4 + a*d^4*f^2 + (a*b^2*f^6 - b^2*d^2*f^4)*x^
2 + 2*(a^2*b*f^6 - a*b*d^2*f^4)*x)*e^3 + 4*(b^3*d*f^6*x^2 + (a*b^2*d*f^6 -
b^2*d^3*f^4)*x)*e^2 - (b^4*f^8*x^2 + a^2*b^2*f^8 - 2*a*b^2*d^2*f^6 + b^2*d^
4*f^4 + 2*(a*b^3*f^8 - b^3*d^2*f^6)*x)*e)*log(-x*e + f*sqrt((b*f^2*x + a*f^
2 + x^2*e^2)/f^2) - d) - 2*(b^4*f^9*x + a*b^3*f^9 + 16*d^3*f*x^2*e^5 - 12*(
2*b*d^2*f^3*x^2 + (3*a*d^2*f^3 - d^4*f)*x)*e^4 + 4*(3*b^2*d*f^5*x^2 + 3*a^2
*d*f^5 - 5*a*d^3*f^3 + (9*a*b*d*f^5 - 2*b*d^3*f^3)*x)*e^3 - (2*b^3*f^7*x^2
+ 6*a^2*b*f^7 - 12*a*b*d^2*f^5 - 6*b*d^4*f^3 + 3*(3*a*b^2*f^7 - b^2*d^2*f^5
)*x)*e^2 - 3*(b^3*d*f^7*x + a*b^2*d*f^7 + b^2*d^3*f^5)*e)*sqrt((b*f^2*x + a
*f^2 + x^2*e^2)/f^2))/(b^6*f^12*x^2 + a^2*b^4*f^12 - 2*a*b^4*d^2*f^10 + b^4
*d^4*f^8 + 64*d^6*x^2*e^6 + 2*(a*b^5*f^12 - b^5*d^2*f^10)*x - 64*(3*b*d^5*f
^2*x^2 + (a*d^5*f^2 - d^7)*x)*e^5 + 16*(15*b^2*d^4*f^4*x^2 + a^2*d^4*f^4 -
2*a*d^6*f^2 + d^8 + 10*(a*b*d^4*f^4 - b*d^6*f^2)*x)*e^4 - 32*(5*b^3*d^3*f^6
*x^2 + a^2*b*d^3*f^6 - 2*a*b*d^5*f^4 + b*d^7*f^2 + 5*(a*b^2*d^3*f^6 - b^2*d
^5*f^4)*x)*e^3 + 4*(15*b^4*d^2*f^8*x^2 + 6*a^2*b^2*d^2*f^8 - 12*a*b^2*d^4*f
^6 + 6*b^2*d^6*f^4 + 20*(a*b^3*d^2*f^8 - b^3*d^4*f^6)*x)*e^2 - 4*(3*b^5*d*f
^10*x^2 + 2*a^2*b^3*d*f^10 - 4*a*b^3*d^3*f^8 + 2*b^3*d^5*f^6 + 5*(a*b^4*d*f
^10 - b^4*d^3*f^8)*x)*e)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3015 vs. 2(319) = 638.
time = 11.77, size = 3015, normalized size = 9.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")

[Out] -4*x*e^3/(b^3*f^6 - 6*b^2*d*f^4*e + 12*b*d^2*f^2*e^2 - 8*d^3*e^3) - 3*(b^3*f^5*abs(f)*e - 2*b^2*d*f^3*abs(f)*e^2 - 4*a*b*f^3*abs(f)*e^3 + 8*a*d*f*abs(f)*e^4)*log(abs((x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b*f^2 - b*d*f^2 + 2*a*f^2*e - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*d*e))/(b^5*f^10 - 10*b^4*d*f^8*e + 40*b^3*d^2*f^6*e^2 - 80*b^2*d^3*f^4*e^3 + 80*b*d^4*f^2*e^4 - 32*d^5*e^5) - 3*(b^2*f^4*e - 4*a*f^2*e^3)*log(abs(-b*f^2*x - a*f^2 + 2*d*x*e + d^2))/(b^4*f^8 - 8*b^3*d*f^6*e + 24*b^2*d^2*f^4*e^2 - 32*b*d^3*f^2*e^3 + 16*d^4*e^4) - 3*(b^2*f^3*abs(f)*e^2 - 4*a*f*abs(f)*e^4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e))/(b^4*f^8*e - 8*b^3*d*f^6*e^2 + 24*b^2*d^2*f^4*e^3 - 32*b*d^3*f^2*e^4 + 16*d^4*e^5) + 3*(b^2*f^3*abs(f)*e - 4*a*f*abs(f)*e^3)*log(abs(-x*e - d + sqrt(b*f^2*x + a*f^2 + x^2*e^2)))/(b^4*f^8 - 8*b^3*d*f^6*e + 24*b^2*d^2*f^4*e^2 - 32*b*d^3*f^2*e^3 + 16*d^4*e^4) + 4*(b^4*f^8*abs(f)*e^2 - 8*b^3*d*f^6*abs(f)*e^3 + 24*b^2*d^2*f^4*abs(f)*e^4 - 32*b*d^3*f^2*abs(f)*e^5 + 16*d^4*abs(f)*e^6)*sqrt(b*f^2*x + a*f^2 + x^2*e^2)/(b^7*f^15 - 14*b^6*d*f^13*e + 84*b^5*d^2*f^11*e^2 - 280*b^4*d^3*f^9*e^3 + 560*b^3*d^4*f^7*e^4 - 672*b^2*d^5*f^5*e^5 + 448*b*d^6*f^3*e^6 - 128*d^7*f*e^7) + 2*((x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*b^5*f^10*abs(f) + 3*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a*b^4*f^10*abs(f)*e + 3*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^2*b^3*f^10*abs(f)*e^2 - 3*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*b^4*d*f^8*abs(f)*e + 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*b^4*d^2*f^8*abs(f)*e - 3*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b^4*d^3*f^8*abs(f)*e - b^4*d^4*f^8*abs(f)*e + a^3*b^2*f^10*abs(f)*e^3 - 7*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*a*b^3*f^8*abs(f)*e^2 - 13*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a*b^3*d*f^8*abs(f)*e^2 + 13*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*b^3*d^2*f^8*abs(f)*e^2 + a*b^3*d^3*f^8*abs(f)*e^2 - 16*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a^2*b^2*f^8*abs(f)*e^3 - 26*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^2*b^2*d*f^8*abs(f)*e^3 + 5*a^2*b^2*d^2*f^8*abs(f)*e^3 + 3*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*b^3*d^2*f^6*abs(f)*e^2 - 11*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*b^3*d^3*f^6*abs(f)*e^2 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b^3*d^4*f^6*abs(f)*e^2 - b^3*d^5*f^6*abs(f)*e^2 - 8*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^3*b*f^8*abs(f)*e^4 - 12*a^3*b*d*f^8*abs(f)*e^4 + 18*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*a*b^2*d*f^6*abs(f)*e^3 + 26*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a*b^2*d^2*f^6*abs(f)*e^3 - 14*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*b^2*d^3*f^6*abs(f)*e^3 + 17*a*b^2*d^4*f^6*abs(f)*e^3 + 12*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*a^2*b*f^6*abs(f)*e^4 + 40*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a^2*b*d*f^6*abs(f)*e^4 + 8*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^2*b*d^2*f^6*abs(f)*e^4 - 44*a^2*b*d^3*f^6*abs(f)*e^4 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*b^2*d^3*f^4*abs(f)*e^3 + 14*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*b^2*d^4*f^4*abs(f)*e^3 - 8*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*b^2*d^5*f^4*abs(f)*e^3 + b^2*d^6*f^4*abs(f)*e^3 + 20*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a^3*f^6*abs(f)*e^5 + 56*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^3*d*f^6*abs(f)*e^5

```
+ 40*a^3*d^2*f^6*abs(f)*e^5 - 12*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*
a*b*d^2*f^4*abs(f)*e^4 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a*b*d^
3*f^4*abs(f)*e^4 + 44*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*b*d^4*f^4*a
bs(f)*e^4 - 8*a*b*d^5*f^4*abs(f)*e^4 - 24*(x*e - sqrt(b*f^2*x + a*f^2 + x^2
*e^2))^3*a^2*d*f^4*abs(f)*e^5 - 76*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^
2*a^2*d^2*f^4*abs(f)*e^5 - 56*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a^2*d
^3*f^4*abs(f)*e^5 + 8*a^2*d^4*f^4*abs(f)*e^5 - 12*(x*e - sqrt(b*f^2*x + a*f
^2 + x^2*e^2))^2*b*d^5*f^2*abs(f)*e^4 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2
*e^2))*b*d^6*f^2*abs(f)*e^4 + 8*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^3*a
*d^3*f^2*abs(f)*e^5 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*a*d^4*f^2
*abs(f)*e^5 - 16*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*a*d^5*f^2*abs(f)*e
^5 + 4*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*d^6*abs(f)*e^5)/((b^4*f^9
- 8*b^3*d*f^7*e + 24*b^2*d^2*f^5*e^2 - 32*b*d^3*f^3*e^3 + 16*d^4*f*e^4)*((x
*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))^2*b*f^2 - b*d^2*f^2 + 2*(x*e - sqrt(b
*f^2*x + a*f^2 + x^2*e^2))*a*f^2*e + 2*a*d*f^2*e - 2*(x*e - sqrt(b*f^2*x +
a*f^2 + x^2*e^2))^2*d*e - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*d^2*e)^
2) + (3*a*b^3*d*f^8 - 3*a^2*b^2*f^8*e - b^3*d^3*f^6 - 6*a*b^2*d^2*f^6*e - 6
*a^2*b*d*f^6*e^2 - 3*b^2*d^4*f^4*e + 10*a^3*f^6*e^3 + 24*a*b*d^3*f^4*e^2 -
6*a^2*d^2*f^4*e^3 + 6*b*d^5*f^2*e^2 - 18*a*d^4*f^2*e^3 - 2*d^6*e^3 + 3*(b^4
*d*f^8 - a*b^3*f^8*e - 3*b^3*d^2*f^6*e - 2*a*b^2*d*f^6*e^2 + 4*a^2*b*f^6*e^
3 + 2*b^2*d^3*f^4*e^2 + 12*a*b*d^2*f^4*e^3 - 8*a^2*d*f^4*e^4 - 8*a*d^3*f^2*
e^4)*x)/((b*f^2*x + a*f^2 - 2*d*x*e - d^2)^2*(b*f^2 - 2*d*e)^4)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)
```

3.479 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx$

Optimal. Leaf size=370

$$\frac{f^2(2de-bf^2)\left(4ae^2-b^2f^2\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4}+\frac{f^2(4ae^2-b^2f^2)\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right.}{12e^3}$$

```
[Out] -5/32*f^2*(-b*f^2+2*d*e)^(3/2)*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(
d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(9/2)*2^(1
/2)+1/12*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e
^3+1/7*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(7/2)/e+1/4*f^2*(-b*f^2+2*d*e)*(
-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^4-1/16*f^2*(-
b*f^2+2*d*e)^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)
/e^4/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.42, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1271, 1824, 214}

$$-\frac{5f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{16\sqrt{2}\,e^{9/2}}+\frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4}-\frac{f^2(4ae^2-b^2f^2)\left(2de-bf^2\right)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2(4ae^2-b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e
^2*x^2)/f^2]])/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x
 + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x
 + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a +
 (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*
f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2
]])/Sqrt[2*d*e - b*f^2]])/(16*Sqrt[2]*e^(9/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2} dx = 2\text{Subst}\left(\int \frac{x^{5/2}(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x,\right.$$

$$= 4\text{Subst}\left(\int \frac{x^6(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx,\right.$$

$$= -\frac{f^2(2de-bf^2)^2\,(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$= -\frac{f^2(2de-bf^2)^2\,(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{16e^4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)}$$

$$= \frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \cdots$$

$$= \frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^4} + \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 985 vs. 2(370) = 740.

time = 5.00, size = 985, normalized size = 2.66

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
```

```
[Out] ((2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(105*b^4*f^8 + 28*b^3*e
*f^6*(-10*d + 3*e*x + 5*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b^2*e^2*f^4*(2
1*d^2 - 119*a*f^2 + 16*e*x*(2*e*x - f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 2*d*
(31*e*x + 49*f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*b*e^3*f^2*(3*d^3 + 79*a
*d*f^2 + 36*d*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 9*d^2*(3*e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*(15*e^3*x^3 - 8*a*f^3*Sqrt[a + x*(b
+ (e^2*x)/f^2)] + 9*e^2*f*x^2*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 16*e^4*(20*
a^2*f^4 + 6*(d + 2*e*x)^3*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(
-3*d^2 + 4*e*x*(19*e*x + 13*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d*(38*e*x
+ 29*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))))/(21*e^4*(b*f^2 + 2*e*(e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)]))) - (20*b^2*d^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(5/
2)*Sqrt[-(d*e) + (b*f^2)/2]) - (80*a*b*d*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d
 + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(3/2)*
Sqrt[-(d*e) + (b*f^2)/2]) + (20*b^3*d*f^6*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d +
e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(7/2)*Sqr
t[-(d*e) + (b*f^2)/2]) + (20*a*b^2*f^6*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x
 + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(5/2)*Sqrt[-
(d*e) + (b*f^2)/2]) - (5*b^4*f^8*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*S
qrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(e^(9/2)*Sqrt[-(d*e)
+ (b*f^2)/2]) + (80*Sqrt[2]*a*d^2*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[e]*Sqrt[-2
*d*e + b*f^2]))/32
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(5/2), x)
```

Fricas [A]
time = 0.55, size = 858, normalized size = 2.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*sqrt(1/2)*(b^3*f^6 - 2*b^2*d*f^4*e - 4*a*b*f^4*e^2 + 8*a*d*f^2*
e^3)*sqrt(-(b*f^2 - 2*d*e)*e^(-1))*log(b^2*f^4 - 4*b*d*f^2*e - 8*d*x*e^3 +
4*(b*f^2*x + a*f^2)*e^2 - 4*(2*sqrt(1/2)*sqrt(-(b*f^2 - 2*d*e)*e^(-1))*f*sq
rt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e^2 - sqrt(1/2)*(b*f^2*e + 2*x*e^3)*sqr
t(-(b*f^2 - 2*d*e)*e^(-1)))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f
^2) + d) - 4*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) +
 2*(105*b^3*f^6 - 280*b^2*d*f^4*e + 192*x^3*e^6 + 288*d*x^2*e^5 + 8*(18*b*f
^2*x^2 + (32*a*f^2 + 39*d^2)*x)*e^4 - 8*(3*b*d*f^2*x - 116*a*d*f^2 - 6*d^3)
*e^3 + 14*(b^2*f^4*x - 24*a*b*f^4 + 6*b*d^2*f^2)*e^2 - 2*(35*b^2*f^5*e - 84
*b*d*f^3*e^2 - 96*f*x^2*e^5 - 144*d*f*x*e^4 - 4*(6*b*f^3*x + 20*a*f^3 - 3*d
^2*f)*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((b*f^2*
x + a*f^2 + x^2*e^2)/f^2) + d))*e^(-4), 1/336*(105*sqrt(1/2)*(b^3*f^6 - 2*b
^2*d*f^4*e - 4*a*b*f^4*e^2 + 8*a*d*f^2*e^3)*sqrt(b*f^2 - 2*d*e)*arctan(-2*s
qrt(1/2)*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d)*e^(1/2)/sq
rt(b*f^2 - 2*d*e))*e^(-1/2) + (105*b^3*f^6 - 280*b^2*d*f^4*e + 192*x^3*e^6
+ 288*d*x^2*e^5 + 8*(18*b*f^2*x^2 + (32*a*f^2 + 39*d^2)*x)*e^4 - 8*(3*b*d*f
^2*x - 116*a*d*f^2 - 6*d^3)*e^3 + 14*(b^2*f^4*x - 24*a*b*f^4 + 6*b*d^2*f^2)
*e^2 - 2*(35*b^2*f^5*e - 84*b*d*f^3*e^2 - 96*f*x^2*e^5 - 144*d*f*x*e^4 - 4*
(6*b*f^3*x + 20*a*f^3 - 3*d^2*f)*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)
)*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))*e^(-4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
```

```
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.480 $\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx$

Optimal. Leaf size=302

$$\frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3}+\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}{5e}-\frac{f^2(2de-bf^2)\,(4ae^2}{8e^3\left(bf^2+2e\right.}$$

```
[Out] -3/16*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^
2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*(-b*f^2+2*d*e)^(1/2)/e^(7/2)*2^(1
/2)+1/5*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2)/e+1/4*f^2*(-b^2*f^2+4*a*e
^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3-1/8*f^2*(-b*f^2+2*d*e)*(-
b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e^3/(b*f^2+2*e*(
e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.30, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1271, 1824, 214}

$$-\frac{3f^2(4ae^2-b^2f^2)\sqrt{2de-bf^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{8\sqrt{2}e^{7/2}}+\frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^3}-\frac{f^2(4ae^2-b^2f^2)(2de-bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{8e^3\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(
4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*
d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) -
(3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqr
t[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt
[2]*e^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2} dx = 2\text{Subst}\left(\int \frac{x^{3/2}(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x,\right.$$

$$= 4\text{Subst}\left(\int \frac{x^4(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx, x\right.$$

$$= -\frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)} -$$

$$= -\frac{f^2(2de-bf^2)\,(4ae^2-b^2f^2)\,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{8e^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)} -$$

$$= \frac{f^2(4ae^2-b^2f^2)\,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\right.}{}$$

$$= \frac{f^2(4ae^2-b^2f^2)\,\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4e^3} + \frac{\left(d+ex+f\right.}{}$$

Mathematica [A]
time = 2.84, size = 443, normalized size = 1.47

$$\frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-15b^3f^6-2b^2ef^4\left(-5d+6ex+10f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4be^2f^2\left(2d^2+17af^2+8ex\left(2ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4d\left(3ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)+8e^3\left(2(d+2ex)^2\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+af^2\left(-d+16ex+12f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)\right)}{40e^3\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)} - \frac{3af^2\sqrt{-de+\frac{bf^2}{2}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{2e^{3/2}} + \frac{3b^2f^4\sqrt{-de+\frac{bf^2}{2}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{8e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-15*b^3*f^6 - 2*b^2*e*f^4
*(-5*d + 6*e*x + 10*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*b*e^2*f^2*(2*d^2 +
 17*a*f^2 + 8*e*x*(2*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*d*(3*e*x +
f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 8*e^3*(2*(d + 2*e*x)^2*(e*x + f*Sqrt[a
+ x*(b + (e^2*x)/f^2)]) + a*f^2*(-d + 16*e*x + 12*f*Sqrt[a + x*(b + (e^2*x)
/f^2)]))))/(40*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) -
 (3*a*f^2*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(2*e^(3/2)) + (3*b^
2*f^4*Sqrt[-(d*e) + (b*f^2)/2]*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqr
t[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(8*e^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(3/2), x)
```

Fricas [A]
time = 0.50, size = 612, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/80*(15*sqrt(1/2)*(b^2*f^4 - 4*a*f^2*e^2)*sqrt(-(b*f^2 - 2*d*e)*e^(-1))*
log(b^2*f^4 - 4*b*d*f^2*e - 8*d*x*e^3 + 4*(b*f^2*x + a*f^2)*e^2 - 4*(2*sqrt
(1/2)*sqrt(-(b*f^2 - 2*d*e)*e^(-1))*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)
*e^2 - sqrt(1/2)*(b*f^2*e + 2*x*e^3)*sqrt(-(b*f^2 - 2*d*e)*e^(-1)))*sqrt(x*
```

```
e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d) - 4*(b*f^3*e - 2*d*f*e^2)*
sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) + 2*(15*b^2*f^4 - 10*b*d*f^2*e - 16*
x^2*e^4 - 36*d*x*e^3 + 2*(b*f^2*x - 24*a*f^2 - 4*d^2)*e^2 - 2*(5*b*f^3*e +
8*f*x*e^3 - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*
sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))*e^(-3), -1/40*(15*sqrt(1/2)*(b^
2*f^4 - 4*a*f^2*e^2)*sqrt(b*f^2 - 2*d*e)*arctan(-2*sqrt(1/2)*sqrt(x*e + f*s
qrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d)*e^(1/2)/sqrt(b*f^2 - 2*d*e))*e^(-
1/2) + (15*b^2*f^4 - 10*b*d*f^2*e - 16*x^2*e^4 - 36*d*x*e^3 + 2*(b*f^2*x -
24*a*f^2 - 4*d^2)*e^2 - 2*(5*b*f^3*e + 8*f*x*e^3 - 2*d*f*e^2)*sqrt((b*f^2*x
 + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2
) + d))*e^(-3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.481 $\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=233

$$\frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e}-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}-\frac{f^2(4ae^2-b^2f^2)\tanh}{}$$

```
[Out] -1/8*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2
/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(5/2)*2^(1/2)/(-b*f^2+2*d*e)^(1/
2)+1/3*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)/e-1/4*f^2*(4*a-b^2*f^2/e^2
)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e
^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.23, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1271, 1167, 214}

$$-\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}-\frac{f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{4\sqrt{2}e^{5/2}\sqrt{2de-bf^2}}+\frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*
f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*
(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*Ar
cTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqr
t[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx = 2\text{Subst}\left(\int \frac{\sqrt{x}\,(d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2)}{(-2de+bf^2+2ex)^2}\,dx, x, d\right.$$

$$= 4\text{Subst}\left(\int \frac{x^2(d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4)}{(-2de+bf^2+2ex^2)^2}\,dx, x,\right.$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \frac{-ef^2}{}\right.}{}$$

$$= -\frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{4\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} - \frac{\text{Subst}\left(\int \left(-4\right.\right.}{}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f}}{4\left(bf^2+2e\left(ex+f\sqrt{a+}\right.\right.}$$

$$= \frac{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2\left(4a-\frac{b^2f^2}{e^2}\right)\sqrt{d+ex+f}}{4\left(bf^2+2e\left(ex+f\sqrt{a+}\right.\right.}$$

Mathematica [A]
time = 1.51, size = 327, normalized size = 1.40

$$\frac{\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(3b^2f^4+4bef^2\left(d+3ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2(d+2ex)\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)\right)}{12e^2\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}+\frac{af^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\sqrt{-2de+bf^2}}-\frac{b^2f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{4e^{5/2}\sqrt{-4de+2bf^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] (Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(3*b^2*f^4 + 4*b*e*f^2*(d
+ 3*e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) + 2*(d + 2*e*x
)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(12*e^2*(b*f^2 + 2*e*(e*x + f*
Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (a*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d +
e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(Sqrt[2]*Sqr
t[e]*Sqrt[-2*d*e + b*f^2]) - (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x
+ f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(4*e^(5/2)*Sqrt[
-4*d*e + 2*b*f^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 0.50, size = 600, normalized size = 2.58

$$\left[\frac{3\,(b^2f^4-4\,af^2e^2)\sqrt{-2\,bf^2e+4\,de^2}\,\log\left(b^2f^4-4\,bdf^2e-8\,dxe^3+4\,(bf^2x+af^2)e^2-2\left(2\,\sqrt{-2\,bf^2e+4\,de^2}\,f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\,e-\sqrt{-2\,bf^2e+4\,de^2}\,(bf^2+2\,xe^2)\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}-4\,(bf^3e-2\,dfe^2)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)+4\left(3\,b^2f^4e-2\,bdf^2e^2-20\,dxe^4+2\,(5\,bf^2x-4\,d^2)e^3-2\,(bf^3e^2-2\,dfe^3)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{48\,(bf^2e^3-2\,de^4)},\ \frac{3\,(b^2f^4-4\,af^2e^2)\sqrt{2\,bf^2e-4\,de^2}\,\arctan\left(-\frac{\sqrt{2\,bf^2e-4\,de^2}\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{bf^2-2\,de}\right)+2\left(3\,b^2f^4e-2\,bdf^2e^2-20\,dxe^4+2\,(5\,bf^2x-4\,d^2)e^3-2\,(bf^3e^2-2\,dfe^3)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{24\,(bf^2e^3-2\,de^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^2*f^4 - 4*a*f^2*e^2)*sqrt(-2*b*f^2*e + 4*d*e^2)*log(b^2*f^4 - 4
*b*d*f^2*e - 8*d*x*e^3 + 4*(b*f^2*x + a*f^2)*e^2 - 2*(2*sqrt(-2*b*f^2*e + 4
*d*e^2)*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - sqrt(-2*b*f^2*e + 4*d*e
```

```
^2)*(b*f^2 + 2*x*e^2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) +
 d) - 4*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) + 4*(3
*b^2*f^4*e - 2*b*d*f^2*e^2 - 20*d*x*e^4 + 2*(5*b*f^2*x - 4*d^2)*e^3 - 2*(b*
f^3*e^2 - 2*d*f*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sq
rt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b*f^2*e^3 - 2*d*e^4), 1/24*(3*(b
^2*f^4 - 4*a*f^2*e^2)*sqrt(2*b*f^2*e - 4*d*e^2)*arctan(-sqrt(2*b*f^2*e - 4*
d*e^2)*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d)/(b*f^2 - 2*d
*e)) + 2*(3*b^2*f^4*e - 2*b*d*f^2*e^2 - 20*d*x*e^4 + 2*(5*b*f^2*x - 4*d^2)*
e^3 - 2*(b*f^3*e^2 - 2*d*f*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt
(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b*f^2*e^3 - 2*d*e^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.482 $\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx$

Optimal. Leaf size=244

$$\frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e}-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}\right)\right)}+\frac{f^2(4ae^2-b^2f}{}$$

```
[Out] 1/4*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/e^(3/2)/(-b*f^2+2*d*e)^(3/2)*2^(1/2
)+(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/e-1/2*f^2*(4*a*e-b^2*f^2/e)*(d+
e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)/(b*f^2+2*e*(e*x+f*(a+
x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]

time = 0.22, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1171, 396, 214}

$$-\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{2\left(2de-bf^2\right)\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{f^2\left(4ae^2-b^2f^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}\left(2de-bf^2\right)^{3/2}}+\frac{\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)
/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*(2*d*e - b*f^2)*(b*
f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b
^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/
f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{\sqrt{x}\,\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+ex+\right.$$

$$= 4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \sqrt{d-}\right.$$

$$= -\frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} + \frac{2\text{Su}}{}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex-}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+\right.\right.}$$

$$= \frac{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{e} - \frac{f^2\left(4ae-\frac{b^2f^2}{e}\right)\sqrt{d+ex-}}{2\left(2de-bf^2\right)\left(bf^2+2e\left(ex+\right.\right.}$$

<u>Mathematica</u> [A]
time = 1.58, size = 341, normalized size = 1.40

$$\frac{\frac{2\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(-b^2f^4-4bef^2\left(-d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+4e^2\left(-af^2+2dex+2df\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{(2de-bf^2)\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}+\frac{4\sqrt{2}\,ae^2f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{3/2}}-\frac{\sqrt{2}\,b^2f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{3/2}}}{4e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]
```

```
[Out] ((2*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]*(-(b^2*f^4) - 4
*b*e*f^2*(-d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 4*e^2*(-(a*f^2) + 2
*d*e*x + 2*d*f*Sqrt[a + x*(b + (e^2*x)/f^2)])))/((2*d*e - b*f^2)*(b*f^2 + 2
*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) + (4*Sqrt[2]*a*e^2*f^2*ArcTan[
(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d
*e + b*f^2]])/(-2*d*e + b*f^2)^(3/2) - (Sqrt[2]*b^2*f^4*ArcTan[(Sqrt[2]*Sqr
t[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]]
)/(-2*d*e + b*f^2)^(3/2))/(4*e^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex+f\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/sqrt(x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d), x)
```

Fricas [A]
time = 0.52, size = 620, normalized size = 2.54

$$\left[-\frac{(b^2f^4-4af^2e^2)\sqrt{-2bf^2e+4de^2}\log\left(b^2f^4-4bdf^2e-8dxe^3+4(bf^2x+af^2)e^2+2\left(2\sqrt{-2bf^2e+4de^2}f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}e-\sqrt{-2bf^2e+4de^2}(bf^2+2xe^2)\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}-4(bf^3e-2dfe^2)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)-4\left(b^2f^4e-6bdf^2e^2+4dxe^4-2(bf^2x-4d^2)e^3+2(bf^3e^2-2dfe^3)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{8(b^2f^4e^2-4bdf^2e^3+4d^2e^4)}, \frac{(b^2f^4-4af^2e^2)\sqrt{2bf^2e-4de^2}\arctan\left(-\frac{\sqrt{2bf^2e-4de^2}\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{bf^2-2de}\right)+2\left(b^2f^4e-6bdf^2e^2+4dxe^4-2(bf^2x-4d^2)e^3+2(bf^3e^2-2dfe^3)\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}\right)\sqrt{xe+f\sqrt{\frac{bf^2x+af^2+x^2e^2}{f^2}}+d}}{4(b^2f^4e^2-4bdf^2e^3+4d^2e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/8*((b^2*f^4 - 4*a*f^2*e^2)*sqrt(-2*b*f^2*e + 4*d*e^2)*log(b^2*f^4 - 4*b
*d*f^2*e - 8*d*x*e^3 + 4*(b*f^2*x + a*f^2)*e^2 + 2*(2*sqrt(-2*b*f^2*e + 4*d
*e^2)*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - sqrt(-2*b*f^2*e + 4*d*e^2
```

```
)*(b*f^2 + 2*x*e^2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d
) - 4*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) - 4*(b^2
*f^4*e - 6*b*d*f^2*e^2 + 4*d*x*e^4 - 2*(b*f^2*x - 4*d^2)*e^3 + 2*(b*f^3*e^2
 - 2*d*f*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((b*f
^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b^2*f^4*e^2 - 4*b*d*f^2*e^3 + 4*d^2*e^4
), 1/4*((b^2*f^4 - 4*a*f^2*e^2)*sqrt(2*b*f^2*e - 4*d*e^2)*arctan(-sqrt(2*b*
f^2*e - 4*d*e^2)*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d)/(b
*f^2 - 2*d*e)) + 2*(b^2*f^4*e - 6*b*d*f^2*e^2 + 4*d*x*e^4 - 2*(b*f^2*x - 4*
d^2)*e^3 + 2*(b*f^3*e^2 - 2*d*f*e^3)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))
*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b^2*f^4*e^2 - 4*
b*d*f^2*e^3 + 4*d^2*e^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d + e\,x + f\,\sqrt{a + b\,x + \frac{e^2\,x^2}{f^2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)
```

3.483 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{4(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}-\frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)}$$

```
[Out] 3/2*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/
f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))/(-b*f^2+2*d*e)^(5/2)*2^(1/2)/e^(1/2
)-4*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(
1/2))^(1/2)-f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2
)/(-b*f^2+2*d*e)^2/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))
```

Rubi [A]
time = 0.28, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1273, 464, 214}

$$-\frac{f^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{(2de-bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}+ex\right)+bf^2\right)}+\frac{3f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(2de-bf^2)^{5/2}}-\frac{4(aef^2-bdf^2+d^2e)}{(2de-bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a
 + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[
a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a
+ (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]
*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^
2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^{3/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+\right.$$

$$= 4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{x^2\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \right.$$

$$= -\frac{f^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^2\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}}\right)\right)} -$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2}{(2de-bf^2)^2}$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{(2de-bf^2)^2\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2}{(2de-bf^2)^2}$$

Mathematica [A]
time = 2.57, size = 395, normalized size = 1.47

$$\frac{b^2f^4\left(5d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)-4bef^2\left(d^2+af^2-2d\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)-4e^2\left(2d^2\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+af^2\left(d+3ex+3f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}{(-2de+bf^2)^2\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}-\frac{6\sqrt{2}\,ae^{3/2}f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{5/2}}+\frac{3b^2f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{\sqrt{2}\sqrt{e}\,(-2de+bf^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2),x]
```

```
[Out] (b^2*f^4*(5*d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - 4*b*e*f^2*(d^2 + a
*f^2 - 2*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 4*e^2*(2*d^2*(e*x + f
*Sqrt[a + x*(b + (e^2*x)/f^2)]) + a*f^2*(d + 3*e*x + 3*f*Sqrt[a + x*(b + (e
^2*x)/f^2)])))/((-2*d*e + b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)
/f^2)]]*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))) - (6*Sqrt[2]
*a*e^(3/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^
2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]])/(-2*d*e + b*f^2)^(5/2) + (3*b^2*f^4*Arc
Tan[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[
-2*d*e + b*f^2]])/(Sqrt[2]*Sqrt[e]*(-2*d*e + b*f^2)^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(242) = 484.
time = 0.65, size = 1312, normalized size = 4.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*(b^3*f^6*x + a*b^2*f^6 - 2*b^2*d*f^4*x*e - b^2*d^2*f^4 + 8*a*d*f^2*
x*e^3 - 4*(a*b*f^4*x + a^2*f^4 - a*d^2*f^2)*e^2)*sqrt(-2*b*f^2*e + 4*d*e^2)
```

```
*log(b^2*f^4 - 4*b*d*f^2*e - 8*d*x*e^3 + 4*(b*f^2*x + a*f^2)*e^2 + 2*(2*sqr
t(-2*b*f^2*e + 4*d*e^2)*f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e - sqrt(-2
*b*f^2*e + 4*d*e^2)*(b*f^2 + 2*x*e^2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 +
 x^2*e^2)/f^2) + d) - 4*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e
^2)/f^2)) + 4*(8*d^2*x^2*e^5 - 4*(2*b*d*f^2*x^2 + (3*a*d*f^2 + d^3)*x)*e^4
+ 2*(b^2*f^4*x^2 - 4*a*d^2*f^2 - 4*d^4 + (3*a*b*f^4 + 7*b*d^2*f^2)*x)*e^3 -
 2*(4*b^2*d*f^4*x - a*b*d*f^4 - 7*b*d^3*f^2)*e^2 + (b^3*f^6*x + a*b^2*f^6 -
 5*b^2*d^2*f^4)*e + 2*(2*b^2*d*f^5*e - 4*d^2*f*x*e^4 + 2*(2*b*d*f^3*x + 3*a
*d*f^3 + d^3*f)*e^3 - (b^2*f^5*x + 3*a*b*f^5 + 5*b*d^2*f^3)*e^2)*sqrt((b*f^
2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/
f^2) + d))/(16*d^4*x*e^5 - 8*(4*b*d^3*f^2*x + a*d^3*f^2 - d^5)*e^4 + 12*(2*
b^2*d^2*f^4*x + a*b*d^2*f^4 - b*d^4*f^2)*e^3 - 2*(4*b^3*d*f^6*x + 3*a*b^2*d
*f^6 - 3*b^2*d^3*f^4)*e^2 + (b^4*f^8*x + a*b^3*f^8 - b^3*d^2*f^6)*e), -1/2*
(3*(b^3*f^6*x + a*b^2*f^6 - 2*b^2*d*f^4*x*e - b^2*d^2*f^4 + 8*a*d*f^2*x*e^3
 - 4*(a*b*f^4*x + a^2*f^4 - a*d^2*f^2)*e^2)*sqrt(2*b*f^2*e - 4*d*e^2)*arcta
n(-sqrt(2*b*f^2*e - 4*d*e^2)*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/
f^2) + d)/(b*f^2 - 2*d*e)) - 2*(8*d^2*x^2*e^5 - 4*(2*b*d*f^2*x^2 + (3*a*d*f
^2 + d^3)*x)*e^4 + 2*(b^2*f^4*x^2 - 4*a*d^2*f^2 - 4*d^4 + (3*a*b*f^4 + 7*b*
d^2*f^2)*x)*e^3 - 2*(4*b^2*d*f^4*x - a*b*d*f^4 - 7*b*d^3*f^2)*e^2 + (b^3*f^
6*x + a*b^2*f^6 - 5*b^2*d^2*f^4)*e + 2*(2*b^2*d*f^5*e - 4*d^2*f*x*e^4 + 2*(
2*b*d*f^3*x + 3*a*d*f^3 + d^3*f)*e^3 - (b^2*f^5*x + 3*a*b*f^5 + 5*b*d^2*f^3
)*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*sqrt(x*e + f*sqrt((b*f^2*x +
a*f^2 + x^2*e^2)/f^2) + d))/(16*d^4*x*e^5 - 8*(4*b*d^3*f^2*x + a*d^3*f^2 -
d^5)*e^4 + 12*(2*b^2*d^2*f^4*x + a*b*d^2*f^4 - b*d^4*f^2)*e^3 - 2*(4*b^3*d*
f^6*x + 3*a*b^2*d*f^6 - 3*b^2*d^3*f^4)*e^2 + (b^4*f^8*x + a*b^3*f^8 - b^3*d
^2*f^6)*e)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)
```

```
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)
```

3.484 $\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx$

Optimal. Leaf size=335

$$-\frac{4(d^2e-bdf^2+aef^2)}{3\left(2de-bf^2\right)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}}-\frac{4f^2(4ae^2-b^2f^2)}{\left(2de-bf^2\right)^3\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}-\frac{2e\ldots}{(2de-\ldots}$$

[Out] 5*f^2*(-b^2*f^2+4*a*e^2)*arctanh(2^(1/2)*e^(1/2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^(1/2))*2^(1/2)*e^(1/2)/(-b*f^2+2*d*e)^(7/2)-4/3*(a*e*f^2-b*d*f^2+d^2*e)/(-b*f^2+2*d*e)^2/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2)-4*f^2*(-b^2*f^2+4*a*e^2)/(-b*f^2+2*d*e)^3/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)-2*e*f^2*(-b^2*f^2+4*a*e^2)*(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2)/(-b*f^2+2*d*e)^3/(b*f^2+2*e*(e*x+f*(a+x*(b*f^2+e^2*x)/f^2)^(1/2)))

Rubi [A]
time = 0.37, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2141, 911, 1273, 1275, 214}

$$-\frac{4f^2(4ae^2-b^2f^2)}{\left(2de-bf^2\right)^3\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}-\frac{2ef^2(4ae^2-b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\left(2de-bf^2\right)^3\left(2e\left(f\sqrt{a+\frac{x\left(bf^2+e^2x\right)}{f^2}}+ex\right)+bf^2\right)}+\frac{5\sqrt{2}\sqrt{e}\,f^2(4ae^2-b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\left(2de-bf^2\right)^{7/2}}-\frac{4(aef^2-bdf^2+d^2e)}{3\left(2de-bf^2\right)^2\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]

[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}}\,dx = 2\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2-(2de-bf^2)\,x+ex^2}{x^{5/2}\left(-2de+bf^2+2ex\right)^2}\,dx, x, d+\right.$$

$$= 4\text{Subst}\left(\int \frac{d^2e-(bd-ae)f^2+(-2de+bf^2)\,x^2+ex^4}{x^4\left(-2de+bf^2+2ex^2\right)^2}\,dx, x, \right.$$

$$= -\frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)} -$$

$$= -\frac{2ef^2(4ae^2-b^2f^2)\sqrt{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}}}{(2de-bf^2)^3\left(bf^2+2e\left(ex+f\sqrt{a+\frac{x\,(bf^2+e^2x)}{f^2}}\right)\right)} -$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{3\,(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{}{(2de-b}$$

$$= -\frac{4(d^2e-bdf^2+aef^2)}{3\,(2de-bf^2)^2\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{}{(2de-b}$$

Mathematica [A]

time = 3.64, size = 557, normalized size = 1.66

$$\frac{2b^3f^6\left(4d+21ex+6f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+2b^2ef^4\left(9d^2+17af^2+14d\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+30ex\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)-8be^2f^2\left(d^3+7adf^2-3d^2\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+5af^2\left(4ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)-8e^3\left(15a^2f^4+2d^3\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+af^2\left(3d^2+20d\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)+30ex\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)\right)}{3\,(2de-bf^2)^3\left(d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)^{3/2}\left(bf^2+2e\left(ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}\right)\right)}+\frac{20\sqrt{2}\,ae^{5/2}f^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}-\frac{5\sqrt{2}\,b^2\sqrt{e}\,f^4\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{e}\,\sqrt{d+ex+f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}}}{\sqrt{-2de+bf^2}}\right)}{(-2de+bf^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
```

```
[Out] (2*b^3*f^6*(4*d + 21*e*x + 6*f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 2*b^2*e*f^4
*(9*d^2 + 17*a*f^2 + 14*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 30*e*x*
(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 8*b*e^2*f^2*(d^3 + 7*a*d*f^2 - 3
*d^2*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 5*a*f^2*(4*e*x + f*Sqrt[a +
x*(b + (e^2*x)/f^2)])) - 8*e^3*(15*a^2*f^4 + 2*d^3*(e*x + f*Sqrt[a + x*(b +
 (e^2*x)/f^2)]) + a*f^2*(3*d^2 + 20*d*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)
]) + 30*e*x*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))))/(3*(2*d*e - b*f^2)^3
*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2)*(b*f^2 + 2*e*(e*x + f*Sq
rt[a + x*(b + (e^2*x)/f^2)]))) + (20*Sqrt[2]*a*e^(5/2)*f^2*ArcTan[(Sqrt[2]*
Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^
2]])/(-2*d*e + b*f^2)^(7/2) - (5*Sqrt[2]*b^2*Sqrt[e]*f^4*ArcTan[(Sqrt[2]*Sq
rt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[-2*d*e + b*f^2]
])/(-2*d*e + b*f^2)^(7/2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^{2}x^{2}}{f^{2}}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

```
[Out] int(1/(d+e*x+f*(a+b*x+1/f^2*e^2*x^2)^(1/2))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1227 vs. 2(302) = 604.
time = 1.49, size = 2523, normalized size = 7.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/6*(15*sqrt(2)*(b^4*f^8*x^2 + a^2*b^2*f^8 - 2*a*b^2*d^2*f^6 + b^2*d^4*f^4
 - 16*a*d^2*f^2*x^2*e^4 + 2*(a*b^3*f^8 - b^3*d^2*f^6)*x + 16*(a*b*d*f^4*x^2
 + (a^2*d*f^4 - a*d^3*f^2)*x)*e^3 - 4*(a^3*f^6 - 2*a^2*d^2*f^4 + a*d^4*f^2
+ (a*b^2*f^6 - b^2*d^2*f^4)*x^2 + 2*(a^2*b*f^6 - a*b*d^2*f^4)*x)*e^2 - 4*(b
^3*d*f^6*x^2 + (a*b^2*d*f^6 - b^2*d^3*f^4)*x)*e)*sqrt(-e/(b*f^2 - 2*d*e))*l
og(b^2*f^4 - 4*b*d*f^2*e - 8*d*x*e^3 + 4*(b*f^2*x + a*f^2)*e^2 - 2*(2*sqrt(
2)*(b*f^3*e - 2*d*f*e^2)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + a*f^2 + x
^2*e^2)/f^2) - sqrt(2)*(b^2*f^4 + 2*b*f^2*x*e^2 - 2*b*d*f^2*e - 4*d*x*e^3)*
sqrt(-e/(b*f^2 - 2*d*e)))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2
) + d) - 4*(b*f^3*e - 2*d*f*e^2)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)) + 4
*(8*b^3*d*f^6*x + 8*a*b^2*d*f^6 - 4*b^2*d^3*f^4 - 24*d^2*x^3*e^5 + 8*(3*b*d
*f^2*x^3 + 2*(5*a*d*f^2 - d^3)*x^2)*e^4 - 2*(3*b^2*f^4*x^3 + 2*(10*a*b*f^4
+ 3*b*d^2*f^2)*x^2 + (15*a^2*f^4 - 46*a*d^2*f^2 - d^4)*x)*e^3 + 4*(b^2*d*f^
4*x^2 - 5*a^2*d*f^4 + 8*a*d^3*f^2 + d^5 - (3*a*b*d*f^4 + 5*b*d^3*f^2)*x)*e^
2 + (3*b^3*f^6*x^2 - 5*a^2*b*f^6 - 2*a*b*d^2*f^4 - 9*b*d^4*f^2 - 2*(a*b^2*f
^6 + 7*b^2*d^2*f^4)*x)*e - 2*(3*b^3*f^7*x + 3*a*b^2*f^7 - b^2*d^2*f^5 - 12*
d^2*f*x^2*e^4 + 4*(3*b*d*f^3*x^2 + 2*(5*a*d*f^3 - d^3*f)*x)*e^3 - (3*b^2*f^
5*x^2 + 20*a*b*f^5*x + 15*a^2*f^5 - 22*a*d^2*f^3 - d^4*f)*e^2 - 2*(2*b^2*d*
f^5*x + a*b*d*f^5 + 3*b*d^3*f^3)*e)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2))*
sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b^5*f^10*x^2 + a^
2*b^3*f^10 - 2*a*b^3*d^2*f^8 + b^3*d^4*f^6 - 32*d^5*x^2*e^5 + 2*(a*b^4*f^10
 - b^4*d^2*f^8)*x + 16*(5*b*d^4*f^2*x^2 + 2*(a*d^4*f^2 - d^6)*x)*e^4 - 8*(1
0*b^2*d^3*f^4*x^2 + a^2*d^3*f^4 - 2*a*d^5*f^2 + d^7 + 8*(a*b*d^3*f^4 - b*d^
5*f^2)*x)*e^3 + 4*(10*b^3*d^2*f^6*x^2 + 3*a^2*b*d^2*f^6 - 6*a*b*d^4*f^4 + 3
*b*d^6*f^2 + 12*(a*b^2*d^2*f^6 - b^2*d^4*f^4)*x)*e^2 - 2*(5*b^4*d*f^8*x^2 +
 3*a^2*b^2*d*f^8 - 6*a*b^2*d^3*f^6 + 3*b^2*d^5*f^4 + 8*(a*b^3*d*f^8 - b^3*d
^3*f^6)*x)*e), 1/3*(15*sqrt(2)*(b^4*f^8*x^2 + a^2*b^2*f^8 - 2*a*b^2*d^2*f^6
 + b^2*d^4*f^4 - 16*a*d^2*f^2*x^2*e^4 + 2*(a*b^3*f^8 - b^3*d^2*f^6)*x + 16*
(a*b*d*f^4*x^2 + (a^2*d*f^4 - a*d^3*f^2)*x)*e^3 - 4*(a^3*f^6 - 2*a^2*d^2*f^
4 + a*d^4*f^2 + (a*b^2*f^6 - b^2*d^2*f^4)*x^2 + 2*(a^2*b*f^6 - a*b*d^2*f^4)
*x)*e^2 - 4*(b^3*d*f^6*x^2 + (a*b^2*d*f^6 - b^2*d^3*f^4)*x)*e)*arctan(-1/2*
(sqrt(2)*(b*f^3 - 2*d*f*e)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2)*e^(1/2)/sq
rt(b*f^2 - 2*d*e) - sqrt(2)*(b*d*f^2 - 2*d*x*e^2 + (b*f^2*x - 2*d^2)*e)*e^(
1/2)/sqrt(b*f^2 - 2*d*e))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2
) + d)/(2*d*x*e^2 - (b*f^2*x + a*f^2 - d^2)*e))*e^(1/2)/sqrt(b*f^2 - 2*d*e)
 + 2*(8*b^3*d*f^6*x + 8*a*b^2*d*f^6 - 4*b^2*d^3*f^4 - 24*d^2*x^3*e^5 + 8*(3
*b*d*f^2*x^3 + 2*(5*a*d*f^2 - d^3)*x^2)*e^4 - 2*(3*b^2*f^4*x^3 + 2*(10*a*b*
f^4 + 3*b*d^2*f^2)*x^2 + (15*a^2*f^4 - 46*a*d^2*f^2 - d^4)*x)*e^3 + 4*(b^2*
d*f^4*x^2 - 5*a^2*d*f^4 + 8*a*d^3*f^2 + d^5 - (3*a*b*d*f^4 + 5*b*d^3*f^2)*x
)*e^2 + (3*b^3*f^6*x^2 - 5*a^2*b*f^6 - 2*a*b*d^2*f^4 - 9*b*d^4*f^2 - 2*(a*b
```

```
^2*f^6 + 7*b^2*d^2*f^4)*x)*e - 2*(3*b^3*f^7*x + 3*a*b^2*f^7 - b^2*d^2*f^5 -
 12*d^2*f*x^2*e^4 + 4*(3*b*d*f^3*x^2 + 2*(5*a*d*f^3 - d^3*f)*x)*e^3 - (3*b^
2*f^5*x^2 + 20*a*b*f^5*x + 15*a^2*f^5 - 22*a*d^2*f^3 - d^4*f)*e^2 - 2*(2*b^
2*d*f^5*x + a*b*d*f^5 + 3*b*d^3*f^3)*e)*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^
2))*sqrt(x*e + f*sqrt((b*f^2*x + a*f^2 + x^2*e^2)/f^2) + d))/(b^5*f^10*x^2
+ a^2*b^3*f^10 - 2*a*b^3*d^2*f^8 + b^3*d^4*f^6 - 32*d^5*x^2*e^5 + 2*(a*b^4*
f^10 - b^4*d^2*f^8)*x + 16*(5*b*d^4*f^2*x^2 + 2*(a*d^4*f^2 - d^6)*x)*e^4 -
8*(10*b^2*d^3*f^4*x^2 + a^2*d^3*f^4 - 2*a*d^5*f^2 + d^7 + 8*(a*b*d^3*f^4 -
b*d^5*f^2)*x)*e^3 + 4*(10*b^3*d^2*f^6*x^2 + 3*a^2*b*d^2*f^6 - 6*a*b*d^4*f^4
 + 3*b*d^6*f^2 + 12*(a*b^2*d^2*f^6 - b^2*d^4*f^4)*x)*e^2 - 2*(5*b^4*d*f^8*x
^2 + 3*a^2*b^2*d*f^8 - 6*a*b^2*d^3*f^6 + 3*b^2*d^5*f^4 + 8*(a*b^3*d*f^8 - b
^3*d^3*f^6)*x)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)
[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")
[Out] integrate((x*e + sqrt(b*x + a + x^2*e^2/f^2)*f + d)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(d+e\,x+f\,\sqrt{a+b\,x+\frac{e^2\,x^2}{f^2}}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)
[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)
```

3.485 $\int (a+x^2)^2 \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=164

$$-\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x+\sqrt{a+x^2}\right)^{1+n}}{16(1+n)}+$$

```
[Out] -1/32*a^5*(x+(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x+(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x+(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x+(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x+(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A]
time = 0.08, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2147, 276}

$$-\frac{a^5\left(\sqrt{a+x^2}+x\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(\sqrt{a+x^2}+x\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(\sqrt{a+x^2}+x\right)^{n+1}}{16(n+1)}+\frac{5a\left(\sqrt{a+x^2}+x\right)^{n+3}}{32(n+3)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/32*(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(5 - n) - (5*a^4*(x + Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^2 \left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{32}\text{Subst}\left(\int x^{-6+n}\left(a+x^2\right)^5 dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{32}\text{Subst}\left(\int \left(a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+x\right.\right.$$
$$= -\frac{a^5\left(x+\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4\left(x+\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3\left(x+\sqrt{a+\right.}{16(1-}$$

Mathematica [A]
time = 0.42, size = 138, normalized size = 0.84

$$\frac{1}{32}\left(x+\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n}+\frac{5a^4\left(x+\sqrt{a+x^2}\right)^2}{-3+n}+\frac{10a^3\left(x+\sqrt{a+x^2}\right)^4}{-1+n}+\frac{10a^2\left(x+\sqrt{a+x^2}\right)^6}{1+n}+\frac{5a\left(x+\sqrt{a+x^2}\right)^8}{3+n}+\frac{\left(x+\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.24, size = 216, normalized size = 1.32

method	result
meijerg	$\frac{2^n x^{5+n}\,\text{hypergeom}\left(\left[-\frac{n}{2},-\frac{5}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2}\right],\left[1-n,-\frac{3}{2}-\frac{n}{2}\right],-\frac{a}{x^2}\right)}{5+n}+\frac{2^{1+n}a\,x^{3+n}\,\text{hypergeom}\left(\left[-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2}\right],\left[1-n,-\frac{1}{2}-\frac{n}{2}\right],-\frac{a}{x^2}\right)}{3+n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n,-5/2-1/2*n,1/2-1/2*n],[1-n,-3/2-1/2*n],
-a/x^2)+2^(1+n)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-
n,-1/2-1/2*n],-a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1
+n)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(
1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1
+n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.38, size = 158, normalized size = 0.96

$$-\frac{\left(5\,(n^4-10\,n^2+9)x^5+10\,(an^4-16\,an^2+15\,a)x^3+5\,(a^2n^4-22\,a^2n^2+45\,a^2)x-(a^2n^5-30\,a^2n^3+(n^5-10\,n^3+9\,n)x^4+149\,a^2n+2\,(an^5-20\,an^3+19\,an)x^2)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2+a\right)}^2\,{\left(x+\sqrt{x^2+a}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)
```

3.486 $\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n\,dx$

Optimal. Leaf size=108

$$-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a\left(x+\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

```
[Out] -1/8*a^3*(x+(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x+(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x+(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x+(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2147, 276}

$$-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n-1}}{8(1-n)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+1}}{8(n+1)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/8*(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(3 - n) - (3*a^2*(x + Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)\left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{8}\text{Subst}\left(\int x^{-4+n}(a+x^2)^3\, dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right) dx, x, x+\sqrt{a+x^2}\right.$$
$$= -\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x+\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x+\sqrt{a+\right.}{8(1+n}$$

Mathematica [A]
time = 0.24, size = 92, normalized size = 0.85

$$\frac{1}{8}\left(x+\sqrt{a+x^2}\right)^{-3+n}\left(\frac{a^3}{-3+n}+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^2}{-1+n}+\frac{3a\left(x+\sqrt{a+x^2}\right)^4}{1+n}+\frac{\left(x+\sqrt{a+x^2}\right)^6}{3+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 3.
time = 0.02, size = 167, normalized size = 1.55

method	result
meijerg	$\frac{2^n x^{3+n}\,\text{hypergeom}\left(\left[-\frac{n}{2},-\frac{3}{2}-\frac{n}{2},\frac{1}{2}-\frac{n}{2}\right],\left[1-n,-\frac{1}{2}-\frac{n}{2}\right],-\frac{a}{x^2}\right)}{3+n}+\frac{a^{\frac{3}{2}+\frac{n}{2}}n\left(\frac{8\sqrt{\pi}\,x^{1+n}a^{-\frac{1}{2}-\frac{n}{2}}\left(\frac{an}{x^2}+n-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+}}{(1+n)n(-2+2n)}\right.}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
```

```
[Out] 2^n/(3+n)*x^(3+n)*hypergeom([-1/2*n,-3/2-1/2*n,1/2-1/2*n],[1-n,-1/2-1/2*n],
-a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/
2*n)*(a/x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x
^(1+n)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)

Fricas [A]
time = 0.35, size = 78, normalized size = 0.72

$$-\frac{\left(3\,(n^2-1)x^3+3\,(an^2-3\,a)x-(an^3+(n^3-n)x^2-7\,an)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^4-10\,n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2149 vs. 2(85) = 170.
time = 15.84, size = 14884, normalized size = 137.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n,x)

[Out] a*Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt
(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/
2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2))
+ a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n*
*2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(
1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sq
rt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(
1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2)
- 2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x
/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a*
*(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*
a**(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamm
a(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1
- n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*

gamma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/
sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma
(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n
/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a
(7/2)*x2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n
*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma
(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2)
- 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sq
rt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) -
2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/
2)*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*as
inh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqr
t(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) -
2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2
)*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh
(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n
/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a
(7/2)*x2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a))
+ asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**
(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**
2*gamma(1 - n/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**
2/a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)
*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*x*c
osh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**
(5/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh
(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n
/2) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh
(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/
2)*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1
- n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)
))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/
2)) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamm
a(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)), Tr
ue)) + Piecewise((2*a**(73/2)*a**(n/2)*n**3*x*cosh(n*asinh(x/sqrt(a)))*gamm
a(-n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2)
+ 18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*
a**(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - 2*
a**(73/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-
n/2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) +
18*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x**2*gamma(1 - n/2) - 20*a**
(69/2)*n**2*x**2*gamma(1 - n/2) + 18*a**(69/2)*x**2*gamma(1 - n/2)) - a**(7

```
1/2)*a**(n/2)*n**4*x**3*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/
2)/(2*a**(71/2)*n**4*gamma(1 - n/2) - 20*a**(71/2)*n**2*gamma(1 - n/2) + 18
*a**(71/2)*gamma(1 - n/2) + 2*a**(69/2)*n**4*x*...
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^2+a\right)\,{\left(x+\sqrt{x^2+a}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)*(x + (a + x^2)^(1/2))^n,x)
[Out] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)
```

3.487 $\int \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=52

$$-\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

```
[Out] -1/2*a*(x+(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x+(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2142, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)}-\frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/2*(a*(x + Sqrt[a + x^2])^(-1 + n))/(1 - n) + (x + Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x+\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)} + \frac{\left(x+\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

Mathematica [A]
time = 0.13, size = 43, normalized size = 0.83

$$\frac{\left(x+\sqrt{a+x^2}\right)^{-1+n}\left(an+(-1+n)x\left(x+\sqrt{a+x^2}\right)\right)}{-1+n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 +
 n^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(44) = 88.
time = 0.01, size = 120, normalized size = 2.31

method	result
meijerg	$\frac{a^{\frac{1}{2}+\frac{n}{2}} n \left(\frac{8\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \left(\frac{an}{x^2}+n-1\right)\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}}{(1+n)n(-2+2n)} + \frac{4\sqrt{\pi}\, x^{1+n} a^{-\frac{1}{2}-\frac{n}{2}} \sqrt{1+\frac{a}{x^2}}\left(\sqrt{1+\frac{a}{x^2}}+1\right)^{-1+n}}{(1+n)n}\right)}{4\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n,x,method=_RETURNVERBOSE)
[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(1+n)/n*x^(1+n)*a^(-1/2-1/2*n)*(a/
x^2*n+n-1)/(-2+2*n)*((1+a/x^2)^(1/2)+1)^(-1+n)+4*Pi^(1/2)/(1+n)/n*x^(1+n)*a
^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(-1+n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.39, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2+a}\,n-x\right)\left(x+\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2147 vs. 2(37) = 74.
time = 1.65, size = 2147, normalized size = 41.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a
)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2)
 + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) +
a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sqrt
(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/s
qrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 -
 n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2
)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*a*
*(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(
1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2
)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*ga
mma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
```

```
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
 2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2
*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)
*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 -
n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*
a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*
x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a)) +
 asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*
gamma(1 - n/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**2/
a)*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n
**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*x*cos
h(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5
/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2
) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh(n
*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)
*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
 n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)
) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)), True
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n,x)
[Out] int((x + (a + x^2)^(1/2))^n, x)
```

3.488 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2} \, dx$

Optimal. Leaf size=59

$$\frac{2\left(x+\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2147, 371}

$$\frac{2\left(\sqrt{a+x^2}+x\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = 2\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{2\left(x+\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.16, size = 61, normalized size = 1.03

$$\frac{2\left(x+\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2),x]
[Out] (2*(x + Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
[Out] integral((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{a+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.489 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2} dx$

Optimal. Leaf size=59

$$\frac{8\left(x+\sqrt{a+x^2}\right)^{3+n} {}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2147, 371}

$$\frac{8\left(\sqrt{a+x^2}+x\right)^{n+3} {}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx = 8\text{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{8\left(x+\sqrt{a+x^2}\right)^{3+n}{}_2F_1\left(3, \frac{3+n}{2}; \frac{5+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Mathematica [**A**]
time = 0.17, size = 61, normalized size = 1.03

$$\frac{8\left(x+\sqrt{a+x^2}\right)^{3+n}{}_2F_1\left(3, \frac{3+n}{2}; 1+\frac{3+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x + Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
[Out] integral((x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**2,x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2,x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.490 $\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=176

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{-1+n}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{1+n}}{16(1+n)}+\frac{5a\cdots}{\cdots}+\cdots$$

```
[Out] -1/32*a^5*(x-(x^2+a)^(1/2))^(-5+n)/(5-n)-5/32*a^4*(x-(x^2+a)^(1/2))^(-3+n)/
(3-n)-5/16*a^3*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+5/16*a^2*(x-(x^2+a)^(1/2))^(1
+n)/(1+n)+5/32*a*(x-(x^2+a)^(1/2))^(3+n)/(3+n)+1/32*(x-(x^2+a)^(1/2))^(5+n)
/(5+n)
```

Rubi [A]
time = 0.07, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 276}

$$-\frac{a^5\left(x-\sqrt{a+x^2}\right)^{n-5}}{32(5-n)}-\frac{5a^4\left(x-\sqrt{a+x^2}\right)^{n-3}}{32(3-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n-1}}{16(1-n)}+\frac{5a^2\left(x-\sqrt{a+x^2}\right)^{n+1}}{16(n+1)}+\frac{5a\left(x-\sqrt{a+x^2}\right)^{n+3}}{32(n+3)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/32*(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(5 - n) - (5*a^4*(x - Sqrt[a + x^2
])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n
)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a +
x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)^2 \left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{32}\text{Subst}\left(\int x^{-6+n}(a+x^2)^5 \, dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{32}\text{Subst}\left(\int (a^5x^{-6+n}+5a^4x^{-4+n}+10a^3x^{-2+n}+10a^2x^n+5ax^{2+n}+$$
$$= -\frac{a^5\left(x-\sqrt{a+x^2}\right)^{-5+n}}{32(5-n)} - \frac{5a^4\left(x-\sqrt{a+x^2}\right)^{-3+n}}{32(3-n)} - \frac{5a^3\left(x-\sqrt{a\right.}}{16(1-}$$

Mathematica [A]
time = 0.45, size = 150, normalized size = 0.85

$$\frac{1}{32}\left(x-\sqrt{a+x^2}\right)^{-5+n}\left(\frac{a^5}{-5+n}+\frac{5a^4\left(x-\sqrt{a+x^2}\right)^2}{-3+n}+\frac{10a^3\left(x-\sqrt{a+x^2}\right)^4}{-1+n}+\frac{10a^2\left(x-\sqrt{a+x^2}\right)^6}{1+n}+\frac{5a\left(x-\sqrt{a+x^2}\right)^8}{3+n}+\frac{\left(x-\sqrt{a+x^2}\right)^{10}}{5+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)
/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a +
 x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2
])^10/(5 + n)))/32
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^2 \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.38, size = 159, normalized size = 0.90

$$-\frac{\left(5\,(n^4-10\,n^2+9)x^5+10\,(an^4-16\,an^2+15\,a)x^3+5\,(a^2n^4-22\,a^2n^2+45\,a^2)x+(a^2n^5-30\,a^2n^3+(n^5-10\,n^3+9\,n)x^4+149\,a^2n+2\,(an^5-20\,an^3+19\,an)x^2)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^6-35\,n^4+259\,n^2-225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4
- 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4
 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(
x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^2\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x-\sqrt{x^2+a}\right)^n\left(x^2+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2,x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)
```

3.491 $\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx$

Optimal. Leaf size=116

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{1+n}}{8(1+n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{3+n}}{8(3+n)}$$

```
[Out] -1/8*a^3*(x-(x^2+a)^(1/2))^(-3+n)/(3-n)-3/8*a^2*(x-(x^2+a)^(1/2))^(-1+n)/(1
-n)+3/8*a*(x-(x^2+a)^(1/2))^(1+n)/(1+n)+1/8*(x-(x^2+a)^(1/2))^(3+n)/(3+n)
```

Rubi [A]
time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2147, 276}

$$-\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-3}}{8(3-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n-1}}{8(1-n)}+\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+1}}{8(n+1)}+\frac{\left(x-\sqrt{a+x^2}\right)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/8*(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(3 - n) - (3*a^2*(x - Sqrt[a + x^2]
)^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (
x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)\left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{8}\text{Subst}\left(\int x^{-4+n}(a+x^2)^3\, dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{8}\text{Subst}\left(\int \left(a^3x^{-4+n}+3a^2x^{-2+n}+3ax^n+x^{2+n}\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-3+n}}{8(3-n)} - \frac{3a^2\left(x-\sqrt{a+x^2}\right)^{-1+n}}{8(1-n)} + \frac{3a\left(x-\sqrt{a+x\right.}}{8(1+n)}$$

Mathematica [A]
time = 0.24, size = 100, normalized size = 0.86

$$\frac{1}{8}\left(x-\sqrt{a+x^2}\right)^{-3+n}\left(\frac{a^3}{-3+n}+\frac{3a^2\left(x-\sqrt{a+x^2}\right)^2}{-1+n}+\frac{3a\left(x-\sqrt{a+x^2}\right)^4}{1+n}+\frac{\left(x-\sqrt{a+x^2}\right)^6}{3+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)
/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3
+ n)))/8
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.35, size = 79, normalized size = 0.68

$$-\frac{\left(3\left(n^2-1\right)x^3+3\left(an^2-3a\right)x+\left(an^3+\left(n^3-n\right)x^2-7an\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^4-10n^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqr
t(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+x^2\right)\left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(x-\sqrt{x^2+a}\right)^n\,\left(x^2+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

3.492 $\int \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=56

$$-\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)}+\frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

```
[Out] -1/2*a*(x-(x^2+a)^(1/2))^(-1+n)/(1-n)+1/2*(x-(x^2+a)^(1/2))^(1+n)/(1+n)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2142, 14}

$$\frac{\left(x-\sqrt{a+x^2}\right)^{n+1}}{2(n+1)}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n,x]
```

```
[Out] -1/2*(a*(x - Sqrt[a + x^2])^(-1 + n))/(1 - n) + (x - Sqrt[a + x^2])^(1 + n)
/(2*(1 + n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(x-\sqrt{a+x^2}\right)^n dx = \frac{1}{2}\text{Subst}\left(\int x^{-2+n}\left(a+x^2\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(ax^{-2+n}+x^n\right) dx, x, x-\sqrt{a+x^2}\right)$$
$$= -\frac{a\left(x-\sqrt{a+x^2}\right)^{-1+n}}{2(1-n)} + \frac{\left(x-\sqrt{a+x^2}\right)^{1+n}}{2(1+n)}$$

Mathematica [A]
time = 0.14, size = 50, normalized size = 0.89

$$\frac{1}{2}\left(x-\sqrt{a+x^2}\right)^{-1+n}\left(\frac{a}{-1+n}+\frac{\left(x-\sqrt{a+x^2}\right)^2}{1+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^(-1 + n)*(a/(-1 + n) + (x - Sqrt[a + x^2])^2/(1 + n)))
/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n,x)
[Out] int((x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.59

$$-\frac{\left(\sqrt{x^2+a}\,n+x\right)\left(x-\sqrt{x^2+a}\right)^n}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x-\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
[Out] Integral((x - sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x-\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n,x)
[Out] int((x - (a + x^2)^(1/2))^n, x)
```

3.493 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2} dx$

Optimal. Leaf size=63

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

```
[Out] 2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(1+n)/a/(1+n)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 371}

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{n+1} {}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2),x]
```

```
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, (3 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{a+x^2}\,dx = 2\text{Subst}\left(\int \frac{x^n}{a+x^2}\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Mathematica [A]
time = 0.16, size = 65, normalized size = 1.03

$$\frac{2\left(x-\sqrt{a+x^2}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};1+\frac{1+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2),x]
[Out] (2*(x - Sqrt[a + x^2])^(1 + n)*Hypergeometric2F1[1, (1 + n)/2, 1 + (1 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a*(1 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="fricas")
[Out] integral((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{a + x^{2}}\right)^{n}}{a + x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(x-\sqrt{x^2+a}\right)}^n}{x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2), x)
```

3.494 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^2}\,dx$

Optimal. Leaf size=63

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}\,{}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

```
[Out] 8*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(
1/2))^(3+n)/a^3/(3+n)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 371}

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{n+3}\,{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
```

```
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, (5 + n)/2, -
((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^2}\,dx = 8\text{Subst}\left(\int \frac{x^{2+n}}{\left(a+x^2\right)^3}\,dx, x, x-\sqrt{a+x^2}\right)$$
$$= \frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}{}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Mathematica [A]
time = 0.17, size = 65, normalized size = 1.03

$$\frac{8\left(x-\sqrt{a+x^2}\right)^{3+n}{}_2F_1\left(3,\frac{3+n}{2};1+\frac{3+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^3(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^2,x]
[Out] (8*(x - Sqrt[a + x^2])^(3 + n)*Hypergeometric2F1[3, (3 + n)/2, 1 + (3 + n)/
2, -((x - Sqrt[a + x^2])^2/a)])/(a^3*(3 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="fricas")
[Out] integral((x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**2,x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^2,x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2,x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^2, x)
```

3.495 $\int (a+x^2)^{5/2} \left(x+\sqrt{a+x^2}\right)^n dx$

Optimal. Leaf size=187

$$-\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}-\frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}-\frac{15a^4\left(x+\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}+\frac{5a^3\left(x+\sqrt{a+x^2}\right)^{n}}{16n}+\frac{1}{}$$

```
[Out] -1/64*a^6*(x+(x^2+a)^(1/2))^(-6+n)/(6-n)-3/32*a^5*(x+(x^2+a)^(1/2))^(-4+n)/
(4-n)-15/64*a^4*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+5/16*a^3*(x+(x^2+a)^(1/2))^n
/n+15/64*a^2*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+3/32*a*(x+(x^2+a)^(1/2))^(4+n)/(
4+n)+1/64*(x+(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A]
time = 0.09, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 276}

$$-\frac{a^6\left(\sqrt{a+x^2}+x\right)^{n-6}}{64(6-n)}-\frac{3a^5\left(\sqrt{a+x^2}+x\right)^{n-4}}{32(4-n)}-\frac{15a^4\left(\sqrt{a+x^2}+x\right)^{n-2}}{64(2-n)}+\frac{5a^3\left(\sqrt{a+x^2}+x\right)^{n}}{16n}+\frac{15a^2\left(\sqrt{a+x^2}+x\right)^{n+2}}{64(n+2)}+\frac{3a\left(\sqrt{a+x^2}+x\right)^{n+4}}{32(n+4)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/64*(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(6 - n) - (3*a^5*(x + Sqrt[a + x^2
])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 -
n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2
 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x +
 Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)^{5/2}\left(x+\sqrt{a+x^2}\right)^n dx = \frac{1}{64}\text{Subst}\left(\int x^{-7+n}(a+x^2)^6 dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{64}\text{Subst}\left(\int (a^6x^{-7+n}+6a^5x^{-5+n}+15a^4x^{-3+n}+20a^3x^{-1+n}+15a^2x$$
$$= -\frac{a^6\left(x+\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} - \frac{3a^5\left(x+\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} - \frac{15a^4\left(x+\sqrt{a}\right.}{64(2}$$

Mathematica [A]
time = 0.38, size = 157, normalized size = 0.84

$$\frac{1}{64}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{20a^3}{n}+\frac{a^6}{(-6+n)\left(x+\sqrt{a+x^2}\right)^6}+\frac{6a^5}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4}+\frac{15a^4}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{15a^2\left(x+\sqrt{a+x^2}\right)^2}{2+n}+\frac{6a\left(x+\sqrt{a+x^2}\right)^4}{4+n}+\frac{\left(x+\sqrt{a+x^2}\right)^6}{6+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) +
 (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a
+ x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^
2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}}\left(x+\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.39, size = 201, normalized size = 1.07

$$\frac{\left(a^3n^6 - 50\,a^3n^4 + (n^6 - 20\,n^4 + 64\,n^2)x^6 + 544\,a^3n^2 + 3\,(an^6 - 30\,an^4 + 104\,an^2)x^4 - 720\,a^3 + 3\,(a^2n^6 - 40\,a^2n^4 + 264\,a^2n^2)x^2 - 6\,((n^5 - 20\,n^3 + 64\,n)x^5 + 2\,(an^5 - 30\,an^3 + 104\,an)x^3 + (a^2n^5 - 40\,a^2n^3 + 264\,a^2n)x)\sqrt{x^2+a}\right)\left(x + \sqrt{x^2+a}\right)^n}{n^7 - 56\,n^5 + 784\,n^3 - 2304\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^
6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2
*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x
^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a
))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2+a\right)}^{5/2}\,{\left(x+\sqrt{x^2+a}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.496 $\int (a+x^2)^{3/2} \left(x+\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=131

$$-\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{3a^2\left(x+\sqrt{a+x^2}\right)^{n}}{8n}+\frac{a\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}+\frac{\left(x+\sqrt{a\right.}{16(}$$

```
[Out] -1/16*a^4*(x+(x^2+a)^(1/2))^(-4+n)/(4-n)-1/4*a^3*(x+(x^2+a)^(1/2))^(-2+n)/(
2-n)+3/8*a^2*(x+(x^2+a)^(1/2))^n/n+1/4*a*(x+(x^2+a)^(1/2))^(2+n)/(2+n)+1/16
*(x+(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A]
time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 276}

$$-\frac{a^4\left(\sqrt{a+x^2}+x\right)^{n-4}}{16(4-n)}-\frac{a^3\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{3a^2\left(\sqrt{a+x^2}+x\right)^{n}}{8n}+\frac{a\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/16*(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(4 - n) - (a^3*(x + Sqrt[a + x^2])
^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt
[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+x^2\right)^{3/2}\left(x+\sqrt{a+x^2}\right)^n\,dx=\frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4\,dx,x,x+\sqrt{a+x^2}\right)$$
$$=\frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right)\,dx$$
$$=-\frac{a^4\left(x+\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}-\frac{a^3\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{3a^2\left(x+\sqrt{a\right.}}{8n}$$

Mathematica [A]
time = 0.22, size = 111, normalized size = 0.85

$$\frac{1}{16}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{6a^2}{n}+\frac{a^4}{(-4+n)\left(x+\sqrt{a+x^2}\right)^4}+\frac{4a^3}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{4a\left(x+\sqrt{a+x^2}\right)^2}{2+n}+\frac{\left(x+\sqrt{a+x^2}\right)^4}{4+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]
[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) +
(4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 +
 n) + (x + Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}}\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.36, size = 110, normalized size = 0.84

$$\frac{\left(a^2n^4+(n^4-4\,n^2)x^4-16\,a^2n^2+2\,(an^4-10\,an^2)x^2+24\,a^2-4\,((n^3-4\,n)x^3+(an^3-10\,an)x)\sqrt{x^2+a}\right)\left(x+\sqrt{x^2+a}\right)^n}{n^5-20\,n^3+64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a
^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2
+ a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+x^2\right)^{\frac{3}{2}}\left(x+\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x^2+a\right)}^{3/2}\,{\left(x+\sqrt{x^2+a}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.497 $\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n dx$

Optimal. Leaf size=75

$$-\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

```
[Out] -1/4*a^2*(x+(x^2+a)^(1/2))^(-2+n)/(2-n)+1/2*a*(x+(x^2+a)^(1/2))^n/n+1/4*(x+
(x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 276}

$$-\frac{a^2\left(\sqrt{a+x^2}+x\right)^{n-2}}{4(2-n)}+\frac{a\left(\sqrt{a+x^2}+x\right)^n}{2n}+\frac{\left(\sqrt{a+x^2}+x\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] -1/4*(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(2 - n) + (a*(x + Sqrt[a + x^2])^n)
/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx = \frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right)\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= -\frac{a^2\left(x+\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}+\frac{a\left(x+\sqrt{a+x^2}\right)^n}{2n}+\frac{\left(x+\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

Mathematica [A]
time = 0.18, size = 65, normalized size = 0.87

$$\frac{1}{4}\left(x+\sqrt{a+x^2}\right)^n\left(\frac{2a}{n}+\frac{a^2}{(-2+n)\left(x+\sqrt{a+x^2}\right)^2}+\frac{\left(x+\sqrt{a+x^2}\right)^2}{2+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x
 + Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.36, size = 48, normalized size = 0.64

$$\frac{\left(n^2x^2 + an^2 - 2\sqrt{x^2+a}\,nx - 2a\right)\left(x + \sqrt{x^2+a}\,\right)^n}{n^3 - 4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 -
4*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2}\left(x+\sqrt{a+x^2}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2+a}\,\left(x+\sqrt{x^2+a}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n,x)
```

```
[Out] int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)
```

3.498 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx$

Optimal. Leaf size=17

$$\frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

[Out] (x+(x^2+a)^(1/2))^n/n

Rubi [A]
time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 30}

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^n/n

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}} \, dx &= \text{Subst}\left(\int x^{-1+n} \, dx, x, x+\sqrt{a+x^2}\right)\\ &= \frac{\left(x+\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\left(x+\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] (x + Sqrt[a + x^2])^n/n
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Fricas [A]
time = 0.35, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] (x + sqrt(x^2 + a))^n/n
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(12) = 24$.

time = 1.54, size = 311, normalized size = 18.29

$$\begin{cases} \frac{\sqrt{a}\, a^{\frac{n}{2}} \sinh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{nx\sqrt{\frac{a}{x^2}+1}} - \frac{2a^{\frac{n}{2}} \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}} x \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, n} + \frac{a^{\frac{n}{2}} x \sinh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, n\sqrt{\frac{a}{x^2}+1}} & \text{for } \left|\frac{x^2}{a}\right| > 1 \\ \frac{a^{\frac{n}{2}} \sinh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n\sqrt{1+\frac{x^2}{a}}} - \frac{2a^{\frac{n}{2}} \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)\Gamma\left(1-\frac{n}{2}\right)}{n^2\Gamma\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}} x^2 \sinh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{an\sqrt{1+\frac{x^2}{a}}} + \frac{a^{\frac{n}{2}} x \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) - \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a}\, n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a)*a**(n/2)*sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(n*
x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/(n
**2*gamma(-n/2)) + a**(n/2)*x*cosh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(
sqrt(a)*n) + a**(n/2)*x*sinh(n*asinh(x/sqrt(a)) - asinh(x/sqrt(a)))/(sqrt(a
)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (a**(n/2)*sinh(n*asinh(x/sqrt(a))
- asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a
)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) + a**(n/2)*x**2*sinh(n*asinh(x/sqrt(a
)) - asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(n*asinh(x/s
qrt(a)) - asinh(x/sqrt(a)))/(sqrt(a)*n), True))
```

Giac [A]

time = 3.35, size = 15, normalized size = 0.88

$$\frac{\left(x + \sqrt{x^2 + a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (x + sqrt(x^2 + a))^n/n
```

Mupad [B]

time = 3.00, size = 15, normalized size = 0.88

$$\frac{\left(x + \sqrt{x^2 + a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
```

```
[Out] (x + (a + x^2)^(1/2))^n/n
```

3.499 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{4\left(x+\sqrt{a+x^2}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

```
[Out] 4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2)
)^(2+n)/a^2/(2+n)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2147, 371}

$$\frac{4\left(\sqrt{a+x^2}+x\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -
((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx = 4\text{Subst}\left(\int \frac{x^{1+n}}{(a+x^2)^2}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{4\left(x+\sqrt{a+x^2}\right)^{2+n} {}_2F_1\left(2, \frac{2+n}{2}; \frac{4+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

Mathematica [**A**]
time = 0.07, size = 61, normalized size = 1.03

$$\frac{4\left(x+\sqrt{a+x^2}\right)^{2+n} {}_2F_1\left(2, \frac{2+n}{2}; 1+\frac{2+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (4*(x + Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)/
2, -((x + Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

```
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(x+\sqrt{x^2+a}\right)}^n}{{\left(x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.500 $\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=59

$$\frac{16\left(x+\sqrt{a+x^2}\right)^{4+n}{}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[Out] 16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x+(x^2+a)^(1/2))^2/a)*(x+(x^2+a)^(1/2
))^(4+n)/a^4/(4+n)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2147, 371}

$$\frac{16\left(\sqrt{a+x^2}+x\right)^{n+4}{}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x+\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
-((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = 16\text{Subst}\left(\int \frac{x^{3+n}}{(a+x^2)^4}\,dx, x, x+\sqrt{a+x^2}\right)$$
$$= \frac{16\left(x+\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4, \frac{4+n}{2}; \frac{6+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

Mathematica [A]
time = 0.09, size = 61, normalized size = 1.03

$$\frac{16\left(x+\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4, \frac{4+n}{2}; 1+\frac{4+n}{2}; -\frac{\left(x+\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
[Out] (16*(x + Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n)
/2, -((x + Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a
^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{a+x^2}\right)^n}{(a+x^2)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
[Out] Integral((x + sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((x + sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{(x^2+a)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x + (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.501 $\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=201

$$\frac{a^6\left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)}+\frac{3a^5\left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{-2+n}}{64(2-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n}}{16n}-15$$

```
[Out] 1/64*a^6*(x-(x^2+a)^(1/2))^(-6+n)/(6-n)+3/32*a^5*(x-(x^2+a)^(1/2))^(-4+n)/(
4-n)+15/64*a^4*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-5/16*a^3*(x-(x^2+a)^(1/2))^n/
n-15/64*a^2*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-3/32*a*(x-(x^2+a)^(1/2))^(4+n)/(4
+n)-1/64*(x-(x^2+a)^(1/2))^(6+n)/(6+n)
```

Rubi [A]
time = 0.08, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 276}

$$\frac{a^6\left(x-\sqrt{a+x^2}\right)^{n-6}}{64(6-n)}+\frac{3a^5\left(x-\sqrt{a+x^2}\right)^{n-4}}{32(4-n)}+\frac{15a^4\left(x-\sqrt{a+x^2}\right)^{n-2}}{64(2-n)}-\frac{5a^3\left(x-\sqrt{a+x^2}\right)^{n}}{16n}-\frac{15a^2\left(x-\sqrt{a+x^2}\right)^{n+2}}{64(n+2)}-\frac{3a\left(x-\sqrt{a+x^2}\right)^{n+4}}{32(n+4)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2]
)^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n
)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2
+ n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x -
Sqrt[a + x^2])^(6 + n)/(64*(6 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)^{5/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = -\left(\frac{1}{64}\text{Subst}\left(\int x^{-7+n}(a+x^2)^6 \, dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\left(\frac{1}{64}\text{Subst}\left(\int (a^6 x^{-7+n} + 6a^5 x^{-5+n} + 15a^4 x^{-3+n} + 20a^3 x^{-1+n} + 15\right.\right.$$
$$= \frac{a^6\left(x-\sqrt{a+x^2}\right)^{-6+n}}{64(6-n)} + \frac{3a^5\left(x-\sqrt{a+x^2}\right)^{-4+n}}{32(4-n)} + \frac{15a^4\left(x-\sqrt{a-\right.}}{64(2-}$$

Mathematica [A]
time = 0.45, size = 173, normalized size = 0.86

$$\frac{1}{64}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{20a^3}{n}-\frac{a^6}{(-6+n)\left(x-\sqrt{a+x^2}\right)^6}-\frac{6a^5}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4}-\frac{15a^4}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{15a^2\left(x-\sqrt{a+x^2}\right)^2}{2+n}-\frac{6a\left(x-\sqrt{a+x^2}\right)^4}{4+n}-\frac{\left(x-\sqrt{a+x^2}\right)^6}{6+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2])^6)
- (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a
 + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x
^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n)))/64
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{5}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.37, size = 204, normalized size = 1.01

$$-\frac{\left(a^3n^6-50\,a^3n^4+\left(n^6-20\,n^4+64\,n^2\right)x^6+544\,a^3n^2+3\,\left(an^6-30\,an^4+104\,an^2\right)x^4-720\,a^3+3\,\left(a^2n^6-40\,a^2n^4+264\,a^2n^2\right)x^2+6\,\left(\left(n^5-20\,n^3+64\,n\right)x^5+2\,\left(an^5-30\,an^3+104\,an\right)x^3+\left(a^2n^5-40\,a^2n^3+264\,a^2n\right)x\right)\sqrt{x^2+a}\right)\left(x-\sqrt{x^2+a}\right)^n}{n^7-56\,n^5+784\,n^3-2304\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n
^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^
2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*
x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 +
a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x-\sqrt{x^2+a}\right)^n\,{\left(x^2+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)
```

3.502 $\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx$

Optimal. Leaf size=141

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}-\frac{\left(x-\sqrt{a+x^2}\right)^{4+n}}{16(4+n)}$$

```
[Out] 1/16*a^4*(x-(x^2+a)^(1/2))^(-4+n)/(4-n)+1/4*a^3*(x-(x^2+a)^(1/2))^(-2+n)/(2
-n)-3/8*a^2*(x-(x^2+a)^(1/2))^n/n-1/4*a*(x-(x^2+a)^(1/2))^(2+n)/(2+n)-1/16*
(x-(x^2+a)^(1/2))^(4+n)/(4+n)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 276}

$$\frac{a^4\left(x-\sqrt{a+x^2}\right)^{n-4}}{16(4-n)}+\frac{a^3\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{3a^2\left(x-\sqrt{a+x^2}\right)^{n}}{8n}-\frac{a\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^
(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[
a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int (a+x^2)^{3/2} \left(x-\sqrt{a+x^2}\right)^n \, dx = -\left(\frac{1}{16}\text{Subst}\left(\int x^{-5+n}\left(a+x^2\right)^4 \, dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\left(\frac{1}{16}\text{Subst}\left(\int \left(a^4x^{-5+n}+4a^3x^{-3+n}+6a^2x^{-1+n}+4ax^{1+n}+x^{3+n}\right.\right.\right.$$
$$= \frac{a^4\left(x-\sqrt{a+x^2}\right)^{-4+n}}{16(4-n)} + \frac{a^3\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} - \frac{3a^2\left(x-\sqrt{a+\right.}}{8n}$$

Mathematica [A]
time = 0.24, size = 123, normalized size = 0.87

$$\frac{1}{16}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{6a^2}{n}-\frac{a^4}{(-4+n)\left(x-\sqrt{a+x^2}\right)^4}-\frac{4a^3}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{4a\left(x-\sqrt{a+x^2}\right)^2}{2+n}-\frac{\left(x-\sqrt{a+x^2}\right)^4}{4+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) -
 (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2
+ n) - (x - Sqrt[a + x^2])^4/(4 + n)))/16
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(x^2+a\right)^{\frac{3}{2}} \left(x-\sqrt{x^2+a}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.37, size = 113, normalized size = 0.80

$$-\frac{\left(a^2n^4 + (n^4 - 4\,n^2)x^4 - 16\,a^2n^2 + 2\,(an^4 - 10\,an^2)x^2 + 24\,a^2 + 4\,((n^3 - 4\,n)x^3 + (an^3 - 10\,an)x)\sqrt{x^2 + a}\right)\left(x - \sqrt{x^2 + a}\right)^n}{n^5 - 20\,n^3 + 64\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*
a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2
 + a))^n/(n^5 - 20*n^3 + 64*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + x^2\right)^{\frac{3}{2}} \left(x - \sqrt{a + x^2}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x-\sqrt{x^2+a}\right)}^n\,{\left(x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2),x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

3.503 $\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx$

Optimal. Leaf size=81

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

```
[Out] 1/4*a^2*(x-(x^2+a)^(1/2))^(-2+n)/(2-n)-1/2*a*(x-(x^2+a)^(1/2))^n/n-1/4*(x-(
x^2+a)^(1/2))^(2+n)/(2+n)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 276}

$$\frac{a^2\left(x-\sqrt{a+x^2}\right)^{n-2}}{4(2-n)}-\frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n}-\frac{\left(x-\sqrt{a+x^2}\right)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/
(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+x^2}\left(x-\sqrt{a+x^2}\right)^n dx = -\left(\frac{1}{4}\text{Subst}\left(\int x^{-3+n}\left(a+x^2\right)^2 dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\left(\frac{1}{4}\text{Subst}\left(\int \left(a^2x^{-3+n}+2ax^{-1+n}+x^{1+n}\right) dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= \frac{a^2\left(x-\sqrt{a+x^2}\right)^{-2+n}}{4(2-n)} - \frac{a\left(x-\sqrt{a+x^2}\right)^n}{2n} - \frac{\left(x-\sqrt{a+x^2}\right)^{2+n}}{4(2+n)}$$

Mathematica [A]
time = 0.19, size = 73, normalized size = 0.90

$$\frac{1}{4}\left(x-\sqrt{a+x^2}\right)^n\left(-\frac{2a}{n}-\frac{a^2}{(-2+n)\left(x-\sqrt{a+x^2}\right)^2}-\frac{\left(x-\sqrt{a+x^2}\right)^2}{2+n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]
[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (
x - Sqrt[a + x^2])^2/(2 + n)))/4
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a}\,\left(x-\sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Fricas [A]
time = 0.38, size = 51, normalized size = 0.63

$$-\frac{\left(n^{2} x^{2}+a n^{2}+2 \sqrt{x^{2}+a}\, n x-2\, a\right) \left(x-\sqrt{x^{2}+a}\right)^{n}}{n^{3}-4\, n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 -
 4*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^{2}} \left(x-\sqrt{a+x^{2}}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n,x)
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(x-\sqrt{x^2+a}\right)}^n\,\sqrt{x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2),x)
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

3.504 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

```
[Out] -(x-(x^2+a)^(1/2))^n/n
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2147, 30}

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
```

```
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\sqrt{a+x^2}}\,dx &= -\text{Subst}\left(\int x^{-1+n}\,dx, x, x-\sqrt{a+x^2}\right)\\ &= -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 20, normalized size = 1.00

$$-\frac{\left(x-\sqrt{a+x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]
[Out] -((x - Sqrt[a + x^2])^n/n)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")
[Out] -(x - sqrt(x^2 + a))^n/n
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.96, size = 36, normalized size = 1.80

$$\begin{cases} -\frac{\left(x-\sqrt{a+x^2}\right)^n}{n} & \text{for } n \neq 0 \\ \begin{cases} \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-(x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asinh(x*sqrt(
1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))
```

Giac [A]
time = 3.55, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(x - sqrt(x^2 + a))^n/n
```

Mupad [B]
time = 3.19, size = 18, normalized size = 0.90

$$-\frac{\left(x-\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2),x)
```

```
[Out] -(x - (a + x^2)^(1/2))^n/n
```

3.505 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{3/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

[Out] -4*hypergeom([2, 1+1/2*n],[2+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/2))^(2+n)/a^2/(2+n)

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 371}

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{n+2}\,{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^2(n+2)}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]

[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{\left(a+x^2\right)^{3/2}}\,dx = -\left(4\text{Subst}\left(\int \frac{x^{1+n}}{\left(a+x^2\right)^2}\,dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

Mathematica [**A**]
time = 0.08, size = 65, normalized size = 1.03

$$-\frac{4\left(x-\sqrt{a+x^2}\right)^{2+n}\,{}_2F_1\left(2,\frac{2+n}{2};1+\frac{2+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(3/2),x]
```

```
[Out] (-4*(x - Sqrt[a + x^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, 1 + (2 + n)
/2, -((x - Sqrt[a + x^2])^2/a)])/(a^2*(2 + n))
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{\left(x^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^4 + 2*a*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(3/2),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(3/2), x)
```

3.506 $\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx$

Optimal. Leaf size=63

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

```
[Out] -16*hypergeom([4, 2+1/2*n],[3+1/2*n],-(x-(x^2+a)^(1/2))^2/a)*(x-(x^2+a)^(1/
2))^(4+n)/a^4/(4+n)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2147, 371}

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{n+4} {}_2F_1\left(4,\frac{n+4}{2};\frac{n+6}{2};-\frac{\left(x-\sqrt{x^2+a}\right)^2}{a}\right)}{a^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, (6 + n)/2,
 -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2147

```
Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_
.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(1/(2^(2*m + 1)*e*f^(2*m)))*(i/c)^m, S
ubst[Int[x^n*((d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1)/(-d + x)^(2*(m + 1))),
x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n},
x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (Integer
Q[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(x-\sqrt{a+x^2}\right)^n}{(a+x^2)^{5/2}}\,dx = -\left(16\text{Subst}\left(\int \frac{x^{3+n}}{(a+x^2)^4}\,dx, x, x-\sqrt{a+x^2}\right)\right)$$
$$= -\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4,\frac{4+n}{2};\frac{6+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

Mathematica [A]
time = 0.09, size = 65, normalized size = 1.03

$$-\frac{16\left(x-\sqrt{a+x^2}\right)^{4+n} {}_2F_1\left(4,\frac{4+n}{2};1+\frac{4+n}{2};-\frac{\left(x-\sqrt{a+x^2}\right)^2}{a}\right)}{a^4(4+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[a + x^2])^n/(a + x^2)^(5/2),x]
```

```
[Out] (-16*(x - Sqrt[a + x^2])^(4 + n)*Hypergeometric2F1[4, (4 + n)/2, 1 + (4 + n
)/2, -((x - Sqrt[a + x^2])^2/a)])/(a^4*(4 + n))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(x-\sqrt{x^2+a}\right)^n}{(x^2+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

```
[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(x^6 + 3*a*x^4 + 3*a^2*x^2 + a
^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{a + x^2}\right)^n}{\left(a + x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(5/2),x)
[Out] Integral((x - sqrt(a + x**2))**n/(a + x**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((x - sqrt(x^2 + a))^n/(x^2 + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\left(x^2 + a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2),x)
[Out] int((x - (a + x^2)^(1/2))^n/(a + x^2)^(5/2), x)
```

3.507 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.$

Optimal. Leaf size=365

$$\frac{(d^2-af^2)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-5+n}}{32ef^4(5-n)}-\frac{5(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{32ef^4(3-n)}$$

```
[Out] 1/32*(-a*f^2+d^2)^5*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-5+n)/e/f^
4/(5-n)-5/32*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3
+n)/e/f^4/(3-n)+5/16*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^(-1+n)/e/f^4/(1-n)+5/16*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^(1+n)/e/f^4/(1+n)-5/32*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^
2*x^2/f^2)^(1/2))^(3+n)/e/f^4/(3+n)+1/32*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^
2)^(1/2))^(5+n)/e/f^4/(5+n)
```

Rubi [A]
time = 0.33, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2146, 12, 276}

$$\frac{(d^2-af^2)^5\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-5}}{32ef^4(5-n)}-\frac{5(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{32ef^4(3-n)}+\frac{5(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{16ef^4(1-n)}+\frac{5(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{16ef^4(n+1)}-\frac{5(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{32ef^4(n+3)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+5}}{32ef^4(n+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5
+ n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(
1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2
)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a
+ (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x +
f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\text{Subst}\left(\int \frac{x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^5}{64e^6}\,d\ldots\right.}{f^4\ldots}$$

$$= \frac{\text{Subst}\left(\int x^{-6+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2\ldots\right.\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int \left(-e^5\left(d^2-af^2\right)^5 x^{-6+n}+5e^5(\ldots\right.\right.}{\ldots}$$

$$= \frac{\left(d^2-af^2\right)^5\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\ldots}\right.}{32ef^4(5-n)}$$

Mathematica [A]

time = 6.92, size = 280, normalized size = 0.77

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-5+n}\left(-\frac{\left(d^2-af^2\right)^5}{-5+n}+\frac{5\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-3+n}-\frac{10\left(d^2-af^2\right)^3\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}{-1+n}+\frac{10\left(d^2-af^2\right)^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^6}{1+n}-\frac{5\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^8}{3+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{10}}{5+n}\right)}{32ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*
e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n)*(-((d^2 - a*f^2)^5/
(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
```

```
)^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
 e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/
(5 + n)))/(32*e*f^4)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2 \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^2*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^
(1/2))^n,x)
```

```
[Out] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^2*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^
(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^2, x)
```

Fricas [A]
time = 0.43, size = 570, normalized size = 1.56

$$-\frac{\left(5\,a^2df^4n^4 + 225\,a^2df^4 + 5\,(n^4 - 10\,n^2 + 9)x^5e^5 - 300\,ad^3f^2 + 25\,(dn^4 - 10\,dn^2 + 9\,d)x^4e^4 + 120\,d^5 + 10\,((af^2 + 4\,d^2)n^4 + 15\,af^2 - 2\,(8\,af^2 + 17\,d^2)n^2 + 30\,d^2)x^3e^3 + 10\,((3\,adf^2 + 2\,d^3)n^4 + 45\,adf^2 - 2\,(24\,adf^2 + d^3)n^2)x^2e^2 - 10\,(11\,a^2df^4 - 6\,ad^3f^2)n^2 + 5\,(45\,a^2f^4 + (a^2f^4 + 4\,ad^2f^2)n^4 - 2\,(11\,a^2f^4 + 26\,ad^2f^2 - 12\,d^4)n^2)xe - (a^2f^5n^5 + (fn^5 - 10\,fn^3 + 9\,fn)x^4e^4 + 4\,(dfn^5 - 10\,dfn^3 + 9\,dfn)x^3e^3 - 10\,(3\,a^2f^5 - 2\,ad^2f^3)n^3 + 2\,((af^3 + 2\,d^2f)n^5 - 10\,(2\,af^3 + d^2f)n^3 + (19\,af^3 + 8\,d^2f)n)x^2e^2 + 4\,(adf^3n^5 - 10\,(2\,adf^3 - d^3f)n^3 + (19\,adf^3 - 10\,d^3f)n)xe + (149\,a^2f^5 - 260\,ad^2f^3 + 120\,d^4f)n)\sqrt{\frac{af^2 + x^2e^2 + 2\,dxe}{f^2}}\right)\left(xe + f\sqrt{\frac{af^2 + x^2e^2 + 2\,dxe}{f^2}} + d\right)^n e^{(-1)}}{f^4n^6 - 35\,f^4n^4 + 259\,f^4n^2 - 225\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 + 5*(n^4 - 10*n^2 + 9)*x^5*e^5 - 300*a*d^
3*f^2 + 25*(d*n^4 - 10*d*n^2 + 9*d)*x^4*e^4 + 120*d^5 + 10*((a*f^2 + 4*d^2)
*n^4 + 15*a*f^2 - 2*(8*a*f^2 + 17*d^2)*n^2 + 30*d^2)*x^3*e^3 + 10*((3*a*d*f
^2 + 2*d^3)*n^4 + 45*a*d*f^2 - 2*(24*a*d*f^2 + d^3)*n^2)*x^2*e^2 - 10*(11*a
^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 5*(45*a^2*f^4 + (a^2*f^4 + 4*a*d^2*f^2)*n^4 -
```

```
2*(11*a^2*f^4 + 26*a*d^2*f^2 - 12*d^4)*n^2)*x*e - (a^2*f^5*n^5 + (f*n^5 -
10*f*n^3 + 9*f*n)*x^4*e^4 + 4*(d*f*n^5 - 10*d*f*n^3 + 9*d*f*n)*x^3*e^3 - 10
*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 2*((a*f^3 + 2*d^2*f)*n^5 - 10*(2*a*f^3 + d
^2*f)*n^3 + (19*a*f^3 + 8*d^2*f)*n)*x^2*e^2 + 4*(a*d*f^3*n^5 - 10*(2*a*d*f^
3 - d^3*f)*n^3 + (19*a*d*f^3 - 10*d^3*f)*n)*x*e + (149*a^2*f^5 - 260*a*d^2*
f^3 + 120*d^4*f)*n)*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2))*(x*e + f*sqrt((a
*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*e^(-1)/(f^4*n^6 - 35*f^4*n^4 + 259*f^
4*n^2 - 225*f^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*
x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.508 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n$

Optimal. Leaf size=239

$$\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-3+n}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{8ef^2(1-n)}$$

```
[Out] 1/8*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-3+n)/e/f^2
/(3-n)-3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n
)/e/f^2/(1-n)-3/8*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(1+n)/e/f^2/(1+n)+1/8*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3+n)/e/f
^2/(3+n)
```

Rubi [A]
time = 0.18, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2146, 12, 276}

$$\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-3}}{8ef^2(3-n)}-\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{8ef^2(1-n)}-\frac{3(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{8ef^2(n+1)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}}{8ef^2(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2
 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3
+ n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d +
 e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)
) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*
(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx=\frac{2\text{Subst}\left(\int\frac{x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}{16e^4}\,dx\right.}{f^2}$$

$$=\frac{\text{Subst}\left(\int x^{-4+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+\right.\right.}{}$$

$$=\frac{\text{Subst}\left(\int\left(-e^3(d^2-af^2)^3x^{-4+n}+3e^3(d^2\right.\right.}{}$$

$$=\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2}{f}}\right.}{8ef^2(3-n)}$$

Mathematica [A]
time = 1.59, size = 186, normalized size = 0.78

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-3+n}\left(-\frac{(d^2-af^2)^3}{-3+n}+\frac{3(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{-1+n}-\frac{3(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}{1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^6}{3+n}\right)}{8ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-((d^2 - a*f^2)^3/
(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^
```

```
2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n)))/
(8*e*f^2)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1
/2))^n,x)
```

```
[Out] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1
/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2), x)
```

Fricas [A]
time = 0.42, size = 224, normalized size = 0.94

$$-\frac{\left(3\,adf^2n^2+3\,(n^2-1)x^3e^3-9\,adf^2+9\,(dn^2-d)x^2e^2+6\,d^3-3\,(3\,af^2-(af^2+2\,d^2)n^2)xe-(af^3n^3+(fn^3-fn)x^2e^2+2\,(dfn^3-dfn)xe-(7\,af^3-6\,d^2f)n)\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}\right)\left(xe+f\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}+d\right)^n e^{(-1)}}{f^2n^4-10\,f^2n^2+9\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(3*a*d*f^2*n^2 + 3*(n^2 - 1)*x^3*e^3 - 9*a*d*f^2 + 9*(d*n^2 - d)*x^2*e^2 +
 6*d^3 - 3*(3*a*f^2 - (a*f^2 + 2*d^2)*n^2)*x*e - (a*f^3*n^3 + (f*n^3 - f*n)
*x^2*e^2 + 2*(d*f*n^3 - d*f*n)*x*e - (7*a*f^3 - 6*d^2*f)*n)*sqrt((a*f^2 + x
^2*e^2 + 2*d*x*e)/f^2))*(x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)
^n*e^(-1)/(f^2*n^4 - 10*f^2*n^2 + 9*f^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int af^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx+\int e^2x^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx+\int 2dex\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**
2/f**2)**(1/2))**n,x)
```

```
[Out] (Integral(a*f**2*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n,
x) + Integral(e**2*x**2*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2
))**n, x) + Integral(2*d*e*x*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2
/f**2))**n, x))/f**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^
(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

3.509 $\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2141, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
```

```
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = 2\text{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}dx, x, d+ex$$

$$= \frac{\text{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex+\right.}{2e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-e(d^2-af^2)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+\right.}{2e^2}$$

$$= \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)} + \frac{\left(d+ex\right.}{}$$

Mathematica [A]
time = 1.04, size = 89, normalized size = 0.83

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n, x)
```

Fricas [A]
time = 0.38, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}-xe-d\right)\left(xe+f\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}+d\right)^ne^{(-1)}}{n^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) - x*e - d)*(x*e + f*sqrt((a*f^2
+ x^2*e^2 + 2*d*x*e)/f^2) + d)^n*e^(-1)/(n^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)
```

3.510 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)\,(1+n)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A]
time = 0.20, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2146, 371}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\,(d^2-af^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx = (2f^2)\,\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx, x, d+ex+f$$

$$= -\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\right.}{e\,(d^2-af^2)\,(1+n)}$$

Mathematica [A]
time = 2.47, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\,(d^2-af^2)\,(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2),x)
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^2/(a*f^2 +
 x^2*e^2 + 2*d*x*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2), x)
```

3.511 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx$

Optimal. Leaf size=122

$$-\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}{}_2F_1\left(3,\frac{3+n}{2};\frac{5+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^3(3+n)}$$

```
[Out] -8*f^4*hypergeom([3, 3/2+1/2*n],[5/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(3
+n)/e/(-a*f^2+d^2)^3/(3+n)
```

Rubi [A]
time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2146, 12, 371}

$$-\frac{8f^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+3}{}_2F_1\left(3,\frac{n+3}{2};\frac{n+5}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+3)\left(d^2-af^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)*Hyper
geometric2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^2}\,dx = (2f^4)\,\text{Subst}\left(\int \frac{4e^2x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}\,dx, x, d+ex$$

$$= (8e^2f^4)\,\text{Subst}\left(\int \frac{x^{2+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^3}\,dx, x, d+$$

$$= -\frac{8f^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{3+n}{}_2F_1\left(3, \frac{3+n}{2}; \frac{5+n}{2}; \right.}{e\left(d^2-af^2\right)^3(3+n)}$$

Mathematica [A]
time = 1.43, size = 112, normalized size = 0.92

$$-\frac{8f^4\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{3+n}{}_2F_1\left(3, \frac{3+n}{2}; \frac{5+n}{2}; \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^3(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^2,x]
```

```
[Out] (-8*f^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(3 + n)*Hypergeometri
c2F1[3, (3 + n)/2, (5 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^3*(3 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^{2}}+\frac{e^{2}x^{2}}{f^{2}}}\right)^{n}}{\left(a+\frac{2dex}{f^{2}}+\frac{e^{2}x^{2}}{f^{2}}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^2,x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="fricas")
```

```
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^4/(a^2*f^4
 + 4*a*d*f^2*x*e + x^4*e^4 + 4*d^2*x^2*e^2 + 2*(a*f^2*x^2 + 2*d*x^3*e)*e^2)
, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^2,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{{\left(a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^2, x)
```

3.512 $\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx$

Optimal. Leaf size=107

$$\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}}{2e(1+n)}$$

```
[Out] 1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-1+n)/e/(1-n)
+1/2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1+n)/e/(1+n)
```

Rubi [A]
time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2143, 2141, 12, 14}

$$\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-1}}{2e(1-n)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 +
n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(
1 + n)/(2*e*(1 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
```

```
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 2143

```
Int[((g_.) + (h_.)*((u_) + (f_.)*Sqrt[v_])^(n_))^(p_.), x_Symbol] :> Int[(g
 + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f,
g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(LinearMatchQ[u, x]
&& QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x,
 2]*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n dx = \int \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

$$= 2\mathrm{Subst}\left(\int \frac{x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)}{4e^2}\,dx, x, d+\right.$$

$$= \frac{\mathrm{Subst}\left(\int x^{-2+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)dx, x, d+ex\right.}{2e^2}$$

$$= \frac{\mathrm{Subst}\left(\int \left(-e(d^2-af^2)x^{-2+n}+ex^n\right)dx, x, d+ex+f\sqrt{a+}\right.}{2e^2}$$

$$= \frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-1+n}}{2e(1-n)}+\frac{\left(d+e\right.}{}$$

Mathematica [A]
time = 0.03, size = 89, normalized size = 0.83

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{-1+n}\left(\frac{-d^2+af^2}{-1+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{1+n}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(d + ex + f\sqrt{\frac{a\,f^2 + ex\,(ex + 2d)}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((x*e + d + sqrt(a*f^2 + (x*e + 2*d)*x*e))^n, x)
```

Fricas [**A**]
time = 0.38, size = 80, normalized size = 0.75

$$\frac{\left(fn\sqrt{\frac{af^2 + x^2e^2 + 2\,dxe}{f^2}} - xe - d\right)\left(xe + f\sqrt{\frac{af^2 + x^2e^2 + 2\,dxe}{f^2}} + d\right)^n e^{(-1)}}{n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="frica
s")
```

```
[Out] (f*n*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) - x*e - d)*(x*e + f*sqrt((a*f^2
+ x^2*e^2 + 2*d*x*e)/f^2) + d)^n*e^(-1)/(n^2 - 1)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac"
)
```

```
[Out] integrate((x*e + f*sqrt((a*f^2 + (x*e + 2*d)*x*e)/f^2) + d)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)
```

3.513 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$

Optimal. Leaf size=122

$$-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

```
[Out] -2*f^2*hypergeom([1, 1/2+1/2*n],[3/2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(1
+n)/e/(-a*f^2+d^2)/(1+n)
```

Rubi [A]
time = 0.33, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2152, 2146, 371}

$$-\frac{2f^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+1}{}_2F_1\left(1,\frac{n+1}{2};\frac{n+3}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+1)\left(d^2-af^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x)/f^2
+ (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)*Hyper
geometric2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
 (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx=\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

$$=\left(2f^2\right)\text{Subst}\left(\int \frac{x^n}{d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2}\,dx,x,d+ex+$$

$$=-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{1+n}{}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2}\right.}{e\left(d^2-af^2\right)(1+n)}$$

Mathematica [**A**]

time = 0.03, size = 112, normalized size = 0.92

$$-\frac{2f^2\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{1+n} {}_2F_1\left(1,\frac{1+n}{2};\frac{3+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/(a + (2*d*e*x
)/f^2 + (e^2*x^2)/f^2),x]
```

```
[Out] (-2*f^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(1 + n)*Hypergeometri
c2F1[1, (1 + n)/2, (3 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)*(1 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^2+ex\,(ex+2d)}{f^2}}\right)^n}{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^2*
x^2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^2*
x^2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d + sqrt(a*f^2 + (x*e + 2*d)*x*e))^n/(a + x^2*e^2/f^2 + 2*
d*x*e/f^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="fricas")
```

```
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^2/(a*f^2 +
 x^2*e^2 + 2*d*x*e), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/(a+2*d*e*x/f**2
+e**2*x**2/f**2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2),x, algorithm="giac")
```

```
[Out] integrate((x*e + f*sqrt((a*f^2 + (x*e + 2*d)*x*e)/f^2) + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)}^n}{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/(a + (e^2*x^2)/f^
2 + (2*d*e*x)/f^2), x)
```

3.514 $\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.$

Optimal. Leaf size=297

$$-\frac{(d^2-af^2)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-4+n}}{16ef^3(4-n)}+\frac{(d^2-af^2)^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef^3(2-n)}$$

```
[Out] -1/16*(-a*f^2+d^2)^4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-4+n)/e/f
^3/(4-n)+1/4*(-a*f^2+d^2)^3*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2
+n)/e/f^3/(2-n)+3/8*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/
2))^n/e/f^3/n-1/4*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^
(2+n)/e/f^3/(2+n)+1/16*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(4+n)/e/
f^3/(4+n)
```

Rubi [A]
time = 0.28, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2146, 12, 276}

$$-\frac{(d^2-af^2)^4\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-4}}{16ef^3(4-n)}+\frac{(d^2-af^2)^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef^3(2-n)}+\frac{3(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{8ef^3n}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef^3(n+2)}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+4}}{16ef^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*
x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/16*((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]
)^(-4 + n))/(e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*
(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^
2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(
4*e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4
 + n)/(16*e*f^3*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\text{Subst}\left(\int \frac{x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^4}{32e^5}\right.}{f}$$

$$= \frac{\text{Subst}\left(\int x^{-5+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f\right.\right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(e^4\left(d^2-af^2\right)^4x^{-5+n}-4e^4(a\right.\right.}{}$$

$$= -\frac{\left(d^2-af^2\right)^4\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}}\right.}{16ef^3(4-n)}$$

Mathematica [A]
time = 6.54, size = 228, normalized size = 0.77

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{6\left(d^2-af^2\right)^2}{n}+\frac{\left(d^2-af^2\right)^4}{(-4+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}-\frac{4\left(d^2-af^2\right)^3}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}-\frac{4\left(d^2-af^2\right)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{2+n}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^4}{4+n}\right)}{16ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (
2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (
d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) -
 (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2]
)^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2
 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n)))/(16*e*f^3
)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a+\frac{2dex}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}\right)^{\frac{3}{2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^{2}}+\frac{e^{2} x^{2}}{f^{2}}}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(3/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x
^2)^(1/2))^n,x)
```

```
[Out] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(3/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x
^2)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^(3/2), x)
```

Fricas [A]
time = 0.39, size = 346, normalized size = 1.16

$$\frac{\left(a^{2} f^{4} n^{4} + 24 \, a^{2} f^{4} + {\left(n^{4} - 4 \, n^{2}\right)} x^{4} e^{4} - 48 \, a d^{2} f^{2} + 4 \, {\left(d n^{4} - 4 \, d n^{2}\right)} x^{3} e^{3} + 24 \, d^{4} + 2 \, {\left({\left(a f^{2} + 2 \, d^{2}\right)} n^{4} - 2 \, {\left(5 \, a f^{2} + d^{2}\right)} n^{2}\right)} x^{2} e^{2} - 4 \, {\left(4 \, a^{2} f^{4} - 3 \, a d^{2} f^{2}\right)} n^{2} + 4 \, {\left(a d f^{2} n^{4} - 2 \, {\left(5 \, a d f^{2} - 3 \, d^{3}\right)} n^{2}\right)} x e - 4 \, {\left(a d f^{3} n^{3} + {\left(f n^{3} - 4 \, f n\right)} x^{3} e^{3} + 3 \, {\left(d f n^{3} - 4 \, d f n\right)} x^{2} e^{2} + {\left({\left(a f^{3} + 2 \, d^{2} f\right)} n^{3} - 2 \, {\left(5 \, a f^{3} + d^{2} f\right)} n\right)} x e - 2 \, {\left(5 \, a d f^{3} - 3 \, d^{3} f\right)} n\right)} \sqrt{\frac{a f^{2} + x^{2} e^{2} + 2 \, d x e}{f^{2}}}\right)} {\left(x e + f \sqrt{\frac{a f^{2} + x^{2} e^{2} + 2 \, d x e}{f^{2}}} + d\right)}^{n} e^{\left(-1\right)}}{f^{3} n^{5} - 20 \, f^{3} n^{3} + 64 \, f^{3} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a^2*f^4*n^4 + 24*a^2*f^4 + (n^4 - 4*n^2)*x^4*e^4 - 48*a*d^2*f^2 + 4*(d*n^4
 - 4*d*n^2)*x^3*e^3 + 24*d^4 + 2*((a*f^2 + 2*d^2)*n^4 - 2*(5*a*f^2 + d^2)*n
^2)*x^2*e^2 - 4*(4*a^2*f^4 - 3*a*d^2*f^2)*n^2 + 4*(a*d*f^2*n^4 - 2*(5*a*d*f
^2 - 3*d^3)*n^2)*x*e - 4*(a*d*f^3*n^3 + (f*n^3 - 4*f*n)*x^3*e^3 + 3*(d*f*n^
```

```
3 - 4*d*f*n)*x^2*e^2 + ((a*f^3 + 2*d^2*f)*n^3 - 2*(5*a*f^3 + d^2*f)*n)*x*e
- 2*(5*a*d*f^3 - 3*d^3*f)*n)*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2))*(x*e +
f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*e^(-1)/(f^3*n^5 - 20*f^3*n^3
 + 64*f^3*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}\right)^{\frac{3}{2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral((a + 2*d*e*x/f**2 + e**2*x**2/f**2)**(3/2)*(d + e*x + f*sqrt(a + 2
*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n \left(a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.515 $\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}} \left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$

Optimal. Leaf size=171

$$-\frac{(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)}-\frac{(d^2-af^2)\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{n}}{2efn}+$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)/e/f/
(2-n)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/f/n+
1/4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/f/(2+n)
```

Rubi [A]
time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.052, Rules used = {2146, 12, 276}

$$-\frac{(d^2-af^2)^2\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)}-\frac{(d^2-af^2)\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn}+\frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)
/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/4*((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
^(-2 + n))/(e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f
^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (
e^2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
```

```
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{2\text{Subst}\left(\int \frac{x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}{8e^3}dx, \ldots}{f}$$

$$= \frac{\text{Subst}\left(\int x^{-3+n}\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ \ldots\right.\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int \left(e^2(d^2-af^2)^2x^{-3+n}-2e^2(d^2- \ldots\right.\right.}{\ldots}$$

$$= -\frac{(d^2-af^2)^2\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+ \ldots}\right.}{4ef(2-n)}$$

Mathematica [A]
time = 2.44, size = 135, normalized size = 0.79

$$\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{2+n}\right)}{4ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d
^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) +
(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x
^2)^(1/2))^n,x)
```

```
[Out] int((a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x
^2)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*sqrt(a + x^2*
e^2/f^2 + 2*d*x*e/f^2), x)
```

Fricas [A]
time = 0.37, size = 124, normalized size = 0.73

$$\frac{\left(af^2n^2+n^2x^2e^2+2\,dn^2xe-2\,af^2+2\,d^2-2\,(fnxe+dfn)\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}\right)\left(xe+f\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}+d\right)^n e^{(-1)}}{fn^3-4\,fn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] (a*f^2*n^2 + n^2*x^2*e^2 + 2*d*n^2*x*e - 2*a*f^2 + 2*d^2 - 2*(f*n*x*e + d*f
*n)*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2))*(x*e + f*sqrt((a*f^2 + x^2*e^2 +
 2*d*x*e)/f^2) + d)^n*e^(-1)/(f*n^3 - 4*f*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e
**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)*(d + e*x + f*sqrt(a + 2*d*
e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2
/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n*sqrt(a + x^2*
e^2/f^2 + 2*d*x*e/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2), x)
```

3.516 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A]
time = 0.17, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2146, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
```

```
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx = (2f)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)$$

$$= \frac{f\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e}$$

$$= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

Mathematica [**A**]
time = 0.23, size = 36, normalized size = 0.88

$$\frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a +
(2*d*e*x)/f^2 + (e^2*x^2)/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [**F**]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/sqrt(a + x^2*
e^2/f^2 + 2*d*x*e/f^2), x)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.00

$$\frac{\left(xe+f\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}+d\right)^n fe^{(-1)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f*e^(-1)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/sqrt(a + x^2*
e^2/f^2 + 2*d*x*e/f^2), x)
```

Mupad [B]
time = 3.08, size = 41, normalized size = 1.00

$$\frac{f\left(d+e\,x+f\,\sqrt{\frac{e^2\,x^2+2\,d\,e\,x+a\,f^2}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)
```

3.517 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2(2+n)}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e
/(-a*f^2+d^2)^2/(2+n)
```

Rubi [A]
time = 0.21, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2146, 12, 371}

$$\frac{4f^3\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{e(n+2)\left(d^2-af^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hyperg
eometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 +
(e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx = (2f^3)\,\text{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+ex$$

$$= (4ef^3)\,\text{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\,dx, x, d+e$$

$$= \frac{4f^3\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n} {}_2F_1\left(2, \frac{2+n}{2}; \frac{4+n}{2}; \right.}{e\left(d^2-af^2\right)^2(2+n)}$$

Mathematica [A]
time = 3.62, size = 112, normalized size = 0.92

$$\frac{4f^3\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n} {}_2F_1\left(2, \frac{2+n}{2}; \frac{4+n}{2}; \frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a + (2*d
*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(2 + n)*Hypergeometric
2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^
2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 + n))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a+2*d*e/f^2*x+1/f^2*e^
2*x^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^4*sqrt((a*
f^2 + x^2*e^2 + 2*d*x*e)/f^2)/(a^2*f^4 + 4*a*d*f^2*x*e + x^4*e^4 + 4*d^2*x^
2*e^2 + 2*(a*f^2*x^2 + 2*d*x^3*e)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f*
*2+e**2*x**2/f**2)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/(a + 2*d*
e*x/f**2 + e**2*x**2/f**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*
x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a + x^2*e^2/
f^2 + 2*d*x*e/f^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + f\sqrt{a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}} + e\,x\right)^n}{\left(a + \frac{e^2\,x^2}{f^2} + \frac{2\,d\,e\,x}{f^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2
)/f^2 + (2*d*e*x)/f^2)^(3/2), x)
```

3.518 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=41

$$\frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}$$

```
[Out] f*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/e/n
```

Rubi [A]
time = 0.31, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2152, 2146, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(
2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
```

```
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}}\,dx &= \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx \\
&= (2f)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right) \\
&= \frac{f\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)}{e} \\
&= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.88

$$\frac{f\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 +
 e*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^{2}+ex\,(ex+2d)}{f^{2}}}\right)^{n}}{\sqrt{\frac{a\,f^{2}+ex\,(ex+2d)}{f^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^
2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((x*e + d + sqrt(a*f^2 + (x*e + 2*d)*x*e))^n/sqrt(a*f^2 + (x*e +
 2*d)*x*e), x)
```

Fricas [A]
time = 0.35, size = 41, normalized size = 1.00

$$\frac{\left(xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}+2\,dxe}{f^{2}}}+d\right)^{n}fe^{(-1)}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f*e^(-1)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2+e*x*(e
*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*
d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + f*sqrt((a*f^2 + (x*e + 2*d)*x*e)/f^2) + d)^n/sqrt((a*f^2 +
 (x*e + 2*d)*x*e)/f^2), x)
```

Mupad [B]
time = 3.08, size = 39, normalized size = 0.95

$$\frac{f\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)^n}{e\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2*
d + e*x))/f^2)^(1/2),x)
```

```
[Out] (f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)
```

3.519 $\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right.$

Optimal. Leaf size=327

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{-2+n}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}}}{2ef}$$

```
[Out] -1/4*(-a*f^2+d^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(-2+n)*(a*g
+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/(2-n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(
1/2)-1/2*(-a*f^2+d^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n*(a*g+2*
d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)/e/f/n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)+1/
4*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)*(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(1/2)/e/f/(2+n)/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2148, 2146, 12, 276}

$$-\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n-2}}{4ef(2-n)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}-\frac{(d^2-af^2)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n}}{2efn\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}+\frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}}{4ef(n+2)\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*
d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -1/4*((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*
x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(e*f*(2 - n)*Sqrt[
a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)
/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2
])^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*
e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^
2)/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2148

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m -
 1/2)*(Sqrt[g + h*x + i*x^2]/Sqrt[a + b*x + c*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && IGtQ[m + 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx = \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\int\sqrt{a+\frac{2de}{f^2}\cdots}}{\sqrt{a\cdots}}$$

$$= \frac{\left(2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\right)\text{Subst}\Big(\cdots}{\cdots}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\,\text{Subst}\Big(\int x^{-3}\cdots}{\cdots}$$

$$= \frac{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\,\text{Subst}\Big(\int\big(e^2\cdots}{\cdots}$$

$$= -\frac{(d^2-af^2)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}\cdots}{4ef(2-n)\sqrt{\cdots}}$$

Mathematica [A]
time = 0.17, size = 175, normalized size = 0.54

$$\frac{\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n\left(\frac{2(-d^2+af^2)}{n}+\frac{(d^2-af^2)^2}{(-2+n)\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}+\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{2+n}\right)}{4ef\sqrt{a+\frac{ex(2d+ex)}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a
 + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x)
)/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sq
rt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))
/f^2])^2/(2 + n)))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{ag+\frac{2degx}{f^{2}}+\frac{e^{2}g\,x^{2}}{f^{2}}}\,\left(d+ex+f\sqrt{a+\frac{2dex}{f^{2}}+\frac{e^{2}x^{2}}{f^{2}}}\right)^{n}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e
^2*x^2)^(1/2))^n,x)
```

```
[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e
^2*x^2)^(1/2))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*g + g*x^2*e^2/f^2 + 2*d*g*x*e/f^2)*(x*e + sqrt(a + x^2*e^2
/f^2 + 2*d*x*e/f^2)*f + d)^n, x)
```

Fricas [A]
time = 0.39, size = 223, normalized size = 0.68

$$-\frac{\left(2\,adf^{2}n+2\,nx^{3}e^{3}+6\,dnx^{2}e^{2}+2\,(af^{2}+2\,d^{2})nxe-(af^{3}n^{2}+fn^{2}x^{2}e^{2}+2\,dfn^{2}xe-2\,af^{3}+2\,d^{2}f)\sqrt{\frac{af^{2}+x^{2}e^{2}+2\,dxe}{f^{2}}}\right)\left(xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}+2\,dxe}{f^{2}}}+d\right)^{n}\sqrt{\frac{af^{2}g+gx^{2}e^{2}+2\,dgxe}{f^{2}}}}{(n^{3}-4\,n)x^{2}e^{3}+2\,(dn^{3}-4\,dn)xe^{2}+(af^{2}n^{3}-4\,af^{2}n)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(2*a*d*f^2*n + 2*n*x^3*e^3 + 6*d*n*x^2*e^2 + 2*(a*f^2 + 2*d^2)*n*x*e - (a*
f^3*n^2 + f*n^2*x^2*e^2 + 2*d*f*n^2*x*e - 2*a*f^3 + 2*d^2*f)*sqrt((a*f^2 +
x^2*e^2 + 2*d*x*e)/f^2))*(x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d
)^n*sqrt((a*f^2*g + g*x^2*e^2 + 2*d*g*x*e)/f^2)/((n^3 - 4*n)*x^2*e^3 + 2*(d
*n^3 - 4*d*n)*x*e^2 + (a*f^2*n^3 - 4*a*f^2*n)*e)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/
f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2))*(d + e*x + f*sqrt(a +
2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e
^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*g + g*x^2*e^2/f^2 + 2*d*g*x*e/f^2)*(x*e + sqrt(a + x^2*e^2
/f^2 + 2*d*x*e/f^2)*f + d)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)}^n\,\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.520 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2150, 2146, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d
*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{\ldots}\right.}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{a\ldots}\right.}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g
+ (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2g\,x^2}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/sqrt(a*g + g*
x^2*e^2/f^2 + 2*d*g*x*e/f^2), x)
```

Fricas [A]
time = 0.37, size = 117, normalized size = 1.26

$$\frac{\left(xe+f\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}+d\right)^n f^3\sqrt{\frac{af^2g+gx^2e^2+2\,dgxe}{f^2}}\,\sqrt{\frac{af^2+x^2e^2+2\,dxe}{f^2}}}{af^2gne+gnx^2e^3+2\,dgnxe^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^3*sqrt((a*f^2*g + g
*x^2*e^2 + 2*d*g*x*e)/f^2)*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2)/(a*f^2*g*n
*e + g*n*x^2*e^3 + 2*d*g*n*x*e^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{g\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(g*(a
 + 2*d*e*x/f**2 + e**2*x**2/f**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/sqrt(a*g + g*
x^2*e^2/f^2 + 2*d*g*x*e/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{\sqrt{a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

3.521 $\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}dx$

Optimal. Leaf size=177

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2g(2+n)\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] 4*f^3*hypergeom([2, 1+1/2*n],[2+1/2*n],(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)
^(1/2))^2/(-a*f^2+d^2))*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e
*x/f^2+e^2*x^2/f^2)^(1/2))^(2+n)/e/(-a*f^2+d^2)^2/g/(2+n)/(a*g+2*d*e*g*x/f^
2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2150, 2146, 12, 371}

$$\frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^{n+2}{}_2F_1\left(2,\frac{n+2}{2};\frac{n+4}{2};\frac{\left(d+ex+f\sqrt{\frac{e^2x^2}{f^2}+\frac{2dex}{f^2}+a}\right)^2}{d^2-af^2}\right)}{eg(n+2)\left(d^2-af^2\right)^2\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2*d*e*g
*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e
*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2
, (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^2/(d^2 - a*f^2)])/(
e*(d^2 - a*f^2)^2*g*(2 + n)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}\right)^{3/2}}\,dx = \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\left(a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}\right)^{3/2}}\,dx}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\mathrm{Subst}\left(\int \frac{2ex^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\right.}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(4ef^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\mathrm{Subst}\left(\int \frac{x^{1+n}}{\left(d^2e-\left(-ae+\frac{2d^2e}{f^2}\right)f^2+ex^2\right)^2}\right.}{g\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{4f^3\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^{2+}}{e\left(d^2-af^2\right)^2 g(2+n)\sqrt{ag}}$$

Mathematica [A]
time = 0.13, size = 152, normalized size = 0.86

$$\frac{4f^3\left(a+\frac{ex(2d+ex)}{f^2}\right)^{3/2}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^{2+n}{}_2F_1\left(2,\frac{2+n}{2};\frac{4+n}{2};\frac{\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^2}{d^2-af^2}\right)}{e\left(d^2-af^2\right)^2(2+n)\left(g\left(a+\frac{ex(2d+ex)}{f^2}\right)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(a*g + (2
*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2)^(3/2),x]
```

```
[Out] (4*f^3*(a + (e*x*(2*d + e*x))/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (e*x*(2*d +
e*x))/f^2])^(2 + n)*Hypergeometric2F1[2, (2 + n)/2, (4 + n)/2, (d + e*x + f
*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(d^2 - a*f^2)])/(e*(d^2 - a*f^2)^2*(2 +
 n)*(g*(a + (e*x*(2*d + e*x))/f^2))^(3/2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^{2}}+\frac{e^{2}x^{2}}{f^{2}}}\right)^{n}}{\left(ag+\frac{2degx}{f^{2}}+\frac{e^{2}g\,x^{2}}{f^{2}}\right)^{\frac{3}{2}}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(3/2),x)
```

```
[Out] int((d+e*x+f*(a+2*d*e/f^2*x+1/f^2*e^2*x^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*
g*x^2/f^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a*g + g*x^2*
e^2/f^2 + 2*d*g*x*e/f^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^4*sqrt((a*
f^2*g + g*x^2*e^2 + 2*d*g*x*e)/f^2)/(a^2*f^4*g^2 + 4*a*d*f^2*g^2*x*e + g^2*
x^4*e^4 + 4*d^2*g^2*x^2*e^2 + 2*(a*f^2*g^2*x^2 + 2*d*g^2*x^3*e)*e^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*
x/f**2+e**2*g*x**2/f**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+
e^2*g*x^2/f^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + sqrt(a + x^2*e^2/f^2 + 2*d*x*e/f^2)*f + d)^n/(a*g + g*x^2*
e^2/f^2 + 2*d*g*x*e/f^2)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+f\,\sqrt{a+\frac{e^2\,x^2}{f^2}+\frac{2\,d\,e\,x}{f^2}}+e\,x\right)^n}{{\left(a\,g+\frac{e^2\,g\,x^2}{f^2}+\frac{2\,d\,e\,g\,x}{f^2}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2),x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g
*x^2)/f^2 + (2*d*e*g*x)/f^2)^(3/2), x)
```

3.522 $\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

```
[Out] f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1
/2))^n/e/n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2152, 2150, 2146, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g
*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/
f^2 + (e^2*x^2)/f^2])^n)/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2146

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(2/f^(2*m)
)*(i/c)^m, Subst[Int[x^n*((d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*
x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1))), x], x, d + e*x + f*Sq
rt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ
[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m]
 && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2150

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*S
qrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] :> Dist[(i/c)^(m +
 1/2)*(Sqrt[a + b*x + c*x^2]/Sqrt[g + h*x + i*x^2]), Int[(a + b*x + c*x^2)^
m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b
*i, 0] && ILtQ[m - 1/2, 0] &&  !GtQ[i/c, 0]
```

Rule 2152

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
 || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}}\,dx = \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}\,dx$$

$$= \frac{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}}\,dx}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(2f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int \frac{x^{-1+n}}{2e}\,dx, x, d+ex+f\sqrt{\cdots}\right.}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)\text{Subst}\left(\int x^{-1+n}\,dx, x, d+ex+f\sqrt{\cdots}\right.}{e\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

$$= \frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Mathematica [A]
time = 0.02, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\left(d+ex+f\sqrt{a+\frac{ex(2d+ex)}{f^2}}\right)^n}{en\sqrt{g\left(a+\frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
 + e*g*x*(2*d + e*x))/f^2],x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex+f\sqrt{\frac{a\,f^{2}+ex\,(ex+2d)}{f^{2}}}\right)^{n}}{\sqrt{\frac{a\,f^{2}g+egx\,(ex+2d)}{f^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

```
[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d)
)/f^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((x*e + d + sqrt(a*f^2 + (x*e + 2*d)*x*e))^n/sqrt(a*f^2*g + (x*e
 + 2*d)*g*x*e), x)
```

Fricas [A]
time = 0.36, size = 117, normalized size = 1.26

$$\frac{\left(xe+f\sqrt{\frac{af^{2}+x^{2}e^{2}+2\,dxe}{f^{2}}}+d\right)^{n}f^{3}\sqrt{\frac{af^{2}g+gx^{2}e^{2}+2\,dgxe}{f^{2}}}\sqrt{\frac{af^{2}+x^{2}e^{2}+2\,dxe}{f^{2}}}}{af^{2}gne+gnx^{2}e^{3}+2\,dgnxe^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x*e + f*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2) + d)^n*f^3*sqrt((a*f^2*g + g
*x^2*e^2 + 2*d*g*x*e)/f^2)*sqrt((a*f^2 + x^2*e^2 + 2*d*x*e)/f^2)/(a*f^2*g*n
*e + g*n*x^2*e^3 + 2*d*g*n*x*e^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*
x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*
x+2*d))/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x*e + f*sqrt((a*f^2 + (x*e + 2*d)*x*e)/f^2) + d)^n/sqrt((a*f^2*g
 + (x*e + 2*d)*g*x*e)/f^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d+e\,x+f\,\sqrt{\frac{a\,f^2+e\,x\,\left(2\,d+e\,x\right)}{f^2}}\right)^n}{\sqrt{\frac{a\,g\,f^2+e\,g\,x\,\left(2\,d+e\,x\right)}{f^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x
*(2*d + e*x))/f^2)^(1/2), x)
```

3.523 $\int \frac{1}{(a+bx)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal. Leaf size=191

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{b^2e+a^2f}\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\sqrt{b^2e+a^2f}}+\frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{b^2c}{a^2d};\sin^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] -b*arctanh((a^2*f+b^2*e)^(1/2)*(d*x^2+c)^(1/2)/(a^2*d+b^2*c)^(1/2)/(f*x^2+e
)^(1/2))/(a^2*d+b^2*c)^(1/2)/(a^2*f+b^2*e)^(1/2)+EllipticPi(x*d^(1/2)/(-c)^
(1/2),-b^2*c/a^2/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2138, 552, 551, 585, 95, 214}

$$\frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\Pi\left(-\frac{b^2c}{a^2d};\text{ArcSin}\left(\frac{\sqrt{d}\,x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^2}\sqrt{a^2f+b^2e}}{\sqrt{e+fx^2}\sqrt{a^2d+b^2c}}\right)}{\sqrt{a^2d+b^2c}\sqrt{a^2f+b^2e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[b^2*e + a^2*f]*Sqrt[c + d*x^2])/(Sqrt[b^2*c + a^2*d]*Sqr
t[e + f*x^2])])/(Sqrt[b^2*c + a^2*d]*Sqrt[b^2*e + a^2*f])) + (Sqrt[-c]*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b^2*c)/(a^2*d)), ArcSin[(
Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2138

```
Int[1/(((a_) + (b_.)*(x_))*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Dist[a, Int[1/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e +
f*x^2]), x], x] - Dist[b, Int[x/((a^2 - b^2*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx = a\int \frac{1}{(a^2-b^2x^2)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx - b\int \frac{x}{(a^2-b^2x^2)\sqrt{c+dx}\cdots}$$

$$= -\left(\frac{1}{2}b\text{Subst}\left(\int \frac{1}{(a^2-b^2x)\sqrt{c+dx}\,\sqrt{e+fx}}\,dx, x, x^2\right)\right) + \cdots$$

$$= -\left(b\text{Subst}\left(\int \frac{1}{b^2c+a^2d-(b^2e+a^2f)\,x^2}\,dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\right)\right) + \cdots$$

$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{b^2e+a^2f}\,\sqrt{c+dx^2}}{\sqrt{b^2c+a^2d}\,\sqrt{e+fx^2}}\right)}{\sqrt{b^2c+a^2d}\,\sqrt{b^2e+a^2f}} + \frac{\sqrt{-c}\,\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\cdots}{a\sqrt{d}\,\sqrt{c+\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.93, size = 772, normalized size = 4.04

$$\frac{2\sqrt{d}\left(\sqrt{c}+i\sqrt{d}\,x\right)\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)\left(i\sqrt{c}+\sqrt{d}\,x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}\,x\right)}}\left(\sqrt{e}+i\sqrt{f}\,x\right)\sqrt{\frac{\sqrt{c}\sqrt{d}\left(i\sqrt{e}+\sqrt{f}\,x\right)}{\left(-\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}\,x\right)}}\left(\left(b\sqrt{c}+ia\sqrt{d}\right)F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)\left(i\sqrt{c}+\sqrt{d}\,x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}\,x\right)}}\right)\middle|\frac{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)^2}{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)^2}\right)-2b\sqrt{c}\,\Pi\left(\frac{\left(b\sqrt{c}-ia\sqrt{d}\right)\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)}{\left(b\sqrt{c}+ia\sqrt{d}\right)\left(-\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)};\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)\left(i\sqrt{c}+\sqrt{d}\,x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(-i\sqrt{c}+\sqrt{d}\,x\right)}}\right)\middle|\frac{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)^2}{\left(\sqrt{d}\sqrt{e}-\sqrt{c}\sqrt{f}\right)^2}\right)\right)}{\left(b\sqrt{c}-ia\sqrt{d}\right)\left(b\sqrt{c}+ia\sqrt{d}\right)\left(-\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\sqrt{\frac{\sqrt{c}\sqrt{d}\left(\sqrt{e}+i\sqrt{f}\,x\right)}{\left(\sqrt{d}\sqrt{e}+\sqrt{c}\sqrt{f}\right)\left(\sqrt{c}+i\sqrt{d}\,x\right)}}\sqrt{c+dx^2}\,\sqrt{e+fx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (2*Sqrt[d]*(Sqrt[c] + I*Sqrt[d]*x)*Sqrt[((Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f]
)*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[
c] + Sqrt[d]*x))]*(Sqrt[e] + I*Sqrt[f]*x)*Sqrt[(Sqrt[c]*Sqrt[d]*(I*Sqrt[e]
+ Sqrt[f]*x))/((-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*((-I)*Sqrt[c] + Sqrt[
d]*x))]*((b*Sqrt[c] + I*a*Sqrt[d])*EllipticF[ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
- Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqr
t[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2/
(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2] - 2*b*Sqrt[c]*EllipticPi[((b*Sqrt[c]
 - I*a*Sqrt[d])*(Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f]))/((b*Sqrt[c] + I*a*Sqrt
[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])), ArcSin[Sqrt[((Sqrt[d]*Sqrt[e]
 - Sqrt[c]*Sqrt[f])*(I*Sqrt[c] + Sqrt[d]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sq
rt[f])*((-I)*Sqrt[c] + Sqrt[d]*x))]], (Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])^2
/(Sqrt[d]*Sqrt[e] - Sqrt[c]*Sqrt[f])^2]))/((b*Sqrt[c] - I*a*Sqrt[d])*(b*Sqr
t[c] + I*a*Sqrt[d])*(-(Sqrt[d]*Sqrt[e]) + Sqrt[c]*Sqrt[f])*Sqrt[(Sqrt[c]*Sq
rt[d]*(Sqrt[e] + I*Sqrt[f]*x))/((Sqrt[d]*Sqrt[e] + Sqrt[c]*Sqrt[f])*(Sqrt[c
] + I*Sqrt[d]*x))]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(164) = 328$.
time = 0.25, size = 339, normalized size = 1.77

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)\,(f\,x^2+e)}\left(-\frac{\operatorname{arctanh}\left(\frac{\frac{(cf+de)a^2}{b^2}+2ce+(cf+de)x^2+\frac{2df\,x^2a^2}{b^2}}{2\sqrt{\frac{df\,a^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}\,\sqrt{df\,x^4+c\,x^2f+de\,x^2+ce}}\right)}{2\sqrt{\frac{df\,a^4}{b^4}+\frac{(cf+de)a^2}{b^2}+ce}}+\frac{b\sqrt{1+}}{}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,b}$
default	$\frac{\left(2b\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\operatorname{EllipticPi}\left(x\sqrt{-\frac{d}{c}},-\frac{b^2c}{d\,a^2},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ce}{b^4}}-\operatorname{arctanh}\left(\frac{2a^2d}{2b^2\sqrt{\frac{a^4df+a^2b}{}}}\right.\right.}{2ba\sqrt{\frac{a^4df+a^2b^2cf+a^2b^2de+b^4ce}{b^4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*b*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),
-b^2*c/d/a^2,(-f/e)^(1/2)/(-d/c)^(1/2))*((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*e+b
^4*c*e)/b^4)^(1/2)-arctanh(1/2*(2*a^2*d*f*x^2+b^2*c*f*x^2+b^2*d*e*x^2+a^2*c
*f+a^2*d*e+2*b^2*c*e)/b^2/((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*e+b^4*c*e)/b^4)^(
1/2)/((d*x^2+c)*(f*x^2+e))^(1/2))*(-d/c)^(1/2)*a*((d*x^2+c)*(f*x^2+e))^(1/2
))/b*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/((a^4*d*f+a^2*b^2*c*f+a^2*b^2*d*e+b^
4*c*e)/b^4)^(1/2)/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt{c+dx^2}\,\sqrt{e+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)*(a + b*x)), x)
```

3.524 $\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\log\left(e-2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}+\frac{\log\left(e+2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)+1/4*ln(e+2*f*x
^2+2*x*(-d)^(1/2)*f^(1/2))/(-d)^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1178, 642}

$$\frac{\log\left(2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{-d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -1/4*Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[-d]*Sqrt[f]) + Log[e + 2
*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2+4dfx^2+4efx^2+4f^2x^4}\,dx = \int \frac{e-2fx^2}{e^2+4(d+e)fx^2+4f^2x^4}\,dx$$
$$= -\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}dx}{4\sqrt{-d}\,\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{-d}\,x}{\sqrt{f}}-x^2}dx}{4\sqrt{-d}\,\sqrt{f}}$$
$$= -\frac{\log\left(e-2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}} + \frac{\log\left(e+2\sqrt{-d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{-d}\,\sqrt{f}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 191 vs. 2(81) = 162.
time = 0.08, size = 191, normalized size = 2.36

$$\frac{-\frac{\left(-d-2e+\sqrt{d}\,\sqrt{d+2e}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{d+e-\sqrt{d}\,\sqrt{d+2e}}}\right)}{\sqrt{d+e-\sqrt{d}\,\sqrt{d+2e}}} - \frac{\left(d+2e+\sqrt{d}\,\sqrt{d+2e}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{d+e+\sqrt{d}\,\sqrt{d+2e}}}\right)}{\sqrt{d+e+\sqrt{d}\,\sqrt{d+2e}}}}{2\sqrt{2}\,\sqrt{d}\,\sqrt{d+2e}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-(((-d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d +
e - Sqrt[d]*Sqrt[d + 2*e]]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2
*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]
*Sqrt[d + 2*e]]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*S
qrt[d + 2*e]*Sqrt[f])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(61) = 122.
time = 0.06, size = 193, normalized size = 2.38

method	result
risch	$-\frac{\ln\left(-2\sqrt{-df}\,f\,x^2-2dfx-\sqrt{-df}\,e\right)}{4\sqrt{-df}}+\frac{\ln\left(-2\sqrt{-df}\,f\,x^2+2dfx-\sqrt{-df}\,e\right)}{4\sqrt{-df}}$

default	$f^{2}\left(-\frac{\left(df+2ef-\sqrt{d\,f^{2}\left(d+2e\right)}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{fx\sqrt{2}}{\sqrt{-df-ef+\sqrt{d\,f^{2}\left(d+2e\right)}}}\right)}{4\sqrt{d\,f^{2}\left(d+2e\right)}\,f^{2}\sqrt{-df-ef+\sqrt{d\,f^{2}\left(d+2e\right)}}}+\frac{\left(-df-2ef-\sqrt{d\,f^{2}\right.}}{4\sqrt{d\,f}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] f^2*(-1/4*(d*f+2*e*f-(d*f^2*(d+2*e))^(1/2))/(d*f^2*(d+2*e))^(1/2)/f^2*2^(1/
2)/(-d*f-e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(-d*f-e*f+(d*
f^2*(d+2*e))^(1/2))^(1/2))+1/4*(-d*f-2*e*f-(d*f^2*(d+2*e))^(1/2))/(d*f^2*(d
+2*e))^(1/2)/f^2*2^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)*arctan(f*x*2
^(1/2)/(d*f+e*f+(d*f^2*(d+2*e))^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*f*x^2*e + e^2), x)
```

Fricas [A]
time = 0.41, size = 150, normalized size = 1.85

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\,f^{2}x^{4}-4\,dfx^{2}+4\,fx^{2}e+4\,(2\,fx^{3}+xe)\,\sqrt{-df}+e^{2}}{4\,f^{2}x^{4}+4\,dfx^{2}+4\,fx^{2}e+e^{2}}\right)}{4\,df},\frac{\sqrt{df}\,\arctan\left(\frac{(2\,fx^{3}+2\,dx+xe)\,\sqrt{df}\,e^{(-1)}}{d}\right)-\sqrt{df}\,\arctan\left(\frac{\sqrt{df}\,x}{d}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*d*f*x^2 + 4*f*x^2*e + 4*(2*f*x^3 + x*e)
*sqrt(-d*f) + e^2)/(4*f^2*x^4 + 4*d*f*x^2 + 4*f*x^2*e + e^2))/(d*f), 1/2*(s
qrt(d*f)*arctan((2*f*x^3 + 2*d*x + x*e)*sqrt(d*f)*e^(-1)/d) - sqrt(d*f)*arc
tan(sqrt(d*f)*x/d))/(d*f)]
```

Sympy [A]
time = 0.33, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**2)/4
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Valueindex.cc index_m operator + Error: Bad Argument Valueindex.
cc ind
```

Mupad [B]
time = 3.11, size = 50, normalized size = 0.62

$$\frac{\operatorname{atan}\left(\frac{2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x}{\sqrt{d}\,e}\right)-\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^(3/2)*x^3 + 2*d*f^(1/2)*x + e*f^(1/2)*x)/(d^(1/2)*e)) - atan((f^
(1/2)*x)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.525 $\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx$

Optimal. Leaf size=73

$$-\frac{\log\left(e-2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}+\frac{\log\left(e+2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

```
[Out] -1/4*ln(e+2*f*x^2-2*x*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)+1/4*ln(e+2*f*x^2+2*x
*d^(1/2)*f^(1/2))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {6, 1178, 642}

$$\frac{\log\left(2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}-\frac{\log\left(-2\sqrt{d}\,\sqrt{f}\,x+e+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] -1/4*Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(Sqrt[d]*Sqrt[f]) + Log[e + 2*S
qrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{e-2fx^2}{e^2-4dfx^2+4efx^2+4f^2x^4}\,dx = \int \frac{e-2fx^2}{e^2+(-4d+4e)fx^2+4f^2x^4}\,dx$$

$$= -\frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}+2x}{-\frac{e}{2f}-\frac{\sqrt{d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\,\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{d}}{\sqrt{f}}-2x}{-\frac{e}{2f}+\frac{\sqrt{d}\,x}{\sqrt{f}}-x^2}\,dx}{4\sqrt{d}\,\sqrt{f}}$$

$$= -\frac{\log\left(e-2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}} + \frac{\log\left(e+2\sqrt{d}\,\sqrt{f}\,x+2fx^2\right)}{4\sqrt{d}\,\sqrt{f}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 233, normalized size = 3.19

$$\frac{-\frac{\left(-id+2ie+\sqrt{d}\,\sqrt{-d+2e}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{-d+e-i\sqrt{d}\,\sqrt{-d+2e}}}\right)}{\sqrt{-d+e-i\sqrt{d}\,\sqrt{-d+2e}}}-\frac{\left(id-2ie+\sqrt{d}\,\sqrt{-d+2e}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{f}\,x}{\sqrt{-d+e+i\sqrt{d}\,\sqrt{-d+2e}}}\right)}{\sqrt{-d+e+i\sqrt{d}\,\sqrt{-d+2e}}}}{2\sqrt{2}\,\sqrt{d}\,\sqrt{-d+2e}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4),x]
```

```
[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/
Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d +
 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]
*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[
-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(53) = 106.
time = 0.05, size = 193, normalized size = 2.64

method	result
risch	$\frac{\ln\left(2\sqrt{df}\,f\,x^2+2dfx+\sqrt{df}\,e\right)}{4\sqrt{df}}-\frac{\ln\left(2\sqrt{df}\,f\,x^2-2dfx+\sqrt{df}\,e\right)}{4\sqrt{df}}$

default	$f^2\left(\frac{\left(df-2ef-\sqrt{d\,f^2\,(d-2e)}\right)\sqrt{2}\,\arctan\left(\frac{fx\sqrt{2}}{\sqrt{-df+ef+\sqrt{d\,f^2\,(d-2e)}}}\right)}{4f^2\sqrt{d\,f^2\,(d-2e)}\,\sqrt{-df+ef+\sqrt{d\,f^2\,(d-2e)}}}-\frac{\left(-df+2ef-\sqrt{d\,f^2\,(d\right.}{4f^2\sqrt{d\,f}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] f^2*(1/4*(d*f-2*e*f-(d*f^2*(d-2*e))^(1/2))/f^2/(d*f^2*(d-2*e))^(1/2)*2^(1/2
)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^
2*(d-2*e))^(1/2))^(1/2))-1/4*(-d*f+2*e*f-(d*f^2*(d-2*e))^(1/2))/f^2/(d*f^2*
(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2^
(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*f*x^2*e + e^2), x)
```

Fricas [A]
time = 0.40, size = 152, normalized size = 2.08

$$\left[\frac{\sqrt{df}\,\log\left(\frac{4\,f^2x^4+4\,dfx^2+4\,fx^2e+4\,(2\,fx^3+xe)\,\sqrt{df}+e^2}{4\,f^2x^4-4\,dfx^2+4\,fx^2e+e^2}\right)}{4\,df},\ \frac{\sqrt{-df}\,\arctan\left(\frac{(2\,fx^3-2\,dx+xe)\,\sqrt{-df}\,e^{(-1)}}{d}\right)-\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*d*f*x^2 + 4*f*x^2*e + 4*(2*f*x^3 + x*e)*s
qrt(d*f) + e^2)/(4*f^2*x^4 - 4*d*f*x^2 + 4*f*x^2*e + e^2))/(d*f), 1/2*(sqrt
(-d*f)*arctan((2*f*x^3 - 2*d*x + x*e)*sqrt(-d*f)*e^(-1)/d) - sqrt(-d*f)*arc
tan(sqrt(-d*f)*x/d))/(d*f)]
```

Sympy [A]
time = 0.32, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Valueindex.cc index_m operator + Error: Bad Argument Valueindex.
cc ind
```

Mupad [B]
time = 3.14, size = 28, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{2fx^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2),x)
```

```
[Out] atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.526 $\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2118, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2+4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[e^2+4df\#1^2+4ef\#1^3+4f^2\#1^6\&,\frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 70, normalized size = 1.84

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4ef\_Z^3+4df\_Z^2+e^2\right)}\frac{\left(-4\_R^3f+e\right)\ln\left(x-\_R\right)}{6f\_R^5+3e\_R^2+2d\_R}}{4f}$
risch	$-\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(-df)^{\frac{3}{2}}-32d^3f^2\right)x+16e(-df)^{\frac{3}{2}}d+27fe^3d\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(32f(-df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(\right.\right.}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*f*x^3*e + 4*d*f*x^2 + e^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(29) = 58$.

time = 0.43, size = 154, normalized size = 4.05

$$\left[-\frac{\sqrt{-df}\,\log\left(\frac{4\,f^2x^6+4\,fx^3e-4\,dfx^2+4\,(2\,fx^4+xe)\,\sqrt{-df}+e^2}{4\,f^2x^6+4\,fx^3e+4\,dfx^2+e^2}\right)}{4\,df},-\frac{\sqrt{df}\,\arctan\left(\frac{\sqrt{df}\,x^2}{d}\right)-\sqrt{df}\,\arctan\left(\frac{(2\,fx^5+x^2e+2\,dx)\,\sqrt{df}\,e^{(-1)}}{d}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*f*x^3*e - 4*d*f*x^2 + 4*(2*f*x^4 + x*e)
*sqrt(-d*f) + e^2)/(4*f^2*x^6 + 4*f*x^3*e + 4*d*f*x^2 + e^2))/(d*f), -1/2*(
sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + x^2*e + 2*d
*x)*sqrt(d*f)*e^(-1)/d))/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(34) = 68$.
time = 0.37, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}}\,\log\left(-dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\,\log\left(dx\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))
*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(29) = 58$.
time = 4.31, size = 69, normalized size = 1.82

$$-\frac{\sqrt{-df}\,\log\left(\left|2\,fx^3+2\,\sqrt{-df}\,x+e\right|\right)}{4\,df}+\frac{\sqrt{-df}\,\log\left(\left|2\,fx^3-2\,\sqrt{-df}\,x+e\right|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] -1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x + e))/(d*f) + 1/4*sqrt(-d*
f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x + e))/(d*f)
```

Mupad [B]
time = 3.32, size = 54, normalized size = 1.42

$$\frac{\mathrm{atan}\left(\frac{2\,f^{3/2}\,x^5+2\,d\,\sqrt{f}\,x+e\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)-\mathrm{atan}\left(\frac{\sqrt{f}\,x^2}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] (atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan((
f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))
```

3.527 $\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2118, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-4fx^3}{e^2-4dfx^2+4efx^3+4f^2x^6}\,dx &= \left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x}{2e+4fx^3}\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 87, normalized size = 2.29

$$-\frac{\text{RootSum}\left[e^2-4df\#1^2+4ef\#1^3+4f^2\#1^6\&,\frac{-e\log(x-\#1)+4f\log(x-\#1)\#1^3}{-2d\#1+3e\#1^2+6f\#1^5}\&\right]}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6),x]
```

```
[Out] -1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) & ]/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 72, normalized size = 1.89

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4ef\_Z^3-4df\_Z^2+e^2\right)}\frac{\left(4\_R^3f-e\right)\ln\left(x-\_R\right)}{-6f\_R^5-3e\_R^2+2d\_R}}{4f}$
risch	$\frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d+54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}-32d^3f^2\right)x-16e(df)^{\frac{3}{2}}d+27fe^3d\right)}{4\sqrt{df}}-\frac{\ln\left(\left(-32f(df)^{\frac{3}{2}}d-54e^2f^2d\right)x^3+\left(54e^2(df)^{\frac{3}{2}}+\right.\right.}{4\sqrt{df}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/f*sum((4*_R^3*f-e)/(-6*_R^5*f-3*_R^2*e+2*_R*d)*ln(x-_R),_R=RootOf(4*_Z^
6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="ma
xima")
```

```
[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*f*x^3*e - 4*d*f*x^2 + e^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(29) = 58.

time = 0.43, size = 156, normalized size = 4.11

$$\left[\frac{\sqrt{df}\,\log\left(\frac{4\,f^2x^6+4\,fx^3e+4\,dfx^2+4\,(2\,fx^4+xe)\,\sqrt{df}+e^2}{4\,f^2x^6+4\,fx^3e-4\,dfx^2+e^2}\right)}{4\,df},\,-\frac{\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x^2}{d}\right)-\sqrt{-df}\,\arctan\left(\frac{(2\,fx^5+x^2e-2\,dx)\,\sqrt{-df}\,e^{(-1)}}{d}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fr
icas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*f*x^3*e + 4*d*f*x^2 + 4*(2*f*x^4 + x*e)*s
qrt(d*f) + e^2)/(4*f^2*x^6 + 4*f*x^3*e - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqr
t(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + x^2*e - 2*d
*x)*sqrt(-d*f)*e^(-1)/d))/(d*f)]
```

Sympy [A]
time = 0.40, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 4.33, size = 65, normalized size = 1.71

$$\frac{\sqrt{df}\,\log\left(\left|2\,fx^3+2\,\sqrt{df}\,x+e\right|\right)}{4\,df}-\frac{\sqrt{df}\,\log\left(\left|2\,fx^3-2\,\sqrt{df}\,x+e\right|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="gi
ac")
```

```
[Out] 1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x + e))/(d*f) - 1/4*sqrt(d*f)*l
og(abs(2*f*x^3 - 2*sqrt(d*f)*x + e))/(d*f)
```

Mupad [B]
time = 3.12, size = 67, normalized size = 1.76

$$-\frac{\mathrm{atanh}\left(\frac{-32\,f\,d^2\,x+27\,e^3+54\,f\,e^2\,x^3}{16\,d^{3/2}\,e\,\sqrt{f}+32\,d^{3/2}\,f^{3/2}\,x^3-54\,\sqrt{d}\,e^2\,\sqrt{f}\,x}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(3
/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))
```

3.528 $\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2118, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx &= -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+n)^2x^2}\,dx, x, \frac{x}{e(-1+n)+\cdots}\right.\right.\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2+4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.
time = 0.04, size = 78, normalized size = 2.05

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_RE
TURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1/4/(
-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*f*x^n*e + e
^2), x)
```

Fricas [A]
time = 0.42, size = 146, normalized size = 3.84

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,dfx^2-4\,f^2x^{2\,n}-4\,\sqrt{-df}\,xe-4\left(2\,\sqrt{-df}\,fx+fe\right)x^n-e^2}{4\,dfx^2+4\,f^2x^{2\,n}+4\,fx^ne+e^2}\right)}{4\,df},\,-\frac{\sqrt{df}\,\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*x*e - 4*(2*
sqrt(-d*f)*f*x + f*e)*x^n - e^2)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*f*x^n*e + e
^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*
f*x))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fnx^n + 2fx^n}{4dfx^2 + e^2 + 4efx^n + 4f^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] Integral((e - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2 + e**2 + 4*e*f*x**n + 4*f*
*2*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*f*x^n*e + e
^2), x)
```

Mupad [B]
time = 3.42, size = 196, normalized size = 5.16

$$\frac{\ln\left(-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}-\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}\right)}{4\,\sqrt{-d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{x\,\left(8\,d\,f\,n^2-16\,d\,f\,n+8\,d\,f\right)}{4\,\sqrt{d}\,\sqrt{f}\,\left(e\,n-e\,n^2\right)}\right)}{2\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{e^2\,n-4\,d\,f\,x^2+4\,d\,f\,n\,x^2+2\,e\,f\,n\,x^n}{8\,\sqrt{-d}\,f^{5/2}\,x}-\frac{e+2\,f\,x^n-2\,f\,n\,x^n}{4\,f^2}\right)}{4\,\sqrt{-d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2
+ 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x*(
8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(1/2
)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1
/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1/2))
```

3.529 $\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2118, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2118

```
Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)
*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)
^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n},
x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B
*c + 2*A*d*(n - 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx &= -\left(\left(e^2(1-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+n)^2x^2}\,dx, x, \frac{}{e(-1+n)+}\right.\right.\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{e-2f(-1+n)x^n}{e^2-4dfx^2+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)),x]
```

```
[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*
n)), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.
time = 0.04, size = 72, normalized size = 1.89

method	result	size
risch	$\frac{\ln\left(x^n+\frac{2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}-\frac{\ln\left(x^n+\frac{-2dfx+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(d*f)
^(1/2)*ln(x^n+1/2*(-2*d*f*x+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="maxima")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*f*x^n*e - e^
2), x)
```

Fricas [A]
time = 0.47, size = 146, normalized size = 3.84

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\,dfx^2+4\,f^2x^{2\,n}+4\,\sqrt{df}\,xe+4\left(2\,\sqrt{df}\,fx+fe\right)x^n+e^2}{4\,dfx^2-4\,f^2x^{2\,n}-4\,fx^ne-e^2}\right)}{4\,df},-\frac{\sqrt{-df}\,\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfx}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*x*e + 4*(2*sqr
t(d*f)*f*x + f*e)*x^n + e^2)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*f*x^n*e - e^2))
/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f
*x))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\,dx - \int \frac{2fx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\,dx - \int \left(-\frac{2fnx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)),
x)
```

```
[Out] -Integral(e/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(2*f*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integr
al(-2*f*n*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, alg
orithm="giac")
```

```
[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*f*x^n*e - e^
2), x)
```

Mupad [B]
time = 5.30, size = 139, normalized size = 3.66

$$\frac{\ln\left(\frac{\left(e+2\,f\,x^n+2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n+2\,\sqrt{d}\,\sqrt{f}\,x-2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\ln\left(\frac{\left(e+2\,f\,x^n-2\,\sqrt{d}\,\sqrt{f}\,x\right)\,\left(e\,n-2\,\sqrt{d}\,\sqrt{f}\,x+2\,\sqrt{d}\,\sqrt{f}\,n\,x\right)}{x}\right)}{4\,\sqrt{d}\,\sqrt{f}}-\frac{\mathrm{atan}\left(\frac{2\,\sqrt{d}\,\sqrt{f}\,x\,\left(n\,1{}\mathrm{i}-\mathrm{i}\right)}{e\,n}\right)\,1{}\mathrm{i}}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)
```

```
[Out] log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^(
1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)*f
^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x))/x)/(4*d^(1/2
)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1/2)*
f^(1/2))
```

3.530 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

```
[Out] 1/4*arctan((e+2*(d+f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6, 1121, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,(2x^2(d+f)+e)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4}\,dx = \int \frac{x}{e^2+4efx^2+4\left(df+f^2\right)x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^2\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4),x]
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^2+4ef}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^2-e\right)}{8\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^2+e\right)}{8\sqrt{-df}\,e}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
[Out] 1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)
```

Maxima [A]
time = 0.52, size = 35, normalized size = 0.83

$$\frac{\arctan\left(\frac{\left(2\left(df+f^2\right)x^2+fe\right)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/4*arctan((2*(d*f + f^2)*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

Fricas [A]
time = 0.36, size = 147, normalized size = 3.50

$$\left[-\frac{\sqrt{-df}\,e^{(-1)}\log\left(\frac{4\,(d^2f+2\,df^2+f^3)x^4+4\,(df+f^2)x^2e-(d-f)e^2-2\,(2\,(d+f)x^2e+e^2)\sqrt{-df}}{4\,(df+f^2)x^4+4\,fx^2e+e^2}\right)}{8\,df}, \frac{\sqrt{df}\,\arctan\left(\frac{(2\,(d+f)x^2+e)\sqrt{df}\,e^{(-1)}}{d}\right)e^{(-1)}}{4\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-d*f)*e^(-1)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 + 4*(d*f + f^2)*
x^2*e - (d - f)*e^2 - 2*(2*(d + f)*x^2*e + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*
x^4 + 4*f*x^2*e + e^2))/(d*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sqr
t(d*f)*e^(-1)/d)*e^(-1)/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.38, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8 + sqrt
(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e
```

Giac [A]
time = 8.13, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{(2\,dfx^2+2\,f^2x^2+fe)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

Mupad [B]
time = 3.10, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^2+2\,d\,\sqrt{f}\,x^2}{\sqrt{d}\,e}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)*
e*f^(1/2))
```

3.531 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2(d-f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

```
[Out] -1/4*arctanh((e-2*(d-f)*x^2)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6, 1121, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2x^2(d-f))}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4}\,dx &= \int \frac{x}{e^2+4efx^2+(-4df+4f^2)\,x^4}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{e^2+4efx+(-4df+4f^2)\,x^2}\,dx,x,x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx,x,4f\left(e-2(d-f)x^2\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e+2(-d+f)x^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2dx^2+2fx^2)}{\sqrt{d}\,e}\right)}{4\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^2\right)x^2-4ef}{4\sqrt{df}\,e}\right)}{4\sqrt{df}\,e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^2-e\right)}{8\sqrt{df}\,e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^2+e\right)}{8\sqrt{df}\,e}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^(1/2)/e)
```

Maxima [A]
time = 0.51, size = 70, normalized size = 1.59

$$\frac{e^{(-1)}\log\left(\frac{2\,(df-f^2)x^2-fe+\sqrt{df}\,e}{2\,(df-f^2)x^2-fe-\sqrt{df}\,e}\right)}{8\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")
```

```
[Out] 1/8*e^(-1)*log((2*(d*f - f^2)*x^2 - f*e + sqrt(d*f)*e)/(2*(d*f - f^2)*x^2 -
 f*e - sqrt(d*f)*e))/sqrt(d*f)
```

Fricas [A]
time = 0.36, size = 160, normalized size = 3.64

$$\left[\frac{\sqrt{df}\,e^{(-1)}\log\left(-\frac{4\,(d^2f-2\,df^2+f^3)x^4-4\,(df-f^2)x^2e+(d+f)e^2+2\,(2\,(d-f)x^2e-e^2)\sqrt{df}}{4\,(df-f^2)x^4-4\,fx^2e-e^2}\right)}{8\,df},\ \frac{\sqrt{-df}\,\arctan\left(-\frac{(2\,(d-f)x^2-e)\sqrt{-df}\,e^{(-1)}}{d}\right)e^{(-1)}}{4\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="fricas")
```

```
[Out] [1/8*sqrt(d*f)*e^(-1)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 - 4*(d*f - f^2)*x
^2*e + (d + f)*e^2 + 2*(2*(d - f)*x^2*e - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^
4 - 4*f*x^2*e - e^2))/(d*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sqr
t(-d*f)*e^(-1)/d)*e^(-1)/(d*f)]
```

Sympy [A]
time = 0.38, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}-\frac{\sqrt{\frac{1}{df}}\log\left(x^2+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**2 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8 - sqrt(1
/(d*f))*log(x**2 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/8)/e
```

Giac [A]
time = 6.92, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{\left(2\,dfx^2-2\,f^2x^2-fe\right)e^{(-1)}}{\sqrt{-df}}\right)e^{(-1)}}{4\,\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="giac")
```

```
[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)*e^(-1)/sqrt(-d*f))*e^(-1)/sqrt(-d
*f)
```

Mupad [B]
time = 3.11, size = 199, normalized size = 4.52

$$\frac{\operatorname{atanh}\left(\frac{16\,d^{3/2}\,f^{3/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}-\frac{32\,\sqrt{d}\,f^{5/2}\,x^2}{\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}}+\frac{16\,f^{7/2}\,x^2}{\sqrt{d}\,\left(\frac{8\,e\,f^3}{d}-32\,f^3\,x^2-16\,e\,f^2+16\,d\,f^2\,x^2+8\,d\,e\,f+\frac{16\,f^4\,x^2}{d}\right)}\right)}{4\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)
```

```
[Out] atanh((16*d^(3/2)*f^(3/2)*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*
f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^(1/2)*f^(5/2)*x^2)/((8*e*f^3)/d
 - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (16*f
^(7/2)*x^2)/(d^(1/2)*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 +
8*d*e*f + (16*f^4*x^2)/d)))/(4*d^(1/2)*e*f^(1/2))
```

3.532 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6}\,dx &= (3e^2)\,\text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4+4df\#1^6\&,\frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2+3d\#1^4}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) & ]/(8*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 74, normalized size = 1.85

method	result	size
default	$\frac{\sum_{\_R=\text{RootOf}\left(4df\_Z^6+4f^2\_Z^4+4ef\_Z^2+e^2\right)}\frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5+2\_R^3f+e\_R}}{8f}$	74
risch	$-\frac{\ln\left(-2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}+\frac{\ln\left(2d^2f^2x^3+2(-df)^{\frac{3}{2}}fx^2+(-df)^{\frac{3}{2}}e\right)}{4\sqrt{-df}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(x-_R),_R=RootOf(4
*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="maxima")
```

```
[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*f*x^2*e + e^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(31) = 62$.
time = 0.36, size = 212, normalized size = 5.30

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4\,dfx^6-4\,f^2x^4-4\,fx^2e-4\,(2\,fx^5+x^3e)\sqrt{-df}-e^2}{4\,dfx^6+4\,f^2x^4+4\,fx^2e+e^2}\right)}{4\,df},\ \frac{\sqrt{df}\arctan\left(-\frac{2\,(2\,dfx^5+2\,f^2x^3-(dx^3-fx)e)\sqrt{df}\,e^{(-2)}}{d}\right)+\sqrt{df}\arctan\left(\frac{\sqrt{df}\,x}{f}\right)-\sqrt{df}\arctan\left(-\frac{(2\,dfx^3+2\,f^2x-dxe)\sqrt{df}\,e^{(-1)}}{df}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*f*x^2*e - 4*(2*f*x^5 + x^3*
e)*sqrt(-d*f) - e^2)/(4*d*f*x^6 + 4*f^2*x^4 + 4*f*x^2*e + e^2))/(d*f), 1/2*
(sqrt(d*f)*arctan(-2*(2*d*f*x^5 + 2*f^2*x^3 - (d*x^3 - f*x)*e)*sqrt(d*f)*e^
(-2)/d) + sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(-(2*d*f*x^3 +
2*f^2*x - d*x*e)*sqrt(d*f)*e^(-1)/(d*f)))/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.
time = 0.69, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}}\log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2}-fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{-\frac{1}{df}}\log\left(\frac{e\sqrt{-\frac{1}{df}}}{2}+fx^2\sqrt{-\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 +
 sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.
time = 5.75, size = 73, normalized size = 1.82

$$-\frac{\sqrt{-df}\log\left(\left|2\sqrt{-df}\,x^3+2\,fx^2+e\right|\right)}{4\,df}+\frac{\sqrt{-df}\log\left(\left|-2\sqrt{-df}\,x^3+2\,fx^2+e\right|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorith
m="giac")
```

```
[Out] -1/4*sqrt(-d*f)*log(abs(2*sqrt(-d*f)*x^3 + 2*f*x^2 + e))/(d*f) + 1/4*sqrt(-
d*f)*log(abs(-2*sqrt(-d*f)*x^3 + 2*f*x^2 + e))/(d*f)
```

Mupad [B]

time = 3.21, size = 278, normalized size = 6.95

$$\frac{\operatorname{atan}\left(\frac{2 f^2 x+2 d f x^3-d e x}{\sqrt{d}\, e \sqrt{f}}\right)-\operatorname{atan}\left(\frac{1984 d^{3/2} f^{9/2} x^3}{432 d^2 e^2 f^2-128 d e f^4}+\frac{1728 d^{5/2} f^{7/2} x^5}{432 d^2 e^2 f^2-128 d e f^4}+\frac{512 \sqrt{d}\, f^{13/2} x^3}{128 d e^2 f^4-432 d^2 e^3 f^2}+\frac{512 d^{3/2} f^{11/2} x^5}{128 d e^2 f^4-432 d^2 e^3 f^2}-\frac{256 \sqrt{d}\, f^{11/2} x}{432 d^2 e^2 f^2-128 d e f^4}+\frac{864 d^{3/2} e f^{7/2} x}{432 d^2 e^2 f^2-128 d e f^4}-\frac{864 d^{5/2} e f^{5/2} x^3}{432 d^2 e^2 f^2-128 d e f^4}\right)+\operatorname{atan}\left(\frac{\sqrt{d}\, x}{\sqrt{f}}\right)}{2 \sqrt{d}\, \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)
```

```
[Out] (atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3/
2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x^5
)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^2*f
^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432*d^2
*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (864
*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*e*f^(5
/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)))/(2*d
^(1/2)*f^(1/2))
```

3.533 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^3*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2119, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6}\,dx = (3e^2)\,\text{Subst}\left(\int \frac{1}{e^2-36de^2fx^2}\,dx, x, \frac{x^3}{3e+6fx^2}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[e^2+4ef\#1^2+4f^2\#1^4-4df\#1^6\&,\frac{3e\log(x-\#1)\#1+2f\log(x-\#1)\#1^3}{e+2f\#1^2-3d\#1^4}\&\right]}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x
]
```

```
[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1
+ 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) & ]/(8*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.10, size = 77, normalized size = 1.92

method	result	size
default	$-\frac{\sum_{\_R=\text{RootOf}\left(4df\_Z^6-4f^2\_Z^4-4ef\_Z^2-e^2\right)}\frac{\left(2\_R^4f+3e\_R^2\right)\ln\left(x-\_R\right)}{3d\_R^5-2\_R^3f-e\_R}}{8f}$	77
risch	$\frac{\ln\left(-2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}-\frac{\ln\left(2d^2f^2x^3-2(df)^{\frac{3}{2}}fx^2-(df)^{\frac{3}{2}}e\right)}{4\sqrt{df}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(x-_R),_R=RootOf(
4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="maxima")
```

```
[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*f*x^2*e - e^2), x
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(31) = 62$.
time = 0.36, size = 215, normalized size = 5.38

$$\left[\frac{\sqrt{df}\log\left(\frac{4\,dfx^6+4\,f^2x^4+4\,fx^2e+4\,(2\,fx^5+x^3e)\sqrt{df}+e^2}{4\,dfx^6-4\,f^2x^4-4\,fx^2e-e^2}\right)}{4\,df}, -\frac{\sqrt{-df}\arctan\left(-\frac{2\,(2\,dfx^5-2\,f^2x^3-(dx^3+fx)e)\sqrt{-df}\,e^{(-2)}}{d}\right)+\sqrt{-df}\arctan\left(\frac{\sqrt{-df}\,x}{f}\right)-\sqrt{-df}\arctan\left(-\frac{(2\,dfx^3-2\,f^2x-dxe)\sqrt{-df}\,e^{(-1)}}{df}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*f*x^2*e + 4*(2*f*x^5 + x^3*e)
*sqrt(d*f) + e^2)/(4*d*f*x^6 - 4*f^2*x^4 - 4*f*x^2*e - e^2))/(d*f), -1/2*(s
qrt(-d*f)*arctan(-2*(2*d*f*x^5 - 2*f^2*x^3 - (d*x^3 + f*x)*e)*sqrt(-d*f)*e^
(-2)/d) + sqrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(-(2*d*f*x^3
 - 2*f^2*x - d*x*e)*sqrt(-d*f)*e^(-1)/(d*f)))/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.
time = 0.70, size = 80, normalized size = 2.00

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-\frac{e\sqrt{\frac{1}{df}}}{2}-fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(\frac{e\sqrt{\frac{1}{df}}}{2}+fx^2\sqrt{\frac{1}{df}}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sq
rt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(31) = 62$.
time = 6.52, size = 69, normalized size = 1.72

$$\frac{\sqrt{df}\log\left(\left|2\sqrt{df}\,x^3+2\,fx^2+e\right|\right)}{4\,df}-\frac{\sqrt{df}\log\left(\left|-2\sqrt{df}\,x^3+2\,fx^2+e\right|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorit
hm="giac")
```

```
[Out] 1/4*sqrt(d*f)*log(abs(2*sqrt(d*f)*x^3 + 2*f*x^2 + e))/(d*f) - 1/4*sqrt(d*f)
*log(abs(-2*sqrt(d*f)*x^3 + 2*f*x^2 + e))/(d*f)
```

Mupad [B]
time = 3.14, size = 30, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^3}{2\,f\,x^2+e}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)
[Out] atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.534 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\,\sqrt{f}\,(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^
2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-1+m)^2(1+m)^2\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A]
time = 0.94, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.
time = 0.06, size = 78, normalized size = 1.86

method	result	size
risch	$-\frac{\ln\left(x^m+\frac{(2f x^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^m-\frac{(2f x^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + (m + 1)*e)*x^m/(4*f^2*x^4 + 4*f*x^2*e + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A]
time = 0.36, size = 148, normalized size = 3.52

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,f^2x^4-4\,dfx^2x^{2\,m}+4\,fx^2e+4\,(2\,fx^3+xe)\sqrt{-df}\,x^m+e^2}{4\,f^2x^4+4\,dfx^2x^{2\,m}+4\,fx^2e+e^2}\right)}{4\,df},-\frac{\sqrt{df}\,\arctan\left(\frac{(2\,fx^2+e)\sqrt{df}}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*f*x^2*e + 4*(2*f*x
^3 + x*e)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*f*x^2*e
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + (m + 1)*e)*x^m/(4*f^2*x^4 + 4*f*x^2*e + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2+4\,d\,f\,x^{2\,m+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*
f*x^(2*m + 2)), x)
```

3.535 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^2+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2119, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x
^2))]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}}\,dx = -\left(\left(e^2(1-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-1+m)^2(1+m)^2\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A]
time = 0.25, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.
time = 0.06, size = 74, normalized size = 1.76

method	result	size
risch	$\frac{\ln\left(x^m+\frac{(2f x^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}-\frac{\ln\left(x^m-\frac{(2f x^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),
x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + (m + 1)*e)*x^m/(4*f^2*x^4 + 4*f*x^2*e - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A]
time = 0.36, size = 148, normalized size = 3.52

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^4+4dfx^2x^{2m}+4fx^2e+4(2fx^3+xe)\sqrt{df}\,x^m+e^2}{4f^2x^4-4dfx^2x^{2m}+4fx^2e+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{(2fx^2+e)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*f*x^2*e + 4*(2*f*x^3
 + x*e)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*f*x^2*e + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + (m + 1)*e)*x^m/(4*f^2*x^4 + 4*f*x^2*e - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-1\right)\,x^2+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^4+4\,e\,f\,x^2-4\,d\,f\,x^{2\,m+2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)),x)
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*
f*x^(2*m + 2)), x)
```

3.536 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6}\,dx &= -\left((2e^2)\,\text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[e^2+4ef\#1^3+4df\#1^4+4f^2\#1^6\&,\frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1+4d\#1^2+6f\#1^4}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) & ]/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 74, normalized size = 1.85

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6+4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(-\_R^4 f+e\_R\right)\ln\left(x-\_R\right)}{6f\_R^5+4d\_R^3+3e\_R^2}}{2f}$
risch	$-\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d+54df^3e\right)x^3+\left(54(-df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2+8e(-df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{-df}}+\frac{\ln\left(\left(16f(-df)^{\frac{3}{2}}d-54df^3e\right)x^3+\left(54(-d\right.\right.}{4\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNVE
RBOSE)
[Out] 1/2/f*sum((-_R^4*f+_R*e)/(6*_R^5*f+4*_R^3*d+3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*f*x^3*e + e^2), x)
```

Fricas [A]
time = 0.37, size = 154, normalized size = 3.85

$$\left[-\frac{\sqrt{-df}\log\left(\frac{4f^2x^6-4dfx^4+4fx^3e+4\left(2fx^5+x^2e\right)\sqrt{-df}+e^2}{4f^2x^6+4dfx^4+4fx^3e+e^2}\right)}{4df},\frac{\sqrt{df}\arctan\left(\frac{\left(2fx^4+2dx^2+xe\right)\sqrt{df}\,e^{(-1)}}{d}\right)-\sqrt{df}\arctan\left(\frac{\sqrt{df}\,x}{d}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*f*x^3*e + 4*(2*f*x^5 + x^2*
e)*sqrt(-d*f) + e^2)/(4*f^2*x^6 + 4*d*f*x^4 + 4*f*x^3*e + e^2))/(d*f), 1/2*
(sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + x*e)*sqrt(d*f)*e^(-1)/d) - sqrt(d*f)
*arctan(sqrt(d*f)*x/d))/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.
time = 0.65, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}}\log\left(-dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}-\frac{\sqrt{-\frac{1}{df}}\log\left(dx^2\sqrt{-\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6+4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] sqrt(-1/(d*f))*log(-d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*
f))*log(d*x**2*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(31) = 62$.
time = 5.18, size = 73, normalized size = 1.82

$$-\frac{\sqrt{-df}\log\left(\left|2fx^3+2\sqrt{-df}x^2+e\right|\right)}{4df}+\frac{\sqrt{-df}\log\left(\left|2fx^3-2\sqrt{-df}x^2+e\right|\right)}{4df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] -1/4*sqrt(-d*f)*log(abs(2*f*x^3 + 2*sqrt(-d*f)*x^2 + e))/(d*f) + 1/4*sqrt(-
d*f)*log(abs(2*f*x^3 - 2*sqrt(-d*f)*x^2 + e))/(d*f)
```

Mupad [B]
time = 3.49, size = 233, normalized size = 5.82

$$\frac{\operatorname{atan}\left(\frac{128\,d^{7/2}\,\sqrt{f}\,x^2}{64\,d^3\,e+729\,f\,e^3}-\frac{216\,d^{3/2}\,e^2\,\sqrt{f}}{64\,d^3\,e+729\,f\,e^3}+\frac{128\,d^{5/2}\,f^{3/2}\,x^4}{64\,d^3\,e+729\,f\,e^3}+\frac{216\,d^{3/2}\,e\,\sqrt{f}}{64\,d^3+729\,f\,e^2}+\frac{729\,d^{3/2}\,e^2\,f^{3/2}\,x}{64\,d^5+729\,f\,d^2\,e^2}+\frac{1458\,d^{3/2}\,e\,f^{5/2}\,x^4}{64\,d^5+729\,f\,d^2\,e^2}+\frac{64\,d^{5/2}\,e\,\sqrt{f}\,x}{64\,d^3\,e+729\,f\,e^3}+\frac{1458\,d^{3/2}\,e\,f^{3/2}\,x^2}{64\,d^4+729\,f\,d\,e^2}\right)-\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] (atan((128*d^(7/2)*f^(1/2)*x^2)/(64*d^3*e + 729*e^3*f) - (216*d^(3/2)*e^2*f
^(1/2))/(64*d^3*e + 729*e^3*f) + (128*d^(5/2)*f^(3/2)*x^4)/(64*d^3*e + 729*
e^3*f) + (216*d^(3/2)*e*f^(1/2))/(729*e^2*f + 64*d^3) + (729*d^(3/2)*e^2*f^
(3/2)*x)/(64*d^5 + 729*d^2*e^2*f) + (1458*d^(3/2)*e*f^(5/2)*x^4)/(64*d^5 +
729*d^2*e^2*f) + (64*d^(5/2)*e*f^(1/2)*x)/(64*d^3*e + 729*e^3*f) + (1458*d^
(3/2)*e*f^(3/2)*x^2)/(64*d^4 + 729*d*e^2*f)) - atan((f^(1/2)*x)/d^(1/2)))/(
2*d^(1/2)*f^(1/2))
```

3.537 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^2*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2119, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6}\,dx &= -\left(\left(2e^2\right)\text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2}\,dx, x, \frac{x^2}{-2e-4fx^3}\right)\right)\\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^2}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 86, normalized size = 2.15

$$-\frac{\text{RootSum}\left[e^2+4ef\#1^3-4df\#1^4+4f^2\#1^6\&,\frac{-e\log(x-\#1)+f\log(x-\#1)\#1^3}{3e\#1-4d\#1^2+6f\#1^4}\&\right]}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]
```

```
[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 & , (-(e*Log[x - #1
]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) & ]/f
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 74, normalized size = 1.85

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(4f^2\_Z^6-4df\_Z^4+4ef\_Z^3+e^2\right)}\frac{\left(\_R^4 f-e\_R\right)\ln\left(x-\_R\right)}{-6f\_R^5+4d\_R^3-3e\_R^2}}{2f}$
risch	$\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d+54d f^3 e\right)x^3+\left(54(df)^{\frac{3}{2}}ef-16d^3f^2\right)x^2-8e(df)^{\frac{3}{2}}d+27e^2f^2d\right)}{4\sqrt{df}}-\frac{\ln\left(\left(-16f(df)^{\frac{3}{2}}d-54d f^3 e\right)x^3+\left(54(df)^{\frac{3}{2}}ef+\right.\right.}{4\sqrt{df}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/2/f*sum((_R^4*f-_R*e)/(-6*_R^5*f+4*_R^3*d-3*_R^2*e)*ln(x-_R),_R=RootOf(4*
_Z^6*f^2-4*_Z^4*d*f+4*_Z^3*e*f+e^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="maxima")
```

```
[Out] -2*integrate((f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*f*x^3*e + e^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(31) = 62.

time = 0.36, size = 156, normalized size = 3.90

$$\left[\frac{\sqrt{df}\,\log\left(\frac{4\,f^2x^6+4\,dfx^4+4\,fx^3e+4\,(2\,fx^5+x^2e)\,\sqrt{df}+e^2}{4\,f^2x^6-4\,dfx^4+4\,fx^3e+e^2}\right)}{4\,df}, \frac{\sqrt{-df}\,\arctan\left(\frac{(2\,fx^4-2\,dx^2+xe)\,\sqrt{-df}\,e^{(-1)}}{d}\right)-\sqrt{-df}\,\arctan\left(\frac{\sqrt{-df}\,x}{d}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*f*x^3*e + 4*(2*f*x^5 + x^2*e)
*sqrt(d*f) + e^2)/(4*f^2*x^6 - 4*d*f*x^4 + 4*f*x^3*e + e^2))/(d*f), 1/2*(sq
rt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + x*e)*sqrt(-d*f)*e^(-1)/d) - sqrt(-d*f)
*arctan(sqrt(-d*f)*x/d))/(d*f)]
```

Sympy [A]
time = 0.66, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}}\log\left(-dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}+\frac{\sqrt{\frac{1}{df}}\log\left(dx^2\sqrt{\frac{1}{df}}+\frac{e}{2f}+x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x**3+2*e)/(4*f**2*x**6-4*d*f*x**4+4*e*f*x**3+e**2),x)
```

```
[Out] -sqrt(1/(d*f))*log(-d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f)
)*log(d*x**2*sqrt(1/(d*f)) + e/(2*f) + x**3)/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(31) = 62.
time = 5.09, size = 69, normalized size = 1.72

$$\frac{\sqrt{df}\,\log\left(\left|2\,fx^3+2\,\sqrt{df}\,x^2+e\right|\right)}{4\,df}-\frac{\sqrt{df}\,\log\left(\left|2\,fx^3-2\,\sqrt{df}\,x^2+e\right|\right)}{4\,df}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2),x, algorithm
="giac")
```

```
[Out] 1/4*sqrt(d*f)*log(abs(2*f*x^3 + 2*sqrt(d*f)*x^2 + e))/(d*f) - 1/4*sqrt(d*f)
*log(abs(2*f*x^3 - 2*sqrt(d*f)*x^2 + e))/(d*f)
```

Mupad [B]
time = 3.38, size = 67, normalized size = 1.68

$$-\frac{\mathrm{atanh}\left(\frac{27\,e^2\,\sqrt{f}+54\,e\,f^{3/2}\,x^3-16\,d^2\,\sqrt{f}\,x^2}{8\,d^{3/2}\,e+16\,d^{3/2}\,f\,x^3-54\,\sqrt{d}\,e\,f\,x^2}\right)}{2\,\sqrt{d}\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2*e - 2*f*x^3))/(e^2 + 4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3),x)
```

```
[Out] -atanh((27*e^2*f^(1/2) + 54*e*f^(3/2)*x^3 - 16*d^2*f^(1/2)*x^2)/(8*d^(3/2)*
e + 16*d^(3/2)*f*x^3 - 54*d^(1/2)*e*f*x^2))/(2*d^(1/2)*f^(1/2))
```

3.538 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6} dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

```
[Out] 1/6*arctan((e+2*(d+f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6, 1366, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{f}\,(2x^3(d+f)+e)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6}\,dx &= \int \frac{x^2}{e^2+4efx^3+4\left(df+f^2\right)x^6}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+4\left(df+f^2\right)x^2}\,dx, x, x^3\right)\\ &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16de^2f-x^2}\,dx, x, 4f\left(e+2(d+f)x^3\right)\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}\,(e+2(d+f)x^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]
```

```
[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])
```

Maple [A]
time = 0.02, size = 42, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2\left(4df+4f^2\right)x^3+4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$-\frac{\ln\left(\left(2\sqrt{-df}-2f\right)x^3-e\right)}{12\sqrt{-df}\,e}+\frac{\ln\left(\left(2\sqrt{-df}+2f\right)x^3+e\right)}{12\sqrt{-df}\,e}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)
```

Maxima [A]
time = 0.52, size = 35, normalized size = 0.83

$$\frac{\arctan\left(\frac{(2\,(df+f^2)x^3+fe)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] 1/6*arctan((2*(d*f + f^2)*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

Fricas [A]
time = 0.36, size = 147, normalized size = 3.50

$$\left[-\frac{\sqrt{-df}\,e^{(-1)}\log\left(\frac{4\,(d^2f+2\,df^2+f^3)x^6+4\,(df+f^2)x^3e-(d-f)e^2-2\,(2\,(d+f)x^3e+e^2)\,\sqrt{-df}}{4\,(df+f^2)x^6+4\,fx^3e+e^2}\right)}{12\,df}, \frac{\sqrt{df}\,\arctan\left(\frac{(2\,(d+f)x^3+e)\,\sqrt{df}\,e^{(-1)}}{d}\right)e^{(-1)}}{6\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-d*f)*e^(-1)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*f + f^2)
*x^3*e - (d - f)*e^2 - 2*(2*(d + f)*x^3*e + e^2)*sqrt(-d*f))/(4*(d*f + f^2)
*x^6 + 4*f*x^3*e + e^2))/(d*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*sq
rt(d*f)*e^(-1)/d)*e^(-1)/(d*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.48, size = 78, normalized size = 1.86

$$\frac{-\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}+\frac{\sqrt{-\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{-\frac{1}{df}}+e}{2d+2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqr
t(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e
```

Giac [A]
time = 4.55, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{(2\,dfx^3+2\,f^2x^3+fe)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
```

```
[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)
```

Mupad [B]
time = 3.43, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\,\sqrt{f}+2\,f^{3/2}\,x^3+2\,d\,\sqrt{f}\,x^3}{\sqrt{d}\,e}\right)}{6\,\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)
```

```
[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)*
e*f^(1/2))
```

3.539 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2(d-f)x^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

[Out] -1/6*arctanh((e-2*(d-f)*x^3)*f^(1/2)/e/d^(1/2))/e/d^(1/2)/f^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6, 1366, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2x^3(d-f))}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]

[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6}\,dx &= \int \frac{x^2}{e^2+4efx^3+(-4df+4f^2)\,x^6}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{e^2+4efx+(-4df+4f^2)\,x^2}\,dx,x,x^3\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{16de^2f-x^2}\,dx,x,4f\left(e-2(d-f)x^3\right)\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2(d-f)x^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.05

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}\,(e-2dx^3+2fx^3)}{\sqrt{d}\,e}\right)}{6\sqrt{d}\,e\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]
[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt
[f])
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.95

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\left(4df-4f^2\right)x^3-4ef}{4\sqrt{df}\,e}\right)}{6\sqrt{df}\,e}$	42
risch	$\frac{\ln\left(\left(-2\sqrt{df}-2f\right)x^3-e\right)}{12\sqrt{df}\,e}-\frac{\ln\left(\left(-2\sqrt{df}+2f\right)x^3+e\right)}{12\sqrt{df}\,e}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x,method=_RETURNVERBOSE)
[Out] 1/6/(d*f)^(1/2)/e*arctanh(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^(1/2)/e)
```

Maxima [A]
time = 0.52, size = 70, normalized size = 1.59

$$\frac{e^{(-1)}\log\left(\frac{2\,(df-f^2)x^3-fe+\sqrt{df}\,e}{2\,(df-f^2)x^3-fe-\sqrt{df}\,e}\right)}{12\,\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")
```

```
[Out] 1/12*e^(-1)*log((2*(d*f - f^2)*x^3 - f*e + sqrt(d*f)*e)/(2*(d*f - f^2)*x^3
- f*e - sqrt(d*f)*e))/sqrt(d*f)
```

Fricas [A]
time = 0.36, size = 160, normalized size = 3.64

$$\left[\frac{\sqrt{df}\,e^{(-1)}\log\left(-\frac{4\,(d^2f-2\,df^2+f^3)x^6-4\,(df-f^2)x^3e+(d+f)e^2+2\,(2\,(d-f)x^3e-e^2)\,\sqrt{df}}{4\,(df-f^2)x^6-4\,fx^3e-e^2}\right)}{12\,df},\ \frac{\sqrt{-df}\,\arctan\left(-\frac{(2\,(d-f)x^3-e)\,\sqrt{-df}\,e^{(-1)}}{d}\right)e^{(-1)}}{6\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="fricas")
```

```
[Out] [1/12*sqrt(d*f)*e^(-1)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*f - f^2)*
x^3*e + (d + f)*e^2 + 2*(2*(d - f)*x^3*e - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x
^6 - 4*f*x^3*e - e^2))/(d*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*sq
rt(-d*f)*e^(-1)/d)*e^(-1)/(d*f)]
```

Sympy [A]
time = 0.48, size = 75, normalized size = 1.70

$$-\frac{\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{-de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}-\frac{\sqrt{\frac{1}{df}}\log\left(x^3+\frac{de\sqrt{\frac{1}{df}}-e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)
```

```
[Out] -(sqrt(1/(d*f))*log(x**3 + (-d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12 - sqrt(
1/(d*f))*log(x**3 + (d*e*sqrt(1/(d*f)) - e)/(2*d - 2*f))/12)/e
```

Giac [A]
time = 5.46, size = 41, normalized size = 0.93

$$-\frac{\arctan\left(\frac{(2\,dfx^3-2\,f^2x^3-fe)e^{(-1)}}{\sqrt{-df}}\right)e^{(-1)}}{6\,\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="giac")
[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)*e^(-1)/sqrt(-d*f))*e^(-1)/sqrt(-d*f)
```

Mupad [B]
time = 3.53, size = 923, normalized size = 20.98

$$\frac{\mathrm{atan}\left(\frac{\frac{\left(x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)+\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3-\frac{\frac{x^3\left(-384d^3e^2f^5+1152d^2e^2f^6-1152de^2f^7+384e^2f^8\right)}{12}+16e^3f^7-48de^3f^6+48d^2e^3f^5-16d^3e^3f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}\right)1i}{\sqrt{d}\,e\,\sqrt{f}}+\frac{\left(x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)-\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3+\frac{\frac{x^3\left(-384d^3e^2f^5+1152d^2e^2f^6-1152de^2f^7+384e^2f^8\right)}{12}+16e^3f^7-48de^3f^6+48d^2e^3f^5-16d^3e^3f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}\right)1i}{\sqrt{d}\,e\,\sqrt{f}}}{\frac{x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)+\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3-\frac{\frac{x^3\left(-384d^3e^2f^5+1152d^2e^2f^6-1152de^2f^7+384e^2f^8\right)}{12}+16e^3f^7-48de^3f^6+48d^2e^3f^5-16d^3e^3f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}-\frac{x^3\left(32d^3f^3-96d^2f^4+96df^5-32f^6\right)-\frac{x^3\left(-64ed^3f^4+192ed^2f^5-192edf^6+64ef^7\right)+16e^2f^6-48de^2f^5+48d^2e^2f^4-16d^3e^2f^3+\frac{\frac{x^3\left(-384d^3e^2f^5+1152d^2e^2f^6-1152de^2f^7+384e^2f^8\right)}{12}+16e^3f^7-48de^3f^6+48d^2e^3f^5-16d^3e^3f^4}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}{\sqrt{d}\,e\,\sqrt{f}}}\right)1i}{6\sqrt{d}\,e\,\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)
[Out] (atan((((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d^(1/2)*e*f^(1/2)) + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))*1i)/(d^(1/2)*e*f^(1/2)))/((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)) - (x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2)))/(d^(1/2)*e*f^(1/2))))*1i)/(6*d^(1/2)*e*f^(1/2))
```

3.540 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(-2+m)^2(1+m)^2x^2}\right.\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [**A**]
time = 2.28, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 + 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.
time = 0.03, size = 78, normalized size = 1.86

method	result	size
risch	$-\frac{\ln\left(x^m+\frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^m-\frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(m-2)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x
,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/
2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + (m + 1)*e)*x^m/(4*f^2*x^6 + 4*f*x^3*e + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [**A**]
time = 0.37, size = 148, normalized size = 3.52

$$\left[-\frac{\sqrt{-df}\log\left(-\frac{4f^2x^6-4dfx^2x^{2m}+4fx^3e+4(2fx^4+xe)\sqrt{-df}x^m+e^2}{4f^2x^6+4dfx^2x^{2m}+4fx^3e+e^2}\right)}{4df}, -\frac{\sqrt{df}\arctan\left(\frac{(2fx^3+e)\sqrt{df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*f*x^3*e + 4*(2*f*x
^4 + x*e)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*f*x^3*e
+ e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m
))/(d*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + (m + 1)*e)*x^m/(4*f^2*x^6 + 4*f*x^3*e + 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-2\right)\,x^3+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3+4\,d\,f\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*
f*x^(2*m + 2)), x)
```

3.541 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(2*f*x^3+e))/d^(1/2)/f^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2119, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d
*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}}\,dx = -\left(\left(e^2(2-m)(1+m)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(-2+m)^2(1+m)^2}\right.\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Mathematica [A]
time = 0.42, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6
 - 4*d*f*x^(2 + 2*m)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(32) = 64.
time = 0.03, size = 74, normalized size = 1.76

method	result	size
risch	$\frac{\ln\left(x^m+\frac{(2fx^3+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}-\frac{\ln\left(x^m-\frac{(2fx^3+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(m-2)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*l
n(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + (m + 1)*e)*x^m/(4*f^2*x^6 + 4*f*x^3*e - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Fricas [A]
time = 0.39, size = 148, normalized size = 3.52

$$\left[\frac{\sqrt{df}\log\left(-\frac{4f^2x^6+4dfx^2x^{2m}+4fx^3e+4(2fx^4+xe)\sqrt{df}x^m+e^2}{4f^2x^6-4dfx^2x^{2m}+4fx^3e+e^2}\right)}{4df}, -\frac{\sqrt{-df}\arctan\left(\frac{(2fx^3+e)\sqrt{-df}}{2dfxx^m}\right)}{2df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*f*x^3*e + 4*(2*f*x^4
 + x*e)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*f*x^3*e + e
^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m)
)/(d*f)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f
*x**(2+2*m)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+
2*m)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + (m + 1)*e)*x^m/(4*f^2*x^6 + 4*f*x^3*e - 4*d*f*
x^(2*m + 2) + e^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(2\,f\,\left(m-2\right)\,x^3+e\,\left(m+1\right)\right)}{e^2+4\,f^2\,x^6+4\,e\,f\,x^3-4\,d\,f\,x^{2\,m+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*
f*x^(2*m + 2)), x)
```

3.542 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*arctan(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2119, 211}

$$\frac{\text{ArcTan}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2+4de^2f(1+m)^2(1+m-\right.$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 + 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(32) = 64.
time = 0.06, size = 84, normalized size = 2.00

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{2dfx\,x^m+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}+\frac{\ln\left(x^n+\frac{-2dfx\,x^m+\sqrt{-df}\,e}{2\sqrt{-df}\,f}\right)}{4\sqrt{-df}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1
/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + (m + 1)*e)*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*f*x^n*e + e^2), x)
```

Fricas [A]
time = 0.39, size = 167, normalized size = 3.98

$$\left[-\frac{\sqrt{-df}\,\log\left(-\frac{4\,dfx^2x^{2\,m}-4\,\sqrt{-df}\,xx^me-4\,f^2x^{2\,n}-4\left(2\,\sqrt{-df}\,fxx^m+fe\right)x^n-e^2}{4\,dfx^2x^{2\,m}+4\,f^2x^{2\,n}+4\,fx^ne+e^2}\right)}{4\,df},\,-\frac{\sqrt{df}\,\arctan\left(\frac{2\,\sqrt{df}\,fx^n+\sqrt{df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*x*x^m*e - 4*f^2*x^(
2*n) - 4*(2*sqrt(-d*f)*f*x*x^m + f*e)*x^n - e^2)/(4*d*f*x^2*x^(2*m) + 4*f^2
*x^(2*n) + 4*f*x^n*e + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*
f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m(em + e + 2fmx^n - 2fnx^n + 2fx^n)}{4dfx^2x^{2m} + e^2 + 4efx^n + 4f^2x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2*x*
*(2*m) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((2*f*(m - n + 1)*x^n + (m + 1)*e)*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*
x^(2*n) + 4*f*x^n*e + e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(e\,\left(m+1\right)+2\,f\,x^n\,\left(m-n+1\right)\right)}{e^2+4\,f^2\,x^{2\,n}+4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

3.543 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

```
[Out] 1/2*arctanh(2*x^(1+m)*d^(1/2)*f^(1/2)/(e+2*f*x^n))/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2119, 214}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*
f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2119

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)
*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*((m - n + 1)/(m + 1)
), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n +
1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2
*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] &
& EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]
```

Rubi steps

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx = \left(e^2(1+m)(1+m-n)\right)\text{Subst}\left(\int \frac{1}{e^2-4de^2f(1+m)^2(1+m-\right.$$

$$= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\,\sqrt{f}\,x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\,\sqrt{f}}$$

Mathematica [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)),x]
```

```
[Out] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m)
+ 4*e*f*x^n + 4*f^2*x^(2*n)), x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.
time = 0.06, size = 78, normalized size = 1.86

method	result	size
risch	$\frac{\ln\left(x^n+\frac{2dfx\,x^m+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}-\frac{\ln\left(x^n+\frac{-2dfx\,x^m+\sqrt{df}\,e}{2\sqrt{df}\,f}\right)}{4\sqrt{df}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2
*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(
d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + (m + 1)*e)*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*f*x^n*e - e^2), x)
```

Fricas [A]
time = 0.36, size = 167, normalized size = 3.98

$$\left[\frac{\sqrt{df}\,\log\left(-\frac{4\,dfx^2x^{2\,m}+4\,\sqrt{df}\,xx^me+4\,f^2x^{2\,n}+4\,\left(2\,\sqrt{df}\,fxx^m+fe\right)x^n+e^2}{4\,dfx^2x^{2\,m}-4\,f^2x^{2\,n}-4\,fx^ne-e^2}\right)}{4\,df},-\frac{\sqrt{-df}\,\arctan\left(\frac{2\,\sqrt{-df}\,fx^n+\sqrt{-df}\,e}{2\,dfxx^m}\right)}{2\,df}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m) + 4*sqrt(d*f)*x*x^m*e + 4*f^2*x^(2*n
) + 4*(2*sqrt(d*f)*f*x*x^m + f*e)*x^n + e^2)/(4*d*f*x^2*x^(2*m) - 4*f^2*x^(
2*n) - 4*f*x^n*e - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*
x^n + sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{ex^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{emx^m}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \frac{2fmx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\,dx-\int \left(-\frac{2fnx^mx^n}{4dfx^2x^{2m}-e^2-4efx^n-4f^2x^{2n}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n
+4*f**2*x**(2*n)),x)
```

```
[Out] -Integral(e*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)
), x) - Integral(e*m*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2
*x**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f
*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**2*x**(2*m
) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(4
*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^
2*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(-(2*f*(m - n + 1)*x^n + (m + 1)*e)*x^m/(4*d*f*x^(2*m + 2) - 4*f^2
*x^(2*n) - 4*f*x^n*e - e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(e\,\left(m+1\right)+2\,f\,x^n\,\left(m-n+1\right)\right)}{e^2+4\,f^2\,x^{2\,n}-4\,d\,f\,x^{2\,m+2}+4\,e\,f\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^
(2*m + 2) + 4*e*f*x^n), x)
```

3.544 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=134

$$-\frac{(2ac^2-d^2)\,x^2}{2b^2c^3}+\frac{d(2ac^2-d^2)\sqrt{a+bx^2}}{b^3c^4}-\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(a+bx^2)^2}{4b^3c}+\frac{(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{b^3c^5}$$

```
[Out] -1/2*(2*a*c^2-d^2)*x^2/b^2/c^3-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2+1/4*(b*x^2+a)^
2/b^3/c+(a*c^2-d^2)^2*ln(d+c*(b*x^2+a)^(1/2))/b^3/c^5+d*(2*a*c^2-d^2)*(b*x^
2+a)^(1/2)/b^3/c^4
```

Rubi [A]
time = 0.25, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2186, 711}

$$-\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(ac^2-d^2)^2\log\left(c\sqrt{a+bx^2}+d\right)}{b^3c^5}+\frac{d\sqrt{a+bx^2}\,(2ac^2-d^2)}{b^3c^4}+\frac{(a+bx^2)^2}{4b^3c}-\frac{x^2(2ac^2-d^2)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*((2*a*c^2 - d^2)*x^2)/(b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/
(b^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + (
(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{(a-x^2)^2}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b^3}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{(2ac^2-d^2)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{(ac^2-d^2)^2}{c^4(d+cx)}\right)dx,x,\sqrt{a+bx^2}\right)}{b^3}$$
$$= -\frac{(2ac^2-d^2)x^2}{2b^2c^3}+\frac{d(2ac^2-d^2)\sqrt{a+bx^2}}{b^3c^4}-\frac{d(a+bx^2)^{3/2}}{3b^3c^2}+\frac{(a+bx^2)^2}{4b^3c}$$

Mathematica [A]
time = 0.16, size = 125, normalized size = 0.93

$$\frac{-4cd\sqrt{a+bx^2}\,(-5ac^2+3d^2+bc^2x^2)+3c^2(-3a^2c^2+2a(d^2-bc^2x^2)+bx^2(2d^2+bc^2x^2))+12(-ac^2+d^2)^2\log\left(d+c\sqrt{a+bx^2}\right)}{12b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (-4*c*d*Sqrt[a + b*x^2]*(-5*a*c^2 + 3*d^2 + b*c^2*x^2) + 3*c^2*(-3*a^2*c^2
+ 2*a*(d^2 - b*c^2*x^2) + b*x^2*(2*d^2 + b*c^2*x^2)) + 12*(-(a*c^2) + d^2)^
2*Log[d + c*Sqrt[a + b*x^2]])/(12*b^3*c^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4946 vs. $2(122) = 244$.
time = 0.06, size = 4947, normalized size = 36.92

method	result	size
default	Expression too large to display	4947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-
(a*c^2-d^2)*b*c^2)^(1/2))/c^4*d^7/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*
c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2
)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2
)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2))-1/2*d/b^2*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2)
)^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)-1/2*d/b^2*c^2*a^2/((-a*b)^(1
```

/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^
(1/2))*(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)
+1/2*d/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-
(-(a*c^2-d^2)*b*c^2)^(1/2))*c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2
/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^
2)^(1/2)*a^2-1/2*d/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((
-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((
-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b
^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)
^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*a^2-1/2/b^(5/2)
/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d
^2)*b*c^2)^(1/2))/c^4*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c^2*(-(a*c^2-d^2)*b
*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*d^5-1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(d^2
/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a^2-1/2/b^2/((-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2
)^(1/2))/c^4*d^7/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(
1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
-1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b
)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2
))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*
(-a*b)^(1/2)))^(1/2))+1/2*d/b^(5/2)*c^2*a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*l
n(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2
*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d
^2/c^2)^(1/2))*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((
-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(d^2/c^2)^(1/2)*ln((2*d
^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^
2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x
+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a-1/b^(5/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-
d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(-(a*c
^2-d^2)*b*c^2)^(1/2)*ln((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/

```
c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2
)^(1/2))*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^
(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(d^2/c^2)^(1/2)*ln((2*d^2/c^2
+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d
^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)
*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1/2/b^2/c^3*x^2*d^2+1/2/b^3/c^5*d^4*ln(b*c^
2*x^2+a*c^2-d^2)-1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b
)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)+d^2/c^2)^(1/2)*a*d^3+1/2/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*(b*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+d^2/c^2)^(1/2)*d^5-1/b^2/((-a*b)^...
```

Maxima [A]
time = 0.27, size = 125, normalized size = 0.93

$$\frac{\frac{3\,(bx^2+a)^2c^3-4\,(bx^2+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^2+a)+12\,(2\,ac^2d-d^3)\sqrt{bx^2+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^5}}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3
```

Fricas [A]
time = 0.38, size = 233, normalized size = 1.74

$$\frac{3\,b^2c^4x^4-6\,(abc^4-bc^2d^2)x^2+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(bc^2x^2+ac^2-d^2\right)+3\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-3\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-4\,(bc^3dx^2-5\,ac^3d+3\,cd^3)\sqrt{bx^2+a}}{12\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a
*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2)
 - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [A]
time = 4.00, size = 155, normalized size = 1.16

$$\frac{(a^2c^4-2ac^2d^2+d^4)\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{b^3c^5}+\frac{3\,(bx^2+a)^2b^9c^3-12\,(bx^2+a)ab^9c^3-4\,(bx^2+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^2+a}\,ab^9c^2d+6\,(bx^2+a)b^9cd^2-12\,\sqrt{bx^2+a}\,b^9d^3}{12\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1
/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/
2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 1
2*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)
```

Mupad [B]
time = 3.64, size = 167, normalized size = 1.25

$$\frac{x^4}{4\,b\,c}-\sqrt{b\,x^2+a}\,\left(\frac{d^3}{b^3\,c^4}-\frac{2\,a\,d}{b^3\,c^2}\right)-\frac{d\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b^3\,c^2}-\frac{x^2\,\left(a\,c^2-d^2\right)}{2\,b^2\,c^3}+\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{b\,x^2+a}}{d}\right)\,{\left(a\,c^2-d^2\right)}^2}{b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,\left(a^2\,c^4-2\,a\,c^2\,d^2+d^4\right)}{2\,b^3\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(a
 + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((c*
(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*c^2
*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)
```

3.545 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=69

$$\frac{x^2}{2bc}-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}$$

```
[Out] 1/2*x^2/b/c-(a*c^2-d^2)*ln(d+c*(b*x^2+a)^(1/2))/b^2/c^3-d*(b*x^2+a)^(1/2)/b
^2/c^2
```

Rubi [A]
time = 0.15, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2}-\frac{(ac^2-d^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3}+\frac{x^2}{2bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt
[a + b*x^2]])/(b^2*c^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{b^2}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)\,dx, x, \sqrt{a+bx^2}\right)}{b^2}$$
$$= \frac{x^2}{2bc} - \frac{d\sqrt{a+bx^2}}{b^2c^2} - \frac{(ac^2-d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{b^2c^3}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.96

$$\frac{c\left(ac+bcx^2-2d\sqrt{a+bx^2}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^2}\right)}{2b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (c*(a*c + b*c*x^2 - 2*d*Sqrt[a + b*x^2]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqr
t[a + b*x^2]])/(2*b^2*c^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3409 vs. 2(63) = 126.
time = 0.04, size = 3410, normalized size = 49.42

method	result	size
default	Expression too large to display	3410

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(
-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/
2)*c^2*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+
2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c
^2)^(1/2)*d^3+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln
```

((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/
b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2
)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*a-1/2/((-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^
2)^(1/2))/b^(3/2)/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((1/c^2*(-(a*c^2-d^2)*b*
c^2)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d
^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*d^3/(d^2/c
^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b
c^2)^(1/2)/b/c^2)+2(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d
^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1/2/((-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)/b*d^5/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*
(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*
d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b
*(-a*b)^(1/2)))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln
(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*
(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(
a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(
b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*c
^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*
b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1
/2)))^(1/2))+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
+d^2/c^2)^(1/2)*c^2*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)+d^2/c^2)^(1/2)*d^3-1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/
2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-(a*c^2-d^2)*b*
c^2)^(1/2)*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*
c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*
a+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c^2*(
-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(
b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))*d^3-1/2/((-a*b)^(1/2)*c

```
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))/b*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a+1/2/(
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^
2)*b*c^2)^(1/2))/b*d^5/c^2/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2
)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x
+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2))-1/2*a/c/b^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*x^2/b/c+1/2/b^2/c^3*d^2*
ln(b*c^2*x^2+a*c^2-d^2)
```

Maxima [A]
time = 0.29, size = 62, normalized size = 0.90

$$\frac{\frac{(bx^2+a)c-2\sqrt{bx^2+a}\,d}{c^2}-\frac{2\,(ac^2-d^2)\log\left(\sqrt{bx^2+a}\,c+d\right)}{c^3}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^2 + a)*c + d)/c^3)/b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(63) = 126.
time = 0.37, size = 161, normalized size = 2.33

$$\frac{2\,bc^2x^2-4\,\sqrt{bx^2+a}\,cd-2\,(ac^2-d^2)\log\left(bc^2x^2+ac^2-d^2\right)-(ac^2-d^2)\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)+(ac^2-d^2)\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)}{4\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 +
a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*
d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c
*d + d^2)/x^2))/(b^2*c^3)
```

Sympy [A]

time = 2.88, size = 88, normalized size = 1.28

$$\begin{cases} \frac{\frac{a+bx^2}{2bc} - \frac{d\sqrt{a+bx^2}}{bc^2} - \frac{(ac^2-d^2)\left(\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c=0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}\right)}{bc^2}}{b} & \text{for } b \neq 0 \\ \frac{x^4}{2\cdot\left(2\sqrt{a}\,d+2ac\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Piecewise((((a + b*x**2)/(2*b*c) - d*sqrt(a + b*x**2)/(b*c**2) - (a*c**2 -
d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(b*c**2))/b, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), True))
```

Giac [A]
time = 4.94, size = 72, normalized size = 1.04

$$-\frac{\frac{2\,(ac^2-d^2)\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{bc^3} - \frac{(bx^2+a)bc-2\sqrt{bx^2+a}\,bd}{b^2c^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a
)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b
```

Mupad [B]
time = 3.50, size = 123, normalized size = 1.78

$$\frac{x^2}{2\,b\,c} - \frac{d\,\sqrt{b\,x^2+a}}{b^2\,c^2} + \frac{\mathrm{atanh}\left(\frac{c\,(a\,c^2-d^2)\,\sqrt{b\,x^2+a}}{d^3-a\,c^2\,d}\right)\,(a\,c^2-d^2)}{b^2\,c^3} - \frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)\,(a\,c^2-d^2)}{2\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a
+ b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 - d^
2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)
```

3.546 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=23

$$\frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}$$

[Out] ln(d+c*(b*x^2+a)^(1/2))/b/c

Rubi [A]
time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2186, 31}

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 26, normalized size = 1.13

$$\frac{\log\left(bd+bc\sqrt{a+bx^2}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] Log[b*d + b*c*Sqrt[a + b*x^2]]/(b*c)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1930 vs. 2(21) = 42.
time = 0.03, size = 1931, normalized size = 83.96

method	result	size
default	Expression too large to display	1931

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2
+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^
2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^
(1/2)-1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^
2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((1/c^2*(-(a*c^2
-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+
(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*
d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*d*c^2/((
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2
)))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))
*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-
a*b)^(1/2)))^(1/2))/b^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2
)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+1/b*(-a*b)^(1
/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2
))*(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*
```

```
(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)-1/2*d*c
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c
^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+1/
2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c
^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c^2*(-(a*c^2-d^2)*b
*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(d^
2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2/b/c*ln(b*c^2*x^
2+a*c^2-d^2)
```

Maxima [**A**]
time = 0.28, size = 21, normalized size = 0.91

$$\frac{\log\left(\sqrt{bx^2+a}\,c+d\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(21) = 42.
time = 0.37, size = 105, normalized size = 4.57

$$\frac{2\log\left(bc^2x^2+ac^2-d^2\right)+\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^
2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d +
d^2)/x^2))/(b*c)
```

Sympy [**A**]

time = 1.94, size = 29, normalized size = 1.26

$$\frac{\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c=0 \\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, T
rue))/b
```

Giac [A]
time = 3.33, size = 22, normalized size = 0.96

$$\frac{\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)
```

Mupad [B]
time = 3.47, size = 45, normalized size = 1.96

$$\frac{\operatorname{atanh}\left(\frac{c\sqrt{b\,x^2+a}}{d}\right)+\frac{\ln\left(b\,c^2\,x^2+a\,c^2-d^2\right)}{2}}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
[Out] (atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)
```

3.547 $\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=88

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}-\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-c*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)+d*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 720, 31, 649, 213, 266}

$$-\frac{c\log\left(c\sqrt{a+bx^2}+d\right)}{ac^2-d^2}+\frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/(
a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$

$$= \text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx, x, \sqrt{a+bx^2}\right)$$

$$= -\frac{c^2\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx, x, \sqrt{a+bx\right.}}{-ac^2+d^2}$$

$$= -\frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2} + \frac{c\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx, x, \sqrt{a+bx^2}\right)}{ac^2-d^2} - \frac{d\text{S}}{}$$

$$= \frac{d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}\left(ac^2-d^2\right)} + \frac{c\log(x)}{ac^2-d^2} - \frac{c\log\left(d+c\sqrt{a+bx^2}\right)}{ac^2-d^2}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.78

$$\frac{\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + c\log\left(bx^2\right) - 2c\log\left(d+c\sqrt{a+bx^2}\right)}{2ac^2-2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] ((2*d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^2] - 2*c*Log[d
+ c*Sqrt[a + b*x^2]])/(2*a*c^2 - 2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2174 vs. $2(80) = 160$.
time = 0.04, size = 2175, normalized size = 24.72

method	result	size
default	Expression too large to display	2175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] c*ln(x)/(a*c^2-d^2)-1/2*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*c
/d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x
-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-
(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/(a*c^2-d^2
)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-
d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((1/c^2*(-(a*c^2-d^2)*b*c^2
)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)
*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(
-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^
3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+d/a^(1/2)/(a*c
^2-d^2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^(1/
2)+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2
)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*
(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-
d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/
2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))
^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b*c^2/a/((-a*b)^(1/2)*
c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2
))*(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-1/2
*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2
)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(
-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)
^(1/2)))^(1/2))-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2
)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b
*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)+1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((-a*b)^(1/
```

```
2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(
1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b
/c^2)+d^2/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)
*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(d^2/c^2)^
(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2
)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-
2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c
^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)
```

Fricas [A]
time = 0.42, size = 316, normalized size = 3.59

$$\left[-\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)+2\,\sqrt{a}\,d\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)}{4\,\left(a^2c^2-ad^2\right)},\ -\frac{2\,ac\log\left(bc^2x^2+ac^2-d^2\right)-4\,ac\log\left(x\right)+ac\log\left(-\frac{bc^2x^2+ac^2+2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)-ac\log\left(-\frac{bc^2x^2+ac^2-2\,\sqrt{bx^2+a}\,cd+d^2}{x^2}\right)+4\,\sqrt{-a}\,d\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{4\,\left(a^2c^2-ad^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*
x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c
^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + 2*sqrt(a)*d*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*log(b*c^2*x^2
 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2
 + a)*c*d + d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d
 + d^2)/x^2) + 4*sqrt(-a)*d*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a^2*c^2 - a*
d^2)]
```

Sympy [A]
time = 3.84, size = 88, normalized size = 1.00

$$-\frac{c^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^2}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{ac^2-d^2}-\frac{-\frac{c\log\left(-bx^2\right)}{2}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{ac^2-d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] -c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d
)/c, True))/(a*c**2 - d**2) - (-c*log(-b*x**2)/2 + d*atan(sqrt(a + b*x**2)/
sqrt(-a))/sqrt(-a))/(a*c**2 - d**2)
```

Giac [A]
time = 3.41, size = 94, normalized size = 1.07

$$-\frac{c^2\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{ac^3-cd^2}+\frac{c\log\left(bx^2\right)}{2\left(ac^2-d^2\right)}-\frac{d\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*
c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))
```

Mupad [B]
time = 3.95, size = 1270, normalized size = 14.43

$$\frac{c\ln(x)}{ac^2-d^2}-\frac{c\,\mathrm{atan}\left(\frac{\frac{c\left(4c^6d^2\sqrt{bx^2+a}+\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}\right)1i}{2\left(ac^2-d^2\right)}+\frac{c\left(4c^6d^2\sqrt{bx^2+a}-\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}\right)1i}{2\left(ac^2-d^2\right)}}{\frac{c\left(4c^6d^2\sqrt{bx^2+a}+\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}-\frac{c\left(4c^6d^2\sqrt{bx^2+a}-\frac{c\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{c\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}\right)}{2\left(ac^2-d^2\right)}}\right)1i}{ac^2-d^2}-\frac{c\ln\left(bc^2x^2+ac^2-d^2\right)}{2ac^2-2d^2}-\frac{d\,\mathrm{atan}\left(\frac{\frac{d\left(4c^6d^2\sqrt{bx^2+a}+\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)1i}{\sqrt{a}\left(2ac^2-2d^2\right)}+\frac{d\left(4c^6d^2\sqrt{bx^2+a}-\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)1i}{\sqrt{a}\left(2ac^2-2d^2\right)}}{\frac{d\left(4c^6d^2\sqrt{bx^2+a}+\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d-\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}-\frac{d\left(4c^6d^2\sqrt{bx^2+a}-\frac{d\left(4c^4d^5-8ac^6d^3+4a^2c^8d+\frac{d\sqrt{bx^2+a}\left(8a^3c^{10}-8a^2c^8d^2-8ac^6d^4+8c^4d^6\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}\right)}{\sqrt{a}\left(2ac^2-2d^2\right)}}\right)1i}{\sqrt{a}\left(ac^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) - (c*atan(((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4
*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8
*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^
2)))*1i)/(2*(a*c^2 - d^2)) + (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^
5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^
6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))*1i
)/(2*(a*c^2 - d^2)))/((c*(4*c^6*d^2*(a + b*x^2)^(1/2) + (c*(4*c^4*d^5 - 8*a
*c^6*d^3 + 4*a^2*c^8*d - (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a
*c^6*d^4 - 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^
2 - d^2)) - (c*(4*c^6*d^2*(a + b*x^2)^(1/2) - (c*(4*c^4*d^5 - 8*a*c^6*d^3 +
 4*a^2*c^8*d + (c*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 -
 8*a^2*c^8*d^2))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2))))/(2*(a*c^2 - d^2)))
)*1i)/(a*c^2 - d^2) - (c*log(a*c^2 - d^2 + b*c^2*x^2))/(2*a*c^2 - 2*d^2) -
(d*atan(((d*(4*c^6*d^2*(a + b*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*
a^2*c^8*d - (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*
a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)))*1i
```

```
)/(a^(1/2)*(2*a*c^2 - 2*d^2)) + (d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4
*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4
*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)
*(2*a*c^2 - 2*d^2)))*1i)/(a^(1/2)*(2*a*c^2 - 2*d^2)))/((d*(4*c^6*d^2*(a + b
*x^2)^(1/2) + (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d - (d*(a + b*x^2)^(1
/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d^2))/(a^(1/2)*(2*a*c
^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2)) -
(d*(4*c^6*d^2*(a + b*x^2)^(1/2) - (d*(4*c^4*d^5 - 8*a*c^6*d^3 + 4*a^2*c^8*d
 + (d*(a + b*x^2)^(1/2)*(8*a^3*c^10 + 8*c^4*d^6 - 8*a*c^6*d^4 - 8*a^2*c^8*d
^2))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(2*a*c^2 - 2*d^2))))/(a^(1/2)*(
2*a*c^2 - 2*d^2))))*1i)/(a^(1/2)*(a*c^2 - d^2))
```

3.548 $\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=151

$$-\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2}-\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}+\frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2}$$

```
[Out] -1/2*b*d*(3*a*c^2-d^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+b*c^3*ln(d+c*(b*x^2+a)^(1/2))/(a*c^2-d^2)^2+1/
2*(-a*c+d*(b*x^2+a)^(1/2))/a/(a*c^2-d^2)/x^2
```

Rubi [A]
time = 0.24, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2186, 755, 815, 649, 212, 266}

$$-\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^2}}{2ax^2\left(ac^2-d^2\right)}+\frac{bc^3\log\left(c\sqrt{a+bx^2}+d\right)}{\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -1/2*(a*c - d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log
[x])/(a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx, x, x^2\right)$$

$$= b\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx, x, \sqrt{a+bx^2}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)(a-x^2)}\,dx, x, \sqrt{a+bx^2}\right)}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{b\text{Subst}\left(\int \left(-\frac{2ac^4}{(ac^2-d^2)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{(ac^2-d^2)(a-x^2)}\right) d\right.}{2a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} - \frac{b\text{Subst}\left(\int \frac{3ac^2d-d^3}{a-}\right.}{2a(}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} + \frac{bc^3\log\left(d+c\sqrt{a+bx^2}\right)}{\left(ac^2-d^2\right)^2} + \frac{\left(bc^3\right)\text{Subst}\left(\int \frac{x}{a-}\right.}{(ac^2}$$

$$= -\frac{ac-d\sqrt{a+bx^2}}{2a\left(ac^2-d^2\right)x^2} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log}{(ac^2-}$$

Mathematica [A]
time = 0.21, size = 139, normalized size = 0.92

$$\frac{bd(-3ac^2+d^2)\,x^2\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\sqrt{a}\left(-\left((ac^2-d^2)\left(ac-d\sqrt{a+bx^2}\right)\right)-abc^3x^2\log\left(bx^2\right)+2abc^3x^2\log\left(d+c\sqrt{a+bx^2}\right)\right)}{2a^{3/2}\left(-ac^2+d^2\right)^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] (b*d*(-3*a*c^2 + d^2)*x^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + Sqrt[a]*(-((a*
c^2 - d^2)*(a*c - d*Sqrt[a + b*x^2])) - a*b*c^3*x^2*Log[b*x^2] + 2*a*b*c^3*
x^2*Log[d + c*Sqrt[a + b*x^2]]))/(2*a^(3/2)*(-(a*c^2) + d^2)^2*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2458 vs. $2(137) = 274$.
time = 0.05, size = 2459, normalized size = 16.28

method	result	size
default	Expression too large to display	2459

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c/(a*c^2-d^2)/x^2-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln
(x)*d^2+1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^2+a*c^2-d^2)+b*c/a/(a*c^2-
d^2)*ln(x)-1/2*b*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^2+a*c^2-d^2)+1/2*d*b^2*c^6/
(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c
^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/
c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2
)^(1/2)+1/2*d*b^(3/2)*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^
2)^(1/2)*ln((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)
/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2
-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))-1/2
*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b
)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2
/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2
/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^
2-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*d/a^2/(a*c^2-d^2)/x^2*(b*x^2+a)^(3/2)+1/2*d
/a^(3/2)/(a*c^2-d^2)*b*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-1/2*d/a^2/(a*c
^2-d^2)*b*(b*x^2+a)^(1/2)-2*d*b/a^(1/2)/(a*c^2-d^2)^2*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)*c^2+b/a^(3/2)/(a*c^2-d^2)^2*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x)*d^3+2*d*b/a/(a*c^2-d^2)^2*(b*x^2+a)^(1/2)*c^2-b/a^2/(a*c^2-d^2)^2*(
b*x^2+a)^(1/2)*d^3-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)
```

```
^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2
))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)-1/2*d*b^(3/2)*c^2/a^2/((-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*
c^2)^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(
b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*
b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2
-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/
b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2
)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*
ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-
2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b^2*c^6/(a*c^2-d^2)^2/((-
a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*
b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)
*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d*b^(
3/2)*c^4/(a*c^2-d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b
)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*ln((-1/c
^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/
2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/
2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))-1/2*b^2*c^4/(a*c^2-
d^2)^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a
*c^2-d^2)*b*c^2)^(1/2))*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^
2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1
/2)/b/c^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)
```

Fricas [A]
time = 0.77, size = 530, normalized size = 3.51

$$\left[\frac{2\,a^{2}bc^{3}x^{2}\log\left(bc^{2}x^{2}+ac^{2}-d^{2}\right)-4\,a^{2}bc^{3}x^{2}\log\left(x\right)+a^{2}bc^{3}x^{2}\log\left(-\frac{bc^{2}x^{2}+ac^{2}+2\,\sqrt{bx^{2}+a}\,cd+d^{2}}{x^{2}}\right)-a^{2}bc^{3}x^{2}\log\left(-\frac{bc^{2}x^{2}+ac^{2}-2\,\sqrt{bx^{2}+a}\,cd+d^{2}}{x^{2}}\right)-2\,a^{3}c^{3}+2\,a^{2}cd^{2}-\left(3\,abc^{2}d-bd^{3}\right)\sqrt{a}\,x^{2}\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(a^{2}c^{2}d-ad^{3}\right)\sqrt{bx^{2}+a}}{4\,\left(a^{4}c^{4}-2\,a^{3}c^{2}d^{2}+a^{2}d^{4}\right)x^{2}},\ \frac{2\,a^{2}bc^{3}x^{2}\log\left(bc^{2}x^{2}+ac^{2}-d^{2}\right)-4\,a^{2}bc^{3}x^{2}\log\left(x\right)+a^{2}bc^{3}x^{2}\log\left(-\frac{bc^{2}x^{2}+ac^{2}+2\,\sqrt{bx^{2}+a}\,cd+d^{2}}{x^{2}}\right)-a^{2}bc^{3}x^{2}\log\left(-\frac{bc^{2}x^{2}+ac^{2}-2\,\sqrt{bx^{2}+a}\,cd+d^{2}}{x^{2}}\right)-2\,a^{3}c^{3}+2\,a^{2}cd^{2}+2\,\left(3\,abc^{2}d-bd^{3}\right)\sqrt{-a}\,x^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+2\,\left(a^{2}c^{2}d-ad^{3}\right)\sqrt{bx^{2}+a}}{4\,\left(a^{4}c^{4}-2\,a^{3}c^{2}d^{2}+a^{2}d^{4}\right)x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
[Out] [1/4*(2*a^2*b*c^3*x^2*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x)
 + a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2
```

```
) - a^2*b*c^3*x^2*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^
2) - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^2*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^
2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*log
(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*log(x) + a^2*b*c^3*x^2*log(-(b*
c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - a^2*b*c^3*x^2*log(-(b
*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*
d^2 + 2*(3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a))
 + 2*(a^2*c^2*d - a*d^3)*sqrt(b*x^2 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d
^4)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Giac [A]
time = 2.89, size = 210, normalized size = 1.39

$$\frac{bc^4\log\left(\left|\sqrt{bx^2+a}\,c+d\right|\right)}{a^2c^5-2\,ac^3d^2+cd^4}-\frac{bc^3\log\left(-bx^2\right)}{2\left(a^2c^4-2\,ac^2d^2+d^4\right)}+\frac{\left(3\,abc^2d-bd^3\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\left(a^3c^4-2\,a^2c^2d^2+ad^4\right)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-\left(abc^2d-bd^3\right)\sqrt{bx^2+a}}{2\left(ac^2-d^2\right)^2abx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] b*c^4*log(abs(sqrt(b*x^2 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/2
*b*c^3*log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d^3
)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*sqrt(
-a)) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^2 + a))/((
a*c^2 - d^2)^2*a*b*x^2)
```

Mupad [B]
time = 5.56, size = 2500, normalized size = 16.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
```

```
[Out] (atan(((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^
5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*
a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c
^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*
d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8
 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2
*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -
4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c
^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(
a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a +
 b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a
^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*
b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6
*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6
*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 -
18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6
 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*
b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5
*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^
6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4
+ a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d
^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(
a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/
(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) -
((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(
32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9
*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^
6*c^6*d^2))^(1/2)*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -
4*a^6*c^6*d^2))/(((b^3*c^8*d^5)/2 - (3*a*b^3*c^10*d^3)/2)/(a^5*c^6 - a^2*d^
6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d
^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*
d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^
(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^
8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16
*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*
c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*
a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d
^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4
- 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4
*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4
+ 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*
d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d
^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*
d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8
*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^1
```

4*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^(1/2)*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2))/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^(1/2)*1i)/(8*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - c/(2*x^2*(a*c^2 - d^2)) - (b*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) + (b*c^3*log(a*c^2 - d^2 + b*c^2*x^2))/(2*d^4 + 2*a^2*c^4 - 4*a*c^2*d^2) - (d*(a + b*x^2)^(1/2))/(2*x^2*(a*d^2 - a^2*c^2)) + (b*c^3*atan(((c^3*(((a + b*x^2)^(1/2)*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^...

3.549 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=147

$$\frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}$$

```
[Out] x/b/c-d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)/c^2-arctan(c*x*b^(1/2)/(
a*c^2-d^2)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2+arctan(d*x*b^(1/2)/(a*c^2-d
^2)^(1/2)/(b*x^2+a)^(1/2))*(a*c^2-d^2)^(1/2)/b^(3/2)/c^2
```

Rubi [A]
time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2187, 327, 211, 494, 223, 212, 385}

$$\frac{\sqrt{ac^2-d^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{\sqrt{ac^2-d^2}\,\text{ArcTan}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3
/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[
a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(
3/2)*c^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx &= (ac)\int \frac{x^2}{a^2c^2-ad^2+abc^2x^2}\,dx-(ad)\int \frac{x^2}{\sqrt{a+bx^2}\,\left(a^2c^2-ad^2+abc^2x^2\right)}\,d\cdots\\
&= \frac{x}{bc}-\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{bc^2}-\frac{\left(a\left(ac^2-d^2\right)\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{bc}+\frac{\left(ad\left(ac^2-d^2\cdots\right.\right.}{\cdots}\\
&= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}-\frac{d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\cdots\right.}{bc^2}\\
&= \frac{x}{bc}-\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2}+\frac{\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{\cdots}}\right.}{b^{3/2}c^2}
\end{aligned}
$$

Mathematica [A]

time = 0.26, size = 99, normalized size = 0.67

$$\frac{\sqrt{b}\,cx+2\sqrt{ac^2-d^2}\,\tan^{-1}\left(\frac{d+c\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)+d\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[b]*c*x + 2*Sqrt[a*c^2 - d^2]*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b
*x^2]))/Sqrt[a*c^2 - d^2]] + d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3/2
)*c^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3484 vs. $2(123) = 246$.
time = 0.04, size = 3485, normalized size = 23.71

method	result	size
default	Expression too large to display	3485

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+
(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)*a-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b
*c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b
*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)*d^3+1/2*d*c
^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^
2-d^2)*b*c^2)^(1/2))*ln((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/
c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2
)^(1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*
b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/
2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^
2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^
(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2)*d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2
-d^2)*b*c^2)^(1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*
c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b
```

```
/c^2))*a+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(d^2/c^2)^(1/
2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c
^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)
^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-1/2*d*c^2*a/(-a*b)^(1/2)/((-a
*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*
b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2))
)^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^
(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/
2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^
(1/2))/b^(1/2)+1/2*d*c^2*a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c
^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+1/b*(-a*b)^
(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/
2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^
(1/2))*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1
/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)-1/2*d*c^4/((-a*b
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)
+d^2/c^2)^(1/2)*a+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(
-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*(b*(
x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(
-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)*d^3+1/2*d*c^2/((-a*b)^(1/2)
*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1
/2))*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/
c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^
2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/b^(1/2
)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-
(a*c^2-d^2)*b*c^2)^(1/2))*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2
)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d
^2/c^2)^(1/2))/b^(1/2)*d^3+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(
1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(
1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c
^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)
^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))*a-1/2/(
(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^
2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^5/(d^2/c^2)^(1/2)*ln((2*d^2/c
^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*
(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^
2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-a/b/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a*
c^2-d^2)*b)^(1/2))+x/b/c+1/b/c^2*d^2/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a
```

```
*c^2-d^2)*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [A]
time = 0.49, size = 1168, normalized size = 7.95

$$\left[\frac{4\,bcx + 2\,\sqrt{b}\,d\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + b\sqrt{-\frac{ac^2 - d^2}{b}}\,\log\left(\frac{a^4c^4 - 2\,a^3c^2d^2 + a^2d^4 + (a^2b^2c^4 - 8\,ab^2c^2d^2 + 8\,b^2d^4)x^4 + 2\,(a^3bc^4 - 5\,a^2bc^2d^2 + 4\,abd^4)x^2 - 4\,((ab^2c^2d - 2\,b^2d^3)x^3 + (a^2bc^2d - abd^3)x)\sqrt{bx^2 + a}\,\sqrt{-\frac{ac^2 - d^2}{b}}}{b^2c^4x^4 + a^2c^4 - 2\,ac^2d^2 + d^4 + 2\,(abc^4 - bc^2d^2)x^2}\right) + 2\,b\sqrt{-\frac{ac^2 - d^2}{b}}\,\log\left(\frac{bc^2x^2 - 2\,bcx\sqrt{-\frac{ac^2 - d^2}{b}} - ac^2 + d^2}{bc^2x^2 + ac^2 - d^2}\right)}{4\,b^2c^2}, \frac{4\,bcx + 4\,\sqrt{-b}\,d\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) + b\sqrt{-\frac{ac^2 - d^2}{b}}\,\log\left(\frac{a^4c^4 - 2\,a^3c^2d^2 + a^2d^4 + (a^2b^2c^4 - 8\,ab^2c^2d^2 + 8\,b^2d^4)x^4 + 2\,(a^3bc^4 - 5\,a^2bc^2d^2 + 4\,abd^4)x^2 - 4\,((ab^2c^2d - 2\,b^2d^3)x^3 + (a^2bc^2d - abd^3)x)\sqrt{bx^2 + a}\,\sqrt{-\frac{ac^2 - d^2}{b}}}{b^2c^4x^4 + a^2c^4 - 2\,ac^2d^2 + d^4 + 2\,(abc^4 - bc^2d^2)x^2}\right) + 2\,b\sqrt{-\frac{ac^2 - d^2}{b}}\,\log\left(\frac{bc^2x^2 - 2\,bcx\sqrt{-\frac{ac^2 - d^2}{b}} - ac^2 + d^2}{bc^2x^2 + ac^2 - d^2}\right)}{4\,b^2c^2}, \frac{2\,bcx + 2\,b\sqrt{\frac{ac^2 - d^2}{b}}\arctan\left(-\frac{bcx\sqrt{\frac{ac^2 - d^2}{b}}}{ac^2 - d^2}\right) - b\sqrt{\frac{ac^2 - d^2}{b}}\arctan\left(\frac{(a^2c^2 - ad^2 + (abc^2 - 2\,bd^2)x^2)\sqrt{bx^2 + a}\,\sqrt{\frac{ac^2 - d^2}{b}}}{2\,((abc^2d - bd^3)x^3 + (a^2c^2d - ad^3)x)}\right) + \sqrt{b}\,d\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right)}{2\,b^2c^2}, \frac{2\,bcx + 2\,b\sqrt{\frac{ac^2 - d^2}{b}}\arctan\left(-\frac{bcx\sqrt{\frac{ac^2 - d^2}{b}}}{ac^2 - d^2}\right) + 2\,\sqrt{-b}\,d\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) - b\sqrt{\frac{ac^2 - d^2}{b}}\arctan\left(\frac{(a^2c^2 - ad^2 + (abc^2 - 2\,bd^2)x^2)\sqrt{bx^2 + a}\,\sqrt{\frac{ac^2 - d^2}{b}}}{2\,((abc^2d - bd^3)x^3 + (a^2c^2d - ad^3)x)}\right)}{2\,b^2c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b
^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2
+ 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^
3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*
c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*
log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 +
 a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^
2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*
a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*
c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 +
a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*
c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2
)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d
^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 -
 d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2
+ a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x
)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2),
1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b
)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt
((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sq
rt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d -
 a*d^3)*x)))/(b^2*c^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a\,c + d\,\sqrt{b\,x^2 + a} + b\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

3.550 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

```
[Out] arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)-arctan(d*x*
b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)/(a*c^2-d^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2187, 211, 385}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTa
n[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 -
d^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx - (ad)\int \frac{1}{\sqrt{a+bx^2}\,(a^2c^2-ad^2+abc^2x^2)}\,d$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}} - (ad)\text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-(-a^2bc^2+b(a^2c^2-ac}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Mathematica [A]
time = 0.20, size = 63, normalized size = 0.61

$$-\frac{2\tan^{-1}\left(\frac{d+c\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\,\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1),x]
```

```
[Out] (-2*ArcTan[(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(Sq
rt[b]*Sqrt[a*c^2 - d^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1994 vs. 2(85) = 170.
time = 0.03, size = 1995, normalized size = 19.37

method	result	size
default	Expression too large to display	1995

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*(b*(x-(-(a*c^2-d^
2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*
b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a
*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln((
1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^
(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^
(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))+1/2*b*c^2/((-a*b
```

```
)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*
c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/
c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/
c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*
c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2
-d^2)*b*c^2)^(1/2)/b/c^2))+1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*
c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x
-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(1/
2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(
a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(
b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*
b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(
1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/
2)*(x+1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2
-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+1
/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)
^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1/2*d*b*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-
d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-
d^2)*b*c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-
d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d
*b^(1/2)*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c
^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c
^2)+d^2/c^2)^(1/2))-1/2*b*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))
/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*
d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+1/((a*c^2-d^
2)*b)^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(85) = 170.
time = 0.38, size = 510, normalized size = 4.95

$$\left[-\frac{\sqrt{-abc^2+bd^2}\log\left(\frac{a^4c^4-2a^3c^2d^2+a^2d^4+(a^2b^2c^4-8ab^2c^2d^2+8b^2d^4)x^4+2(a^3bc^4-5a^2bc^2d^2+4abd^4)x^2-4\sqrt{-abc^2+bd^2}((abc^2d-2bd^3)x^3+(a^2c^2d-ad^3)x)\sqrt{bx^2+a}}{b^2c^4x^4+a^2c^4-2ac^2d^2+d^4+2(abc^4-bc^2d^2)x^2}\right)+2\sqrt{-abc^2+bd^2}\log\left(\frac{bc^2x^2-ac^2-2\sqrt{-abc^2+bd^2}cx+d^2}{bc^2x^2+ac^2-d^2}\right)}{4(abc^2-bd^2)}, -\frac{2\sqrt{abc^2-bd^2}\arctan\left(-\frac{\sqrt{abc^2-bd^2}cx}{ac^2-d^2}\right)-\sqrt{abc^2-bd^2}\arctan\left(\frac{(a^2c^2-ad^2+(abc^2-2bd^2)x^2)\sqrt{abc^2-bd^2}\sqrt{bx^2+a}}{2((ab^2c^2d-b^2d^3)x^3+(a^2bc^2d-abd^3)x)}\right)}{2(abc^2-bd^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2
*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^
2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 +
(a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^
2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c
^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d
^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2
 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 -
a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*
b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

Giac [A]
time = 3.74, size = 107, normalized size = 1.04

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2 - bd^2}}\right)}{\sqrt{abc^2 - bd^2}} + \frac{\arctan\left(\frac{\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 c^2 + ac^2 - 2\,d^2}{2\,\sqrt{ac^2 - d^2}\,d}\right)}{\sqrt{ac^2 - d^2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sq
rt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(s
qrt(a*c^2 - d^2)*sqrt(b))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\begin{cases} \frac{\operatorname{atan}\left(\frac{b c x}{\sqrt{a b c^2 - b d^2}}\right)}{\sqrt{a b c^2 - b d^2}} - \frac{d x}{\sqrt{a}\,(a c^2 - d^2)} & \text{if } b = 0 \vee d = 0 \\ \frac{\operatorname{atan}\left(\frac{b c x}{\sqrt{a b c^2 - b d^2}}\right)}{\sqrt{a b c^2 - b d^2}} - \frac{d \operatorname{atan}\left(\frac{x \sqrt{a b c^2 - b (a c^2 - d^2)}}{\sqrt{a c^2 - d^2}\,\sqrt{b x^2 + a}}\right)}{\sqrt{-(a c^2 - d^2)\,(b (a c^2 - d^2) - a b c^2)}} & \text{if } 0 < b d^2 \\ \frac{\operatorname{atan}\left(\frac{b c x}{\sqrt{a b c^2 - b d^2}}\right)}{\sqrt{a b c^2 - b d^2}} - \frac{d \ln\left(\frac{\sqrt{(a c^2 - d^2)\,(b x^2 + a)} + x \sqrt{b (a c^2 - d^2) - a b c^2}}{\sqrt{(a c^2 - d^2)\,(b x^2 + a)} - x \sqrt{b (a c^2 - d^2) - a b c^2}}\right)}{2 \sqrt{(a c^2 - d^2)\,(b (a c^2 - d^2) - a b c^2)}} & \text{if } b d^2 < 0 \\ \int \frac{1}{a c + d \sqrt{b x^2 + a} + b c x^2}\, dx & \text{if } b d^2 \notin \mathbb{R} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)
```

```
[Out] piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2
 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x)/
(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(a*c
^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(-(a*
c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)/(- b
*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 - d^2)*(
a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 - d^2)*(a
 + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^2 - d^2)
*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a*c + d*(a
 + b*x^2)^(1/2) + b*c*x^2), x))
```

3.551 $\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx$

Optimal. Leaf size=160

$$-\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{a+bx^2}}{a\,(ac^2-d^2)\,x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}+\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,dx}{\sqrt{ac^2-d^2}\,\sqrt{a+bx^2}}\right)}{(ac^2-d^2)^{3/2}}$$

```
[Out] -c/(a*c^2-d^2)/x-c^2*arctan(c*x*b^(1/2)/(a*c^2-d^2)^(1/2))*b^(1/2)/(a*c^2-d
^2)^(3/2)+c^2*arctan(d*x*b^(1/2)/(a*c^2-d^2)^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)
/(a*c^2-d^2)^(3/2)+d*(b*x^2+a)^(1/2)/a/(a*c^2-d^2)/x
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2187, 331, 211, 491, 12, 385}

$$\frac{\sqrt{b}\,c^2\text{ArcTan}\left(\frac{\sqrt{b}\,dx}{\sqrt{a+bx^2}\,\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}-\frac{\sqrt{b}\,c^2\text{ArcTan}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}+\frac{d\sqrt{a+bx^2}}{ax\,(ac^2-d^2)}-\frac{c}{x\,(ac^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]
*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b
]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d
^2)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 491

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q
 + 1)/(a*c*e*(m + 1))), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^2+d\sqrt{a+bx^2}\right)}\,dx = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^2\right)}\,dx-(ad)\int \frac{1}{x^2\sqrt{a+bx^2}\,\left(a^2c^2-a\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^2}\,dx}{ac^2-d^2}+\frac{d\int -}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{(ab}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{(ab}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x}+\frac{d\sqrt{a+bx^2}}{a\left(ac^2-d^2\right)x}-\frac{\sqrt{b}\,c^2\tan^{-1}\left(\frac{\sqrt{b}\,cx}{\sqrt{ac^2-d^2}}\right)}{\left(ac^2-d^2\right)^{3/2}}+\frac{\sqrt{b}}{}$$

Mathematica [A]
time = 0.35, size = 105, normalized size = 0.66

$$-\frac{ac-d\sqrt{a+bx^2}}{a^2c^2x-ad^2x}+\frac{2\sqrt{b}\,c^2\tan^{-1}\left(\frac{d+c\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]
```

```
[Out] -((a*c - d*Sqrt[a + b*x^2])/(a^2*c^2*x - a*d^2*x)) + (2*Sqrt[b]*c^2*ArcTan[
(d + c*(-(Sqrt[b]*x) + Sqrt[a + b*x^2]))/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^
(3/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2288 vs. 2(140) = 280.
time = 0.04, size = 2289, normalized size = 14.31

method	result	size
default	Expression too large to display	2289

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] b*c^2/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))-c/(a*c^
2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a*c^2
-d^2)*b)^(1/2))-1/2*d*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*
c^2)^(1/2)*(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*
c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d*b^(3/2
)*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/
2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln((1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x-
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x-(-(a*c^2-d^2)*b*c^2)^(1/
2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/
b/c^2)+d^2/c^2)^(1/2))+1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d
^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^
2)*b*c^2)^(1/2)*d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2+2/c^2*(-(a*c^2-d^2)*b*c^2
)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x-(-(a*c
^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-
d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2))/(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^
2))+d/a^2/(a*c^2-d^2)/x*(b*x^2+a)^(3/2)-d/a^2/(a*c^2-d^2)*b*x*(b*x^2+a)^(1/
2)-d/a/(a*c^2-d^2)*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*d*b^2*c^2/a/(-
a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-
(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b
```

```
*(-a*b)^(1/2)))^(1/2)+1/2*d*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b
*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-1/b*(-a*b
)^(1/2))*b+(-a*b)^(1/2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(
x-1/b*(-a*b)^(1/2)))^(1/2))-1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+
(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(
b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2)+1/2*d*b
^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^
2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1
/2)+(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))+1
/2*d*b^2*c^6/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a
*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*(b*(x+
(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(
a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^(1/2)-1/2*d*b^(3/2)*c^4/(a*c^2-d^2)
/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d
^2)*b*c^2)^(1/2))*ln((-1/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)+(x+(-(a*c^2-d^2)*b*
c^2)^(1/2)/b/c^2)*b)/b^(1/2)+(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2/c^
2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+d^2/c^2)^
(1/2))-1/2*b^2*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)
)/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(a*c^2-d^2)*b*c^2)^(1/2)*
d^3/(d^2/c^2)^(1/2)*ln((2*d^2/c^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*
c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(d^2/c^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(
1/2)/b/c^2)^2-2/c^2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2
)/b/c^2)+d^2/c^2)^(1/2))/(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)
```

Fricas [A]
time = 0.42, size = 581, normalized size = 3.63

$$\left[-\frac{ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{a^4c^4-2a^3c^2d^2+a^2d^4+(a^2b^2c^4-8ab^2c^2d^2+8b^2d^4)x^4+2(a^3bc^4-5a^2bc^2d^2+4abd^4)x^2+4((a^2bc^4d-3abc^2d^3+2bd^5)x^3+(a^3c^4d-2a^2c^2d^3+ad^5)x)\sqrt{bx^2+a}\sqrt{-\frac{b}{ac^2-d^2}}}{b^2c^4x^4+a^2c^4-2ac^2d^2+d^4+2(abc^4-bc^2d^2)x^2}\right)+2ac^2x\sqrt{-\frac{b}{ac^2-d^2}}\log\left(\frac{bc^2x^2-ac^2+2(ac^3-cd^2)x\sqrt{-\frac{b}{ac^2-d^2}}+d^2}{bc^2x^2+ac^2-d^2}\right)+4ac-4\sqrt{bx^2+a}\,d}{4(a^2c^2-ad^2)x}, -\frac{2ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(cx\sqrt{\frac{b}{ac^2-d^2}}\right)-ac^2x\sqrt{\frac{b}{ac^2-d^2}}\arctan\left(-\frac{(a^2c^2-ad^2+(abc^2-2bd^2)x^2)\sqrt{bx^2+a}\sqrt{\frac{b}{ac^2-d^2}}}{2(b^2dx^3+abdx)}\right)+2ac-2\sqrt{bx^2+a}\,d}{2(a^2c^2-ad^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^
4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*
b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3
```

```
+ (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 -
d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)
*x^2)) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3
 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*
c - 4*sqrt(b*x^2 + a)*d)/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c
^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2)
)*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*s
qrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d)/
((a^2*c^2 - a*d^2)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(ac + bcx^2 + d\sqrt{a + bx^2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)
```

Giac [A]
time = 3.14, size = 211, normalized size = 1.32

$$-b^{\frac{3}{2}}d\left(\frac{c^2\arctan\left(\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2c^2+ac^2-2\,d^2}{2\,\sqrt{ac^2-d^2}\,d}\right)}{(abc^2-bd^2)\sqrt{ac^2-d^2}\,d}+\frac{2}{(abc^2-bd^2)\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)}\right)-\frac{bc^2\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}\,(ac^2-d^2)}-\frac{c}{(ac^2-d^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2
*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/((
a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c
*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2
 - d^2)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^2+a}+b\,c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)
[Out] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)
```

3.552 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=140

$$-\frac{(2ac^2-d^2)\,x^3}{3b^2c^3}+\frac{2d(2ac^2-d^2)\,\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{(a+bx^3)^2}{6b^3c}+\frac{2(ac^2-d^2)^2\log\left(d+c\sqrt{a+bx^3}\right)}{3b^3c^5}$$

```
[Out] -1/3*(2*a*c^2-d^2)*x^3/b^2/c^3-2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+1/6*(b*x^3+a)^
2/b^3/c+2/3*(a*c^2-d^2)^2*ln(d+c*(b*x^3+a)^(1/2))/b^3/c^5+2/3*d*(2*a*c^2-d^
2)*(b*x^3+a)^(1/2)/b^3/c^4
```

Rubi [A]
time = 0.21, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2186, 711}

$$-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{2(ac^2-d^2)^2\log\left(c\sqrt{a+bx^3}+d\right)}{3b^3c^5}+\frac{2d\sqrt{a+bx^3}\,(2ac^2-d^2)}{3b^3c^4}+\frac{(a+bx^3)^2}{6b^3c}-\frac{x^3(2ac^2-d^2)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/3*((2*a*c^2 - d^2)*x^3)/(b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3]
)/(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*
c) + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^3\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{(a-x^2)^2}{d+cx}\,dx,x,\sqrt{a+bx^3}\right)}{3b^3}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{2ac^2d-d^3}{c^4}-\frac{(2ac^2-d^2)x}{c^3}-\frac{dx^2}{c^2}+\frac{x^3}{c}+\frac{(ac^2-d^2)^2}{c^4(d+cx)}\right)\,dx,x,\sqrt{a+bx^3}\right)}{3b^3}$$
$$= -\frac{(2ac^2-d^2)\,x^3}{3b^2c^3}+\frac{2d(2ac^2-d^2)\sqrt{a+bx^3}}{3b^3c^4}-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2}+\frac{(a+bx^3)^2}{6b^3c}$$

Mathematica [A]
time = 0.16, size = 125, normalized size = 0.89

$$\frac{-4cd\sqrt{a+bx^3}\,(-5ac^2+3d^2+bc^2x^3)+3c^2(-3a^2c^2+2a(d^2-bc^2x^3)+bx^3(2d^2+bc^2x^3))+12(-ac^2+d^2)^2\log\left(d+c\sqrt{a+bx^3}\right)}{18b^3c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*c*d*Sqrt[a + b*x^3]*(-5*a*c^2 + 3*d^2 + b*c^2*x^3) + 3*c^2*(-3*a^2*c^2
+ 2*a*(d^2 - b*c^2*x^3) + b*x^3*(2*d^2 + b*c^2*x^3)) + 12*(-(a*c^2) + d^2)^
2*Log[d + c*Sqrt[a + b*x^3]])/(18*b^3*c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 647, normalized size = 4.62 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-2/9/b^3/c^2*(b*x^3+a)^(3/2)+2/3*a^2/b^3/d^2*(b*x^3+a)^(1/2)-(a^2*c^4-2*
a*c^2*d^2+d^4)/d^2/c^2/b^2*(2/3/c^2/b*(b*x^3+a)^(1/2)+1/3*I/b^3/c^2*2^(1/2)
*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1
/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*
3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)
*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3
^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alp
ha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I
*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2
)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3
*b*c^2+a*c^2-d^2)))-1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*ln(b*c^2*x^3+a*c^2-d^2)-
```

```
2/3*a/c^3*d^2/b^3*ln(b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d^2+1
/3/b^3/c^5*d^4*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A]
time = 0.29, size = 125, normalized size = 0.89

$$\frac{\frac{3\,(bx^3+a)^2c^3-4\,(bx^3+a)^{\frac{3}{2}}c^2d-6\,(2\,ac^3-cd^2)\,(bx^3+a)+12\,(2\,ac^2d-d^3)\,\sqrt{bx^3+a}}{c^4}+\frac{12\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^5}}{18\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5)/b^3
```

Fricas [A]
time = 0.39, size = 191, normalized size = 1.36

$$\frac{3\,b^2c^4x^6-6\,(abc^4-bc^2d^2)x^3+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(bc^2x^3+ac^2-d^2\right)+6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^3+a}\,c+d\right)-6\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\sqrt{bx^3+a}\,c-d\right)-4\,(bc^3dx^3-5\,ac^3d+3\,cd^3)\sqrt{bx^3+a}}{18\,b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^
2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log
(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 +
a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(x**8/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [A]
time = 3.19, size = 156, normalized size = 1.11

$$\frac{2\,(a^2c^4-2\,ac^2d^2+d^4)\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\,b^3c^5}+\frac{3\,(bx^3+a)^2b^9c^3-12\,(bx^3+a)ab^9c^3-4\,(bx^3+a)^{\frac{3}{2}}b^9c^2d+24\,\sqrt{bx^3+a}\,ab^9c^2d+6\,(bx^3+a)b^9cd^2-12\,\sqrt{bx^3+a}\,b^9d^3}{18\,b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5)
 + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)
^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2
 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)
```

Mupad [B]
time = 3.74, size = 200, normalized size = 1.43

$$\frac{\left(\frac{2\,d\,\left(a\,c^2-d^2\right)}{b^2\,c^4}+\frac{4\,a\,d}{3\,b^2\,c^2}\right)\,\sqrt{b\,x^3+a}}{3\,b}+\frac{x^6}{6\,b\,c}-\frac{x^3\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}+\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)\,{\left(a\,c^2-d^2\right)}^2}{3\,b^3\,c^5}+\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a^2\,c^4-2\,a\,c^2\,d^2+d^4\right)}{3\,b^3\,c^5}-\frac{2\,d\,x^3\,\sqrt{b\,x^3+a}}{9\,b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/(
3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b*x
^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (log(a
*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2*d*x
^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

3.553 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$\frac{x^3}{3bc}-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2(ac^2-d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

```
[Out] 1/3*x^3/b/c-2/3*(a*c^2-d^2)*ln(d+c*(b*x^3+a)^(1/2))/b^2/c^3-2/3*d*(b*x^3+a)
^(1/2)/b^2/c^2
```

Rubi [A]
time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2186, 711}

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2}-\frac{2(ac^2-d^2)\log\left(c\sqrt{a+bx^3}+d\right)}{3b^2c^3}+\frac{x^3}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d +
c*Sqrt[a + b*x^3]])/(3*b^2*c^3)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{-a+x^2}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b^2}$$
$$= \frac{2\text{Subst}\left(\int \left(-\frac{d}{c^2}+\frac{x}{c}+\frac{-ac^2+d^2}{c^2(d+cx)}\right)\,dx, x, \sqrt{a+bx^3}\right)}{3b^2}$$
$$= \frac{x^3}{3bc} - \frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2(ac^2-d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Mathematica [A]
time = 0.05, size = 66, normalized size = 0.90

$$\frac{c\left(ac+bcx^3-2d\sqrt{a+bx^3}\right)+(-2ac^2+2d^2)\log\left(d+c\sqrt{a+bx^3}\right)}{3b^2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (c*(a*c + b*c*x^3 - 2*d*Sqrt[a + b*x^3]) + (-2*a*c^2 + 2*d^2)*Log[d + c*Sqr
t[a + b*x^3]])/(3*b^2*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 555, normalized size = 7.60

method	result

default	d	$-\frac{2a\sqrt{b\,x^3+a}}{3d^2b^2}+\frac{(c^2a-d^2)\left(\frac{2\sqrt{b\,x^3+a}}{3c^2b}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^2\_Z^3+c^2a-d^2\right)}\frac{(-a\,b^2)^{\frac{1}{3}}\sqrt{2}\sqrt{\frac{ib\left(2x+\frac{(-a\,b^2 \cdots}{\cdots}\right.}{\cdots}}}{\cdots}\right.}{\cdots}\right.}{\cdots}$

elliptic	$\frac{\sqrt{b x^3+a}\,\left(d+c\sqrt{b x^3+a}\right)\left(\frac{x^3}{3bc}-\frac{a\ln\left(b c^2 x^3+c^2 a-d^2\right)}{3c b^2}+\frac{d^2\ln\left(b c^2 x^3+c^2 a-d^2\right)}{3b^2c^3}-\frac{2d\sqrt{b x^3+a}}{3b^2c^2}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b c^2 \ldots\right.}\ldots\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-2/3*a/d^2/b^2*(b*x^3+a)^(1/2)+(a*c^2-d^2)/d^2/b*(2/3/c^2/b*(b*x^3+a)^(1
/2)+1/3*I/b^3/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1
/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/
3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*
((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^
(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^
2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/
3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3
)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2
)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^
```

```
(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))))-1/3*a/c/b^2*ln(b*c^2*x^3+a*c^
2-d^2)+1/3*x^3/b/c+1/3/b^2/c^3*d^2*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A]
time = 0.27, size = 62, normalized size = 0.85

$$\frac{\frac{(bx^3+a)c-2\sqrt{bx^3+a}\,d}{c^2}-\frac{2\,(ac^2-d^2)\log\left(\sqrt{bx^3+a}\,c+d\right)}{c^3}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b
*x^3 + a)*c + d)/c^3)/b^2
```

Fricas [A]
time = 0.35, size = 118, normalized size = 1.62

$$\frac{bc^2x^3-2\sqrt{bx^3+a}\,cd-(ac^2-d^2)\log\left(bc^2x^3+ac^2-d^2\right)-(ac^2-d^2)\log\left(\sqrt{bx^3+a}\,c+d\right)+(ac^2-d^2)\log\left(\sqrt{bx^3+a}\,c-d\right)}{3\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^3 - 2*sqrt(b*x^3 + a)*c*d - (a*c^2 - d^2)*log(b*c^2*x^3 + a*c^
2 - d^2) - (a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d) + (a*c^2 - d^2)*log(sqr
t(b*x^3 + a)*c - d))/(b^2*c^3)
```

Sympy [A]
time = 3.21, size = 95, normalized size = 1.30

$$\begin{cases} \frac{2\left(\frac{a+bx^3}{6bc}-\frac{d\sqrt{a+bx^3}}{3bc^2}-\frac{(ac^2-d^2)\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for}\ c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3bc^2}\right)}{b} & \text{for}\ b\neq 0\\ \frac{x^6}{2\cdot\left(3\sqrt{a}\,d+3ac\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Piecewise((2*((a + b*x**3)/(6*b*c) - d*sqrt(a + b*x**3)/(3*b*c**2) - (a*c**
2 - d**2)*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3)
 + d)/c, True))/(3*b*c**2))/b, Ne(b, 0)), (x**6/(2*(3*sqrt(a)*d + 3*a*c)),
True))
```

Giac [A]
time = 3.69, size = 72, normalized size = 0.99

$$-\frac{\frac{2\,(ac^2-d^2)\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{bc^3}-\frac{(bx^3+a)bc-2\,\sqrt{bx^3+a}\,bd}{b^2c^2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c^3) - ((b*x^3 + a
)*b*c - 2*sqrt(b*x^3 + a)*b*d)/(b^2*c^2))/b
```

Mupad [B]
time = 3.56, size = 119, normalized size = 1.63

$$\frac{x^3}{3\,b\,c}-\frac{2\,d\,\sqrt{b\,x^3+a}}{3\,b^2\,c^2}+\frac{\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}-\frac{\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)\,\left(a\,c^2-d^2\right)}{3\,b^2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3)
^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^2
- d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)
```

3.554 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=26

$$\frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}$$

```
[Out] 2/3*ln(d+c*(b*x^3+a)^(1/2))/b/c
```

Rubi [A]
time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2186, 31}

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx, x, x^3\right)\\ &= \frac{2\text{Subst}\left(\int \frac{1}{d+cx}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\ &= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 29, normalized size = 1.12

$$\frac{2\log\left(bd + bc\sqrt{a + bx^3}\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[b*d + b*c*Sqrt[a + b*x^3]])/(3*b*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 501, normalized size = 19.27

method	result

default	$d\left(\frac{2\sqrt{b\,x^3+a}}{3d^2b}-\frac{c^2\left(\frac{2\sqrt{b\,x^3+a}}{3c^2b}+\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b\,c^2\_Z^3+c^2a-d^2\right)}\frac{\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}}{b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{}\right.}{}\right)}{}\right.$

elliptic	$\frac{\sqrt{b x^3+a}\,\left(d+c\sqrt{b x^3+a}\right)\left(\frac{\ln\left(b c^2 x^3+c^2 a-d^2\right)}{3bc}-\frac{i\sqrt{2}\left(\sum_{\_\alpha=\mathrm{RootOf}\left(b c^2 \_Z^3+c^2 a-d^2\right)}\frac{\left(-a b^2\right)^{\frac{1}{3}}\sqrt{2}\,\sqrt{\frac{ib\left(2x+\frac{\left(-a b^2\right)\cdots}{\cdots}\right)}{\left(\cdots\right.}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(2/3/d^2/b*(b*x^3+a)^(1/2)-c^2/d^2*(2/3/c^2/b*(b*x^3+a)^(1/2)+1/3*I/b^3/c
^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(
-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^
2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^
(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^
(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/
b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alph
a+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1
/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=R
```

```
ootOf(_Z^3*b*c^2+a*c^2-d^2))))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.85

$$\frac{2 \log \left(\sqrt{bx^3+a}\,c+d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 0.38, size = 61, normalized size = 2.35

$$\frac{\log\left(bc^2x^3+ac^2-d^2\right)+\log\left(\sqrt{bx^3+a}\,c+d\right)-\log\left(\sqrt{bx^3+a}\,c-d\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b
*x^3 + a)*c - d))/(b*c)
```

Sympy [A]
time = 2.00, size = 32, normalized size = 1.23

$$\frac{2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] 2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) + d)/c,
 True))/(3*b)
```

Giac [A]
time = 3.24, size = 23, normalized size = 0.88

$$\frac{2 \log \left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

Mupad [B]
time = 3.51, size = 60, normalized size = 2.31

$$\frac{\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)+\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] (log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^2
 + b*c^2*x^3))/(3*b*c)
```

3.555 $\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=93

$$\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}-\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\,(ac^2-d^2)}$$

```
[Out] c*ln(x)/(a*c^2-d^2)-2/3*c*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)+2/3*d*arctanh
((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2186, 720, 31, 649, 213, 266}

$$-\frac{2c\log\left(c\sqrt{a+bx^3}+d\right)}{3\,(ac^2-d^2)}+\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\,(ac^2-d^2)}+\frac{c\log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x
])/(a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{(d+cx)\left(-a+x^2\right)}\,dx,x,\sqrt{a+bx^3}\right)\\
&= -\frac{2\text{Subst}\left(\int \frac{d-cx}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}+\frac{(2c)\text{Subst}\left(\int \frac{x}{-a+x^2}\,dx,x,\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}-\\
&= \frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\left(ac^2-d^2\right)}+\frac{c\log(x)}{ac^2-d^2}-\frac{2c\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.74

$$
\frac{\frac{2d\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}+c\log\left(bx^3\right)-2c\log\left(d+c\sqrt{a+bx^3}\right)}{3ac^2-3d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] ((2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/Sqrt[a] + c*Log[b*x^3] - 2*c*Log[d
+ c*Sqrt[a + b*x^3]])/(3*a*c^2 - 3*d^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 621, normalized size = 6.68

method	result

default	$-\frac{a\,c^3\ln\left(b\,c^2x^3+c^2a-d^2\right)}{3(c^2a-d^2)d^2}+\frac{c\ln(x)}{c^2a-d^2}+\frac{c\ln\left(b\,c^2x^3+c^2a-d^2\right)}{3d^2}-d\left(\frac{2\sqrt{b\,x^3+a}}{3a\,d^2}-\frac{b\,c^4\left(\frac{2\sqrt{b\,x^3+a}}{3c^2b}+\frac{i\sqrt{2}\left(\sum_{\alpha=\mathrm{Ro}}\right.}{}\right.}{}\right.$

elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+c*ln(x)/(a*c^2-d^2)+1/3*
c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-d*(2/3/a/d^2*(b*x^3+a)^(1/2)-b*c^4/(a*c^2-d^2
)/d^2*(2/3/c^2/b*(b*x^3+a)^(1/2)+1/3*I/b^3/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(
1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))
^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3
)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a
*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a
*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/
2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3
)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1
/a/(a*c^2-d^2)*(2/3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(
1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

Fricas [A]
time = 0.38, size = 232, normalized size = 2.49

$$\left[-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}\,c+d\right)-ac\log\left(\sqrt{bx^3+a}\,c-d\right)-3\,ac\log\left(x\right)-\sqrt{a}\,d\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)}{3\,\left(a^2c^2-ad^2\right)},\,-\frac{ac\log\left(bc^2x^3+ac^2-d^2\right)+ac\log\left(\sqrt{bx^3+a}\,c+d\right)-ac\log\left(\sqrt{bx^3+a}\,c-d\right)-3\,ac\log\left(x\right)+2\,\sqrt{-a}\,d\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)}{3\,\left(a^2c^2-ad^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

Sympy [A]
time = 4.23, size = 97, normalized size = 1.04

$$-\frac{2c^2\left(\begin{cases}\frac{\sqrt{a+bx^3}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^3}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{3\left(ac^2-d^2\right)}-\frac{2\left(-\frac{c\log\left(-bx^3\right)}{2}+\frac{d\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] -2*c**2*Piecewise((sqrt(a + b*x**3)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**3) +
 d)/c, True))/(3*(a*c**2 - d**2)) - 2*(-c*log(-b*x**3)/2 + d*atan(sqrt(a +
b*x**3)/sqrt(-a))/sqrt(-a))/(3*(a*c**2 - d**2))
```

Giac [A]
time = 3.73, size = 94, normalized size = 1.01

$$-\frac{2\,c^2\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\left(ac^3-cd^2\right)}+\frac{c\log\left(bx^3\right)}{3\left(ac^2-d^2\right)}-\frac{2\,d\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\left(ac^2-d^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

Mupad [B]
time = 4.31, size = 156, normalized size = 1.68

$$\frac{c\ln\left(x\right)}{a\,c^2-d^2}+\frac{c\ln\left(\frac{d-c\,\sqrt{b\,x^3+a}}{d+c\,\sqrt{b\,x^3+a}}\right)}{3\,\left(a\,c^2-d^2\right)}-\frac{c\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a\,c^2-3\,d^2}+\frac{d\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\,{\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}^3}{x^6}\right)}{3\,\sqrt{a}\,\left(a\,c^2-d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x
^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^2
- 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/
2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

3.556 $\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=154

$$-\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3}-\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}+\frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2}$$

```
[Out] -1/3*b*d*(3*a*c^2-d^2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)/(a*c^2-d^2)
^2-b*c^3*ln(x)/(a*c^2-d^2)^2+2/3*b*c^3*ln(d+c*(b*x^3+a)^(1/2))/(a*c^2-d^2)^
2+1/3*(-a*c+d*(b*x^3+a)^(1/2))/a/(a*c^2-d^2)/x^3
```

Rubi [A]
time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2186, 755, 815, 649, 212, 266}

$$-\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2}-\frac{ac-d\sqrt{a+bx^3}}{3ax^3\left(ac^2-d^2\right)}+\frac{2bc^3\log\left(c\sqrt{a+bx^3}+d\right)}{3\left(ac^2-d^2\right)^2}-\frac{bc^3\log(x)}{\left(ac^2-d^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -1/3*(a*c - d*Sqrt[a + b*x^3])/(a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)
*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log
[x])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2
)^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\int \frac{1}{x^4\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(ac+bcx+d\sqrt{a+bx}\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}(2b)\text{Subst}\left(\int \frac{1}{(d+cx)\left(a-x^2\right)^2}\,dx,x,\sqrt{a+bx^3}\right)$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\text{Subst}\left(\int \frac{-2ac^2+d^2+cdx}{(d+cx)(a-x^2)}\,dx,x,\sqrt{a+bx^3}\right)}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{b\text{Subst}\left(\int \left(-\frac{2ac^4}{(ac^2-d^2)(d+cx)} + \frac{3ac^2d-d^3-2ac^3x}{(ac^2-d^2)(a-x^2)}\right)\,dx\right.}{3a\left(ac^2-d^2\right)}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} - \frac{b\text{Subst}\left(\int \frac{3ac^2d-d^3}{a-}\right.}{3a(}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} + \frac{2bc^3\log\left(d+c\sqrt{a+bx^3}\right)}{3\left(ac^2-d^2\right)^2} + \frac{\left(2bc^3\right)\text{Subst}\left(\int \frac{}{a-}\right.}{3(ac}$$

$$= -\frac{ac-d\sqrt{a+bx^3}}{3a\left(ac^2-d^2\right)x^3} - \frac{bd\left(3ac^2-d^2\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}\left(ac^2-d^2\right)^2} - \frac{bc^3\log(}{(ac^2-d}$$

Mathematica [A]
time = 0.20, size = 139, normalized size = 0.90

$$\frac{bd(-3ac^2+d^2)\,x^3\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\sqrt{a}\left(-\left((ac^2-d^2)\left(ac-d\sqrt{a+bx^3}\right)\right)-abc^3x^3\log\left(bx^3\right)+2abc^3x^3\log\left(d+c\sqrt{a+bx^3}\right)\right)}{3a^{3/2}\left(-ac^2+d^2\right)^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b*d*(-3*a*c^2 + d^2)*x^3*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]] + Sqrt[a]*(-((a*
c^2 - d^2)*(a*c - d*Sqrt[a + b*x^3])) - a*b*c^3*x^3*Log[b*x^3] + 2*a*b*c^3*
x^3*Log[d + c*Sqrt[a + b*x^3]]))/(3*a^(3/2)*(-(a*c^2) + d^2)^2*x^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 774, normalized size = 5.03

method	result

default	$\frac{a\,c^5 b\ln\left(b\,c^2x^3+c^2a-d^2\right)}{3\left(c^2a-d^2\right)^2 d^2}-\frac{c}{3\left(c^2a-d^2\right)x^3}-\frac{2b\,c^3\ln\left(x\right)}{\left(c^2a-d^2\right)^2}+\frac{cb\ln\left(x\right)d^2}{a\left(c^2a-d^2\right)^2}-\frac{b\,c^3\ln\left(b\,c^2x^3+c^2a-d^2\right)}{3\left(c^2a-d^2\right)d^2}+\frac{bc\ln\left(x\right)}{a\left(c^2a-d^2\right)}-d\left(-\frac{2b\sqrt{\ }}{}\right.$

elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*ln(b*c^2*x^3+a*c^2-d^2)-1/3*c/(a*c^2-d^2)/x^3
-2*b*c^3*ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*ln(x)*d^2-1/3*b*c^3/(a*c
^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+b*c/a/(a*c^2-d^2)*ln(x)-d*(-2/3*b/a^2/d
^2*(b*x^3+a)^(1/2)+b^2*c^6/(a*c^2-d^2)^2/d^2*(2/3/c^2/b*(b*x^3+a)^(1/2)+1/3
*I/b^3/c^2*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3
^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3
*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^
2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(
-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(
-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/
2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a
*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),
_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2)))-b*(2*a*c^2-d^2)/a^2/(a*c^2-d^2)^2*(2/
3*(b*x^3+a)^(1/2)-2/3*a^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+1/a/(a*c^2-
d^2)*(-1/3*(b*x^3+a)^(1/2)/x^3-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/
2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)
```

Fricas [A]
time = 0.45, size = 445, normalized size = 2.89

$$\left[\frac{2\,a^{2}bc^{3}x^{3}\log\left(bc^{2}x^{3}+ac^{2}-d^{2}\right)+2\,a^{2}bc^{3}x^{3}\log\left(\sqrt{bx^{3}+a}\,c+d\right)-2\,a^{2}bc^{3}x^{3}\log\left(\sqrt{bx^{3}+a}\,c-d\right)-6\,a^{2}bc^{3}x^{3}\log\left(x\right)-2\,a^{3}c^{3}-\left(3\,abc^{2}d-bd^{3}\right)\sqrt{a}\,x^{3}\log\left(\frac{bx^{3}+2\,\sqrt{bx^{3}+a}\,\sqrt{a}+2\,a}{x^{3}}\right)+2\,a^{2}cd^{2}+2\,\left(a^{2}c^{2}d-ad^{3}\right)\sqrt{bx^{3}+a}}{6\,\left(a^{4}c^{4}-2\,a^{3}c^{2}d^{2}+a^{2}d^{4}\right)x^{3}},\ \frac{a^{2}bc^{3}x^{3}\log\left(bc^{2}x^{3}+ac^{2}-d^{2}\right)+a^{2}bc^{3}x^{3}\log\left(\sqrt{bx^{3}+a}\,c+d\right)-a^{2}bc^{3}x^{3}\log\left(\sqrt{bx^{3}+a}\,c-d\right)-3\,a^{2}bc^{3}x^{3}\log\left(x\right)-a^{3}c^{3}+\left(3\,abc^{2}d-bd^{3}\right)\sqrt{-a}\,x^{3}\arctan\left(\frac{\sqrt{bx^{3}+a}\,\sqrt{-a}}{a}\right)+a^{2}cd^{2}+\left(a^{2}c^{2}d-ad^{3}\right)\sqrt{bx^{3}+a}}{3\,\left(a^{4}c^{4}-2\,a^{3}c^{2}d^{2}+a^{2}d^{4}\right)x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b*c^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*log(sq
rt(b*x^3 + a)*c + d) - 2*a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 6*a^2*b
*c^3*x^3*log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*sqrt(a)*x^3*log((b*x^3
```

```
+ 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^
3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c
^3*x^3*log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c +
 d) - a^2*b*c^3*x^3*log(sqrt(b*x^3 + a)*c - d) - 3*a^2*b*c^3*x^3*log(x) - a
^3*c^3 + (3*a*b*c^2*d - b*d^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*sqrt(b*x^3 + a))/((a^4*c^4 - 2*a^3*c^
2*d^2 + a^2*d^4)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**4*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [A]
time = 4.20, size = 211, normalized size = 1.37

$$\frac{2\,bc^4\log\left(\left|\sqrt{bx^3+a}\,c+d\right|\right)}{3\,(a^2c^5-2\,ac^3d^2+cd^4)}-\frac{bc^3\log\left(-bx^3\right)}{3\,(a^2c^4-2\,ac^2d^2+d^4)}+\frac{(3\,abc^2d-bd^3)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,(a^3c^4-2\,a^2c^2d^2+ad^4)\sqrt{-a}}-\frac{a^2bc^3-abcd^2-(abc^2d-bd^3)\sqrt{bx^3+a}}{3\,(ac^2-d^2)^2abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*b*c^4*log(abs(sqrt(b*x^3 + a)*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) -
 1/3*b*c^3*log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d - b
*d^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*s
qrt(-a)) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*sqrt(b*x^3 + a)
)/((a*c^2 - d^2)^2*a*b*x^3)
```

Mupad [B]
time = 5.46, size = 248, normalized size = 1.61

$$\frac{b\,c^3\,\ln\left(b\,c^2\,x^3+a\,c^2-d^2\right)}{3\,a^2\,c^4-6\,a\,c^2\,d^2+3\,d^4}-\frac{b\,c^3\,\ln\left(x\right)}{a^2\,c^4-2\,a\,c^2\,d^2+d^4}-\frac{c}{3\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,c^3\,\ln\left(\frac{d+c\,\sqrt{b\,x^3+a}}{d-c\,\sqrt{b\,x^3+a}}\right)}{3\,{\left(a\,c^2-d^2\right)}^2}+\frac{d\,\sqrt{b\,x^3+a}}{3\,a\,x^3\,\left(a\,c^2-d^2\right)}+\frac{b\,d\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,a\,c^2-d^2\right)}{6\,a^{3/2}\,{\left(a\,c^2-d^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (b
*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b*c
^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^
2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(3*a*c^2 - d^2))
/(6*a^(3/2)*(a*c^2 - d^2)^2)
```

3.557 $\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=311

$$\frac{x}{bc}-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}}+\frac{\sqrt[3]{ac^2-d^2}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{\ldots}\right.}{3b\ldots}$$

```
[Out] x/b/c-1/3*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(4/3)
/c^(5/3)+1/6*(a*c^2-d^2)^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-
d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/b^(4/3)/c^(5/3)+1/3*(a*c^2-d^2)^(1/3)*arc
tan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(4/3)/c^(5/3)*
3^(1/2)-1/4*d*x^4*AppellF1(4/3,1/2,1,7/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(
1+b*x^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2187, 327, 206, 31, 648, 631, 210, 642, 525, 524}

$$-\frac{dx^4\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\sqrt{a+bx^3}\left(ac^2-d^2\right)}+\frac{\sqrt[3]{ac^2-d^2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}c^{5/3}}+\frac{\sqrt[3]{ac^2-d^2}\,\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}-\frac{\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3b^{4/3}c^{5/3}}+\frac{x}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] x/(b*c) - (d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a
), -((b*c^2*x^3)/(a*c^2 - d^2))])/(4*(a*c^2 - d^2)*Sqrt[a + b*x^3]) + ((a*c
^2 - d^2)^(1/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(4/3)*c^(5/3)) - ((a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1
/3) + b^(1/3)*c^(2/3)*x])/(3*b^(4/3)*c^(5/3)) + ((a*c^2 - d^2)^(1/3)*Log[(a
*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)
*x^2])/(6*b^(4/3)*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^3}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x^3}{a^2c^2-ad^2+abc^2x^3}\,dx - (ad)\int \frac{x^3}{\sqrt{a+bx^3}\,\left(a^2c^2-ad^2+abc^2x^3\right)}$$

$$= \frac{x}{bc} - \frac{(a(ac^2-d^2))\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{bc} - \frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{}{\sqrt{1+\frac{bx^3}{a}}\ (}}{\sqrt{a+bx^3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\ F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\left(\sqrt[3]{a}\ \sqrt[3]{ac^2-d^2}\right)\int}{}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\ F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\sqrt[3]{ac^2-d^2}\ \log\left(\sqrt[3]{ac^2}\right.}{3b^{4/3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\ F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} - \frac{\sqrt[3]{ac^2-d^2}\ \log\left(\sqrt[3]{ac^2}\right.}{3b^{4/3}}$$

$$= \frac{x}{bc} - \frac{dx^4\sqrt{1+\frac{bx^3}{a}}\ F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{4\left(ac^2-d^2\right)\sqrt{a+bx^3}} + \frac{\sqrt[3]{ac^2-d^2}\ \tan^{-1}\left(\frac{1}{}\right.}{\sqrt{3}\ b^{4/3}c}$$

Mathematica [A]

time = 10.32, size = 296, normalized size = 0.95

$$-\frac{dx^4\sqrt{1+\frac{bx^3}{a}}\ F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(4ac^2-4d^2\right)\sqrt{a+bx^3}} + \frac{6\sqrt[3]{b}\,c^{2/3}x - 2\sqrt{3}\,\sqrt[3]{ac^2-d^2}\,\tan^{-1}\left(\frac{-1+\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right) - 2\sqrt[3]{ac^2-d^2}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right) + \sqrt[3]{ac^2-d^2}\,\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}\,c^{2/3}\sqrt[3]{ac^2-d^2}\,x+b^{2/3}c^{4/3}x^2\right)}{6b^{4/3}c^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] -((d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((4*a*c^2 - 4*d^2)*Sqrt[a + b*x^3])) + (6*b^(1/3)*c^(2/3)*x - 2*Sqrt[3]*(a*c^2 - d^2)^(1/3)*ArcTan[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*(a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + (a*c^2 - d^2)^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(4/3)*c^(5/3))

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.09, size = 1421, normalized size = 4.57

method	result	size
elliptic	Expression too large to display	1174
default	Expression too large to display	1421

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)

[Out] d*(2/3*I*a/b^2/d^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-(-a*c^2+d^2)/d^2/b*(-2/3*I/c^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/c^2/b^3*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^

```
(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2)
)))-1/3*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1
/6*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a
*c^2-d^2)/b/c^2)^(2/3))-1/3*a/c/b^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))+x/b/c+1/3/b^2/c^3*d^2/((a*
c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b^2/c^3*d^2/((a*c
^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)
^(2/3))+1/3/b^2/c^3*d^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(x**3/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] int(x^3/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.558 $\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=304

$$-\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}+\cdots$$

```
[Out] -1/3*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1
/3)+1/6*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^
(4/3)*x^2)/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)-1/3*arctan(1/3*(1-2*b^(1/3)*c^
(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(2/3)/c^(1/3)/(a*c^2-d^2)^(1/3)*3^(1/
2)-1/2*d*x^2*AppellF1(2/3,1/2,1,5/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x
^3/a)^(1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2187, 298, 31, 648, 631, 210, 642, 525, 524}

$$-\frac{dx^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\sqrt{a+bx^3}\left(ac^2-d^2\right)}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}+\frac{\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3b^{2/3}\sqrt[3]{c}\,\sqrt[3]{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] -1/2*(d*x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -(
(b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3]) - ArcTan[(1 -
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*c^(1/3
)*(a*c^2 - d^2)^(1/3)) - Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x]/(3*b^
(2/3)*c^(1/3)*(a*c^2 - d^2)^(1/3)) + Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2
/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]/(6*b^(2/3)*c^(1/3)*(a*c^2
- d^2)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{x}{\sqrt{a+bx^3}\,\left(a^2c^2-ad^2+abc^2x^3\right)}$$

$$= -\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\sqrt[3]{b}\,c^{2/3}x}\,dx}{3\sqrt[3]{b}\sqrt[3]{ac^2-d^2}}+\frac{\left(\sqrt[3]{a}\sqrt[3]{c}\right)\int \frac{}{a^{2/3}(ac^2-d^2)^2}}{}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^2\right.}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^2\right.}{3b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

$$= -\frac{dx^2\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)\sqrt{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}\sqrt[3]{c}\sqrt[3]{ac^2-d^2}}$$

Mathematica [F]
time = 10.10, size = 0, normalized size = 0.00

$$\int \frac{x}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] Integrate[x/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.09, size = 1505, normalized size = 4.95

method	result

elliptic	$\frac{\sqrt{b x^3+a}\left(d+c\sqrt{b x^3+a}\right)\left(-\frac{\ln\left(x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}}+\frac{\ln\left(x^2-\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c^2 a-d^2}{b c^2}\right.}\right.}{3}\right.}{3bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-2/3*I/d^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1
/3))*EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b
^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/
3)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2
```

```
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-c^2/d^2*(-2/3*I/c
^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*
b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b
*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*
x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2
)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^
(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/3*I/c^2/b^3*2^(1/2)*sum(1/
_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^
(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+
I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/
2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)
*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_a
lpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2
*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1
/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z
^3*b*c^2+a*c^2-d^2))))-1/3/b/c/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/
b/c^2)^(1/3))+1/6/b/c/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^
(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3/b/c*3^(1/2)/((a*c^2-d^2)/b/c^2)^(1/3
)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{ac + bcx^3 + d\sqrt{a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(x/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{a\,c + d\,\sqrt{b\,x^3 + a} + b\,c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] int(x/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.559 $\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=300

$$-\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}+\frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}\right.}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

```
[Out] 1/3*c^(1/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x)/b^(1/3)/(a*c^2-d^2)^(2/
3)-1/6*c^(1/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(
2/3)*c^(4/3)*x^2)/b^(1/3)/(a*c^2-d^2)^(2/3)-1/3*c^(1/3)*arctan(1/3*(1-2*b^(
1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/b^(1/3)/(a*c^2-d^2)^(2/3)*3^(1/2
)-d*x*AppellF1(1/3,1/2,1,4/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2))*(1+b*x^3/a)^(
1/2)/(a*c^2-d^2)/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {2187, 206, 31, 648, 631, 210, 642, 441, 440}

$$-\frac{dx\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\,(ac^2-d^2)}-\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}-\frac{\sqrt[3]{c}\,\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}+\frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^3)/a]*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^3])) - (c^(1/3)*ArcTan[
(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)*
(a*c^2 - d^2)^(2/3)) + (c^(1/3)*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x
])/(3*b^(1/3)*(a*c^2 - d^2)^(2/3)) - (c^(1/3)*Log[(a*c^2 - d^2)^(2/3) - b^(
1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2])/(6*b^(1/3)*(a*c^
2 - d^2)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 210

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 440

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 441

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 631

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*Simplify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 642

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 648

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 2187

Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/

```
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^3}\,(a^2c^2-ad^2+abc^2x^3)}$$

$$= \frac{\left(\sqrt[3]{a}\,c\right)\int \frac{1}{\sqrt[3]{a}\,\sqrt[3]{ac^2-d^2}+\sqrt[3]{a}\,\sqrt[3]{b}\,c^{2/3}x}\,dx}{3\left(ac^2-d^2\right)^{2/3}}+\frac{\left(\sqrt[3]{a}\,c\right)\int \frac{2\sqrt[3]{a}\,\sqrt[3]{ac}}{a^{2/3}(ac^2-d^2)^{2/3}-a^{2/3}\sqrt[3]{}}}{3\,(ac}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\right.}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}+\frac{\sqrt[3]{c}\,\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\right.}{3\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^3}}-\frac{\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right.}{\sqrt{3}\,\sqrt[3]{b}\,(ac^2-d^2)^{2/3}}$$

Mathematica [F]
time = 10.05, size = 0, normalized size = 0.00

$$\int \frac{1}{ac+bcx^3+d\sqrt{a+bx^3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1),x]
```

```
[Out] Integrate[(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])^(-1), x]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.07, size = 1201, normalized size = 4.00

method	result

elliptic	$\frac{\sqrt{b x^3+a}\,\left(d+c\sqrt{b x^3+a}\right)\left(\frac{\ln\left(x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}\right)}{3bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{1}{3}}x+\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{2}{3}}\right)}{6bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c^2 a-d^2}{b c^2}\right)}\right.}{3}\right.}{3bc\left(\frac{c^2 a-d^2}{b c^2}\right)^{\frac{2}{3}}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-2/3*I/d^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(
1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/
3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3)
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/
2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*
(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2
))-c^2/d^2*(-2/3*I/c^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-
1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a
```

```
*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*
(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b
^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)
^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3
^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3)))^(1/2))+1/3*I/c^2/b^3*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2
*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*
(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*
(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3
))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3
)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(
1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^
2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3
*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))))+1/3/b/c/((
a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6/b/c/((a*c^2-d^2
)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3)
)+1/3/b/c/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d
^2)/b/c^2)^(1/3)*x-1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac + bcx^3 + d\sqrt{a + bx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(a*c + b*c*x**3 + d*sqrt(a + b*x**3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^3 + a*c + sqrt(b*x^3 + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
```

```
[Out] int(1/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)
```

3.560 $\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=319

$$-\frac{c}{(ac^2-d^2)\,x}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{(ac^2-d^2)\,x\sqrt{a+bx^3}}+\frac{\sqrt[3]{b}\,c^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}\,c^{5/3}\log}{}$$

```
[Out] -c/(a*c^2-d^2)/x+1/3*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(2/3)*x
)/(a*c^2-d^2)^(4/3)-1/6*b^(1/3)*c^(5/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*c^(2/3
)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(4/3)+1/3*b^(1/3)*c^
(5/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(a*c^2-
d^2)^(4/3)*3^(1/2)+d*AppellF1(-1/3,1/2,1,2/3,-b*x^3/a,-b*c^2*x^3/(a*c^2-d^2
))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x/(b*x^3+a)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2187, 331, 298, 31, 648, 631, 210, 642, 525, 524}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{x\sqrt{a+bx^3}\,(ac^2-d^2)}+\frac{\sqrt[3]{b}\,c^{5/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,(ac^2-d^2)^{4/3}}-\frac{\sqrt[3]{b}\,c^{5/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+(ac^2-d^2)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\,(ac^2-d^2)^{4/3}}+\frac{\sqrt[3]{b}\,c^{5/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3\,(ac^2-d^2)^{4/3}}-\frac{c}{x\,(ac^2-d^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-1/3, 1/2, 1, 2/3,
 -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/((a*c^2 - d^2)*x*Sqrt[a + b*x
^3]) + (b^(1/3)*c^(5/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/
3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(4/3)) + (b^(1/3)*c^(5/3)*Log[(a*c^2 -
 d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(4/3)) - (b^(1/3)*c^(5/3
)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(2/3)
*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx = (ac)\int \frac{1}{x^2\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx - (ad)\int \frac{1}{x^2\sqrt{a+bx^3}\,\left(a^2c^2-a\right.}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x} - \frac{\left(abc^3\right)\int \frac{x}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2} - \frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int -}{\sqrt{}}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}} + \frac{\left(\sqrt[3]{a}\right.}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}} + \frac{\sqrt[3]{b}\,c}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}} + \frac{\sqrt[3]{b}\,c}{}$$

$$= -\frac{c}{\left(ac^2-d^2\right)x} + \frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{2},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)x\sqrt{a+bx^3}} + \frac{\sqrt[3]{b}\,c}{}$$

Mathematica [A]
time = 10.44, size = 496, normalized size = 1.55

$$\frac{15bd\sqrt[3]{ac^2-d^2}\left(ac^2+d^2\right)x^3\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right) - 6b^2c^2d\sqrt[3]{ac^2-d^2}\,x^6\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right) - 10(ac^2-d^2)\left(-6ad\sqrt[3]{ac^2-d^2} - 6bd\sqrt[3]{ac^2-d^2}\,x^3 + 6ac\sqrt[3]{ac^2-d^2}\sqrt{a+bx^3} + 2\sqrt{3}\,a\sqrt[3]{b}\,c^{5/3}x\sqrt{a+bx^3}\tan^{-1}\left(\frac{-1+\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right) - 2a\sqrt[3]{b}\,c^{5/3}x\sqrt{a+bx^3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right) + a\sqrt[3]{b}\,c^{5/3}x\sqrt{a+bx^3}\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}\,c^{2/3}\sqrt[3]{ac^2-d^2}\,x+b^{2/3}c^{4/3}x^2\right)\right)}{60a\left(ac^2-d^2\right)^{7/3}x\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (15*b*d*(a*c^2 - d^2)^(1/3)*(a*c^2 + d^2)*x^3*Sqrt[1 + (b*x^3)/a]*AppellF1[
2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 6*b^2*c^2*d
*(a*c^2 - d^2)^(1/3)*x^6*Sqrt[1 + (b*x^3)/a]*AppellF1[5/3, 1/2, 1, 8/3, -((
b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))] - 10*(a*c^2 - d^2)*(-6*a*d*(a*c^2
- d^2)^(1/3) - 6*b*d*(a*c^2 - d^2)^(1/3)*x^3 + 6*a*c*(a*c^2 - d^2)^(1/3)*Sq
rt[a + b*x^3] + 2*Sqrt[3]*a*b^(1/3)*c^(5/3)*x*Sqrt[a + b*x^3]*ArcTan[(-1 +
(2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(1/3)*c^(5/3)*x
*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(1/3)*c
^(5/3)*x*Sqrt[a + b*x^3]*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 -
 d^2)^(1/3)*x + b^(2/3)*c^(4/3)*x^2]))/(60*a*(a*c^2 - d^2)^(7/3)*x*Sqrt[a +
 b*x^3])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.14, size = 2227, normalized size = 6.98

method	result	size
elliptic	Expression too large to display	1202
default	Expression too large to display	2227

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^2-d^2)/b/c^2
)^(1/3))-1/6*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2
-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2*3
^(1/2)/((a*c^2-d^2)/b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^
(1/3)*x-1))-c/(a*c^2-d^2)/x-1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x+((a*c^
2-d^2)/b/c^2)^(1/3))+1/6*c/d^2/((a*c^2-d^2)/b/c^2)^(1/3)*ln(x^2-((a*c^2-d^2
)/b/c^2)^(1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))+1/3*c/d^2*3^(1/2)/((a*c^2-d^2)/
b/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-d*(-2/3*
I/a/d^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(
-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/
2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/
(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3
))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))
*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^
2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))-b*c^4/(a*c^2-d^2)/d^2*(-2
```

```
/3*I/c^2*3^(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/
b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/
2)/(b*x^3+a)^(1/2)*((-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*
EllipticE(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(
1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(
-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/
3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a
*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)))+1/3*I/c^2/b^3*2^(1/2)*
sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a
*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-3*(-a*b^2)^
(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I
*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)
^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1
/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1
/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),-1/2/b*
c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+
I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-a*b^2)^(1/3
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)),_alpha=Roo
tOf(_Z^3*b*c^2+a*c^2-d^2)))+1/a/(a*c^2-d^2)*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(
-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^
(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3
)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(
(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*EllipticE(1/3*3^(1/2
)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*
b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*
3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/b*(-a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^
2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^
(1/2)/b*(-a*b^2)^(1/3)))^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(ac + bcx^3 + d\sqrt{a + bx^3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(a\,c+d\,\sqrt{b\,x^3+a}+b\,c\,x^3\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] int(1/(x^2*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.561 $\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx$

Optimal. Leaf size=324

$$-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\;F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}+\frac{b^{2/3}c^{7/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}\,\mathrm{l}}{}$$

```
[Out] -1/2*c/(a*c^2-d^2)/x^2-1/3*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(1/3)+b^(1/3)*c^(
2/3)*x)/(a*c^2-d^2)^(5/3)+1/6*b^(2/3)*c^(7/3)*ln((a*c^2-d^2)^(2/3)-b^(1/3)*
c^(2/3)*(a*c^2-d^2)^(1/3)*x+b^(2/3)*c^(4/3)*x^2)/(a*c^2-d^2)^(5/3)+1/3*b^(2
/3)*c^(7/3)*arctan(1/3*(1-2*b^(1/3)*c^(2/3)*x/(a*c^2-d^2)^(1/3))*3^(1/2))/(
a*c^2-d^2)^(5/3)*3^(1/2)+1/2*d*AppellF1(-2/3,1/2,1,1/3,-b*x^3/a,-b*c^2*x^3/
(a*c^2-d^2))*(1+b*x^3/a)^(1/2)/(a*c^2-d^2)/x^2/(b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2187, 331, 206, 31, 648, 631, 210, 642, 525, 524}

$$\frac{d\sqrt{\frac{bx^3}{a}+1}\;F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2x^2\sqrt{a+bx^3}\,\left(ac^2-d^2\right)}+\frac{b^{2/3}c^{7/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)}{\sqrt{3}\,\left(ac^2-d^2\right)^{5/3}}+\frac{b^{2/3}c^{7/3}\log\left(-\sqrt[3]{b}\,c^{2/3}x\sqrt[3]{ac^2-d^2}+\left(ac^2-d^2\right)^{2/3}+b^{2/3}c^{4/3}x^2\right)}{6\left(ac^2-d^2\right)^{5/3}}-\frac{b^{2/3}c^{7/3}\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)}{3\left(ac^2-d^2\right)^{5/3}}-\frac{c}{2x^2\left(ac^2-d^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] -1/2*c/((a*c^2 - d^2)*x^2) + (d*Sqrt[1 + (b*x^3)/a]*AppellF1[-2/3, 1/2, 1,
1/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))])/(2*(a*c^2 - d^2)*x^2*Sqrt
[a + b*x^3]) + (b^(2/3)*c^(7/3)*ArcTan[(1 - (2*b^(1/3)*c^(2/3)*x)/(a*c^2 -
d^2)^(1/3))/Sqrt[3]])/(Sqrt[3]*(a*c^2 - d^2)^(5/3)) - (b^(2/3)*c^(7/3)*Log[
(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x])/(3*(a*c^2 - d^2)^(5/3)) + (b^(2/3
)*c^(7/3)*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x +
 b^(2/3)*c^(4/3)*x^2])/(6*(a*c^2 - d^2)^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(ac+bcx^3+d\sqrt{a+bx^3}\right)}\,dx=(ac)\int \frac{1}{x^3\left(a^2c^2-ad^2+abc^2x^3\right)}\,dx-(ad)\int \frac{1}{x^3\sqrt{a+bx^3}\,\left(a^2c^2-a\cdots\right.}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}-\frac{\left(abc^3\right)\int \frac{1}{a^2c^2-ad^2+abc^2x^3}\,dx}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^3}{a}}\right)\int\cdots}{\cdots}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{b^2\cdots}{\cdots}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}-\frac{b^2\cdots}{\cdots}$$

$$=-\frac{c}{2\left(ac^2-d^2\right)x^2}+\frac{d\sqrt{1+\frac{bx^3}{a}}\,F_1\left(-\frac{2}{3};\frac{1}{2},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{2\left(ac^2-d^2\right)x^2\sqrt{a+bx^3}}+\frac{b^2\cdots}{\cdots}$$

Mathematica [**A**]
time = 13.35, size = 604, normalized size = 1.86

$$\frac{b^2c^2dx^4\sqrt{1+\frac{bx^3}{a}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{16a\left(-ac^2+d^2\right)^2\sqrt{a+bx^3}}+\frac{2bd\left(-5ac^2+d^2\right)xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)}{\sqrt{a+bx^3}\left(ac^2-d^2+bc^2x^3\right)\left(8a\left(-ac^2+d^2\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+3bx^3\left(2ac^2F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)+\left(ac^2-d^2\right)F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{bc^2x^3}{ac^2-d^2}\right)\right)\right)}+\frac{-3ac\left(ac^2-d^2\right)^{2/3}+3d\left(ac^2-d^2\right)^{2/3}\sqrt{a+bx^3}-2\sqrt{3}\,ab^{2/3}c^{7/3}x^2\tan^{-1}\left(\frac{-1+\frac{2\sqrt[3]{b}\,c^{2/3}x}{\sqrt[3]{ac^2-d^2}}}{\sqrt{3}}\right)-2ab^{2/3}c^{7/3}x^2\log\left(\sqrt[3]{ac^2-d^2}+\sqrt[3]{b}\,c^{2/3}x\right)+ab^{2/3}c^{7/3}x^2\log\left(\left(ac^2-d^2\right)^{2/3}-\sqrt[3]{b}\,c^{2/3}\sqrt[3]{ac^2-d^2}\,x+b^{2/3}c^{4/3}x^2\right)}{6a\left(ac^2-d^2\right)^{5/3}x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]
```

```
[Out] (b^2*c^2*d*x^4*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a),
 -((b*c^2*x^3)/(a*c^2 - d^2))])/(16*a*(-(a*c^2) + d^2)^2*Sqrt[a + b*x^3]) +
 (2*b*d*(-5*a*c^2 + d^2)*x*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^
2*x^3)/(a*c^2 - d^2))])/(Sqrt[a + b*x^3]*(a*c^2 - d^2 + b*c^2*x^3)*(8*a*(-(
a*c^2) + d^2)*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*c^2*x^3)/(a*c^2
 - d^2))] + 3*b*x^3*(2*a*c^2*AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*
c^2*x^3)/(a*c^2 - d^2))] + (a*c^2 - d^2)*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^
3)/a), -((b*c^2*x^3)/(a*c^2 - d^2))]))) + (-3*a*c*(a*c^2 - d^2)^(2/3) + 3*d
*(a*c^2 - d^2)^(2/3)*Sqrt[a + b*x^3] - 2*Sqrt[3]*a*b^(2/3)*c^(7/3)*x^2*ArcT
an[(-1 + (2*b^(1/3)*c^(2/3)*x)/(a*c^2 - d^2)^(1/3))/Sqrt[3]] - 2*a*b^(2/3)*
c^(7/3)*x^2*Log[(a*c^2 - d^2)^(1/3) + b^(1/3)*c^(2/3)*x] + a*b^(2/3)*c^(7/3
)*x^2*Log[(a*c^2 - d^2)^(2/3) - b^(1/3)*c^(2/3)*(a*c^2 - d^2)^(1/3)*x + b^(
2/3)*c^(4/3)*x^2])/(6*a*(a*c^2 - d^2)^(5/3)*x^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 6.
time = 0.09, size = 1771, normalized size = 5.47

method	result	size
elliptic	Expression too large to display	1051
default	Expression too large to display	1771

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))-1/6*c/d
^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(1/3)*x+((a*c^2-d^2
)/b/c^2)^(2/3))+1/3*c/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-1/2*c/(a*c^2-d^2)/x^2-1/3*a*c^3/(a*c
^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x+((a*c^2-d^2)/b/c^2)^(1/3))+1/6*a
*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/c^2)^(2/3)*ln(x^2-((a*c^2-d^2)/b/c^2)^(
1/3)*x+((a*c^2-d^2)/b/c^2)^(2/3))-1/3*a*c^3/(a*c^2-d^2)/d^2/((a*c^2-d^2)/b/
c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*c^2-d^2)/b/c^2)^(1/3)*x-1))-d*
(-2/3*I/a/d^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/
2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3)
)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/
b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
+1/a/(a*c^2-d^2)*(-1/2/x^2*(b*x^3+a)^(1/2)-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*
(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^
```

```
(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b
*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2
)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1
/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-
a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*
I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))-1/(a*c^2-d^2)*b*c^4/d^2*(-2/3*I/c^2*3^
(1/2)/b*(-a*b^2)^(1/3)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^
(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)*((x-1/b*(-a*b^2)^(1/3))/(-3/2/b*(-a*
b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2)*(-I*(x+1/2/b*(-a*b^2)^(1/
3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/b*(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-
a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1/2),(I*3^(1/2)/b*(-a*b^2)^(1/3)/(
-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))+1/3*I/c^2/b^3
*2^(1/2)*sum(1/_alpha^2*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*
3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(-a*b^2)^(1/3))/(-
3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*
(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-
(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*
(-a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(-a*b^2)^(1/3))*3^(1/2)*b/(-a*b^2)^(1/3))^(1
/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1
/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)/d^2,(I*3^(1/2)/b*(-
a*b^2)^(1/3)/(-3/2/b*(-a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(-a*b^2)^(1/3)))^(1/2))
,_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(ac+bcx^{3}+d\sqrt{a+bx^{3}}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
[Out] Integral(1/(x**3*(a*c + b*c*x**3 + d*sqrt(a + b*x**3))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\left(a\,c+d\,\sqrt{b\,x^{3}+a}+b\,c\,x^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
[Out] int(1/(x^3*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)), x)
```

3.562 $\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=135

$$-\frac{dx\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

```
[Out] c*x*hypergeom([1, 1/n],[1+1/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2-d^2)-d*x*Appe
llF1(1/n,1/2,1,1+1/n,-b*x^n/a,-b*c^2*x^n/(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*
c^2-d^2)/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2187, 251, 441, 440}

$$\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{dx\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] -((d*x*Sqrt[1 + (b*x^n)/a]*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a
), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*Sqrt[a + b*x^n])) + (c*x*H
ypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c^2*x^n)/(a*c^2 - d^2))])/(a*c
^2 - d^2)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{1}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = (ac)\int \frac{1}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{1}{\sqrt{a+bx^n}\,(a^2c^2-ad^2+abc^2x^n)}$$

$$= \frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{1}{\sqrt{1+\frac{bx^n}{a}}\,(a^2c^2-ad^2+abc}}{\sqrt{a+bx^n}}$$

$$= -\frac{dx\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\sqrt{a+bx^n}}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b}{a}\right.}{ac^2-d^2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 320 vs. 2(135) = 270.
time = 0.49, size = 320, normalized size = 2.37

$$-\frac{2ad(ac^2-d^2)(1+n)xF_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\sqrt{a+bx^n}\,(ac^2-d^2+bc^2x^n)\left(-2abc^2nx^nF_1\left(1+\frac{1}{n};\frac{1}{2},2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+(ac^2-d^2)\left(-bnx^nF_1\left(1+\frac{1}{n};\frac{3}{2},1;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+2a(1+n)F_1\left(\frac{1}{n};\frac{1}{2},1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)\right)\right)}+\frac{cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{ac^2-d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*c + b*c*x^n + d*Sqrt[a + b*x^n])^(-1),x]
```

```
[Out] (-2*a*d*(a*c^2 - d^2)*(1 + n)*x*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x
^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/(Sqrt[a + b*x^n]*(a*c^2 - d^2 + b*c^
2*x^n)*(-2*a*b*c^2*n*x^n*AppellF1[1 + n^(-1), 1/2, 2, 2 + n^(-1), -((b*x^n)
/a), -((b*c^2*x^n)/(a*c^2 - d^2))] + (a*c^2 - d^2)*(-(b*n*x^n*AppellF1[1 +
n^(-1), 3/2, 1, 2 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) +
2*a*(1 + n)*AppellF1[n^(-1), 1/2, 1, 1 + n^(-1), -((b*x^n)/a), -((b*c^2*x^n
)/(a*c^2 - d^2))]))) + (c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*c
^2*x^n)/(a*c^2 - d^2))])/(a*c^2 - d^2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac+bc\,x^{n}+d\sqrt{a+b\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
[Out] int(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
[Out] integral((b*c*x^n + a*c - sqrt(b*x^n + a)*d)/(b^2*c^2*x^(2*n) + a^2*c^2 - a
*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac+bcx^{n}+d\sqrt{a+bx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
[Out] Integral(1/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a\,c+d\,\sqrt{a+b\,x^n}+b\,c\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

```
[Out] int(1/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.563 $\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=167

$$-\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},1;\frac{1+m+n}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)\sqrt{a+bx^n}}+\frac{cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(ac^2-d^2)\,(1+m)}$$

```
[Out] c*x^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*c^2*x^n/(a*c^2-d^2))/(a*c^2
-d^2)/(1+m)-d*x^(1+m)*AppellF1((1+m)/n,1/2,1,(1+m+n)/n,-b*x^n/a,-b*c^2*x^n/
(a*c^2-d^2))*(1+b*x^n/a)^(1/2)/(a*c^2-d^2)/(1+m)/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2187, 371, 525, 524}

$$\frac{cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)}-\frac{dx^{m+1}\sqrt{\frac{bx^n}{a}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},1;\frac{m+n+1}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{(m+1)\,(ac^2-d^2)\,\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] -((d*x^(1 + m)*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)/
n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m)*Sqrt
[a + b*x^n])) + (c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
 -((b*c^2*x^n)/(a*c^2 - d^2))])/((a*c^2 - d^2)*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{x^m}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx = (ac)\int \frac{x^m}{a^2c^2-ad^2+abc^2x^n}\,dx-(ad)\int \frac{x^m}{\sqrt{a+bx^n}\,\left(a^2c^2-ad^2+abc^2x^n\right)}$$

$$= \frac{cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)}-\frac{\left(ad\sqrt{1+\frac{bx^n}{a}}\right)\int \frac{x^m}{\sqrt{1+\frac{bx^n}{a}}\,\left(a^2c^2\right.}}{\sqrt{a+bx^n}}$$

$$= -\frac{dx^{1+m}\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},1;\frac{1+m+n}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)}{\left(ac^2-d^2\right)(1+m)\sqrt{a+bx^n}}+\frac{cx^{1+m}\,{}_2F_1\left(1,\frac{1}{}\right.}{\left(ac^2-\right.}$$

Mathematica [A]
time = 0.36, size = 156, normalized size = 0.93

$$\frac{x^{1+m}\left(-d\sqrt{1+\frac{bx^n}{a}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},1;\frac{1+m+n}{n};-\frac{bx^n}{a},-\frac{bc^2x^n}{ac^2-d^2}\right)+c\sqrt{a+bx^n}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bc^2x^n}{ac^2-d^2}\right)\right)}{\left(ac^2-d^2\right)(1+m)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (x^(1 + m)*(-(d*Sqrt[1 + (b*x^n)/a]*AppellF1[(1 + m)/n, 1/2, 1, (1 + m + n)
/n, -((b*x^n)/a), -((b*c^2*x^n)/(a*c^2 - d^2))]) + c*Sqrt[a + b*x^n]*Hyperg
eometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*c^2*x^n)/(a*c^2 - d^2))]))/((
a*c^2 - d^2)*(1 + m)*Sqrt[a + b*x^n])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{ac+bc\,x^{n}+d\sqrt{a+b\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
[Out] int(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
[Out] integral((b*c*x^m*x^n + a*c*x^m - sqrt(b*x^n + a)*d*x^m)/(b^2*c^2*x^(2*n) +
 a^2*c^2 - a*d^2 + (2*a*b*c^2 - b*d^2)*x^n), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{ac+bcx^{n}+d\sqrt{a+bx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
[Out] Integral(x**m/(a*c + b*c*x**n + d*sqrt(a + b*x**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(x^m/(b*c*x^n + a*c + sqrt(b*x^n + a)*d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{a\,c+d\,\sqrt{a+b\,x^n}+b\,c\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
```

```
[Out] int(x^m/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n), x)
```

3.564 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=27

$$\frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}$$

```
[Out] 2*ln(d+c*(a+b*x^n)^(1/2))/b/c/n
```

Rubi [A]
time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2186, 31}

$$\frac{2\log\left(c\sqrt{a+bx^n}+d\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
```

```
[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}}\,dx,x,x^n\right)}{n}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{d+cx}\,dx,x,\sqrt{a+bx^n}\right)}{bn}\\
&= \frac{2\log\left(d+c\sqrt{a+bx^n}\right)}{bcn}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 1.19

$$\frac{2\log\left(bdn+bcn\sqrt{a+bx^n}\right)}{bcn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]
[Out] (2*Log[b*d*n + b*c*n*Sqrt[a + b*x^n]])/(b*c*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+n}}{ac+bc\,x^n+d\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
[Out] int(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(25) = 50$.
time = 0.28, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn}+\frac{2\,\log\left(\frac{bcx^n+ac+\sqrt{bx^n+a}\,d}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")
[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/
(b*c*n)
```

Fricas [A]
time = 0.33, size = 25, normalized size = 0.93

$$\frac{2\,\log\left(\sqrt{bx^n+a}\,c+d\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")
[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)
```

Sympy [A]
time = 15.59, size = 32, normalized size = 1.19

$$\frac{2\left(\begin{cases}\frac{\sqrt{a+bx^n}}{d} & \text{for } c=0\\ \frac{\log\left(c\sqrt{a+bx^n}+d\right)}{c} & \text{otherwise}\end{cases}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)
[Out] 2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**n) + d)/c,
 True))/(b*n)
```

Giac [A]
time = 3.31, size = 26, normalized size = 0.96

$$\frac{2\log\left(\left|\sqrt{bx^n+a}\,c+d\right|\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)
```

Mupad [B]
time = 3.35, size = 25, normalized size = 0.93

$$\frac{2\ln\left(d+c\sqrt{a+b\,x^n}\right)}{b\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)
[Out] (2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)
```

3.565 $\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(2\sqrt{x}\right)$$

```
[Out] arctan(2*x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1607, 65, 209}

$$\text{ArcTan}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}+4x^{3/2}}\,dx=\int \frac{1}{\sqrt{x}\,(1+4x)}\,dx$$
$$=2\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx,x,\sqrt{x}\right)$$
$$=\tan^{-1}\left(2\sqrt{x}\right)$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$\tan^{-1}\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1),x]
```

```
[Out] ArcTan[2*Sqrt[x]]
```

Maple [A]
time = 0.20, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$\arctan\left(2\sqrt{x}\right)$	7
default	$\arctan\left(2\sqrt{x}\right)$	7
meijerg	$\arctan\left(2\sqrt{x}\right)$	7
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{4x\operatorname{RootOf}\left(\_Z^2+1\right)-\operatorname{RootOf}\left(\_Z^2+1\right)-4\sqrt{x}}{1+4x}\right)}{2}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^(3/2)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(2*x^(1/2))
```

Maxima [A]
time = 0.48, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] arctan(2*sqrt(x))
```

Fricas [A]
time = 0.35, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(2*sqrt(x))
```

Sympy [A]
time = 0.07, size = 7, normalized size = 0.88

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**(3/2)+x**(1/2)),x)
[Out] atan(2*sqrt(x))
```

Giac [A]
time = 4.21, size = 6, normalized size = 0.75

$$\arctan\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(2*sqrt(x))
```

Mupad [B]
time = 0.05, size = 6, normalized size = 0.75

$$\operatorname{atan}\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 4*x^(3/2)),x)
[Out] atan(2*x^(1/2))
```

3.566 $\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {1607, 335, 218, 212, 209}

$$\text{ArcTan}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - x^(5/2))^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}-x^{5/2}}\,dx &= \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - x^(5/2))^(-1),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A]
time = 0.20, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\,\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)-2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{1}{4}}}$	40
trager	$-\frac{\ln\left(\frac{-1-x+2\sqrt{x}}{-1+x}\right)}{2}-\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{x\,\text{RootOf}\left(\_Z^2+1\right)-\text{RootOf}\left(\_Z^2+1\right)+2\sqrt{x}}{1+x}\right)}{2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(5/2)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.49, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.
time = 0.36, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.
time = 0.12, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(5/2)+x**(1/2)),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 4.14, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 3.08, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - x^(5/2)),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.567 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} \, dx$

Optimal. Leaf size=27

$$4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)$$

[Out] 4*x^(1/4)+4*ln(1-x^(1/4))+2*x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1607, 272, 45}

$$2\sqrt{x}+4\sqrt[4]{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(-x^(1/4) + Sqrt[x])^(-1),x]

[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(-1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx$$
$$= 4\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1-\sqrt[4]{x}\right)$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 0.93

$$2\left(2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(-1+\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*(2 + x^(1/4))*x^(1/4) + 4*Log[-1 + x^(1/4)]
```

Maple [A]
time = 0.20, size = 20, normalized size = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(-1+x^{\frac{1}{4}}\right)$	20
default	$2\sqrt{x}+4x^{\frac{1}{4}}+4\ln\left(-1+x^{\frac{1}{4}}\right)$	20
meijerg	$\frac{2x^{\frac{1}{4}}\left(3x^{\frac{1}{4}}+6\right)}{3}+4\ln\left(1-x^{\frac{1}{4}}\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(-1+x^(1/4))
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.70

$$2\sqrt{x}+4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Fricas [A]
time = 0.33, size = 19, normalized size = 0.70

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.81

$$4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/4)+x**(1/2)),x)
[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)
```

Giac [A]
time = 4.44, size = 20, normalized size = 0.74

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(\left|x^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))
```

Mupad [B]
time = 3.10, size = 19, normalized size = 0.70

$$4\ln\left(x^{1/4} - 1\right) + 2\sqrt{x} + 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - x^(1/4)),x)
[Out] 4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)
```

3.568 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)-6*ln(1+x^(1/6))+2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx$$
$$= 6\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.03

$$\left(6-3\sqrt[6]{x}+2\sqrt[3]{x}\right)\sqrt[6]{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] (6 - 3*x^(1/6) + 2*x^(1/3))*x^(1/6) - 6*Log[1 + x^(1/6)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(24) = 48.
time = 0.20, size = 92, normalized size = 2.88

method	result
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}-6\ln\left(1+x^{\frac{1}{6}}\right)+2\sqrt{x}$
meijerg	$\frac{x^{\frac{1}{6}}\left(4x^{\frac{1}{3}}-6x^{\frac{1}{6}}+12\right)}{2}-6\ln\left(1+x^{\frac{1}{6}}\right)$
default	$-\ln\left(x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+2\ln\left(-1+x^{\frac{1}{6}}\right)-2\ln\left(1+x^{\frac{1}{6}}\right)+\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{x}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x^(1/3)+x^(1/6)+1)+2*ln(-1+x^(1/6))-2*ln(1+x^(1/6))+ln(1-x^(1/6)+x^(1/3
))+2*x^(1/2)+ln(-1+x^(1/2))-ln(1+x^(1/2))+6*x^(1/6)-ln(-1+x)+ln(x^(2/3)+x^(
1/3)+1)-2*ln(-1+x^(1/3))-3*x^(1/3)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.75

$$2\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A]
time = 3.90, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3\,x^{\frac{1}{3}} + 6\,x^{\frac{1}{6}} - 6\,\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\,\ln\left(x^{1/6} + 1\right) - 3\,x^{1/3} + 6\,x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/3)),x)
[Out] 2*x^(1/2) - 6*log(x^(1/6) + 1) - 3*x^(1/3) + 6*x^(1/6)
```

3.569 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=25

$$-4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)$$

```
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$2\sqrt{x}-4\sqrt[4]{x}+4\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx$$
$$= 4\mathrm{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{x}\right)$$
$$= 4\mathrm{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx, x, \sqrt[4]{x}\right)$$
$$= -4\sqrt[4]{x}+2\sqrt{x}+4\log\left(1+\sqrt[4]{x}\right)$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$2\left(-2+\sqrt[4]{x}\right)\sqrt[4]{x}+4\log\left(1+\sqrt[4]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + Sqrt[x])^(-1),x]
[Out] 2*(-2 + x^(1/4))*x^(1/4) + 4*Log[1 + x^(1/4)]
```

Maple [A]
time = 0.20, size = 20, normalized size = 0.80

method	result	size
derivativedivides	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
default	$-4x^{\frac{1}{4}}+4\ln\left(1+x^{\frac{1}{4}}\right)+2\sqrt{x}$	20
meijerg	$-\frac{2x^{\frac{1}{4}}\left(-3x^{\frac{1}{4}}+6\right)}{3}+4\ln\left(1+x^{\frac{1}{4}}\right)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.76

$$2\sqrt{x}-4\,x^{\frac{1}{4}}+4\,\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Fricas [A]
time = 0.32, size = 19, normalized size = 0.76

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Sympy [A]
time = 0.07, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x} + 2\sqrt{x} + 4\log\left(\sqrt[4]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/2)),x)
[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)
```

Giac [A]
time = 5.04, size = 19, normalized size = 0.76

$$2\sqrt{x} - 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$4\ln\left(x^{1/4} + 1\right) + 2\sqrt{x} - 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(1/4)),x)
[Out] 4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)
```

3.570 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx$

Optimal. Leaf size=20

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)+3*ln(1-x^(1/3))
```

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1607, 272, 45}

$$3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt[3]{x}+x^{2/3}}\,dx &= \int \frac{1}{\left(-1+\sqrt[3]{x}\right)\sqrt[3]{x}}\,dx\\ &= 3\text{Subst}\left(\int \frac{x}{-1+x}\,dx, x, \sqrt[3]{x}\right)\\ &= 3\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)\,dx, x, \sqrt[3]{x}\right)\\ &= 3\sqrt[3]{x}+3\log\left(1-\sqrt[3]{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.90

$$3\sqrt[3]{x}+3\log\left(-1+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/3) + x^(2/3))^(-1),x]
```

```
[Out] 3*x^(1/3) + 3*Log[-1 + x^(1/3)]
```

Maple [A]
time = 0.20, size = 15, normalized size = 0.75

method	result	size
derivativedivides	$3x^{\frac{1}{3}}+3\ln\left(-1+x^{\frac{1}{3}}\right)$	15
default	$3x^{\frac{1}{3}}+3\ln\left(-1+x^{\frac{1}{3}}\right)$	15
meijerg	$3x^{\frac{1}{3}}+3\ln\left(1-x^{\frac{1}{3}}\right)$	17
trager	$3x^{\frac{1}{3}}+\ln\left(-3x^{\frac{2}{3}}+3x^{\frac{1}{3}}+x-1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/3)+x^(2/3)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*x^(1/3)+3*ln(-1+x^(1/3))
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.70

$$3\,x^{\frac{1}{3}}+3\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Fricas [A]
time = 0.34, size = 14, normalized size = 0.70

$$3x^{\frac{1}{3}} + 3\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.75

$$3\sqrt[3]{x} + 3\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/3)+x**(2/3)),x)
[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)
```

Giac [A]
time = 4.53, size = 15, normalized size = 0.75

$$3x^{\frac{1}{3}} + 3\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x^(2/3)),x, algorithm="giac")
[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))
```

Mupad [B]
time = 0.08, size = 14, normalized size = 0.70

$$3\ln\left(x^{1/3} - 1\right) + 3x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/3) - x^(2/3)),x)
[Out] 3*log(x^(1/3) - 1) + 3*x^(1/3)
```

3.571 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{4}{3}\log\left(1+\sqrt[4]{x}\right)-\frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

```
[Out] 4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))+4/3*arctan(1/3*(1-2*x^(1/4))*3^
(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1607, 348, 327, 298, 31, 648, 632, 210, 642}

$$\frac{4\text{ArcTan}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}+2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx$$
$$= 4\text{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} - 4\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right) - 2\text{Subst}\left(\int \frac{1}{1-x+\right.$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right) + 4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+\right.$$
$$= 2\sqrt{x} + \frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 1.00

$$\frac{2}{3}\left(3\sqrt{x} + 2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right) + 2\log\left(1+\sqrt[4]{x}\right) - \log\left(1-\sqrt[4]{x}+\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] (2*(3*Sqrt[x] + 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]] + 2*Log[1 + x^(1/
4)] - Log[1 - x^(1/4) + Sqrt[x]]))/3
```

Maple [A]
time = 0.20, size = 46, normalized size = 0.74

method	result	size
derivativedivides	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46
default	$2\sqrt{x}+\frac{4\ln\left(1+x^{\frac{1}{4}}\right)}{3}-\frac{2\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{3}-\frac{4\sqrt{3}\,\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}$	46

meijerg	$2\sqrt{x}-\frac{4\sqrt{x}\left(-\frac{\ln\left(1+x^{\frac{1}{4}}\right)}{\sqrt{x}}+\frac{\ln\left(1-x^{\frac{1}{4}}+\sqrt{x}\right)}{2\sqrt{x}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,x^{\frac{1}{4}}}{2-x^{\frac{1}{4}}}\right)}{\sqrt{x}}\right)}{3}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/
3*(2*x^(1/4)-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.76

$$-\frac{4}{3}\sqrt{3}\,\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Sympy [A]
time = 0.18, size = 68, normalized size = 1.10

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.07, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Mupad [B]
time = 3.08, size = 73, normalized size = 1.18

$$\frac{4\ln\left(16x^{1/4}+16\right)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)-\ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)^2+16x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}\,2\mathrm{i}}{3}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + 1/x^(1/4)),x)
```

```
[Out] (4*log(16*x^(1/4) + 16))/3 + log(9*((3^(1/2)*2i)/3 - 2/3)^2 + 16*x^(1/4))*(
(3^(1/2)*2i)/3 - 2/3) - log(9*((3^(1/2)*2i)/3 + 2/3)^2 + 16*x^(1/4))*((3^(1
/2)*2i)/3 + 2/3) + 2*x^(1/2)
```

3.572 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=73

$$-12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\sqrt[12]{x}\right)$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[12]{x}\right)\sqrt[4]{x}}\,dx \\ &= 12\text{Subst}\left(\int \frac{x^8}{1+x}\,dx, x, \sqrt[12]{x}\right) \\ &= 12\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7+\frac{1}{1+x}\right)dx, x, \sqrt[12]{x}\right) \\ &= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}+12\log\left(1+\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 72, normalized size = 0.99

$$\frac{1}{70}\left(-840\sqrt[12]{x}+420\sqrt[6]{x}-280\sqrt[4]{x}+210\sqrt[3]{x}-168x^{5/12}+140\sqrt{x}-120x^{7/12}+105x^{2/3}\right)+12\log\left(1+\sqrt[12]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/4) + x^(1/3))^(-1),x]
```

```
[Out] (-840*x^(1/12) + 420*x^(1/6) - 280*x^(1/4) + 210*x^(1/3) - 168*x^(5/12) + 1
40*Sqrt[x] - 120*x^(7/12) + 105*x^(2/3))/70 + 12*Log[1 + x^(1/12)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(49) = 98$.
time = 0.21, size = 173, normalized size = 2.37

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}+12\ln\left(1+x^{\frac{1}{12}}\right)+2\sqrt{x}$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-315x^{\frac{7}{12}}+360\sqrt{x}-420x^{\frac{5}{12}}+504x^{\frac{1}{3}}-630x^{\frac{1}{4}}+840x^{\frac{1}{6}}-1260x^{\frac{1}{12}}+2520\right)}{210}+12\ln\left(1+x^{\frac{1}{12}}\right)$
default	$-4\ln\left(-1+x^{\frac{1}{12}}\right)+\frac{3x^{\frac{2}{3}}}{2}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}+\ln\left(-1+x\right)-12x^{\frac{1}{12}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}-\ln\left(x^{\frac{2}{3}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*ln(-1+x^(1/12))+3/2*x^(2/3)-4*x^(1/4)+6*x^(1/6)+ln(-1+x)-12*x^(1/12)-12/
5*x^(5/12)-12/7*x^(7/12)-ln(x^(2/3)+x^(1/3)+1)+2*x^(1/2)+3*x^(1/3)+2*ln(-1+
x^(1/6))-ln(x^(1/3)+x^(1/6)+1)+ln(-1+x^(1/2))-ln(1+x^(1/2))+4*ln(1+x^(1/12)
)+2*ln(x^(1/6)+x^(1/12)+1)-2*ln(-1+x^(1/4))+2*ln(-1+x^(1/3))-2*ln(1+x^(1/6)
)+2*ln(1+x^(1/4))+ln(1-x^(1/6)+x^(1/3))-2*ln(1-x^(1/12)+x^(1/6))
```

Maxima [A]
time = 0.30, size = 49, normalized size = 0.67

$$\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Fricas [A]
time = 0.34, size = 49, normalized size = 0.67

$$\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(1/(x**(1/4) + x**(1/3)), x)
```

Giac [A]
time = 4.36, size = 49, normalized size = 0.67

$$\frac{3}{2}\,x^{\frac{2}{3}}-\frac{12}{7}\,x^{\frac{7}{12}}+2\,\sqrt{x}-\frac{12}{5}\,x^{\frac{5}{12}}+3\,x^{\frac{1}{3}}-4\,x^{\frac{1}{4}}+6\,x^{\frac{1}{6}}-12\,x^{\frac{1}{12}}+12\,\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(
1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Mupad [B]
time = 0.04, size = 49, normalized size = 0.67

$$12\,\ln\left(x^{1/12}+1\right)+2\,\sqrt{x}+3\,x^{1/3}-4\,x^{1/4}+\frac{3\,x^{2/3}}{2}+6\,x^{1/6}-12\,x^{1/12}-\frac{12\,x^{5/12}}{5}-\frac{12\,x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3) + x^(1/4)),x)
```

```
[Out] 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/2 +
6*x^(1/6) - 12*x^(1/12) - (12*x^(5/12))/5 - (12*x^(7/12))/7
```

3.573 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

Optimal. Leaf size=130

$$12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x^{13/12}}{13}-\frac{6x^{7/}}{7}$$

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1607, 272, 45}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{x}-12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx=\int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx$$
$$=12\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx,x,\sqrt[12]{x}\right)$$
$$=12\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}\right.\right.$$
$$=12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^5}{5}$$

Mathematica [A]
time = 0.04, size = 117, normalized size = 0.90

$$\frac{360360\sqrt[12]{x}-180180\sqrt[6]{x}+120120\sqrt[4]{x}-90090\sqrt[3]{x}+72072x^{5/12}-60060\sqrt{x}+51480x^{7/12}-45045x^{2/3}+40040x^{3/4}-36036x^{5/6}+32760x^{11/12}-30030x+27720x^{13/12}-25740x^{7/6}+24024x^{5/4}}{30030}-12\log\left(1+\sqrt[12]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
[Out] (360360*x^(1/12) - 180180*x^(1/6) + 120120*x^(1/4) - 90090*x^(1/3) + 72072*
x^(5/12) - 60060*Sqrt[x] + 51480*x^(7/12) - 45045*x^(2/3) + 40040*x^(3/4) -
 36036*x^(5/6) + 32760*x^(11/12) - 30030*x + 27720*x^(13/12) - 25740*x^(7/6
) + 24024*x^(5/4))/30030 - 12*Log[1 + x^(1/12)]
```

Maple [A]
time = 0.22, size = 83, normalized size = 0.64

method	result
derivativedivides	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-6$
default	$12x^{\frac{1}{12}}-6x^{\frac{1}{6}}+4x^{\frac{1}{4}}-3x^{\frac{1}{3}}+\frac{12x^{\frac{5}{12}}}{5}+\frac{12x^{\frac{7}{12}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+\frac{4x^{\frac{3}{4}}}{3}-\frac{6x^{\frac{5}{6}}}{5}+\frac{12x^{\frac{11}{12}}}{11}-x+\frac{12x^{\frac{13}{12}}}{13}-6$
meijerg	$\frac{x^{\frac{1}{12}}\left(48048x^{\frac{7}{6}}-51480x^{\frac{13}{12}}+55440x-60060x^{\frac{11}{12}}+65520x^{\frac{5}{6}}-72072x^{\frac{3}{4}}+80080x^{\frac{2}{3}}-90090x^{\frac{7}{12}}+102960\sqrt{x}-120120x^{\frac{5}{12}}\right.}{60060}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x,method=_RETURNVERBOSE)
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Maxima [A]
time = 0.28, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Fricas [A]
time = 0.36, size = 76, normalized size = 0.58

$$\frac{4}{5}(x+5)x^{\frac{1}{4}} - \frac{6}{7}(x+7)x^{\frac{1}{6}} + \frac{12}{13}(x+13)x^{\frac{1}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} - 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{12}}}{\sqrt[4]{x} + \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] Integral(x**(7/12)/(x**(1/4) + x**(1/3)), x)
```

Giac [A]
time = 4.48, size = 82, normalized size = 0.63

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Mupad [B]
time = 0.15, size = 82, normalized size = 0.63

$$4x^{1/4} - 12\ln\left(x^{1/12}+1\right) - 2\sqrt{x} - 3x^{1/3} - x - \frac{3x^{2/3}}{2} - 6x^{1/6} + \frac{4x^{3/4}}{3} + \frac{4x^{5/4}}{5} - \frac{6x^{5/6}}{5} + 12x^{1/12} - \frac{6x^{7/6}}{7} + \frac{12x^{5/12}}{5} + \frac{12x^{7/12}}{7} + \frac{12x^{11/12}}{11} + \frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)
```

```
[Out] 4*x^(1/4) - 12*log(x^(1/12) + 1) - 2*x^(1/2) - 3*x^(1/3) - x - (3*x^(2/3))/
2 - 6*x^(1/6) + (4*x^(3/4))/3 + (4*x^(5/4))/5 - (6*x^(5/6))/5 + 12*x^(1/12)
 - (6*x^(7/6))/7 + (12*x^(5/12))/5 + (12*x^(7/12))/7 + (12*x^(11/12))/11 +
(12*x^(13/12))/13
```

3.574 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}\right.\right.$$

```
[Out] 6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(-5^(1/2)+1)
-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)+2*x^(1/2)+3/5*arc
tan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-3/5*ar
ctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1607, 348, 327, 300, 648, 632, 210, 642, 31}

$$\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\text{ArcTan}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\text{ArcTan}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)+2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 300

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*
```

k - 1)*m*(Pi/n)] + s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r^(m + 1)/(a*n*s^m))*Int[1/(r - s*x), x] - Dist[2*((-r)^(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=\int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx$$

$$=6\text{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}+6\text{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$

$$=2\sqrt{x}-\frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[6]{x}\right)-\frac{12}{5}\text{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\right.$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)+\frac{3\text{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)}{\sqrt{5}}-\frac{3\text{Subst}\left(\int \frac{}{1+\frac{1}{2}}\right.}{}$$

$$=2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1\right.$$

$$=2\sqrt{x}+6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 126, normalized size = 0.63

$$2\sqrt{x}+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{-\log\left(\sqrt[6]{x}-\#1\right)-2\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1^
4 & , (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1
^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A]
time = 0.21, size = 172, normalized size = 0.86

method	result

meijerg	$-\frac{6(-1)^{\frac{2}{5}}\left(\frac{5\sqrt{x}\,(-1)^{\frac{3}{5}}}{3}+(-1)^{\frac{3}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)-\cos\left(\frac{\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sin\left(\frac{\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}{1-\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}}\right)\right.\right.}{5}$
derivativedivides	$2\sqrt{x}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(\sqrt{5}+1\right)}{10}-\frac{12\left(-\sqrt{5}-1-\frac{\left(\sqrt{5}+1\right)\left(-\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x}{\sqrt{10}}\right.}{5\sqrt{10+2\sqrt{5}}}$
default	$2\sqrt{x}-\frac{3\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{5}\right)\left(\sqrt{5}+1\right)}{10}-\frac{12\left(-\sqrt{5}-1-\frac{\left(\sqrt{5}+1\right)\left(-\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x}{\sqrt{10}}\right.}{5\sqrt{10+2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*(5^(1/2)+1)-12/5*(-5
^(1/2)-1-1/4*(5^(1/2)+1)*(-5^(1/2)+1))/(10+2*5^(1/2))^(1/2)*arctan((1+4*x^(
1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))+3/10*(5^(1/2)-1)*ln(2+x^(1/6)+2*x^(1/3)
+x^(1/6)*5^(1/2))+12/5*(-5^(1/2)+1-1/4*(5^(1/2)-1)*(5^(1/2)+1))/(10-2*5^(1/
2))^(1/2)*arctan((1+4*x^(1/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5*ln(-1+x^(1
/6))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(133) = 266$.
time = 0.52, size = 272, normalized size = 1.36

$$-\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}+2\sqrt{x}+\frac{6\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}+(-1)^{\frac{2}{5}}\right)}-\frac{6\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{2}{5}}-(-1)^{\frac{2}{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(133) = 266$.

time = 1.15, size = 638, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Giac [A]
time = 4.25, size = 139, normalized size = 0.70

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)-\frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+2\sqrt{x}-\frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

Mupad [B]
time = 0.12, size = 223, normalized size = 1.12

$$\frac{6\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\sqrt{5}}{10}+\frac{3}{10}\right)+\ln\left(750\,x^{1/6}\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\sqrt{5}}{10}-\frac{3}{10}\right)-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{5}}{10}-\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{5}}{10}-\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)-\ln\left(-750\,x^{1/6}\left(\frac{3\sqrt{5}}{10}+\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)^3-1296\right)\left(\frac{3\sqrt{5}}{10}+\frac{3\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] (6*log(1296*x^(1/6) - 1296))/5 - log(- 750*x^(1/6)*((3*2^(1/2)*(- 5^(1/2) -
 5)^(1/2))/10 - (3*5^(1/2))/10 + 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2) - 5
)^(1/2))/10 - (3*5^(1/2))/10 + 3/10) + log(750*x^(1/6)*((3*2^(1/2)*(- 5^(1/
2) - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10)^3 - 1296)*((3*2^(1/2)*(- 5^(1/2)
 - 5)^(1/2))/10 + (3*5^(1/2))/10 - 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))/1
0 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10 -
(3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) - log(- 750*x^(1/6)*((3*5^(1/2))
/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)^3 - 1296)*((3*5^(1/2))/10
+ (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10) + 2*x^(1/2)
```

3.575 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arctan(x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {661, 65, 209}

$$2\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 661

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = \int \frac{1}{\sqrt{x}\,(1+x)}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\tan^{-1}\left(\sqrt{x}\right)$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.20, size = 7, normalized size = 0.88

method	result	size
derivativedivides	$2\arctan\left(\sqrt{x}\right)$	7
default	$2\arctan\left(\sqrt{x}\right)$	7
meijerg	$2\arctan\left(\sqrt{x}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x,method=_RETURNVERBOSE)
[Out] 2*arctan(x^(1/2))
```

Maxima [A]
time = 0.51, size = 6, normalized size = 0.75

$$2\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Fricas [A]
time = 0.34, size = 6, normalized size = 0.75

$$2 \arctan \left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [A]
time = 0.17, size = 7, normalized size = 0.88

$$2 \operatorname{atan} \left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
```

```
[Out] 2*atan(sqrt(x))
```

Giac [A]
time = 3.61, size = 6, normalized size = 0.75

$$2 \arctan \left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

Mupad [B]
time = 0.14, size = 6, normalized size = 0.75

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^2),x)
```

```
[Out] 2*atan(x^(1/2))
```

3.576 $\int \frac{x}{4\sqrt{x}+x}\,dx$

Optimal. Leaf size=19

$$-8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1598, 272, 45}

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{4\sqrt{x}+x}\,dx &= \int \frac{\sqrt{x}}{4+\sqrt{x}}\,dx \\
&= 2\text{Subst}\left(\int \frac{x^2}{4+x}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(-4+x+\frac{16}{4+x}\right)\,dx, x, \sqrt{x}\right) \\
&= -8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$-8\sqrt{x}+x+32\log\left(4+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4*Sqrt[x] + x),x]
```

```
[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]
```

Maple [A]
time = 0.23, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
default	$x+32\ln\left(4+\sqrt{x}\right)-8\sqrt{x}$	16
trager	$-1+x-8\sqrt{x}+16\ln\left(8\sqrt{x}+16+x\right)$	20
meijerg	$-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+32\ln\left(1+\frac{\sqrt{x}}{4}\right)$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.79

$$x-8\sqrt{x}+32\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Fricas [A]
time = 0.39, size = 15, normalized size = 0.79

$$x - 8\sqrt{x} + 32 \log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Sympy [A]
time = 0.06, size = 17, normalized size = 0.89

$$-8\sqrt{x} + x + 32\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x**(1/2)),x)
[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)
```

Giac [A]
time = 3.30, size = 15, normalized size = 0.79

$$x - 8\sqrt{x} + 32 \log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")
[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)
```

Mupad [B]
time = 0.04, size = 15, normalized size = 0.79

$$x + 32 \ln\left(\sqrt{x} + 4\right) - 8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + 4*x^(1/2)),x)
[Out] x + 32*log(x^(1/2) + 4) - 8*x^(1/2)
```

3.577 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$

Optimal. Leaf size=108

$$2\sqrt{x}+\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}\right.}{2\sqrt{2}}$$

```
[Out] -3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/
2)-3/4*ln(1+x^(1/3)-x^(1/6)*2^(1/2))*2^(1/2)+3/4*ln(1+x^(1/3)+x^(1/6)*2^(1/
2))*2^(1/2)+2*x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1598, 348, 327, 335, 303, 1176, 631, 210, 1179, 642}

$$\frac{3\text{ArcTan}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}}-\frac{3\text{ArcTan}\left(\sqrt{2}\sqrt[6]{x}+1\right)}{\sqrt{2}}+2\sqrt{x}-\frac{3\log\left(\sqrt[3]{x}-\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}+\frac{3\log\left(\sqrt[3]{x}+\sqrt{2}\sqrt[6]{x}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)])/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]
*x^(1/6)])/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2]) + (
3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)])/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
```

```
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx &= \int \frac{\sqrt[6]{x}}{1+x^{2/3}}\,dx \\
&= 3\text{Subst}\left(\int \frac{x^{5/2}}{1+x^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 2\sqrt{x} - 3\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \sqrt[3]{x}\right) \\
&= 2\sqrt{x} - 6\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x} + 3\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) - 3\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt[6]{x}\right) \\
&= 2\sqrt{x} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}} - \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, \right.}{\sqrt{2}} \\
&= 2\sqrt{x} + \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\tan^{-1}\left(1+\sqrt{2}\sqrt[6]{x}\right)}{\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\sqrt[6]{x}+\sqrt[3]{x}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 62, normalized size = 0.57

$$
2\sqrt{x} - \frac{3\tan^{-1}\left(\frac{-1+\sqrt[3]{x}}{\sqrt{2}\sqrt[6]{x}}\right)}{\sqrt{2}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[6]{x}}{1+\sqrt[3]{x}}\right)}{\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/3) + x),x]
```

```
[Out] 2*Sqrt[x] - (3*ArcTan[(-1 + x^(1/3))/(Sqrt[2]*x^(1/6))])/Sqrt[2] + (3*ArcTa
nh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]
```

Maple [A]
time = 0.30, size = 66, normalized size = 0.61

method	result
derivativedivides	$2\sqrt{x}-\frac{3\sqrt{2}\,\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$
default	$2\sqrt{x}-\frac{3\sqrt{2}\,\left(\ln\left(\frac{1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}}{1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}}\right)+2\arctan\left(1+x^{\frac{1}{6}}\sqrt{2}\right)+2\arctan\left(-1+x^{\frac{1}{6}}\sqrt{2}\right)\right)}{4}$
meijerg	$2\sqrt{x}-\frac{3\sqrt{x}\,\left(\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}-x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\arctan\left(\frac{x^{\frac{1}{6}}\sqrt{2}}{2-x^{\frac{1}{6}}\sqrt{2}}\right)}{\sqrt{x}}-\frac{\sqrt{2}\,\ln\left(1+x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{2}\right)}{2\sqrt{x}}+\frac{\sqrt{2}\,\mathrm{arct}}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3)+x),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1/2)-3/4*2^(1/2)*(ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2^
(1/2)))+2*arctan(1+x^(1/6)*2^(1/2))+2*arctan(-1+x^(1/6)*2^(1/2)))
```

Maxima [A]
time = 0.50, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

Fricas [A]
time = 0.35, size = 120, normalized size = 1.11

$$3\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1}-\sqrt{2}x^{\frac{1}{6}}-1\right)+3\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4}-\sqrt{2}x^{\frac{1}{6}}+1\right)+\frac{3}{4}\sqrt{2}\log\left(4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4\right)-\frac{3}{4}\sqrt{2}\log\left(-4\sqrt{2}x^{\frac{1}{6}}+4x^{\frac{1}{3}}+4\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(1
/6) - 1) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3)
 + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/3
) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 2*sqrt(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt[3]{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/3)+x),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/3) + x), x)
```

Giac [A]
time = 3.43, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="giac")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^
(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)
```

Mupad [B]
time = 0.08, size = 42, normalized size = 0.39

$$2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}+\frac{3}{2}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x^{1/6}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{3}{2}-\frac{3}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/3)),x)
```

```
[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1/
2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)
```

3.578 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=76

$$-12\sqrt[12]{x}+3\sqrt[3]{x}-\frac{12x^{7/12}}{7}+\frac{6x^{5/6}}{5}-4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+6\log\left(1+\sqrt[12]{x}\right)-2\log\left(1+\sqrt[4]{x}\right)$$

```
[Out] -12*x^(1/12)+3*x^(1/3)-12/7*x^(7/12)+6/5*x^(5/6)+6*ln(1+x^(1/12))-2*ln(1+x^
(1/4))-4*arctan(1/3*(1-2*x^(1/12))*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {1598, 348, 52, 60, 632, 210, 31}

$$-4\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+\frac{6x^{5/6}}{5}-\frac{12x^{7/12}}{7}+3\sqrt[3]{x}-12\sqrt[12]{x}+6\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[4]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2)
, x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/
3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx &= \int \frac{\sqrt[12]{x}}{1+\sqrt[4]{x}}\,dx \\
&= 4\text{Subst}\left(\int \frac{x^{10/3}}{1+x}\,dx, x, \sqrt[4]{x}\right) \\
&= \frac{6x^{5/6}}{5} - 4\text{Subst}\left(\int \frac{x^{7/3}}{1+x}\,dx, x, \sqrt[4]{x}\right) \\
&= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\text{Subst}\left(\int \frac{x^{4/3}}{1+x}\,dx, x, \sqrt[4]{x}\right) \\
&= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\text{Subst}\left(\int \frac{\sqrt[3]{x}}{1+x}\,dx, x, \sqrt[4]{x}\right) \\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4\text{Subst}\left(\int \frac{1}{x^{2/3}(1+x)}\,dx, x, \sqrt[4]{x}\right) \\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2\log\left(1+\sqrt[4]{x}\right) + 6\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[12]{x} \right. \\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1+\sqrt[4]{x}\right) - 12\text{Subst}\left(\right. \\
&= -12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 6\log\left(1+\sqrt[12]{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 1.09

$$
-12\sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right) + 4\log\left(1+\sqrt[12]{x}\right) - 2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]
```

```
[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcT
an[(1 - 2*x^(1/12))/Sqrt[3]] + 4*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x
^(1/6)]
```

Maple [A]
time = 0.31, size = 61, normalized size = 0.80

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5} - \frac{12x^{\frac{7}{12}}}{7} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} + 4\ln\left(1+x^{\frac{1}{12}}\right) - 2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right) + 4\sqrt{3}\ \arctan\left(\right.$

default	$\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}+4\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)+4\sqrt{3}\ \arctan\left(\frac{(2x\ldots}{}\right.$
meijerg	$-\frac{3x^{\frac{1}{12}}\left(-182x^{\frac{3}{4}}+260\sqrt{x}-455x^{\frac{1}{4}}+1820\right)}{455}+4x^{\frac{1}{12}}\left(\frac{\ln\left(1+x^{\frac{1}{12}}\right)}{x^{\frac{1}{12}}}-\frac{\ln\left(1-x^{\frac{1}{12}}+x^{\frac{1}{6}}\right)}{2x^{\frac{1}{12}}}+\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\ x^{\frac{1}{12}}}{2-x^{\frac{1}{12}}}\right)}{x^{\frac{1}{12}}}\ldots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)+4*ln(1+x^(1/12))-2*ln(1-x^(
1/12)+x^(1/6))+4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 60, normalized size = 0.79

$$4\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 0.82

$$4\sqrt{3}\ \arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x
^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x
^(1/12) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)
```

Giac [A]
time = 4.30, size = 60, normalized size = 0.79

$$4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}-\frac{12}{7}x^{\frac{7}{12}}+3x^{\frac{1}{3}}-12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)+4\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12
) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12
) + 1)
```

Mupad [B]
time = 3.08, size = 78, normalized size = 1.03

$$4\ln\left(144x^{1/12}+144\right)-\ln\left(18-36x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)\left(2+\sqrt{3}\,2\mathrm{i}\right)+\ln\left(36x^{1/12}-18+\sqrt{3}\,18\mathrm{i}\right)\left(-2+\sqrt{3}\,2\mathrm{i}\right)+3x^{1/3}+\frac{6x^{5/6}}{5}-12x^{1/12}-\frac{12x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(x^(1/2) + x^(1/4)),x)
```

```
[Out] 4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2i
 + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3) +
(6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7
```

3.579 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=119

$$-12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x^{5/6}}{5}-\frac{12x^{11/12}}{11}+x-\frac{12x^{13/12}}{13}+\frac{6x$$

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1598, 272, 45}

$$\frac{6x^{7/6}}{7}-\frac{12x^{13/12}}{13}-\frac{12x^{11/12}}{11}+\frac{6x^{5/6}}{5}-\frac{4x^{3/4}}{3}+\frac{3x^{2/3}}{2}-\frac{12x^{7/12}}{7}-\frac{12x^{5/12}}{5}+x+2\sqrt{x}+3\sqrt[3]{x}-4\sqrt[4]{x}+6\sqrt[6]{x}-12\sqrt[12]{x}+12\log\left(\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt
[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12
*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12
)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx &= \int \frac{\sqrt[4]{x}}{1+\sqrt[12]{x}}\,dx\\
&= 12\text{Subst}\left(\int \frac{x^{14}}{1+x}\,dx, x, \sqrt[12]{x}\right)\\
&= 12\text{Subst}\left(\int \left(-1+x-x^2+x^3-x^4+x^5-x^6+x^7-x^8+x^9-x^{10}+x^{11}-x^{12}+x^1\right.\right.\\
&= -12\sqrt[12]{x}+6\sqrt[6]{x}-4\sqrt[4]{x}+3\sqrt[3]{x}-\frac{12x^{5/12}}{5}+2\sqrt{x}-\frac{12x^{7/12}}{7}+\frac{3x^{2/3}}{2}-\frac{4x^{3/4}}{3}+\frac{6x}{}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 110, normalized size = 0.92

$$\frac{-360360\sqrt[12]{x}+180180\sqrt[6]{x}-120120\sqrt[4]{x}+90090\sqrt[3]{x}-72072x^{5/12}+60060\sqrt{x}-51480x^{7/12}+45045x^{2/3}-40040x^{3/4}+36036x^{5/6}-32760x^{11/12}+30030x-27720x^{13/12}+25740x^{7/6}}{30030}+12\log\left(1+\sqrt[12]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]
```

```
[Out] (-360360*x^(1/12) + 180180*x^(1/6) - 120120*x^(1/4) + 90090*x^(1/3) - 72072
*x^(5/12) + 60060*Sqrt[x] - 51480*x^(7/12) + 45045*x^(2/3) - 40040*x^(3/4)
+ 36036*x^(5/6) - 32760*x^(11/12) + 30030*x - 27720*x^(13/12) + 25740*x^(7/
6))/30030 + 12*Log[1 + x^(1/12)]
```

Maple [A]
time = 0.30, size = 76, normalized size = 0.64

method	result
derivativedivides	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
default	$-12x^{\frac{1}{12}}+6x^{\frac{1}{6}}-4x^{\frac{1}{4}}+3x^{\frac{1}{3}}-\frac{12x^{\frac{5}{12}}}{5}-\frac{12x^{\frac{7}{12}}}{7}+\frac{3x^{\frac{2}{3}}}{2}-\frac{4x^{\frac{3}{4}}}{3}+\frac{6x^{\frac{5}{6}}}{5}-\frac{12x^{\frac{11}{12}}}{11}+x-\frac{12x^{\frac{13}{12}}}{13}+$
meijerg	$-\frac{x^{\frac{1}{12}}\left(-25740x^{\frac{13}{12}}+27720x-30030x^{\frac{11}{12}}+32760x^{\frac{5}{6}}-36036x^{\frac{3}{4}}+40040x^{\frac{2}{3}}-45045x^{\frac{7}{12}}+51480\sqrt{x}-60060x^{\frac{5}{12}}+72072\right.}{30030}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*
x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/
6)+12*ln(1+x^(1/12))+2*x^(1/2)
```

Maxima [A]
time = 0.29, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{12}{13}x^{\frac{13}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Fricas [A]
time = 0.35, size = 71, normalized size = 0.60

$$\frac{6}{7}(x+7)x^{\frac{1}{6}}-\frac{12}{13}(x+13)x^{\frac{1}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")
```

```
[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x
^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/
12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)
```

```
[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)
```

Giac [A]
time = 4.17, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{12}{13}x^{\frac{13}{12}}+x-\frac{12}{11}x^{\frac{11}{12}}+\frac{6}{5}x^{\frac{5}{6}}-\frac{4}{3}x^{\frac{3}{4}}+\frac{3}{2}x^{\frac{2}{3}}-\frac{12}{7}x^{\frac{7}{12}}+2\sqrt{x}-\frac{12}{5}x^{\frac{5}{12}}+3x^{\frac{1}{3}}-4x^{\frac{1}{4}}+6x^{\frac{1}{6}}-12x^{\frac{1}{12}}+12\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(
3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3)
- 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)
```

Mupad [B]
time = 0.15, size = 75, normalized size = 0.63

$$x+12\,\ln\left(x^{1/12}+1\right)+2\,\sqrt{x}+3\,x^{1/3}-4\,x^{1/4}+\frac{3\,x^{2/3}}{2}+6\,x^{1/6}-\frac{4\,x^{3/4}}{3}+\frac{6\,x^{5/6}}{5}-12\,x^{1/12}+\frac{6\,x^{7/6}}{7}-\frac{12\,x^{5/12}}{5}-\frac{12\,x^{7/12}}{7}-\frac{12\,x^{11/12}}{11}-\frac{12\,x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + x^(1/4)),x)
```

```
[Out] x + 12*log(x^(1/12) + 1) + 2*x^(1/2) + 3*x^(1/3) - 4*x^(1/4) + (3*x^(2/3))/
2 + 6*x^(1/6) - (4*x^(3/4))/3 + (6*x^(5/6))/5 - 12*x^(1/12) + (6*x^(7/6))/7
 - (12*x^(5/12))/5 - (12*x^(7/12))/7 - (12*x^(11/12))/11 - (12*x^(13/12))/1
3
```

3.580 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} \, dx$

Optimal. Leaf size=201

$$6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\right.\right.$$

```
[Out] 6*x^(1/6)+x+6/5*ln(1-x^(1/6))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*
(-5^(1/2)+1)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*(5^(1/2)+1)-3/5*a
rctan(1/20*(1+4*x^(1/6)+5^(1/2))*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)-3/5*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/
2)
```

Rubi [A]
time = 0.16, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1598, 348, 308, 208, 648, 632, 210, 642, 31}

$$-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\text{ArcTan}\left(\frac{4\sqrt[6]{x}-\sqrt{5}+1}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\text{ArcTan}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(4\sqrt[6]{x}+\sqrt{5}+1\right)\right)+x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k
- 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]; (r/(a*
n))*Int[1/(r - s*x), x] + Dist[2*(r/(a*n)), Sum[u, {k, 1, (n - 1)/2}], x],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx = \int \frac{x^{5/6}}{-1+x^{5/6}}\,dx$$
$$= 6\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}+x+6\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}+x-\frac{6}{5}\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,d$$
$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\right.$$
$$= 6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}$$
$$= 6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 127, normalized size = 0.63

$$6\sqrt[6]{x}+x+\frac{6}{5}\log\left(-1+\sqrt[6]{x}\right)-\frac{6}{5}\text{RootSum}\left[1+\#1+\#1^2+\#1^3+\#1^4\&,\frac{4\log\left(\sqrt[6]{x}-\#1\right)+3\log\left(\sqrt[6]{x}-\#1\right)\#1+2\log\left(\sqrt[6]{x}-\#1\right)\#1^2+\log\left(\sqrt[6]{x}-\#1\right)\#1^3}{1+2\#1+3\#1^2+4\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 +
 #1^4 & , (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #
1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) & ])/5
```

Maple [A]
time = 0.32, size = 166, normalized size = 0.83

method	result
meijerg	$\frac{6(-1)^{\frac{4}{5}}\left(-\frac{5x^{\frac{1}{6}}(-1)^{\frac{1}{5}}\left(11x^{\frac{5}{6}}+66\right)}{66}-(-1)^{\frac{1}{5}}\left(\ln\left(1-x^{\frac{1}{6}}\right)+\cos\left(\frac{2\pi}{5}\right)\ln\left(1-2\cos\left(\frac{2\pi}{5}\right)x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)-2\sin\left(\frac{2\pi}{5}\right)\arctan\left(\frac{\sin\left(\frac{2}{}\right.}{1-\cos\left(\right.}\right.\right.\right.}{5}$
derivativedivides	$x+6x^{\frac{1}{6}}+\frac{3\left(-\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-4-\frac{\left(-\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-}{\sqrt{10-}}\right.}{5\sqrt{10-2\sqrt{5}}}$
default	$x+6x^{\frac{1}{6}}+\frac{3\left(-\sqrt{5}-1\right)\ln\left(2+x^{\frac{1}{6}}+2x^{\frac{1}{3}}+x^{\frac{1}{6}}\sqrt{5}\right)}{10}+\frac{12\left(-4-\frac{\left(-\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}{4}\right)\arctan\left(\frac{1+4x^{\frac{1}{6}}-}{\sqrt{10-}}\right.}{5\sqrt{10-2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x+6*x^(1/6)+3/10*(-5^(1/2)-1)*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))+12/5*
(-4-1/4*(-5^(1/2)-1)*(5^(1/2)+1))/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)+
5^(1/2))/(10-2*5^(1/2))^(1/2))-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))
*(-5^(1/2)+1)-12/5*(4-1/4*(-5^(1/2)+1)^2)/(10+2*5^(1/2))^(1/2)*arctan((1+4*
x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))+6/5*ln(-1+x^(1/6))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(134) = 268$.

time = 0.52, size = 293, normalized size = 1.46

$$-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{-2\sqrt{5}-10}}-\frac{6}{5}(-1)^{\frac{1}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)+x-\frac{3\left(\sqrt{5}+3\right)\log\left(-x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}+(-1)^{\frac{4}{5}}\right)}-\frac{3\left(\sqrt{5}-3\right)\log\left(x^{\frac{1}{6}}\left(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\right)+2(-1)^{\frac{2}{5}}+2x^{\frac{1}{3}}\right)}{5\left(\sqrt{5}(-1)^{\frac{4}{5}}-(-1)^{\frac{4}{5}}\right)}+6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
4/5)) + 6*x^(1/6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(134) = 268$.

time = 1.16, size = 547, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{6}}}{\left(\sqrt[6]{x} - 1\right)\left(\sqrt[6]{x} + x^{\frac{2}{3}} + \sqrt[3]{x} + \sqrt{x} + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(5/6)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

Giac [A]

time = 4.47, size = 140, normalized size = 0.70

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+1\right)+\frac{3}{10}\sqrt{5}\log\left(-\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}-1\right)+x^{\frac{1}{3}}+1\right)+x+6x^{\frac{1}{6}}-\frac{3}{10}\log\left(x^{\frac{2}{3}}+\sqrt{x}+x^{\frac{1}{3}}+x^{\frac{1}{6}}+1\right)+\frac{6}{5}\log\left(\left|x^{\frac{1}{6}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

Mupad [B]
time = 0.06, size = 208, normalized size = 1.03

$$x+\frac{6\,\ln\left(1296\,x^{1/6}-1296\right)}{5}-\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}-270\,\sqrt{5}+1080\,x^{1/6}+270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}-\frac{3\,\sqrt{5}}{10}+\frac{3}{10}\right)+\ln\left(270\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}+270\,\sqrt{5}-1080\,x^{1/6}-270\right)\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-5}}{10}+\frac{3\,\sqrt{5}}{10}-\frac{3}{10}\right)+6\,x^{1/6}-\ln\left(270\,\sqrt{5}+1080\,x^{1/6}-270\,\sqrt{2}\,\sqrt{\sqrt{5}-5}+270\right)\,\left(\frac{3\,\sqrt{5}}{10}-\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)-\ln\left(270\,\sqrt{5}+1080\,x^{1/6}+270\,\sqrt{2}\,\sqrt{\sqrt{5}-5}+270\right)\,\left(\frac{3\,\sqrt{5}}{10}+\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-5}}{10}+\frac{3}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)
```

```
[Out] x + (6*log(1296*x^(1/6) - 1296))/5 - log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2)
- 270*5^(1/2) + 1080*x^(1/6) + 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 -
 (3*5^(1/2))/10 + 3/10) + log(270*2^(1/2)*(- 5^(1/2) - 5)^(1/2) + 270*5^(1/
2) - 1080*x^(1/6) - 270)*((3*2^(1/2)*(- 5^(1/2) - 5)^(1/2))/10 + (3*5^(1/2)
)/10 - 3/10) + 6*x^(1/6) - log(270*5^(1/2) + 1080*x^(1/6) - 270*2^(1/2)*(5^
(1/2) - 5)^(1/2) + 270)*((3*5^(1/2))/10 - (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/1
0 + 3/10) - log(270*5^(1/2) + 1080*x^(1/6) + 270*2^(1/2)*(5^(1/2) - 5)^(1/2
) + 270)*((3*5^(1/2))/10 + (3*2^(1/2)*(5^(1/2) - 5)^(1/2))/10 + 3/10)
```

3.581 $\int \frac{\sqrt{b-\frac{a}{x}}\,x^m}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=36

$$\frac{2\sqrt{b-\frac{a}{x}}\,x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

[Out] 2*x^(1+m)*(b-a/x)^(1/2)/(1+2*m)/(-b*x+a)^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 30}

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\, x^m}{\sqrt{a-bx}}\, dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{x^{-\frac{1}{2}+m}\sqrt{-a+bx}}{\sqrt{a-bx}}\, dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int x^{-\frac{1}{2}+m}\, dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\, x^{1+m}}{(1+2m)\sqrt{a-bx}}$$

Mathematica [A]
time = 1.01, size = 35, normalized size = 0.97

$$\frac{\sqrt{b-\frac{a}{x}}\, x^{1+m}}{\left(\frac{1}{2}+m\right)\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])
```

Maple [A]
time = 0.31, size = 36, normalized size = 1.00

method	result	size
gosper	$\frac{2x^{1+m}\sqrt{-\frac{-bx+a}{x}}}{(1+2m)\sqrt{-bx+a}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x^(1+m)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 15, normalized size = 0.42

$$\frac{2\sqrt{x}\, x^m}{2i\, m + i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(x)*x^m/(2*I*m + I)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.22

$$\frac{2\sqrt{-bx+a}\,xx^m\sqrt{\frac{bx-a}{x}}}{2\,am-(2\,bm+b)x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^m\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 3.26, size = 32, normalized size = 0.89

$$\frac{2\,x^{m+1}\,\sqrt{b-\frac{a}{x}}}{(2\,m+1)\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))
```

3.582 $\int \frac{\sqrt{b-\frac{a}{x}}\,x^2}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{b-\frac{a}{x}}\,x^3}{5\sqrt{a-bx}}$$

```
[Out] 2/5*x^3*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 30}

$$\frac{2x^3\sqrt{b-\frac{a}{x}}}{5\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x^2}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{x^{3/2}\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int x^{3/2}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^3}{5\sqrt{a-bx}}$$

Mathematica [A]
time = 9.31, size = 29, normalized size = 1.00

$$-\frac{2x^2\sqrt{a-bx}}{5\sqrt{b-\frac{a}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]
[Out] (-2*x^2*Sqrt[a - b*x])/(5*Sqrt[b - a/x])
```

Maple [A]
time = 0.34, size = 27, normalized size = 0.93

method	result	size
gosper	$\frac{2x^3\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
default	$\frac{2x^3\sqrt{-\frac{-bx+a}{x}}}{5\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x^3}{5(-bx+a)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 5, normalized size = 0.17

$$-\frac{2}{5} i x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/5*I*x^(5/2)
```

Fricas [A]
time = 0.33, size = 35, normalized size = 1.21

$$-\frac{2 \sqrt{-bx+a}\, x^3 \sqrt{\frac{bx-a}{x}}}{5 (bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(23) = 46.
time = 5.05, size = 76, normalized size = 2.62

$$\frac{2 \sqrt{-ab}\, a^2 |b| \mathrm{sgn}(x)}{5 b^4} - \frac{2 \left(\sqrt{-ab}\, a^2 - \frac{((bx-a)b+ab)^2 \sqrt{-(bx-a)b-ab}}{b^2}\right) |b| \mathrm{sgn}(x)}{5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b +
 a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4
```

Mupad [B]
time = 3.10, size = 23, normalized size = 0.79

$$\frac{2\,x^3\,\sqrt{b-\frac{a}{x}}}{5\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
```

```
[Out] (2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))
```

3.583 $\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{b-\frac{a}{x}}\,x^2}{3\sqrt{a-bx}}$$

```
[Out] 2/3*x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {529, 23, 30}

$$\frac{2x^2\sqrt{b-\frac{a}{x}}}{3\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
```

```
[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{x}\,\sqrt{-a+bx}}{\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \sqrt{x}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x^2}{3\sqrt{a-bx}}$$

Mathematica [A]
time = 5.94, size = 27, normalized size = 0.93

$$-\frac{2x\sqrt{a-bx}}{3\sqrt{b-\frac{a}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]
[Out] (-2*x*Sqrt[a - b*x])/(3*Sqrt[b - a/x])
```

Maple [A]
time = 0.30, size = 27, normalized size = 0.93

method	result	size
gosper	$\frac{2x^2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
default	$\frac{2x^2\sqrt{-\frac{-bx+a}{x}}}{3\sqrt{-bx+a}}$	27
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-(-bx+a)\,x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x^2}{3(-bx+a)^{\frac{3}{2}}\sqrt{(bx-a)\,x}\,\sqrt{-x}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 5, normalized size = 0.17

$$-\frac{2}{3} i x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*I*x^(3/2)
```

Fricas [A]
time = 0.34, size = 35, normalized size = 1.21

$$-\frac{2 \sqrt{-bx + a}\, x^2 \sqrt{\frac{bx - a}{x}}}{3\,(bx - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(23) = 46.
time = 4.68, size = 56, normalized size = 1.93

$$\frac{2\sqrt{-ab}\, a|b|\mathrm{sgn}(x)}{3\, b^3} - \frac{2\left(\sqrt{-ab}\, a + \frac{(-(bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\mathrm{sgn}(x)}{3\, b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-(b*x - a)*b - a*
b)^(3/2)/b)*abs(b)*sgn(x)/b^3
```

Mupad [B]
time = 3.06, size = 23, normalized size = 0.79

$$\frac{2\,x^2\,\sqrt{b-\frac{a}{x}}}{3\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)
[Out] (2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))
```

3.584 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}$$

[Out] 2*x*(b-a/x)^(1/2)/(-b*x+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {446, 23, 30}

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{\sqrt{x}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\sqrt{x}\right)\int \frac{1}{\sqrt{x}}\,dx}{\sqrt{a-bx}}$$
$$= \frac{2\sqrt{b-\frac{a}{x}}\,x}{\sqrt{a-bx}}$$

Mathematica [A]
time = 4.78, size = 24, normalized size = 0.96

$$-\frac{2\sqrt{a-bx}}{\sqrt{b-\frac{a}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]
[Out] (-2*Sqrt[a - b*x])/Sqrt[b - a/x]
```

Maple [A]
time = 0.29, size = 25, normalized size = 1.00

method	result	size
gosper	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
default	$\frac{2x\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	25
risch	$-\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)x}{(-bx+a)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 5, normalized size = 0.20

$$-2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*sqrt(x)
```

Fricas [A]
time = 0.33, size = 33, normalized size = 1.32

$$-\frac{2\sqrt{-bx+a}\,x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.
time = 4.30, size = 51, normalized size = 2.04

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\mathrm{sgn}(x)}{b^2}+\frac{2\sqrt{-ab}\,|b|\mathrm{sgn}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*
abs(b)*sgn(x)/b^2
```

Mupad [B]
time = 3.04, size = 21, normalized size = 0.84

$$\frac{2\,x\,\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(a - b*x)^(1/2),x)
[Out] (2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.585 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx$

Optimal. Leaf size=24

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

```
[Out] -2*(b-a/x)^(1/2)/(-b*x+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{1}{x^{3/2}}\,dx}{\sqrt{a-bx}}$$
$$= -\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Mathematica [A]
time = 1.68, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]
```

Maple [A]
time = 0.30, size = 24, normalized size = 1.00

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$	24
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{(-bx+a)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.31, size = 5, normalized size = 0.21

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I/sqrt(x)
```

Fricas [A]
time = 0.32, size = 32, normalized size = 1.33

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.
time = 4.08, size = 42, normalized size = 1.75

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}}-\frac{b^3}{\sqrt{-ab}}\right)|b|\mathrm{sgn}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3
```

Mupad [B]
time = 3.08, size = 20, normalized size = 0.83

$$-\frac{2\sqrt{b-\frac{a}{x}}}{\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)),x)
[Out] -(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)
```

3.586 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

```
[Out] -2/3*(b-a/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 30}

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{-a+bx}}{x^{5/2}\sqrt{a-bx}}\,dx}{\sqrt{-a+bx}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x}}\,\sqrt{x}\right)\int \frac{1}{x^{5/2}}\,dx}{\sqrt{a-bx}}$$
$$= -\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Mathematica [A]
time = 3.45, size = 26, normalized size = 0.90

$$\frac{2\left(b-\frac{a}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]
```

```
[Out] (2*(b - a/x)^(3/2))/(3*(a - b*x)^(3/2))
```

Maple [A]
time = 0.32, size = 27, normalized size = 0.93

method	result	size
gosper	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
default	$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{3x\sqrt{-bx+a}}$	27
risch	$\frac{2\sqrt{-\frac{-bx+a}{x}}\,\sqrt{-\left(-bx+a\right)x}\,\sqrt{\frac{x(-bx+a)}{bx-a}}\,(bx-a)}{3(-bx+a)^{\frac{3}{2}}\sqrt{\left(bx-a\right)x}\,\sqrt{-x}\,x}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.32, size = 5, normalized size = 0.17

$$\frac{2i}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*I/x^(3/2)
```

Fricas [A]
time = 0.33, size = 35, normalized size = 1.21

$$\frac{2\,\sqrt{-bx+a}\,\sqrt{\frac{bx-a}{x}}}{3\,(bx^2-ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x}+b}}{x^2\sqrt{a-bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.
time = 3.43, size = 60, normalized size = 2.07

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}}-\frac{b^4}{\sqrt{-ab}\,a}\right)|b|\mathrm{sgn}(x)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a
))*abs(b)*sgn(x)/b^3
```

Mupad [B]
time = 3.10, size = 23, normalized size = 0.79

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3\,x\,\sqrt{a-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)),x)
[Out] -(2*(b - a/x)^(1/2))/(3*x*(a - b*x)^(1/2))
```

3.587 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$

Optimal. Leaf size=80

$$\frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m}(c+dx)^n\left(1+\frac{dx}{c}\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

```
[Out] (a+b/x)^m*x*(d*x+c)^n*AppellF1(1-m,-m,-n,2-m,-a*x/b,-d*x/c)/(1-m)/((1+a*x/b
)^m)/((1+d*x/c)^n)
```

Rubi [A]
time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {446, 140, 138}

$$\frac{x\left(a+\frac{b}{x}\right)^m\left(\frac{ax}{b}+1\right)^{-m}(c+dx)^n\left(\frac{dx}{c}+1\right)^{-n}F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\frac{dx}{c}\right)}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] ((a + b/x)^m*x*(c + d*x)^n*AppellF1[1 - m, -m, -n, 2 - m, -((a*x)/b), -((d*
x)/c)])/((1 - m)*(1 + (a*x)/b)^m*(1 + (d*x)/c)^n)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 140

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[
n]), Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx = \left(\left(a+\frac{b}{x}\right)^m x^m (b+ax)^{-m}\right)\int x^{-m}(b+ax)^m (c+dx)^n\,dx$$
$$= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m (c+dx)^n\,dx$$
$$= \left(\left(a+\frac{b}{x}\right)^m x^m \left(1+\frac{ax}{b}\right)^{-m} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n}\right)\int x^{-m}\left(1+\frac{ax}{b}\right)^m \left(1+\right.$$
$$= \frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} (c+dx)^n \left(1+\frac{dx}{c}\right)^{-n} F_1\left(1-m;-m,-n;2-m;-\frac{ax}{b},-\right.}{1-m}$$

Mathematica [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m (c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^n,x]
```

```
[Out] Integrate[(a + b/x)^m*(c + d*x)^n, x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^n,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="fricas")
[Out] integral((d*x + c)^n*((a*x + b)/x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^{m} \left(c + dx\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c)**n,x)
[Out] Integral((a + b/x)**m*(c + d*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^n,x, algorithm="giac")
[Out] integrate((d*x + c)^n*(a + b/x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{x}\right)}^m\,{\left(c+d\,x\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x)^n,x)
[Out] int((a + b/x)^m*(c + d*x)^n, x)
```

3.588 $\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx$

Optimal. Leaf size=138

$$\frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m}x^2}{6a^2}+\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}x^3}{3a}-\frac{b(6a^2c^2-6abcd(1-m)+b^2d^2(2-3m+m^2))\left(a+\frac{b}{x}\right)^{1+m}}{6a^4(1+m)}$$

```
[Out] 1/6*d*(6*a*c-b*d*(2-m))*(a+b/x)^(1+m)*x^2/a^2+1/3*d^2*(a+b/x)^(1+m)*x^3/a-1
/6*b*(6*a^2*c^2-6*a*b*c*d*(1-m)+b^2*d^2*(m^2-3*m+2))*(a+b/x)^(1+m)*hypergeo
m([2, 1+m],[2+m],1+b/a/x)/a^4/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 91, 79, 67}

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(2-m))}{6a^2}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(6a^2c^2-6abcd(1-m)+b^2d^2(m^2-3m+2))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{6a^4(m+1)}+\frac{d^2x^3\left(a+\frac{b}{x}\right)^{m+1}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] (d*(6*a*c - b*d*(2 - m))*(a + b/x)^(1 + m)*x^2)/(6*a^2) + (d^2*(a + b/x)^(1
 + m)*x^3)/(3*a) - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 - m) + b^2*d^2*(2 - 3*m + m
^2))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(6*
a^4*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
```

```
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x}\right)^m (c+dx)^2\,dx &= \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right)^2 x^2\,dx \\
&= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)^2}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^m(d(6ac-bd(2-m))+3ac^2x)}{x^3}\,dx, x, \frac{1}{x}\right)}{3a} \\
&= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{1}{6}\left(6c^2 - \frac{bd(6ac-b
\end{aligned}
$$

$$
= \frac{d(6ac-bd(2-m))\left(a+\frac{b}{x}\right)^{1+m} x^2}{6a^2} + \frac{d^2\left(a+\frac{b}{x}\right)^{1+m} x^3}{3a} - \frac{b(6a^2c^2-6abcd(1-
$$

Mathematica [A]
time = 0.16, size = 112, normalized size = 0.81

$$
\frac{\left(a+\frac{b}{x}\right)^m (b+ax)\left(a^2d(1+m)x^2(bd(-2+m)+2a(3c+dx)) - b(6a^2c^2+6abcd(-1+m)+b^2d^2(2-3m+m^2))\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)\right)}{6a^4(1+m)x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2*(b*d*(-2 + m) + 2*a*(3*c + d*x))
- b*(6*a^2*c^2 + 6*a*b*c*d*(-1 + m) + b^2*d^2*(2 - 3*m + m^2))*Hypergeometr
ic2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(6*a^4*(1 + m)*x)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^{m} (dx+c)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c)^2,x)
```

```
[Out] int((a+b/x)^m*(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^2 + 2*c*d*x + c^2)*((a*x + b)/x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 12.60, size = 121, normalized size = 0.88

$$\frac{b^{m}c^{2}xx^{-m}\Gamma(1-m)\,{}_{2}F_{1}\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{2b^{m}cdx^{2}x^{-m}\Gamma(2-m)\,{}_{2}F_{1}\left(\begin{matrix}-m,2-m\\3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}+\frac{b^{m}d^{2}x^{3}x^{-m}\Gamma(3-m)\,{}_{2}F_{1}\left(\begin{matrix}-m,3-m\\4-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(4-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c)**2,x)
```

```
[Out] b**m*c**2*x*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi)/b
)/(x**m*gamma(2 - m)) + 2*b**m*c*d*x**2*gamma(2 - m)*hyper((-m, 2 - m), (3
- m,), a*x*exp_polar(I*pi)/b)/(x**m*gamma(3 - m)) + b**m*d**2*x**3*gamma(3
```

```
- m)*hyper((-m, 3 - m), (4 - m,), a*x*exp_polar(I*pi)/b)/(x**m*gamma(4 - m)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*(a + b/x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x}\right)^m\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x)^2,x)
[Out] int((a + b/x)^m*(c + d*x)^2, x)
```

3.589 $\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx$

Optimal. Leaf size=79

$$\frac{d\left(a+\frac{b}{x}\right)^{1+m}x^2}{2a}-\frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{2a^3(1+m)}$$

```
[Out] 1/2*d*(a+b/x)^(1+m)*x^2/a-1/2*b*(2*a*c-b*d*(1-m))*(a+b/x)^(1+m)*hypergeom([
2, 1+m],[2+m],1+b/a/x)/a^3/(1+m)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {445, 457, 79, 67}

$$\frac{dx^2\left(a+\frac{b}{x}\right)^{m+1}}{2a}-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(1-m))\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{2a^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m*(c + d*x),x]
```

```
[Out] (d*(a + b/x)^(1 + m)*x^2)/(2*a) - (b*(2*a*c - b*d*(1 - m))*(a + b/x)^(1 + m
)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(2*a^3*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^m (c+dx)\,dx &= \int \left(a+\frac{b}{x}\right)^m \left(d+\frac{c}{x}\right) x\,dx\\ &= -\text{Subst}\left(\int \frac{(a+bx)^m(d+cx)}{x^3}\,dx, x, \frac{1}{x}\right)\\ &= \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{(2ac+bd(-1+m))\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx, x, \frac{1}{x}\right)}{2a}\\ &= \frac{d\left(a+\frac{b}{x}\right)^{1+m} x^2}{2a} - \frac{b(2ac-bd(1-m))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{2a^3(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 73, normalized size = 0.92

$$\frac{\left(a+\frac{b}{x}\right)^m (b+ax)\left(a^2d(1+m)x^2+b(-2ac-bd(-1+m))\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)\right)}{2a^3(1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m*(c + d*x),x]
[Out] ((a + b/x)^m*(b + a*x)*(a^2*d*(1 + m)*x^2 + b*(-2*a*c - b*d*(-1 + m))*Hyper
geometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)]))/(2*a^3*(1 + m)*x)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m (dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m*(d*x+c),x)
[Out] int((a+b/x)^m*(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)*((a*x + b)/x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.67, size = 75, normalized size = 0.95

$$\frac{b^m c x x^{-m} \Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m, 1-m\\ 2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}+\frac{b^m d x^2 x^{-m} \Gamma(2-m)\,{}_2F_1\left(\begin{matrix}-m, 2-m\\ 3-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(3-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m*(d*x+c),x)
```

```
[Out] b**m*c*x*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi)/b)/(
x**m*gamma(2 - m)) + b**m*d*x**2*gamma(2 - m)*hyper((-m, 2 - m), (3 - m,),
a*x*exp_polar(I*pi)/b)/(x**m*gamma(3 - m))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*(a + b/x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x}\right)^m\,(c+d\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m*(c + d*x),x)
```

```
[Out] int((a + b/x)^m*(c + d*x), x)
```

3.590 $\int \left(a+\frac{b}{x}\right)^m\,dx$

Optimal. Leaf size=40

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{a^2(1+m)}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],1+b/a/x)/a^2/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {248, 67}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(2,m+1;m+2;\frac{b}{ax}+1\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, 1 + b/(a*x)])/(a^
2*(1 + m)))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x}\right)^m\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^m}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{b\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(2,1+m;2+m;1+\frac{b}{ax}\right)}{a^2(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 1.25

$$-\frac{\left(a+\frac{b}{x}\right)^m x\left(1+\frac{ax}{b}\right)^{-m} {}_2F_1\left(1-m,-m;2-m;-\frac{ax}{b}\right)}{-1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m,x]
```

```
[Out] -(((a + b/x)^m*x*Hypergeometric2F1[1 - m, -m, 2 - m, -((a*x)/b)])/((-1 + m)
*(1 + (a*x)/b)^m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m,x)
```

```
[Out] int((a+b/x)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.65, size = 34, normalized size = 0.85

$$\frac{b^m x x^{-m}\Gamma(1-m)\,{}_2F_1\left(\begin{matrix}-m,1-m\\2-m\end{matrix}\middle|\frac{axe^{i\pi}}{b}\right)}{\Gamma(2-m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m,x)
```

```
[Out] b**m*x*gamma(1 - m)*hyper((-m, 1 - m), (2 - m,), a*x*exp_polar(I*pi)/b)/(x*
*m*gamma(2 - m))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m, x)
```

Mupad [B]
time = 3.13, size = 51, normalized size = 1.28

$$-\frac{x\,{\left(a+\frac{b}{x}\right)}^m\,{}_2\mathrm{F}_1\left(1-m,-m;\ 2-m;\ -\frac{a\,x}{b}\right)}{{\left(\frac{a\,x}{b}+1\right)}^m\,\left(m-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m,x)
```

```
[Out] -(x*(a + b/x)^m*hypergeom([1 - m, -m], 2 - m, -(a*x)/b))/(((a*x)/b + 1)^m*(
m - 1))
```

3.591 $\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx$

Optimal. Leaf size=101

$$-\frac{c\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)}+\frac{\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)}{ad(1+m)}$$

```
[Out] -c*(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/d/(a*c-b*d)/
(1+m)+(a+b/x)^(1+m)*hypergeom([1, 1+m],[2+m],1+b/a/x)/a/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 88, 67, 70}

$$\frac{\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{b}{ax}+1\right)}{ad(m+1)}-\frac{c\left(a+\frac{b}{x}\right)^{m+1}\,{}_2F_1\left(1,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(m+1)(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x),x]
```

```
[Out] -((c*(a + b/x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/(d*(a*c - b*d)*(1 + m))) + ((a + b/x)^(1 + m)*Hypergeometric2F1[
1, 1 + m, 2 + m, 1 + b/(a*x)])/(a*d*(1 + m))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
```

```
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{c+dx}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)x}\,dx\\
&= -\text{Subst}\left(\int \frac{(a+bx)^m}{x(d+cx)}\,dx, x, \frac{1}{x}\right)\\
&= -\frac{\text{Subst}\left(\int \frac{(a+bx)^m}{x}\,dx, x, \frac{1}{x}\right)}{d}+\frac{c\text{Subst}\left(\int \frac{(a+bx)^m}{d+cx}\,dx, x, \frac{1}{x}\right)}{d}\\
&= -\frac{c\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{d(ac-bd)(1+m)}+\frac{\left(a+\frac{b}{x}\right)^{1+m}\,{}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)}{ad(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 97, normalized size = 0.96

$$\frac{\left(a+\frac{b}{x}\right)^m(b+ax)\left(ac\,{}_2F_1\left(1,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)+(-ac+bd)\,{}_2F_1\left(1,1+m;2+m;1+\frac{b}{ax}\right)\right)}{ad(-ac+bd)(1+m)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x),x]
```

```
[Out] ((a + b/x)^m*(b + a*x)*(a*c*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(a + b/x)
)/(a*c - b*d)] + (-(a*c) + b*d)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + b/(a
*x)]))/(a*d*(-(a*c) + b*d)*(1 + m)*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c),x)
```

```
[Out] int((a+b/x)^m/(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c),x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x),x)
```

```
[Out] int((a + b/x)^m/(c + d*x), x)
```

3.592 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}$$

```
[Out] -b*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/(a*c-b*d)^2/
(1+m)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {445, 455, 70}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1} {}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(m+1)(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^2,x]
```

```
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*
c - b*d)])/((a*c - b*d)^2*(1 + m)))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^2}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^2 x^2}\,dx \\
&= -\mathrm{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{(ac-bd)^2(1+m)}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 57, normalized size = 1.02

$$
-\frac{b\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;-\frac{c\left(a+\frac{b}{x}\right)}{-ac+bd}\right)}{(-ac+bd)^2(1+m)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^2,x]
[Out] -((b*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, -((c*(a + b/x))/(
-(a*c) + b*d))])/((-(a*c) + b*d)^2*(1 + m)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^2}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^2,x)
[Out] int((a+b/x)^m/(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(((a*x + b)/x)^m/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**2,x)
[Out] Integral((a + b/x)**m/(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a + b/x)^m/(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a+\frac{b}{x}\right)}^m}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^2,x)
[Out] int((a + b/x)^m/(c + d*x)^2, x)
```

3.593 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx$

Optimal. Leaf size=112

$$-\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2}-\frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m}{}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}$$

```
[Out] -1/2*d*(a+b/x)^(1+m)/c/(a*c-b*d)/(d+c/x)^2-1/2*b*(2*a*c-b*d*(1+m))*(a+b/x)^
(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c/(a*c-b*d)^3/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {445, 457, 79, 70}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}(2ac-bd(m+1))\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(m+1)(ac-bd)^3}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}}{2c\left(\frac{c}{x}+d\right)^2(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^3,x]
```

```
[Out] -1/2*(d*(a + b/x)^(1 + m))/(c*(a*c - b*d)*(d + c/x)^2) - (b*(2*a*c - b*d*(1
 + m))*(a + b/x)^(1 + m)*Hypergeometric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(
a*c - b*d)])/(2*c*(a*c - b*d)^3*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^3}\,dx &= \int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^3 x^3}\,dx \\
&= -\text{Subst}\left(\int \frac{x(a+bx)^m}{(d+cx)^3}\,dx, x, \frac{1}{x}\right) \\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{(2ac-bd(1+m))\text{Subst}\left(\int \frac{(a+bx)^m}{(d+cx)^2}\,dx, x, \frac{1}{x}\right)}{2c(ac-bd)} \\
&= -\frac{d\left(a+\frac{b}{x}\right)^{1+m}}{2c(ac-bd)\left(d+\frac{c}{x}\right)^2} - \frac{b(2ac-bd(1+m))\left(a+\frac{b}{x}\right)^{1+m} {}_2F_1\left(2,1+m;2+m;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{2c(ac-bd)^3(1+m)}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 99, normalized size = 0.88

$$\frac{\left(a+\frac{b}{x}\right)^{1+m}\left(-\frac{dx^2}{(c+dx)^2}+\frac{b(-2ac+bd(1+m))\,{}_2F_1\left(2,1+m;2+m;\frac{bc+acx}{acx-bdx}\right)}{(ac-bd)^2(1+m)}\right)}{2c(ac-bd)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^3,x]
[Out] ((a + b/x)^(1 + m)*(-((d*x^2)/(c + d*x)^2) + (b*(-2*a*c + b*d*(1 + m))*Hype
rgeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d*x)])/((a*c - b*d)
^2*(1 + m))))/(2*c*(a*c - b*d))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^3,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{\left(c + dx\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**3,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^3,x)
[Out] int((a + b/x)^m/(c + d*x)^3, x)
```

3.594 $\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx$

Optimal. Leaf size=185

$$\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3}-\frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2}-\frac{b(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2))\left(a\right.}{6c^2(ac-bd)^4(1}$$

```
[Out] 1/3*d^2*(a+b/x)^(1+m)/c^2/(a*c-b*d)/(d+c/x)^3-1/6*d*(6*a*c-b*d*(4+m))*(a+b/
x)^(1+m)/c^2/(a*c-b*d)^2/(d+c/x)^2-1/6*b*(6*a^2*c^2-6*a*b*c*d*(1+m)+b^2*d^2
*(m^2+3*m+2))*(a+b/x)^(1+m)*hypergeom([2, 1+m],[2+m],c*(a+b/x)/(a*c-b*d))/c
^2/(a*c-b*d)^4/(1+m)
```

Rubi [A]
time = 0.13, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {445, 457, 91, 79, 70}

$$-\frac{b\left(a+\frac{b}{x}\right)^{m+1}\left(6a^2c^2-6abcd(m+1)+b^2d^2\left(m^2+3m+2\right)\right)\,{}_2F_1\left(2,m+1;m+2;\frac{c\left(a+\frac{b}{x}\right)}{ac-bd}\right)}{6c^2(m+1)(ac-bd)^4}+\frac{d^2\left(a+\frac{b}{x}\right)^{m+1}}{3c^2\left(\frac{c}{x}+d\right)^3(ac-bd)}-\frac{d\left(a+\frac{b}{x}\right)^{m+1}(6ac-bd(m+4))}{6c^2\left(\frac{c}{x}+d\right)^2(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] (d^2*(a + b/x)^(1 + m))/(3*c^2*(a*c - b*d)*(d + c/x)^3) - (d*(6*a*c - b*d*(
4 + m))*(a + b/x)^(1 + m))/(6*c^2*(a*c - b*d)^2*(d + c/x)^2) - (b*(6*a^2*c^
2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2 + 3*m + m^2))*(a + b/x)^(1 + m)*Hypergeo
metric2F1[2, 1 + m, 2 + m, (c*(a + b/x))/(a*c - b*d)])/(6*c^2*(a*c - b*d)^4
*(1 + m))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 91

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)
/(d^2*(d*e - c*f)*(n + 1))), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 445

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^m}{(c+dx)^4}\,dx=\int \frac{\left(a+\frac{b}{x}\right)^m}{\left(d+\frac{c}{x}\right)^4x^4}\,dx$$

$$=-\text{Subst}\left(\int \frac{x^2(a+bx)^m}{(d+cx)^4}\,dx,x,\frac{1}{x}\right)$$

$$=\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3}-\frac{\text{Subst}\left(\int \frac{(a+bx)^m(-d(3ac-bd(1+m))+3c(ac-bd)x)}{(d+cx)^3}\,dx,x,\frac{1}{x}\right)}{3c^2(ac-bd)}$$

$$=\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3}-\frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2}-\frac{(6a^2c^2-6abcd(1+m)+b^2c}{}$$

$$=\frac{d^2\left(a+\frac{b}{x}\right)^{1+m}}{3c^2(ac-bd)\left(d+\frac{c}{x}\right)^3}-\frac{d(6ac-bd(4+m))\left(a+\frac{b}{x}\right)^{1+m}}{6c^2(ac-bd)^2\left(d+\frac{c}{x}\right)^2}-\frac{b(6a^2c^2-6abcd(1+m)+b^2}{}$$

Mathematica [A]
time = 0.22, size = 155, normalized size = 0.84

$$\frac{\left(a+\frac{b}{x}\right)^{1+m}\left(\frac{2d^2(ac-bd)x^3}{(c+dx)^3}+\frac{d(-6ac+bd(4+m))x^2}{(c+dx)^2}-\frac{b\left(6a^2c^2-6abcd(1+m)+b^2d^2(2+3m+m^2)\right)\,{}_2F_1\left(2,1+m;2+m;\frac{bc+acx}{acx-bdx}\right)}{(ac-bd)^2(1+m)}\right)}{6c^2(ac-bd)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^m/(c + d*x)^4,x]
```

```
[Out] ((a + b/x)^(1 + m)*((2*d^2*(a*c - b*d)*x^3)/(c + d*x)^3 + (d*(-6*a*c + b*d*
(4 + m))*x^2)/(c + d*x)^2 - (b*(6*a^2*c^2 - 6*a*b*c*d*(1 + m) + b^2*d^2*(2
+ 3*m + m^2))*Hypergeometric2F1[2, 1 + m, 2 + m, (b*c + a*c*x)/(a*c*x - b*d
*x)])/((a*c - b*d)^2*(1 + m))))/(6*c^2*(a*c - b*d)^2)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x}\right)^{m}}{\left(dx+c\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^m/(d*x+c)^4,x)
```

```
[Out] int((a+b/x)^m/(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^m/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x
 + c^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{\left(c + dx\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**m/(d*x+c)**4,x)
```

```
[Out] Integral((a + b/x)**m/(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^m/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^m/(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x}\right)^m}{\left(c + d\,x\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^m/(c + d*x)^4,x)
```

```
[Out] int((a + b/x)^m/(c + d*x)^4, x)
```

3.595 $\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^m}{\sqrt{a-bx^2}}\, dx$

Optimal. Leaf size=33

$$\frac{\sqrt{b-\frac{a}{x^2}}\, x^{1+m}}{m\sqrt{a-bx^2}}$$

```
[Out] x^(1+m)*(b-a/x^2)^(1/2)/m/(-b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {529, 23, 30}

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^m}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{x^{-1+m}\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int x^{-1+m}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x^{1+m}}{m\sqrt{a-bx^2}}$$

Mathematica [A]
time = 0.06, size = 33, normalized size = 1.00

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x^{1+m}}{m\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])
```

Maple [A]
time = 0.39, size = 35, normalized size = 1.06

method	result	size
gosper	$\frac{x^{1+m}\sqrt{-\frac{-b\,x^2+a}{x^2}}}{m\sqrt{-b\,x^2+a}}$	35
risch	$\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)x\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}\,x^m}{(-b\,x^2+a)^{\frac{3}{2}}m}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1+m)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.29, size = 8, normalized size = 0.24

$$-\frac{i\,x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*x^m/m
```

Fricas [A]
time = 0.37, size = 44, normalized size = 1.33

$$-\frac{\sqrt{-bx^2+a}\,xx^m\sqrt{\frac{bx^2-a}{x^2}}}{bmx^2-am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)
```

Mupad [B]
time = 3.36, size = 29, normalized size = 0.88

$$\frac{x^{m+1}\sqrt{b-\frac{a}{x^2}}}{m\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

```
[Out] (x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))
```

3.596 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x^3}{2\sqrt{a-bx^2}}$$

```
[Out] 1/2*x^3*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {529, 23, 30}

$$\frac{x^3\sqrt{b-\frac{a}{x^2}}}{2\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\, x^2}{\sqrt{a-bx^2}}\, dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int \frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\, dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\, x\right)\int x\, dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\, x^3}{2\sqrt{a-bx^2}}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 1.03

$$-\frac{\sqrt{b-\frac{a}{x^2}}\, x\sqrt{a-bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2],x]
```

```
[Out] -1/2*(Sqrt[b - a/x^2]*x*Sqrt[a - b*x^2])/b
```

Maple [A]
time = 0.34, size = 31, normalized size = 1.00

method	result	size
gosper	$\frac{x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2 \sqrt{-b\,x^{2}+a}}$	31
default	$\frac{x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{2 \sqrt{-b\,x^{2}+a}}$	31
risch	$\frac{i x^{3} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right) \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{2 \left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.30, size = 5, normalized size = 0.16

$$-\frac{1}{2}i\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*x^2
```

Fricas [A]
time = 0.36, size = 41, normalized size = 1.32

$$-\frac{\sqrt{-bx^2+a}\,x^3\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^2-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [C] Result contains complex when optimal does not.
time = 4.73, size = 15, normalized size = 0.48

$$-\frac{i\,bx^2-i\,a}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(I*b*x^2 - I*a)/b
```

Mupad [B]
time = 3.14, size = 25, normalized size = 0.81

$$\frac{x^3\,\sqrt{b-\frac{a}{x^2}}}{2\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
[Out] (x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))
```

3.597 $\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}$$

```
[Out] x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {529, 23, 8}

$$\frac{x^2\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int 1\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}$$

Mathematica [**A**]
time = 0.04, size = 28, normalized size = 1.00

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x^2}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]
```

Maple [**A**]
time = 0.33, size = 30, normalized size = 1.07

method	result	size
default	$\frac{x^{2} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}}{\sqrt{-b\,x^{2}+a}}$	30
risch	$\frac{i x^{2} \sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right) \sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [**C**] Result contains complex when optimal does not.
time = 0.28, size = 7, normalized size = 0.25

$$-i\sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*sqrt(x^2)
```

Fricas [A]
time = 0.38, size = 41, normalized size = 1.46

$$-\frac{\sqrt{-bx^2+a}\,x^2\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)
```

Mupad [B]
time = 3.10, size = 27, normalized size = 0.96

$$\frac{\sqrt{b\,x^2-a}\,\sqrt{x^2}}{\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

```
[Out] ((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)
```

3.598 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

[Out] x*ln(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {446, 23, 29}

$$\frac{x\log(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x}\,dx}{\sqrt{a-bx^2}}$$
$$= \frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

Mathematica [A]
time = 0.05, size = 28, normalized size = 1.00

$$\frac{\sqrt{b-\frac{a}{x^2}}\,x\log(x)}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2],x]
[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]
```

Maple [A]
time = 0.33, size = 30, normalized size = 1.07

method	result	size
default	$\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}\,x\ln(x)}{\sqrt{-b\,x^2+a}}$	30
risch	$\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)x\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}\,\ln(x)}{(-b\,x^2+a)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(-b*x^2+a)/x^2)^(1/2)*x/(-b*x^2+a)^(1/2)*ln(x)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.29, size = 4, normalized size = 0.14

$$-i\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -I*log(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(24) = 48$.
time = 0.35, size = 51, normalized size = 1.82

$$-\arctan\left(\frac{\sqrt{-bx^2+a}\,(x^3+x)\sqrt{\frac{bx^2-a}{x^2}}}{bx^4-(a+b)x^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x
^2 + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{a}{x^2}+b}}{\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.599 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

```
[Out] -(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {529, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^2\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^2}\,dx}{\sqrt{a-bx^2}}$$
$$= -\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Mathematica [A]
time = 0.05, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]
```

```
[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])
```

Maple [A]
time = 0.30, size = 28, normalized size = 1.08

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$	28
default	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{\sqrt{-b\,x^2+a}}$	28
risch	$-\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{(-b\,x^2+a)^{\frac{3}{2}}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.29, size = 7, normalized size = 0.27

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] I/sqrt(x^2)
```

Fricas [A]
time = 0.33, size = 41, normalized size = 1.58

$$-\frac{\sqrt{-bx^2+a}\,(x-1)\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)
```

Mupad [B]
time = 3.34, size = 22, normalized size = 0.85

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)),x)
```

```
[Out] -(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)
```

3.600 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx$

Optimal. Leaf size=31

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

```
[Out] -1/2*(b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {529, 23, 30}

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
```

```
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 529

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p
_.), x_Symbol] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^
n)^FracPart[q]), Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; Fre
eQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !Intege
rQ[p]
```

Rubi steps

$$\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}}\,dx = \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{\sqrt{-a+bx^2}}{x^3\sqrt{a-bx^2}}\,dx}{\sqrt{-a+bx^2}}$$
$$= \frac{\left(\sqrt{b-\frac{a}{x^2}}\,x\right)\int \frac{1}{x^3}\,dx}{\sqrt{a-bx^2}}$$
$$= -\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.00

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2x\sqrt{a-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]
[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])
```

Maple [A]
time = 0.34, size = 31, normalized size = 1.00

method	result	size
gosper	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2x\sqrt{-b\,x^2+a}}$	31
default	$-\frac{\sqrt{-\frac{-b\,x^2+a}{x^2}}}{2x\sqrt{-b\,x^2+a}}$	31
risch	$-\frac{i\sqrt{-\frac{-b\,x^2+a}{x^2}}\,(b\,x^2-a)\sqrt{\frac{-b\,x^2+a}{b\,x^2-a}}}{2x(-b\,x^2+a)^{\frac{3}{2}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)
```

Maxima [C] Result contains complex when optimal does not.
time = 0.28, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*I/x^2
```

Fricas [A]
time = 0.42, size = 44, normalized size = 1.42

$$-\frac{\sqrt{-bx^2+a}\,(x^2-1)\sqrt{\frac{bx^2-a}{x^2}}}{2\,(bx^3-ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2}+b}}{x^2\sqrt{a-bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.70, size = 5, normalized size = 0.16

$$\frac{i}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*I/x^2
```

Mupad [B]
time = 3.46, size = 25, normalized size = 0.81

$$-\frac{\sqrt{b-\frac{a}{x^2}}}{2\,x\,\sqrt{a-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)),x)
[Out] -(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))
```

3.601 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$

Optimal. Leaf size=406

$$\frac{2c\sqrt{c+dx}\,(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}\,x}+\frac{2(c+dx)^{3/2}\,(b+ax^2)}{5a\sqrt{a+\frac{b}{x^2}}\,x}+\frac{2\sqrt{b}\,(ac^2-3bd^2)\sqrt{c+dx}\,\sqrt{1+\frac{ax^2}{b}}\,E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{-\cdots}}{\sqrt{\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{5(-a)^{3/2}d\sqrt{a+\frac{b}{x^2}}\,x\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\,\sqrt{\cdots}}}}\cdots$$

```
[Out] 2/5*(d*x+c)^(3/2)*(a*x^2+b)/a/x/(a+b/x^2)^(1/2)+2/5*c*(a*x^2+b)*(d*x+c)^(1/
2)/a/x/(a+b/x^2)^(1/2)+2/5*(a*c^2-3*b*d^2)*EllipticE(1/2*(1-x*(-a)^(1/2)/b^
(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(
1/2))*b^(1/2)*(d*x+c)^(1/2)*(1+a*x^2/b)^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2
)/(a*(d*x+c)/(a*c-d*(-a)^(1/2)*b^(1/2)))^(1/2)-2/5*c*(a*c^2+b*d^2)*Elliptic
F(1/2*(1-x*(-a)^(1/2)/b^(1/2))^(1/2)*2^(1/2),(-2*d*(-a)^(1/2)*b^(1/2)/(a*c-
d*(-a)^(1/2)*b^(1/2)))^(1/2))*b^(1/2)*(1+a*x^2/b)^(1/2)*(a*(d*x+c)/(a*c-d*(
-a)^(1/2)*b^(1/2)))^(1/2)/(-a)^(3/2)/d/x/(a+b/x^2)^(1/2)/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1464, 847, 858, 733, 435, 430}

$$-\frac{2\sqrt{b}\,c\sqrt{\frac{ax^2}{b}+1}\,(ac^2+bd^2)\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\,\sqrt{b}\,d}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{-a}\,x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\,\sqrt{b}\,d}{ac-\sqrt{-a}\,\sqrt{b}\,d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+dx}}+\frac{2\sqrt{b}\,\sqrt{\frac{ax^2}{b}+1}\,\sqrt{c+dx}\,(ac^2-3bd^2)\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\frac{\sqrt{-a}\,x}{\sqrt{b}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{-a}\,\sqrt{b}\,d}{ac-\sqrt{-a}\,\sqrt{b}\,d}\right)}{5(-a)^{3/2}dx\sqrt{a+\frac{b}{x^2}}\,\sqrt{\frac{a(c+dx)}{ac-\sqrt{-a}\,\sqrt{b}\,d}}}+\frac{2(ax^2+b)(c+dx)^{3/2}}{5ax\sqrt{a+\frac{b}{x^2}}}+\frac{2c(ax^2+b)\sqrt{c+dx}}{5ax\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (2*c*Sqrt[c + d*x]*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*(c + d*x)^(3/2
)*(b + a*x^2))/(5*a*Sqrt[a + b/x^2]*x) + (2*Sqrt[b]*(a*c^2 - 3*b*d^2)*Sqrt[
c + d*x]*Sqrt[1 + (a*x^2)/b]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]
]/Sqrt[2]], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3
/2)*d*Sqrt[a + b/x^2]*x*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]) - (
2*Sqrt[b]*c*(a*c^2 + b*d^2)*Sqrt[(a*(c + d*x))/(a*c - Sqrt[-a]*Sqrt[b]*d)]*
Sqrt[1 + (a*x^2)/b]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[-a]*x)/Sqrt[b]]/Sqrt[2]
], (-2*Sqrt[-a]*Sqrt[b]*d)/(a*c - Sqrt[-a]*Sqrt[b]*d)])/(5*(-a)^(3/2)*d*Sqr
t[a + b/x^2]*x*Sqrt[c + d*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1464

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Sy
mbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(c + a*x^(2*
n))^FracPart[p]), Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x], x] /
; FreeQ[{a, c, d, e, n, p, q}, x] && EqQ[mn2, -2*n] &&  !IntegerQ[p] &&  !I
ntegerQ[q] && PosQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx &= \frac{\sqrt{b+ax^2}\int \frac{x(c+dx)^{3/2}}{\sqrt{b+ax^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x} \\
&= \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{b+ax^2}\right)\int \frac{\left(-\frac{3bd}{2}+\frac{3acx}{2}\right)\sqrt{c+dx}}{\sqrt{b+ax^2}}\,dx}{5a\sqrt{a+\frac{b}{x^2}}\,x} \\
&= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(4\sqrt{b+ax^2}\right)\int \frac{-3abcd+\frac{3}{4}a\left(ac^2-3bd^2\right)x}{\sqrt{c+dx}\sqrt{b+ax^2}}\,d\cdots}{15a^2\sqrt{a+\frac{b}{x^2}}\,x} \\
&= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(\left(ac^2-3bd^2\right)\sqrt{b+ax^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{b+ax^2}}\cdots}{5ad\sqrt{a+\frac{b}{x^2}}\,x} \\
&= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{\left(2\sqrt{-a}\sqrt{b}\left(ac^2-3bd^2\right)\sqrt{c+dx}\sqrt{1\cdots}\right.}{5a^{\cdots}} \\
&= \frac{2c\sqrt{c+dx}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2(c+dx)^{3/2}\left(b+ax^2\right)}{5a\sqrt{a+\frac{b}{x^2}}\,x} + \frac{2\sqrt{b}\left(ac^2-3bd^2\right)\sqrt{c+dx}\sqrt{1+\frac{ax^2}{b}}\cdots}{5(-a)^{3/2}d\sqrt{a+\cdots}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 21.94, size = 540, normalized size = 1.33

$$
\frac{\sqrt{c+dx}\left(\frac{2(2c+dx)\left(b+ax^2\right)}{a}+\frac{2\left(d^2\sqrt{-c-\frac{i\sqrt{b}\,d}{\sqrt{a}}}\left(-3b^2d^2+a^2c^2x^2+ab\left(c^2-3d^2x^2\right)\right)+\sqrt{a}\left(-ia^{3/2}c^3+a\sqrt{b}\,c^2d+3i\sqrt{a}\,bcd^2-3b^{3/2}d^3\right)\sqrt{\frac{d\left(\frac{i\sqrt{b}}{\sqrt{a}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{i\sqrt{b}\,d}{\sqrt{a}}-dx}{c+dx}}\,(c+dx)^{3/2}E\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{i\sqrt{b}\,d}{\sqrt{a}}}}{\sqrt{c+dx}}\right)\middle|\frac{\sqrt{a}\,c-i\sqrt{b}\,d}{\sqrt{a}\,c+i\sqrt{b}\,d}\right)-\sqrt{a}\sqrt{b}\,d\left(ac^2+4i\sqrt{a}\sqrt{b}\,cd-3bd^2\right)\sqrt{\frac{d\left(\frac{i\sqrt{b}}{\sqrt{a}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{i\sqrt{b}\,d}{\sqrt{a}}-dx}{c+dx}}\,(c+dx)^{3/2}F\left(i\sinh^{-1}\left(\frac{\sqrt{-c-\frac{i\sqrt{b}\,d}{\sqrt{a}}}}{\sqrt{c+dx}}\right)\middle|\frac{\sqrt{a}\,c-i\sqrt{b}\,d}{\sqrt{a}\,c+i\sqrt{b}\,d}\right)\right)}{a^2d^2\sqrt{-c-\frac{i\sqrt{b}\,d}{\sqrt{a}}}\,(c+dx)}\right)}{5\sqrt{a+\frac{b}{x^2}}\,x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b/x^2],x]
```

```
[Out] (Sqrt[c + d*x]*((2*(2*c + d*x)*(b + a*x^2))/a + (2*(d^2*Sqrt[-c - (I*Sqrt[b
]*d)/Sqrt[a]]*(-3*b^2*d^2 + a^2*c^2*x^2 + a*b*(c^2 - 3*d^2*x^2)) + Sqrt[a]*
((-I)*a^(3/2)*c^3 + a*Sqrt[b]*c^2*d + (3*I)*Sqrt[a]*b*c*d^2 - 3*b^(3/2)*d^3
)*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[
a] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt
[b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sq
rt[b]*d)] - Sqrt[a]*Sqrt[b]*d*(a*c^2 + (4*I)*Sqrt[a]*Sqrt[b]*c*d - 3*b*d^2)
*Sqrt[(d*((I*Sqrt[b])/Sqrt[a] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[b]*d)/Sqrt[a
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[
b]*d)/Sqrt[a]]/Sqrt[c + d*x]], (Sqrt[a]*c - I*Sqrt[b]*d)/(Sqrt[a]*c + I*Sqr
t[b]*d)]))/(a^2*d^2*Sqrt[-c - (I*Sqrt[b]*d)/Sqrt[a]]*(c + d*x))))/(5*Sqrt[a
 + b/x^2]*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1144 vs. $2(330) = 660$.

time = 0.36, size = 1145, normalized size = 2.82

method	result
risch	$\frac{2(dx+2c)(a\,x^2+b)\sqrt{dx+c}}{5a\sqrt{\frac{a\,x^2+b}{x^2}}\,x}+\frac{\left(\frac{2\left(c^2a-3d^2b\right)\left(\frac{c}{d}-\frac{\sqrt{-ab}}{a}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{a}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{a}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{a}}}\left(\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$\frac{2\sqrt{-\frac{(dx+c)a}{\sqrt{-ab}\,d-ac}}\sqrt{\frac{\left(-ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d+ac}}\sqrt{\frac{\left(ax+\sqrt{-ab}\right)d}{\sqrt{-ab}\,d-ac}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)a}{\sqrt{-ab}\,d-ac}},\sqrt{-\frac{\sqrt{-ab}\,d-ac}{\sqrt{-ab}\,d+ac}}\cdots\right.}{5\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)
^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*Elli
pticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b
)^(1/2)*d+a*c))^(1/2))*(-a*b)^(1/2)*a*c^3*d+(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c
))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1
/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*
c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*(-a*b)^(1/2)*
b*c*d^3-3*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((
-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)
*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/(
(-a*b)^(1/2)*d+a*c))^(1/2))*a*b*c^2*d^2-3*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))
^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2
))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticF((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c)
)^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*b^2*d^4-(-(d*x+
c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c
))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticE((-(d*x
+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*
c))^(1/2))*a^2*c^4+2*(-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^
(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-
a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(1/2)*d-a*c))^(1/2),(-((-a*b)^(1/
2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*a*b*c^2*d^2+3*(-(d*x+c)*a/((-a*b)^(1
/2)*d-a*c))^(1/2)*((-a*x+(-a*b)^(1/2))*d/((-a*b)^(1/2)*d+a*c))^(1/2)*((a*x+
(-a*b)^(1/2))*d/((-a*b)^(1/2)*d-a*c))^(1/2)*EllipticE((-(d*x+c)*a/((-a*b)^(
1/2)*d-a*c))^(1/2),(-((-a*b)^(1/2)*d-a*c)/((-a*b)^(1/2)*d+a*c))^(1/2))*b^2*
d^4+a^2*d^4*x^4+3*a^2*c*d^3*x^3+2*a^2*c^2*d^2*x^2+a*b*d^4*x^2+3*a*b*c*d^3*x
+2*a*b*c^2*d^2)/(d*x+c)^(1/2)/d^2/a^2/x/((a*x^2+b)/x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 235, normalized size = 0.58

$$-\frac{2\left((ac^3+9\,bcd^2)\sqrt{ad}\ \text{weierstrassPInverse}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\frac{3\,dx+c}{3\,d}\right)+3\,(ac^2d-3\,bd^3)\sqrt{ad}\ \text{weierstrassZeta}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\text{weierstrassPInverse}\left(\frac{4\,(ac^2-3\,bd^2)}{3\,ad^2},-\frac{8\,(ac^3+9\,bcd^2)}{27\,ad^3},\frac{3\,dx+c}{3\,d}\right)\right)-3\,(ad^3x^2+2\,acd^2x)\sqrt{dx+c}\,\sqrt{\frac{ax^2+b}{x^2}}\right)}{15\,a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*((a*c^3 + 9*b*c*d^2)*sqrt(a*d)*weierstrassPInverse(4/3*(a*c^2 - 3*b*d
^2)/(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d) + 3*(a*c
^2*d - 3*b*d^3)*sqrt(a*d)*weierstrassZeta(4/3*(a*c^2 - 3*b*d^2)/(a*d^2), -8
/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), weierstrassPInverse(4/3*(a*c^2 - 3*b*d^2)/
(a*d^2), -8/27*(a*c^3 + 9*b*c*d^2)/(a*d^3), 1/3*(3*d*x + c)/d)) - 3*(a*d^3*
x^2 + 2*a*c*d^2*x)*sqrt(d*x + c)*sqrt((a*x^2 + b)/x^2))/(a^2*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(a+b/x**2)**(1/2),x)
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b/x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(a+b/x^2)^(1/2),x, algorithm="giac")
[Out] integrate((d*x + c)^(3/2)/sqrt(a + b/x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{(c+d\,x)}^{3/2}}{\sqrt{a+\frac{b}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2),x)
[Out] int((c + d*x)^(3/2)/(a + b/x^2)^(1/2), x)
```

3.602 $\int \frac{-1+x^3}{(-4x+x^4)^{2/3}} dx$

Optimal. Leaf size=15

$$\frac{3}{4}\sqrt[3]{-4x+x^4}$$

```
[Out] 3/4*(x^4-4*x)^(1/3)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$\frac{3}{4}\sqrt[3]{x^4-4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

```
[Out] (3*(-4*x + x^4)^(1/3))/4
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}}\,dx = \frac{3}{4}\sqrt[3]{-4x+x^4}$$

Mathematica [A]
time = 10.04, size = 15, normalized size = 1.00

$$\frac{3}{4}\sqrt[3]{x\left(-4+x^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3),x]
```

[Out] (3*(x*(-4 + x^3))^(1/3))/4

Maple [A]
time = 0.32, size = 12, normalized size = 0.80

method	result
default	$\frac{3\left(x^{4}-4x\right)^{\frac{1}{3}}}{4}$
trager	$\frac{3\left(x^{4}-4x\right)^{\frac{1}{3}}}{4}$
gosper	$\frac{3x\left(x^{3}-4\right)}{4\left(x^{4}-4x\right)^{\frac{2}{3}}}$
risch	$\frac{3x\left(x^{3}-4\right)}{4\left(x\left(x^{3}-4\right)\right)^{\frac{2}{3}}}$
meijerg	$-\frac{3\,2^{\frac{2}{3}}\left(-\mathrm{signum}\left(-1+\frac{x^{3}}{4}\right)\right)^{\frac{2}{3}}x^{\frac{1}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{9},\frac{2}{3}\right],\left[\frac{10}{9}\right],\frac{x^{3}}{4}\right)}{4\mathrm{signum}\left(-1+\frac{x^{3}}{4}\right)^{\frac{2}{3}}}+\frac{3\,2^{\frac{2}{3}}\left(-\mathrm{signum}\left(-1+\frac{x^{3}}{4}\right)\right)^{\frac{2}{3}}x^{\frac{10}{3}}\,\mathrm{hypergeom}\left(\left[\frac{2}{3},\frac{10}{9}\right],\left[\frac{19}{9}\right],\frac{x^{3}}{4}\right)}{40\mathrm{signum}\left(-1+\frac{x^{3}}{4}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x^4-4*x)^(2/3),x,method=_RETURNVERBOSE)

[Out] 3/4*(x^4-4*x)^(1/3)

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")

[Out] 3/4*(x^4 - 4*x)^(1/3)

Fricas [A]
time = 0.33, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^{4}-4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")

[Out] 3/4*(x^4 - 4*x)^(1/3)

Sympy [A]
time = 0.07, size = 12, normalized size = 0.80

$$\frac{3\sqrt[3]{x^{4}-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)
```

```
[Out] 3*(x**4 - 4*x)**(1/3)/4
```

Giac [A]
time = 2.81, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^4 - 4\,x\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^4 - 4*x)^(1/3)
```

Mupad [B]
time = 3.52, size = 11, normalized size = 0.73

$$\frac{3\left(x^4 - 4\,x\right)^{1/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)
```

```
[Out] (3*(x^4 - 4*x)^(1/3))/4
```

3.603 $\int (2-x^2) \sqrt[4]{6x-x^3}\, dx$

Optimal. Leaf size=17

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

```
[Out] 4/15*(-x^3+6*x)^(5/4)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (4*(6*x - x^3)^(5/4))/15
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(2-x^2\right) \sqrt[4]{6x-x^3}\, dx = \frac{4}{15}\left(6x-x^3\right)^{5/4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.
time = 10.04, size = 72, normalized size = 4.24

$$-\frac{4\sqrt[4]{-x\left(-6+x^2\right)}\left(-26x\,{}_2F_1\left(-\frac{1}{4},\frac{5}{8};\frac{13}{8};\frac{x^2}{6}\right)+5x^3\,{}_2F_1\left(-\frac{1}{4},\frac{13}{8};\frac{21}{8};\frac{x^2}{6}\right)\right)}{65\sqrt[4]{1-\frac{x^2}{6}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4),x]
```

```
[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6
] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4
))
```

Maple [A]
time = 0.32, size = 14, normalized size = 0.82

method	result	size
default	$\frac{4\left(-x^{3}+6x\right)^{\frac{5}{4}}}{15}$	14
risch	$-\frac{4\left(-x\left(x^{2}-6\right)\right)^{\frac{1}{4}} x\left(x^{2}-6\right)}{15}$	19
gosper	$-\frac{4\left(-x^{3}+6x\right)^{\frac{1}{4}} x\left(x^{2}-6\right)}{15}$	20
trager	$-\frac{4\left(-x^{3}+6x\right)^{\frac{1}{4}} x\left(x^{2}-6\right)}{15}$	20
meijerg	$\frac{8\,6^{\frac{1}{4}} x^{\frac{5}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{5}{8}\right],\left[\frac{13}{8}\right],\frac{x^{2}}{6}\right)}{5}-\frac{4\,6^{\frac{1}{4}} x^{\frac{13}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{13}{8}\right],\left[\frac{21}{8}\right],\frac{x^{2}}{6}\right)}{13}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*(-x^3+6*x)^(5/4)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^{3}+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Fricas [A]
time = 0.35, size = 20, normalized size = 1.18

$$-\frac{4}{15}\left(x^{3}-6\,x\right)\left(-x^{3}+6\,x\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.09, size = 31, normalized size = 1.82

$$-\frac{4x^3\sqrt[4]{-x^3+6x}}{15}+\frac{8x\sqrt[4]{-x^3+6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)
```

```
[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5
```

Giac [A]
time = 3.33, size = 13, normalized size = 0.76

$$\frac{4}{15}\left(-x^3+6\,x\right)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")
```

```
[Out] 4/15*(-x^3 + 6*x)^(5/4)
```

Mupad [B]
time = 3.16, size = 19, normalized size = 1.12

$$-\frac{4\,x\,\left(x^2-6\right)\,{\left(6\,x-x^3\right)}^{1/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)
```

```
[Out] -(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15
```

3.604 $\int (1+x^4) \sqrt{5x+x^5}\, dx$

Optimal. Leaf size=15

$$\frac{2}{15}\left(5x+x^5\right)^{3/2}$$

```
[Out] 2/15*(x^5+5*x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1602}

$$\frac{2}{15}\left(x^5+5x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(1+x^4\right) \sqrt{5x+x^5}\, dx = \frac{2}{15}\left(5x+x^5\right)^{3/2}$$

Mathematica [A]
time = 10.03, size = 15, normalized size = 1.00

$$\frac{2}{15}\left(x\left(5+x^4\right)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]
```

```
[Out] (2*(x*(5 + x^4))^(3/2))/15
```

Maple [A]
time = 0.35, size = 12, normalized size = 0.80

method	result	size
default	$\frac{2(x^5+5x)^{\frac{3}{2}}}{15}$	12
gosper	$\frac{2x(x^4+5)\sqrt{x^5+5x}}{15}$	18
trager	$\frac{2x(x^4+5)\sqrt{x^5+5x}}{15}$	18
risch	$\frac{2x^2(x^4+5)^2}{15\sqrt{x(x^4+5)}}$	22
meijerg	$\frac{2\sqrt{5}\, x^{\frac{3}{2}} \operatorname{hypergeom}\left([-\frac{1}{2},\frac{3}{8}],[\frac{11}{8}],-\frac{x^4}{5}\right)}{3}+\frac{2\sqrt{5}\, x^{\frac{11}{2}} \operatorname{hypergeom}\left([-\frac{1}{2},\frac{11}{8}],[\frac{19}{8}],-\frac{x^4}{5}\right)}{11}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(x^5+5*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/15*(x^5+5*x)^(3/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^5+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.
time = 0.08, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+5x}}{15}+\frac{2x\sqrt{x^5+5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)
```

```
[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3
```

Giac [A]
time = 3.70, size = 11, normalized size = 0.73

$$\frac{2}{15}\left(x^{5}+5\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(x^5 + 5*x)^(3/2)
```

Mupad [B]
time = 3.10, size = 11, normalized size = 0.73

$$\frac{2\left(x^{5}+5\,x\right)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x + x^5)^(1/2)*(x^4 + 1),x)
```

```
[Out] (2*(5*x + x^5)^(3/2))/15
```

3.605 $\int (2+5x^4)\sqrt{2x+x^5}\,dx$

Optimal. Leaf size=15

$$\frac{2}{3}(2x+x^5)^{3/2}$$

```
[Out] 2/3*(x^5+2*x)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{1602\}$

$$\frac{2}{3}(x^5+2x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(2*x + x^5)^(3/2))/3
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int (2+5x^4)\sqrt{2x+x^5}\,dx = \frac{2}{3}(2x+x^5)^{3/2}$$

Mathematica [A]
time = 10.04, size = 15, normalized size = 1.00

$$\frac{2}{3}(x(2+x^4))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]
[Out] (2*(x*(2 + x^4))^(3/2))/3
```

Maple [A]
time = 0.38, size = 12, normalized size = 0.80

method	result	size
derivativedivides	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
default	$\frac{2(x^5+2x)^{\frac{3}{2}}}{3}$	12
gosper	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
trager	$\frac{2x(x^4+2)\sqrt{x^5+2x}}{3}$	18
risch	$\frac{2x^2(x^4+2)^2}{3\sqrt{x(x^4+2)}}$	22
meijerg	$\frac{4\sqrt{2}\, x^{\frac{3}{2}} \operatorname{hypergeom}\left([-\frac{1}{2},\frac{3}{8}],[\frac{11}{8}],-\frac{x^4}{2}\right)}{3}+\frac{10\sqrt{2}\, x^{\frac{11}{2}} \operatorname{hypergeom}\left([-\frac{1}{2},\frac{11}{8}],[\frac{19}{8}],-\frac{x^4}{2}\right)}{11}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x^5+2*x)^(3/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Fricas [A]
time = 0.33, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^5+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.08, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5+2x}}{3}+\frac{4x\sqrt{x^5+2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)
[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3
```

Giac [A]
time = 3.63, size = 11, normalized size = 0.73

$$\frac{2}{3}\left(x^{5}+2\,x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")
[Out] 2/3*(x^5 + 2*x)^(3/2)
```

Mupad [B]
time = 3.11, size = 11, normalized size = 0.73

$$\frac{2\left(x^{5}+2\,x\right)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)
[Out] (2*(2*x + x^5)^(3/2))/3
```

3.606 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx$

Optimal. Leaf size=13

$$\sqrt{x^2+2x^3}$$

[Out] (2*x^3+x^2)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1602}

$$\sqrt{2x^3+x^2}$$

Antiderivative was successfully verified.

[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]

[Out] Sqrt[x^2 + 2*x^3]

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x+3x^2}{\sqrt{x^2+2x^3}}\,dx = \sqrt{x^2+2x^3}$$

Mathematica [A]
time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{x^2(1+2x)}$$

Antiderivative was successfully verified.

[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3],x]

[Out] Sqrt[x^2*(1 + 2*x)]

Maple [A]
time = 0.32, size = 21, normalized size = 1.62

method	result	size
trager	$\sqrt{2x^3+x^2}$	12
gosper	$\frac{x^2(2x+1)}{\sqrt{2x^3+x^2}}$	21
default	$\frac{x^2(2x+1)}{\sqrt{2x^3+x^2}}$	21
risch	$\frac{x^2(2x+1)}{\sqrt{x^2\left(2x+1\right)}}$	21
meijerg	$\frac{\sqrt{\pi}-\frac{\sqrt{\pi}\,(-8x+8)\sqrt{2x+1}}{8}}{\sqrt{\pi}}+\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{2x+1}}{2\sqrt{\pi}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x)/(2*x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] x^2*(2*x+1)/(2*x^3+x^2)^(1/2)
```

Maxima [A]
time = 0.27, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(2*x^3 + x^2)
```

Fricas [A]
time = 0.33, size = 11, normalized size = 0.85

$$\sqrt{2\,x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(2*x^3 + x^2)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.77

$$\sqrt{2x^3+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)
```

```
[Out] sqrt(2*x**3 + x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.
time = 3.33, size = 23, normalized size = 1.77

$$\frac{(2x+1)^{\frac{3}{2}} - \sqrt{2x+1}}{2\,\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((2*x + 1)^(3/2) - sqrt(2*x + 1))/sgn(x)
```

Mupad [B]
time = 3.23, size = 10, normalized size = 0.77

$$|x|\,\sqrt{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)
```

```
[Out] abs(x)*(2*x + 1)^(1/2)
```

3.607 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx$

Optimal. Leaf size=44

$$-\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)$$

```
[Out] -9/5*(1-5*x)^(1/3)+3/10*(1-5*x)^(2/3)+x+27/5*ln(3+(1-5*x)^(1/3))
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {442, 383, 78}

$$x+\frac{3}{10}(1-5x)^{2/3}-\frac{9}{5}\sqrt[3]{1-5x}+\frac{27}{5}\log\left(\sqrt[3]{1-5x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
```

```
[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)
^(1/3)])/5
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}}\,dx &= -\left(\frac{1}{5}\mathrm{Subst}\left(\int \frac{2+\sqrt[3]{x}}{3+\sqrt[3]{x}}\,dx, x, 1-5x\right)\right)\\
&= -\left(\frac{3}{5}\mathrm{Subst}\left(\int \frac{x^2(2+x)}{3+x}\,dx, x, \sqrt[3]{1-5x}\right)\right)\\
&= -\left(\frac{3}{5}\mathrm{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right)\,dx, x, \sqrt[3]{1-5x}\right)\right)\\
&= -\frac{9}{5}\sqrt[3]{1-5x}+\frac{3}{10}(1-5x)^{2/3}+x+\frac{27}{5}\log\left(3+\sqrt[3]{1-5x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.02

$$
\frac{1}{10}\left(-2-18\sqrt[3]{1-5x}+3(1-5x)^{2/3}+10x+54\log\left(3+\sqrt[3]{1-5x}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]
[Out] (-2 - 18*(1 - 5*x)^(1/3) + 3*(1 - 5*x)^(2/3) + 10*x + 54*Log[3 + (1 - 5*x)^
(1/3)])/10
```

Maple [A]
time = 0.32, size = 34, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
default	$-\frac{1}{5}+x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{27\ln\left(3+(1-5x)^{\frac{1}{3}}\right)}{5}$	34
trager	$x-\frac{9(1-5x)^{\frac{1}{3}}}{5}+\frac{3(1-5x)^{\frac{2}{3}}}{10}+\frac{9\ln\left(9(1-5x)^{\frac{2}{3}}+27(1-5x)^{\frac{1}{3}}-5x+28\right)}{5}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x,method=_RETURNVERBOSE)
[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.75

$$
x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

Fricas [A]
time = 0.33, size = 32, normalized size = 0.73

$$x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3)
```

Sympy [A]
time = 0.06, size = 39, normalized size = 0.89

$$x+\frac{3(1-5x)^{\frac{2}{3}}}{10}-\frac{9\sqrt[3]{1-5x}}{5}+\frac{27\log\left(\sqrt[3]{1-5x}+3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)
```

```
[Out] x + 3*(1 - 5*x)**(2/3)/10 - 9*(1 - 5*x)**(1/3)/5 + 27*log((1 - 5*x)**(1/3)
+ 3)/5
```

Giac [A]
time = 3.36, size = 33, normalized size = 0.75

$$x+\frac{3}{10}\,(-5\,x+1)^{\frac{2}{3}}-\frac{9}{5}\,(-5\,x+1)^{\frac{1}{3}}+\frac{27}{5}\,\log\left((-5\,x+1)^{\frac{1}{3}}+3\right)-\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")
```

```
[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3
) + 3) - 1/5
```

Mupad [B]
time = 0.11, size = 32, normalized size = 0.73

$$x+\frac{27\,\ln\left((1-5\,x)^{1/3}+3\right)}{5}-\frac{9\,(1-5\,x)^{1/3}}{5}+\frac{3\,(1-5\,x)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)
```

```
[Out] x + (27*log((1 - 5*x)^(1/3) + 3))/5 - (9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^
(2/3))/10
```

3.608 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx$

Optimal. Leaf size=21

$$4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)$$

```
[Out] x+4*ln(1-x^(1/2))+4*x^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {383, 78}

$$x+4\sqrt{x}+4\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} \, dx &= 2\text{Subst}\left(\int \frac{x(1+x)}{-1+x} \, dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right) \, dx,x,\sqrt{x}\right)\\
&= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 0.90

$$4\sqrt{x}+x+4\log\left(-1+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
[Out] 4*Sqrt[x] + x + 4*Log[-1 + Sqrt[x]]
```

Maple [A]
time = 0.33, size = 16, normalized size = 0.76

method	result	size
derivativedivides	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
default	$x+4\sqrt{x}+4\ln\left(-1+\sqrt{x}\right)$	16
trager	$x-2+4\sqrt{x}+2\ln\left(-1-x+2\sqrt{x}\right)$	22
meijerg	$2\sqrt{x}+4\ln\left(1-\sqrt{x}\right)+\frac{\sqrt{x}\,\left(3\sqrt{x}+6\right)}{3}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(-1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+4*x^(1/2)+4*ln(-1+x^(1/2))
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.71

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Fricas [A]
time = 0.33, size = 15, normalized size = 0.71

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

Giac [A]
time = 3.35, size = 16, normalized size = 0.76

$$x + 4\sqrt{x} + 4\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 3.01, size = 15, normalized size = 0.71

$$x + 4\ln\left(\sqrt{x} - 1\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)
[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)
```

3.609 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx$

Optimal. Leaf size=33

$$-x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)$$

```
[Out] -x-4/3*ln(1+(2+3*x)^(1/2))+4/3*(2+3*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {442, 383, 78}

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
```

```
[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx, x, 2+3x\right) \\
&= \frac{2}{3}\text{Subst}\left(\int \frac{(1-x)x}{1+x}\,dx, x, \sqrt{2+3x}\right) \\
&= \frac{2}{3}\text{Subst}\left(\int \left(2-x-\frac{2}{1+x}\right)dx, x, \sqrt{2+3x}\right) \\
&= -x+\frac{4}{3}\sqrt{2+3x}-\frac{4}{3}\log\left(1+\sqrt{2+3x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.03

$$\frac{1}{3}\left(-2-3x+4\sqrt{2+3x}-4\log\left(1+\sqrt{2+3x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]),x]
[Out] (-2 - 3*x + 4*Sqrt[2 + 3*x] - 4*Log[1 + Sqrt[2 + 3*x]])/3
```

Maple [A]
time = 0.27, size = 27, normalized size = 0.82

method	result	size
derivativedivides	$-\frac{2}{3}-x+\frac{4\sqrt{2+3x}}{3}-\frac{4\ln\left(1+\sqrt{2+3x}\right)}{3}$	27
default	$-\frac{2}{3}-x+\frac{4\sqrt{2+3x}}{3}-\frac{4\ln\left(1+\sqrt{2+3x}\right)}{3}$	27
trager	$-x+\frac{4\sqrt{2+3x}}{3}-\frac{2\ln\left(2\sqrt{2+3x}+3+3x\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2/3-x+4/3*(2+3*x)^(1/2)-4/3*ln(1+(2+3*x)^(1/2))
```

Maxima [A]
time = 0.27, size = 26, normalized size = 0.79

$$-x+\frac{4}{3}\sqrt{3x+2}-\frac{4}{3}\log\left(\sqrt{3x+2}+1\right)-\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.76

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log\left(\sqrt{3x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)
```

Sympy [A]
time = 0.06, size = 27, normalized size = 0.82

$$-x + \frac{4\sqrt{3x+2}}{3} - \frac{4\log\left(\sqrt{3x+2}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)
```

```
[Out] -x + 4*sqrt(3*x + 2)/3 - 4*log(sqrt(3*x + 2) + 1)/3
```

Giac [A]
time = 3.11, size = 26, normalized size = 0.79

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log\left(\sqrt{3x+2}+1\right) - \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3
```

Mupad [B]
time = 3.10, size = 25, normalized size = 0.76

$$\frac{4\sqrt{3x+2}}{3} - \frac{4\ln\left(\sqrt{3x+2}+1\right)}{3} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)
```

```
[Out] (4*(3*x + 2)^(1/2))/3 - (4*log((3*x + 2)^(1/2) + 1))/3 - x
```

3.610 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx$

Optimal. Leaf size=33

$$x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b}$$

```
[Out] x+4*ln(1+(b*x+a)^(1/2))/b-4*(b*x+a)^(1/2)/b
```

Rubi [A]

time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {442, 383, 78}

$$-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(\sqrt{a+bx}+1\right)}{b}+x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
```

```
[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}}\,dx &= \frac{\text{Subst}\left(\int \frac{-1+\sqrt{x}}{1+\sqrt{x}}\,dx, x, a+bx\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \frac{(-1+x)x}{1+x}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= \frac{2\text{Subst}\left(\int \left(-2+x+\frac{2}{1+x}\right)\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= x-\frac{4\sqrt{a+bx}}{b}+\frac{4\log\left(1+\sqrt{a+bx}\right)}{b}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 1.09

$$\frac{a+bx-4\sqrt{a+bx}+4\log\left(b\left(1+\sqrt{a+bx}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]
[Out] (a + b*x - 4*Sqrt[a + b*x] + 4*Log[b*(1 + Sqrt[a + b*x])])/b
```

Maple [A]
time = 0.31, size = 35, normalized size = 1.06

method	result	size
derivativedivides	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35
default	$\frac{bx+a-4\sqrt{bx+a}+4\ln\left(1+\sqrt{bx+a}\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/b*(1/2*b*x+1/2*a-2*(b*x+a)^(1/2)+2*ln(1+(b*x+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 30, normalized size = 0.91

$$\frac{bx+a-4\sqrt{bx+a}+4\log\left(\sqrt{bx+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*x + a - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Fricas [A]
time = 0.33, size = 29, normalized size = 0.88

$$\frac{bx - 4\sqrt{bx + a} + 4\log\left(\sqrt{bx + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b
```

Sympy [A]
time = 0.20, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a + bx}}{b} + \frac{4\log\left(\sqrt{a + bx} + 1\right)}{b} & \text{for } b \neq 0 \\ \frac{x\left(\sqrt{a} - 1\right)}{\sqrt{a} + 1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)),x)
```

```
[Out] Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (
x*(sqrt(a) - 1)/(sqrt(a) + 1), True))
```

Giac [A]
time = 3.41, size = 38, normalized size = 1.15

$$\frac{4\log\left(\sqrt{bx + a} + 1\right)}{b} + \frac{(bx + a)b - 4\sqrt{bx + a}\,b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2
```

Mupad [B]
time = 3.02, size = 29, normalized size = 0.88

$$x + \frac{4\ln\left(\sqrt{a + b\,x} + 1\right)}{b} - \frac{4\sqrt{a + b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1),x)
[Out] x + (4*log((a + b*x)^(1/2) + 1))/b - (4*(a + b*x)^(1/2))/b
```

3.611 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$

Optimal. Leaf size=10

$$\log(ax + bx^n)$$

```
[Out] ln(a*x+b*x^n)
```

Rubi [A]
time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1607, 528, 457, 78}

$$\log(ax^{1-n} + b) + n\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
```

```
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bnx^{-1+n}}{ax+bx^n}\,dx &= \int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx\\ &= \int \frac{bn+ax^{1-n}}{x\,(b+ax^{1-n})}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx, x, x^{1-n}\right)}{1-n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx, x, x^{1-n}\right)}{1-n}\\ &= n\log(x)+\log\left(b+ax^{1-n}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n),x]
[Out] Log[a*x + b*x^n]
```

Maple [A]
time = 0.32, size = 12, normalized size = 1.20

method	result	size
risch	$\ln\left(x^n+\frac{ax}{b}\right)$	12
norman	$\ln\left(ax+b\,\mathrm{e}^{n\ln(x)}\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(-1+n))/(a*x+b*x^n),x,method=_RETURNVERBOSE)
[Out] ln(x^n+a*x/b)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 1.00

$$\log\left(ax+bx^n\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] log(a*x + b*x^n)
```

Fricas [A]
time = 0.34, size = 10, normalized size = 1.00

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] log(a*x + b*x^n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(8) = 16.
time = 1.67, size = 36, normalized size = 3.60

$$\begin{cases} \log\left(x + \frac{bx^n}{a}\right) & \text{for}\ a \neq 0 \\ n\left(-\frac{n\log(x^{-n})}{n^2-n} + \frac{\log(x^{-n})}{n^2-n}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(a*x+b*x**n),x)
```

```
[Out] Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(-n*log(x**(-n))/(n**2 - n) + l
og(x**(-n))/(n**2 - n)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)
```

Mupad [B]
time = 3.27, size = 10, normalized size = 1.00

$$\ln(b\,x^n + a\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(b*x^n + a*x),x)
```

```
[Out] log(b*x^n + a*x)
```

3.612 $\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx$

Optimal. Leaf size=17

$$n\log(x)+\log\left(b+ax^{1-n}\right)$$

```
[Out] n*ln(x)+ln(b+a*x^(1-n))
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {528, 457, 78}

$$\log\left(ax^{1-n}+b\right)+n\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] n*Log[x] + Log[b + a*x^(1 - n)]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}}\,dx &= \int \frac{bn+ax^{1-n}}{x\,(b+ax^{1-n})}\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{bn+ax}{x(b+ax)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{n}{x}+\frac{a-an}{b+ax}\right)\,dx, x, x^{1-n}\right)}{1-n} \\
&= n\log(x)+\log\left(b+ax^{1-n}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.59

$$\log\left(ax+bx^n\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]
```

```
[Out] Log[a*x + b*x^n]
```

Maple [A]
time = 0.33, size = 12, normalized size = 0.71

method	result	size
risch	$\ln\left(x^n+\frac{ax}{b}\right)$	12
norman	$\ln\left(ax+b\,e^{n\ln(x)}\right)$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x^n+a*x/b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(17) = 34.
time = 0.29, size = 86, normalized size = 5.06

$$bn\left(\frac{\log(x)}{b}-\frac{n\log(x)}{b(n-1)}+\frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)}\right)+a\left(\frac{n\log(x)}{a(n-1)}-\frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")
```

```
[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) +
a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))
```

Fricas [A]
time = 0.37, size = 10, normalized size = 0.59

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")
[Out] log(a*x + b*x^n)
```

Sympy [A]
time = 31.22, size = 8, normalized size = 0.47

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)),x)
[Out] log(a*x + b*x**n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")
[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)
```

Mupad [B]
time = 3.37, size = 39, normalized size = 2.29

$$-\frac{\ln\left(b + a\,x^{1-n}\right) - 2\,n\,\mathrm{atanh}\left(\frac{2\,a\,x^{1-n}}{b} + 1\right)}{n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))),x)
[Out] -(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

3.613 $\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3$

Optimal. Leaf size=37

$$x^2(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}$$

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A]
time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1604}

$$x^2(a+bx+cx^2)^{m+1}(d+ex+fx^2+gx^3)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
 c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2cdm$$

Mathematica [A]
time = 4.04, size = 34, normalized size = 0.92

$$x^2(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
 + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A]
time = 0.44, size = 38, normalized size = 1.03

method	result
gosper	$x^{2} \left(c\,x^{2}+b x +a \right)^{1+m} \left(g\,x^{3}+f\,x^{2}+e x +d \right)^{1+n}$
risch	$x^{2} \left(c g\,x^{5}+b g\,x^{4}+c f\,x^{4}+a g\,x^{3}+b f\,x^{3}+c e\,x^{3}+a f\,x^{2}+b e\,x^{2}+c d\,x^{2}+a e x +b d x +a d \right) \left(c\,x^{2}\right.$ -

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

```
[Out] x^2*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(38) = 76.
time = 0.39, size = 101, normalized size = 2.73

$$\left(cgx^{7}+(cf+bg)x^{6}+(bf+ag+ce)x^{5}+(cd+af+be)x^{4}+adx^{2}+(bd+ae)x^{3}\right)e^{\left(n\log\left(gx^{3}+fx^{2}+xe+d\right)+m\log\left(cx^{2}+bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (b*f + a*g + c*e)*x^5 + (c*d + a*f + b*e)*x^4
 + a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + x*e + d) + m*log(c*x^
2 + b*x + a))
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+
3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*
n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*
m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g +
 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*d
+ b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n) +
x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) +
 c*g*x^5*(2*m + 3*n + 7)),x)
```

```
[Out] \text{Hanged}
```

3.614 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd$

Optimal. Leaf size=35

$$x\left(a+bx+cx^2\right)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A]
time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = $\{1604\}$

$$x\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e +
 b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*
f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a
*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g
*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m \left(d+ex+fx^2+gx^3\right)^n \left(ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm+b\right.$$

Mathematica [A]
time = 10.25, size = 32, normalized size = 0.91

$$x(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2
*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n
+ 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
 + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4
 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [**A**]
time = 0.43, size = 36, normalized size = 1.03

method	result
gosper	$x(c\,x^2+bx+a)^{1+m}\,(g\,x^3+f\,x^2+ex+d)^{1+n}$
risch	$x(cg\,x^5+bg\,x^4+cf\,x^4+ag\,x^3+bf\,x^3+ce\,x^3+af\,x^2+be\,x^2+cd\,x^2+aex+bdx+ad)\,(c\,x^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(36) = 72.
time = 0.38, size = 99, normalized size = 2.83

$$(cgx^6+(cf+bg)x^5+(bf+ag+ce)x^4+(cd+af+be)x^3+adx+(bd+ae)x^2)e^{(n\log(gx^3+fx^2+xe+d)+m\log(cx^2+bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (b*f + a*g + c*e)*x^4 + (c*d + a*f + b*e)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + x*e + d) + m*log(c*x^2
+ b*x + a))
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F(-1)**]
time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*c
*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d + b*
e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x^3*
(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g
*x^5*(2*m + 3*n + 6)),x)
```

```
[Out] \text{Hanged}
```

3.615 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+$

Optimal. Leaf size=34

$$\left(a+bx+cx^2\right)^{1+m}\left(d+ex+fx^2+gx^3\right)^{1+n}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Rubi [A]
time = 0.08, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1604}

$$\left(a+bx+cx^2\right)^{m+1}\left(d+ex+fx^2+gx^3\right)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*
e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*
e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c
*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)
*x^4),x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1604

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+ben\right.$$

Mathematica [A]
time = 7.17, size = 31, normalized size = 0.91

$$(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*
m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x +
 (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2
+ (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m
+ 3*n)*x^4),x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

Maple [A]
time = 0.42, size = 35, normalized size = 1.03

method	result
gosper	$\left(c\,x^{2}+bx+a\right)^{1+m} \left(g\,x^{3}+f\,x^{2}+ex+d\right)^{1+n}$
risch	$\left(cg\,x^{5}+bg\,x^{4}+cf\,x^{4}+ag\,x^{3}+bf\,x^{3}+ce\,x^{3}+af\,x^{2}+be\,x^{2}+cd\,x^{2}+aex+bdx+ad\right) \left(c\,x^{2}+b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(35) = 70$.
time = 0.38, size = 96, normalized size = 2.82

$$(cgx^5 + (cf + bg)x^4 + (bf + ag + ce)x^3 + (cd + af + be)x^2 + ad + (bd + ae)x)e^{(n\log(gx^3+fx^2+xe+d)+m\log(cx^2+bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (b*f + a*g + c*e)*x^3 + (c*d + a*f + b*e)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + x*e + d) + m*log(c*x^2 + b*
x + a))
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c
*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c
*f)*x**3+c*g*(5+2*m+3*n)*x**4),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.78, size = 148, normalized size = 4.35

$$(g x^3+f x^2+e x+d)^n \left(x^4 (b g+c f) \left(c x^2+b x+a\right)^m+x^2 \left(c x^2+b x+a\right)^m (a f+b e+c d)+x^3 \left(c x^2+b x+a\right)^m (a g+b f+c e)+a d \left(c x^2+b x+a\right)^m+x (a e+b d) \left(c x^2+b x+a\right)^m+c g x^5 \left(c x^2+b x+a\right)^m\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*g
 + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*c*d
 + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m + 3*n
+ 5)),x)
```

```
[Out] (d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a +
 b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f + c*
e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x^5*
(a + b*x + c*x^2)^m)
```

3.616 $\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{1+m}(d+ex+fx^2+gx^3)^{1+n}}{x}$$

[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x

Rubi [F]
time = 2.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfn+3agn)x^3+(3cf+3bg+2cfm+bgm+2cfn+3bgn)x^4+cg(4+2m+3n)x^5)}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x^5))/x^2,x]

[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (b*d*m + a*e*n)*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x, x] + (c*e*(2 + 2*m + n) + b*f*(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3*n)*Defer[Int][x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]

Rubi steps

$$\int \frac{(a+bx+cx^2)^m(d+ex+fx^2+gx^3)^n(-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2a$$

Mathematica [A]
time = 9.06, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m
 + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (
2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m +
3*n)*x^5))/x^2,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x
```

Maple [A]
time = 0.40, size = 38, normalized size = 1.03

method	result	size
gosper	$\frac{\left(c\,x^{2}+b x+a\right)^{1+m} \left(g\,x^{3}+f\,x^{2}+e x+d\right)^{1+n}}{x}$	38
risch	$\frac{\left(c g\,x^{5}+b g\,x^{4}+c f\,x^{4}+a g\,x^{3}+b f\,x^{3}+c e\,x^{3}+a f\,x^{2}+b e\,x^{2}+c d\,x^{2}+a e x+b d x+a d\right) \left(c\,x^{2}+b x+a\right)^{m} \left(g\,x^{3}+f\,x^{2}+e x+d\right)^{n}}{x}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a
*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+
2*m+3*n)*x^5)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(38) = 76$.
time = 0.38, size = 99, normalized size = 2.68

$$\frac{(cgx^5 + (cf + bg)x^4 + (bf + ag + ce)x^3 + (cd + af + be)x^2 + ad + (bd + ae)x)e^{\left(n\log\left(gx^3+fx^2+xe+d\right)+m\log\left(cx^2+bx+a\right)\right)}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (b*f + a*g + c*e)*x^3 + (c*d + a*f + b*e)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + x*e + d) + m*log(c*x^2 + b*
x + a))/x
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f
)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.68, size = 37, normalized size = 1.00

$$\frac{\left(c\,x^2+b\,x+a\right)^{m+1}\,\left(g\,x^3+f\,x^2+e\,x+d\right)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m +
 2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 4))
)/x^2,x)
```

```
[Out] ((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

3.617 $\int \frac{\left(a+bx+cx^2\right)^m\left(d+ex+fx^2+gx^3\right)^n\left(-2ad+(-bd-ae+bdm+aen)x+(2\right.}{}$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{1+m}\,(d+ex+fx^2+gx^3)^{1+n}}{x^2}$$

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Rubi [F]
time = 2.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx+cx^2)^m\,(d+ex+fx^2+gx^3)^n\,(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfn+3agn)x^3+(2cf+2bg+2cfm+bgm+2cfn+3bgn)x^4+cg(3+2m+3n)x^5)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d) - a*
e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f
 + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g
+ 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3,
x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x
 + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][((a + b*x +
c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))
*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x^2, x] + (2*
c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][((a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int]
[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n
)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{(a+bx+cx^2)^m\,(d+ex+fx^2+gx^3)^n\,(-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2af}{}$$

Mathematica [A]
time = 9.51, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{1+m}(d+x(e+x(f+gx)))^{1+n}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-(b*d
) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e
 + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f +
2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5)
)/x^3,x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

Maple [A]
time = 0.39, size = 38, normalized size = 1.03

method	result	size
gosper	$\frac{\left(c\,x^{2}+b x+a\right)^{1+m} \left(g\,x^{3}+f\,x^{2}+e x+d\right)^{1+n}}{x^{2}}$	38
risch	$\frac{\left(cg\,x^{5}+bg\,x^{4}+cf\,x^{4}+ag\,x^{3}+bf\,x^{3}+ce\,x^{3}+af\,x^{2}+be\,x^{2}+cd\,x^{2}+aex+bdx+ad\right) \left(c\,x^{2}+bx+a\right)^{m} \left(g\,x^{3}+f\,x^{2}+ex+d\right)^{n}}{x^{2}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1+m)*(g*x^3+f*x^2+e*x+d)^(1+n)/x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(38) = 76$.
time = 0.39, size = 99, normalized size = 2.68

$$\frac{(cgx^5 + (cf + bg)x^4 + (bf + ag + ce)x^3 + (cd + af + be)x^2 + ad + (bd + ae)x)e^{(n\log(gx^3+fx^2+xe+d)+m\log(cx^2+bx+a))}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (b*f + a*g + c*e)*x^3 + (c*d + a*f + b*e)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + x*e + d) + m*log(c*x^2 + b*
x + a))/x^2
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.22, size = 146, normalized size = 3.95

$$(c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n\,(a\,f+b\,e+c\,d+c\,g\,x^3+a\,g\,x+b\,f\,x+c\,e\,x+b\,g\,x^2+c\,f\,x^2)+\frac{(a\,e+b\,d)\,(c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n}{x}+\frac{a\,d\,(c\,x^2+b\,x+a)^m\,(g\,x^3+f\,x^2+e\,x+d)^n}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e*n
) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f*m
+ 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
+ a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*x^
2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x^2
```

3.618 $\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=185

$$-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2(3a^2-b^2c)\,(c+dx)^2}{2d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-\frac{c(a^2-b^2c)\,(c+dx)^3}{d^4}-\frac{12abc(c+d}{7d^4}$$

```
[Out] -a^2*c^3*x/d^3-4/3*a*b*c^3*(d*x+c)^(3/2)/d^4+1/2*c^2*(-b^2*c+3*a^2)*(d*x+c)
^2/d^4+12/5*a*b*c^2*(d*x+c)^(5/2)/d^4-c*(-b^2*c+a^2)*(d*x+c)^3/d^4-12/7*a*b
*c*(d*x+c)^(7/2)/d^4+1/4*(-3*b^2*c+a^2)*(d*x+c)^4/d^4+4/9*a*b*(d*x+c)^(9/2)
/d^4+1/5*b^2*(d*x+c)^5/d^4
```

Rubi [A]

time = 0.18, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$\frac{c^2(3a^2-b^2c)\,(c+dx)^2}{2d^4}+\frac{(a^2-3b^2c)\,(c+dx)^4}{4d^4}-\frac{c(a^2-b^2c)\,(c+dx)^3}{d^4}-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}+\frac{4ab(c+dx)^{9/2}}{9d^4}-\frac{12abc(c+dx)^{7/2}}{7d^4}+\frac{b^2(c+dx)^5}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^
2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2
- b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*
b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d
*x)^5)/(5*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^2\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\text{Subst}\left(\int\left(-a^2c^3x-2abc^3x^2-c^2\left(-3a^2+b^2c\right)x^3+6abc^2x^4+3c\left(-a^2+b^2c\right)\right.\right.}{d^4}$$
$$=-\frac{a^2c^3x}{d^3}-\frac{4abc^3(c+dx)^{3/2}}{3d^4}+\frac{c^2\left(3a^2-b^2c\right)(c+dx)^2}{2d^4}+\frac{12abc^2(c+dx)^{5/2}}{5d^4}-$$

Mathematica [A]
time = 0.08, size = 105, normalized size = 0.57

$$-\frac{16ab\sqrt{c+dx}\left(16c^4-8c^3dx+6c^2d^2x^2-5cd^3x^3-35d^4x^4\right)+315a^2\left(c^4-d^4x^4\right)+63b^2\left(c^5-5cd^4x^4-4d^5x^5\right)}{1260d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]
[Out] -1/1260*(16*a*b*Sqrt[c + d*x]*(16*c^4 - 8*c^3*d*x + 6*c^2*d^2*x^2 - 5*c*d^3
*x^3 - 35*d^4*x^4) + 315*a^2*(c^4 - d^4*x^4) + 63*b^2*(c^5 - 5*c*d^4*x^4 -
4*d^5*x^5))/d^4
```

Maple [A]
time = 0.36, size = 78, normalized size = 0.42

method	result
default	$b^2\left(\frac{1}{5}d\,x^5+\frac{1}{4}c\,x^4\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{9}{2}}}{9}-\frac{3c(dx+c)^{\frac{7}{2}}}{7}+\frac{3c^2(dx+c)^{\frac{5}{2}}}{5}-\frac{c^3(dx+c)^{\frac{3}{2}}}{3}\right)}{d^4}+\frac{a^2x^4}{4}$
trager	$\frac{\left(4b^2dx+5b^2c+5a^2\right)x^4}{20}-\frac{4ab\left(-35d^4x^4-5c\,d^3x^3+6c^2x^2d^2-8c^3dx+16c^4\right)\sqrt{dx+c}}{315d^4}$
derivativedivides	$\frac{\frac{b^2(dx+c)^5}{5}+\frac{4ab(dx+c)^{\frac{9}{2}}}{9}+\frac{\left(-3b^2c+a^2\right)(dx+c)^4}{4}-\frac{12cab(dx+c)^{\frac{7}{2}}}{7}+\frac{\left(3b^2c^2-3a^2c\right)(dx+c)^3}{3}+\frac{12c^2ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^2c^3+3a^2c^2\right)(}{2}}{d^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+
3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4
```

Maxima [A]
time = 0.26, size = 151, normalized size = 0.82

$$\frac{252\,(dx+c)^5b^2+560\,(dx+c)^{\frac{9}{2}}ab-2160\,(dx+c)^{\frac{7}{2}}abc+3024\,(dx+c)^{\frac{5}{2}}abc^2-1680\,(dx+c)^{\frac{3}{2}}abc^3-1260\,(dx+c)a^2c^3-315\,(3\,b^2c-a^2)(dx+c)^4+1260\,(b^2c^2-a^2c)(dx+c)^3-630\,(b^2c^3-3\,a^2c^2)(dx+c)^2}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2
)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 126
0*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2
*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4
```

Fricas [A]
time = 0.39, size = 94, normalized size = 0.51

$$\frac{252\,b^2d^5x^5+315\,(b^2c+a^2)d^4x^4+16\,(35\,abd^4x^4+5\,abcd^3x^3-6\,abc^2d^2x^2+8\,abc^3dx-16\,abc^4)\sqrt{dx+c}}{1260\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 +
5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x
+ c))/d^4
```

Sympy [A]
time = 3.73, size = 139, normalized size = 0.75

$$\begin{cases} \frac{\frac{a^2dx^4}{4}+\frac{4ab\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{2b^2\left(-\frac{c^3(c+dx)^2}{4}+\frac{c^2(c+dx)^3}{2}-\frac{3c(c+dx)^4}{8}+\frac{(c+dx)^5}{10}\right)}{d^3}}{d} & \text{for}\ d\neq 0 \\ \frac{x^4\left(a+b\sqrt{c}\right)^2}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d
*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**2*(
-c**3*(c + d*x)**2/4 + c**2*(c + d*x)**3/2 - 3*c*(c + d*x)**4/8 + (c + d*x)
**5/10)/d**3)/d, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))
```

Giac [A]
time = 4.63, size = 151, normalized size = 0.82

$$\frac{252\,b^2d^2x^5+315\,b^2cdx^4+315\,a^2dx^4+\frac{144\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)abc}{d^3}+\frac{16\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab}{d^3}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x + c
)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*
c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x
+ c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d^3)/
d
```

Mupad [B]
time = 3.37, size = 167, normalized size = 0.90

$$\frac{b^2\,{\left(c+d\,x\right)}^5}{5\,d^4}-\frac{\left(6\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^4}{8\,d^4}+\frac{\left(6\,a^2\,c^2-2\,b^2\,c^3\right)\,{\left(c+d\,x\right)}^2}{4\,d^4}-\frac{a^2\,c^3\,x}{d^3}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{c\,\left(b^2\,c-a^2\right)\,{\left(c+d\,x\right)}^3}{d^4}-\frac{4\,a\,b\,c^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{12\,a\,b\,c^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}-\frac{12\,a\,b\,c\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*a
^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c + d*
x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c + d*x
)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c + d*
x)^(7/2))/(7*d^4)
```

3.619 $\int x^2\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=138

$$\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c(2a^2-b^2c)\,(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{(a^2-2b^2c)\,(c+dx)^3}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}+$$

```
[Out] a^2*c^2*x/d^2+4/3*a*b*c^2*(d*x+c)^(3/2)/d^3-1/2*c*(-b^2*c+2*a^2)*(d*x+c)^2/
d^3-8/5*a*b*c*(d*x+c)^(5/2)/d^3+1/3*(-2*b^2*c+a^2)*(d*x+c)^3/d^3+4/7*a*b*(d
*x+c)^(7/2)/d^3+1/4*b^2*(d*x+c)^4/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$\frac{(a^2-2b^2c)\,(c+dx)^3}{3d^3}-\frac{c(2a^2-b^2c)\,(c+dx)^2}{2d^3}+\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}+\frac{4ab(c+dx)^{7/2}}{7d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{b^2(c+dx)^4}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*
(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)
*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)
/(4*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\left(a+b\sqrt{c+dx}\right)^2dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^2(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$
$$=\frac{2\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$=\frac{2\text{Subst}\left(\int\left(a^2c^2x+2abc^2x^2+c\left(-2a^2+b^2c\right)x^3-4abcx^4+\left(a^2-2b^2c\right)x^5+2\right.\right.}{d^3}$$
$$=\frac{a^2c^2x}{d^2}+\frac{4abc^2(c+dx)^{3/2}}{3d^3}-\frac{c\left(2a^2-b^2c\right)(c+dx)^2}{2d^3}-\frac{8abc(c+dx)^{5/2}}{5d^3}+\frac{(a^2-}{}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.67

$$\frac{140a^2(c^3+d^3x^3)+16ab\sqrt{c+dx}\,(8c^3-4c^2dx+3cd^2x^2+15d^3x^3)+35b^2(c^4+4cd^3x^3+3d^4x^4)}{420d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (140*a^2*(c^3 + d^3*x^3) + 16*a*b*Sqrt[c + d*x]*(8*c^3 - 4*c^2*d*x + 3*c*d^
2*x^2 + 15*d^3*x^3) + 35*b^2*(c^4 + 4*c*d^3*x^3 + 3*d^4*x^4))/(420*d^3)
```

Maple [A]
time = 0.30, size = 66, normalized size = 0.48

method	result	siz
default	$b^2\left(\frac{1}{4}d\,x^4+\frac{1}{3}c\,x^3\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{7}{2}}}{7}-\frac{2c(dx+c)^{\frac{5}{2}}}{5}+\frac{c^2(dx+c)^{\frac{3}{2}}}{3}\right)}{d^3}+\frac{a^2x^3}{3}$	66
trager	$\frac{(3b^2dx+4b^2c+4a^2)x^3}{12}+\frac{4ab(15d^3x^3+3c\,d^2x^2-4c^2dx+8c^3)\sqrt{dx+c}}{105d^3}$	70
derivativedivides	$\frac{\frac{b^2(dx+c)^4}{4}+\frac{4ab(dx+c)^{\frac{7}{2}}}{7}+\frac{\left(-2b^2c+a^2\right)(dx+c)^3}{3}-\frac{8cab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(b^2c^2-2a^2c\right)(dx+c)^2}{2}+\frac{4c^2ab(dx+c)^{\frac{3}{2}}}{3}+c^2a^2(dx+c)}{d^3}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/4*d*x^4+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^(7/2)-2/5*c*(d*x+c)^(5/2)+
1/3*c^2*(d*x+c)^(3/2))+1/3*a^2*x^3
```

Maxima [A]
time = 0.27, size = 112, normalized size = 0.81

$$\frac{105\,(dx+c)^4b^2+240\,(dx+c)^{\frac{7}{2}}ab-672\,(dx+c)^{\frac{5}{2}}abc+560\,(dx+c)^{\frac{3}{2}}abc^2+420\,(dx+c)a^2c^2-140\,(2\,b^2c-a^2)(dx+c)^3+210\,(b^2c^2-2\,a^2c)(dx+c)^2}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^(7/2)*a*b - 672*(d*x + c)^(5/2)*
a*b*c + 560*(d*x + c)^(3/2)*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c
- a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3
```

Fricas [A]
time = 0.38, size = 81, normalized size = 0.59

$$\frac{105\,b^2d^4x^4+140\,(b^2c+a^2)d^3x^3+16\,(15\,abd^3x^3+3\,abcd^2x^2-4\,abc^2dx+8\,abc^3)\sqrt{dx+c}}{420\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3
*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3
```

Sympy [A]
time = 3.20, size = 110, normalized size = 0.80

$$\begin{cases} \frac{\frac{a^2dx^3}{3}+\frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{2b^2\left(\frac{c^2(c+dx)^2}{4}-\frac{c(c+dx)^3}{3}+\frac{(c+dx)^4}{8}\right)}{d^2}}{d} & \text{for}\ d\neq 0 \\ \frac{x^3\left(a+b\sqrt{c}\right)^2}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((a**2*d*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)*
*(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*(c**2*(c + d*x)**2/4 - c*(c +
d*x)**3/3 + (c + d*x)**4/8)/d**2)/d, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3,
 True))
```

Giac [A]
time = 5.57, size = 127, normalized size = 0.92

$$\frac{105\,b^2d^2x^4+140\,b^2cdx^3+140\,a^2dx^3+\frac{112\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)abc}{d^2}+\frac{48\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab}{d^2}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(d*
x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)*a*b/d^2)/d
```

Mupad [B]
time = 0.06, size = 124, normalized size = 0.90

$$\frac{b^2\,{\left(c+d\,x\right)}^4}{4\,d^3}-\frac{\left(4\,a^2\,c-2\,b^2\,c^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^3}-\frac{\left(4\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^3}{6\,d^3}+\frac{a^2\,c^2\,x}{d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{7/2}}{7\,d^3}+\frac{4\,a\,b\,c^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^3}-\frac{8\,a\,b\,c\,{\left(c+d\,x\right)}^{5/2}}{5\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) - (
(4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*x)
^(7/2))/(7*d^3) + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (8*a*b*c*(c + d*x)^
(5/2))/(5*d^3)
```

3.620 $\int x\left(a+b\sqrt{c+dx}\right)^2\,dx$

Optimal. Leaf size=89

$$-\frac{a^2cx}{d}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}+\frac{4ab(c+dx)^{5/2}}{5d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

```
[Out] -a^2*c*x/d-4/3*a*b*c*(d*x+c)^(3/2)/d^2+1/2*(-b^2*c+a^2)*(d*x+c)^2/d^2+4/5*a
*b*(d*x+c)^(5/2)/d^2+1/3*b^2*(d*x+c)^3/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$\frac{(a^2-b^2c)\,(c+dx)^2}{2d^2}-\frac{a^2cx}{d}+\frac{4ab(c+dx)^{5/2}}{5d^2}-\frac{4abc(c+dx)^{3/2}}{3d^2}+\frac{b^2(c+dx)^3}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*
x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^2\,dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2(-c+x)\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int x(a+bx)^2\left(-c+x^2\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \left(-a^2cx-2abcx^2+\left(a^2-b^2c\right)x^3+2abx^4+b^2x^5\right)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{a^2cx}{d} - \frac{4abc(c+dx)^{3/2}}{3d^2} + \frac{\left(a^2-b^2c\right)(c+dx)^2}{2d^2} + \frac{4ab(c+dx)^{5/2}}{5d^2} + \frac{b^2(c+d}{3d^2}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.76

$$\frac{(c+dx)\left(-15a^2(c-dx)-8ab(2c-3dx)\sqrt{c+dx}+5b^2\left(-c^2+cdx+2d^2x^2\right)\right)}{30d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((c + d*x)*(-15*a^2*(c - d*x) - 8*a*b*(2*c - 3*d*x)*Sqrt[c + d*x] + 5*b^2*(
-c^2 + c*d*x + 2*d^2*x^2)))/(30*d^2)
```

Maple [A]
time = 0.29, size = 54, normalized size = 0.61

method	result	size
default	$b^2\left(\frac{1}{3}d\,x^3+\frac{1}{2}c\,x^2\right)+\frac{4ab\left(\frac{(dx+c)^{\frac{5}{2}}}{5}-\frac{c(dx+c)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{a^2x^2}{2}$	54
trager	$\frac{\left(2b^2dx+3b^2c+3a^2\right)x^2}{6}-\frac{4ab\left(-3d^2x^2-cdx+2c^2\right)\sqrt{dx+c}}{15d^2}$	59
derivativedivides	$\frac{\frac{b^2(dx+c)^3}{3}+\frac{4ab(dx+c)^{\frac{5}{2}}}{5}+\frac{\left(-b^2c+a^2\right)(dx+c)^2}{2}-\frac{4cab(dx+c)^{\frac{3}{2}}}{3}-c\,a^2(dx+c)}{d^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*c*(d*x+c)^(3/2))
+1/2*a^2*x^2
```

Maxima [A]
time = 0.28, size = 72, normalized size = 0.81

$$\frac{10\,(dx+c)^3b^2+24\,(dx+c)^{\frac{5}{2}}ab-40\,(dx+c)^{\frac{3}{2}}abc-30\,(dx+c)a^2c-15\,(b^2c-a^2)(dx+c)^2}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*
c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2
```

Fricas [A]
time = 0.51, size = 67, normalized size = 0.75

$$\frac{10\,b^2d^3x^3+15\,(b^2c+a^2)d^2x^2+8\,(3\,abd^2x^2+abcdx-2\,abc^2)\sqrt{dx+c}}{30\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*
d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2
```

Sympy [A]
time = 2.49, size = 94, normalized size = 1.06

$$\begin{cases} \frac{\frac{2a^2\left(-\frac{c(c+dx)}{2}+\frac{(c+dx)^2}{4}\right)}{d}+\frac{4ab\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{2b^2\left(-\frac{c(c+dx)^2}{4}+\frac{(c+dx)^3}{6}\right)}{d}}{d} & \text{for } d\neq 0 \\ \frac{x^2\left(a+b\sqrt{c}\right)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise(((2*a**2*(-c*(c + d*x)/2 + (c + d*x)**2/4)/d + 4*a*b*(-c*(c + d*x
)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*b**2*(-c*(c + d*x)**2/4 + (c + d*x)*
*3/6)/d)/d, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))
```

Giac [A]
time = 4.24, size = 131, normalized size = 1.47

$$\frac{10\,b^2d^2x^3+\frac{40\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)abc}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)b^2c}{d}+\frac{15\left((dx+c)^2-2\,(dx+c)c\right)a^2}{d}+\frac{8\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ab}{d}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 1
5*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*
a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b/d)/d
```

Mupad [B]
time = 0.03, size = 79, normalized size = 0.89

$$\frac{b^2\,{\left(c+d\,x\right)}^3}{3\,d^2}-\frac{\left(2\,b^2\,c-2\,a^2\right)\,{\left(c+d\,x\right)}^2}{4\,d^2}+\frac{4\,a\,b\,{\left(c+d\,x\right)}^{5/2}}{5\,d^2}-\frac{a^2\,c\,x}{d}-\frac{4\,a\,b\,c\,{\left(c+d\,x\right)}^{3/2}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a*
b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2
)
```

3.621 $\int \left(a+b\sqrt{c+dx}\right)^2 \, dx$

Optimal. Leaf size=41

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

```
[Out] a^2*x+4/3*a*b*(d*x+c)^(3/2)/d+1/2*b^2*(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$a^2x+\frac{4ab(c+dx)^{3/2}}{3d}+\frac{b^2(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \left(a+b\sqrt{c+dx}\right)^2 dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^2 dx, x, c+dx\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int x(a+bx)^2 dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right) dx, x, \sqrt{c+dx}\right)}{d}$$
$$= a^2x + \frac{4ab(c+dx)^{3/2}}{3d} + \frac{b^2(c+dx)^2}{2d}$$

Mathematica [A]
time = 0.02, size = 41, normalized size = 1.00

$$\frac{(c+dx)\left(6a^2+8ab\sqrt{c+dx}+3b^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]
[Out] ((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)
```

Maple [A]
time = 0.26, size = 35, normalized size = 0.85

method	result	size
default	$b^2\left(cx+\frac{1}{2}d\,x^2\right)+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}+a^2x$	35
trager	$\frac{\left(b^2dx+2b^2c+2a^2\right)x}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3d}$	37
derivativedivides	$\frac{\frac{b^2(dx+c)^2}{2}+\frac{4ab(dx+c)^{\frac{3}{2}}}{3}+a^2(dx+c)}{d}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] b^2*(c*x+1/2*d*x^2)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.85

$$\frac{1}{2}\left(dx^2+2\,cx\right)b^2+a^2x+\frac{4\,(dx+c)^{\frac{3}{2}}ab}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d
```

Fricas [A]
time = 0.37, size = 49, normalized size = 1.20

$$\frac{3\,b^2d^2x^2+6\,(b^2c+a^2)dx+8\,(abdx+abc)\sqrt{dx+c}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c
))/d
```

Sympy [A]
time = 0.07, size = 68, normalized size = 1.66

$$\begin{cases} a^2x+\frac{4abc\sqrt{c+dx}}{3d}+\frac{4abx\sqrt{c+dx}}{3}+b^2cx+\frac{b^2dx^2}{2} & \text{for } d\neq 0\\ x\left(a+b\sqrt{c}\right)^2 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 +
 b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(35) = 70.
time = 4.09, size = 82, normalized size = 2.00

$$\frac{6\,(dx+c)b^2c+24\,\sqrt{dx+c}\,abc+6\,(dx+c)a^2+8\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ab+3\,\left((dx+c)^2-2\,(dx+c)c\right)b^2}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*x
 + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^2)
/d
```

Mupad [B]
time = 0.05, size = 36, normalized size = 0.88

$$\frac{3\,b^2\,{\left(c+d\,x\right)}^2+8\,a\,b\,{\left(c+d\,x\right)}^{3/2}+6\,a^2\,d\,x}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2,x)
[Out] (3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)
```

3.622 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x} dx$

Optimal. Leaf size=57

$$b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)$$

```
[Out] b^2*d*x+(b^2*c+a^2)*ln(x)-4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*
b*(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {378, 1412, 815, 649, 213, 266}

$$\log(x)\left(a^2+b^2c\right)+4ab\sqrt{c+dx}-4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+b^2dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]
] + (a^2 + b^2*c)*Log[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{-c+x}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x(a+bx)^2}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(2ab+b^2x+\frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\right)dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+2\text{Subst}\left(\int \frac{2abc+\left(a^2+b^2c\right)x}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= b^2dx+4ab\sqrt{c+dx}+(4abc)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)+\left(2\left(a^2+b^2\right.\right.\\
&= b^2dx+4ab\sqrt{c+dx}-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(x)
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 1.11

$$b\left(bc+bdx+4a\sqrt{c+dx}\right)-4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\left(a^2+b^2c\right)\log(-dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]
```

```
[Out] b*(b*c + b*d*x + 4*a*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/S
qrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]
```

Maple [A]
time = 0.30, size = 51, normalized size = 0.89

method	result	siz
default	$b^{2}\left(dx+c\ln\left(x\right)\right)+2ab\left(2\sqrt{dx+c}\,-2\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\right)+a^{2}\ln\left(x\right)$	51
derivativedivides	$\left(dx+c\right)b^{2}+4ab\sqrt{dx+c}\,-\left(-b^{2}c-a^{2}\right)\ln\left(-dx\right)-4ab\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)\sqrt{c}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(d*x+c*ln(x))+2*a*b*(2*(d*x+c)^(1/2)-2*c^(1/2)*arctanh((d*x+c)^(1/2)/c^
(1/2)))+a^2*ln(x)
```

Maxima [A]
time = 0.51, size = 70, normalized size = 1.23

$$2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}\,-\sqrt{c}}{\sqrt{dx+c}\,+\sqrt{c}}\right)+(dx+c)b^{2}+4\,\sqrt{dx+c}\,ab+\left(b^{2}c+a^{2}\right)\log\left(dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] 2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + (d
*x + c)*b^2 + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Fricas [A]
time = 0.34, size = 118, normalized size = 2.07

$$\left[b^{2}dx+2\,ab\sqrt{c}\,\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}\,+2\,c}{x}\right)+4\,\sqrt{dx+c}\,ab+\left(b^{2}c+a^{2}\right)\log\left(x\right),b^{2}dx+4\,ab\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)+4\,\sqrt{dx+c}\,ab+\left(b^{2}c+a^{2}\right)\log\left(x\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*s
qrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sq
rt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]
```

Sympy [A]
time = 13.60, size = 65, normalized size = 1.14

$$a^2\log(x) - 2ab\left(-\frac{2c\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{c+dx}\right) + b^2c\log(x) + b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)
```

```
[Out] a**2*log(x) - 2*a*b*(-2*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*sqrt(c
+ d*x)) + b**2*c*log(x) + b**2*d*x
```

Giac [A]
time = 4.00, size = 59, normalized size = 1.04

$$\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx+c)b^2 + 4\sqrt{dx+c}\,ab + \left(b^2c+a^2\right)\log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] 4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) + (d*x + c)*b^2 + 4*sqrt(d*
x + c)*a*b + (b^2*c + a^2)*log(d*x)
```

Mupad [B]
time = 0.09, size = 130, normalized size = 2.28

$$\ln\left(\left(2a^2+2cb^2\right)\sqrt{c+dx} - 2\left(a+b\sqrt{c}\right)^2\sqrt{c+dx} + 4abc\right)\left(a+b\sqrt{c}\right)^2 + \ln\left(\left(2a^2+2cb^2\right)\sqrt{c+dx} - 2\left(a-b\sqrt{c}\right)^2\sqrt{c+dx} + 4abc\right)\left(a-b\sqrt{c}\right)^2 + 4ab\sqrt{c+dx} + b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x,x)
```

```
[Out] log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2)
 + 4*a*b*c)*(a + b*c^(1/2))^2 + log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(
a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c +
d*x)^(1/2) + b^2*d*x
```

3.623 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

```
[Out] b^2*d*ln(x)-2*a*b*d*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(1/2)-(a+b*(d*x+c)^(1/
2))^2/x
```

Rubi [A]
time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {378, 1412, 833, 649, 213, 266}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^2}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqr
t[c] + b^2*d*Log[x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^2}\,dx &= d\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^2}\,dx, x, c+dx\right)\\
&= (2d)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{d\text{Subst}\left(\int \frac{-2abc-2b^2cx}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)}{c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} + (2abd)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+dx}\right) + \left(2b^2d\right)\text{Su}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{x} - \frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d\log(x)
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 64, normalized size = 1.19

$$-\frac{a^2+b^2c+2ab\sqrt{c+dx}}{x}-\frac{2abd\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}}+b^2d\log(-dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2,x]
```

```
[Out] -((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/S
qrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

Maple [A]
time = 0.30, size = 63, normalized size = 1.17

method	result	size
default	$b^2\left(-\frac{c}{x}+d\ln(x)\right)+4abd\left(-\frac{\sqrt{dx+c}}{2dx}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2\sqrt{c}}\right)-\frac{a^2}{x}$	63
derivativedivides	$2d\left(-\frac{ab\sqrt{dx+c}+\frac{b^2c}{2}+\frac{a^2}{2}}{dx}+b\left(\frac{b\ln(-dx)}{2}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right)$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-c/x+d*ln(x))+4*a*b*d*(-1/2*(d*x+c)^(1/2)/d/x-1/2/c^(1/2)*arctanh((d*x
+c)^(1/2)/c^(1/2)))-a^2/x
```

Maxima [A]
time = 0.50, size = 73, normalized size = 1.35

$$\left(b^2\log(dx)+\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}}-\frac{b^2c+2\sqrt{dx+c}\,ab+a^2}{dx}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*log(d*x) + a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))
)/sqrt(c) - (b^2*c + 2*sqrt(d*x + c)*a*b + a^2)/(d*x))*d
```

Fricas [A]
time = 0.36, size = 147, normalized size = 2.72

$$\left[\frac{b^{2}cdx\log\left(x\right)+ab\sqrt{c}\,dx\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-b^{2}c^{2}-2\,\sqrt{dx+c}\,abc-a^{2}c}{cx},\frac{b^{2}cdx\log\left(x\right)+2\,ab\sqrt{-c}\,dx\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)-b^{2}c^{2}-2\,\sqrt{dx+c}\,abc-a^{2}c}{cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] [(b^2*c*d*x*log(x) + a*b*sqrt(c)*d*x*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2
*c)/x) - b^2*c^2 - 2*sqrt(d*x + c)*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*log(x)
+ 2*a*b*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) - b^2*c^2 - 2*sqrt(d*
x + c)*a*b*c - a^2*c)/(c*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(49) = 98$.
time = 35.46, size = 139, normalized size = 2.57

$$-\frac{a^{2}}{x}-abcd\sqrt{\frac{1}{c^{3}}}\log\left(-c^{2}\sqrt{\frac{1}{c^{3}}}+\sqrt{c+dx}\right)+abcd\sqrt{\frac{1}{c^{3}}}\log\left(c^{2}\sqrt{\frac{1}{c^{3}}}+\sqrt{c+dx}\right)+\frac{4abd\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{2ab\sqrt{c+dx}}{x}-\frac{b^{2}c}{x}+b^{2}d\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) +
a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + 4*a*b*d*ata
n(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**
2*d*log(x)
```

Giac [A]
time = 3.85, size = 80, normalized size = 1.48

$$\frac{b^{2}d^{2}\log\left(dx\right)+\frac{2\,abd^{2}\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{b^{2}cd^{2}+2\,\sqrt{dx+c}\,abd^{2}+a^{2}d^{2}}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*log(d*x) + 2*a*b*d^2*arctan(sqrt(d*x + c)/sqrt(-c))/sqrt(-c) - (b^
2*c*d^2 + 2*sqrt(d*x + c)*a*b*d^2 + a^2*d^2)/(d*x))/d
```

Mupad [B]
time = 0.12, size = 131, normalized size = 2.43

$$b\,d\,\ln\left(2\,b\,d\,\left(b+\frac{a}{\sqrt{c}}\right)\,\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\,\left(b+\frac{a}{\sqrt{c}}\right)-\frac{a^2\,d+b^2\,c\,d+2\,a\,b\,d\,\sqrt{c+d\,x}}{d\,x}+b\,d\,\ln\left(2\,b\,d\,\left(b-\frac{a}{\sqrt{c}}\right)\,\sqrt{c+d\,x}-2\,b^2\,d\,\sqrt{c+d\,x}-2\,a\,b\,d\right)\,\left(b-\frac{a}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^2,x)
```

```
[Out] b*d*log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2
*a*b*d)*(b + a/c^(1/2)) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*x)
 + b*d*log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2)
- 2*a*b*d)*(b - a/c^(1/2))
```

3.624 $\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3} \, dx$

Optimal. Leaf size=80

$$-\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}+\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

```
[Out] 1/2*a*b*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3/2)-1/2*b*d*(b*c+a*(d*x+c)^(
1/2))/c/x-1/2*(a+b*(d*x+c)^(1/2))^2/x^2
```

Rubi [A]

time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {378, 1412, 835, 12, 653, 213}

$$\frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2}-\frac{bd\left(a\sqrt{c+dx}+bc\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -1/2*(b*d*(b*c + a*Sqrt[c + d*x]))/(c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2)
+ (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\sqrt{c+dx}\right)^2}{x^3}\,dx &= d^2\text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^2}{(-c+x)^3}\,dx, x, c+dx\right)\\
&= \left(2d^2\right)\text{Subst}\left(\int \frac{x(a+bx)^2}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{d^2\text{Subst}\left(\int -\frac{2bc(a+bx)}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}\\
&= -\frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \left(bd^2\right)\text{Subst}\left(\int \frac{a+bx}{\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} - \frac{\left(abd^2\right)\text{Subst}\left(\int \frac{1}{-c+x^2}\,dx, x, \sqrt{c+\right.}{2c}\\
&= -\frac{bd\left(bc+a\sqrt{c+dx}\right)}{2cx} - \frac{\left(a+b\sqrt{c+dx}\right)^2}{2x^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.17, size = 77, normalized size = 0.96

$$-\frac{a^2c + ab\sqrt{c+dx}\,(2c+dx) + b^2c(c+2dx)}{2cx^2} + \frac{abd^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c + a*b*Sqrt[c + d*x]*(2*c + d*x) + b^2*c*(c + 2*d*x))/(c*x^2) +
(a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))
```

Maple [A]
time = 0.31, size = 82, normalized size = 1.02

method	result	size
derivativedivides	$2d^2\left(-\frac{\frac{ab(dx+c)^{\frac{3}{2}}}{4c}+\frac{(dx+c)b^2}{2}+\frac{ab\sqrt{dx+c}}{4}-\frac{b^2c}{4}+\frac{a^2}{4}}{d^2x^2}+\frac{ab\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)$	81
default	$b^2\left(-\frac{c}{2x^2}-\frac{d}{x}\right)+4ab\,d^2\left(-\frac{\frac{(dx+c)^{\frac{3}{2}}}{8c}+\frac{\sqrt{dx+c}}{8}}{d^2x^2}+\frac{\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}}\right)-\frac{a^2}{2x^2}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(-1/2*c/x^2-d/x)+4*a*b*d^2*(-(1/8/c*(d*x+c)^(3/2)+1/8*(d*x+c)^(1/2))/d^
2/x^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a^2/x^2
```

Maxima [A]
time = 0.51, size = 113, normalized size = 1.41

$$-\frac{1}{4}\left(\frac{ab\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2\left(2\,(dx+c)b^2c - b^2c^2 + (dx+c)^{\frac{3}{2}}ab + \sqrt{dx+c}\,abc + a^2c\right)}{(dx+c)^2c - 2\,(dx+c)c^2 + c^3}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2)
+ 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*b*
c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2
```

Fricas [A]
time = 0.36, size = 181, normalized size = 2.26

$$\left[\frac{ab\sqrt{c}\,d^2x^2\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-4\,b^2c^2dx-2\,b^2c^3-2\,a^2c^2-2\,(abcdx+2\,abc^2)\sqrt{dx+c}}{4\,c^2x^2},\,-\frac{ab\sqrt{-c}\,d^2x^2\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)+2\,b^2c^2dx+b^2c^3+a^2c^2+(abcdx+2\,abc^2)\sqrt{dx+c}}{2\,c^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*
b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x +
c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c)
+ 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c)
)/(c^2*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(68) = 136$.
time = 114.49, size = 292, normalized size = 3.65

$$-\frac{a^2}{2x^2}-\frac{20abc^2d^2\sqrt{c+dx}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{12abcd^2(c+dx)^{\frac{3}{2}}}{-8c^4-16c^3dx+8c^2(c+dx)^2}+\frac{3abcd^2\sqrt{\frac{1}{c^5}}\log\left(-c^3\sqrt{\frac{1}{c^5}}+\sqrt{c+dx}\right)}{4}-\frac{3abcd^2\sqrt{\frac{1}{c^5}}\log\left(c^3\sqrt{\frac{1}{c^5}}+\sqrt{c+dx}\right)}{4}-abd^2\sqrt{\frac{1}{c^3}}\log\left(-c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)+abd^2\sqrt{\frac{1}{c^3}}\log\left(c^2\sqrt{\frac{1}{c^3}}+\sqrt{c+dx}\right)-\frac{2abd\sqrt{c+dx}}{cx}-\frac{b^2c}{2x^2}-\frac{b^2d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) - 20*a*b*c**2*d**2*sqrt(c + d*x)/(-8*c**4 - 16*c**3*d*x + 8*
c**2*(c + d*x)**2) + 12*a*b*c*d**2*(c + d*x)**(3/2)/(-8*c**4 - 16*c**3*d*x
+ 8*c**2*(c + d*x)**2) + 3*a*b*c*d**2*sqrt(c**(-5))*log(-c**3*sqrt(c**(-5))
 + sqrt(c + d*x))/4 - 3*a*b*c*d**2*sqrt(c**(-5))*log(c**3*sqrt(c**(-5)) + s
qrt(c + d*x))/4 - a*b*d**2*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c +
 d*x)) + a*b*d**2*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) - 2
*a*b*d*sqrt(c + d*x)/(c*x) - b**2*c/(2*x**2) - b**2*d/x
```

Giac [A]
time = 3.00, size = 105, normalized size = 1.31

$$-\frac{\frac{abd^3\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}\,c}+\frac{2\,(dx+c)b^2cd^3-b^2c^2d^3+(dx+c)^{\frac{3}{2}}abd^3+\sqrt{dx+c}\,abcd^3+a^2cd^3}{cd^2x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b^
2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 +
 a^2*c*d^3)/(c*d^2*x^2))/d
```

Mupad [B]
time = 3.40, size = 80, normalized size = 1.00

$$\frac{a\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+d\,x}}{\sqrt{c}}\right)}{2\,c^{3/2}}-\frac{b^2\,c}{2\,x^2}-\frac{b^2\,d}{x}-\frac{a\,b\,\sqrt{c+d\,x}}{2\,x^2}-\frac{a\,b\,{(c+d\,x)}^{3/2}}{2\,c\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^2/x^3,x)
[Out] (a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (b
^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2) -
 a^2/(2*x^2)
```

3.625 $\int x^3 \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=326

$$-\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}-\frac{12a(7a^2-3b^2c)(a^2-b^2}{7b^8}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4+4/5*(-b^2*c+a^2)^2*
(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4-12/7*a*(-3*b^2*c+7*a^2)*(-
b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4+4/9*(3*b^4*c^2-30*a^2*b^2*c+35
*a^4)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4-20/11*a*(-3*b^2*c+7*a^2)*(a+b*(d*x+
c)^(1/2))^(11/2)/b^8/d^4+12/13*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)/b^
8/d^4-28/15*a*(a+b*(d*x+c)^(1/2))^(15/2)/b^8/d^4+4/17*(a+b*(d*x+c)^(1/2))^(
17/2)/b^8/d^4
```

Rubi [A]
time = 0.18, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {378, 1412, 786}

$$\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}-\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}-\frac{4a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{17/2}}{17b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) + (4*(a^2 -
b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) - (12*a*(
7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) + (
4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d
^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) +
(12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) - (28*a*(a +
 b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(17/2))/(
17*b^8*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int\sqrt{a+b\sqrt{x}}\,(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\,\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^3\sqrt{a+bx}}{b^7}-\frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{3/2}}{b^7}-\frac{3\left(7a^5-10a^3b^2c+\right.}{}\right.\right.}{}$$

$$= -\frac{4a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}+\frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right.}{5b^8d^4}$$

Mathematica [A]
time = 0.22, size = 232, normalized size = 0.71

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-14336a^7+3840a^5b^2(10c-7dx)+21504a^6b\sqrt{c+dx}-640a^4b^3(104c-49dx)\sqrt{c+dx}-48a^3b^4\left(616c^2-1080cdx+735d^2x^2\right)+24a^2b^5\sqrt{c+dx}\left(2960c^2-2716cdx+1617d^2x^2\right)+6ab^6\left(320c^3-3936c^2dx+5754cd^2x^2-7007d^3x^3\right)-231b^7\sqrt{c+dx}\left(128c^3-160c^2dx+180cd^2x^2-195d^3x^3\right)\right)}{765765b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) +
21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48
*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2
960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 57
54*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x
 + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

Maple [A]
time = 0.10, size = 383, normalized size = 1.17

method	result
derivativedivides	$\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}-\frac{28a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{4\left(-3b^2c+21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2a\left(-2b^2c\right.}{}$
default	$\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{17}{2}}}{17}-\frac{28a\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}+\frac{4\left(-3b^2c+21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2a\left(-2b^2c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1/2))^(15/2
)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(-8*(-b^2*c+a^2)*a
-2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(
(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c
+a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+
a^2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*
a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b
*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A]
time = 0.48, size = 268, normalized size = 0.82

$$\frac{4\left(45045\left(\sqrt{dx+c}\,b+a\right)^{\frac{17}{2}}-357357\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}a-176715\left(b^2c-7a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}+348075\left(3ab^2c-7a^3\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}+85085\left(3b^4c^2-30a^2b^2c+35a^4\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}-328185\left(3ab^4c^2-10a^3b^2c+7a^5\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-153153\left(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+255255\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}\right)}{765765\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)
^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3
*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*
b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*
b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^
2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 -
 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

Fricas [A]
time = 0.39, size = 286, normalized size = 0.88

$$\frac{4\left(45045\,b^8d^4x^4-29568\,b^8c^4+72960\,a^2b^6c^3-96128\,a^4b^4c^2+59904\,a^6b^2c-14336\,a^8+231\left(15\,b^8c-14\,a^2b^6\right)d^3x^3-28\left(165\,b^8c^2-291\,a^2b^6c+140\,a^4b^4\right)d^2x^2+32\left(231\,b^8c^3-555\,a^2b^6c^2+520\,a^4b^4c-168\,a^6b^2\right)dx+\left(3003\,ab^7d^3x^3-27648\,ab^7c^3+41472\,a^3b^5c^2-28160\,a^5b^3c+7168\,a^7b-3528\left(2\,ab^7c-a^3b^5\right)d^2x^2+32\left(417\,ab^7c^2-417\,a^3b^5c+140\,a^5b^3\right)dx\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{765765\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4
*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^
3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^
3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^
3 - 27648*a*b^7*c^3 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*
a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(279) = 558.
time = 3.24, size = 915, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(
d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 810
81*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/
2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt
(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c +
 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a
)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*s
qrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24
255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2
 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7
/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b
+ a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)*a/(b^7*d^3) + (153153
*(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b + a)^(3/2)*a
*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255255*(sqrt(d*x
+ c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7/2)*a*b^4*c^2 -
 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*(sqrt(d*x + c)*b
 + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a)*a^4*b^4*c^2 + 1
76715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(d*x + c)*b + a)^(1
```

```
1/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b^2*c - 6563700*(sqr
t(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*x + c)*b + a)^(5/2)*a^4
*b^2*c - 4594590*(sqrt(d*x + c)*b + a)^(3/2)*a^5*b^2*c + 2297295*sqrt(sqrt(
d*x + c)*b + a)*a^6*b^2*c - 45045*(sqrt(d*x + c)*b + a)^(17/2) + 408408*(sq
rt(d*x + c)*b + a)^(15/2)*a - 1649340*(sqrt(d*x + c)*b + a)^(13/2)*a^2 + 38
98440*(sqrt(d*x + c)*b + a)^(11/2)*a^3 - 5955950*(sqrt(d*x + c)*b + a)^(9/2
)*a^4 + 6126120*(sqrt(d*x + c)*b + a)^(7/2)*a^5 - 4288284*(sqrt(d*x + c)*b
+ a)^(5/2)*a^6 + 2042040*(sqrt(d*x + c)*b + a)^(3/2)*a^7 - 765765*sqrt(sqrt
(d*x + c)*b + a)*a^8)/(b^7*d^3))/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + b\sqrt{c + d\,x}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2),x)
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.626 $\int x^2 \sqrt{a+b\sqrt{c+dx}}\, dx$

Optimal. Leaf size=224

$$-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{\right.}}{7b^6d^3}$$

```
[Out] -4/3*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3+4/5*(b^4*c^2-6*a^2*
b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3-8/7*a*(-3*b^2*c+5*a^2)*(a+b*
(d*x+c)^(1/2))^(7/2)/b^6/d^3+8/9*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b
^6/d^3-20/11*a*(a+b*(d*x+c)^(1/2))^(11/2)/b^6/d^3+4/13*(a+b*(d*x+c)^(1/2))^
(13/2)/b^6/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {378, 1412, 786}

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4
 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5
*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(
11/2))/(11*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
```

```
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx=\frac{\text{Subst}\left(\int\sqrt{a+b\sqrt{x}}\,(-c+x)^2\,dx,x,c+dx\right)}{d^3}$$
$$=\frac{2\text{Subst}\left(\int x\sqrt{a+bx}\,(-c+x^2)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}$$
$$=\frac{2\text{Subst}\left(\int\left(-\frac{a(a^2-b^2c)^2\sqrt{a+bx}}{b^5}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+bx)^{3/2}}{b^5}-\frac{2(5a^3-3ab^2c)(a+bx)^{5/}}{b^5}\right.\right.}{d^3}$$
$$=-\frac{4a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)}{5b^6d^3}$$

Mathematica [A]
time = 0.13, size = 147, normalized size = 0.66

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-1280a^5+32a^3b^2(68c-75dx)+1920a^4b\sqrt{c+dx}+16a^2b^3\sqrt{c+dx}\,(-254c+175dx)+77b^5\sqrt{c+dx}\,(32c^2-40cdx+45d^2x^2)-6ab^4(96c^2-380cdx+525d^2x^2)\right)}{45045b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 19
20*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b
^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c
*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)
```

Maple [A]
time = 0.10, size = 183, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}-\frac{20a\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{4\left(-2b^2c+10a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{4\left(-4\left(-b^2c+a^2\right)a-a\left(-2b^2c+6\right.\right.}{7}}{d^3 b}$
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}-\frac{20a\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{4\left(-2b^2c+10a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{4\left(-4\left(-b^2c+a^2\right)a-a\left(-2b^2c+6\right.\right.}{7}}{d^3 b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^3/b^6*(1/13*(a+b*(d*x+c)^(1/2))^(13/2)-5/11*a*(a+b*(d*x+c)^(1/2))^(11/2
)+1/9*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-4*(-b^2*c+a^2)*a-a*
(-2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^2+4*a^2*(-b^2
*c+a^2))*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))
^(3/2))
```

Maxima [A]
time = 0.27, size = 167, normalized size = 0.75

$$\frac{4\left(3465\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}-20475\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a-10010\,(b^2c-5\,a^2)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+12870\,(3\,ab^2c-5\,a^3)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+9009\,(b^4c^2-6\,a^2b^2c+5\,a^4)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-15015\,(ab^4c^2-2\,a^3b^2c+a^5)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}\right)}{45045\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3465*(sqrt(d*x + c)*b + a)^(13/2) - 20475*(sqrt(d*x + c)*b + a)^(1
1/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(9/2) + 12870*(3*a*b^2
*c - 5*a^3)*(sqrt(d*x + c)*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a
^4)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sq
rt(d*x + c)*b + a)^(3/2))/(b^6*d^3)
```

Fricas [A]
time = 0.41, size = 184, normalized size = 0.82

$$\frac{4\left(3465\,b^6d^3x^3+2464\,b^6c^3-4640\,a^2b^4c^2+4096\,a^4b^2c-1280\,a^6+35\,(11\,b^6c-10\,a^2b^4)d^2x^2-8\,(77\,b^6c^2-127\,a^2b^4c+60\,a^4b^2)dx+(315\,ab^5d^2x^2+1888\,ab^5c^2-1888\,a^3b^3c+640\,a^5b-400\,(2\,ab^5c-a^3b^3)dx)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{45045\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*
c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2
*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b
^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(
d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(188) = 376.
time = 3.13, size = 549, normalized size = 2.45

$$\frac{4\left(\frac{13\left(1155\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}\,b+a}\,ab^4c^2-990\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^2c+6930\sqrt{\sqrt{dx+c}\,b+a}\,a^3b^2c+315\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a+4950\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^2-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^3+5775\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^4-3465\sqrt{\sqrt{dx+c}\,b+a}\,a^5\right)a}{b^5d^2}+\frac{9009\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}b^4c^2-30030\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}ab^4c^2+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^2b^4c^2-10010\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}b^2c+51480\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}ab^2c-108108\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^2b^2c+120120\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^3b^2c-90090\sqrt{\sqrt{dx+c}\,b+a}\,a^4b^2c+3465\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}-24570\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a+75075\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a^2-128700\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^3+135135\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^4-90090\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^5+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^6}{b^5d^2}\right)}{45045\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/ 2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)* a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/ 2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sqrt(d*x + c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4*c^2 + 4 5045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*b + a)^(9 /2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^2*c - 90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)*b + a)^(13/ 2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x + c)*b + a)^(9/ 2)*a^2 - 128700*(sqrt(d*x + c)*b + a)^(7/2)*a^3 + 135135*(sqrt(d*x + c)*b + a)^(5/2)*a^4 - 90090*(sqrt(d*x + c)*b + a)^(3/2)*a^5 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^6)/(b^5*d^2))/(b*d)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2),x)

[Out] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)

3.627 $\int x\sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=133

$$-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}+\frac{4\left(a+b\right.}{9}$$

```
[Out] -4/3*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2+4/5*(-b^2*c+3*a^2)*(a
+b*(d*x+c)^(1/2))^(5/2)/b^4/d^2-12/7*a*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2+4/
9*(a+b*(d*x+c)^(1/2))^(9/2)/b^4/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 -
b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x
])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,(-c+x)\,dx, x, c+dx\right)}{d^2}$$

$$= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\,(-c+x^2)\,dx, x, \sqrt{c+dx}\right)}{d^2}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{(-a^3+ab^2c)\sqrt{a+bx}}{b^3}+\frac{(3a^2-b^2c)(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx,\right.}{d^2}$$

$$= -\frac{4a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}-\frac{12a\left(a\right.}{}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.63

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}\left(-16a^3+6ab^2(2c-5dx)+24a^2b\sqrt{c+dx}+7b^3\sqrt{c+dx}\,(-4c+5dx)\right)}{315b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*
Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)
```

Maple [A]
time = 0.09, size = 94, normalized size = 0.71

method	result
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}-\frac{12a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{4\left(-b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4\left(-b^2c+a^2\right)a\left(a+b\sqrt{dx+c}\right)}{3}}{b^4d^2}$
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}-\frac{12a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{4\left(-b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4\left(-b^2c+a^2\right)a\left(a+b\sqrt{dx+c}\right)}{3}}{b^4d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^2/b^4*(1/9*(a+b*(d*x+c)^(1/2))^(9/2)-3/7*a*(a+b*(d*x+c)^(1/2))^(7/2)+1/
5*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)*a*(a+b*(d*x+c)^
(1/2))^(3/2))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 0.70

$$\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}-135\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a-63\,(b^2c-3\,a^2)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+105\,(ab^2c-a^3)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}\right)}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/315*(35*(sqrt(d*x + c)*b + a)^(9/2) - 135*(sqrt(d*x + c)*b + a)^(7/2)*a -
 63*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(sqrt
(d*x + c)*b + a)^(3/2))/(b^4*d^2)
```

Fricas [A]
time = 0.44, size = 103, normalized size = 0.77

$$\frac{4\left(35\,b^4d^2x^2-28\,b^4c^2+36\,a^2b^2c-16\,a^4+(7\,b^4c-6\,a^2b^2)dx+(5\,ab^3dx-16\,ab^3c+8\,a^3b)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{315\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a
^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\sqrt{c+dx}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(109) = 218.
time = 3.62, size = 279, normalized size = 2.10

$$-\frac{4\left(\frac{3\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}\,b+a}\,ab^2c-15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}\,b+a}\,a^3\right)a}{b^3d}+\frac{63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}b^2c-210\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}ab^2c+315\sqrt{\sqrt{dx+c}\,b+a}\,a^2b^2c-35\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+180\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a-378\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^2+420\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^3-315\sqrt{\sqrt{dx+c}\,b+a}\,a^4}{b^3d}\right)}{315\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b
+ a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5
/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)
*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x + c
)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*(sqrt
(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqrt(d*x
+ c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sqrt(sqrt
(d*x + c)*b + a)*a^4)/(b^3*d))/(b*d)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{a+b\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.628 $\int \sqrt{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

```
[Out] -4/3*a*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d+4/5*(a+b*(d*x+c)^(1/2))^(5/2)/b^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {253, 196, 45}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/
2))/(5*b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \sqrt{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int x\sqrt{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{4a\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^2d}$$

Mathematica [**A**]
time = 0.03, size = 55, normalized size = 0.98

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-2a^2+ab\sqrt{c+dx}+3b^2(c+dx)\right)}{15b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-2*a^2 + a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x))
)/(15*b^2*d)
```

Maple [**A**]
time = 0.07, size = 41, normalized size = 0.73

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^2d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}-\frac{4a\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}}{b^2d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d/b^2*(1/5*(a+b*(d*x+c)^(1/2))^(5/2)-1/3*a*(a+b*(d*x+c)^(1/2))^(3/2))
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.77

$$\frac{4\left(\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a}{b^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b
^2)/d
```

Fricas [A]
time = 0.42, size = 50, normalized size = 0.89

$$\frac{4\left(3\,b^2dx+3\,b^2c+\sqrt{dx+c}\,ab-2\,a^2\right)\sqrt{\sqrt{dx+c}\,b+a}}{15\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b
 + a)/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(44) = 88.
time = 5.40, size = 99, normalized size = 1.77

$$\frac{4\left(\frac{5\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\,\sqrt{\sqrt{dx+c}\,b+a}\,a\right)a}{b}+\frac{3\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-10\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a+15\,\sqrt{\sqrt{dx+c}\,b+a}\,a^2}{b}\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b +
 (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*sqr
t(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)
```

Mupad [B]
time = 3.39, size = 44, normalized size = 0.79

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{5/2}}{5\,b^2\,d}-\frac{4\,a\,\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] (4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^
(3/2))/(3*b^2*d)
```

3.629 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$

Optimal. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))*(a+b*c^(1/2))^(
1/2)+4*(a+b*(d*x+c)^(1/2))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {378, 1412, 839, 841, 1180, 213}

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{a-b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)-2\sqrt{a+b\sqrt{c}}\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*
Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
```

```
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{-c+x}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 2\text{Subst}\left(\int \frac{bc+ax}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + 4\text{Subst}\left(\int \frac{-a^2+b^2c+ax^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 4\sqrt{a+b\sqrt{c+dx}} + \left(2\left(a-b\sqrt{c}\right)\right)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c}}\right.\\
&= 4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 124, normalized size = 1.07

$$4\sqrt{a+b\sqrt{c+dx}}-2\sqrt{-a-b\sqrt{c}}\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)-2\sqrt{-a+b\sqrt{c}}\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]
```

```
[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[-a - b*Sqrt[c]]*ArcTan[Sqrt[a + b*Sqrt
[c + d*x]]/Sqrt[-a - b*Sqrt[c]]] - 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[Sqrt[a + b
*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]]
```

Maple [A]
time = 0.12, size = 152, normalized size = 1.31

method	result
derivativedivides	$4\sqrt{a+b\sqrt{dx+c}}+\frac{2\left(-b^2c+a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\sqrt{\sqrt{b^2c}-a}}+\frac{2\left(b^2c+a\sqrt{b^2c}\right)\arctan}{\sqrt{b^2c}\sqrt{-}}$
default	$4\sqrt{a+b\sqrt{dx+c}}+\frac{2\left(-b^2c+a\sqrt{b^2c}\right)\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{b^2c}\sqrt{\sqrt{b^2c}-a}}+\frac{2\left(b^2c+a\sqrt{b^2c}\right)\arctan}{\sqrt{b^2c}\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 4*(a+b*(d*x+c)^(1/2))^(1/2)+2*(-b^2*c+a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/((b^2*
c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))
+2*(b^2*c+a*(b^2*c)^(1/2))/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a
+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(88) = 176$.
time = 0.40, size = 194, normalized size = 1.67

$$-\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}+2\sqrt{a+\sqrt{b^2c}}\right)+\sqrt{a+\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}-2\sqrt{a+\sqrt{b^2c}}\right)-\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}+2\sqrt{a-\sqrt{b^2c}}\right)+\sqrt{a-\sqrt{b^2c}}\log\left(2\sqrt{\sqrt{dx+c}\,b+a}-2\sqrt{a-\sqrt{b^2c}}\right)+4\sqrt{\sqrt{dx+c}\,b+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^
2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a +
 sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*
sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b +
 a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x, x)
```

Giac [A]
time = 3.94, size = 150, normalized size = 1.29

$$\frac{2\left(2\sqrt{\sqrt{dx+c}\,b+a}\,b-\frac{(b^3c-a^2b)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{(b^3c-a^2b)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c
)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b
^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*
sqrt(c) - a)*sqrt(-b*sqrt(c) - a)))/b
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,\sqrt{c+d\,x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)
```

3.630 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx$

Optimal. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\,\sqrt{c}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{a+b\sqrt{c}}\,\sqrt{c}}$$

```
[Out] 1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-b
*c^(1/2))^(1/2)-1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/
2))/c^(1/2)/(a+b*c^(1/2))^(1/2)-(a+b*(d*x+c)^(1/2))^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {378, 1412, 835, 12, 722, 1107, 213}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\,\sqrt{a-b\sqrt{c}}}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\,\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqr
t[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a +
b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2}\,dx &= d\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^2}\,dx, x, c+dx\right) \\
&= (2d)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right) \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} - \frac{d\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{1}{2}(bd)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right) \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \left(b^2d\right)\text{Subst}\left(\int \frac{1}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+}}\right. \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{(bd)\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)}{2\sqrt{c}} - \frac{(bd}{} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{a-b\sqrt{c}}\,\sqrt{c}} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{}}}{\sqrt{a+}}\right.}{2\sqrt{a+b\sqrt{c}}}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 0.39, size = 144, normalized size = 1.05

$$
\frac{1}{2}\left(-\frac{2\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}\,\sqrt{c}} - \frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}\,\sqrt{c}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] ((-2*Sqrt[a + b*Sqrt[c + d*x]])/x + (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/S
qrt[-a - b*Sqrt[c]]])/(Sqrt[-a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTan[Sqrt[a +
 b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(Sqrt[-a + b*Sqrt[c]]*Sqrt[c]))/2
```

Maple [A]
time = 0.14, size = 166, normalized size = 1.21

method	result
derivativedivides	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}-$
default	$4d\,b^{2}\left(-\frac{\sqrt{a+b\sqrt{dx+c}}}{4\left(\left(a+b\sqrt{dx+c}\right)^{2}-2a\left(a+b\sqrt{dx+c}\right)-b^{2}c+a^{2}\right)}+\frac{\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^{2}c}-a}}\right)}{8\sqrt{b^{2}c}\,\sqrt{-\sqrt{b^{2}c}-a}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*d*b^2*(-1/4*(a+b*(d*x+c)^(1/2))^(1/2)/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*
x+c)^(1/2))-b^2*c+a^2)+1/8/(b^2*c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a
+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))-1/8/(b^2*c)^(1/2)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(101) = 202$.
time = 0.40, size = 1003, normalized size = 7.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*
(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2
```

```
- a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)
)*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^
4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*
log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3
- 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*
d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)
)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(
b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3
+ (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 -
 a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))
*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4
/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*l
og(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d
^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))
) + 4*sqrt(sqrt(d*x + c)*b + a))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(101) = 202.
time = 3.65, size = 232, normalized size = 1.69

$$\frac{\frac{2\sqrt{\sqrt{dx+c}\,b+a}\,b^3d^2}{b^2c-\left(\sqrt{dx+c}\,b+a\right)^2+2\left(\sqrt{dx+c}\,b+a\right)a-a^2}-\frac{\left(b^3cd^2|b|+ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}+ac\right)\sqrt{b\sqrt{c}-a}\,|b|}+\frac{\left(b^3cd^2|b|-ab^3\sqrt{c}\,d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(bc^{\frac{3}{2}}-ac\right)\sqrt{-b\sqrt{c}-a}\,|b|}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2 +
 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2)*
arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a*c)
*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*arcta
n(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)*sqrt
(-b*sqrt(c) - a)*abs(b)))/(b*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)
```

3.631 $\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$

Optimal. Leaf size=224

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}+\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x}-\frac{b\left(2a-3b\sqrt{c}\right)d^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right.}{16\left(a-b\sqrt{c}\right)^{3/2}c^{3/2}}$$

```
[Out] -1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-3*b
*c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(3/2)+1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2)
)^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+3*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(3/2)-
1/2*(a+b*(d*x+c)^(1/2))^(1/2)/x^2+1/8*b*d*(b*c-a*(d*x+c)^(1/2))*(a+b*(d*x+c
)^(1/2))^(1/2)/c/(-b^2*c+a^2)/x
```

Rubi [A]
time = 0.31, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {378, 1412, 835, 12, 755, 841, 1180, 213}

$$\frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8cx\left(a^2-b^2c\right)}-\frac{bd^2\left(2a-3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd^2\left(2a+3b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{3/2}}-\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + b*Sqrt[c + d*x]]/x^2 + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a +
b*Sqrt[c + d*x]])/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan
h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2)
*c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sq
rt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx = d^2\text{Subst}\left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2\text{Subst}\left(\int -\frac{bc}{2\sqrt{a+bx}\,(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4}\left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(bd^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} + \frac{\left(b\left(2a-3b\sqrt{c}\right.\right.}{}$$

$$= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd\left(bc-a\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{8c\left(a^2-b^2c\right)x} - \frac{b\left(2a-3b\sqrt{c}\right)}{16}$$

Mathematica [A]
time = 1.81, size = 217, normalized size = 0.97

$$\frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^2c+abdx\sqrt{c+dx}-b^2c(4c+dx)\right)}{(a^2-b^2c)x^2}+\frac{b\left(2a+3b\sqrt{c}\right)d^2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}}+\frac{b\left(-2a+3b\sqrt{c}\right)d^2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}}}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] ((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^2*c + a*b*d*x*Sqrt[c + d*x] - b
^2*c*(4*c + d*x)))/((a^2 - b^2*c)*x^2) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTan[
Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(3/2) + (
```

```
b*(-2*a + 3*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqr
t[c]]])/(-a + b*Sqrt[c])^(3/2))/(16*c^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(174) = 348$.
time = 0.14, size = 373, normalized size = 1.67

method	result
derivativedivides	$4d^2b^4\left(-\frac{\frac{a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{32b^2c\left(-b^2c+a^2\right)}-\frac{\left(b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{32b^2c\left(-b^2c+a^2\right)}+\frac{a\left(b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{32b^2c\left(-b^2c+a^2\right)}-\frac{\left(-3b^2c+a^2\right)}{}}{\left(\left(a+b\sqrt{dx+c}\right)^2-2a\left(a+b\sqrt{dx+c}\right)-b^2c+a^2\right)^2}\right.$
default	$4d^2b^4\left(-\frac{\frac{a\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{32b^2c\left(-b^2c+a^2\right)}-\frac{\left(b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{32b^2c\left(-b^2c+a^2\right)}+\frac{a\left(b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{32b^2c\left(-b^2c+a^2\right)}-\frac{\left(-3b^2c+a^2\right)}{}}{\left(\left(a+b\sqrt{dx+c}\right)^2-2a\left(a+b\sqrt{dx+c}\right)-b^2c+a^2\right)^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 4*d^2*b^4*(-(1/32*a/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(7/2)-1/32*(b^2*
c+3*a^2)/b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)+1/32*a*(b^2*c+3*a^2)/
b^2/c/(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-1/32*(-3*b^2*c+a^2)/b^2/c*(a+b
*(d*x+c)^(1/2))^(1/2))/((a+b*(d*x+c)^(1/2))^2-2*a*(a+b*(d*x+c)^(1/2))-b^2*c
+a^2)^2-1/32/b^2/c/(-b^2*c+a^2)*(1/2*(-3*b^2*c+a*(b^2*c)^(1/2)+2*a^2)/(b^2*
c)^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c
)^(1/2)-a)^(1/2))+1/2*(3*b^2*c+a*(b^2*c)^(1/2)-2*a^2)/(b^2*c)^(1/2)/((b^2*c
)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2)))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(175) = 350$.
time = 0.62, size = 2856, normalized size = 12.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/32*((b^2*c^2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2
)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c
^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b
^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2
*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2
*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*
c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b
^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*
a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^
4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*
sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10
*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 +
a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^
2 - a^2*c)*x^2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*
c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b
^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a
^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a
^4*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*
a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b
^2*c^4 + a^9*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^1
2*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 -
 6*a^10*b^2*c^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b
^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14
*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4
*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(
b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) + (b^2*c^2 - a^2*c)*x^
2*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b
^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^
4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 +
15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*
a^4*b^2*c^4 - a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(
sqrt(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*
d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*
c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2
```

*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c
^4 + a^12*c^3)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^
6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2
*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20
*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a
^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))) - (b^2*c^2 - a^2*c)*x^2*sqrt(-((15*
a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^
4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(
b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^
5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 -
 a^6*c^3))*log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*sqrt(sqrt(d*x + c)
*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*
b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81
*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 1
5*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3
)))*sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2
*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^14*c^2 - 90*a^2*b^12*c + 25*
a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6
+ 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3)))/(b^6*c^6 - 3*a^2*b^4*c^5 +
3*a^4*b^2*c^4 - a^6*c^3))) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c
^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a))/((b^2*c^2 - a^2*c)*x^2)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)

[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 895 vs. 2(175) = 350.
time = 3.33, size = 895, normalized size = 4.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")

[Out] 1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^2
 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c^(
7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x + c

```
)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 -
 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))/(b^2*c^2 - a^2*c)))/((b^5*c^(9/
2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) - a^5*c^
2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*c)^2*a*
b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3*abs(b^3
*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2*a^7*b^3*
c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c - sqrt((a
*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c)))
/(b^2*c^2 - a^2*c)))/((b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3 - 2*a^3*b^2*
c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*
c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x + c)*b + a)^(5
/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*sqrt(sqrt(d*x +
 c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^3*d^3 + 3*(sqrt(
d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^3*d
^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/((b^2*c^2 - a^2*c)*(b^2*c - (sq
rt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\sqrt{c+d\,x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)
[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)
```

3.632 $\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=230

$$-\frac{a(a^4-3a^2b^2c+3b^4c^2)\,x}{b^6d^3}+\frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4}+\frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4}-\frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4}$$

```
[Out] -a*(3*b^4*c^2-3*a^2*b^2*c+a^4)*x/b^6/d^3+2/3*(3*b^4*c^2-3*a^2*b^2*c+a^4)*(d
*x+c)^(3/2)/b^5/d^4-1/2*a*(-3*b^2*c+a^2)*(d*x+c)^2/b^4/d^4+2/5*(-3*b^2*c+a^
2)*(d*x+c)^(5/2)/b^3/d^4-1/3*a*(d*x+c)^3/b^2/d^4+2/7*(d*x+c)^(7/2)/b/d^4-2*
a*(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2))/b^8/d^4+2*(-b^2*c+a^2)^3*(d*x+c)^(1/
2)/b^7/d^4
```

Rubi [A]
time = 0.19, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}+\frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4}-\frac{a(a^2-3b^2c)\,(c+dx)^2}{2b^4d^4}+\frac{2(a^2-3b^2c)\,(c+dx)^{5/2}}{5b^3d^4}-\frac{ax(a^4-3a^2b^2c+3b^4c^2)}{b^6d^3}+\frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx)^{3/2}}{3b^5d^4}-\frac{a(c+dx)^3}{3b^2d^4}+\frac{2(c+dx)^{7/2}}{7bd^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqr
t[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))
/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^
2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d
*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*
d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^3}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{(-a^2+b^2c)^3}{b^7} - \frac{a(a^4-3a^2b^2c+3b^4c^2)x}{b^6} + \frac{(a^4-3a^2b^2c+3b^4c^2)x^2}{b^5} - \frac{a(a^2-3b^2c)x^3}{b^4} - \frac{(-a^2}{} \right.\right.}{d^4}$$

$$= -\frac{a(a^4-3a^2b^2c+3b^4c^2)\,x}{b^6d^3} + \frac{2(a^2-b^2c)^3\sqrt{c+dx}}{b^7d^4} + \frac{2(a^4-3a^2b^2c+3b^4c^2)\,(c+dx}{3b^5d^4}$$

Mathematica [A]
time = 0.17, size = 235, normalized size = 1.02

$$\frac{b\left(420a^6\sqrt{c+dx} - 140a^4b^2(8c-dx)\sqrt{c+dx} - 210a^5b(c+dx) + 105a^3b^3(5c^2+4cdx-d^2x^2) + 84a^2b^4\sqrt{c+dx}\,(11c^2-3cdx+d^2x^2) - 35ab^5(11c^3+6c^2dx-3cd^2x^2+2d^3x^3) + 12b^6\sqrt{c+dx}\,(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3)\right) - 420a(a^2-b^2c)^3\log\left(a+b\sqrt{c+dx}\right)}{210b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(420*a^6*Sqrt[c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] - 210*a^5
*b*(c + d*x) + 105*a^3*b^3*(5*c^2 + 4*c*d*x - d^2*x^2) + 84*a^2*b^4*Sqrt[c
+ d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*(11*c^3 + 6*c^2*d*x - 3*c*d^
2*x^2 + 2*d^3*x^3) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^
2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*
d^4)
```

Maple [A]
time = 0.03, size = 288, normalized size = 1.25

method	result
derivativedivides	$\frac{2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7} - \frac{a(dx+c)^3b^5}{6} - \frac{3b^6c(dx+c)^{\frac{5}{2}}}{5} + \frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5} + \frac{3a\,b^5c(dx+c)^2}{4} + b^6c^2(dx+c)^{\frac{3}{2}} - \frac{a^3b^3(dx+c)^2}{4} - a^2b^4c(dx+c)^{\frac{3}{2}} - \frac{3a\,b^5c}{}\right.}{}$

default	$2\left(\frac{(dx+c)^{\frac{7}{2}}b^6}{7}-\frac{a(dx+c)^3b^5}{6}-\frac{3b^6c(dx+c)^{\frac{5}{2}}}{5}+\frac{a^2b^4(dx+c)^{\frac{5}{2}}}{5}+\frac{3a\,b^5c(dx+c)^2}{4}+b^6c^2(dx+c)^{\frac{3}{2}}-\frac{a^3b^3(dx+c)^2}{4}-a^2b^4c(dx+c)^{\frac{3}{2}}-\frac{3a\,b\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(1/b^7*(1/7*(d*x+c)^(7/2)*b^6-1/6*a*(d*x+c)^3*b^5-3/5*b^6*c*(d*x+c)^(
5/2)+1/5*a^2*b^4*(d*x+c)^(5/2)+3/4*a*b^5*c*(d*x+c)^2+b^6*c^2*(d*x+c)^(3/2)-
1/4*a^3*b^3*(d*x+c)^2-a^2*b^4*c*(d*x+c)^(3/2)-3/2*a*b^5*c^2*(d*x+c)-b^6*c^3
*(d*x+c)^(1/2)+1/3*a^4*b^2*(d*x+c)^(3/2)+3/2*a^3*b^3*c*(d*x+c)+3*a^2*b^4*c^
2*(d*x+c)^(1/2)-1/2*a^5*b*(d*x+c)-3*a^4*b^2*c*(d*x+c)^(1/2)+a^6*(d*x+c)^(1/
2))-a*(-b^6*c^3+3*a^2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.28, size = 243, normalized size = 1.06

$$\frac{\frac{60\,(dx+c)^{\frac{7}{2}}b^6-70\,(dx+c)^3ab^5-84\,(3\,b^6c-a^2b^4)\,(dx+c)^{\frac{5}{2}}+105\,(3\,ab^5c-a^3b^3)\,(dx+c)^2+140\,(3\,b^6c^2-3\,a^2b^4c+a^4b^2)\,(dx+c)^{\frac{3}{2}}-210\,(3\,ab^5c^2-3\,a^3b^3c+a^5b)\,(dx+c)-420\,(b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6)\,\sqrt{dx+c}}{b^7}+\frac{420\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)\log\left(\sqrt{dx+c}\,b+a\right)}{b^8}}{210\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
 + a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqr
t(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

Fricas [A]
time = 0.33, size = 228, normalized size = 0.99

$$-\frac{70\,ab^6d^3x^3-105\,(ab^6c-a^3b^4)d^2x^2+210\,(ab^6c^2-2\,a^3b^4c+a^5b^2)dx-420\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)\log\left(\sqrt{dx+c}\,b+a\right)-4\,(15\,b^7d^3x^3-48\,b^7c^3+231\,a^2b^5c^2-280\,a^4b^3c+105\,a^6b-3\,(6\,b^7c-7\,a^2b^5)d^2x^2+(24\,b^7c^2-63\,a^2b^5c+35\,a^4b^3)dx)\sqrt{dx+c}}{210\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2
 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*
c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^
2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (
24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 3.62, size = 341, normalized size = 1.48

$$\frac{2\,(ab^6c^3 - 3\,a^3b^4c^2 + 3\,a^5b^2c - a^7)\log\left(\left|\sqrt{dx+c}\,b + a\right|\right)}{b^8d^4} + \frac{60\,(dx+c)^{\frac{7}{2}}b^6d^{24} - 252\,(dx+c)^{\frac{5}{2}}b^6cd^{24} + 420\,(dx+c)^{\frac{3}{2}}b^6c^2d^{24} - 420\,\sqrt{dx+c}\,b^6c^3d^{24} - 70\,(dx+c)^3ab^5d^{24} + 315\,(dx+c)^2ab^5cd^{24} - 630\,(dx+c)ab^5c^2d^{24} + 84\,(dx+c)^{\frac{5}{2}}a^2b^4d^{24} - 420\,(dx+c)^{\frac{3}{2}}a^2b^4cd^{24} + 1260\,\sqrt{dx+c}\,a^2b^4c^2d^{24} - 105\,(dx+c)^2a^3b^3d^{24} + 630\,(dx+c)a^3b^3cd^{24} + 140\,(dx+c)^{\frac{3}{2}}a^4b^2d^{24} - 1260\,\sqrt{dx+c}\,a^4b^2cd^{24} - 210\,(dx+c)a^5bd^{24} + 420\,\sqrt{dx+c}\,a^6d^{24}}{210\,b^7d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b +
 a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b
^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^
24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*x +
c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a
^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b
^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 -
 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*
x + c)*a^6*d^24)/(b^7*d^28)
```

Mupad [B]
time = 0.07, size = 317, normalized size = 1.38

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,b\,d^4}-\left(\frac{a^2\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{b^2}+\frac{2\,c^3}{b\,d^4}\right)\,\sqrt{c+d\,x}-\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{3\,b^2}-\frac{2\,c^2}{b\,d^4}\right)\,{\left(c+d\,x\right)}^{3/2}-\left(\frac{6\,c}{5\,b\,d^4}-\frac{2\,a^2}{5\,b^3\,d^4}\right)\,{\left(c+d\,x\right)}^{5/2}+\frac{a\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)\,{\left(c+d\,x\right)}^2}{4\,b}-\frac{a\,{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^7-6\,a^5\,b^2\,c+6\,a^3\,b^4\,c^2-2\,a\,b^6\,c^3\right)}{b^8\,d^4}+\frac{a\,d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b\,d^4}-\frac{2\,a^2}{b^3\,d^4}\right)}{b^2}-\frac{6\,c^2}{b\,d^4}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d
^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((a^
2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d*x)
^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*((6*c
)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3*b^2*
d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3 + 6*a
^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/
b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

3.633 $\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=151

$$-\frac{a(a^2-2b^2c)\,x}{b^4d^2}+\frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3}+\frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}-\frac{2a(a^2-b^2c$$

```
[Out] -a*(-2*b^2*c+a^2)*x/b^4/d^2+2/3*(-2*b^2*c+a^2)*(d*x+c)^(3/2)/b^3/d^3-1/2*a*
(d*x+c)^2/b^2/d^3+2/5*(d*x+c)^(5/2)/b/d^3-2*a*(-b^2*c+a^2)^2*ln(a+b*(d*x+c)
^(1/2))/b^6/d^3+2*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^5/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}+\frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3}-\frac{ax(a^2-2b^2c)}{b^4d^2}+\frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3}-\frac{a(c+dx)^2}{2b^2d^3}+\frac{2(c+dx)^{5/2}}{5bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5
*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(
2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b
*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d^3}$$
$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^2}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{(-a^2+b^2c)^2}{b^5} - \frac{a(a^2-2b^2c)x}{b^4} - \frac{(-a^2+2b^2c)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)}\right) dx, x, \sqrt{c\cdot}\right.}{d^3}$$
$$= -\frac{a(a^2-2b^2c)\,x}{b^4d^2} + \frac{2(a^2-b^2c)^2\sqrt{c+dx}}{b^5d^3} + \frac{2(a^2-2b^2c)\,(c+dx)^{3/2}}{3b^3d^3} - \frac{a(c+dx)^2}{2b^2d^3} +$$

Mathematica [A]
time = 0.11, size = 151, normalized size = 1.00

$$\frac{b\left(60a^4\sqrt{c+dx} - 20a^2b^2(5c-dx)\sqrt{c+dx} - 30a^3b(c+dx) + 15ab^3(3c^2+2cdx-d^2x^2) + 4b^4\sqrt{c+dx}\,(8c^2-4cdx+3d^2x^2)\right) - 60a(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{30b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x]),x]
[Out] (b*(60*a^4*Sqrt[c + d*x] - 20*a^2*b^2*(5*c - d*x)*Sqrt[c + d*x] - 30*a^3*b*
(c + d*x) + 15*a*b^3*(3*c^2 + 2*c*d*x - d^2*x^2) + 4*b^4*Sqrt[c + d*x]*(8*c
^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])
/(30*b^6*d^3)
```

Maple [A]
time = 0.03, size = 166, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^4}{5} - \frac{a(dx+c)^2b^3}{4} - \frac{2b^4c(dx+c)^{\frac{3}{2}}}{3} + \frac{a^2b^2(dx+c)^{\frac{3}{2}}}{3} + a\,b^3c(dx+c) + b^4c^2\sqrt{dx+c} - \frac{a^3b(dx+c)}{2} - 2a^2b^2c\sqrt{dx+c} + \right.}{b^5}}{d^3}$
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^4}{5} - \frac{a(dx+c)^2b^3}{4} - \frac{2b^4c(dx+c)^{\frac{3}{2}}}{3} + \frac{a^2b^2(dx+c)^{\frac{3}{2}}}{3} + a\,b^3c(dx+c) + b^4c^2\sqrt{dx+c} - \frac{a^3b(dx+c)}{2} - 2a^2b^2c\sqrt{dx+c} + \right.}{b^5}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(1/b^5*(1/5*(d*x+c)^(5/2)*b^4-1/4*a*(d*x+c)^2*b^3-2/3*b^4*c*(d*x+c)^(
3/2)+1/3*a^2*b^2*(d*x+c)^(3/2)+a*b^3*c*(d*x+c)+b^4*c^2*(d*x+c)^(1/2)-1/2*a^
3*b*(d*x+c)-2*a^2*b^2*c*(d*x+c)^(1/2)+a^4*(d*x+c)^(1/2))-a*(b^4*c^2-2*a^2*b
^2*c+a^4)/b^6*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.28, size = 148, normalized size = 0.98

$$\frac{\frac{12\,(dx+c)^{\frac{5}{2}}b^4-15\,(dx+c)^2ab^3-20\,(2\,b^4c-a^2b^2)(dx+c)^{\frac{3}{2}}+30\,(2\,ab^3c-a^3b)(dx+c)+60\,(b^4c^2-2\,a^2b^2c+a^4)\sqrt{dx+c}}{b^5}-\frac{60\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\sqrt{dx+c}\,b+a\right)}{b^6}}{30\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^
2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2
*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(s
qrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A]
time = 0.36, size = 138, normalized size = 0.91

$$-\frac{15\,ab^4d^2x^2-30\,(ab^4c-a^3b^2)dx+60\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\sqrt{dx+c}\,b+a\right)-4\,(3\,b^5d^2x^2+8\,b^5c^2-25\,a^2b^3c+15\,a^4b-(4\,b^5c-5\,a^2b^3)dx)\sqrt{dx+c}}{30\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^
3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25
*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{a+b\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 3.90, size = 198, normalized size = 1.31

$$-\frac{2\,(ab^4c^2-2\,a^3b^2c+a^5)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^6d^3}+\frac{12\,(dx+c)^{\frac{5}{2}}b^4d^{12}-40\,(dx+c)^{\frac{3}{2}}b^4cd^{12}+60\,\sqrt{dx+c}\,b^4c^2d^{12}-15\,(dx+c)^2ab^3d^{12}+60\,(dx+c)ab^3cd^{12}+20\,(dx+c)^{\frac{3}{2}}a^2b^2d^{12}-120\,\sqrt{dx+c}\,a^2b^2cd^{12}-30\,(dx+c)a^3bd^{12}+60\,\sqrt{dx+c}\,a^4d^{12}}{30\,b^5d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3)
+ 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*sq
rt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c
*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12
- 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)
```

Mupad [B]
time = 3.21, size = 184, normalized size = 1.22

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b\,d^3}-\left(\frac{a^2\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{b^2}-\frac{2\,c^2}{b\,d^3}\right)\,\sqrt{c+d\,x}-\left(\frac{4\,c}{3\,b\,d^3}-\frac{2\,a^2}{3\,b^3\,d^3}\right)\,{\left(c+d\,x\right)}^{3/2}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^5-4\,a^3\,b^2\,c+2\,a\,b^4\,c^2\right)}{b^6\,d^3}-\frac{a\,{\left(c+d\,x\right)}^2}{2\,b^2\,d^3}+\frac{a\,d\,x\,\left(\frac{4\,c}{b\,d^3}-\frac{2\,a^2}{b^3\,d^3}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/
b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^3*
d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c +
2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(b*d^
3) - (2*a^2)/(b^3*d^3)))/(2*b)
```

3.634 $\int \frac{x}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=90

$$-\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2}-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

```
[Out] -a*x/b^2/d+2/3*(d*x+c)^(3/2)/b/d^2-2*a*(-b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/b
^4/d^2+2*(-b^2*c+a^2)*(d*x+c)^(1/2)/b^3/d^2
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}-\frac{ax}{b^2d}+\frac{2(c+dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)
^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}}\,dx,x,c+dx\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{a+bx}\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}+\frac{-a^3+ab^2c}{b^3(a+bx)}\right)dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= -\frac{ax}{b^2d}+\frac{2(a^2-b^2c)\sqrt{c+dx}}{b^3d^2}+\frac{2(c+dx)^{3/2}}{3bd^2}-\frac{2a(a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Mathematica [A]
time = 0.06, size = 85, normalized size = 0.94

$$\frac{b\left(6a^2\sqrt{c+dx}+2b^2(-2c+dx)\sqrt{c+dx}-3ab(c+dx)\right)-6(a^3-ab^2c)\log\left(a+b\sqrt{c+dx}\right)}{3b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x] - 3*a*b*(c + d*x
)) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)
```

Maple [A]
time = 0.02, size = 85, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85
default	$\frac{\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2}{3}-\frac{a(dx+c)b}{2}-b^2c\sqrt{dx+c}+a^2\sqrt{dx+c}\right)}{b^3}-\frac{2a\left(-b^2c+a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(1/b^3*(1/3*(d*x+c)^(3/2)*b^2-1/2*a*(d*x+c)*b-b^2*c*(d*x+c)^(1/2)+a^2
*(d*x+c)^(1/2))-a*(-b^2*c+a^2)/b^4*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.29, size = 81, normalized size = 0.90

$$\frac{\frac{2\,(dx+c)^{\frac{3}{2}}b^2-3\,(dx+c)ab-6\,(b^2c-a^2)\,\sqrt{dx+c}}{b^3}+\frac{6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x +
c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Fricas [A]
time = 0.36, size = 71, normalized size = 0.79

$$-\frac{3\,ab^2dx-6\,(ab^2c-a^3)\log\left(\sqrt{dx+c}\,b+a\right)-2\,(b^3dx-2\,b^3c+3\,a^2b)\sqrt{dx+c}}{3\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x
 - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)
```

Sympy [A]
time = 2.25, size = 109, normalized size = 1.21

$$\begin{cases}\frac{2\left(-\frac{a(c+dx)}{2b^2d}-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3d}+\frac{(c+dx)^{\frac{3}{2}}}{3bd}+\frac{\left(a^2-b^2c\right)\sqrt{c+dx}}{b^3d}\right)}{d} & \text{for}\ d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2*d) - a*(a**2 - b**2*c)*Piecewise((sqrt(c
 + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d) + (c + d
```

```
*x)**(3/2)/(3*b*d) + (a**2 - b**2*c)*sqrt(c + d*x)/(b**3*d))/d, Ne(d, 0)),
(x**2/(2*(a + b*sqrt(c))), True))
```

Giac [A]
time = 4.04, size = 105, normalized size = 1.17

$$\frac{\frac{6\,(ab^2c-a^3)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}+\frac{2\,(dx+c)^{\frac{3}{2}}b^2d^2-6\,\sqrt{dx+c}\,b^2cd^2-3\,(dx+c)abd^2+6\,\sqrt{dx+c}\,a^2d^2}{b^3d^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c)
^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d
*x + c)*a^2*d^2)/(b^3*d^3))/d
```

Mupad [B]
time = 0.05, size = 89, normalized size = 0.99

$$\frac{2\,{(c+d\,x)}^{3/2}}{3\,b\,d^2}-\left(\frac{2\,c}{b\,d^2}-\frac{2\,a^2}{b^3\,d^2}\right)\,\sqrt{c+d\,x}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(2\,a^3-2\,a\,b^2\,c\right)}{b^4\,d^2}-\frac{a\,x}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d*
x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (a*
x)/(b^2*d)
```

3.635 $\int \frac{1}{a+b\sqrt{c+dx}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

```
[Out] -2*a*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*(d*x+c)^(1/2)/b/d
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {253, 196, 45}

$$\frac{2\sqrt{c+dx}}{bd}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^(-1),x]
```

```
[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{a+b\sqrt{c+dx}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx, x, c+dx\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\sqrt{c+dx}}{bd} - \frac{2a\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.98

$$\frac{2b\sqrt{c+dx}-2a\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]
[Out] (2*b*Sqrt[c + d*x] - 2*a*Log[b*d*(a + b*Sqrt[c + d*x])])/(b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(37) = 74.
time = 0.02, size = 87, normalized size = 2.12

method	result	size
derivativedivides	$\frac{\frac{2\sqrt{dx+c}}{b}-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{b^2}}{d}$	36
default	$\frac{2\sqrt{dx+c}}{bd}-\frac{a\ln\left(a+b\sqrt{dx+c}\right)}{b^2d}+\frac{a\ln\left(-a+b\sqrt{dx+c}\right)}{b^2d}-\frac{a\ln\left(b^2dx+b^2c-a^2\right)}{b^2d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2*(d*x+c)^(1/2)/b/d-a*ln(a+b*(d*x+c)^(1/2))/b^2/d+1/b^2/d*a*ln(-a+b*(d*x+c)
^(1/2))-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d
```

Maxima [A]
time = 0.31, size = 35, normalized size = 0.85

$$-\frac{2\left(\frac{a\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}-\frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.80

$$-\frac{2\left(a\log\left(\sqrt{dx+c}\,b+a\right)-\sqrt{dx+c}\,b\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)
```

Sympy [A]
time = 0.26, size = 49, normalized size = 1.20

$$\begin{cases}\frac{x}{a} & \text{for } b=0\wedge(b=0\vee d=0)\\ \frac{x}{a+b\sqrt{c}} & \text{for } d=0\\ -\frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{b^2d}+\frac{2\sqrt{c+dx}}{bd} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d
, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), Tru
e))
```

Giac [A]
time = 3.89, size = 38, normalized size = 0.93

$$-\frac{2\,a\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2d}+\frac{2\,\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)
```

Mupad [B]
time = 0.05, size = 33, normalized size = 0.80

$$-\frac{2\left(a\,\ln\left(a+b\,\sqrt{c+d\,x}\right)-b\,\sqrt{c+d\,x}\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] -(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)
```

3.636 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=82

$$\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

```
[Out] a*ln(x)/(-b^2*c+a^2)-2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2*b*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {378, 1412, 815, 649, 212, 266}

$$-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c) + (a*Log[x])/(a^
2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)}\,dx, x, \sqrt{c+dx}\right)\\
&= 2\text{Subst}\left(\int \left(-\frac{ab}{\left(a^2-b^2c\right)(a+bx)}+\frac{bc-ax}{\left(a^2-b^2c\right)\left(c-x^2\right)}\right)dx, x, \sqrt{c+dx}\right)\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}+\frac{2\text{Subst}\left(\int \frac{bc-ax}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}\\
&= -\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}-\frac{(2a)\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x, \sqrt{c+dx}\right)}{a^2-b^2c}+\frac{(2bc)\text{Subst}\left(\right.}{}\\
&= \frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c}+\frac{a\log(x)}{a^2-b^2c}-\frac{2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.76

$$\frac{2b\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+a\log(-dx)-2a\log\left(a+b\sqrt{c+dx}\right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[-(d*x)] - 2*a*Log[a + b
*Sqrt[c + d*x]])/(a^2 - b^2*c)
```

Maple [A]
time = 0.04, size = 69, normalized size = 0.84

method	result	size
derivativedivides	$-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}+\frac{a\ln(-dx)+2b\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2}$	69
default	$-\frac{2a\ln\left(a+b\sqrt{dx+c}\right)}{-b^2c+a^2}+\frac{a\ln(-dx)+2b\sqrt{c}\ \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)+2/(-b^2*c+a^2)*(1/2*a*ln(-d*x)+b*c^
(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))
```

Maxima [A]
time = 0.53, size = 95, normalized size = 1.16

$$\frac{b\sqrt{c}\ \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2}-\frac{a\log(dx)}{b^2c-a^2}+\frac{2\,a\log\left(\sqrt{dx+c}\,b+a\right)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c -
 a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c - a^
2)
```

Fricas [A]
time = 0.38, size = 125, normalized size = 1.52

$$\left[\frac{b\sqrt{c}\ \log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\,a\log\left(\sqrt{dx+c}\,b+a\right)-a\log(x)}{b^2c-a^2},\ \frac{2\,b\sqrt{-c}\ \arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2\,a\log\left(\sqrt{dx+c}\,b+a\right)-a\log(x)}{b^2c-a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a*log(sqrt(d*x
 + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + 2*a*log(sqrt(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]
```

Sympy [A]
time = 5.89, size = 85, normalized size = 1.04

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{a^2-b^2c}-\frac{2\left(-\frac{a\log\left(-dx\right)}{2}+\frac{bc\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{a^2-b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b,
True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d*x)/sqrt(-c
))/sqrt(-c))/(a**2 - b**2*c)
```

Giac [A]
time = 4.83, size = 88, normalized size = 1.07

$$\frac{2\,ab\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^3c-a^2b}+\frac{2\,bc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^2c-a^2\right)\sqrt{-c}}-\frac{a\log\left(dx\right)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x
 + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)
```

Mupad [B]
time = 3.28, size = 181, normalized size = 2.21

$$\frac{\ln\left(2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c+6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a+b\,\sqrt{c}}+\frac{\ln\left(-2\,b^3\,c^{3/2}-2\,b^3\,c\,\sqrt{c+d\,x}-6\,a\,b^2\,c-6\,a\,b^2\,\sqrt{c}\,\sqrt{c+d\,x}\right)}{a-b\,\sqrt{c}}+\frac{2\,a\,\ln\left(4\,b^5\,c^2\,\sqrt{c+d\,x}-36\,a^3\,b^2\,c+4\,a\,b^4\,c^2-36\,a^2\,b^3\,c\,\sqrt{c+d\,x}\right)}{b^2\,c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*(
c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(
1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2*a*
log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3*c*(
c + d*x)^(1/2)))/(b^2*c - a^2)
```

3.637 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x}+\frac{b(a^2+b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2-b^2c\right)^2}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}$$

```
[Out] a*b^2*d*ln(x)/(-b^2*c+a^2)^2-2*a*b^2*d*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^2
+b*(b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2))/(-b^2*c+a^2)^2/c^(1/2)+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x
```

Rubi [A]

time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$-\frac{a-b\sqrt{c+dx}}{x\,(a^2-b^2c)}+\frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2}-\frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2}+\frac{bd(a^2+b^2c)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2-b^2c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqr
t[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^
2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx = d\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (2d)\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{d\text{Subst}\left(\int \frac{-abc+b^2cx}{(a+bx)(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{d\text{Subst}\left(\int \left(-\frac{2ab^3c}{(a^2-b^2c)(a+bx)} - \frac{bc(a^2+b^2c-2abx)}{(-a^2+b^2c)(c-x^2)}\right)\,dx, x, \sqrt{c+c}\right.}{c\,(a^2-b^2c)}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} + \frac{(bd)\text{Subst}\left(\int \frac{a^2+b^2c-2abx}{c-x^2}\,dx\right.}{\left(a^2-b^2c\right)^2}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} - \frac{2ab^2d\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^2} - \frac{(2ab^2d)\,\text{Subst}\left(\int \frac{x}{c-x^2}\,dx, x,\right.}{\left(a^2-b^2c\right)^2}$$
$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x} + \frac{b(a^2+b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,\left(a^2-b^2c\right)^2} + \frac{ab^2d\log(x)}{\left(a^2-b^2c\right)^2} - \frac{2ab^2d}{}$$

Mathematica [A]
time = 0.18, size = 118, normalized size = 0.91

$$\frac{b(a^2+b^2c)\,dx\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(-\left((a^2-b^2c)\left(a-b\sqrt{c+dx}\right)\right)+ab^2dx\log(-dx)-2ab^2dx\log\left(a+b\sqrt{c+dx}\right)\right)}{\sqrt{c}\,\left(a^2-b^2c\right)^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (b*(a^2 + b^2*c)*d*x*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*(-((a^2 - b^2
*c)*(a - b*Sqrt[c + d*x])) + a*b^2*d*x*Log[-(d*x)] - 2*a*b^2*d*x*Log[a + b*
Sqrt[c + d*x]]))/(Sqrt[c]*(a^2 - b^2*c)^2*x)
```

Maple [A]
time = 0.04, size = 128, normalized size = 0.98

method	result

derivativedivides	$2d\left(-\frac{a\,b^2\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}+\frac{-\frac{\left(\frac{1}{2}b^3c-\frac{1}{2}a^2b\right)\sqrt{dx+c}-\frac{a\,b^2c}{2}+\frac{a^3}{2}}{dx}+\frac{b\left(ab\ln(-dx)+\frac{\left(b^2c+a^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx}}{\sqrt{}}\right.}{\sqrt{c}}\right.}{2}}{\left(-b^2c+a^2\right)^2}\right.$
default	$2d\left(-\frac{a\,b^2\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^2c+a^2\right)^2}+\frac{-\frac{\left(\frac{1}{2}b^3c-\frac{1}{2}a^2b\right)\sqrt{dx+c}-\frac{a\,b^2c}{2}+\frac{a^3}{2}}{dx}+\frac{b\left(ab\ln(-dx)+\frac{\left(b^2c+a^2\right)\operatorname{arctanh}\left(\frac{\sqrt{dx}}{\sqrt{}}\right.}{\sqrt{c}}\right.}{2}}{\left(-b^2c+a^2\right)^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(-a*b^2/(-b^2*c+a^2)^2*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^2*(-((1/2*b
^3*c-1/2*a^2*b)*(d*x+c)^(1/2)-1/2*a*b^2*c+1/2*a^3)/d/x+1/2*b*(a*b*ln(-d*x)+
(b^2*c+a^2)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Maxima [A]

time = 0.51, size = 191, normalized size = 1.47

$$\frac{1}{2}\left(\frac{2\,ab^2\log\left(dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{4\,ab^2\log\left(\sqrt{dx+c}\,b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3c+a^2b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{c}}+\frac{2\left(\sqrt{dx+c}\,b-a\right)}{b^2c^2-a^2c-\left(b^2c-a^2\right)\left(dx+c\right)}\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*b^2*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 4*a*b^2*log(sqrt(d*x
+ c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c + a^2*b)*log((sqrt(d*x +
 c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sq
rt(c)) + 2*(sqrt(d*x + c)*b - a)/(b^2*c^2 - a^2*c - (b^2*c - a^2)*(d*x + c)
))*d
```

Fricas [A]

time = 0.40, size = 283, normalized size = 2.18

$$\left[-\frac{4\,ab^2cdx\log\left(\sqrt{dx+c}\,b+a\right)-2\,ab^2cdx\log\left(x\right)-2\,ab^2c^2-\left(b^3c+a^2b\right)\sqrt{c}\,dx\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\,a^3c+2\left(b^3c^2-a^2bc\right)\sqrt{dx+c}}{2\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x},\ -\frac{2\,ab^2cdx\log\left(\sqrt{dx+c}\,b+a\right)-ab^2cdx\log\left(x\right)-ab^2c^2+\left(b^3c+a^2b\right)\sqrt{-c}\,dx\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)+a^3c+\left(b^3c^2-a^2bc\right)\sqrt{dx+c}}{\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*
b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) +
2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*
b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x
*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqr
t(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2
*c^2 + a^4*c)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)
```

Giac [A]
time = 3.67, size = 191, normalized size = 1.47

$$-\frac{2\,ab^3d\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^5c^2-2\,a^2b^3c+a^4b}+\frac{ab^2d\log\left(-dx\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{\left(b^3cd+a^2bd\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\left(b^4c^2-2\,a^2b^2c+a^4\right)\sqrt{-c}}+\frac{ab^2cd-a^3d-\left(b^3cd-a^2bd\right)\sqrt{dx+c}}{\left(b^2c-a^2\right)^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) +
a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arcta
n(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*b^2
*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)
```

Mupad [B]
time = 3.59, size = 220, normalized size = 1.69

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d-b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3}+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)\,\left(4\,a\,b^2\,c\,d+b\,\sqrt{c}\,d\,\left(2\,a^2+2\,c\,b^2\right)\right)}{4\,a^4\,c-8\,a^2\,b^2\,c^2+4\,b^4\,c^3}+\frac{\frac{a\,d}{b^2\,c-a^2}-\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}}{d\,x}-\frac{2\,a\,b^2\,d\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{{\left(b^2\,c-a^2\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((c + d*x)^(1/2) - c^(1/2))*(4*a*b^2*c*d - b*c^(1/2)*d*(2*b^2*c + 2*a^2
)))/(4*a^4*c + 4*b^4*c^3 - 8*a^2*b^2*c^2) + (log((c + d*x)^(1/2) + c^(1/2))
*(4*a*b^2*c*d + b*c^(1/2)*d*(2*b^2*c + 2*a^2)))/(4*a^4*c + 4*b^4*c^3 - 8*a^
2*b^2*c^2) + ((a*d)/(b^2*c - a^2) - (b*d*(c + d*x)^(1/2))/(b^2*c - a^2))/(d
*x) - (2*a*b^2*d*log(a + b*(c + d*x)^(1/2)))/(b^2*c - a^2)^2
```

3.638 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$

Optimal. Leaf size=204

$$-\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x}-\frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}+\frac{ab^4d^2}{(a^2-}$$

```
[Out] -1/4*b*(-3*b^4*c^2-6*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/c^(3
/2)/(-b^2*c+a^2)^3+a*b^4*d^2*ln(x)/(-b^2*c+a^2)^3-2*a*b^4*d^2*ln(a+b*(d*x+c
)^(1/2))/(-b^2*c+a^2)^3+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x^2-1/4*b*d*(
4*a*b*c-(3*b^2*c+a^2)*(d*x+c)^(1/2))/c/(-b^2*c+a^2)^2/x
```

Rubi [A]
time = 0.20, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$-\frac{a-b\sqrt{c+dx}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4cx\left(a^2-b^2c\right)^2}+\frac{ab^4d^2\log(x)}{\left(a^2-b^2c\right)^3}-\frac{2ab^4d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^3}-\frac{bd^2\left(a^4-6a^2b^2c-3b^4c^2\right)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}\left(a^2-b^2c\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -1/2*(a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b
^2*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b
^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (
a*b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx = d^2\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)(-c+x)^3}\,dx, x, c+dx\right)$$

$$= (2d^2)\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\text{Subst}\left(\int \frac{-abc+3b^2cx}{(a+bx)(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\text{Subst}\left(\int \frac{abc(a^2-5b^2c)\cdot}{(a+bx)}\right.}{4c^2}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} + \frac{d^2\text{Subst}\left(\int \left(-\frac{8ab^5}{(a^2-b^2c)(}\right.\right.}{}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+}\right.}{\left(a^2-b^2c\right)^3}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{2ab^4d^2\log\left(a+b\sqrt{c+}\right.}{\left(a^2-b^2c\right)^3}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\left(4abc-\left(a^2+3b^2c\right)\sqrt{c+dx}\right)}{4c\left(a^2-b^2c\right)^2x} - \frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)}{4c^{3/2}\left(a\right.}$$

Mathematica [A]
time = 0.38, size = 198, normalized size = 0.97

$$\frac{b\left(a^4-6a^2b^2c-3b^4c^2\right)d^2x^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\sqrt{c}\left(\left(a^2-b^2c\right)\left(2a^3c-2ab^2c(c-2dx)+b^3c(2c-3dx)\sqrt{c+dx}-a^2b\sqrt{c+dx}\,(2c+dx)\right)-4ab^4cd^2x^2\log(-dx)+8ab^4cd^2x^2\log\left(a+b\sqrt{c+dx}\right)\right)}{4c^{3/2}\left(-a^2+b^2c\right)^3x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] (b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] +
 Sqrt[c]*((a^2 - b^2*c)*(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d
*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Lo
g[-(d*x)] + 8*a*b^4*c*d^2*x^2*Log[a + b*Sqrt[c + d*x]]))/(4*c^(3/2)*(-a^2 +
 b^2*c)^3*x^2)
```

Maple [A]
time = 0.05, size = 227, normalized size = 1.11

method	result
derivativedivides	$2d^2\left(-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}b^2a^3\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{dx+c}+\frac{3a\,b^4c^2}{4}-a^3}{d^2x^2}}{\left(-b^2c+a^2\right)^3}\right.$
default	$2d^2\left(-\frac{\frac{-\frac{b\left(-3b^4c^2+2a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{8c}+\left(-\frac{1}{2}a\,b^4c+\frac{1}{2}b^2a^3\right)(dx+c)+\left(\frac{3}{4}a^2b^3c-\frac{1}{8}b\,a^4-\frac{5}{8}c^2b^5\right)\sqrt{dx+c}+\frac{3a\,b^4c^2}{4}-a^3}{d^2x^2}}{\left(-b^2c+a^2\right)^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/(-b^2*c+a^2)^3*((-1/8*b*(-3*b^4*c^2+2*a^2*b^2*c+a^4)/c*(d*x+c)^(3
/2)+(-1/2*a*b^4*c+1/2*b^2*a^3)*(d*x+c)+(3/4*a^2*b^3*c-1/8*b*a^4-5/8*c^2*b^5
)*(d*x+c)^(1/2)+3/4*a*b^4*c^2-a^3*b^2*c+1/4*a^5)/d^2/x^2+1/8*b/c*(-4*a*b^3*
c*ln(-d*x)+(-3*b^4*c^2-6*a^2*b^2*c+a^4)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/
2))))-a*b^4/(-b^2*c+a^2)^3*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]

time = 0.52, size = 367, normalized size = 1.80

$$-\frac{1}{8}\left(\frac{8\,ab^4\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{16\,ab^4\log\left(\sqrt{dx+c}\,b+a\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}-\frac{\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{c}}+\frac{2\left(4\left(dx+c\right)ab^2c-6\,ab^2c^2+2\,a^3c-\left(3\,b^3c+a^2b\right)\left(dx+c\right)^{\frac{3}{2}}+\left(5\,b^3c^2-a^2bc\right)\sqrt{dx+c}\right)}{b^4c^5-2\,a^2b^2c^4+a^4c^3+\left(b^4c^3-2\,a^2b^2c^2+a^4c\right)\left(dx+c\right)^2-2\left(b^4c^4-2\,a^2b^2c^3+a^4c^2\right)\left(dx+c\right)}\right)d^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*a
*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)
 - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*
x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(
c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*b)*(d
*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2*b^2*c
^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^4*c^4 -
 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c)))*d^2
```

Fricas [A]
time = 0.74, size = 534, normalized size = 2.62

$$\left[\frac{16\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}\,b+a\right)-8\,ab^4c^2d^2x^2\log\left(x\right)+4\,ab^4c^4-8\,a^3b^2c^3+4\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{c}\,d^2x^2\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-8\,\left(ab^4c^3-a^3b^2c^2\right)dx-2\,\left(2\,b^5c^4-4\,a^2b^3c^3+2\,a^4bc^2-\left(3\,b^5c^3-2\,a^2b^3c^2-a^4bc\right)dx\right)\sqrt{dx+c}}{8\,\left(b^6c^5-3\,a^2b^4c^4+3\,a^4b^2c^3-a^6c^2\right)x^2},\ \frac{8\,ab^4c^2d^2x^2\log\left(\sqrt{dx+c}\,b+a\right)-4\,ab^4c^2d^2x^2\log\left(x\right)+2\,ab^4c^4-4\,a^3b^2c^3+2\,a^5c^2+\left(3\,b^5c^2+6\,a^2b^3c-a^4b\right)\sqrt{-c}\,d^2x^2\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)-4\,\left(ab^4c^3-a^3b^2c^2\right)dx-\left(2\,b^5c^4-4\,a^2b^3c^3+2\,a^4bc^2-\left(3\,b^5c^3-2\,a^2b^3c^2-a^4bc\right)dx\right)\sqrt{dx+c}}{4\,\left(b^6c^5-3\,a^2b^4c^4+3\,a^4b^2c^3-a^6c^2\right)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*l
og(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c
- a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(
a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 -
 (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^
2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqr
t(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^
3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(s
qrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4
*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqr
t(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))), x)
```

Giac [A]
time = 4.13, size = 375, normalized size = 1.84

$$\frac{2\,ab^5d^2\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^7c^3-3\,a^2b^5c^2+3\,a^4b^3c-a^6b}-\frac{ab^4d^2\log\left(dx\right)}{b^6c^3-3\,a^2b^4c^2+3\,a^4b^2c-a^6}+\frac{\left(3\,b^5c^2d^2+6\,a^2b^3cd^2-a^4bd^2\right)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{4\,\left(b^6c^4-3\,a^2b^4c^3+3\,a^4b^2c^2-a^6c\right)\sqrt{-c}}+\frac{6\,ab^4c^3d^2-8\,a^3b^2c^2d^2+2\,a^5cd^2+\left(3\,b^5c^2d^2-2\,a^2b^3cd^2-a^4bd^2\right)\left(dx+c\right)^{\frac{3}{2}}-4\,\left(ab^4c^2d^2-a^3b^2cd^2\right)\left(dx+c\right)-\left(5\,b^5c^3d^2-6\,a^2b^3c^2d^2+a^4bcd^2\right)\sqrt{dx+c}}{4\,\left(b^2c-a^2\right)^3cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*
b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c
- a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x
+ c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c))
 + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2*d^2
- 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a^3*b^2
```

```
*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(
d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)
```

Mupad [B]
time = 5.01, size = 1094, normalized size = 5.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)
```

```
[Out] (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2*(b^2*c - a^2)^4) -
 (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b
*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*
(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(3*b^
4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^3) -
 (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c - a^2)^2))*(3*b^4*
c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(b^2*c - a^2)^3))*(8
*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^(1/2) + 3*b^5*c^2*d^2*(c^3)^(1/2) + 6*a^2*
b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c
^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - ((a^2
*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^(3/2))/(4*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
+ (b*d^2*(5*b^2*c - a^2)*(c + d*x)^(1/2))/(4*(a^4 + b^4*c^2 - 2*a^2*b^2*c))
 + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c + d*x)^2 - 2*c*
(c + d*x) + c^2) + (log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^(1/2))/(16*c^2
*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b
^2*c - a^2)^4) - (b*d^2*(c^3)^(1/2)*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c
- a^2)) + (b^2*d^2*(c^3)^(1/2)*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c
+ d*x)^(1/2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(4*c^3
*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^(1/2))/(2*c*(b^2*c
 - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^(1/2)))/(8*c^3*(
b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 + a^4*b*d^2*(c^3)^(1/2) - 3*b^5*c^2*d^2*(
c^3)^(1/2) - 6*a^2*b^3*c*d^2*(c^3)^(1/2)))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^
2*c^4 + 3*a^2*b^4*c^5)) + (2*a*b^4*d^2*log(a + b*(c + d*x)^(1/2)))/(b^2*c -
 a^2)^3
```

3.639 $\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=240

$$\frac{(5a^4-9a^2b^2c+3b^4c^2)\,x}{b^6d^3}-\frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a(2a^2-3b^2c)\,(c+dx)^{3/2}}{3b^5d^4}+\frac{3(a^2-b^2c)\,(c+dx)^2}{2b^4d^4}-4a($$

```
[Out] (3*b^4*c^2-9*a^2*b^2*c+5*a^4)*x/b^6/d^3-4/3*a*(-3*b^2*c+2*a^2)*(d*x+c)^(3/2
)/b^5/d^4+3/2*(-b^2*c+a^2)*(d*x+c)^2/b^4/d^4-4/5*a*(d*x+c)^(5/2)/b^3/d^4+1/
3*(d*x+c)^3/b^2/d^4+2*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*ln(a+b*(d*x+c)^(1/2))/b
^8/d^4-12*a*(-b^2*c+a^2)^2*(d*x+c)^(1/2)/b^7/d^4+2*a*(-b^2*c+a^2)^3/b^8/d^4
/(a+b*(d*x+c)^(1/2))
```

Rubi [A]
time = 0.20, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {378, 1412, 786}

$$\frac{2a(a^2-b^2c)^3}{b^8d^4\left(a+b\sqrt{c+dx}\right)}+\frac{2(7a^2-b^2c)\,(a^2-b^2c)^2\log\left(a+b\sqrt{c+dx}\right)}{b^8d^4}-\frac{12a(a^2-b^2c)^2\sqrt{c+dx}}{b^7d^4}-\frac{4a(2a^2-3b^2c)\,(c+dx)^{3/2}}{3b^5d^4}+\frac{3(a^2-b^2c)\,(c+dx)^2}{2b^4d^4}+\frac{x(5a^4-9a^2b^2c+3b^4c^2)}{b^6d^3}-\frac{4a(c+dx)^{5/2}}{5b^3d^4}+\frac{(c+dx)^3}{3b^2d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqr
t[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4)
 + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3
*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqr
t[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])
/(b^8*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{6a\left(a^2-b^2c\right)^2}{b^7} + \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6} - \frac{2a\left(2a^2-3b^2c\right)x^2}{b^5} - \frac{3\left(-a^2+b^2c\right)x^3}{b^4} - \frac{2ax}{b^3}\right.\right.}{d^4}$$

$$= \frac{\left(5a^4-9a^2b^2c+3b^4c^2\right)x}{b^6d^3} - \frac{12a\left(a^2-b^2c\right)^2\sqrt{c+dx}}{b^7d^4} - \frac{4a\left(2a^2-3b^2c\right)(c+dx)^{3}}{3b^5d^4}$$

Mathematica [A]
time = 0.20, size = 283, normalized size = 1.18

$$\frac{60a^7-360a^6b\sqrt{c+dx}-30a^5b^2(13c+7dx)+10a^4b^3\sqrt{c+dx}\,(79c+7dx)-3a^2b^5\sqrt{c+dx}\,\left(163c^2+36cdx-7d^2x^2\right)+5a^3b^4\left(119c^2+76cdx-7d^2x^2\right)+5b^7\sqrt{c+dx}\,\left(11c^3+6c^2dx-3cd^2x^2+2d^3x^3\right)-ab^6\left(269c^3+162c^2dx-33cd^2x^2+14d^3x^3\right)+60\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)}{30b^8d^4\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (60*a^7 - 360*a^6*b*Sqrt[c + d*x] - 30*a^5*b^2*(13*c + 7*d*x) + 10*a^4*b^3*
Sqrt[c + d*x]*(79*c + 7*d*x) - 3*a^2*b^5*Sqrt[c + d*x]*(163*c^2 + 36*c*d*x
- 7*d^2*x^2) + 5*a^3*b^4*(119*c^2 + 76*c*d*x - 7*d^2*x^2) + 5*b^7*Sqrt[c +
d*x]*(11*c^3 + 6*c^2*d*x - 3*c*d^2*x^2 + 2*d^3*x^3) - a*b^6*(269*c^3 + 162*
c^2*d*x - 33*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c +
d*x]))
```

Maple [A]
time = 0.04, size = 277, normalized size = 1.15

method	result
derivativedivides	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2 b^3 (dx+c)^2}{4}-2a\,b^4 c(dx+c)^{\frac{3}{2}}-\frac{3b^5 c^2 (dx+c)}{2}+\frac{4a^3 b^2 (dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2 b^3 c(dx+c)}{2}+\right.}{b^7}$
default	$-\frac{2\left(-\frac{(dx+c)^3 b^5}{6}+\frac{2a(dx+c)^{\frac{5}{2}} b^4}{5}+\frac{3b^5 c(dx+c)^2}{4}-\frac{3a^2 b^3 (dx+c)^2}{4}-2a\,b^4 c(dx+c)^{\frac{3}{2}}-\frac{3b^5 c^2 (dx+c)}{2}+\frac{4a^3 b^2 (dx+c)^{\frac{3}{2}}}{3}+\frac{9a^2 b^3 c(dx+c)}{2}+\right.}{b^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^4*(-1/b^7*(-1/6*(d*x+c)^3*b^5+2/5*a*(d*x+c)^(5/2)*b^4+3/4*b^5*c*(d*x+c)
^2-3/4*a^2*b^3*(d*x+c)^2-2*a*b^4*c*(d*x+c)^(3/2)-3/2*b^5*c^2*(d*x+c)+4/3*a^
3*b^2*(d*x+c)^(3/2)+9/2*a^2*b^3*c*(d*x+c)+6*a*c^2*b^4*(d*x+c)^(1/2)-5/2*a^4
*b*(d*x+c)-12*a^3*c*b^2*(d*x+c)^(1/2)+6*a^5*(d*x+c)^(1/2))+a*(-b^6*c^3+3*a^
2*b^4*c^2-3*a^4*b^2*c+a^6)/b^8/(a+b*(d*x+c)^(1/2))+1/b^8*(-b^6*c^3+9*a^2*b^
4*c^2-15*a^4*b^2*c+7*a^6)*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]

time = 0.28, size = 251, normalized size = 1.05

$$-\frac{\frac{60\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)}{\sqrt{dx+c}\,b^9+ab^8}-\frac{10(dx+c)^3b^5-24(dx+c)^{\frac{5}{2}}ab^4-45\left(b^5c-a^2b^3\right)(dx+c)^2+40\left(3ab^4c-2a^3b^2\right)(dx+c)^{\frac{3}{2}}+30\left(3b^5c^2-9a^2b^3c+5a^4b\right)(dx+c)-360\left(ab^4c^2-2a^3b^2c+a^5\right)\sqrt{dx+c}}{b^7}+\frac{60\left(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^8}}{30\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^
9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a
^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^
5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a
^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6
)*log(sqrt(d*x + c)*b + a)/b^8)/d^4
```

Fricas [A]

time = 0.36, size = 392, normalized size = 1.63

$$\frac{10\,b^8d^4x^4+55\,b^8c^4-220\,a^2b^6c^3+195\,a^4b^4c^2+30\,a^6b^2c-60\,a^8-5\left(b^8c-7\,a^2b^6\right)d^3x^3+15\left(b^8c^2-8\,a^2b^6c+7\,a^4b^4\right)d^2x^2+5\left(17\,b^8c^3-87\,a^2b^6c^2+96\,a^4b^4c-30\,a^6b^2\right)dx-60\left(b^8c^4-10\,a^2b^6c^3+24\,a^4b^4c^2-22\,a^6b^2c+7\,a^8+\left(b^8c^3-9\,a^2b^6c^2+15\,a^4b^4c-7\,a^6b^2\right)dx\right)\log\left(\sqrt{dx+c}\,b+a\right)-4\left(6\,ab^7d^3x^3+81\,ab^7c^3-271\,a^3b^5c^2+295\,a^5b^3c-105\,a^7b-2\left(6\,ab^7c-7\,a^3b^5\right)d^2x^2+2\left(24\,ab^7c^2-61\,a^3b^5c+35\,a^5b^3\right)dx\right)\sqrt{dx+c}}{30\left(b^{10}d^5x+\left(b^{10}c-a^2b^8\right)d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*
a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^
```

```
6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -
30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^
2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log
(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2
 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*
b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^1
0*c - a^2*b^8)*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(a + b*sqrt(c + d*x))**2, x)
```

Giac [A]
time = 3.73, size = 324, normalized size = 1.35

$$-\frac{2\,(b^6c^3-9\,a^2b^4c^2+15\,a^4b^2c-7\,a^6)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^8d^4}-\frac{2\,(ab^6c^3-3\,a^3b^4c^2+3\,a^5b^2c-a^7)}{\left(\sqrt{dx+c}\,b+a\right)b^8d^4}+\frac{10\,(dx+c)^3b^{10}d^{20}-45\,(dx+c)^2b^{10}cd^{20}+90\,(dx+c)b^{10}c^2d^{20}-24\,(dx+c)^{\frac{5}{2}}ab^9d^{20}+120\,(dx+c)^{\frac{3}{2}}ab^9cd^{20}-360\,\sqrt{dx+c}\,ab^9c^2d^{20}+45\,(dx+c)^2a^2b^8d^{20}-270\,(dx+c)a^2b^8cd^{20}-80\,(dx+c)^{\frac{3}{2}}a^3b^7d^{20}+720\,\sqrt{dx+c}\,a^3b^7cd^{20}+150\,(dx+c)a^4b^6d^{20}-360\,\sqrt{dx+c}\,a^5b^5d^{20}}{30\,b^{12}d^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*b
 + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sqrt
(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c)^2
*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^20 +
 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 + 45*(
d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)^(3/2)
*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4*b^6*d^
20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)
```

Mupad [B]
time = 0.09, size = 461, normalized size = 1.92

$$\left(\frac{4\,a^3}{3\,b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{3\,b}\right)\,{\left(c+d\,x\right)}^{3/2}-\left(\frac{3\,c}{2\,b^2\,d^4}-\frac{3\,a^2}{2\,b^4\,d^4}\right)\,{\left(c+d\,x\right)}^2-\left(\frac{2\,a\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b^2}-\frac{2\,a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{6\,c^2}{b^2\,d^4}\right)}{b}+\frac{a^2\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b^2}\right)\,\sqrt{c+d\,x}+\frac{{\left(c+d\,x\right)}^3}{3\,b^2\,d^4}+\frac{2\,\left(a^7-3\,a^5\,b^2\,c+3\,a^3\,b^4\,c^2-a\,b^6\,c^3\right)}{b\,\left(b^8\,d^4\,\sqrt{c+d\,x}+a\,b^7\,d^4\right)}+d\,x\,\left(\frac{a^2\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{2\,b^2}-\frac{a\,\left(\frac{4\,a^3}{b^5\,d^4}+\frac{2\,a\,\left(\frac{6\,c}{b^2\,d^4}-\frac{6\,a^2}{b^4\,d^4}\right)}{b}\right)}{b}+\frac{3\,c^2}{b^2\,d^4}\right)+\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(14\,a^6-30\,a^4\,b^2\,c+18\,a^2\,b^4\,c^2-2\,b^6\,c^3\right)}{b^8\,d^4}-\frac{4\,a\,{\left(c+d\,x\right)}^{5/2}}{5\,b^3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(3*b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(3*b))*(
c + d*x)^(3/2) - ((3*c)/(2*b^2*d^4) - (3*a^2)/(2*b^4*d^4))*(c + d*x)^2 - ((
2*a*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b^2 - (2*a*((4*a^3)/(b^5*d
^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (6*c^2)/(b^2*d^4)
))/b + (a^2*((4*a^3)/(b^5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4))
)/b))/b^2)*(c + d*x)^(1/2) + (c + d*x)^3/(3*b^2*d^4) + (2*(a^7 - 3*a^5*b^2*
c - a*b^6*c^3 + 3*a^3*b^4*c^2))/(b*(b^8*d^4*(c + d*x)^(1/2) + a*b^7*d^4)) +
 d*x*((a^2*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/(2*b^2) - (a*((4*a^3)/(b^
5*d^4) + (2*a*((6*c)/(b^2*d^4) - (6*a^2)/(b^4*d^4)))/b))/b + (3*c^2)/(b^2*d
^4)) + (log(a + b*(c + d*x)^(1/2))*(14*a^6 - 2*b^6*c^3 - 30*a^4*b^2*c + 18*
a^2*b^4*c^2))/(b^8*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4)
```

3.640 $\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(3a^2-2b^2c)\,x}{b^4d^2}-\frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}+\frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}+\frac{2(5a^4-6a^2b^2c}{}$$

```
[Out] (-2*b^2*c+3*a^2)*x/b^4/d^2-4/3*a*(d*x+c)^(3/2)/b^3/d^3+1/2*(d*x+c)^2/b^2/d^
3+2*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a+b*(d*x+c)^(1/2))/b^6/d^3-8*a*(-b^2*c+a
^2)*(d*x+c)^(1/2)/b^5/d^3+2*a*(-b^2*c+a^2)^2/b^6/d^3/(a+b*(d*x+c)^(1/2))
```

Rubi [A]

time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$\frac{2a(a^2-b^2c)^2}{b^6d^3\left(a+b\sqrt{c+dx}\right)}-\frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3}+\frac{x(3a^2-2b^2c)}{b^4d^2}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\log\left(a+b\sqrt{c+dx}\right)}{b^6d^3}-\frac{4a(c+dx)^{3/2}}{3b^3d^3}+\frac{(c+dx)^2}{2b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^
3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^
2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b
^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
```

```
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^2}{(a+bx)^2}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{4a\left(a^2-b^2c\right)}{b^5} - \frac{\left(-3a^2+2b^2c\right)x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a\left(a^2-b^2c\right)^2}{b^5(a+bx)^2} + \frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right) d\right.}{d^3}$$

$$= \frac{\left(3a^2-2b^2c\right)x}{b^4d^2} - \frac{8a\left(a^2-b^2c\right)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a(}{b^6d^3\left(a\right.}$$

Mathematica [A]
time = 0.12, size = 194, normalized size = 1.17

$$\frac{12a^5-48a^4b\sqrt{c+dx}-6a^3b^2(9c+5dx)+2a^2b^3\sqrt{c+dx}\,(29c+5dx)+ab^4\left(43c^2+26cdx-5d^2x^2\right)+3b^5\sqrt{c+dx}\left(-3c^2-2cdx+d^2x^2\right)+12\left(5a^4-6a^2b^2c+b^4c^2\right)\left(a+b\sqrt{c+dx}\right)\log\left(a+b\sqrt{c+dx}\right)}{6b^6d^3\left(a+b\sqrt{c+dx}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (12*a^5 - 48*a^4*b*Sqrt[c + d*x] - 6*a^3*b^2*(9*c + 5*d*x) + 2*a^2*b^3*Sqrt
[c + d*x]*(29*c + 5*d*x) + a*b^4*(43*c^2 + 26*c*d*x - 5*d^2*x^2) + 3*b^5*Sq
rt[c + d*x]*(-3*c^2 - 2*c*d*x + d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^
2)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c
 + d*x]))
```

Maple [A]
time = 0.03, size = 161, normalized size = 0.97

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)^2b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}}b^2}{3}+b^3c(dx+c)-\frac{3a^2b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}+}{d^3}$

default	$\frac{-\frac{2\left(-\frac{(dx+c)^2 b^3}{4}+\frac{2a(dx+c)^{\frac{3}{2}} b^2}{3}+b^3 c(dx+c)-\frac{3a^2 b(dx+c)}{2}-4acb^2\sqrt{dx+c}+4a^3\sqrt{dx+c}\right)}{b^5}+\frac{2a\left(b^4c^2-2a^2b^2c+a^4\right)}{b^6\left(a+b\sqrt{dx+c}\right)}}{d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^3*(-1/b^5*(-1/4*(d*x+c)^2*b^3+2/3*a*(d*x+c)^(3/2)*b^2+b^3*c*(d*x+c)-3/2
*a^2*b*(d*x+c)-4*a*c*b^2*(d*x+c)^(1/2)+4*a^3*(d*x+c)^(1/2))+a*(b^4*c^2-2*a^
2*b^2*c+a^4)/b^6/(a+b*(d*x+c)^(1/2))+1/b^6*(b^4*c^2-6*a^2*b^2*c+5*a^4)*ln(a
+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.28, size = 158, normalized size = 0.95

$$\frac{\frac{12\left(ab^4c^2-2a^3b^2c+a^5\right)}{\sqrt{dx+c}\,b^7+ab^6}+\frac{3(dx+c)^2b^3-8(dx+c)^{\frac{3}{2}}ab^2-6\left(2b^3c-3a^2b\right)(dx+c)+48\left(ab^2c-a^3\right)\sqrt{dx+c}}{b^5}+\frac{12\left(b^4c^2-6a^2b^2c+5a^4\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^6}}{6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(sqrt(d*x + c)*b^7 + a*b^6) + (3*(d
*x + c)^2*b^3 - 8*(d*x + c)^(3/2)*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) +
 48*(a*b^2*c - a^3)*sqrt(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)
*log(sqrt(d*x + c)*b + a)/b^6)/d^3
```

Fricas [A]
time = 0.35, size = 269, normalized size = 1.62

$$\frac{3b^6d^3x^3-9b^6c^3+15a^2b^4c^2+6a^4b^2c-12a^6-3\left(b^6c-5a^2b^4\right)d^2x^2-3\left(5b^6c^2-14a^2b^4c+6a^4b^2\right)dx+12\left(b^6c^3-7a^2b^4c^2+11a^4b^2c-5a^6+\left(b^6c^2-6a^2b^4c+5a^4b^2\right)dx\right)\log\left(\sqrt{dx+c}\,b+a\right)-4\left(2ab^5d^2x^2-13ab^5c^2+28a^3b^3c-15a^5b-2\left(4ab^5c-5a^3b^3\right)dx\right)\sqrt{dx+c}}{6\left(b^8d^4x+\left(b^8c-a^2b^6\right)d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*
(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x
+ 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4
*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b
^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x
+ c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```

Giac [A]
time = 3.14, size = 191, normalized size = 1.15

$$\frac{2\,(b^4c^2-6\,a^2b^2c+5\,a^4)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^6d^3}+\frac{2\,(ab^4c^2-2\,a^3b^2c+a^5)}{\left(\sqrt{dx+c}\,b+a\right)b^6d^3}+\frac{3\,(dx+c)^2b^6d^9-12\,(dx+c)b^6cd^9-8\,(dx+c)^{\frac{3}{2}}ab^5d^9+48\,\sqrt{dx+c}\,ab^5cd^9+18\,(dx+c)a^2b^4d^9-48\,\sqrt{dx+c}\,a^3b^3d^9}{6\,b^8d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3) +
 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((sqrt(d*x + c)*b + a)*b^6*d^3) + 1/6*(3
*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^(3/2)*a*b^5*d^9
 + 48*sqrt(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*sqrt(d*x +
c)*a^3*b^3*d^9)/(b^8*d^12)
```

Mupad [B]
time = 3.20, size = 197, normalized size = 1.19

$$\left(\frac{4\,a^3}{b^5\,d^3}+\frac{2\,a\,\left(\frac{4\,c}{b^2\,d^3}-\frac{6\,a^2}{b^4\,d^3}\right)}{b}\right)\,\sqrt{c+d\,x}+\frac{2\,(a^5-2\,a^3\,b^2\,c+a\,b^4\,c^2)}{b\,\left(b^6\,d^3\,\sqrt{c+d\,x}+a\,b^5\,d^3\right)}+\frac{(c+d\,x)^2}{2\,b^2\,d^3}-d\,x\,\left(\frac{2\,c}{b^2\,d^3}-\frac{3\,a^2}{b^4\,d^3}\right)-\frac{4\,a\,(c+d\,x)^{3/2}}{3\,b^3\,d^3}+\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,(10\,a^4-12\,a^2\,b^2\,c+2\,b^4\,c^2)}{b^6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] ((4*a^3)/(b^5*d^3) + (2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3)))/b)*(c + d*
x)^(1/2) + (2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2))/(b*(b^6*d^3*(c + d*x)^(1/2)
+ a*b^5*d^3)) + (c + d*x)^2/(2*b^2*d^3) - d*x*((2*c)/(b^2*d^3) - (3*a^2)/(b
^4*d^3)) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (log(a + b*(c + d*x)^(1/2))*
(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c))/(b^6*d^3)
```

3.641 $\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2} dx$

Optimal. Leaf size=95

$$\frac{x}{b^2d} - \frac{4a\sqrt{c+dx}}{b^3d^2} + \frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)} + \frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

```
[Out] x/b^2/d+2*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2))/b^4/d^2-4*a*(d*x+c)^(1/2)/b^
3/d^2+2*a*(-b^2*c+a^2)/b^4/d^2/(a+b*(d*x+c)^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)} + \frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2} - \frac{4a\sqrt{c+dx}}{b^3d^2} + \frac{x}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a
 + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^
2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
```

```
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+b\sqrt{c+dx}\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{\left(a+b\sqrt{x}\right)^2}\,dx,x,c+dx\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{(a+bx)^2}\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{2\text{Subst}\left(\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}+\frac{-a^3+ab^2c}{b^3(a+bx)^2}+\frac{3a^2-b^2c}{b^3(a+bx)}\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}\\
&= \frac{x}{b^2d}-\frac{4a\sqrt{c+dx}}{b^3d^2}+\frac{2a(a^2-b^2c)}{b^4d^2\left(a+b\sqrt{c+dx}\right)}+\frac{2(3a^2-b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 108, normalized size = 1.14

$$\frac{2a^3-2ab^2c-4a^2b\sqrt{c+dx}-3ab^2(c+dx)+b^3(c+dx)^{3/2}}{b^4d^2\left(a+b\sqrt{c+dx}\right)}-\frac{2(-3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]
```

```
[Out] (2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c + d
*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) - (2*(-3*a^2 + b^2*c)*Log[a + b*
Sqrt[c + d*x]])/(b^4*d^2)
```

Maple [A]
time = 0.03, size = 87, normalized size = 0.92

method	result	size
derivativedivides	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^3}+\frac{2a\left(-b^2c+a^2\right)}{b^4\left(a+b\sqrt{dx+c}\right)}+\frac{2\left(-b^2c+3a^2\right)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	87

default	$\frac{-\frac{2\left(-\frac{(dx+c)b}{2}+2a\sqrt{dx+c}\right)}{b^3}+\frac{2a(-b^2c+a^2)}{b^4\left(a+b\sqrt{dx+c}\right)}+\frac{2(-b^2c+3a^2)\ln\left(a+b\sqrt{dx+c}\right)}{b^4}}{d^2}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d^2*(-1/b^3*(-1/2*(d*x+c)*b+2*a*(d*x+c)^(1/2))+a*(-b^2*c+a^2)/b^4/(a+b*(d
*x+c)^(1/2))+1/b^4*(-b^2*c+3*a^2)*ln(a+b*(d*x+c)^(1/2)))
```

Maxima [A]
time = 0.28, size = 90, normalized size = 0.95

$$-\frac{\frac{2\,(ab^2c-a^3)}{\sqrt{dx+c}\,b^5+ab^4}-\frac{(dx+c)b-4\sqrt{dx+c}\,a}{b^3}+\frac{2\,(b^2c-3\,a^2)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a*b^2*c - a^3)/(sqrt(d*x + c)*b^5 + a*b^4) - ((d*x + c)*b - 4*sqrt(d*x
 + c)*a)/b^3 + 2*(b^2*c - 3*a^2)*log(sqrt(d*x + c)*b + a)/b^4)/d^2
```

Fricas [A]
time = 0.36, size = 163, normalized size = 1.72

$$\frac{b^4d^2x^2+b^4c^2+a^2b^2c-2\,a^4+(2\,b^4c-a^2b^2)dx-2\,(b^4c^2-4\,a^2b^2c+3\,a^4+(b^4c-3\,a^2b^2)dx)\log\left(\sqrt{dx+c}\,b+a\right)-2\,(2\,ab^3dx+3\,ab^3c-3\,a^3b)\sqrt{dx+c}}{b^6d^3x+(b^6c-a^2b^4)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b
^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b
 + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (
b^6*c - a^2*b^4)*d^2)
```

Sympy [A]
time = 18.60, size = 131, normalized size = 1.38

$$\begin{cases} \frac{2\left(-\frac{a\left(a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a^2} & \text{for } b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{b^3d}-\frac{2a\sqrt{c+dx}}{b^3d}+\frac{c+dx}{2b^2d}+\frac{\left(3a^2-b^2c\right)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for } b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{b^3d}\right)}{d} & \text{for } d\neq 0\\ \frac{x^2}{2\left(a+b\sqrt{c}\right)^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((2*(-a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)),
(-1/(b*(a + b*sqrt(c + d*x))), True))/(b**3*d) - 2*a*sqrt(c + d*x)/(b**3*d)
 + (c + d*x)/(2*b**2*d) + (3*a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(
b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(b**3*d))/d, Ne(d, 0)), (x**2/(
2*(a + b*sqrt(c))**2), True))
```

Giac [A]
time = 3.14, size = 102, normalized size = 1.07

$$-\frac{\frac{2\,(b^2c-3\,a^2)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^4d}-\frac{(dx+c)b^2d-4\,\sqrt{dx+c}\,abd}{b^4d^2}+\frac{2\,(ab^2c-a^3)}{\left(\sqrt{dx+c}\,b+a\right)b^4d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) - ((d*x + c)*b^2*
d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*b
+ a)*b^4*d))/d
```

Mupad [B]
time = 0.06, size = 98, normalized size = 1.03

$$\frac{x}{b^2\,d}+\frac{2\,(a^3-a\,b^2\,c)}{b\,\left(b^4\,d^2\,\sqrt{c+d\,x}+a\,b^3\,d^2\right)}-\frac{\ln\left(a+b\,\sqrt{c+d\,x}\right)\,(2\,b^2\,c-6\,a^2)}{b^4\,d^2}-\frac{4\,a\,\sqrt{c+d\,x}}{b^3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^(1/2) + a*b^3*d^2)) -
 (log(a + b*(c + d*x)^(1/2))*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x)^
(1/2))/(b^3*d^2)
```

3.642 $\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=47

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

[Out] 2*ln(a+b*(d*x+c)^(1/2))/b^2/d+2*a/b^2/d/(a+b*(d*x+c)^(1/2))

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^(-2),x]

[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sqrt{c+dx}\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx,x,c+dx\right)}{d}$$
$$=\frac{2\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx,x,\sqrt{c+dx}\right)}{d}$$
$$=\frac{2\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)\,dx,x,\sqrt{c+dx}\right)}{d}$$
$$=\frac{2a}{b^2d\left(a+b\sqrt{c+dx}\right)}+\frac{2\log\left(a+b\sqrt{c+dx}\right)}{b^2d}$$

Mathematica [A]
time = 0.04, size = 43, normalized size = 0.91

$$\frac{2\left(\frac{a}{a+b\sqrt{c+dx}}+\log\left(bd\left(a+b\sqrt{c+dx}\right)\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^(-2),x]
[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[b*d*(a + b*Sqrt[c + d*x])]))/(b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(43)=86$.
time = 0.03, size = 219, normalized size = 4.66

method	result
derivativedivides	$\frac{\frac{2\ln\left(a+b\sqrt{dx+c}\right)}{b^2}+\frac{2a}{b^2\left(a+b\sqrt{dx+c}\right)}}{d}$
default	$\frac{a^2}{(-b^2dx-b^2c+a^2)b^2d}+\frac{c}{(-b^2dx-b^2c+a^2)d}+\frac{\ln(b^2dx+b^2c-a^2)}{b^2d}+\frac{c}{d(b^2dx+b^2c-a^2)}-\frac{a^2}{b^2d(b^2dx+b^2c-a^2)}+\frac{}{b^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/(-b^2*d*x-b^2*c+a^2)/b^2/d+c/(-b^2*d*x-b^2*c+a^2)/d+ln(b^2*d*x+b^2*c-a^
2)/b^2/d+1/d/(b^2*d*x+b^2*c-a^2)*c-1/b^2/d/(b^2*d*x+b^2*c-a^2)*a^2+a/b^2/d/
(a+b*(d*x+c)^(1/2))+ln(a+b*(d*x+c)^(1/2))/b^2/d+a/b^2/d/(-a+b*(d*x+c)^(1/2)
)-1/b^2/d*ln(-a+b*(d*x+c)^(1/2))
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.91

$$\frac{2\left(\frac{a}{\sqrt{dx+c}\,b^3+ab^2}+\frac{\log\left(\sqrt{dx+c}\,b+a\right)}{b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.60

$$\frac{2\left(\sqrt{dx+c}\,ab-a^2+(b^2dx+b^2c-a^2)\log\left(\sqrt{dx+c}\,b+a\right)\right)}{b^4d^2x+(b^4c-a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b +
a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(39) = 78.
time = 0.62, size = 124, normalized size = 2.64

$$\begin{cases} \frac{x}{a^2} & \text{for } b=0 \wedge (b=0 \vee d=0) \\ \frac{x}{\left(a+b\sqrt{c}\right)^2} & \text{for } d=0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2a}{ab^2d+b^3d\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}\,\log\left(\frac{a}{b}+\sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2
, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)
```

```
) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqr
t(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))
```

Giac [A]
time = 3.60, size = 44, normalized size = 0.94

$$\frac{2 \log \left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^2 d}+\frac{2\,a}{\left(\sqrt{dx+c}\,b+a\right)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d)
```

Mupad [B]
time = 0.05, size = 43, normalized size = 0.91

$$\frac{2\,\ln\left(a+b\,\sqrt{c+d\,x}\right)}{b^2\,d}+\frac{2\,a}{b^2\,\left(a\,d+b\,d\,\sqrt{c+d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(c + d*x)^(1/2))^2,x)
```

```
[Out] (2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(1
/2)))
```

3.643 $\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=129

$$\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{(a^2+b^2c)\log(x)}{(a^2-b^2c)^2}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

```
[Out] (b^2*c+a^2)*ln(x)/(-b^2*c+a^2)^2-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-b^2*
c+a^2)^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-
b^2*c+a^2)/(a+b*(d*x+c)^(1/2))
```

Rubi [A]
time = 0.09, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {378, 1412, 815, 649, 212, 266}

$$\frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}+\frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2}+\frac{\log(x)\,(a^2+b^2c)}{(a^2-b^2c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2
- (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x\left(a+b\sqrt{c+dx}\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)}\,dx, x, c+dx\right)$$

$$= 2\text{Subst}\left(\int \frac{x}{(a+bx)^2\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)$$

$$= 2\text{Subst}\left(\int \left(-\frac{ab}{(a^2-b^2c)\,(a+bx)^2} - \frac{b(a^2+b^2c)}{(a^2-b^2c)^2\,(a+bx)} + \frac{2abc-(a^2+b^2c)}{(a^2-b^2c)^2\,(c-x}\right.\right.$$

$$= \frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)} - \frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2} + \frac{2\text{Subst}\left(\int \frac{2a}{}\right.}{}$$

$$= \frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)} - \frac{2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2} + \frac{(4abc)\text{Subst}}{}$$

$$= \frac{2a}{(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)} + \frac{4ab\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2-b^2c)^2} + \frac{(a^2+b^2c)\log(x}{(a^2-b^2c)^2}$$

Mathematica [A]
time = 0.16, size = 107, normalized size = 0.83

$$\frac{\frac{2a(a^2-b^2c)}{a+b\sqrt{c+dx}}+4ab\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+(a^2+b^2c)\log(-dx)-2(a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] ((2*a*(a^2 - b^2*c))/(a + b*Sqrt[c + d*x]) + 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)] - 2*(a^2 + b^2*c)*Log[a + b*Sqrt
[c + d*x]])/(a^2 - b^2*c)^2
```

Maple [A]
time = 0.05, size = 118, normalized size = 0.91

method	result
derivativedivides	$\frac{2a}{(-b^2c+a^2)\left(a+b\sqrt{dx+c}\right)}-\frac{2(b^2c+a^2)\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\,\mathrm{arctanh}\left(\frac{\sqrt{dx}}{\sqrt{}}\right.}{(-b^2c+a^2)^2}$
default	$\frac{2a}{(-b^2c+a^2)\left(a+b\sqrt{dx+c}\right)}-\frac{2(b^2c+a^2)\ln\left(a+b\sqrt{dx+c}\right)}{(-b^2c+a^2)^2}+\frac{-(-b^2c-a^2)\ln(-dx)+4ab\,\mathrm{arctanh}\left(\frac{\sqrt{dx}}{\sqrt{}}\right.}{(-b^2c+a^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*a/(-b^2*c+a^2)/(a+b*(d*x+c)^(1/2))-2*(b^2*c+a^2)*ln(a+b*(d*x+c)^(1/2))/(-
b^2*c+a^2)^2+2/(-b^2*c+a^2)^2*(-1/2*(-b^2*c-a^2)*ln(-d*x)+2*a*b*arctanh((d*
x+c)^(1/2)/c^(1/2))*c^(1/2))
```

Maxima [A]
time = 0.54, size = 176, normalized size = 1.36

$$-\frac{2\,ab\sqrt{c}\,\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2-2\,a^2b^2c+a^4}+\frac{(b^2c+a^2)\log(dx)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\,(b^2c+a^2)\log\left(\sqrt{dx+c}\,b+a\right)}{b^4c^2-2\,a^2b^2c+a^4}-\frac{2\,a}{ab^2c-a^3+(b^3c-a^2b)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^
4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c
```

```
+ a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c +
a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))
```

Fricas [A]
time = 0.38, size = 444, normalized size = 3.44

$$\left[\frac{2\,a^{2}b^{2}c-2\,a^{4}+2\,(ab^{3}dx+ab^{3}c-a^{3}b)\sqrt{c}\,\log\left(\frac{dx+2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)-2\,(b^{4}c^{2}-a^{4}+(b^{4}c+a^{2}b^{2})dx)\log\left(\sqrt{dx+c}\,b+a\right)+(b^{4}c^{2}-a^{4}+(b^{4}c+a^{2}b^{2})dx)\log\left(x\right)-2\,(ab^{3}c-a^{3}b)\sqrt{dx+c}}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}+(b^{6}c^{2}-2\,a^{2}b^{4}c+a^{4}b^{2})dx},\ \frac{2\,a^{2}b^{2}c-2\,a^{4}-4\,(ab^{3}dx+ab^{3}c-a^{3}b)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)-2\,(b^{4}c^{2}-a^{4}+(b^{4}c+a^{2}b^{2})dx)\log\left(\sqrt{dx+c}\,b+a\right)+(b^{4}c^{2}-a^{4}+(b^{4}c+a^{2}b^{2})dx)\log\left(x\right)-2\,(ab^{3}c-a^{3}b)\sqrt{dx+c}}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}+(b^{6}c^{2}-2\,a^{2}b^{4}c+a^{4}b^{2})dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x +
2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*
x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x
) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2
*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4
*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) -
2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c
^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c
))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c +
a^4*b^2)*d*x)]
```

Sympy [A]
time = 22.06, size = 153, normalized size = 1.19

$$-\frac{2ab\left(\begin{cases}\frac{\sqrt{c+dx}}{a^{2}} & \text{for}\ b=0\\ -\frac{1}{b\left(a+b\sqrt{c+dx}\right)} & \text{otherwise}\end{cases}\right)}{a^{2}-b^{2}c}-\frac{2b(a^{2}+b^{2}c)\left(\begin{cases}\frac{\sqrt{c+dx}}{a} & \text{for}\ b=0\\ \frac{\log\left(a+b\sqrt{c+dx}\right)}{b} & \text{otherwise}\end{cases}\right)}{\left(a^{2}-b^{2}c\right)^{2}}-\frac{2\cdot\left(\frac{2abc\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+\left(-\frac{a^{2}}{2}-\frac{b^{2}c}{2}\right)\log\left(-dx\right)\right)}{\left(a^{2}-b^{2}c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x
))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqrt(c + d*x)/
a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c)**2 - 2*(2
*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c/2)*log(-d*
x))/(a**2 - b**2*c)**2
```

Giac [A]
time = 4.70, size = 174, normalized size = 1.35

$$-\frac{4\,abc\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^{4}c^{2}-2\,a^{2}b^{2}c+a^{4})\sqrt{-c}}+\frac{(b^{2}c+a^{2})\log\left(-dx\right)}{b^{4}c^{2}-2\,a^{2}b^{2}c+a^{4}}-\frac{2\,(b^{3}c+a^{2}b)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b}-\frac{2\,(ab^{2}c-a^{3})}{(b^{2}c-a^{2})^{2}\left(\sqrt{dx+c}\,b+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt
(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c +
a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2*(a
*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))
```

Mupad [B]
time = 3.51, size = 125, normalized size = 0.97

$$\frac{\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{{\left(a+b\,\sqrt{c}\right)}^2}+\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{2}{b^2\,c-a^2}-\frac{4\,b^2\,c}{{\left(b^2\,c-a^2\right)}^2}\right)+\frac{\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{{\left(a-b\,\sqrt{c}\right)}^2}-\frac{2\,a}{\left(b^2\,c-a^2\right)\,\left(a+b\,\sqrt{c+d\,x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/2
))*(2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) + c^
(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))
```

3.644 $\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)}+\frac{2ab(a^2+3b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3}+\frac{b^2(3a^2}{(}$$

```
[Out] b^2*(b^2*c+3*a^2)*d*ln(x)/(-b^2*c+a^2)^3-2*b^2*(b^2*c+3*a^2)*d*ln(a+b*(d*x+
c)^(1/2))/(-b^2*c+a^2)^3+2*a*b*(3*b^2*c+a^2)*d*arctanh((d*x+c)^(1/2)/c^(1/2
))/(-b^2*c+a^2)^3/c^(1/2)+4*a*b^2*d/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))+(-a+
b*(d*x+c)^(1/2))/(-b^2*c+a^2)/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A]

time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{x\,(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}+\frac{b^2d\log(x)\,(3a^2+b^2c)}{(a^2-b^2c)^3}-\frac{2b^2d(3a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}+\frac{2abd(a^2+3b^2c)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}\,(a^2-b^2c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/(a^2 - b^2*c)^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
```

```
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+b\sqrt{c+dx}\right)^2}\,dx = d\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^2}\,dx, x, c+dx\right)$$

$$= (2d)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^2}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{d\text{Subst}\left(\int \frac{-2abc+2b^2cx}{(a+bx)^2(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)}{c\,(a^2-b^2c)}$$

$$= -\frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{d\text{Subst}\left(\int \left(-\frac{4ab^3c}{(a^2-b^2c)(a+bx)^2} - \frac{2b^3c(3a^2+b^2c)}{(-a^2+b^2c)^2(a+bx}\right.\right.}{c\,(a}$$

$$= \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2(3a^2+b^2c}{}$$

$$= \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} - \frac{2b^2(3a^2+b^2c}{}$$

$$= \frac{4ab^2d}{(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{(a^2-b^2c)\,x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2}{\sqrt{}}$$

Mathematica [A]
time = 0.31, size = 177, normalized size = 0.88

$$\frac{\frac{(a^2-b^2c)\left(-a^3+a^2b\sqrt{c+dx}-b^3c\sqrt{c+dx}+ab^2(c+4dx)\right)}{x\left(a+b\sqrt{c+dx}\right)} + \frac{2ab(a^2+3b^2c)\,d\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2(3a^2+b^2c)\,d\log(-dx) - 2b^2(3a^2+b^2c)\,d\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (((a^2 - b^2*c)*(-a^3 + a^2*b*Sqrt[c + d*x] - b^3*c*Sqrt[c + d*x] + a*b^2*(
c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh[S
qrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*(3*a^2 + b^2*c)*d*Log[-(d*x)] - 2*b^2*
(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3
```

Maple [A]
time = 0.06, size = 182, normalized size = 0.90

method	result
derivativedivides	$2d\left(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+}{dx}}{}\right.$
default	$2d\left(\frac{a\,b^{2}}{\left(-b^{2}c+a^{2}\right)^{2}\left(a+b\sqrt{dx+c}\right)}-\frac{b^{2}\left(b^{2}c+3a^{2}\right)\ln\left(a+b\sqrt{dx+c}\right)}{\left(-b^{2}c+a^{2}\right)^{3}}+\frac{-\frac{\left(a\,b^{3}c-a^{3}b\right)\sqrt{dx+c}-\frac{b^{4}c^{2}}{2}+}{dx}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d*(a*b^2/(-b^2*c+a^2)^2/(a+b*(d*x+c)^(1/2))-b^2*(b^2*c+3*a^2)/(-b^2*c+a^2
)^3*ln(a+b*(d*x+c)^(1/2))+1/(-b^2*c+a^2)^3*(-((a*b^3*c-a^3*b)*(d*x+c)^(1/2)
-1/2*b^4*c^2+1/2*a^4)/d/x+b*(-1/2*(-b^3*c-3*a^2*b)*ln(-d*x)+(3*a*b^2*c+a^3)
/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))))
```

Maxima [A]

time = 0.66, size = 367, normalized size = 1.82

$$-d\left(\frac{\left(b^{4}c+3\,a^{2}b^{2}\right)\log\left(dx\right)}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}}-\frac{2\left(b^{4}c+3\,a^{2}b^{2}\right)\log\left(\sqrt{dx+c}\,b+a\right)}{b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}}-\frac{\left(3\,ab^{3}c+a^{3}b\right)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\left(b^{6}c^{3}-3\,a^{2}b^{4}c^{2}+3\,a^{4}b^{2}c-a^{6}\right)\sqrt{c}}+\frac{4\,(dx+c)ab^{2}-3\,ab^{2}c-a^{3}-\left(b^{3}c-a^{2}b\right)\sqrt{dx+c}}{ab^{4}c^{3}-2\,a^{3}b^{2}c^{2}+a^{5}c-\left(b^{5}c^{2}-2\,a^{2}b^{3}c+a^{4}b\right)(dx+c)^{\frac{3}{2}}-\left(ab^{4}c^{2}-2\,a^{3}b^{2}c+a^{5}\right)(dx+c)+\left(b^{5}c^{3}-2\,a^{2}b^{3}c^{2}+a^{4}bc\right)\sqrt{dx+c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c
^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(c))
/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*
sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt(d*x
+ c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*
(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c^3 - 2*
a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(195) = 390$.

time = 0.56, size = 854, normalized size = 4.23

$$\left[-\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx - ((3ab^5c + a^3b^3)d^2x^2 + (3ab^5c^2 - 2a^3b^3c - a^5b)dx)\sqrt{c}\log\left(\frac{dx - 2\sqrt{dx+c}\sqrt{c} + 2c}{x}\right) - 2((b^6c^2 + 3a^2b^4c)d^2x^2 + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx)\log\left(\sqrt{dx+c}\,b + a\right) + ((b^6c^2 + 3a^2b^4c)d^2x^2 + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx)\log(x) - 2(ab^5c^3 - 2a^3b^3c^2 + a^5bc + 2(ab^5c^2 - a^3b^3c)dx)\sqrt{dx+c}}{(b^8c^4 - 3a^2b^6c^3 + 3a^4b^4c^2 - a^6b^2c)dx^2 + (b^8c^5 - 4a^2b^6c^4 + 6a^4b^4c^3 - 4a^6b^2c^2 + a^8c)x}, -\frac{b^6c^4 - a^2b^4c^3 - a^4b^2c^2 + a^6c + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx - 2((3ab^5c + a^3b^3)d^2x^2 + (3ab^5c^2 - 2a^3b^3c - a^5b)dx)\sqrt{-c}\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) - 2((b^6c^2 + 3a^2b^4c)d^2x^2 + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx)\log\left(\sqrt{dx+c}\,b + a\right) + ((b^6c^2 + 3a^2b^4c)d^2x^2 + (b^6c^3 + 2a^2b^4c^2 - 3a^4b^2c)dx)\log(x) - 2(ab^5c^3 - 2a^3b^3c^2 + a^5bc + 2(ab^5c^2 - a^3b^3c)dx)\sqrt{dx+c}}{(b^8c^4 - 3a^2b^6c^3 + 3a^4b^4c^2 - a^6b^2c)dx^2 + (b^8c^5 - 4a^2b^6c^4 + 6a^4b^4c^3 - 4a^6b^2c^2 + a^8c)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 -
 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b
^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2
*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)
*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2
 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^
2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6
*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2
*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c
+ a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arct
an(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^
3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2
+ 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x
) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)
*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^
2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a + b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))**2), x)
```

Giac [A]
time = 4.92, size = 311, normalized size = 1.54

$$-\frac{(b^4cd + 3a^2b^2d)\log(-dx)}{b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6} + \frac{2(b^5cd + 3a^2b^3d)\log\left(\left|-\sqrt{dx+c}\,b - a\right|\right)}{b^7c^3 - 3a^2b^5c^2 + 3a^4b^3c - a^6b} + \frac{2(3ab^3cd + a^3bd)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6)\sqrt{-c}} - \frac{\sqrt{dx+c}\,b^3cd - 4(dx+c)ab^2d + 3ab^2cd - \sqrt{dx+c}\,a^2bd + a^3d}{(b^4c^2 - 2a^2b^2c + a^4)\left((dx+c)^{\frac{3}{2}}b - \sqrt{dx+c}\,bc + (dx+c)a - ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c -
 a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3 -
 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan(sq
rt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(-
c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqrt(d*x
 + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2)*b -
sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))
```

Mupad [B]
time = 0.73, size = 275, normalized size = 1.36

$$\frac{b\,d\,\ln\left(\sqrt{c+d\,x}+\sqrt{c}\right)}{a^3\,\sqrt{c}-b^3\,c^2+3\,a\,b^2\,c^{3/2}-3\,a^2\,b\,c}-\frac{\frac{a\,d\,\left(a^2+3\,c\,b^2\right)}{{\left(b^2\,c-a^2\right)}^2}+\frac{b\,d\,\sqrt{c+d\,x}}{b^2\,c-a^2}-\frac{4\,a\,b^2\,d\,\left(c+d\,x\right)}{a^4-2\,a^2\,b^2\,c+b^4\,c^2}}{b\,{\left(c+d\,x\right)}^{3/2}-a\,c+a\,\left(c+d\,x\right)-b\,c\,\sqrt{c+d\,x}}-\frac{b\,d\,\ln\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{a^3\,\sqrt{c}+b^3\,c^2+3\,a\,b^2\,c^{3/2}+3\,a^2\,b\,c}-\ln\left(a+b\,\sqrt{c+d\,x}\right)\,\left(\frac{6\,b^2\,d}{{\left(b^2\,c-a^2\right)}^2}-\frac{8\,b^4\,c\,d}{{\left(b^2\,c-a^2\right)}^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3/
2) - 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)^(
1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/
(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*log((c
 + d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3*a^2*
b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4*c*d)/
(b^2*c - a^2)^3)
```

3.645 $\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$

Optimal. Leaf size=306

$$\frac{ab^2(a^2+11b^2c)\,d^2}{2c\,(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2\,(a^2-b^2c)\,x^2\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-(a^2+2b^2c)\sqrt{c+dx}\right)}{2c\,(a^2-b^2c)^2\,x\left(a+b\sqrt{c+dx}\right)}-\frac{ab(a}{}$$

```
[Out] -1/2*a*b*(-15*b^4*c^2-10*a^2*b^2*c+a^4)*d^2*arctanh((d*x+c)^(1/2)/c^(1/2))/
c^(3/2)/(-b^2*c+a^2)^4+b^4*(b^2*c+5*a^2)*d^2*ln(x)/(-b^2*c+a^2)^4-2*b^4*(b^
2*c+5*a^2)*d^2*ln(a+b*(d*x+c)^(1/2))/(-b^2*c+a^2)^4+1/2*a*b^2*(11*b^2*c+a^2
)*d^2/c/(-b^2*c+a^2)^3/(a+b*(d*x+c)^(1/2))+1/2*(-a+b*(d*x+c)^(1/2))/(-b^2*c
+a^2)/x^2/(a+b*(d*x+c)^(1/2))-1/2*b*d*(3*a*b*c-(2*b^2*c+a^2)*(d*x+c)^(1/2))
/c/(-b^2*c+a^2)^2/x/(a+b*(d*x+c)^(1/2))
```

Rubi [A]
time = 0.29, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$,
Rules used = {378, 1412, 837, 815, 649, 212, 266}

$$\frac{ab^2d^2(a^2+11b^2c)}{2c\,(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)}-\frac{a-b\sqrt{c+dx}}{2x^2\,(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)}-\frac{bd\left(3abc-(a^2+2b^2c)\sqrt{c+dx}\right)}{2cx\,(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)}+\frac{b^4d^2\log(x)\,(5a^2+b^2c)}{(a^2-b^2c)^4}-\frac{2b^4d^2(5a^2+b^2c)\log\left(a+b\sqrt{c+dx}\right)}{(a^2-b^2c)^4}-\frac{abd^2(a^4-10a^2b^2c-15b^4c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}\,(a^2-b^2c)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) -
(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3
*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt
[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d
*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x]
)/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a
^2 - b^2*c)^4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx = d^2\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(-c+x)^3}\,dx, x, c+dx\right)$$

$$= \left(2d^2\right)\text{Subst}\left(\int \frac{x}{(a+bx)^2\left(-c+x^2\right)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} + \frac{d^2\text{Subst}\left(\int \frac{-2abc+4b^2cx}{(a+bx)^2(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right.}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + d^2\text{S}$$

$$= -\frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc-\left(a^2+2b^2c\right)\sqrt{c+dx}\right)}{2c\left(a^2-b^2c\right)^2x\left(a+b\sqrt{c+dx}\right)} + d^2\text{S}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc\right.}{2c\left(a^2-\right.}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc\right.}{2c\left(a^2-\right.}$$

$$= \frac{ab^2\left(a^2+11b^2c\right)d^2}{2c\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)} - \frac{a-b\sqrt{c+dx}}{2\left(a^2-b^2c\right)x^2\left(a+b\sqrt{c+dx}\right)} - \frac{bd\left(3abc\right.}{2c\left(a^2-\right.}$$

Mathematica [A]
time = 0.60, size = 301, normalized size = 0.98

$$\frac{1}{2}\left(\frac{a^5c-b^5c^2(c-2dx)\sqrt{c+dx}+a^2b^3c(2c-dx)\sqrt{c+dx}-a^4b(c+dx)^{3/2}+ab^4c\left(c^2-3cdx-11d^2x^2\right)-a^3b^2\left(2c^2-3cdx+d^2x^2\right)}{c\left(-a^2+b^2c\right)^3x^2\left(a+b\sqrt{c+dx}\right)}+\frac{\left(-a^5b+10a^3b^3c+15ab^5c^2\right)d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{c^{3/2}\left(a^2-b^2c\right)^4}+\frac{2b^4\left(5a^2+b^2c\right)d^2\log(-dx)}{\left(a^2-b^2c\right)^4}-\frac{4b^4\left(5a^2+b^2c\right)d^2\log\left(a+b\sqrt{c+dx}\right)}{\left(a^2-b^2c\right)^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]
```

```
[Out] ((a^5*c - b^5*c^2*(c - 2*d*x)*Sqrt[c + d*x] + a^2*b^3*c*(2*c - d*x)*Sqrt[c
+ d*x] - a^4*b*(c + d*x)^(3/2) + a*b^4*c*(c^2 - 3*c*d*x - 11*d^2*x^2) - a^3
*b^2*(2*c^2 - 3*c*d*x + d^2*x^2))/(c*(-a^2 + b^2*c)^3*x^2*(a + b*Sqrt[c + d
*x])) + ((-(a^5*b) + 10*a^3*b^3*c + 15*a*b^5*c^2)*d^2*ArcTanh[Sqrt[c + d*x]
/Sqrt[c]])/(c^(3/2)*(a^2 - b^2*c)^4) + (2*b^4*(5*a^2 + b^2*c)*d^2*Log[-(d*x
```

```
)])/(a^2 - b^2*c)^4 - (4*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/
(a^2 - b^2*c)^4)/2
```

Maple [A]
time = 0.07, size = 303, normalized size = 0.99

method	result
derivativedivides	$2d^2\left(-\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}a^4b^2\right)(dx+c)+\left(-\frac{9}{4}ab^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+3}{d^2x^2}\right.$ $(-b^2$
default	$2d^2\left(-\frac{-\frac{ab\left(-7b^4c^2+6a^2b^2c+a^4\right)(dx+c)^{\frac{3}{2}}}{4c}+\left(-\frac{1}{2}c^2b^6-a^2b^4c+\frac{3}{2}a^4b^2\right)(dx+c)+\left(-\frac{9}{4}ab^5c^2+\frac{5}{2}a^3b^3c-\frac{1}{4}a^5b\right)\sqrt{dx+c}+3}{d^2x^2}\right.$ $(-b^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d^2*(-1/(-b^2*c+a^2)^4*((-1/4*a*b*(-7*b^4*c^2+6*a^2*b^2*c+a^4)/c*(d*x+c)^
(3/2)+(-1/2*c^2*b^6-a^2*b^4*c+3/2*a^4*b^2)*(d*x+c)+(-9/4*a*b^5*c^2+5/2*a^3*
b^3*c-1/4*a^5*b)*(d*x+c)^(1/2)+3/4*c^3*b^6+3/4*a^2*b^4*c^2-7/4*a^4*b^2*c+1/
4*a^6)/d^2/x^2+1/4*b/c*(-1/2*(4*b^5*c^2+20*a^2*b^3*c)*ln(-d*x)+(-15*a*b^4*c
^2-10*a^3*b^2*c+a^5)/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))))+b^4/(-b^2*c+a
^2)^3*a/(a+b*(d*x+c)^(1/2))-b^4*(b^2*c+5*a^2)/(-b^2*c+a^2)^4*ln(a+b*(d*x+c)
^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(289) = 578$.
time = 0.57, size = 659, normalized size = 2.15

$$\frac{1}{4}d^2\left(\frac{4\,(b^6c+5\,a^2b^4)\log(dx)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{8\,(b^6c+5\,a^2b^4)\log\left(\sqrt{dx+c}\,b+a\right)}{b^8c^4-4\,a^2b^6c^3+6\,a^4b^4c^2-4\,a^6b^2c+a^8}-\frac{(15\,ab^5c^2+10\,a^3b^3c-a^5b)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{(b^8c^5-4\,a^2b^6c^4+6\,a^4b^4c^3-4\,a^6b^2c^2+a^8c)\sqrt{c}}-\frac{2\left(7\,ab^4c^3+6\,a^3b^2c^2-a^5c+(11\,ab^4c+a^3b^2)(dx+c)^2-(2\,b^5c^2-a^2b^3c-a^4b)(dx+c)^{\frac{3}{2}}-(19\,ab^4c^2+5\,a^3b^2c)(dx+c)+3\,(b^5c^3-a^2b^3c^2)\sqrt{dx+c}\right)}{ab^6c^6-3\,a^3b^4c^5+3\,a^5b^2c^4-a^7c^3+(b^7c^4-3\,a^2b^5c^3+3\,a^4b^3c^2-a^6bc)(dx+c)^{\frac{5}{2}}+(ab^6c^4-3\,a^3b^4c^3+3\,a^5b^2c^2-a^7c)(dx+c)^2-2\,(b^7c^5-3\,a^2b^5c^4+3\,a^4b^3c^3-a^6bc^2)(dx+c)^{\frac{3}{2}}-2\,(ab^6c^5-3\,a^3b^4c^4+3\,a^5b^2c^3-a^7c^2)(dx+c)+(b^7c^6-3\,a^2b^5c^5+3\,a^4b^3c^4-a^6bc^3)\sqrt{dx+c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^
4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b + a)
/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b^5*
c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) +
sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c
)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a^3*b^2
)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (19*a*b^4
*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x + c))/(a
*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*a^2*b^5*c
^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^3*b^4*c^3
+ 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^4 + 3*a^4*b
^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*c^4 + 3*a^5*
b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^4*b^3*c^4 - a
^6*b*c^3)*sqrt(d*x + c)))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(289) = 578$.
time = 1.18, size = 1252, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 +
 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^
4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d
^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*sqr
t(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 8*((b^8*c^3 + 5*a^2*b^6
*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(sqrt
(d*x + c)*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*
b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 +
 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*
d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*sqrt
(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8
*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c
^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*
b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (
b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c
^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a
^5*b^3*c + a^7*b)*d^2*x^2)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*((
b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2
)*d^2*x^2)*log(sqrt(d*x + c)*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3
+ (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*log(x) - (2*a*b^7*c^5
- 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*
```

```
c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a
^7*b*c)*d*x)*sqrt(d*x + c))/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*
a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c
^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\sqrt{c+dx}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*sqrt(c + d*x))**2), x)
```

Giac [A]
time = 3.24, size = 521, normalized size = 1.70

$$\frac{(b^6cd^2+5a^2b^4d^2)\log(-dx)}{b^8c^4-4a^2b^6c^3+6a^4b^4c^2-4a^6b^2c+a^8}-\frac{2(b^7cd^2+5a^2b^5d^2)\log\left(\left|\sqrt{dx+c}\,b+a\right|\right)}{b^9c^4-4a^2b^7c^3+6a^4b^5c^2-4a^6b^3c+a^8b}-\frac{(15ab^5c^2d^2+10a^3b^3cd^2-a^5bd^2)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{2(b^8c^5-4a^2b^6c^4+6a^4b^4c^3-4a^6b^2c^2+a^8c)\sqrt{-c}}-\frac{7ab^6c^4d^2-a^3b^4c^3d^2-7a^5b^2c^2d^2+a^7cd^2+(11ab^6c^2d^2-10a^3b^4cd^2-a^5b^2d^2)(dx+c)^2-(2b^7c^3d^2-3a^2b^5c^2d^2+a^6bd^2)(dx+c)^{\frac{3}{2}}-(19ab^6c^3d^2-14a^3b^4c^2d^2-5a^5b^2cd^2)(dx+c)+3(b^7c^4d^2-2a^2b^5c^3d^2+a^4b^3c^2d^2)\sqrt{dx+c}}{2(b^2c-a^2)^4\left(\sqrt{dx+c}\,b+a\right)cd^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] (b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*
c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x +
 c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8*b)
 - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*x +
c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^
8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c^2*d^2
 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(d*x + c
)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/2) - (19
*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) + 3*(b^7*c
^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/((b^2*c - a^2)
^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)
```

Mupad [B]
time = 5.96, size = 1441, normalized size = 4.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*c
^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a^6
```

- b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2 + 6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*(a^2*d^2 + 2*b^2*c*d^2)*(c + d*x)^(3/2))/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - (3*b^3*c*d^2*(c + d*x)^(1/2))/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(c + d*x)^2 + b*(c + d*x)^(5/2) + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)^(3/2) + b*c^2*(c + d*x)^(1/2)) + log(a + b*(c + d*x)^(1/2))*((10*b^4*d^2)/(b^2*c - a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (log((a*b^4*d^4*(a^6 - 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^(1/2) + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^(1/2) - 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^(1/2) - 10*a^3*b^3*c*d^2*(c^3)^(1/2) - 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (log((a*b^4*d^4*(a^6 - 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^2*d^2*(a^2*(c + d*x)^(1/2) + 4*a*b*c + 3*b^2*c*(c + d*x)^(1/2))*(4*b^5*c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^4*c^2*(c^3)^(1/2)))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^(1/2)*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(4*b^5*c^4 - a^5*(c^3)^(1/2) + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^(1/2) + 15*a*b^4*c^2*(c^3)^(1/2)))/(4*c^3*(b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^(1/2))/(4*c^2*(b^2*c - a^2)^6))*(4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 - a^5*b*d^2*(c^3)^(1/2) + 10*a^3*b^3*c*d^2*(c^3)^(1/2) + 15*a*b^5*c^2*d^2*(c^3)^(1/2)))/(4*(a^8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6))

3.646 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=324

$$-\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4}+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}-\frac{12a(7a^2-3b^2c)(a^2-b^2c)}{5b^8d^4}$$

```
[Out] 4/3*(-b^2*c+a^2)^2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^8/d^4-12/5*a*
(-3*b^2*c+7*a^2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^8/d^4+4/7*(3*b^4*
c^2-30*a^2*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(7/2)/b^8/d^4-20/9*a*(-3*b^2*c
+7*a^2)*(a+b*(d*x+c)^(1/2))^(9/2)/b^8/d^4+12/11*(-b^2*c+7*a^2)*(a+b*(d*x+c)
^(1/2))^(11/2)/b^8/d^4-28/13*a*(a+b*(d*x+c)^(1/2))^(13/2)/b^8/d^4+4/15*(a+b
*(d*x+c)^(1/2))^(15/2)/b^8/d^4-4*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1/2)
/b^8/d^4
```

Rubi [A]
time = 0.17, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {378, 1412, 786}

$$\frac{12(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^8d^4}-\frac{20a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^8d^4}-\frac{12a(7a^2-3b^2c)(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^8d^4}+\frac{4(a^2-b^2c)^2(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}-\frac{4a(a^2-b^2c)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4}+\frac{4(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^8d^4}+\frac{4\left(a+b\sqrt{c+dx}\right)^{15/2}}{15b^8d^4}-\frac{28a\left(a+b\sqrt{c+dx}\right)^{13/2}}{13b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]])/(b^8*d^4) + (4*(a^2 - b^2*
c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^
2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3
5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4)
- (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7
*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqr
t[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8
*d^4)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \frac{x\left(-c+x^2\right)^3}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^4}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a\left(a^2-b^2c\right)^3}{b^7\sqrt{a+bx}} - \frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2\sqrt{a+bx}}{b^7} - \frac{3\left(7a^5-10a^3b^2c+3ab^4c^2\right)(a+}{b^7}\right.\right.}{}$$

$$= -\frac{4a\left(a^2-b^2c\right)^3\sqrt{a+b\sqrt{c+dx}}}{b^8d^4} + \frac{4\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^8d^4}$$

Mathematica [A]
time = 0.20, size = 232, normalized size = 0.72

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-14336a^7+768a^5b^2(58c-7dx)+7168a^6b\sqrt{c+dx}-640a^4b^3(32c-7dx)\sqrt{c+dx}+24a^2b^5\sqrt{c+dx}\left(784c^2-356cdx+147d^2x^2\right)-16a^3b^4\left(2936c^2-680cdx+245d^2x^2\right)+6ab^6\left(2880c^3-928c^2dx+658cd^2x^2-539d^3x^3\right)-39b^7\sqrt{c+dx}\left(128c^3-96c^2dx+84cd^2x^2-77d^3x^3\right)\right)}{45045b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 716
8*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b
^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2
 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x
^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x
^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

Maple [A]
time = 0.10, size = 383, normalized size = 1.18

method	result
derivativedivides	$\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}-\frac{28a\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{4\left(-3b^2c+21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2a\left(-2b\right.\right.}{}$
default	$\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{15}{2}}}{15}-\frac{28a\left(a+b\sqrt{dx+c}\right)^{\frac{13}{2}}}{13}+\frac{4\left(-3b^2c+21a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}+\frac{4\left(-8\left(-b^2c+a^2\right)a-2a\left(-2b\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^4/b^8*(1/15*(a+b*(d*x+c)^(1/2))^(15/2)-7/13*a*(a+b*(d*x+c)^(1/2))^(13/2
)+1/11*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(-8*(-b^2*c+a^2)*a-
2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*((-
b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c+a
^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-6*(-b^2*c+a^
2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)*a)
*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b*(
d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A]

time = 0.34, size = 268, normalized size = 0.83

$$\frac{4\left(3003\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}-24255\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}a-12285\left(b^2c-7a^2\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}+25025\left(3ab^2c-7a^3\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}+6435\left(3b^4c^2-30a^2b^2c+35a^4\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-27027\left(3ab^4c^2-10a^3b^2c+7a^5\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}-15015\left(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6\right)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}+45045\left(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7\right)\sqrt{\sqrt{dx+c}\,b+a}\right)}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(1
3/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^
2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c +
 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c +
7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^
4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4
*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)
```

Fricas [A]

time = 0.43, size = 231, normalized size = 0.71

$$-\frac{4\left(3234\,ab^6d^3x^3-17280\,ab^6c^3+46976\,a^3b^4c^2-44544\,a^5b^2c+14336\,a^7-28\left(141\,ab^6c-140\,a^3b^4\right)d^2x^2+64\left(87\,ab^6c^2-170\,a^3b^4c+84\,a^5b^2\right)dx-\left(3003\,b^7d^3x^3-4992\,b^7c^3+18816\,a^2b^5c^2-20480\,a^4b^3c+7168\,a^6b-252\left(13\,b^7c-14\,a^2b^5\right)d^2x^2+32\left(117\,b^7c^2-267\,a^2b^5c+140\,a^4b^3\right)dx\right)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/45045*(3234*a*b^6*d^3*x^3 - 17280*a*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*
a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b
^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
 + 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^
2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d
*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 3.60, size = 409, normalized size = 1.26

$$-\frac{4\left(15015\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^6c^3-45045\sqrt{\sqrt{dx+c}\,b+a}\,ab^6c^3-19305\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^4c^2+81081\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^4c^2-135135\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^4c^2+135135\sqrt{\sqrt{dx+c}\,b+a}\,a^3b^4c^2+12285\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}b^2c-75075\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}ab^2c+193050\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^2b^2c-270270\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^3b^2c+225225\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^4b^2c-135135\sqrt{\sqrt{dx+c}\,b+a}\,a^5b^2c-3003\left(\sqrt{dx+c}\,b+a\right)^{\frac{15}{2}}+24255\left(\sqrt{dx+c}\,b+a\right)^{\frac{13}{2}}a-85995\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}a^2+175175\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a^3-225225\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^4+189189\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^5-105105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^6+45045\sqrt{\sqrt{dx+c}\,b+a}\,a^7\right)}{45045\,b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x +
 c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*(s
qrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^
2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x
+ c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + 1930
50*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(s
qrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24255*(
sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 17
5175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a
^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^
(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.647 $\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=222

$$-\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c\right.}}{5b^6d^3}$$

```
[Out] 4/3*(b^4*c^2-6*a^2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(3/2)/b^6/d^3-8/5*a*(-3
*b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2))^(5/2)/b^6/d^3+8/7*(-b^2*c+5*a^2)*(a+b*(d*
x+c)^(1/2))^(7/2)/b^6/d^3-20/9*a*(a+b*(d*x+c)^(1/2))^(9/2)/b^6/d^3+4/11*(a+
b*(d*x+c)^(1/2))^(11/2)/b^6/d^3-4*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1/2
)/b^6/d^3
```

Rubi [A]
time = 0.12, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {378, 1412, 786}

$$\frac{8(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^6d^3}-\frac{8a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^6d^3}-\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3}+\frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3}+\frac{4\left(a+b\sqrt{c+dx}\right)^{11/2}}{11b^6d^3}-\frac{20a\left(a+b\sqrt{c+dx}\right)^{9/2}}{9b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]])/(b^6*d^3) + (4*(5*a^4 - 6*
a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2
 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(
a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2)
)/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)^2}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^3}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^2}{b^5\sqrt{a+bx}} + \frac{(5a^4-6a^2b^2c+b^4c^2)\sqrt{a+bx}}{b^5} - \frac{2(5a^3-3ab^2c)(a+bx)^{3/2}}{b^5} - \frac{2(-\cdots}{\cdots}\right.\right.}{d^3}$$

$$= -\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^6d^3} \cdots$$

Mathematica [A]

time = 0.12, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-1280a^5+96a^3b^2(28c-5dx)+640a^4b\sqrt{c+dx}-16a^2b^3(74c-25dx)\sqrt{c+dx}+15b^5\sqrt{c+dx}(32c^2-24cdx+21d^2x^2)-2ab^4(736c^2-244cdx+175d^2x^2)\right)}{3465b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a
^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt
[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x +
 175*d^2*x^2)))/(3465*b^6*d^3)
```

Maple [A]

time = 0.10, size = 183, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11} - \frac{20a\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9} + \frac{4\left(-2b^2c+10a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7} + \frac{4\left(-4\left(-b^2c+a^2\right)a-a\left(-2b^2c+6a\cdots\right.\right.}{5}\cdots}{d^{\cdots}}$

default	$\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{11}{2}}}{11}-\frac{20a\left(a+b\sqrt{dx+c}\right)^{\frac{9}{2}}}{9}+\frac{4\left(-2b^{2}c+10a^{2}\right)\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}+\frac{4\left(-4\left(-b^{2}c+a^{2}\right)a-a\left(-2b^{2}c+\right.\right.}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^3/b^6*(1/11*(a+b*(d*x+c)^(1/2))^(11/2)-5/9*a*(a+b*(d*x+c)^(1/2))^(9/2)+
1/7*(-2*b^2*c+10*a^2)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*(-4*(-b^2*c+a^2)*a-a*(-
2*b^2*c+6*a^2))*(a+b*(d*x+c)^(1/2))^(5/2)+1/3*((-b^2*c+a^2)^2+4*a^2*(-b^2*c
+a^2))*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)^2*a*(a+b*(d*x+c)^(1/2))^(1/2)
)
```

Maxima [A]
time = 0.29, size = 167, normalized size = 0.75

$$\frac{4\left(315\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a-990\,(b^{2}c-5\,a^{2})\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+1386\,(3\,ab^{2}c-5\,a^{3})\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}+1155\,(b^{4}c^{2}-6\,a^{2}b^{2}c+5\,a^{4})\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3465\,(ab^{4}c^{2}-2\,a^{3}b^{2}c+a^{5})\sqrt{\sqrt{dx+c}\,b+a}\right)}{3465\,b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

Fricas [A]
time = 0.40, size = 140, normalized size = 0.63

$$-\frac{4\left(350\,ab^{4}d^{2}x^{2}+1472\,ab^{4}c^{2}-2688\,a^{3}b^{2}c+1280\,a^{5}-8\,(61\,ab^{4}c-60\,a^{3}b^{2})dx-(315\,b^{5}d^{2}x^{2}+480\,b^{5}c^{2}-1184\,a^{2}b^{3}c+640\,a^{4}b-40\,(9\,b^{5}c-10\,a^{2}b^{3})dx)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{3465\,b^{6}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8
*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*
b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt
(d*x + c)*b + a)/(b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 3.84, size = 238, normalized size = 1.07

$$\frac{4\left(1155\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4c^2-3465\sqrt{\sqrt{dx+c}\,b+a}\,ab^4c^2-990\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}b^2c+4158\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}ab^2c-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2b^2c+6930\sqrt{\sqrt{dx+c}\,b+a}\,a^3b^2c+315\left(\sqrt{dx+c}\,b+a\right)^{\frac{11}{2}}-1925\left(\sqrt{dx+c}\,b+a\right)^{\frac{9}{2}}a+4950\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}a^2-6930\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a^3+5775\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^4-3465\sqrt{\sqrt{dx+c}\,b+a}\,a^5\right)}{3465\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*
b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x +
 c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 693
0*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) -
1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 -
 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^
4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.648 $\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=131

$$-\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}+\frac{4\left(a+b\sqrt{c\right.}{7b^4}$$

```
[Out] 4/3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)/b^4/d^2-12/5*a*(a+b*(d*x+c)^(1
/2))^(5/2)/b^4/d^2+4/7*(a+b*(d*x+c)^(1/2))^(7/2)/b^4/d^2-4*a*(-b^2*c+a^2)*(
a+b*(d*x+c)^(1/2))^(1/2)/b^4/d^2
```

Rubi [A]
time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$\frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2}-\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2}+\frac{4\left(a+b\sqrt{c+dx}\right)^{7/2}}{7b^4d^2}-\frac{12a\left(a+b\sqrt{c+dx}\right)^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (4*(3*a^2 - b^2*
c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(
5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}} + \frac{(3a^2-b^2c)\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \sqrt{c-\right.}{d^2}$$
$$= -\frac{4a(a^2-b^2c)\sqrt{a+b\sqrt{c+dx}}}{b^4d^2} + \frac{4(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^4d^2} - \frac{12a\left(a+\right.}{}$$

Mathematica [A]
time = 0.06, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}\left(-48a^3+2ab^2(26c-9dx)+24a^2b\sqrt{c+dx}+5b^3\sqrt{c+dx}\,(-4c+3dx)\right)}{105b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*S
qrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)
```

Maple [A]
time = 0.07, size = 94, normalized size = 0.72

method	result
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}-\frac{12a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{4\left(-b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4\left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx-}}}{b^4d^2}$
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{7}{2}}}{7}-\frac{12a\left(a+b\sqrt{dx+c}\right)^{\frac{5}{2}}}{5}+\frac{4\left(-b^2c+3a^2\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4\left(-b^2c+a^2\right)a\sqrt{a+b\sqrt{dx-}}}{b^4d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/d^2/b^4*(1/7*(a+b*(d*x+c)^(1/2))^(7/2)-3/5*a*(a+b*(d*x+c)^(1/2))^(5/2)+1/
3*(-b^2*c+3*a^2)*(a+b*(d*x+c)^(1/2))^(3/2)-(-b^2*c+a^2)*a*(a+b*(d*x+c)^(1/2
))^(1/2))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 0.71

$$\frac{4\left(15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}-63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-35\,(b^2c-3\,a^2)\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}+105\,(ab^2c-a^3)\sqrt{\sqrt{dx+c}\,b+a}\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/105*(15*(sqrt(d*x + c)*b + a)^(7/2) - 63*(sqrt(d*x + c)*b + a)^(5/2)*a -
35*(b^2*c - 3*a^2)*(sqrt(d*x + c)*b + a)^(3/2) + 105*(a*b^2*c - a^3)*sqrt(s
qrt(d*x + c)*b + a))/(b^4*d^2)
```

Fricas [A]
time = 0.40, size = 71, normalized size = 0.54

$$-\frac{4\left(18\,ab^2dx-52\,ab^2c+48\,a^3-(15\,b^3dx-20\,b^3c+24\,a^2b)\sqrt{dx+c}\right)\sqrt{\sqrt{dx+c}\,b+a}}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^
2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 3.84, size = 115, normalized size = 0.88

$$-\frac{4\left(35\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+c}\,b+a}\,ab^2c-15\left(\sqrt{dx+c}\,b+a\right)^{\frac{7}{2}}+63\left(\sqrt{dx+c}\,b+a\right)^{\frac{5}{2}}a-105\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+c}\,b+a}\,a^3\right)}{105\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a
)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)
*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^
3)/(b^4*d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+b\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

3.649 $\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=54

$$-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

```
[Out] 4/3*(a+b*(d*x+c)^(1/2))^(3/2)/b^2/d-4*a*(a+b*(d*x+c)^(1/2))^(1/2)/b^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {253, 196, 45}

$$\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}-\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]])/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/
(3*b^2*d)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}}\,dx, x, c+dx\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}}\,dx, x, \sqrt{c+dx}\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx, x, \sqrt{c+dx}\right)}{d}$$

$$= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}+\frac{4\left(a+b\sqrt{c+dx}\right)^{3/2}}{3b^2d}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.78

$$\frac{4\left(-2a+b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)
```

Maple [A]
time = 0.04, size = 41, normalized size = 0.76

method	result	size
derivativedivides	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^2d}$	41
default	$\frac{\frac{4\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{3}-4a\sqrt{a+b\sqrt{dx+c}}}{b^2d}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/d/b^2*(1/3*(a+b*(d*x+c)^(1/2))^(3/2)-a*(a+b*(d*x+c)^(1/2))^(1/2))
```

Maxima [A]
time = 0.28, size = 42, normalized size = 0.78

$$\frac{4\left(\frac{\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}\,b+a}\,a}{b^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d
```

Fricas [A]
time = 0.40, size = 34, normalized size = 0.63

$$\frac{4\sqrt{\sqrt{dx+c}\,b+a}\left(\sqrt{dx+c}\,b-2\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)
```

Giac [A]
time = 4.50, size = 38, normalized size = 0.70

$$\frac{4\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}\,b+a}\,a\right)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)

Mupad [B]
time = 3.26, size = 44, normalized size = 0.81

$$\frac{4\left(a+b\sqrt{c+d\,x}\right)^{3/2}}{3\,b^2\,d}-\frac{4\,a\,\sqrt{a+b\sqrt{c+d\,x}}}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)

[Out] (4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^(1/2))/(b^2*d)

3.650 $\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=97

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

```
[Out] -2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/(a-b*c^(1/2))^(1/
2)-2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1/2))/(a+b*c^(1/2))^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {378, 1412, 841, 1180, 213}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt
[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a +
b*Sqrt[c]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)}\,dx, x, c+dx\right)\\
&= 2\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+dx}\right)\\
&= 4\text{Subst}\left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)\\
&= 2\text{Subst}\left(\int \frac{1}{-a-b\sqrt{c}+x^2}\,dx, x, \sqrt{a+b\sqrt{c+dx}}\right)+2\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}}\right.\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}
\end{aligned}$$

Mathematica [A]

time = 0.15, size = 105, normalized size = 1.08

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\sqrt{-a-b\sqrt{c}}}+\frac{2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\sqrt{-a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/Sqrt[-a - b*Sqrt
[c]] + (2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/Sqrt[-a +
 b*Sqrt[c]]
```

Maple [A]
time = 0.12, size = 92, normalized size = 0.95

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92
default	$\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{-\sqrt{b^2c}-a}}\right)}{\sqrt{-\sqrt{b^2c}-a}}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\sqrt{\sqrt{b^2c}-a}}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(-(b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)
-a)^(1/2))+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2
*c)^(1/2)-a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(73) = 146$.
time = 0.42, size = 743, normalized size = 7.66

$$\sqrt{-\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a}{b^{2} c-a^{2}}} \log \left(4\left(\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a\right) \sqrt{-\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a}{b^{2} c-a^{2}}}+4 \sqrt{\sqrt{d x+c}\, b+a}\right)-\sqrt{-\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a}{b^{2} c-a^{2}}} \log \left(-4\left(\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a\right) \sqrt{-\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a}{b^{2} c-a^{2}}}+4 \sqrt{\sqrt{d x+c}\, b+a}\right)-\sqrt{\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a}{b^{2} c-a^{2}}} \log \left(4\left(\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a\right) \sqrt{\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a}{b^{2} c-a^{2}}}+4 \sqrt{\sqrt{d x+c}\, b+a}\right)+\sqrt{\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a}{b^{2} c-a^{2}}} \log \left(-4\left(\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}+a\right) \sqrt{\frac{\left(b^{2} c-a^{2}\right) \sqrt{\frac{b^{2} c}{b^{4} c^{2}-2 a^{2} b^{2} c+a^{4}}}-a}{b^{2} c-a^{2}}}+4 \sqrt{\sqrt{d x+c}\, b+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)
*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c
 - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(
b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqr
t(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c
/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*
b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a
^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a
)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(
b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a
^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt
(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x
 + c)*b + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [A]
time = 4.26, size = 140, normalized size = 1.44

$$\frac{2\left(\frac{\left(b^{2}\sqrt{c}\,|b|+ab^{2}\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a+\sqrt{b^{2}c}}}\right)}{\left(b\sqrt{c}+a\right)\sqrt{b\sqrt{c}-a}}+\frac{\left(b^{2}\sqrt{c}\,|b|-ab^{2}\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-a-\sqrt{b^{2}c}}}\right)}{\left(b\sqrt{c}-a\right)\sqrt{-b\sqrt{c}-a}}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a +
sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b) -
 a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c
) - a)*sqrt(-b*sqrt(c) - a)))/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.651 $\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=163

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\left(a+b\sqrt{c}\right)^{3/2}\sqrt{c}}$$

```
[Out] -1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))/c^(1/2)/(a-
b*c^(1/2))^(3/2)+1/2*b*d*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a+b*c^(1/2))^(1
/2))/c^(1/2)/(a+b*c^(1/2))^(3/2)-(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1
/2)/(-b^2*c+a^2)/x
```

Rubi [A]
time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {378, 1412, 837, 841, 1180, 213}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}\left(a-b\sqrt{c+dx}\right)}{x\left(a^2-b^2c\right)}-\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\left(a-b\sqrt{c}\right)^{3/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\left(a+b\sqrt{c}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b
*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c
])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[
c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx = d\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^2}\,dx, x, c+dx\right)$$
$$= (2d)\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,(-c+x^2)^2}\,dx, x, \sqrt{c+dx}\right)$$
$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{d\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{1}{2}b^2cx}{\sqrt{a+bx}\,(-c+x^2)}\,dx, x, \sqrt{c+\right.}{c\,(a^2-b^2c)}$$
$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{(2d)\text{Subst}\left(\int \frac{-ab^2c+\frac{1}{2}b^2cx^2}{a^2-b^2c-2ax^2+x^4}\,dx, x, \sqrt{a\right.}{c\,(a^2-b^2c)}$$
$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{(bd)\text{Subst}\left(\int \frac{1}{-a+b\sqrt{c}+x^2}\,dx, x, \sqrt{a+\right.}{2\left(a-b\sqrt{c}\right)\sqrt{c}}$$
$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} - \frac{bd\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\left(a-b\sqrt{c}\right)^{3/2}\sqrt{c}} + \frac{bd}{}$$

Mathematica [A]
time = 0.52, size = 170, normalized size = 1.04

$$\frac{1}{2}\left(-\frac{2\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{(a^2-b^2c)\,x} + \frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{3/2}\sqrt{c}} - \frac{bd\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{3/2}\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] ((-2*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x) + (
b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a - b*Sqrt[c]]])/((-a - b*Sqrt[c
])^(3/2)*Sqrt[c]) - (b*d*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[
c]]])/((-a + b*Sqrt[c])^(3/2)*Sqrt[c]))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(125) = 250.
time = 0.13, size = 259, normalized size = 1.59

method	result
derivativedivides	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}}{}\right.$
default	$4d\,b^2\left(-\frac{\sqrt{b^2c}\left(\frac{2\sqrt{a+b\sqrt{dx+c}}}{\left(4\sqrt{b^2c}-4a\right)\left(b\sqrt{dx+c}+\sqrt{b^2c}\right)}+\frac{2\arctan\left(\frac{\sqrt{a+b\sqrt{dx+c}}}{\sqrt{\sqrt{b^2c}-a}}\right)}{\left(4\sqrt{b^2c}-4a\right)\sqrt{\sqrt{b^2c}-a}}\right)}{4b^2c}+\frac{\sqrt{b^2c}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d*b^2*(-1/4*(b^2*c)^(1/2)/b^2/c*(2*(a+b*(d*x+c)^(1/2))^(1/2)/(4*(b^2*c)^(
1/2)-4*a)/(b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(4*(b^2*c)^(1/2)-4*a)/((b^2*c)^
(1/2)-a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2)))+1
/4*(b^2*c)^(1/2)/b^2/c*(-2*(a+b*(d*x+c)^(1/2))^(1/2)/(-4*(b^2*c)^(1/2)-4*a)
/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))+2/(-4*(b^2*c)^(1/2)-4*a)/(-(b^2*c)^(1/2)-
a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2493 vs. $2(127) = 254$.
time = 0.48, size = 2493, normalized size = 15.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

```
[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^
4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^
4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4
*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2
- a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6
*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)
*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 +
 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c
)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2
*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a
^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2
*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2
*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3
*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c
^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*
a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))
*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*
a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^
10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b
^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(
-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^
6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c
^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^
12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)
*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*
c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^
2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*
c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6
*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c
)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 +
6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 -
20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^
4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt
((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a
^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(
b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-
((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^
6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^1
2*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^
2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (
```

```
b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8
*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^
6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3
*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^
2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^
5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 -
 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sq
rt(d*x + c)*b - a))/((b^2*c - a^2)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(127) = 254.
time = 3.80, size = 654, normalized size = 4.01

$$\frac{\frac{\left(\left(b^3c-a^2b\right)^2b^4c^{\frac{3}{2}}d^2-2\left(ab^6c^2-a^3b^4c\right)d^2\left|-b^3c+a^2b\right|+\left(a^2b^8c^{\frac{5}{2}}-2a^4b^6c^{\frac{3}{2}}+a^6b^4\sqrt{c}\right)d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-\frac{ab^2c-a^3+\sqrt{\left(ab^2c-a^3\right)^2+\left(b^4c^2-2a^2b^2c+a^4\right)\left(b^2c-a^2\right)}}{b^2c-a^2}}}\right)}{\left(b^5c^{\frac{7}{2}}+ab^4c^3-2a^2b^3c^{\frac{5}{2}}-2a^3b^2c^2+a^4bc^{\frac{3}{2}}+a^5c\right)\sqrt{b\sqrt{c}-a}\left|-b^3c+a^2b\right|}+\frac{\left(\left(b^3c-a^2b\right)^2b^4c^{\frac{3}{2}}d^2+2\left(ab^6c^2-a^3b^4c\right)d^2\left|-b^3c+a^2b\right|+\left(a^2b^8c^{\frac{5}{2}}-2a^4b^6c^{\frac{3}{2}}+a^6b^4\sqrt{c}\right)d^2\right)\arctan\left(\frac{\sqrt{\sqrt{dx+c}\,b+a}}{\sqrt{-\frac{ab^2c-a^3-\sqrt{\left(ab^2c-a^3\right)^2+\left(b^4c^2-2a^2b^2c+a^4\right)\left(b^2c-a^2\right)}}{b^2c-a^2}}}\right)}{\left(b^5c^{\frac{7}{2}}-ab^4c^3-2a^2b^3c^{\frac{5}{2}}+2a^3b^2c^2+a^4bc^{\frac{3}{2}}-a^5c\right)\sqrt{-b\sqrt{c}-a}\left|-b^3c+a^2b\right|}+\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^{\frac{3}{2}}b^4d^2-2\sqrt{\sqrt{dx+c}\,b+a}\,ab^4d^2\right)}{\left(b^2c-\left(\sqrt{dx+c}\,b+a\right)^2+2\left(\sqrt{dx+c}\,b+a\right)a-a^2\right)\left(b^2c-a^2\right)}}{2b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^3*c - a^2*b)^2*b^4*c^(3/2)*d^2 - 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs
(-b^3*c + a^2*b) + (a^2*b^8*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*
d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c
- a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((
b^5*c^(7/2) + a*b^4*c^3 - 2*a^2*b^3*c^(5/2) - 2*a^3*b^2*c^2 + a^4*b*c^(3/2)
 + a^5*c)*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4
*c^(3/2)*d^2 + 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8
*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*d^2)*arctan(sqrt(sqrt(d*x +
 c)*b + a)/sqrt(-(a*b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2
*b^2*c + a^4)*(b^2*c - a^2)))/(b^2*c - a^2)))/((b^5*c^(7/2) - a*b^4*c^3 - 2
*a^2*b^3*c^(5/2) + 2*a^3*b^2*c^2 + a^4*b*c^(3/2) - a^5*c)*sqrt(-b*sqrt(c) -
 a)*abs(-b^3*c + a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(
sqrt(d*x + c)*b + a)*a*b^4*d^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt
(d*x + c)*b + a)*a - a^2)*(b^2*c - a^2)))/(b^2*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)
[Out] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.652 $\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$

Optimal. Leaf size=261

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8c\left(a^2-b^2c\right)^2x}+\frac{b\left(2a-5b\sqrt{c}\right.}{}$$

```
[Out] 1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))^(1/2)/(a-b*c^(1/2))^(1/2))*(2*a-5*b*
c^(1/2))/c^(3/2)/(a-b*c^(1/2))^(5/2)-1/16*b*d^2*arctanh((a+b*(d*x+c)^(1/2))
^(1/2)/(a+b*c^(1/2))^(1/2))*(2*a+5*b*c^(1/2))/c^(3/2)/(a+b*c^(1/2))^(5/2)-1
/2*(a-b*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/(-b^2*c+a^2)/x^2-1/8*b*d*(
6*a*b*c-(5*b^2*c+a^2)*(d*x+c)^(1/2))*(a+b*(d*x+c)^(1/2))^(1/2)/c/(-b^2*c+a^
2)^2/x
```

Rubi [A]
time = 0.35, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {378, 1412, 837, 841, 1180, 213}

$$-\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2x^2\left(a^2-b^2c\right)}-\frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-\left(a^2+5b^2c\right)\sqrt{c+dx}\right)}{8cx\left(a^2-b^2c\right)^2}+\frac{bd^2\left(2a-5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}\left(a-b\sqrt{c}\right)^{5/2}}-\frac{bd^2\left(2a+5b\sqrt{c}\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}\left(a+b\sqrt{c}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] -1/2*((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x^2)
- (b*d*Sqrt[a + b*Sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*Sqrt[c + d*x]))
/(8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sq
rt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(5/2)*c^(3/2)) - (b*
(2*a + 5*b*Sqrt[c])*d^2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c
]]])/(16*(a + b*Sqrt[c])^(5/2)*c^(3/2))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
```

```
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx = d^2\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}\,(-c+x)^3}\,dx, x, c+dx\right)$$

$$= (2d^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\,(-c+x^2)^3}\,dx, x, \sqrt{c+dx}\right)$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} + \frac{d^2\text{Subst}\left(\int \frac{-\frac{1}{2}abc+\frac{5}{2}b^2cx}{\sqrt{a+bx}\,(-c+x^2)^2}\,dx, x, \cdots\right.}{2c\left(a^2-b^2c\right)}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-(a^2+5\cdots\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-(a^2+5\cdots\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-(a^2+5\cdots\right.}{8c\left(a^2-b^2c\right)^2x}$$

$$= -\frac{\left(a-b\sqrt{c+dx}\right)\sqrt{a+b\sqrt{c+dx}}}{2\left(a^2-b^2c\right)x^2} - \frac{bd\sqrt{a+b\sqrt{c+dx}}\left(6abc-(a^2+5\cdots\right.}{8c\left(a^2-b^2c\right)^2x}$$

Mathematica [A]
time = 1.34, size = 249, normalized size = 0.95

$$\frac{-\frac{2\sqrt{c}\sqrt{a+b\sqrt{c+dx}}\left(4a^3c+b^3c(4c-5dx)\sqrt{c+dx}-a^2b\sqrt{c+dx}\,(4c+dx)+2ab^2c(-2c+3dx)\right)}{(a^2-b^2c)^2x^2}+\frac{b\left(2a+5b\sqrt{c}\right)d^2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a-b\sqrt{c}}}\right)}{\left(-a-b\sqrt{c}\right)^{5/2}}+\frac{b\left(-2a+5b\sqrt{c}\right)d^2\tan^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{-a+b\sqrt{c}}}\right)}{\left(-a+b\sqrt{c}\right)^{5/2}}}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

```
[Out] ((-2*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]]*(4*a^3*c + b^3*c*(4*c - 5*d*x)*Sqrt[
c + d*x] - a^2*b*Sqrt[c + d*x]*(4*c + d*x) + 2*a*b^2*c*(-2*c + 3*d*x)))/((a
^2 - b^2*c)^2*x^2) + (b*(2*a + 5*b*Sqrt[c])*d^2*ArcTan[Sqrt[a + b*Sqrt[c +
d*x]]/Sqrt[-a - b*Sqrt[c]]])/(-a - b*Sqrt[c])^(5/2) + (b*(-2*a + 5*b*Sqrt[c
])*d^2*ArcTan[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[-a + b*Sqrt[c]]])/(-a + b*Sqrt
[c])^(5/2))/(16*c^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(209) = 418$.

time = 0.21, size = 427, normalized size = 1.64

method	result
derivativedivides	$4d^2b^4\left(-\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\cdots\right.}{4\left(b^2c+2a\sqrt{b^2c}+a\cdots\right.}}{16b^2c\sqrt{b^2c}}\cdots\right.$
default	$4d^2b^4\left(-\frac{\frac{-\frac{\left(5\sqrt{b^2c}+2a\right)\left(a+b\sqrt{dx+c}\right)^{\frac{3}{2}}}{4\left(b^2c+2a\sqrt{b^2c}+a^2\right)}+\frac{\left(7\sqrt{b^2c}+2a\right)\sqrt{a+b\sqrt{dx+c}}}{4\sqrt{b^2c}+4a}}{\left(-b\sqrt{dx+c}+\sqrt{b^2c}\right)^2}-\frac{\left(5\sqrt{b^2c}+2a\right)\arctan\left(\cdots\right.}{4\left(b^2c+2a\sqrt{b^2c}+a\cdots\right.}}{16b^2c\sqrt{b^2c}}\cdots\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*d^2*b^4*(-1/16/b^2/c/(b^2*c)^(1/2)*((-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*
a*(b^2*c)^(1/2)+a^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(7*(b^2*c)^(1/2)+2*a)/((
b^2*c)^(1/2)+a)*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)+(b^2*c)^(1/2))
^2-1/4*(5*(b^2*c)^(1/2)+2*a)/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-(b^2*c)^(1/2)-
a)^(1/2)*arctan((a+b*(d*x+c)^(1/2))^(1/2)/(-(b^2*c)^(1/2)-a)^(1/2)))+1/16/b
^2/c/(b^2*c)^(1/2)*((-1/4*(-5*(b^2*c)^(1/2)+2*a)/(b^2*c-2*a*(b^2*c)^(1/2)+a
^2)*(a+b*(d*x+c)^(1/2))^(3/2)+1/4*(-7*(b^2*c)^(1/2)+2*a)/(-(b^2*c)^(1/2)+a)
*(a+b*(d*x+c)^(1/2))^(1/2))/(-b*(d*x+c)^(1/2)-(b^2*c)^(1/2))^2-1/4*(5*(b^2*
c)^(1/2)-2*a)/(-b^2*c+2*a*(b^2*c)^(1/2)-a^2)/((b^2*c)^(1/2)-a)^(1/2)*arctan
((a+b*(d*x+c)^(1/2))^(1/2)/((b^2*c)^(1/2)-a)^(1/2))))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^3), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4390 vs. $2(212) = 424$.
time = 1.28, size = 4390, normalized size = 16.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3
*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a
^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4
+ 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10
)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10
+ 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^
6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c
^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*log((625*
b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x
+ c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 +
35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*
a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c
^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6
*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^
11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^
8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))*
sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b
^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 -
 a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10
780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*
b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*
a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*
c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*
b^2*c^4 - a^10*c^3))) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*sqrt(-((105*a
*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a
^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sq
rt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*
c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 1
20*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7
- 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3)))/(b^10*
c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^1
0*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^
6)*sqrt(sqrt(d*x + c)*b + a)*d^6 - ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 6
```

```
09*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a
^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b
^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^1
4*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12
 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*
c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c
^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*
a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5
+ 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*
a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^
18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^1
0*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^1
8*b^2*c^4 + a^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6
*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))) + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*
x^2*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4
- (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c
^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2
- 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*
a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 +
210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a
^20*c^3)))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*
a^8*b^2*c^4 - a^10*c^3))*log((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b
^8*c + 140*a^6*b^6)*sqrt(sqrt(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977
*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 + (5*b^14*c^10 - 16*a
^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^
4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*b^16*c^
3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 -
 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9
 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^
5 - 10*a^18*b^2*c^4 + a^20*c^3)))*sqrt(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 -
35*a^5*b^4*c + 4*a^7*b^2)*d^4 - (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6
- 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*sqrt((625*b^18*c^4 + 7700*a^2*
b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20
*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 1...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1303 vs. $2(212) = 424$.

time = 3.97, size = 1303, normalized size = 4.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13*
a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))
*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^
12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^
3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*
c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 +
 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^
2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4*a^3*b^6
*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a^7*b^2*c^
(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b
^3*c^2 + a^4*b*c)) + ((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*
b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a
^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*
c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 -
 a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3
*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 -
3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/
(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(11/2) - 4*a^2*b^7*
c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3
 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^5
*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) - 2*(9*(sqrt(d*x + c)*b + a)^(3/2)*b^8*c^2
*d^3 - 19*sqrt(sqrt(d*x + c)*b + a)*a*b^8*c^2*d^3 - 5*(sqrt(d*x + c)*b + a)
^(7/2)*b^6*c*d^3 + 21*(sqrt(d*x + c)*b + a)^(5/2)*a*b^6*c*d^3 - 30*(sqrt(d*
x + c)*b + a)^(3/2)*a^2*b^6*c*d^3 + 18*sqrt(sqrt(d*x + c)*b + a)*a^3*b^6*c*
d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a^2*b^4*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/
2)*a^3*b^4*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^4*d^3 + sqrt(sqrt(d*x
+ c)*b + a)*a^5*b^4*d^3)/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c - (sqrt(
d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2))/(b^2*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a+b\,\sqrt{c+d\,x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)
[Out] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)
```

3.653 $\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=350

$$-\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)}+\frac{2(a^2-b^2c)^2\,(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^8d^4(2+p)}-\frac{6a(7a^2-3b^2c)\,(a^2-}{b^8d^4}$$

```
[Out] -2*a*(-b^2*c+a^2)^3*(a+b*(d*x+c)^(1/2))^(1+p)/b^8/d^4/(1+p)+2*(-b^2*c+a^2)^
2*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/2))^(2+p)/b^8/d^4/(2+p)-6*a*(-3*b^2*c+7*a^
2)*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(3+p)/b^8/d^4/(3+p)+2*(3*b^4*c^2-30*a^2
*b^2*c+35*a^4)*(a+b*(d*x+c)^(1/2))^(4+p)/b^8/d^4/(4+p)-10*a*(-3*b^2*c+7*a^2
)*(a+b*(d*x+c)^(1/2))^(5+p)/b^8/d^4/(5+p)+6*(-b^2*c+7*a^2)*(a+b*(d*x+c)^(1/
2))^(6+p)/b^8/d^4/(6+p)-14*a*(a+b*(d*x+c)^(1/2))^(7+p)/b^8/d^4/(7+p)+2*(a+b
*(d*x+c)^(1/2))^(8+p)/b^8/d^4/(8+p)
```

Rubi [A]
time = 0.20, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$,
Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)^3\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^8d^4(p+1)}+\frac{2(a^2-b^2c)^2\,(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^8d^4(p+2)}-\frac{6a(7a^2-3b^2c)\,(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^8d^4(p+3)}-\frac{10a(7a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^8d^4(p+5)}+\frac{6(7a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^8d^4(p+6)}+\frac{2(35a^4-30a^2b^2c+3b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^8d^4(p+4)}-\frac{14a\left(a+b\sqrt{c+dx}\right)^{p+7}}{b^8d^4(p+7)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+8}}{b^8d^4(p+8)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2
*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2
 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p)
)/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c
+ d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c
+ d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x
])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d
^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x^3\left(a+b\sqrt{c+dx}\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)^3\,dx,x,c+dx\right)}{d^4}$$
$$=\frac{2\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^3\,dx,x,\sqrt{c+dx}\right)}{d^4}$$
$$=\frac{2\text{Subst}\left(\int\left(-\frac{a\left(a^2-b^2c\right)^3(a+bx)^p}{b^7}-\frac{\left(-7a^2+b^2c\right)\left(-a^2+b^2c\right)^2(a+bx)^{1+p}}{b^7}-\frac{3\left(7a^5-10a^3b^2c+3ab^4c\right.}{b^7}\right.\right.}{}$$
$$=-\frac{2a\left(a^2-b^2c\right)^3\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^8d^4(1+p)}+\frac{2\left(a^2-b^2c\right)^2\left(7a^2-b^2c\right)\left(a+b\sqrt{c+dx}\right.}{b^8d^4(2+p)}$$

Mathematica [A]
time = 0.79, size = 554, normalized size = 1.58

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(5040a^7-5040a^6b(1+p)\sqrt{c+dx}-360a^5b^2\left(-6c\left(-7+p+p^2\right)-7d\left(2+3p+p^2\right)x\right)+120a^4b^3(1+p)\sqrt{c+dx}\left(c\left(126+10p-4p^2\right)-7d\left(6+5p+p^2\right)x\right)+6a^3b^4\left(8c^2\left(315-124p-139p^2-14p^3+p^4\right)+40cd\left(-42-61p-16p^2+4p^3+p^4\right)x+35d^2\left(24+50p+35p^2+10p^3+p^4\right)x^2\right)-6a^2b^5(1+p)\sqrt{c+dx}\left(-24c^2\left(-105-24p+5p^2+p^3\right)+4cd\left(-420-386p-94p^2-p^3+p^4\right)x+7d^2\left(120+154p+71p^2+14p^3+p^4\right)x^2\right)+b^7\left(105+176p+86p^2+16p^3+p^4\right)\sqrt{c+dx}\left(48c^3-24c^2d(2+p)x+6cd^2\left(8+6p+p^2\right)x^2-d^3\left(48+44p+12p^2+p^3\right)x^3\right)+ab^6\left(48c^3\left(-105+103p+138p^2+38p^3+3p^4\right)-24c^2d\left(-210-283p-21p^2+74p^3+24p^4+2p^5\right)x+6cd^2\left(-840-1726p-1151p^2-265p^3+10p^4+11p^5+p^6\right)x^2+7d^3\left(720+1764p+1624p^2+735p^3+175p^4+21p^5+p^6\right)x^3\right)\right)}{b^8d^4(1+p)(2+p)(3+p)(4+p)(5+p)(6+p)(7+p)(8+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d
*x] - 360*a^5*b^2*(-6*c*(-7 + p + p^2) - 7*d*(2 + 3*p + p^2)*x) + 120*a^4*b
^3*(1 + p)*Sqrt[c + d*x]*(c*(126 + 10*p - 4*p^2) - 7*d*(6 + 5*p + p^2)*x) +
 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61
*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x
^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3)
+ 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2
 + 14*p^3 + p^4)*x^2) + b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c +
d*x]*(48*c^3 - 24*c^2*d*(2 + p)*x + 6*c*d^2*(8 + 6*p + p^2)*x^2 - d^3*(48 +
 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^3
 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x +
6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 +
```

```
7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/(
b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b\sqrt{dx + c}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^3*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(335) = 670.
time = 0.29, size = 728, normalized size = 2.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^
4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b
^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c^2/((p^4 + 1
0*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 27
4*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x +
 c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(
p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2
 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p*c/((p^6 + 2
1*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6
+ 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b
^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x
+ c)^(7/2)*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d
*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5
/2)*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p
^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^
2 + 5040*sqrt(d*x + c)*a^7*b*p - 5040*a^8)*(sqrt(d*x + c)*b + a)^p/((p^8 +
36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p
 + 40320)*b^8))/d^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. 2(335) = 670.
time = 0.50, size = 1416, normalized size = 4.05

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c + 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7 + 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 + 13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 + 7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6)*d^3*p^6 + 5*(38*b^8*c - 21*a^2*b^6)*d^3*p^5 + 5*(164*b^8*c - 119*a^2*b^6) *d^3*p^4 + (1849*b^8*c - 1575*a^2*b^6)*d^3*p^3 + 2*(1019*b^8*c - 959*a^2*b^ 6)*d^3*p^2 + 840*(b^8*c - a^2*b^6)*d^3*p)*x^3 + 48*(86*b^8*c^4 + 81*a^2*b^6 *c^3 - 124*a^4*b^4*c^2 + 45*a^6*b^2*c)*p^2 + 6*(18*b^8*c^2*d^2*p^5 + (b^8*c ^2 + a^2*b^6*c)*d^2*p^6 + (118*b^8*c^2 - 95*a^2*b^6*c + 35*a^4*b^4)*d^2*p^4 + 6*(58*b^8*c^2 - 80*a^2*b^6*c + 35*a^4*b^4)*d^2*p^3 + (457*b^8*c^2 - 806* a^2*b^6*c + 385*a^4*b^4)*d^2*p^2 + 210*(b^8*c^2 - 2*a^2*b^6*c + a^4*b^4)*d^ 2*p)*x^2 + 192*(44*b^8*c^4 - 71*a^2*b^6*c^3 + 54*a^4*b^4*c^2 - 15*a^6*b^2*c)*p - 24*((b^8*c^3 + 3*a^2*b^6*c^2)*d*p^5 + 2*(8*b^8*c^3 + 9*a^2*b^6*c^2 - 5*a^4*b^4*c)*d*p^4 + (86*b^8*c^3 - 57*a^2*b^6*c^2 + 15*a^4*b^4*c)*d*p^3 + (176*b^8*c^3 - 387*a^2*b^6*c^2 + 340*a^4*b^4*c - 105*a^6*b^2)*d*p^2 + 105*(b ^8*c^3 - 3*a^2*b^6*c^2 + 3*a^4*b^4*c - a^6*b^2)*d*p)*x + (192*(a*b^7*c^3 + a^3*b^5*c^2)*p^4 + 96*(27*a*b^7*c^3 + 2*a^3*b^5*c^2 - 5*a^5*b^3*c)*p^3 - (a *b^7*d^3*p^7 + 21*a*b^7*d^3*p^6 + 175*a*b^7*d^3*p^5 + 735*a*b^7*d^3*p^4 + 1 624*a*b^7*d^3*p^3 + 1764*a*b^7*d^3*p^2 + 720*a*b^7*d^3*p)*x^3 + 192*(56*a*b ^7*c^3 - 49*a^3*b^5*c^2 + 15*a^5*b^3*c)*p^2 + 6*(2*a*b^7*c*d^2*p^6 + (33*a* b^7*c - 7*a^3*b^5)*d^2*p^5 + 10*(20*a*b^7*c - 7*a^3*b^5)*d^2*p^4 + 5*(111*a *b^7*c - 49*a^3*b^5)*d^2*p^3 + 2*(349*a*b^7*c - 175*a^3*b^5)*d^2*p^2 + 24*(13*a*b^7*c - 7*a^3*b^5)*d^2*p)*x^2 + 48*(279*a*b^7*c^3 - 511*a^3*b^5*c^2 + 385*a^5*b^3*c - 105*a^7*b)*p - 24*((3*a*b^7*c^2 + a^3*b^5*c)*d*p^5 + 2*(21* a*b^7*c^2 - 5*a^3*b^5*c)*d*p^4 + (192*a*b^7*c^2 - 135*a^3*b^5*c + 35*a^5*b^ 3)*d*p^3 + (327*a*b^7*c^2 - 320*a^3*b^5*c + 105*a^5*b^3)*d*p^2 + 2*(87*a*b^ 7*c^2 - 98*a^3*b^5*c + 35*a^5*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^8*d^4*p^8 + 36*b^8*d^4*p^7 + 546*b^8*d^4*p^6 + 4536*b^8*d^4*p^5 + 22449*b^8*d^4*p^4 + 67284*b^8*d^4*p^3 + 118124*b^8*d^4*p^2 + 109584*b^8*d^ 4*p + 40320*b^8*d^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b\sqrt{c + dx}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)

[Out] Integral(x**3*(a + b*sqrt(c + d*x))**p, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5699 vs. $2(335) = 670$.
time = 4.13, size = 5699, normalized size = 16.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^7 - (sqrt(d*x
 + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^7 + 34*(sqrt(d*x + c)*b +
a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^6 - 35*(sqrt(d*x + c)*b + a)*(sqrt(d
*x + c)*b + a)^p*a*b^6*c^3*p^6 - 3*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
 + a)^p*b^4*c^2*p^7 + 9*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b
^4*c^2*p^7 - 9*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*
p^7 + 3*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^7 + 478
*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^5 - 511*(sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^5 - 96*(sqrt(d*x + c)*b +
 a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^6 + 297*(sqrt(d*x + c)*b + a)^3*(sq
rt(d*x + c)*b + a)^p*a*b^4*c^2*p^6 - 306*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*b^4*c^2*p^6 + 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b
+ a)^p*a^3*b^4*c^2*p^6 + 3*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*
b^2*c*p^7 - 15*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^7
+ 30*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^7 - 30*(sq
rt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^7 + 15*(sqrt(d*x +
 c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^7 - 3*(sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^7 + 3580*(sqrt(d*x + c)*b + a)^2*(sqr
t(d*x + c)*b + a)^p*b^6*c^3*p^4 - 4025*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a*b^6*c^3*p^4 - 1254*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a
)^p*b^4*c^2*p^5 + 4023*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^
4*c^2*p^5 - 4302*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^
2*p^5 + 1533*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^5
+ 90*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^6 - 465*(sqrt(
d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^6 + 960*(sqrt(d*x + c)*
b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^6 - 990*(sqrt(d*x + c)*b + a)^
3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^6 + 510*(sqrt(d*x + c)*b + a)^2*(sqrt
(d*x + c)*b + a)^p*a^4*b^2*c*p^6 - 105*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)
*b + a)^p*a^5*b^2*c*p^6 - (sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p
^7 + 7*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^7 - 21*(sqrt(d*x
 + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^7 + 35*(sqrt(d*x + c)*b + a)^5
*(sqrt(d*x + c)*b + a)^p*a^3*p^7 - 35*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c
)*b + a)^p*a^4*p^7 + 21*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^5
*p^7 - 7*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^6*p^7 + (sqrt(d*
x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^7*p^7 + 15289*(sqrt(d*x + c)*b + a)
^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^3 - 18424*(sqrt(d*x + c)*b + a)*(sqrt(
```

```
d*x + c)*b + a)^p*a*b^6*c^3*p^3 - 8592*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x +
c)*b + a)^p*b^4*c^2*p^4 + 28755*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a*b^4*c^2*p^4 - 32220*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*
a^2*b^4*c^2*p^4 + 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b
^4*c^2*p^4 + 1098*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5
 - 5865*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^5 + 12540
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 - 13410*(sqr
t(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 7170*(sqrt(d*x
+ c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*b^2*c*p^5 - 1533*(sqrt(d*x + c)*b
 + a)*(sqrt(d*x + c)*b + a)^p*a^5*b^2*c*p^5 - 28*(sqrt(d*x + c)*b + a)^8*(s
qrt(d*x + c)*b + a)^p*p^6 + 203*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b +
a)^p*a*p^6 - 630*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^6 +
1085*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a^3*p^6 - 1120*(sqrt(d
*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^4*p^6 + 693*(sqrt(d*x + c)*b + a
)^3*(sqrt(d*x + c)*b + a)^p*a^5*p^6 - 238*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
 + c)*b + a)^p*a^6*p^6 + 35*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a
^7*p^6 + 36706*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^6*c^3*p^2
- 48860*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^6*c^3*p^2 - 32979
*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 + 115776*(sqrt
(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 137601*(sqrt(d*x
 + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^4*c^2*p^3 + 55272*(sqrt(d*x +
c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^4*c^2*p^3 + 7020*(sqrt(d*x + c)*b +
 a)^6*(sqrt(d*x + c)*b + a)^p*b^2*c*p^4 - 38715*(sqrt(d*x + c)*b + a)^5*(sq
rt(d*x + c)*b + a)^p*a*b^2*c*p^4 + 85920*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x
+ c)*b + a)^p*a^2*b^2*c*p^4 - 95850*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a^3*b^2*c*p^4 + 53700*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*b^2*c*p^4 - 12075*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5
*b^2*c*p^4 - 322*(sqrt(d*x + c)*b + a)^8*(sqrt(d*x + c)*b + a)^p*p^5 + 2401
*(sqrt(d*x + c)*b + a)^7*(sqrt(d*x + c)*b + a)^p*a*p^5 - 7686*(sqrt(d*x + c
)*b + a)^6*(sqrt(d*x + c)*b + a)^p*a^2*p^5 + 13685*(sqrt(d*x + c)*b + a)^5*
(sqrt(d*x + c)*b + a)^p*a^3*p^5 - 14630*(sqrt(d...
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b\sqrt{c + d\,x}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^p, x)
```

3.654 $\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=242

$$-\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^6d^3(2+p)}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{\right.}\right.}{b^6d^3(3+p}$$

```
[Out] -2*a*(-b^2*c+a^2)^2*(a+b*(d*x+c)^(1/2))^(1+p)/b^6/d^3/(1+p)+2*(b^4*c^2-6*a^
2*b^2*c+5*a^4)*(a+b*(d*x+c)^(1/2))^(2+p)/b^6/d^3/(2+p)-4*a*(-3*b^2*c+5*a^2)
*(a+b*(d*x+c)^(1/2))^(3+p)/b^6/d^3/(3+p)+4*(-b^2*c+5*a^2)*(a+b*(d*x+c)^(1/2
))^(4+p)/b^6/d^3/(4+p)-10*a*(a+b*(d*x+c)^(1/2))^(5+p)/b^6/d^3/(5+p)+2*(a+b*
(d*x+c)^(1/2))^(6+p)/b^6/d^3/(6+p)
```

Rubi [A]
time = 0.13, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^6d^3(p+1)}-\frac{4a(5a^2-3b^2c)\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^6d^3(p+3)}+\frac{4(5a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^6d^3(p+4)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^6d^3(p+2)}-\frac{10a\left(a+b\sqrt{c+dx}\right)^{p+5}}{b^6d^3(p+5)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+6}}{b^6d^3(p+6)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2
*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2
 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3
+ p)) + (4*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p))
 - (10*a*(a + b*Sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*Sqrt[
c + d*x])^(6 + p))/(b^6*d^3*(6 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx &= \frac{\text{Subst}\left(\int\left(a+b\sqrt{x}\right)^p(-c+x)^2\,dx,x,c+dx\right)}{d^3}\\ &= \frac{2\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)^2\,dx,x,\sqrt{c+dx}\right)}{d^3}\\ &= \frac{2\text{Subst}\left(\int\left(-\frac{a(a^2-b^2c)^2(a+bx)^p}{b^5}+\frac{(5a^4-6a^2b^2c+b^4c^2)(a+bx)^{1+p}}{b^5}-\frac{2(5a^3-3ab^2c)(a+bx)^{2+p}}{b^5}-\right.\right.}{d^3}\\ &= -\frac{2a(a^2-b^2c)^2\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^6d^3(1+p)}+\frac{2(5a^4-6a^2b^2c+b^4c^2)\left(a+b\sqrt{c+dx}\right.}{b^6d^3(2+p)}\end{aligned}$$

Mathematica [A]
time = 0.45, size = 285, normalized size = 1.18

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(-120a^5+120a^4b(1+p)\sqrt{c+dx}+12a^3b^2(-4c(-5+p+p^2)-5d(2+3p+p^2)x)-4a^2b^3(1+p)\sqrt{c+dx}\,(c(60+8p-2p^2)-5d(6+5p+p^2)x)+b^5(15+23p+9p^2+p^3)\sqrt{c+dx}\,(8c^2-4cd(2+p)x+d^2(8+6p+p^2)x^2)-ab^4(-8c^2(-15+10p+12p^2+2p^3)+4cd(-30-43p-10p^2+4p^3+p^4)x+5d^2(24+50p+35p^2+10p^3+p^4)x^2)\right)}{b^6d^3(1+p)(2+p)(3+p)(4+p)(5+p)(6+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-120*a^5 + 120*a^4*b*(1 + p)*Sqrt[c + d*x
] + 12*a^3*b^2*(-4*c*(-5 + p + p^2) - 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1
 + p)*Sqrt[c + d*x]*(c*(60 + 8*p - 2*p^2) - 5*d*(6 + 5*p + p^2)*x) + b^5*(1
5 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6
*p + p^2)*x^2) - a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 -
 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)
*x^2)))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int(x^2*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A]
time = 0.30, size = 402, normalized size = 1.66

$$\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^pc^2}{(p^2+3p+2)b^2}-\frac{2\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+c}\,a^3bp-6a^4\right)\left(\sqrt{dx+c}\,b+a\right)^pc}{(p^4+10p^3+35p^2+50p+24)b^4}+\frac{\left((p^5+15p^4+85p^3+225p^2+274p+120)(dx+c)^3b^6+(p^5+10p^4+35p^3+50p^2+24p)(dx+c)^{\frac{5}{2}}ab^5-5(p^4+6p^3+11p^2+6p)(dx+c)^2a^2b^4+20(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}a^3b^3-60(p^2+p)(dx+c)a^4b^2+120\sqrt{dx+c}\,a^5bp-120a^6\right)\left(\sqrt{dx+c}\,b+a\right)^p}{(p^6+21p^5+175p^4+735p^3+1624p^2+1764p+720)b^6}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a
)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4
 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^
2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p
^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p +
 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(
5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 +
 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 12
0*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 +
 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(230) = 460$.
time = 0.42, size = 712, normalized size = 2.94

$$\frac{2\left(120b^6c^3-360a^2b^4c^2+360a^4b^2c-120a^6+8(b^6c^3+3a^2b^4c^2)p^3+(b^6d^3p^5+15b^6d^3p^4+85b^6d^3p^3+225b^6d^3p^2+274b^6d^3p+120b^6d^3)x^3+24(3b^6c^3+3a^2b^4c^2-2a^4b^2c)p^2+(b^6cd^2p^5+(11b^6c-5a^2b^4)d^2p^4+(41b^6c-30a^2b^4)d^2p^3+(61b^6c-55a^2b^4)d^2p^2+30(b^6c-a^2b^4)d^2p)x^2+8(23b^6c^3-24a^2b^4c^2+9a^4b^2c)p-4((b^6c^2+a^2b^4c)dp^4+3(3b^6c^2-a^2b^4c)dp^3+(23b^6c^2-34a^2b^4c+15a^4b^2)dp^2+15(b^6c^2-2a^2b^4c+a^4b^2)dp)x+(8(3ab^5c^2+a^3b^3c)p^3+24(7ab^5c^2-3a^3b^3c)p^2+(ab^5d^2p^5+10ab^5d^2p^4+35ab^5d^2p^3+50ab^5d^2p^2+24ab^5d^2p)x^2+8(33ab^5c^2-40a^3b^3c+15a^5b)p-4(2ab^5cdp^4+5(3ab^5c-a^3b^3)dp^3+(31ab^5c-15a^3b^3)dp^2+2(9ab^5c-5a^3b^3)dp)x)\sqrt{dx+c}\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^6d^3p^6+21b^6d^3p^5+175b^6d^3p^4+735b^6d^3p^3+1624b^6d^3p^2+1764b^6d^3p+720b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3
*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^
6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^
2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (4
1*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c
 - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p -
4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*
c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^
2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3*
c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^
2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p
- 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3
*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x +
c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p
^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x**2*(a + b*sqrt(c + d*x))**p, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2511 vs. 2(230) = 460.
time = 3.25, size = 2511, normalized size = 10.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b + a
)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b
+ a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*
c*p^5 - 6*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c*p^5 + 2
*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^5 + 137*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(sqrt(d*x + c)*b
+ a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*b^2*c*p^4 + 108*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)
*b + a)^p*a*b^2*c*p^4 - 114*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p
*a^2*b^2*c*p^4 + 40*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c
*p^4 + (sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^5 - 5*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^5 + 10*(sqrt(d*x + c)*b + a)^4*(sqr
t(d*x + c)*b + a)^p*a^2*p^5 - 10*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a^3*p^5 + 5*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^5 -
 (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^5 + 461*(sqrt(d*x + c)
*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(sqrt(d*x + c)*b + a)*(
sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(sqrt(d*x + c)*b + a)^4*(sqrt(d*
x + c)*b + a)^p*b^2*c*p^3 + 726*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a*b^2*c*p^3 - 822*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*
b^2*c*p^3 + 310*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2*c*p^3
 + 15*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^4 - 80*(sqrt(d*x +
c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^4 + 170*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^2*p^4 - 180*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b
```

```
 + a)^p*a^3*p^4 + 95*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4*p^
4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^4 + 702*(sqrt(d*
x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p - 1044*(sqrt(d*x + c)*b +
 a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p - 614*(sqrt(d*x + c)*b + a)^4*(sqrt
(d*x + c)*b + a)^p*b^2*c*p^2 + 2232*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*
b + a)^p*a*b^2*c*p^2 - 2766*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p
*a^2*b^2*c*p^2 + 1160*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2
*c*p^2 + 85*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^3 - 475*(sqrt
(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^3 + 1070*(sqrt(d*x + c)*b +
a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^3 - 1210*(sqrt(d*x + c)*b + a)^3*(sqrt(d
*x + c)*b + a)^p*a^3*p^3 + 685*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a
)^p*a^4*p^3 - 155*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^3 + 3
60*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2 - 720*(sqrt(d*x
+ c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2 - 792*(sqrt(d*x + c)*b + a)^4
*(sqrt(d*x + c)*b + a)^p*b^2*c*p + 3048*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x +
 c)*b + a)^p*a*b^2*c*p - 4212*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)
^p*a^2*b^2*c*p + 2088*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*b^2
*c*p + 225*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p*p^2 - 1300*(sqrt
(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p^2 + 3070*(sqrt(d*x + c)*b +
a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p^2 - 3720*(sqrt(d*x + c)*b + a)^3*(sqrt(d
*x + c)*b + a)^p*a^3*p^2 + 2305*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p*a^4*p^2 - 580*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5*p^2 -
360*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c + 1440*(sqrt(d*x
+ c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 2160*(sqrt(d*x + c)*b + a)^
2*(sqrt(d*x + c)*b + a)^p*a^2*b^2*c + 1440*(sqrt(d*x + c)*b + a)*(sqrt(d*x
+ c)*b + a)^p*a^3*b^2*c + 274*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)
^p*p - 1620*(sqrt(d*x + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a*p + 3960*(sqr
t(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*a^2*p - 5080*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a^3*p + 3510*(sqrt(d*x + c)*b + a)^2*(sqrt(d*
x + c)*b + a)^p*a^4*p - 1044*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*
a^5*p + 120*(sqrt(d*x + c)*b + a)^6*(sqrt(d*x + c)*b + a)^p - 720*(sqrt(d*x
 + c)*b + a)^5*(sqrt(d*x + c)*b + a)^p*a + 1800*(sqrt(d*x + c)*b + a)^4*(sq
rt(d*x + c)*b + a)^p*a^2 - 2400*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
a)^p*a^3 + 1800*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^4 - 720*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^5)/((b^4*d^2*p^6 + 21*b^4*d^
2*p^5 + 175*b^4*d^2*p^4 + 735*b^4*d^2*p^3 + 1624*b^4*d^2*p^2 + 1764*b^4*d^2
*p + 720*b^4*d^2)*b^2*d)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x^2*(a + b*(c + d*x)^(1/2))^p, x)
```

3.655 $\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$

Optimal. Leaf size=145

$$-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{3+p}}{b^4d^2(3+p)}+\frac{2\left(a+b\right.}{b^4d}$$

```
[Out] -2*a*(-b^2*c+a^2)*(a+b*(d*x+c)^(1/2))^(1+p)/b^4/d^2/(1+p)+2*(-b^2*c+3*a^2)*
(a+b*(d*x+c)^(1/2))^(2+p)/b^4/d^2/(2+p)-6*a*(a+b*(d*x+c)^(1/2))^(3+p)/b^4/d
^2/(3+p)+2*(a+b*(d*x+c)^(1/2))^(4+p)/b^4/d^2/(4+p)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$-\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^4d^2(p+1)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^4d^2(p+2)}-\frac{6a\left(a+b\sqrt{c+dx}\right)^{p+3}}{b^4d^2(p+3)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{p+4}}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(1 + p))/(b^4*d^2*(1 + p)) + (2*(
3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^4*d^2*(2 + p)) - (6*a*(a +
 b*Sqrt[c + d*x])^(3 + p))/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^(4
+ p))/(b^4*d^2*(4 + p))
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p(-c+x)\,dx,x,c+dx\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int x(a+bx)^p\left(-c+x^2\right)\,dx,x,\sqrt{c+dx}\right)}{d^2}$$
$$= \frac{2\text{Subst}\left(\int \left(\frac{(-a^3+ab^2c)(a+bx)^p}{b^3}+\frac{(3a^2-b^2c)(a+bx)^{1+p}}{b^3}-\frac{3a(a+bx)^{2+p}}{b^3}+\frac{(a+bx)^{3+p}}{b^3}\right)\,dx,x,\right.}{d^2}$$
$$= -\frac{2a(a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^4d^2(1+p)}+\frac{2(3a^2-b^2c)\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^4d^2(2+p)}-\frac{6a\left(\right.}{}$$

Mathematica [A]
time = 0.27, size = 128, normalized size = 0.88

$$-\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(6a^3-6a^2b(1+p)\sqrt{c+dx}-b^3(3+4p+p^2)\sqrt{c+dx}\,(-2c+d(2+p)x)+ab^2(2c(-3+p+p^2)+3d(2+3p+p^2)x)\right)}{b^4d^2(1+p)(2+p)(3+p)(4+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]
[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] -
b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p
 + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p)
)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\sqrt{dx+c}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*(d*x+c)^(1/2))^p,x)
[Out] int(x*(a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A]
time = 0.30, size = 187, normalized size = 1.29

$$-\frac{2\left(\frac{\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p c}{(p^2+3p+2)b^2}-\frac{\left((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+c}\,a^3bp-6a^4\right)\left(\sqrt{dx+c}\,b+a\right)^p}{(p^4+10p^3+35p^2+50p+24)b^4}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] -2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b +
a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 +
(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 +
 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 +
35*p^2 + 50*p + 24)*b^4))/d^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(137) = 274$.
time = 0.40, size = 294, normalized size = 2.03

$$-\frac{2\left(6\,b^4c^2-12\,a^2b^2c+6\,a^4+2\,(b^4c^2+a^2b^2c)p^2-(b^4d^2p^3+6\,b^4d^2p^2+11\,b^4d^2p+6\,b^4d^2)x^2+4\,(2\,b^4c^2-a^2b^2c)p-(b^4cdp^3+(4\,b^4c-3\,a^2b^2)dp^2+3\,(b^4c-a^2b^2)dp)x+(4\,ab^3cp^2+2\,(5\,ab^3c-3\,a^3b)p-(ab^3dp^3+3\,ab^3dp^2+2\,ab^3dp)x)\sqrt{dx+c}\right)\left(\sqrt{dx+c}\,b+a\right)^p}{b^4d^2p^4+10\,b^4d^2p^3+35\,b^4d^2p^2+50\,b^4d^2p+24\,b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d
^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2
*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2
)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*
b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c))*(sqrt(d*x + c)*b + a)^p/(b^4*d^2
*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\sqrt{c+dx}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral(x*(a + b*sqrt(c + d*x))**p, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(137) =$ 274.
time = 3.21, size = 806, normalized size = 5.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x +
 c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^2*
```

```
(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*
b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^3
+ 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*x + c
)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(sqrt(d*
x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^
p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 6*
(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x + c)*b +
 a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x
+ c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3
*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c - 24*(sqrt(
d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b + a)^
4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p + 26*(
sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p - 6*(sqrt(d*x + c)*b + a
)^4*(sqrt(d*x + c)*b + a)^p + 24*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b +
 a)^p*a - 36*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2 + 24*(sqrt
(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3)/((b^2*p^4 + 10*b^2*p^3 + 35*b
^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\sqrt{c+d\,x}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] int(x*(a + b*(c + d*x)^(1/2))^p, x)
```

3.656 $\int \left(a+b\sqrt{c+dx}\right)^p\, dx$

Optimal. Leaf size=62

$$-\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

```
[Out] -2*a*(a+b*(d*x+c)^(1/2))^(1+p)/b^2/d/(1+p)+2*(a+b*(d*x+c)^(1/2))^(2+p)/b^2/
d/(2+p)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2d(p+2)}-\frac{2a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2d(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d
*x])^(2 + p))/(b^2*d*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \left(a+b\sqrt{c+dx}\right)^p dx = \frac{\text{Subst}\left(\int \left(a+b\sqrt{x}\right)^p dx, x, c+dx\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int x(a+bx)^p dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{2\text{Subst}\left(\int \left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right) dx, x, \sqrt{c+dx}\right)}{d}$$
$$= -\frac{2a\left(a+b\sqrt{c+dx}\right)^{1+p}}{b^2d(1+p)}+\frac{2\left(a+b\sqrt{c+dx}\right)^{2+p}}{b^2d(2+p)}$$

Mathematica [A]
time = 0.09, size = 53, normalized size = 0.85

$$\frac{2\left(a+b\sqrt{c+dx}\right)^{1+p}\left(-a+b(1+p)\sqrt{c+dx}\right)}{b^2d(1+p)(2+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]
```

```
[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1
+ p)*(2 + p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sqrt{dx+c}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^p,x)
```

```
[Out] int((a+b*(d*x+c)^(1/2))^p,x)
```

Maxima [A]
time = 0.28, size = 60, normalized size = 0.97

$$\frac{2\left((dx+c)b^2(p+1)+\sqrt{dx+c}\,abp-a^2\right)\left(\sqrt{dx+c}\,b+a\right)^p}{(p^2+3\,p+2)b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)
^p/((p^2 + 3*p + 2)*b^2*d)
```

Fricas [A]
time = 0.41, size = 81, normalized size = 1.31

$$\frac{2\left(b^{2}cp+\sqrt{dx+c}\,abp+b^{2}c-a^{2}+(b^{2}dp+b^{2}d)x\right)\left(\sqrt{dx+c}\,b+a\right)^{p}}{b^{2}dp^{2}+3\,b^{2}dp+2\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt
(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sqrt{c+dx}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p,x)
```

```
[Out] Integral((a + b*sqrt(c + d*x))**p, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(58) = 116.
time = 2.68, size = 129, normalized size = 2.08

$$\frac{2\left(\left(\sqrt{dx+c}\,b+a\right)^{2}\left(\sqrt{dx+c}\,b+a\right)^{p}p-\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^{p}ap+\left(\sqrt{dx+c}\,b+a\right)^{2}\left(\sqrt{dx+c}\,b+a\right)^{p}-2\left(\sqrt{dx+c}\,b+a\right)\left(\sqrt{dx+c}\,b+a\right)^{p}a\right)}{(p^{2}+3\,p+2)b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b + a
)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b +
a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p + 2)*
b^2*d)
```

Mupad [B]

time = 3.58, size = 146, normalized size = 2.35

$$\begin{cases} -\frac{2a\ln\left(a+b\sqrt{c+dx}\right)-2b\sqrt{c+dx}}{b^2 d} & \text{if } p=-1 \\ \frac{2\left(\ln\left(a+b\sqrt{c+dx}\right)+\frac{a}{a+b\sqrt{c+dx}}\right)}{b^2 d} & \text{if } p=-2 \\ \frac{4\left(a+b\sqrt{c+dx}\right)^{p+2}}{b^2 d\,(2p+4)}-\frac{4a\left(a+b\sqrt{c+dx}\right)^{p+1}}{b^2 d\,(2p+2)} & \text{if } p\neq -1\wedge p\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^p,x)
```

```
[Out] piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))/
(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/2)
)))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2*d*
(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))
```

3.657 $\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx$

Optimal. Leaf size=139

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)}-\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{}}{a+}\right.}{\left(a+b\sqrt{c}\right)(1+p)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],(a+b*(d*x+c)^(1/2))/(a-b*c^(1/2)))*(a+b*(d*x+c)^(
1/2))^(1+p)/(1+p)/(a-b*c^(1/2))-hypergeom([1, 1+p],[2+p],(a+b*(d*x+c)^(1/2)
)/(a+b*c^(1/2)))*(a+b*(d*x+c)^(1/2))^(1+p)/(1+p)/(a+b*c^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {378, 1412, 845, 70}

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{(p+1)\left(a-b\sqrt{c}\right)}-\frac{\left(a+b\sqrt{c+dx}\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)}{(p+1)\left(a+b\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*
Sqrt[c + d*x])/(a - b*Sqrt[c])])/((a - b*Sqrt[c])*(1 + p))) - ((a + b*Sqrt[
c + d*x])^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/
(a + b*Sqrt[c])])/((a + b*Sqrt[c])*(1 + p))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 845

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a + c*x^2), x], x
] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !RationalQ[m
]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\sqrt{c+dx}\right)^p}{x}\,dx &= \text{Subst}\left(\int \frac{\left(a+b\sqrt{x}\right)^p}{-c+x}\,dx, x, c+dx\right)\\ &= 2\text{Subst}\left(\int \frac{x(a+bx)^p}{-c+x^2}\,dx, x, \sqrt{c+dx}\right)\\ &= 2\text{Subst}\left(\int \left(-\frac{(a+bx)^p}{2\left(\sqrt{c}-x\right)}+\frac{(a+bx)^p}{2\left(\sqrt{c}+x\right)}\right)dx, x, \sqrt{c+dx}\right)\\ &= -\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}-x}\,dx, x, \sqrt{c+dx}\right)+\text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{c}+x}\,dx, x, \sqrt{c+dx}\right)\\ &= -\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)}{\left(a-b\sqrt{c}\right)(1+p)}-\frac{\left(a+b\sqrt{c+dx}\right)^{1+\dots}}{(\dots}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 136, normalized size = 0.98

$$-\frac{\left(a+b\sqrt{c+dx}\right)^{1+p}\left(\left(a+b\sqrt{c}\right){}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a-b\sqrt{c}}\right)+\left(a-b\sqrt{c}\right){}_2F_1\left(1,1+p;2+p;\frac{a+b\sqrt{c+dx}}{a+b\sqrt{c}}\right)\right)}{\left(a-b\sqrt{c}\right)\left(a+b\sqrt{c}\right)(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^p/x,x]
```

```
[Out] -(((a + b*Sqrt[c + d*x])^(1 + p)*((a + b*Sqrt[c])*Hypergeometric2F1[1, 1 +
p, 2 + p, (a + b*Sqrt[c + d*x])/(a - b*Sqrt[c])] + (a - b*Sqrt[c])*Hypergeo
metric2F1[1, 1 + p, 2 + p, (a + b*Sqrt[c + d*x])/(a + b*Sqrt[c])]))/((a - b
*Sqrt[c])*(a + b*Sqrt[c])*(1 + p)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sqrt{dx+c}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(d*x+c)^(1/2))^p/x,x)
[Out] int((a+b*(d*x+c)^(1/2))^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="maxima")
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="fricas")
[Out] integral((sqrt(d*x + c)*b + a)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\sqrt{c+dx}\right)^{p}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**p/x,x)
[Out] Integral((a + b*sqrt(c + d*x))**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^p/x,x, algorithm="giac")
```

```
[Out] integrate((sqrt(d*x + c)*b + a)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\sqrt{c+d\,x}\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c + d*x)^(1/2))^p/x,x)
```

```
[Out] int((a + b*(c + d*x)^(1/2))^p/x, x)
```

3.658 $\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx$

Optimal. Leaf size=93

$$\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*a*(a+b*(c*x)^n)^(3/2)/n+2/5*(a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctanh((a+
b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]
time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 52, 65, 214}

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{a+b(cx)^n}}{n}+\frac{2a(a+b(cx)^n)^{3/2}}{3n}+\frac{2(a+b(cx)^n)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a +
 b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])
/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{(a+b(cx)^n)^{5/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$
$$= \text{Subst}\left(\int \frac{(a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{a^3\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{n}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{b}$$
$$= \frac{2a^2\sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.}{n}$$

Mathematica [**A**]
time = 0.16, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a+b(cx)^n}\,(23a^2+11ab(cx)^n+3b^2(cx)^{2n}) - 30a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a
^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [**A**]
time = 0.64, size = 70, normalized size = 0.75

method	result	size

derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\,(cx)^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	70
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}+\frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{a+b\,(cx)^n}-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	70
risch	$\frac{2(3b^2\mathrm{e}^{2n\ln(cx)}+11a\,\mathrm{e}^{n\ln(cx)}b+23a^2)\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{15n}-\frac{2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*a^2*(a+b*(c*x)^n)^
(1/2)-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Fricas [A]
time = 0.41, size = 164, normalized size = 1.76

$$\left[\frac{15\,a^{\frac{5}{2}}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n\,b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\,(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2)\sqrt{(cx)^n\,b+a}}{15\,n},\ \frac{2\left(15\sqrt{-a}\,a^2\arctan\left(\frac{\sqrt{(cx)^n\,b+a}\,\sqrt{-a}}{a}\right)+(11\,(cx)^n\,ab+3\,(cx)^{2\,n}\,b^2+23\,a^2)\sqrt{(cx)^n\,b+a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*
x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)
)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*
x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A]

time = 49.96, size = 122, normalized size = 1.31

$$\begin{cases} \frac{\frac{2a^3 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2a^2\sqrt{a+b(cx)^n} + \frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3} + \frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}}{n} & \text{for } n \neq 0 \\ -\left(-a^2\sqrt{a+b} - 2ab\sqrt{a+b} - b^2\sqrt{a+b}\right)\log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) + 2*a**2*sq
rt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)**n)**(5
/2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a
 + b))*log(c*x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,{\left(c\,x\right)}^n\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(5/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(5/2)/x, x)
```

3.659 $\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$

Optimal. Leaf size=70

$$\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2(a+b(cx)^n)^{3/2}}{3n}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*(a+b*(c*x)^n)^(3/2)/n-2*a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n+
2*a*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {374, 12, 272, 52, 65, 214}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}+\frac{2a\sqrt{a+b(cx)^n}}{n}+\frac{2(a+b(cx)^n)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*
ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{(a+bx^n)^{3/2}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a+b(cx)^n}\,(4a+b(cx)^n)-6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*
x)^n]/Sqrt[a]])/(3*n)
```

Maple [A]
time = 0.62, size = 54, normalized size = 0.77

method	result	size
derivativedivides	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	54
default	$\frac{\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3}+2a\sqrt{a+b\,(cx)^n}-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	54
risch	$\frac{2(b\,e^{n\ln(cx)}+4a)\sqrt{a+b\,e^{n\ln(cx)}}}{3n}-\frac{2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/3*(a+b*(c*x)^n)^(3/2)+2*a*(a+b*(c*x)^n)^(1/2)-2*a^(3/2)*arctanh((a+b
*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Fricas [A]
time = 0.40, size = 130, normalized size = 1.86

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n\,b+a}\,\sqrt{a}+2\,a}{(cx)^n}\right)+2\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}}{3\,n},\ \frac{2\left(3\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{(cx)^n\,b+a}\,\sqrt{-a}}{a}\right)+\left((cx)^n\,b+4\,a\right)\sqrt{(cx)^n\,b+a}\right)}{3\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)
^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(
sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]
```

Sympy [A]
time = 36.80, size = 102, normalized size = 1.46

$$\begin{cases} \frac{-a\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+b(cx)^n}\right)-b\left(\begin{cases}-\sqrt{a}\,(cx)^n & \text{for}\ b=0\\ -\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for}\ n\neq 0\\ \left(a\sqrt{a+b}+b\sqrt{a+b}\right)\log(x) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((-a*(-2*a*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) - 2*sqrt(
a + b*(c*x)**n)) - b*Piecewise((-sqrt(a)*(c*x)**n, Eq(b, 0)), (-2*(a + b*(c
*x)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b)
)*log(x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,{\left(c\,x\right)}^n\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(3/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(3/2)/x, x)
```

3.660 $\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 52, 65, 214}

$$\frac{2\sqrt{a+b(cx)^n}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*(c*x)^n]/x,x]

[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]
)/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+b(cx)^n}}{x}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{a\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{(2a)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}
$$

Mathematica [A]

time = 0.10, size = 46, normalized size = 0.94

$$\frac{2\left(\sqrt{a+b(cx)^n}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*(c*x)^n]/x,x]
[Out] (2*(Sqrt[a + b*(c*x)^n] - Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]))/n
```

Maple [A]
time = 0.57, size = 40, normalized size = 0.82

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,(cx)^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{n}$	40
default	$\frac{2\sqrt{a+b\,(cx)^{n}}-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{n}$	40
risch	$\frac{2\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{n}-\frac{2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(2*(a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

Fricas [A]
time = 0.36, size = 103, normalized size = 2.10

$$\left[\frac{\sqrt{a}\,\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b+a}\,\sqrt{a}+2a}{(cx)^n}\right)+2\sqrt{(cx)^n b+a}}{n},\ \frac{2\left(\sqrt{-a}\,\arctan\left(\frac{\sqrt{(cx)^n b+a}\,\sqrt{-a}}{a}\right)+\sqrt{(cx)^n b+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) +
2*sqrt((c*x)^n*b + a))/n, 2*(sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a
) + sqrt((c*x)^n*b + a))/n]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\,(cx)^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*(c*x)**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b\,(c\,x)^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*(c*x)^n)^(1/2)/x,x)
```

```
[Out] int((a + b*(c*x)^n)^(1/2)/x, x)
```

3.661 $\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {374, 12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{bn} \\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 30, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A]
time = 0.60, size = 25, normalized size = 0.83

method	result	size

derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{n\sqrt{a}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Fricas [A]
time = 0.38, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^{n}b-2\sqrt{(cx)^{n}\,b+a}\,\sqrt{a}+2\,a}{(cx)^{n}}\right)}{\sqrt{a}\,n},\ \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{(cx)^{n}\,b+a}\,\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n)
, 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{a+b\,(cx)^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,x\right)}^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(1/2)), x)
```

3.662 $\int \frac{1}{x(a+b(cx)^n)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n+2/a/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {374, 12, 272, 53, 65, 214}

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{3/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{an}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{abn}\\
&= \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 52, normalized size = 1.00

$$\frac{2}{an\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(
3/2)*n)
```

Maple [A]
time = 0.45, size = 43, normalized size = 0.83

method	result	size
derivativedivides	$\frac{\frac{2}{a\sqrt{a+b\left(cx\right)^{n}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43
default	$\frac{\frac{2}{a\sqrt{a+b\left(cx\right)^{n}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}}{n}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/a/(a+b*(c*x)^n)^(1/2)-2/a^(3/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

Fricas [A]
time = 0.36, size = 164, normalized size = 3.15

$$\left[\frac{\left(\left(cx\right)^{n}\sqrt{a}\,b+a^{\frac{3}{2}}\right)\log\left(\frac{\left(cx\right)^{n}b-2\sqrt{\left(cx\right)^{n}b+a}\,\sqrt{a}+2\,a}{\left(cx\right)^{n}}\right)+2\sqrt{\left(cx\right)^{n}b+a}\,a}{\left(cx\right)^{n}a^{2}bn+a^{3}n},\ \frac{2\left(\left(\left(cx\right)^{n}\sqrt{-a}\,b+\sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{\left(cx\right)^{n}b+a}\,\sqrt{-a}}{a}\right)+\sqrt{\left(cx\right)^{n}b+a}\,a\right)}{\left(cx\right)^{n}a^{2}bn+a^{3}n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt
(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2
*(((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)
+ sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]
```

Sympy [A]
time = 5.55, size = 48, normalized size = 0.92

$$\frac{2}{an\sqrt{a+b\,(cx)^n}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)
```

```
[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a*n*s
qrt(-a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,{\left(a+b\,{\left(c\,x\right)}^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(3/2)), x)
```

3.663 $\int \frac{1}{x(a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

```
[Out] 2/3/a/n/(a+b*(c*x)^n)^(3/2)-2*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2)/
n+2/a^2/n/(a+b*(c*x)^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {374, 12, 272, 53, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{a+b(cx)^n}}+\frac{2}{3an\left(a+b(cx)^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan
h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(a+bx^n\right)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^n\right)}{a^2n}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^n}\right)}{a^2bn}\\
&= \frac{2}{3an\left(a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 67, normalized size = 0.89

$$\frac{2(a+3(a+b(cx)^n))}{3a^2n\left(a+b(cx)^n\right)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[Sq
rt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Maple [A]
time = 0.39, size = 59, normalized size = 0.79

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+b\,(cx)^{n}}}+\frac{2}{3a(a+b(cx)^{n})^{\frac{3}{2}}}}{n}$	59
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{a^{2}\sqrt{a+b\,(cx)^{n}}}+\frac{2}{3a(a+b(cx)^{n})^{\frac{3}{2}}}}{n}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-2/a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))+2/a^2/(a+b*(c*x)^n)^(1
/2)+2/3/a/(a+b*(c*x)^n)^(3/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.
time = 0.36, size = 262, normalized size = 3.49

$$\left[\frac{3\left(2\,(cx)^{n}a^{\frac{3}{2}}b+(cx)^{2n}\sqrt{a}\,b^{2}+a^{\frac{5}{2}}\right)\log\left(\frac{(cx)^{n}b-2\sqrt{(cx)^{n}b+a}\,\sqrt{a}+2\,a}{(cx)^{n}}\right)+2\,(3\,(cx)^{n}ab+4\,a^{2})\sqrt{(cx)^{n}b+a}}{3\left(2\,(cx)^{n}a^{4}bn+(cx)^{2n}a^{3}b^{2}n+a^{5}n\right)},\ \frac{2\left(3\left(2\,(cx)^{n}\sqrt{-a}\,ab+(cx)^{2n}\sqrt{-a}\,b^{2}+\sqrt{-a}\,a^{2}\right)\arctan\left(\frac{\sqrt{(cx)^{n}b+a}\,\sqrt{-a}}{a}\right)+(3\,(cx)^{n}ab+4\,a^{2})\sqrt{(cx)^{n}b+a}\right)}{3\left(2\,(cx)^{n}a^{4}bn+(cx)^{2n}a^{3}b^{2}n+a^{5}n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x
)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b +
4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^
5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*
a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt(
(c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]
```

Sympy [A]
time = 7.77, size = 70, normalized size = 0.93

$$\frac{2}{3an\left(a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}n\sqrt{a+b\left(cx\right)^{n}}}+\frac{2\operatorname{atan}\left(\frac{\sqrt{a+b\left(cx\right)^{n}}}{\sqrt{-a}}\right)}{a^{2}n\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n)) + 2*ata
n(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+b\,{\left(c\,x\right)}^n\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^n)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^n)^(5/2)), x)
```

3.664 $\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$

Optimal. Leaf size=101

$$\frac{2a^2\sqrt{-a+b(cx)^n}}{n}-\frac{2a(-a+b(cx)^n)^{3/2}}{3n}+\frac{2(-a+b(cx)^n)^{5/2}}{5n}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((
-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]

time = 0.05, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {374, 12, 272, 52, 65, 211}

$$-\frac{2a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}+\frac{2a^2\sqrt{b(cx)^n-a}}{n}-\frac{2a(b(cx)^n-a)^{3/2}}{3n}+\frac{2(b(cx)^n-a)^{5/2}}{5n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*a^2*Sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-
a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a
]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{(-a+b(cx)^n)^{5/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)}{c}$$

$$= \text{Subst}\left(\int \frac{(-a+bx^n)^{5/2}}{x}\,dx, x, cx\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= -\frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} + \frac{a^2\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{a^3\text{Subst}\left(\int \frac{}{x\sqrt{-}}\right.}{}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{(2a^3)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2a^2\sqrt{-a+b(cx)^n}}{n} - \frac{2a(-a+b(cx)^n)^{3/2}}{3n} + \frac{2(-a+b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2}\tan^{-1}\left(\sqrt{}\right.}{n}$$

Mathematica [A]
time = 0.16, size = 81, normalized size = 0.80

$$\frac{2\sqrt{-a+b(cx)^n}\,(23a^2 - 11ab(cx)^n + 3b^2(cx)^{2n}) - 30a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*
a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)
```

Maple [A]
time = 0.61, size = 78, normalized size = 0.77

method	result	size

derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b\,(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	78
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+2a^2\sqrt{-a+b\,(cx)^n}-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	78
risch	$-\frac{2(3b^2\mathrm{e}^{2n\ln(cx)}-11a\,\mathrm{e}^{n\ln(cx)}b+23a^2)\,(a-b\,\mathrm{e}^{n\ln(cx)})}{15n\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}-\frac{2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/5*(-a+b*(c*x)^n)^(5/2)-2/3*a*(-a+b*(c*x)^n)^(3/2)+2*a^2*(-a+b*(c*x)^
n)^(1/2)-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Fricas [A]

time = 0.38, size = 169, normalized size = 1.67

$$\left[\frac{15\sqrt{-a}\,a^2\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\,\sqrt{-a}-2a}{(cx)^n}\right)-2\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}}{15\,n},\ -\frac{2\left(15\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\left(11\,(cx)^n ab-3\,(cx)^{2n}b^2-23\,a^2\right)\sqrt{(cx)^n b-a}\right)}{15\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*
a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*
b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x
)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]
```

Sympy [A]

time = 45.40, size = 114, normalized size = 1.13

$$\begin{cases}\frac{-2a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)+2a^2\sqrt{-a+b\,(cx)^n}-\frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3}+\frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}}{n} & \text{for } n\neq 0\\ -\left(-a^2\sqrt{-a+b}+2ab\sqrt{-a+b}-b^2\sqrt{-a+b}\right)\log\left(cx\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((-2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*a**2*sqrt(-
a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*x)**n)**(5/
2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(
-a + b))*log(c*x), True))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\left(c\,x\right)}^n-a\right)}^{5/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(5/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(5/2)/x, x)
```

3.665 $\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx$

Optimal. Leaf size=76

$$-\frac{2a\sqrt{-a+b(cx)^n}}{n}+\frac{2(-a+b(cx)^n)^{3/2}}{3n}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] 2/3*(-a+b*(c*x)^n)^(3/2)/n+2*a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n
-2*a*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {374, 12, 272, 52, 65, 211}

$$\frac{2a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}-\frac{2a\sqrt{b(cx)^n-a}}{n}+\frac{2(b(cx)^n-a)^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]
```

```
[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/
2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+b(cx)^n)^{3/2}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{(-a+bx^n)^{3/2}}{x}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x}\,dx, x, (cx)^n\right)}{n} \\
&= \frac{2(-a+b(cx)^n)^{3/2}}{3n} - \frac{a\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n} \\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n} \\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{(2a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn} \\
&= -\frac{2a\sqrt{-a+b(cx)^n}}{n} + \frac{2(-a+b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 66, normalized size = 0.87

$$\frac{-2(4a-b(cx)^n)\sqrt{-a+b(cx)^n}+6a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x,x]
[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(
c*x)^n]/Sqrt[a]])/(3*n)
```

Maple [A]
time = 0.60, size = 60, normalized size = 0.79

method	result	size
derivativedivides	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b\,(cx)^n}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	60
default	$\frac{\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3}-2a\sqrt{-a+b\,(cx)^n}+2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,(cx)^n}}{\sqrt{a}}\right)}{n}$	60
risch	$\frac{2\left(-b\,e^{n\ln(cx)}+4a\right)\left(a-b\,e^{n\ln(cx)}\right)}{3n\sqrt{-a+b\,e^{n\ln(cx)}}}+\frac{2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-a+b\,e^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(2/3*(-a+b*(c*x)^n)^(3/2)-2*a*(-a+b*(c*x)^n)^(1/2)+2*a^(3/2)*arctan((-a
+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Fricas [A]
time = 0.35, size = 135, normalized size = 1.78

$$\left[\frac{3\sqrt{-a}\,a\log\left(\frac{(cx)^n b+2\sqrt{(cx)^n b-a}\,\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\,((cx)^n b-4a)}{3n},\ \frac{2\left(3a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}\,((cx)^n b-4a)\right)}{3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(
c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan
(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]
```

Sympy [A]
time = 32.38, size = 95, normalized size = 1.25

$$\begin{cases} \frac{a\left(2\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{-a+b\,(cx)^{n}}}{\sqrt{a}}\right)-2\sqrt{-a+b\,(cx)^{n}}\right)-b\left(\begin{cases}-\sqrt{-a}\,(cx)^{n} & \text{for}\: b=0\\ -\frac{2\left(-a+b(cx)^{n}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{n} & \text{for}\: n\neq 0\\ \left(-a\sqrt{-a+b}+b\sqrt{-a+b}\right)\log\left(x\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((a*(2*sqrt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*sqrt(-a +
b*(c*x)**n)) - b*Piecewise((-sqrt(-a)*(c*x)**n, Eq(b, 0)), (-2*(-a + b*(c*x
)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,{\left(c\,x\right)}^n-a\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(3/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(3/2)/x, x)
```

3.666 $\int \frac{\sqrt{-a+b(cx)^n}}{x} \, dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{-a+b(cx)^n}}{n}-\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(-a+b*(c*x)^n)^(1/2)/n
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {374, 12, 272, 52, 65, 211}

$$\frac{2\sqrt{b(cx)^n-a}}{n}-\frac{2\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]
```

```
[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]
])/n
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+b(cx)^n}}{x}\,dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{\sqrt{-a+bx^n}}{x}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\
&= \frac{2\sqrt{-a+b(cx)^n}}{n} - \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 50, normalized size = 0.94

$$\frac{2\left(\sqrt{-a+b(cx)^n}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]
[Out] (2*(Sqrt[-a + b*(c*x)^n] - Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]))/n
```

Maple [A]
time = 0.61, size = 44, normalized size = 0.83

method	result	size
derivativedivides	$\frac{2\sqrt{-a+b\left(cx\right)^{n}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{n}$	44
default	$\frac{2\sqrt{-a+b\left(cx\right)^{n}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{n}$	44
risch	$-\frac{2\left(a-b\,\mathrm{e}^{n\ln(cx)}\right)}{n\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{n\ln(cx)}}}{\sqrt{a}}\right)}{n}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+b*(c*x)^n)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/n*(2*(-a+b*(c*x)^n)^(1/2)-2*a^(1/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

Fricas [A]
time = 0.39, size = 110, normalized size = 2.08

$$\left[\frac{\sqrt{-a}\,\log\left(\frac{(cx)^n b-2\,\sqrt{(cx)^n\,b-a}\,\sqrt{-a}\,-2\,a}{(cx)^n}\right)+2\,\sqrt{(cx)^n\,b-a}}{n},\,-\frac{2\left(\sqrt{a}\,\arctan\left(\frac{\sqrt{(cx)^n\,b-a}}{\sqrt{a}}\right)-\sqrt{(cx)^n\,b-a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)
+ 2*sqrt((c*x)^n*b - a))/n, -2*(sqrt(a)*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
 - sqrt((c*x)^n*b - a))/n]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a + b\,(cx)^n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-a + b*(c*x)**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*x)^n*b - a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,{\left(c\,x\right)}^n-a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*(c*x)^n - a)^(1/2)/x,x)
```

```
[Out] int((b*(c*x)^n - a)^(1/2)/x, x)
```

3.667 $\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {374, 12, 272, 65, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]
```

```
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)}{c}\\ &= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}}\,dx, x, cx\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{n}\\ &= \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{bn}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 32, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]),x]
[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A]
time = 0.65, size = 27, normalized size = 0.84

method	result	size

derivativedivides	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27
default	$\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n\sqrt{a}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Fricas [A]
time = 0.37, size = 80, normalized size = 2.50

$$\left[-\frac{\sqrt{-a}\log\left(\frac{(cx)^n b-2\sqrt{(cx)^n b-a}\sqrt{-a}-2a}{(cx)^n}\right)}{an}, \frac{2\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)}{\sqrt{a}\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/
(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-a+b(cx)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{b\,{\left(c\,x\right)}^n-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(1/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(1/2)), x)
```

3.668 $\int \frac{1}{x(-a+b(cx)^n)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

```
[Out] -2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(3/2)/n-2/a/n/(-a+b*(c*x)^n)^(1/2
)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {374, 12, 272, 53, 65, 211}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{3/2}n}-\frac{2}{an\sqrt{b(cx)^n-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}}\,dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{3/2}}\,dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{n} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx)^n\right)}{an} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b(cx)^n}\right)}{abn} \\
&= -\frac{2}{an\sqrt{-a+b(cx)^n}} - \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 56, normalized size = 1.00

$$-\frac{2}{an\sqrt{-a+b(cx)^n}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)),x]
```

```
[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a
^(3/2)*n)
```

Maple [A]
time = 0.39, size = 47, normalized size = 0.84

method	result	size
derivativedivides	$\frac{-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{-a+b(cx)^n}}}{n}$	47
default	$\frac{-\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{a\sqrt{-a+b(cx)^n}}}{n}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(-2/a^(3/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))-2/a/(-a+b*(c*x)^n)^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Fricas [A]
time = 0.38, size = 175, normalized size = 3.12

$$\left[-\frac{((cx)^n\sqrt{-a}\,b-\sqrt{-a}\,a)\log\left(\frac{(cx)^nb+2\sqrt{(cx)^n b-a}\,\sqrt{-a}-2a}{(cx)^n}\right)+2\sqrt{(cx)^n b-a}\,a}{(cx)^n a^2bn-a^3n},\ -\frac{2\left(\left((cx)^n\sqrt{a}\,b-a^{\frac{3}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^n b-a}}{\sqrt{a}}\right)+\sqrt{(cx)^n b-a}\,a\right)}{(cx)^n a^2bn-a^3n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(((c*x)^n*sqrt(-a)*b - sqrt(-a)*a)*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)
*sqrt(-a) - 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3
*n), -2*(((c*x)^n*sqrt(a)*b - a^(3/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a))
+ sqrt((c*x)^n*b - a)*a)/((c*x)^n*a^2*b*n - a^3*n)]
```

Sympy [A]
time = 7.43, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a+b\left(cx\right)^{n}}}-\frac{2\operatorname{atan}\left(\frac{\sqrt{-a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)
```

```
[Out] -2/(a*n*sqrt(-a + b*(c*x)**n)) - 2*atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**
(3/2)*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\left(b\left(c\,x\right)^{n}-a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(3/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(3/2)), x)
```

3.669 $\int \frac{1}{x(-a+b(cx)^n)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}}+\frac{2}{a^2n\sqrt{-a+b(cx)^n}}+\frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

```
[Out] -2/3/a/n/(-a+b*(c*x)^n)^(3/2)+2*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/a^(5/2
)/n+2/a^2/n/(-a+b*(c*x)^n)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {374, 12, 272, 53, 65, 211}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{a^{5/2}n}+\frac{2}{a^2n\sqrt{b(cx)^n-a}}-\frac{2}{3an\left(b(cx)^n-a\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*Arc
Tan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-a+b(cx)^n\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\left(-a+bx^n\right)^{5/2}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}}\,dx, x, (cx)^n\right)}{n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}}\,dx, x, (cx)^n\right)}{an}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}}\,dx, x, (cx\right.}{a^2n}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+b\right.}{a^2bn}\\
&= -\frac{2}{3an\left(-a+b(cx)^n\right)^{3/2}} + \frac{2}{a^2n\sqrt{-a+b(cx)^n}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
\end{aligned}$$

Mathematica [A]
time = 0.19, size = 70, normalized size = 0.86

$$\frac{2\left(\frac{\sqrt{a}\,(-4a+3b(cx)^n)}{(-a+b(cx)^n)^{3/2}}+3\tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)\right)}{3a^{5/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)),x]
```

```
[Out] (2*((Sqrt[a]*(-4*a + 3*b*(c*x)^n))/(-a + b*(c*x)^n)^(3/2) + 3*ArcTan[Sqrt[-
a + b*(c*x)^n]/Sqrt[a]]))/(3*a^(5/2)*n)
```

Maple [A]
time = 0.43, size = 65, normalized size = 0.80

method	result	size
derivativedivides	$\frac{\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}}{n}$	65
default	$\frac{\frac{2\arctan\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2}{3a(-a+b(cx)^n)^{\frac{3}{2}}}+\frac{2}{a^2\sqrt{-a+b(cx)^n}}}{n}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a+b*(c*x)^n)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(2/a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))-2/3/a/(-a+b*(c*x)^n)^(3
/2)+2/a^2/(-a+b*(c*x)^n)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

Fricas [A]
time = 0.36, size = 277, normalized size = 3.42

$$\left[-\frac{3\left(2(cx)^n\sqrt{-a}\,ab-(cx)^{2n}\sqrt{-a}\,b^2-\sqrt{-a}\,a^2\right)\log\left(\frac{(cx)^nb-2\sqrt{(cx)^nb-a}\,\sqrt{-a}-2a}{(cx)^n}\right)+2\left(3(cx)^nab-4a^2\right)\sqrt{(cx)^nb-a}}{3\left(2(cx)^na^4bn-(cx)^{2n}a^3b^2n-a^5n\right)}, \frac{2\left(3\left(2(cx)^na^{\frac{3}{2}}b-(cx)^{2n}\sqrt{a}\,b^2-a^{\frac{5}{2}}\right)\arctan\left(\frac{\sqrt{(cx)^nb-a}}{\sqrt{a}}\right)-\left(3(cx)^nab-4a^2\right)\sqrt{(cx)^nb-a}\right)}{3\left(2(cx)^na^4bn-(cx)^{2n}a^3b^2n-a^5n\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)
*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*
x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3
*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^
(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((
c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]
```

Sympy [A]
time = 7.00, size = 63, normalized size = 0.78

$$-\frac{2}{3an\left(-a+b\left(cx\right)^{n}\right)^{\frac{3}{2}}}+\frac{2}{a^{2}n\sqrt{-a+b\left(cx\right)^{n}}}+\frac{2\operatorname{atan}\left(\frac{\sqrt{-a+b\left(cx\right)^{n}}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)**n)) + 2*
atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\left(c\,x\right)^{n}-a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*(c*x)^n - a)^(5/2)),x)
```

```
[Out] int(1/(x*(b*(c*x)^n - a)^(5/2)), x)
```

3.670 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx}}\,dx &= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.36, size = 18, normalized size = 0.78

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [A]
time = 0.49, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

Fricas [A]
time = 0.36, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A]
time = 0.58, size = 24, normalized size = 1.04

$$-\frac{2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A]
time = 3.81, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 3.11, size = 17, normalized size = 0.74

$$-\frac{2\operatorname{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

3.671 $\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {374, 12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(cx)^m}}\,dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}}\,dx, x, cx\right)}{c}\\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}}\,dx, x, cx\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, (cx)^m\right)}{m}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b(cx)^m}\right)}{bm}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 30, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a}\,m}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)
```

Maple [A]
time = 0.61, size = 25, normalized size = 0.83

method	result	size

derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{m}}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\,(cx)^{m}}}{\sqrt{a}}\right)}{m\sqrt{a}}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*x)^m)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Fricas [A]
time = 0.39, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^{m}b-2\sqrt{(cx)^{m}b+a}\,\sqrt{a}+2\,a}{(cx)^{m}}\right)}{\sqrt{a}\,m},\ \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{(cx)^{m}b+a}\,\sqrt{-a}}{a}\right)}{am}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m)
, 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\,(cx)^{m}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,x\right)}^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*x)^m)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*x)^m)^(1/2)), x)
```

3.672 $\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(dx)^m\right)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

```
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {374, 12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(dx)^m\right)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c(dx)^m\right)^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}}\,dx, x, (dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{cm}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, c(dx)^m\right)}{m}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c(dx)^m\right)^n\right)}{mn}\\
&= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c(dx)^m\right)^n}\right)}{bmn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(dx)^m\right)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 37, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c(dx)^m\right)^n}}{\sqrt{a}}\right)}{\sqrt{a}\,mn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]])/(Sqrt[a]*m*n)
```

Maple [A]
time = 0.48, size = 32, normalized size = 0.86

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(dx\right)^{m}\right)^{n}}}{\sqrt{a}}\right)}{mn\sqrt{a}}$	32
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(c\left(dx\right)^{m}\right)^{n}}}{\sqrt{a}}\right)}{mn\sqrt{a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Fricas [A]
time = 0.40, size = 116, normalized size = 3.14

$$\left[\frac{\log\left(\left(be^{(mn\log(dx)+n\log(c))}-2\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\,\sqrt{a}+2a\right)e^{(-mn\log(dx)-n\log(c))}\right)}{\sqrt{a}\,mn},\ \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{be^{(mn\log(dx)+n\log(c))}+a}\,\sqrt{-a}}{a}\right)}{amn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*e^(m*n*log(d*x) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x) + n*log(c))
+ a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c)))/(sqrt(a)*m*n), 2*sqrt(-a)
*arctan(sqrt(b*e^(m*n*log(d*x) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(dx\right)^{m}\right)^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,x\right)}^m\right)}^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)
```

3.673 $\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx$

Optimal. Leaf size=44

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

```
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {374, 12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c(dx)^n\right)^p}}\,dx, x, (ex)^m\right)}{m}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}}\,dx, x, \left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}}\,dx, x, c\left(d(ex)^m\right)^n\right)}{cmn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}}\,dx, x, c\left(d(ex)^m\right)^n\right)}{mn}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d(ex)^m\right)^n\right)^p\right)}{mnp}\\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}\right)}{bmnp}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}
\end{aligned}
$$

Mathematica [A]

time = 0.18, size = 44, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d(ex)^m\right)^n\right)^p}}{\sqrt{a}}\right)}{\sqrt{a}\,mnp}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n)^p]/Sqrt[a]])/(Sqrt[a]*m*n*p)
```

Maple [A]
time = 0.43, size = 39, normalized size = 0.89

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\,(ex)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\,(ex)^{m}\right)^{n}\right)^{p}}}{\sqrt{a}}\right)}{mnp\sqrt{a}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt((((x*e)^m*d)^n*c)^p*b + a)*x), x)
```

Fricas [A]
time = 0.39, size = 151, normalized size = 3.43

$$\left[\frac{\log\left(\left(be^{(mnp\log(xe)+np\log(d)+p\log(c))}-2\sqrt{be^{(mnp\log(xe)+np\log(d)+p\log(c))}+a}\,\sqrt{a}+2\,a\right)e^{(-mnp\log(xe)-np\log(d)-p\log(c))}\right)}{\sqrt{a}\,mnp},\ \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{be^{(mnp\log(xe)+np\log(d)+p\log(c))}+a}\,\sqrt{-a}}{a}\right)}{amnp}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*log(x*e) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(
x*e) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(x*e) - n*p*
log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(
x*e) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(ex\right)^{m}\right)^{n}\right)^{p}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((((x*e)^m*d)^n*c)^p*b + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,x\right)}^m\right)}^n\right)}^p}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)
```

3.674 $\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)

Rubi [A]
time = 0.46, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {374, 12, 272, 65, 214}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*q)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 374

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_)*(x_))^(n_))^(p_.), x_Symbol] :>
Dist[1/c, Subst[Int[(d*(x/c))^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,
 b, c, d, m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c\left(d(ex)^n\right)^p\right)^q}}\,dx, x, (fx)^m\right)}{m}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b\left(c(dx)^p\right)^q}}\,dx, x, \left(e(fx)^m\right)^n\right)}{mn}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^q}}\,dx, x, \left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{cmnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^q}}\,dx, x, c\left(d\left(e(fx)^m\right)^n\right)^p\right)}{mnp}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q\right)}{mnpq}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}\right)}{bmnpq}$$

$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Mathematica [A]
time = 0.23, size = 51, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\left(c\left(d\left(e(fx)^m\right)^n\right)^p\right)^q}}{\sqrt{a}}\right)}{\sqrt{a}\,mnpq}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p)^q]/Sqrt[a]])/(Sqrt[a]*m*n*p*
q)
```

Maple [A]
time = 0.44, size = 46, normalized size = 0.90

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}{\sqrt{a}}\right)}{mnpq\sqrt{a}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

Fricas [A]
time = 0.36, size = 174, normalized size = 3.41

$$\left[\frac{\log\left(\left(be^{\left(mnpq\log\left(fx\right)+npq+pq\log\left(d\right)+q\log\left(c\right)\right)}-2\,\sqrt{be^{\left(mnpq\log\left(fx\right)+npq+pq\log\left(d\right)+q\log\left(c\right)\right)}+a}\,\sqrt{a}+2\,a\right)e^{\left(-mnpq\log\left(fx\right)-npq-pq\log\left(d\right)-q\log\left(c\right)\right)}\right)}{\sqrt{a}\,mnpq},\ \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{be^{\left(mnpq\log\left(fx\right)+npq+pq\log\left(d\right)+q\log\left(c\right)\right)}+a}\,\sqrt{-a}}{a}\right)}{amnpq}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q + p*q*log(d) + q*log(c)) - 2*sqrt(b*e^(
m*n*p*q*log(f*x) + n*p*q + p*q*log(d) + q*log(c)) + a)*sqrt(a) + 2*a)*e^(-m
*n*p*q*log(f*x) - n*p*q - p*q*log(d) - q*log(c)))/(sqrt(a)*m*n*p*q), 2*sqrt
(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q + p*q*log(d) + q*log(c)) + a
)*sqrt(-a)/a)/(a*m*n*p*q)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\left(c\left(d\left(e\left(fx\right)^{m}\right)^{n}\right)^{p}\right)^{q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\,{\left(c\,{\left(d\,{\left(e\,{\left(f\,x\right)}^m\right)}^n\right)}^p\right)}^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)
```

3.675 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^3}{x}\,dx$

Optimal. Leaf size=76

$$\frac{35}{16}\sqrt{-1+\frac{1}{x^2}}-\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2-\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4-\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2}x^6-\frac{35}{16}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] -35/48*(-1+1/x^2)^(3/2)*x^2-7/24*(-1+1/x^2)^(5/2)*x^4-1/6*(-1+1/x^2)^(7/2)*
x^6-35/16*arctan((-1+1/x^2)^(1/2))+35/16*(-1+1/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {25, 272, 43, 52, 65, 209}

$$-\frac{35}{16}\text{ArcTan}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{35}{48}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{35}{16}\sqrt{\frac{1}{x^2}-1}-\frac{1}{6}\left(\frac{1}{x^2}-1\right)^{7/2}x^6-\frac{7}{24}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-
2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-
2)]])/16
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^3}{x}\,dx = -\int \left(-1+\frac{1}{x^2}\right)^{7/2} x^5\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{7/2}}{x^4}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{7}{12}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{48}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,d\right.$$
$$= -\frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{32}\text{S}$$
$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\right.$$
$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\right.$$
$$= \frac{35}{16}\sqrt{-1+\frac{1}{x^2}} - \frac{35}{48}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6}\left(-1+\right.$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 0.92

$$\frac{1}{48}\sqrt{-1+\frac{1}{x^2}}\,(48+87x^2-38x^4+8x^6) - \frac{35\sqrt{-1+\frac{1}{x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{8\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*Sqrt[-1 + x^(-2
)]*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/(8*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.38, size = 83, normalized size = 1.09

method	result	siz
trager	$2\left(\frac{1}{12}x^6-\frac{19}{48}x^4+\frac{29}{32}x^2+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{35\,\text{RootOf}(\_Z^2+1)\ln\left(-\left(\text{RootOf}(\_Z^2+1)-\sqrt{-\frac{x^2-1}{x^2}}\right)x\right)}{16}$	66

risch	$\frac{(8x^8-46x^6+125x^4-39x^2-48)\sqrt{-\frac{x^2-1}{x^2}}}{48x^2-48}-\frac{35\arcsin(x)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{16(x^2-1)}$	78
default	$-\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(8x^4(-x^2+1)^{\frac{3}{2}}-30x^2(-x^2+1)^{\frac{3}{2}}-48(-x^2+1)^{\frac{3}{2}}-105x^2\sqrt{-x^2+1}-105\arcsin(x)x\right)}{48\sqrt{-x^2+1}}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(-(x^2-1)/x^2)^(1/2)*(8*x^4*(-x^2+1)^(3/2)-30*x^2*(-x^2+1)^(3/2)-48*(
-x^2+1)^(3/2)-105*x^2*(-x^2+1)^(1/2)-105*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(56) = 112$.
time = 0.50, size = 120, normalized size = 1.58

$$\frac{3}{2}x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}}+8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3+3\left(\frac{1}{x^2}-1\right)^2+\frac{3}{x^2}-2\right)}+\frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}-\frac{35}{16}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(
1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/
x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2
 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))
```

Fricas [A]
time = 0.36, size = 55, normalized size = 0.72

$$\frac{1}{48}\left(8x^6-38x^4+87x^2+48\right)\sqrt{-\frac{x^2-1}{x^2}}-\frac{35}{8}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*s
qrt(-(x^2 - 1)/x^2) - 1)/x)
```

Sympy [A]
time = 168.85, size = 80, normalized size = 1.05

$$-\frac{x^6\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}{6}-\frac{5x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{16}+\frac{3x^2\sqrt{-1+\frac{1}{x^2}}}{2}+\sqrt{-1+\frac{1}{x^2}}-\frac{35\operatorname{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] -x**6*(-1 + x**(-2))**(3/2)/6 - 5*x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/16 +
 3*x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 35*atan(sqrt(-1 + x**(-
2)))/16
```

Giac [A]
time = 2.51, size = 77, normalized size = 1.01

$$\frac{1}{48}\left(2\left(4x^{2}\operatorname{sgn}(x)-19\operatorname{sgn}(x)\right)x^{2}+87\operatorname{sgn}(x)\right)\sqrt{-x^{2}+1}\,x+\frac{35}{16}\arcsin\left(x\right)\operatorname{sgn}(x)-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^{2}+1}-1\right)}+\frac{\left(\sqrt{-x^{2}+1}-1\right)\operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/1
6*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1
) - 1)*sgn(x)/x
```

Mupad [B]
time = 3.50, size = 54, normalized size = 0.71

$$\sqrt{\frac{1}{x^{2}}-1}-\frac{35\operatorname{atan}\left(\sqrt{\frac{1}{x^{2}}-1}\right)}{16}+\frac{19x^{6}\sqrt{\frac{1}{x^{2}}-1}}{16}+\frac{17x^{6}\left(\frac{1}{x^{2}}-1\right)^{3/2}}{6}+\frac{29x^{6}\left(\frac{1}{x^{2}}-1\right)^{5/2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^(
1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

3.676 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^2}{x}\,dx$

Optimal. Leaf size=60

$$-\frac{15}{8}\sqrt{-1+\frac{1}{x^2}}+\frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2+\frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2}x^4+\frac{15}{8}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] 5/8*(-1+1/x^2)^(3/2)*x^2+1/4*(-1+1/x^2)^(5/2)*x^4+15/8*arctan((-1+1/x^2)^(1
/2))-15/8*(-1+1/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {25, 272, 43, 52, 65, 209}

$$\frac{15}{8}\text{ArcTan}\left(\sqrt{\frac{1}{x^2}-1}\right)+\frac{5}{8}\left(\frac{1}{x^2}-1\right)^{3/2}x^2-\frac{15}{8}\sqrt{\frac{1}{x^2}-1}+\frac{1}{4}\left(\frac{1}{x^2}-1\right)^{5/2}x^4$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^
(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
```

```
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)^2}{x}\,dx = \int \left(-1+\frac{1}{x^2}\right)^{5/2} x^3\,dx$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{5/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{5}{8}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)$$

$$= \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 - \frac{15}{16}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x,\right.$$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{16}\text{Subst}\Bigg($$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\text{Subst}\Bigg($$

$$= -\frac{15}{8}\sqrt{-1+\frac{1}{x^2}} + \frac{5}{8}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4}\left(-1+\frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8}\tan^{-1}\Bigg($$

Mathematica [A]
time = 0.08, size = 65, normalized size = 1.08

$$\frac{1}{8}\sqrt{-1+\frac{1}{x^2}}\left(-8-9x^2+2x^4\right)+\frac{15\sqrt{-1+\frac{1}{x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{4\sqrt{-1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(-8 - 9*x^2 + 2*x^4))/8 + (15*Sqrt[-1 + x^(-2)]*x*ArcTan
h[Sqrt[-1 + x^2]/(-1 + x)])/(4*Sqrt[-1 + x^2])
```

Maple [A]
time = 0.36, size = 69, normalized size = 1.15

method	result	size
trager	$2\left(\frac{1}{8}x^4-\frac{9}{16}x^2-\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\left(\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-\frac{x^2-1}{x^2}}\right)x\right)}{8}$	58
default	$-\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2x^2\left(-x^2+1\right)^{\frac{3}{2}}+8\left(-x^2+1\right)^{\frac{3}{2}}+15x^2\sqrt{-x^2+1}+15\arcsin\left(x\right)x\right)}{8\sqrt{-x^2+1}}$	69
risch	$\frac{\left(2x^6-11x^4+x^2+8\right)\sqrt{-\frac{x^2-1}{x^2}}}{8x^2-8}+\frac{15\arcsin\left(x\right)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{8\left(x^2-1\right)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*x^2*(-x^2+1)^(3/2)+8*(-x^2+1)^(3/2)+15*x^2*(-x
^2+1)^(1/2)+15*arcsin(x)*x)/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.51, size = 67, normalized size = 1.12

$$-x^2\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}-1}-\frac{\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}}-\sqrt{\frac{1}{x^2}-1}}{8\left(\left(\frac{1}{x^2}-1\right)^2+\frac{2}{x^2}-1\right)}+\frac{15}{8}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^
2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))
```

Fricas [A]
time = 0.39, size = 50, normalized size = 0.83

$$\frac{1}{8}\left(2x^4-9x^2-8\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{15}{4}\arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 -
1)/x^2) - 1)/x)
```

Sympy [A]
time = 65.07, size = 60, normalized size = 1.00

$$\frac{x^4\sqrt{-1+\frac{1}{x^2}}\cdot\left(2-\frac{1}{x^2}\right)}{8}-x^2\sqrt{-1+\frac{1}{x^2}}-\sqrt{-1+\frac{1}{x^2}}+\frac{15\,\mathrm{atan}\left(\sqrt{-1+\frac{1}{x^2}}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1
+ x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

Giac [A]
time = 1.76, size = 67, normalized size = 1.12

$$\frac{1}{8}\left(2x^2\mathrm{sgn}(x)-9\,\mathrm{sgn}(x)\right)\sqrt{-x^2+1}\,x-\frac{15}{8}\arcsin(x)\,\mathrm{sgn}(x)+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/
2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B]
time = 3.36, size = 44, normalized size = 0.73

$$\frac{15\,\mathrm{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{8}-\sqrt{\frac{1}{x^2}-1}-\frac{7x^4\sqrt{\frac{1}{x^2}-1}}{8}-\frac{9x^4\left(\frac{1}{x^2}-1\right)^{3/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)
```

```
[Out] (15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1/
2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8
```

3.677 $\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)}{x}\,dx$

Optimal. Leaf size=44

$$\frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2}x^2-\frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)$$

```
[Out] -1/2*(-1+1/x^2)^(3/2)*x^2-3/2*arctan((-1+1/x^2)^(1/2))+3/2*(-1+1/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {25, 272, 43, 52, 65, 209}

$$-\frac{3}{2}\text{ArcTan}\left(\sqrt{\frac{1}{x^2}-1}\right)-\frac{1}{2}\left(\frac{1}{x^2}-1\right)^{3/2}x^2+\frac{3}{2}\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 +
 x^(-2)]])/2
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}\,(-1+x^2)}{x}\,dx &= -\int \left(-1+\frac{1}{x^2}\right)^{3/2} x\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(-1+x)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2+\frac{3}{4}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x^2}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x^2}}\right)\\
&= \frac{3}{2}\sqrt{-1+\frac{1}{x^2}}-\frac{1}{2}\left(-1+\frac{1}{x^2}\right)^{3/2} x^2-\frac{3}{2}\tan^{-1}\left(\sqrt{-1+\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [A]

time = 0.06, size = 46, normalized size = 1.05

$$\frac{1}{2}\sqrt{-1+\frac{1}{x^2}}\left(2+x^2-\frac{6x\tanh^{-1}\left(\frac{\sqrt{-1+x^2}}{-1+x}\right)}{\sqrt{-1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(2 + x^2 - (6*x*ArcTanh[Sqrt[-1 + x^2]/(-1 + x)])/Sqrt[-
1 + x^2]))/2
```

Maple [A]
time = 0.11, size = 55, normalized size = 1.25

method	result	size
default	$\frac{\sqrt{-\frac{x^2-1}{x^2}}\left(2(-x^2+1)^{\frac{3}{2}}+3x^2\sqrt{-x^2+1}+3\arcsin(x)x\right)}{2\sqrt{-x^2+1}}$	55
trager	$2\left(\frac{x^2}{4}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2}}+\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\left(\mathrm{RootOf}\left(\_Z^2+1\right)-\sqrt{-\frac{x^2-1}{x^2}}\right)x\right)}{2}$	56
risch	$\frac{(x^4+x^2-2)\sqrt{-\frac{x^2-1}{x^2}}}{2x^2-2}-\frac{3\arcsin(x)\sqrt{-\frac{x^2-1}{x^2}}\,x\sqrt{-x^2+1}}{2(x^2-1)}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*x^2*(-x^2+1)^(1/2)+3*arcsin(x)
*x)/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.53, size = 30, normalized size = 0.68

$$\frac{1}{2}x^2\sqrt{\frac{1}{x^2}-1}+\sqrt{\frac{1}{x^2}-1}-\frac{3}{2}\arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))
```

Fricas [A]
time = 0.34, size = 43, normalized size = 0.98

$$\frac{1}{2}\left(x^{2}+2\right)\sqrt{-\frac{x^{2}-1}{x^{2}}}-3\arctan\left(\frac{x\sqrt{-\frac{x^{2}-1}{x^{2}}}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/
x)
```

Sympy [A]
time = 22.77, size = 39, normalized size = 0.89

$$\frac{x^{2}\sqrt{-1+\frac{1}{x^{2}}}}{2}+\sqrt{-1+\frac{1}{x^{2}}}-\frac{3\operatorname{atan}\left(\sqrt{-1+\frac{1}{x^{2}}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))
/2
```

Giac [A]
time = 2.26, size = 57, normalized size = 1.30

$$\frac{1}{2}\sqrt{-x^{2}+1}\,x\operatorname{sgn}(x)+\frac{3}{2}\arcsin\left(x\right)\operatorname{sgn}(x)-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^{2}+1}-1\right)}+\frac{\left(\sqrt{-x^{2}+1}-1\right)\operatorname{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^
2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B]
time = 3.57, size = 30, normalized size = 0.68

$$\sqrt{\frac{1}{x^{2}}-1}-\frac{3\operatorname{atan}\left(\sqrt{\frac{1}{x^{2}}-1}\right)}{2}+\frac{x^{2}\,\sqrt{\frac{1}{x^{2}}-1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2))
/2
```

3.678 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)}\,dx$

Optimal. Leaf size=9

$$\sqrt{-1+\frac{1}{x^2}}$$

```
[Out] (-1+1/x^2)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {25, 267}

$$\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)}\,dx &= -\int \frac{1}{\sqrt{-1+\frac{1}{x^2}}\,x^3}\,dx\\ &= \sqrt{-1+\frac{1}{x^2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 9, normalized size = 1.00

$$\sqrt{-1+\frac{1}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]
```

```
[Out] Sqrt[-1 + x^(-2)]
```

Maple [A]
time = 0.41, size = 13, normalized size = 1.44

method	result	size
gosper	$\sqrt{-\frac{x^2-1}{x^2}}$	13
default	$\sqrt{-\frac{x^2-1}{x^2}}$	13
trager	$\sqrt{-\frac{x^2-1}{x^2}}$	13
risch	$\sqrt{-\frac{x^2-1}{x^2}}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] (-(x^2-1)/x^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(7) = 14.
time = 0.27, size = 16, normalized size = 1.78

$$\frac{\sqrt{x+1}\,\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*sqrt(-x + 1)/x
```

Fricas [A]
time = 0.36, size = 12, normalized size = 1.33

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")
```

```
[Out] sqrt(-(x^2 - 1)/x^2)
```

Sympy [A]
time = 1.27, size = 8, normalized size = 0.89

$$\sqrt{-1+\frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

```
[Out] sqrt(-1 + x**(-2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(7) = 14.
time = 1.89, size = 37, normalized size = 4.11

$$-\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}+\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B]
time = 3.18, size = 14, normalized size = 1.56

$$\frac{\sqrt{1-x^2}}{|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)
```

```
[Out] (1 - x^2)^(1/2)/abs(x)
```

3.679 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2} dx$

Optimal. Leaf size=21

$$\frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}$$

```
[Out] 1/(-1+1/x^2)^(1/2)-(-1+1/x^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {25, 272, 45}

$$\frac{1}{\sqrt{\frac{1}{x^2}-1}}-\sqrt{\frac{1}{x^2}-1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2}\,dx &= \int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{3/2}x^5}\,dx\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{x}{(-1+x)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{1}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{1}{\sqrt{-1+\frac{1}{x^2}}}-\sqrt{-1+\frac{1}{x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 24, normalized size = 1.14

$$
\frac{\sqrt{-1+\frac{1}{x^2}}\,(1-2x^2)}{-1+x^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)
```

Maple [A]
time = 0.38, size = 32, normalized size = 1.52

method	result	size
gosper	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
trager	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
risch	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$	29
default	$-\frac{(2x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{(1+x)(-1+x)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(1+x)/(-1+x)
```

Maxima [A]
time = 0.28, size = 30, normalized size = 1.43

$$-\frac{(2\,x^2-1)\sqrt{x+1}\,\sqrt{-x+1}}{x^3-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")
[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)
```

Fricas [A]
time = 0.35, size = 28, normalized size = 1.33

$$-\frac{(2\,x^2-1)\sqrt{-\frac{x^2-1}{x^2}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")
[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)
```

Sympy [A]
time = 2.06, size = 20, normalized size = 0.95

$$-\sqrt{-1+\frac{1}{x^2}}+\frac{1}{\sqrt{-1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)
[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(17) = 34.
time = 2.34, size = 58, normalized size = 2.76

$$-\frac{\sqrt{-x^2+1}\,x\mathrm{sgn}(x)}{x^2-1}+\frac{x\mathrm{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)}-\frac{\left(\sqrt{-x^2+1}-1\right)\mathrm{sgn}(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/
2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x
```

Mupad [B]
time = 3.11, size = 25, normalized size = 1.19

$$\frac{x\,\sqrt{\frac{1}{x^2}-1}\,\left(2\,x^2-1\right)}{x-x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)
[Out] (x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)
```

3.680 $\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}+\sqrt{-1+\frac{1}{x^2}}$$

```
[Out] -1/3/(-1+1/x^2)^(3/2)-2/(-1+1/x^2)^(1/2)+(-1+1/x^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {25, 272, 45}

$$\sqrt{\frac{1}{x^2}-1}-\frac{2}{\sqrt{\frac{1}{x^2}-1}}-\frac{1}{3\left(\frac{1}{x^2}-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
```

```
[Out] -1/3*1/(-1 + x^(-2))^(3/2) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x\left(-1+x^2\right)^3}\,dx &= -\int \frac{1}{\left(-1+\frac{1}{x^2}\right)^{5/2}x^7}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-1+x)^{5/2}}\,dx,x,\frac{1}{x^2}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^{5/2}}+\frac{2}{(-1+x)^{3/2}}+\frac{1}{\sqrt{-1+x}}\right)dx,x,\frac{1}{x^2}\right)\\ &= -\frac{1}{3\left(-1+\frac{1}{x^2}\right)^{3/2}}-\frac{2}{\sqrt{-1+\frac{1}{x^2}}}+\sqrt{-1+\frac{1}{x^2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.94

$$\frac{\sqrt{-1+\frac{1}{x^2}}\left(3-12x^2+8x^4\right)}{3\left(-1+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3),x]
[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)
```

Maple [A]
time = 0.36, size = 37, normalized size = 1.09

method	result	size
gosper	$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^2-1\right)^2}$	34
trager	$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^2-1\right)^2}$	34
risch	$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3\left(x^2-1\right)^2}$	34
default	$\frac{\left(8x^4-12x^2+3\right)\sqrt{-\frac{x^2-1}{x^2}}}{3(1+x)^2(-1+x)^2}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(1+x)^2/(-1+x)^2
```

Maxima [A]
time = 0.30, size = 38, normalized size = 1.12

$$\frac{(8x^4 - 12x^2 + 3)\sqrt{x+1}\sqrt{-x+1}}{3(x^5 - 2x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)
```

Fricas [A]
time = 0.35, size = 38, normalized size = 1.12

$$\frac{(8x^4 - 12x^2 + 3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)
```

Sympy [A]
time = 3.01, size = 34, normalized size = 1.00

$$\sqrt{-1+\frac{1}{x^2}} - \frac{2}{\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{3\left(-1+\frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)
```

```
[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.
time = 1.83, size = 68, normalized size = 2.00

$$-\frac{x\text{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\text{sgn}(x)}{2x} - \frac{(5x^2\text{sgn}(x) - 6\,\text{sgn}(x))x}{3(x^2-1)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")
```

```
[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/
3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))
```

Mupad [B]
time = 3.09, size = 28, normalized size = 0.82

$$\frac{\sqrt{\frac{1}{x^2}-1}\,\left(8\,x^4-12\,x^2+3\right)}{3\,{\left(x^2-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)
```

```
[Out] ((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)
```

3.681 $\int \frac{\sqrt{1+\frac{1}{x^2}}\,x}{(1+x^2)^2}\,dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

[Out] 1/(1+1/x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {25, 267}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]

[Out] 1/Sqrt[1 + x^(-2)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x^2}}\,x}{(1+x^2)^2}\,dx = \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2}x^3}\,dx$$
$$= \frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.02, size = 20, normalized size = 2.22

$$\frac{\sqrt{1+\frac{1}{x^2}}\,x^2}{1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]
```

```
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.
time = 0.34, size = 23, normalized size = 2.56

method	result	size
gosper	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
default	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
risch	$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}}{x^2+1}$	23
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)
```

Maxima [A]
time = 0.49, size = 11, normalized size = 1.22

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/sqrt((x^2 + 1)/x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(7) = 14$.
time = 0.37, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A]
time = 1.56, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] x/sqrt(x**2 + 1)
```

Giac [A]
time = 2.12, size = 11, normalized size = 1.22

$$\frac{x\operatorname{sgn}(x)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] x*sgn(x)/sqrt(x^2 + 1)
```

Mupad [B]
time = 3.11, size = 18, normalized size = 2.00

$$\frac{x^2\,\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.682 $\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x(1+x^2)}\,dx$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

[Out] 1/(1+1/x^2)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {25, 267}

$$\frac{1}{\sqrt{\frac{1}{x^2}+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] 1/Sqrt[1 + x^(-2)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\frac{1}{x^2}}\,x\,(1+x^2)}\,dx &= \int \frac{1}{\left(1+\frac{1}{x^2}\right)^{3/2}x^3}\,dx\\ &= \frac{1}{\sqrt{1+\frac{1}{x^2}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 0.00, size = 20, normalized size = 2.22

$$\frac{\sqrt{1+\frac{1}{x^2}}\,x^2}{1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]
[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)
```

Maple [A]
time = 0.42, size = 12, normalized size = 1.33

method	result	size
gosper	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
default	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
risch	$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$	12
trager	$\frac{x^2\sqrt{-\frac{-x^2-1}{x^2}}}{x^2+1}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((x^2+1)/x^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(7) = 14.
time = 0.35, size = 28, normalized size = 3.11

$$\frac{x^2\sqrt{\frac{x^2+1}{x^2}}+x^2+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)
```

Sympy [A]
time = 1.57, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1+\frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
```

```
[Out] 1/sqrt(1 + x**(-2))
```

Giac [A]
time = 1.71, size = 13, normalized size = 1.44

$$\frac{x}{\sqrt{x^2+1}\,\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(x^2 + 1)*sgn(x))
```

Mupad [B]
time = 3.10, size = 18, normalized size = 2.00

$$\frac{x^2\sqrt{\frac{1}{x^2}+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)
```

```
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

3.683 $\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}$$

```
[Out] ln(1+(b*x^2+a)^(1/2))/b
```

Rubi [A]
time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2186, 31}

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[1 + Sqrt[a + b*x^2]]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2186

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^2+\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 20, normalized size = 1.11

$$\frac{\log\left(b+b\sqrt{a+bx^2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]
```

```
[Out] Log[b + b*Sqrt[a + b*x^2]]/b
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(16) = 32$.
time = 0.05, size = 1059, normalized size = 58.83

method	result
default	$\frac{\sqrt{b\left(x+\frac{\sqrt{-(a-1)\,b}}{b}\right)^2-2\sqrt{-(a-1)\,b}\,\left(x+\frac{\sqrt{-(a-1)\,b}}{b}\right)+1}}{2\left(\sqrt{-(a-1)\,b}+\sqrt{-ab}\right)\left(-\sqrt{-(a-1)\,b}+\sqrt{-ab}\right)} - \frac{\sqrt{-(a-1)\,b}\,\ln\Bigg(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^2+(b*x^2+a)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x+
(-(a-1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2)-1/
2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*
b)^(1/2)*ln(((x+(-(a-1)*b)^(1/2)/b)*b-(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x+(-(a-
1)*b)^(1/2)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2
)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arct
anh(1/2*(2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b))/(b*(x+(-(a-1)*b)^(1/2
)/b)^2-2*(-(a-1)*b)^(1/2)*(x+(-(a-1)*b)^(1/2)/b)+1)^(1/2))+1/2/((-(a-1)*b)^
(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x-(-(a-1)*b)^(1/2)
/b)^2+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2)+1/2/((-(a-1)*b)^(1
/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-(a-1)*b)^(1/2)*ln(((x-
(-(a-1)*b)^(1/2)/b)*b+(-(a-1)*b)^(1/2))/b^(1/2)+(b*(x-(-(a-1)*b)^(1/2)/b)^2
+2*(-(a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))/b^(1/2)-1/2/((-(a-1)*b
)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*arctanh(1/2*(2+2*(-(
a-1)*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b))/(b*(x-(-(a-1)*b)^(1/2)/b)^2+2*(-(a-1)
*b)^(1/2)*(x-(-(a-1)*b)^(1/2)/b)+1)^(1/2))-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/
2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/
2)*(x-1/b*(-a*b)^(1/2)))^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1
```

```
)*b)^(1/2)+(-a*b)^(1/2))*(-a*b)^(1/2)*ln(((x-1/b*(-a*b)^(1/2))*b+(-a*b)^(1/
2))/b^(1/2)+(b*(x-1/b*(-a*b)^(1/2))^2+2*(-a*b)^(1/2)*(x-1/b*(-a*b)^(1/2)))^
(1/2))/b^(1/2)-1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b
)^(1/2))*(b*(x+1/b*(-a*b)^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/
2)+1/2/((-(a-1)*b)^(1/2)+(-a*b)^(1/2))/(-(-(a-1)*b)^(1/2)+(-a*b)^(1/2))*(-a
*b)^(1/2)*ln(((x+1/b*(-a*b)^(1/2))*b-(-a*b)^(1/2))/b^(1/2)+(b*(x+1/b*(-a*b)
^(1/2))^2-2*(-a*b)^(1/2)*(x+1/b*(-a*b)^(1/2)))^(1/2))/b^(1/2)+1/2/b*ln(b*x^
2+a-1)
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.89

$$\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(16) = 32$.
time = 0.37, size = 67, normalized size = 3.72

$$\frac{2\log\left(bx^2+a-1\right)+\log\left(\frac{bx^2+a+2\sqrt{bx^2+a}+1}{x^2}\right)-\log\left(\frac{bx^2+a-2\sqrt{bx^2+a}+1}{x^2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(b*x^2 + a - 1) + log((b*x^2 + a + 2*sqrt(b*x^2 + a) + 1)/x^2) -
log((b*x^2 + a - 2*sqrt(b*x^2 + a) + 1)/x^2))/b
```

Sympy [A]
time = 1.66, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)
```

```
[Out] log(sqrt(a + b*x**2) + 1)/b
```

Giac [A]
time = 2.16, size = 16, normalized size = 0.89

$$\frac{\log\left(\sqrt{bx^2+a}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="giac")
[Out] log(sqrt(b*x^2 + a) + 1)/b
```

Mupad [B]
time = 3.38, size = 26, normalized size = 1.44

$$\frac{\operatorname{atanh}\left(\sqrt{b\,x^2+a}\right)+\frac{\ln\left(b\,x^2+a-1\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)
[Out] (atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b
```

3.684 $\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

```
[Out] 3/4*ln(1-(x^2)^(2/3))
```

Rubi [A]

time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6847, 1607, 266}

$$\frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x^2 - (x^2)^(1/3)),x]
```

```
[Out] (3*Log[1 - (x^2)^(2/3)])/4
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x^2-\sqrt[3]{x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-\sqrt[3]{x}+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}}\,dx,x,x^2\right)\\ &= \frac{3}{4}\log\left(1-\left(x^2\right)^{2/3}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 29, normalized size = 1.81

$$\frac{3}{4}\log\left(-1+\sqrt[3]{x^2}\right)+\frac{3}{4}\log\left(1+\sqrt[3]{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x^2 - (x^2)^(1/3)),x]
[Out] (3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(12) = 24$.
time = 0.41, size = 70, normalized size = 4.38

method	result
meijerg	$\frac{3\ln\left(1-\frac{x^2}{\left(x^2\right)^{\frac{1}{3}}}\right)}{4}$
derivativedivides	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}$
default	$\frac{\ln(x^2-1)}{4}+\frac{\ln(x^2+1)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}-1\right)}{2}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}+\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}-\frac{\ln\left(\left(x^2\right)^{\frac{2}{3}}-\left(x^2\right)^{\frac{1}{3}}+1\right)}{4}+\frac{\ln\left(\left(x^2\right)^{\frac{1}{3}}+1\right)}{2}$
trager	$-\frac{\ln\left(-\frac{x^8+3\left(x^2\right)^{\frac{1}{3}}x^6+6\left(x^2\right)^{\frac{2}{3}}x^4+7x^4+6x^2\left(x^2\right)^{\frac{1}{3}}+3\left(x^2\right)^{\frac{2}{3}}+1}{\left(x^2+1\right)^3(1+x)^3(-1+x)^3}\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-(x^2)^(1/3)),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)+1/2*ln((x^2)^(1/3)-1)-1/4*ln((x^2)^(2/3)+(x^2)^
(1/3)+1)-1/4*ln((x^2)^(2/3)-(x^2)^(1/3)+1)+1/2*ln((x^2)^(1/3)+1)
```

Maxima [A]
time = 0.27, size = 21, normalized size = 1.31

$$\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}+1\right)+\frac{3}{4}\log\left(\left(x^2\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")
[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

time = 0.39, size = 32, normalized size = 2.00

$$-3 \log\left(\frac{(x^2)^{\frac{1}{3}}}{x}\right) + \frac{3}{4}\log\left(-\frac{x^2 - (x^2)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")
[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)
```

Sympy [A]
time = 0.08, size = 19, normalized size = 1.19

$$-\frac{\log(x)}{2} + \frac{3\log\left(x^2 - \sqrt[3]{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-(x**2)**(1/3)),x)
[Out] -log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4
```

Giac [A]
time = 1.88, size = 16, normalized size = 1.00

$$\frac{3}{4}\log\left(\left|(x\mathrm{sgn}(x))^{\frac{1}{3}}\, x\mathrm{sgn}(x) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")
[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))
```

Mupad [B]
time = 3.34, size = 10, normalized size = 0.62

$$\frac{3\ln\left((x^2)^{2/3} - 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((x^2)^(1/3) - x^2),x)
[Out] (3*log((x^2)^(2/3) - 1))/4
```

3.685 $\int x(1+x^2)^3 \sqrt{2+2x^2+x^4}\, dx$

Optimal. Leaf size=44

$$-\frac{1}{15}\left(2+2x^2+x^4\right)^{3/2}+\frac{1}{10}\left(1+x^2\right)^2\left(2+2x^2+x^4\right)^{3/2}$$

```
[Out] -1/15*(x^4+2*x^2+2)^(3/2)+1/10*(x^2+1)^2*(x^4+2*x^2+2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1261, 706, 643}

$$\frac{1}{10}\left(x^2+1\right)^2\left(x^4+2x^2+2\right)^{3/2}-\frac{1}{15}\left(x^4+2x^2+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] -1/15*(2 + 2*x^2 + x^4)^(3/2) + ((1 + x^2)^2*(2 + 2*x^2 + x^4)^(3/2))/10
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x(1+x^2)^3\sqrt{2+2x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int (1+x)^3\sqrt{2+2x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{10}(1+x^2)^2(2+2x^2+x^4)^{3/2} - \frac{1}{5}\text{Subst}\left(\int (1+x)\sqrt{2+2x+x^2}\,dx, x,\right.$$
$$= -\frac{1}{15}(2+2x^2+x^4)^{3/2} + \frac{1}{10}(1+x^2)^2(2+2x^2+x^4)^{3/2}$$

Mathematica [A]
time = 0.12, size = 40, normalized size = 0.91

$$\frac{1}{30}\sqrt{2+2x^2+x^4}\,(2+14x^2+19x^4+12x^6+3x^8)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[2 + 2*x^2 + x^4]*(2 + 14*x^2 + 19*x^4 + 12*x^6 + 3*x^8))/30
```

Maple [A]
time = 0.38, size = 50, normalized size = 1.14

method	result	size
gosper	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
elliptic	$\frac{(x^4+2x^2+2)^{\frac{3}{2}}(3x^4+6x^2+1)}{30}$	27
trager	$\left(\frac{1}{10}x^8+\frac{2}{5}x^6+\frac{19}{30}x^4+\frac{7}{15}x^2+\frac{1}{15}\right)\sqrt{x^4+2x^2+2}$	36
risch	$\frac{(3x^8+12x^6+19x^4+14x^2+2)\sqrt{x^4+2x^2+2}}{30}$	37
default	$\frac{x^4(x^4+2x^2+2)^{\frac{3}{2}}}{10}+\frac{x^2(x^4+2x^2+2)^{\frac{3}{2}}}{5}+\frac{(x^4+2x^2+2)^{\frac{3}{2}}}{30}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^4*(x^4+2*x^2+2)^(3/2)+1/5*x^2*(x^4+2*x^2+2)^(3/2)+1/30*(x^4+2*x^2+2)
^(3/2)
```

Maxima [A]
time = 0.51, size = 49, normalized size = 1.11

$$\frac{1}{10}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^4+\frac{1}{5}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}x^2+\frac{1}{30}\left(x^4+2\,x^2+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(
x^4 + 2*x^2 + 2)^(3/2)
```

Fricas [A]
time = 0.35, size = 36, normalized size = 0.82

$$\frac{1}{30}\left(3\,x^{8}+12\,x^{6}+19\,x^{4}+14\,x^{2}+2\right)\sqrt{x^{4}+2\,x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(36) = 72$.
time = 0.15, size = 94, normalized size = 2.14

$$\frac{x^{8}\sqrt{x^{4}+2x^{2}+2}}{10}+\frac{2x^{6}\sqrt{x^{4}+2x^{2}+2}}{5}+\frac{19x^{4}\sqrt{x^{4}+2x^{2}+2}}{30}+\frac{7x^{2}\sqrt{x^{4}+2x^{2}+2}}{15}+\frac{\sqrt{x^{4}+2x^{2}+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2),x)
[Out] x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**
4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**
4 + 2*x**2 + 2)/15
```

Giac [A]
time = 2.04, size = 29, normalized size = 0.66

$$\frac{1}{10}\left(x^{4}+2\,x^{2}+2\right)^{\frac{5}{2}}-\frac{1}{6}\left(x^{4}+2\,x^{2}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2),x, algorithm="giac")
[Out] 1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)
```

Mupad [B]
time = 0.09, size = 26, normalized size = 0.59

$$\frac{{\left(x^{4}+2\,x^{2}+2\right)}^{3/2}\,\left(3\,x^{4}+6\,x^{2}+1\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2),x)
[Out] ((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30
```

3.686 $\int x^5 \sqrt{1-x^3} \left(1+x^9\right)^2 \, dx$

Optimal. Leaf size=121

$$-\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{14}{45}\left(1-x^3\right.$$

```
[Out] -8/9*(-x^3+1)^(3/2)+32/15*(-x^3+1)^(5/2)-22/7*(-x^3+1)^(7/2)+86/27*(-x^3+1)
^(9/2)-74/33*(-x^3+1)^(11/2)+14/13*(-x^3+1)^(13/2)-14/45*(-x^3+1)^(15/2)+2/
51*(-x^3+1)^(17/2)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1835, 1634}

$$\frac{2}{51}\left(1-x^3\right)^{17/2}-\frac{14}{45}\left(1-x^3\right)^{15/2}+\frac{14}{13}\left(1-x^3\right)^{13/2}-\frac{74}{33}\left(1-x^3\right)^{11/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{8}{9}\left(1-x^3\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
```

```
[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 +
 (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))
/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1835

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{1-x^3} \left(1+x^9\right)^2 \, dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{1-x}\, x\left(1+x^3\right)^2 \, dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(4\sqrt{1-x}-16(1-x)^{3/2}+33(1-x)^{5/2}-43(1-x)^{7/2}+37(1-\right.\right.\\
&= -\frac{8}{9}\left(1-x^3\right)^{3/2}+\frac{32}{15}\left(1-x^3\right)^{5/2}-\frac{22}{7}\left(1-x^3\right)^{7/2}+\frac{86}{27}\left(1-x^3\right)^{9/2}-\frac{74}{33}\left(1-x\right.
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3}\left(-173014-86507x^3+126561x^6-22160x^9-19390x^{12}+135702x^{15}-3234x^{18}-3003x^{21}+45045x^{24}\right)}{2297295}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]
[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12
 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295
```

Maple [A]
time = 0.50, size = 125, normalized size = 1.03

method	result
trager	$\left(\frac{2}{51}x^{24}-\frac{2}{765}x^{21}-\frac{28}{9945}x^{18}+\frac{1436}{12155}x^{15}-\frac{1108}{65637}x^{12}-\frac{8864}{459459}x^{9}+\frac{84374}{765765}x^{6}-\frac{173014}{2297295}x^{3}-\frac{346028}{2297295}\right)\sqrt{-}$
gosper	$\frac{2\sqrt{-x^3+1}\,(45045x^{21}+42042x^{18}+38808x^{15}+174510x^{12}+155120x^{9}+132960x^{6}+259521x^{3}+173014)(-1+x)(x^2+x+1)}{2297295}$
risch	$-\frac{2(45045x^{24}-3003x^{21}-3234x^{18}+135702x^{15}-19390x^{12}-22160x^{9}+126561x^{6}-86507x^{3}-173014)(x^3-1)}{2297295\sqrt{-x^3+1}}$
default	$\frac{2x^{24}\sqrt{-x^3+1}}{51}-\frac{2x^{21}\sqrt{-x^3+1}}{765}-\frac{28x^{18}\sqrt{-x^3+1}}{9945}+\frac{1436x^{15}\sqrt{-x^3+1}}{12155}-\frac{1108x^{12}\sqrt{-x^3+1}}{65637}-$
elliptic	$\frac{2x^{24}\sqrt{-x^3+1}}{51}-\frac{2x^{21}\sqrt{-x^3+1}}{765}-\frac{28x^{18}\sqrt{-x^3+1}}{9945}+\frac{1436x^{15}\sqrt{-x^3+1}}{12155}-\frac{1108x^{12}\sqrt{-x^3+1}}{65637}-$
meijerg	$-\frac{-\frac{8192\sqrt{\pi}}{109395}+\frac{4\sqrt{\pi}\,(-x^3+1)^{\frac{3}{2}}(6435x^{21}+6006x^{18}+5544x^{15}+5040x^{12}+4480x^{9}+3840x^{6}+3072x^{3}+2048)}{109395}}{6\sqrt{\pi}}+\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\,(-x^3+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/51*x^24*(-x^3+1)^(1/2)-2/765*x^21*(-x^3+1)^(1/2)-28/9945*x^18*(-x^3+1)^(1
/2)+1436/12155*x^15*(-x^3+1)^(1/2)-1108/65637*x^12*(-x^3+1)^(1/2)-8864/4594
59*x^9*(-x^3+1)^(1/2)+84374/765765*x^6*(-x^3+1)^(1/2)-173014/2297295*x^3*(-
x^3+1)^(1/2)-346028/2297295*(-x^3+1)^(1/2)
```

Maxima [A]
time = 0.50, size = 89, normalized size = 0.74

$$\frac{2}{51}\left(-x^3+1\right)^{\frac{17}{2}}-\frac{14}{45}\left(-x^3+1\right)^{\frac{15}{2}}+\frac{14}{13}\left(-x^3+1\right)^{\frac{13}{2}}-\frac{74}{33}\left(-x^3+1\right)^{\frac{11}{2}}+\frac{86}{27}\left(-x^3+1\right)^{\frac{9}{2}}-\frac{22}{7}\left(-x^3+1\right)^{\frac{7}{2}}+\frac{32}{15}\left(-x^3+1\right)^{\frac{5}{2}}-\frac{8}{9}\left(-x^3+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2)
- 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2)
+ 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)
```

Fricas [A]
time = 0.36, size = 53, normalized size = 0.44

$$\frac{2}{2297295}\left(45045\,x^{24}-3003\,x^{21}-3234\,x^{18}+135702\,x^{15}-19390\,x^{12}-22160\,x^{9}+126561\,x^{6}-86507\,x^{3}-173014\right)\sqrt{-x^{3}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 -
22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)
```

Sympy [A]
time = 1.18, size = 133, normalized size = 1.10

$$\frac{2x^{24}\sqrt{1-x^{3}}}{51}-\frac{2x^{21}\sqrt{1-x^{3}}}{765}-\frac{28x^{18}\sqrt{1-x^{3}}}{9945}+\frac{1436x^{15}\sqrt{1-x^{3}}}{12155}-\frac{1108x^{12}\sqrt{1-x^{3}}}{65637}-\frac{8864x^{9}\sqrt{1-x^{3}}}{459459}+\frac{84374x^{6}\sqrt{1-x^{3}}}{765765}-\frac{173014x^{3}\sqrt{1-x^{3}}}{2297295}-\frac{346028\sqrt{1-x^{3}}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

Giac [A]
time = 2.30, size = 138, normalized size = 1.14

$$\frac{2}{51}\,{\left(x^{3}-1\right)}^{8}\sqrt{-x^{3}+1}+\frac{14}{45}\,{\left(x^{3}-1\right)}^{7}\sqrt{-x^{3}+1}+\frac{14}{13}\,{\left(x^{3}-1\right)}^{6}\sqrt{-x^{3}+1}+\frac{74}{33}\,{\left(x^{3}-1\right)}^{5}\sqrt{-x^{3}+1}+\frac{86}{27}\,{\left(x^{3}-1\right)}^{4}\sqrt{-x^{3}+1}+\frac{22}{7}\,{\left(x^{3}-1\right)}^{3}\sqrt{-x^{3}+1}+\frac{32}{15}\,{\left(x^{3}-1\right)}^{2}\sqrt{-x^{3}+1}-\frac{8}{9}\,{\left(-x^{3}+1\right)}^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*
(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3
- 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2
*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)
```

Mupad [B]
time = 3.27, size = 124, normalized size = 1.02

$$\frac{84374\,x^{6}\,\sqrt{1-x^{3}}}{765765}-\frac{173014\,x^{3}\,\sqrt{1-x^{3}}}{2297295}-\frac{8864\,x^{9}\,\sqrt{1-x^{3}}}{459459}-\frac{1108\,x^{12}\,\sqrt{1-x^{3}}}{65637}+\frac{1436\,x^{15}\,\sqrt{1-x^{3}}}{12155}-\frac{28\,x^{18}\,\sqrt{1-x^{3}}}{9945}-\frac{2\,x^{21}\,\sqrt{1-x^{3}}}{765}+\frac{2\,x^{24}\,\sqrt{1-x^{3}}}{51}-\frac{346028\,\sqrt{1-x^{3}}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)
```

```
[Out] (84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295 -
 (8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 + (1
436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^21*
(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^(1/2
))/2297295
```

3.687 $\int \left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right) dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {267, 455, 65, 214}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int\left(\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx&=\int\frac{x}{(a+bx^2)^{3/2}}\,dx+\int\frac{x}{(1+x^2)\sqrt{a+bx^2}}\,dx\\&=-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\text{Subst}\left(\int\frac{1}{(1+x)\sqrt{a+bx}}\,dx,x,x^2\right)\\&=-\frac{1}{b\sqrt{a+bx^2}}+\frac{\text{Subst}\left(\int\frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\&=-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.98

$$-\frac{1}{b\sqrt{a+bx^2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a + b
]
```

Maple [A]
time = 0.41, size = 42, normalized size = 0.84

method	result	size
default	$-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(42) = 84$.
time = 0.42, size = 268, normalized size = 5.36

$$\left[\frac{(b^2x^2+ab)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,\sqrt{bx^2+a}\,(a-b)}{4\,(a^2b-ab^2+(ab^2-b^3)x^2)}, -\frac{(b^2x^2+ab)\sqrt{-a+b}\,\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,\sqrt{bx^2+a}\,(a-b)}{2\,(a^2b-ab^2+(ab^2-b^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A]
time = 1.96, size = 49, normalized size = 0.98

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)
```

Giac [A]

time = 2.30, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B]

time = 3.76, size = 42, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{1}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2))
```

3.688 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6, 585, 79, 65, 214}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
```

```
[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
```

```
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx &= \int \frac{x(1+a+(1+b)x^2)}{(1+x^2)(a+bx^2)^{3/2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b}\\ &= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.98

$$-\frac{1}{b\sqrt{a+bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]
[Out] -(1/(b*Sqrt[a + b*x^2])) + ArcTan[Sqrt[a + b*x^2]/Sqrt[-a + b]]/Sqrt[-a + b
]
```

Maple [A]
time = 0.40, size = 76, normalized size = 1.52

method	result	size
default	$-\frac{b+1}{b\sqrt{b\,x^{2}+a}}+\left(a-b\right)\left(\frac{1}{\left(a-b\right)\sqrt{b\,x^{2}+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{-a+b}}\right)}{\left(a-b\right)\sqrt{-a+b}}\right)$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b+1)/b/(b*x^2+a)^(1/2)+(a-b)*(1/(a-b)/(b*x^2+a)^(1/2)+1/(a-b)/(-a+b)^(1/2
)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(42) = 84.
time = 0.41, size = 268, normalized size = 5.36

$$\left[\frac{{\left(b^{2} x^{2} + a b\right)} \sqrt{a - b} \log\left(\frac{b^{2} x^{4} + 2 \, {\left(4 \, a b - 3 \, b^{2}\right)} x^{2} - 4 \, {\left(b x^{2} + 2 \, a - b\right)} \sqrt{b x^{2} + a} \sqrt{a - b} + 8 \, a^{2} - 8 \, a b + b^{2}}{x^{4} + 2 \, x^{2} + 1}\right) - 4 \, \sqrt{b x^{2} + a} {\left(a - b\right)}}{4 \, {\left(a^{2} b - a b^{2} + {\left(a b^{2} - b^{3}\right)} x^{2}\right)}}, -\frac{{\left(b^{2} x^{2} + a b\right)} \sqrt{-a + b} \arctan\left(-\frac{{\left(b x^{2} + 2 \, a - b\right)} \sqrt{b x^{2} + a} \sqrt{-a + b}}{2 \, {\left({\left(a b - b^{2}\right)} x^{2} + a^{2} - a b\right)}}\right) + 2 \, \sqrt{b x^{2} + a} {\left(a - b\right)}}{2 \, {\left(a^{2} b - a b^{2} + {\left(a b^{2} - b^{3}\right)} x^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*
(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 +
 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^
2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b
*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(
a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]
```

Sympy [A]
time = 35.72, size = 37, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2))
```

Giac [A]
time = 1.56, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)
```

Mupad [B]
time = 4.52, size = 96, normalized size = 1.92

$$\frac{1}{\sqrt{b\,x^2+a}\,\left(a-b\right)}-\frac{a}{\sqrt{b\,x^2+a}\,\left(a\,b-b^2\right)}-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{{\left(a-b\right)}^{3/2}}+\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)
[Out] 1/((a + b*x^2)^(1/2)*(a - b)) - a/((a + b*x^2)^(1/2)*(a*b - b^2)) - (a*atan
h((a + b*x^2)^(1/2)/(a - b)^(1/2)))/(a - b)^(3/2) + (b*atanh((a + b*x^2)^(1
/2)/(a - b)^(1/2)))/(a - b)^(3/2)
```

3.689 $\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx$

Optimal. Leaf size=68

$$-\frac{1}{3b(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.085, Rules used = {267, 455, 65, 214}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b(a+bx^2)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2
]),x]
```

```
[Out] -1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \left(\frac{x}{(a+bx^2)^{5/2}}+\frac{x}{(a+bx^2)^{3/2}}+\frac{x}{(1+x^2)\sqrt{a+bx^2}}\right)dx=\int \frac{x}{(a+bx^2)^{5/2}}\,dx+\int \frac{x}{(a+bx^2)^{3/2}}\,dx+\int$$

$$=-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{1}{2}\mathrm{Subst}\Bigg($$

$$=-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}+\frac{\mathrm{Subst}\Big(\int}{}$$

$$=-\frac{1}{3b\,(a+bx^2)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\Big(}{\sqrt{}}$$

Mathematica [A]
time = 0.09, size = 62, normalized size = 0.91

$$\frac{-1-3a-3bx^2}{3b\,(a+bx^2)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a +
 b*x^2]),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt[
-a + b]]/Sqrt[-a + b]
```

Maple [A]
time = 0.47, size = 56, normalized size = 0.82

---

method	result	size
default	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}-\frac{1}{b\sqrt{b\,x^2+a}}+\frac{\arctan\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-1/b/(b*x^2+a)^(1/2)+1/(-a+b)^(1/2)*arctan((b*x^2+a)^
(1/2)/(-a+b)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(56) = 112$.
time = 0.38, size = 382, normalized size = 5.62

$$\left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b+a)\sqrt{bx^2+a}}{12\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}, -\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{-a+b}\,\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b+a)\sqrt{bx^2+a}}{6\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [A]
time = 2.13, size = 97, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1
/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
 Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(5/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sq
rt(-a + b)
```

Giac [A]
time = 2.04, size = 55, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}\,b} - \frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x,
algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) -
 1/3/((b*x^2 + a)^(3/2)*b)
```

Mupad [B]
time = 3.69, size = 56, normalized size = 0.82

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{1}{b\,\sqrt{b\,x^2+a}} - \frac{1}{3\,b\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(1
/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(
1/2)) - 1/(3*b*(a + b*x^2)^(3/2))
```

3.690 $\int \frac{x\left(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4\right)}{\left(1+x^2\right)\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] -1/3/b/(b*x^2+a)^(3/2)-arctanh((b*x^2+a)^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)-1/b
/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {6, 6847, 911, 1275, 213}

$$-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\left(a+bx^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/(
(1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] -1/3*1/(b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a - b]]/Sqrt[a - b]
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 6847

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{x(1+a+a^2+(1+a)x^2+bx^2+2abx^2+}{(1+x^2)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x(1+a+a^2+2abx^2+(1+a+b)x^2+b}{(1+x^2)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+bx^4}{(1+x^2)\left(a+bx^2\right)^{5/2}}$$

$$= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+(b-}{(1+x^2)\left(a+bx^2\right)^{5/2}}$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1+a+a^2+(1+a+b+2ab)x}{(1+x)(a+bx)^{5/2}}\right.$$

$$= \frac{\text{Subst}\left(\int \frac{\frac{\left(1+a+a^2\right)b^2-ab(1+a+b+2ab)+a^2\left(b+b^2\right)}{b^2}-\frac{\left(-b(1-\right.}{}}{x^4\left(\frac{-a+b}{b}+\frac{x^2}{b}\right.}\right.}{}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x^4}+\frac{1}{x^2}+\frac{b}{-a+b+x^2}\right)\,dx, x, \sqrt{a+bx}\right.}{b}$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \text{Subst}\left(\int\right.$$

$$= -\frac{1}{3b\left(a+bx^2\right)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\right.}{\sqrt{a}}$$

Mathematica [**A**]
time = 0.05, size = 62, normalized size = 0.91

$$\frac{-1-3a-3bx^2}{3b\,(a+bx^2)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x
^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) + ArcTan[Sqrt[a + b*x^2]/Sqrt[
-a + b]]/Sqrt[-a + b]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(56) = 112$.
time = 0.39, size = 148, normalized size = 2.18

method	result
default	$(b^2+b)\left(-\frac{x^2}{b(b\,x^2+a)^{\frac{3}{2}}}-\frac{2a}{3b^2(b\,x^2+a)^{\frac{3}{2}}}\right)-\frac{2ab-b^2+a+1}{3b(b\,x^2+a)^{\frac{3}{2}}}+(a^2-2ab+b^2)\left(\frac{1}{3(a-b)(b\,x^2+a)^{\frac{3}{2}}}+\frac{1}{(a-b)^2\sqrt{b\,x^2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(
5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b^2+b)*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))-1/3*(2*a*b-b^2+a
+1)/b/(b*x^2+a)^(3/2)+(a^2-2*a*b+b^2)*(1/3/(a-b)/(b*x^2+a)^(3/2)+1/(a-b)^2/
(b*x^2+a)^(1/2)+1/(a-b)^2/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(56) = 112$.
time = 0.42, size = 382, normalized size = 5.62

$$\left[\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{a-b}\,\log\left(\frac{b^2x^4+2\,(4\,ab-3\,b^2)x^2-4\,(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{a-b}+8\,a^2-8\,ab+b^2}{x^4+2\,x^2+1}\right)-4\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b+a)\sqrt{bx^2+a}}{12\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}, -\frac{3\,(b^3x^4+2\,ab^2x^2+a^2b)\sqrt{-a+b}\,\arctan\left(-\frac{(bx^2+2\,a-b)\sqrt{bx^2+a}\,\sqrt{-a+b}}{2\,((ab-b^2)x^2+a^2-ab)}\right)+2\,(3\,(ab-b^2)x^2+3\,a^2-(3\,a+1)b+a)\sqrt{bx^2+a}}{6\,((ab^3-b^4)x^4+a^3b-a^2b^2+2\,(a^2b^2-ab^3)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*
b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 -
8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*
b + a)*sqrt(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -
 a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-
1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 -
 a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a))/(
(a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]
```

Sympy [A]
time = 150.73, size = 53, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{1}{b\sqrt{a+bx^2}}-\frac{1}{3b\,(a+bx^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2
+1)/(b*x**2+a)**(5/2),x)
```

```
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2)) -
 1/(3*b*(a + b*x**2)**(3/2))
```

Giac [A]
time = 1.49, size = 52, normalized size = 0.76

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}-\frac{3\,bx^2+3\,a+1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^
2+a)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)
/((b*x^2 + a)^(3/2)*b)
```

Mupad [B]
time = 3.90, size = 50, normalized size = 0.74

$$-\frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\frac{b\,x^2+a+\frac{1}{3}}{b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/(
(x^2 + 1)*(a + b*x^2)^(5/2)),x)
```

```
[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)/
(b*(a + b*x^2)^(3/2))
```

3.691 $\int \frac{1}{\sqrt{\sqrt{x}+x}} \, dx$

Optimal. Leaf size=34

$$2\sqrt{\sqrt{x}+x} - 2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] -2*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))+2*(x+x^(1/2))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2035, 2038, 634, 212}

$$2\sqrt{x+\sqrt{x}} - 2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Sqrt[x] + x],x]
```

```
[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2035

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[-2*(Sqrt
[a*x^j + b*x^n]/(b*(n - 2)*x^(n - 1))), x] - Dist[a*((2*n - j - 2)/(b*(n -
2))), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2038

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\sqrt{x}+x}}\,dx &= 2\sqrt{\sqrt{x}+x}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}}\,dx\\ &= 2\sqrt{\sqrt{x}+x}-\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{\sqrt{x}+x}-2\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\\ &= 2\sqrt{\sqrt{x}+x}-2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 37, normalized size = 1.09

$$2\sqrt{\sqrt{x}+x}+\log\left(-1-2\sqrt{x}+2\sqrt{\sqrt{x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Sqrt[x] + x],x]
[Out] 2*Sqrt[Sqrt[x] + x] + Log[-1 - 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]
```

Maple [A]
time = 0.38, size = 45, normalized size = 1.32

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)$	26
meijerg	$\frac{2\sqrt{\pi}\,x^{\frac{1}{4}}\sqrt{1+\sqrt{x}}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{\sqrt{\pi}}$	30
default	$\frac{\sqrt{x+\sqrt{x}}\,\left(2\sqrt{x+\sqrt{x}}-\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)\right)}{\sqrt{\sqrt{x}\,\left(1+\sqrt{x}\right)}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x+x^(1/2))^(1/2)/(x^(1/2)*(1+x^(1/2)))^(1/2)*(2*(x+x^(1/2))^(1/2)-ln(1/2+x
^(1/2)+(x+x^(1/2))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x)), x)
```

Fricas [A]
time = 0.56, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x}}\,\left(2\sqrt{x}+1\right)-8\,x-8\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8
*sqrt(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sqrt{x}+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(x) + x), x)
```

Giac [A]
time = 2.08, size = 27, normalized size = 0.79

$$2\sqrt{x+\sqrt{x}}+\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [B]
time = 3.33, size = 39, normalized size = 1.15

$$\frac{2\sqrt{x}\,\left(\sqrt{x}+1\right)+x^{1/4}\,\mathrm{asin}\left(x^{1/4}\,1\mathrm{i}\right)\,\sqrt{\sqrt{x}+1}\,2\mathrm{i}}{\sqrt{x+\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(1/2))^(1/2),x)
[Out] (2*x^(1/2)*(x^(1/2) + 1) + x^(1/4)*asin(x^(1/4)*1i)*(x^(1/2) + 1)^(1/2)*2i)
/(x + x^(1/2))^(1/2)
```

3.692 $\int \sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=74

$$-\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] 1/4*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-1/4*(x+x^(1/2))^(1/2)+2/3*x*(x+x^(1/
2))^(1/2)+1/6*x^(1/2)*(x+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2029, 2043, 684, 654, 634, 212}

$$\frac{2}{3}\sqrt{x+\sqrt{x}}\,x+\frac{1}{6}\sqrt{x+\sqrt{x}}\,\sqrt{x}-\frac{\sqrt{x+\sqrt{x}}}{4}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[x] + x],x]
```

```
[Out] -1/4*Sqrt[Sqrt[x] + x] + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x]
+ x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{6}\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\,dx \\
&= \frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}-\frac{1}{4}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^2}}\,dx,x,\sqrt{x}\right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}}\,dx,x,\sqrt{\ }\right. \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{\ }}{\sqrt{\sqrt{\ }}}\right. \\
&= -\frac{1}{4}\sqrt{\sqrt{x}+x}+\frac{1}{6}\sqrt{x}\sqrt{\sqrt{x}+x}+\frac{2}{3}x\sqrt{\sqrt{x}+x}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [**A**]

time = 0.08, size = 50, normalized size = 0.68

$$\frac{1}{12}\sqrt{\sqrt{x}+x}\,\left(-3+2\sqrt{x}+8x\right)+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + ArcTanh[Sqrt[Sqrt[x] + x]/S
qrt[x]]/4
```

Maple [A]
time = 0.38, size = 42, normalized size = 0.57

method	result	size
meijerg	$-\frac{\frac{\sqrt{\pi}\,x^{\frac{1}{4}}\left(-40x-10\sqrt{x}+15\right)\sqrt{1+\sqrt{x}}}{60}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{4}}{\sqrt{\pi}}$	41
derivativedivides	$\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42
default	$\frac{2\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{8}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(1/2+x^(1/2
)+(x+x^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x)), x)
```

Fricas [A]
time = 0.62, size = 49, normalized size = 0.66

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}-3\right)\sqrt{x+\sqrt{x}}+\frac{1}{16}\,\log\left(4\,\sqrt{x+\sqrt{x}}\,\left(2\,\sqrt{x}+1\right)+8\,x+8\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x}+x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x), x)
```

Giac [A]
time = 2.39, size = 43, normalized size = 0.58

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-3\right)\sqrt{x+\sqrt{x}}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x
+ sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [B]
time = 3.16, size = 27, normalized size = 0.36

$$\frac{4\,x\,\sqrt{x+\sqrt{x}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\sqrt{x}\right)}{5\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/2))^(1/2),x)
```

```
[Out] (4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2)
 + 1)^(1/2))
```

3.693 $\int \sqrt{-x} \left(\sqrt{-x}+x\right) \, dx$

Optimal. Leaf size=19

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

```
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {14}

$$\frac{2}{5}(-x)^{5/2}-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \sqrt{-x} \left(\sqrt{-x}+x\right) \, dx &= \int \left(-(-x)^{3/2}-x\right) \, dx\\ &= \frac{2}{5}(-x)^{5/2}-\frac{x^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.95

$$\frac{1}{10}\left(-5+4\sqrt{-x}\right)x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]
```

```
[Out] ((-5 + 4*Sqrt[-x])*x^2)/10
```

Maple [A]
time = 0.02, size = 14, normalized size = 0.74

method	result	size
derivativedivides	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^2}{2}$	14
default	$\frac{2(-x)^{\frac{5}{2}}}{5}-\frac{x^2}{2}$	14
trager	$-\frac{(1+x)(-1+x)}{2}+\frac{2x^2\sqrt{-x}}{5}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/5*(-x)^(5/2)-1/2*x^2
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.68

$$\frac{2}{5}\,(-x)^{\frac{5}{2}}-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")
[Out] 2/5*(-x)^(5/2) - 1/2*x^2
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.84

$$\frac{2}{5}\,\sqrt{-x}\,x^2-\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Sympy [A]
time = 0.07, size = 17, normalized size = 0.89

$$\frac{2x^2\sqrt{-x}}{5}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)
```

```
[Out] 2*x**2*sqrt(-x)/5 - x**2/2
```

Giac [A]
time = 2.16, size = 16, normalized size = 0.84

$$\frac{2}{5}\sqrt{-x}\,x^2 - \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2
```

Mupad [B]
time = 0.03, size = 13, normalized size = 0.68

$$\frac{2\,(-x)^{5/2}}{5} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x)^(1/2)*(x + (-x)^(1/2)),x)
```

```
[Out] (2*(-x)^(5/2))/5 - x^2/2
```

3.694 $\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx$

Optimal. Leaf size=54

$$4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)$$

```
[Out] 4*x^(1/4)-2*6^(1/4)*arctan(1/6*x^(1/4)*6^(3/4))-2*6^(1/4)*arctanh(1/6*x^(1/
4)*6^(3/4))+5*ln(6-x)
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1845, 266, 327, 218, 212, 209}

$$-2\sqrt[4]{6}\,\text{ArcTan}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+4\sqrt[4]{x}+5\log(6-x)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^(1/4))/(-6 + x),x]
```

```
[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6
^(1/4)] + 5*Log[6 - x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1845

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[(c*x)^(m + ii)*((Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
)/(c^ii*(a + b*x^n))), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rubi steps

$$
\begin{aligned}
\int \frac{5+\sqrt[4]{x}}{-6+x}\,dx &= 4\text{Subst}\left(\int \frac{x^3(5+x)}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\text{Subst}\left(\int \left(\frac{5x^3}{-6+x^4}+\frac{x^4}{-6+x^4}\right)\,dx, x, \sqrt[4]{x}\right)\\
&= 4\text{Subst}\left(\int \frac{x^4}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)+20\text{Subst}\left(\int \frac{x^3}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)+24\text{Subst}\left(\int \frac{1}{-6+x^4}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\sqrt[4]{x}+5\log(6-x)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{6}-x^2}\,dx, x, \sqrt[4]{x}\right)-\left(2\sqrt{6}\right)\text{Subst}\left(\int \frac{}{\sqrt{6}}\right.\\
&= 4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(6-x)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 52, normalized size = 0.96

$$4\sqrt[4]{x}-2\sqrt[4]{6}\tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)-2\sqrt[4]{6}\tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)+5\log(-6+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^(1/4))/(-6 + x),x]
```

[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6^(1/4)] + 5*Log[-6 + x]

Maple [A]
time = 5.08, size = 50, normalized size = 0.93

method	result
derivativedivides	$4x^{\frac{1}{4}}-6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right)+5\ln\left(-6+x\right)$
default	$4x^{\frac{1}{4}}-6^{\frac{1}{4}}\left(\ln\left(\frac{x^{\frac{1}{4}}+6^{\frac{1}{4}}}{x^{\frac{1}{4}}-6^{\frac{1}{4}}}\right)+2\arctan\left(\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)\right)+5\ln\left(-6+x\right)$
meijerg	$5\ln\left(1-\frac{x}{6}\right)-6^{\frac{1}{4}}(-1)^{\frac{3}{4}}\left(\frac{2\,8^{\frac{1}{4}}3^{\frac{3}{4}}x^{\frac{1}{4}}(-1)^{\frac{1}{4}}}{3}+(-1)^{\frac{1}{4}}\left(\ln\left(1-\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)-\ln\left(1+\frac{x^{\frac{1}{4}}6^{\frac{3}{4}}}{6}\right)-2\,\mathrm{ar}\right.\right.$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+x^(1/4))/(-6+x),x,method=_RETURNVERBOSE)

[Out] 4*x^(1/4)-6^(1/4)*(ln((x^(1/4)+6^(1/4))/(x^(1/4)-6^(1/4)))+2*arctan(1/6*x^(1/4)*6^(3/4)))+5*ln(-6+x)

Maxima [A]
time = 0.51, size = 67, normalized size = 1.24

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+6^{\frac{1}{4}}\log\left(-\frac{6^{\frac{1}{4}}-x^{\frac{1}{4}}}{6^{\frac{1}{4}}+x^{\frac{1}{4}}}\right)+4\,x^{\frac{1}{4}}+5\,\log\left(\sqrt{6}+\sqrt{x}\right)+5\,\log\left(-\sqrt{6}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")

[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) + 6^(1/4)*log(-(6^(1/4) - x^(1/4))/(6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(sqrt(6) + sqrt(x)) + 5*log(-sqrt(6) + sqrt(x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. 2(42) = 84.
time = 0.40, size = 86, normalized size = 1.59

$$-\left(6^{\frac{1}{4}}-5\right)\log\left(2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+\left(6^{\frac{1}{4}}+5\right)\log\left(-2\cdot 6^{\frac{1}{4}}+2\,x^{\frac{1}{4}}\right)+4\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}\sqrt{\sqrt{6}+\sqrt{x}}-\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right)+4\,x^{\frac{1}{4}}+5\,\log\left(4\,\sqrt{6}+4\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="fricas")

[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) + 2*x^(1/4)) + 4*6^(1/4)*arctan(1/6*6^(3/4)*sqrt(sqrt(6) + sqrt(x)) - 1/6*6^(3/4)*x^(1/4)) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))

Sympy [A]
time = 0.64, size = 100, normalized size = 1.85

$$4\sqrt[4]{x} + \sqrt[4]{6}\log\left(\sqrt[4]{x} - \sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x} - \sqrt[4]{6}\right) - \sqrt[4]{6}\log\left(\sqrt[4]{x} + \sqrt[4]{6}\right) + 5\log\left(\sqrt[4]{x} + \sqrt[4]{6}\right) + 5\log\left(\sqrt{x} + \sqrt{6}\right) - 2\cdot\sqrt[4]{6}\operatorname{atan}\left(\frac{6^{\frac{3}{4}}\sqrt[4]{x}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x**(1/4))/(-6+x),x)
```

```
[Out] 4*x**(1/4) + 6**(1/4)*log(x**(1/4) - 6**(1/4)) + 5*log(x**(1/4) - 6**(1/4))
 - 6**(1/4)*log(x**(1/4) + 6**(1/4)) + 5*log(x**(1/4) + 6**(1/4)) + 5*log(s
qrt(x) + sqrt(6)) - 2*6**(1/4)*atan(6**(3/4)*x**(1/4)/6)
```

Giac [A]
time = 2.32, size = 55, normalized size = 1.02

$$-2\cdot 6^{\frac{1}{4}}\arctan\left(\frac{1}{6}\cdot 6^{\frac{3}{4}}x^{\frac{1}{4}}\right) - 6^{\frac{1}{4}}\log\left(6^{\frac{1}{4}} + x^{\frac{1}{4}}\right) + 6^{\frac{1}{4}}\log\left(\left|-6^{\frac{1}{4}} + x^{\frac{1}{4}}\right|\right) + 4\,x^{\frac{1}{4}} + 5\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")
```

```
[Out] -2*6^(1/4)*arctan(1/6*6^(3/4)*x^(1/4)) - 6^(1/4)*log(6^(1/4) + x^(1/4)) + 6
^(1/4)*log(abs(-6^(1/4) + x^(1/4))) + 4*x^(1/4) + 5*log(abs(x - 6))
```

Mupad [B]
time = 0.09, size = 162, normalized size = 3.00

$$\ln\left(11520\,x^{1/4} - \left(6^{1/4}+5\right)\left(2304\,x^{1/4} - 2304\,6^{1/4} + 11520\right) + 57600\right)\left(6^{1/4}+5\right) - \ln\left(\left(6^{1/4}-5\right)\left(2304\,6^{1/4} + 2304\,x^{1/4} + 11520\right) + 11520\,x^{1/4} + 57600\right)\left(6^{1/4}-5\right) - \ln\left(11520\,x^{1/4} + \left(\sqrt{-\sqrt{6}}-5\right)\left(2304\sqrt{-\sqrt{6}} + 2304\,x^{1/4} + 11520\right) + 57600\right)\left(\sqrt{-\sqrt{6}}-5\right) + \ln\left(11520\,x^{1/4} - \left(\sqrt{-\sqrt{6}}+5\right)\left(2304\,x^{1/4} - 2304\sqrt{-\sqrt{6}} + 11520\right) + 57600\right)\left(\sqrt{-\sqrt{6}}+5\right) + 4\,x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4) + 5)/(x - 6),x)
```

```
[Out] log(11520*x^(1/4) - (6^(1/4) + 5)*(2304*x^(1/4) - 2304*6^(1/4) + 11520) + 5
7600)*(6^(1/4) + 5) - log((6^(1/4) - 5)*(2304*6^(1/4) + 2304*x^(1/4) + 1152
0) + 11520*x^(1/4) + 57600)*(6^(1/4) - 5) - log(11520*x^(1/4) + ((-6^(1/2))
^(1/2) - 5)*(2304*(-6^(1/2))^(1/2) + 2304*x^(1/4) + 11520) + 57600)*((-6^(1
/2))^(1/2) - 5) + log(11520*x^(1/4) - ((-6^(1/2))^(1/2) + 5)*(2304*x^(1/4)
- 2304*(-6^(1/2))^(1/2) + 11520) + 57600)*((-6^(1/2))^(1/2) + 5) + 4*x^(1/4
)
```

3.695 $\int \frac{1}{4+\sqrt{4-x}-x} \, dx$

Optimal. Leaf size=14

$$-2\log\left(1+\sqrt{4-x}\right)$$

[Out] -2*ln(1+(4-x)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {31}

$$-2\log\left(\sqrt{4-x}+1\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + Sqrt[4 - x] - x)^(-1),x]

[Out] -2*Log[1 + Sqrt[4 - x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+\sqrt{4-x}-x} \, dx &= -\left(2\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, \sqrt{4-x}\right)\right)\\ &= -2\log\left(1+\sqrt{4-x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$-2\log\left(1+\sqrt{4-x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1),x]

[Out] -2*Log[1 + Sqrt[4 - x]]

Maple [A]
time = 0.05, size = 18, normalized size = 1.29

method	result	size
derivativedivides	$-2\ln\left(1+\sqrt{4-x}\right)$	13
default	$-\ln\left(x-3\right)-2\,\mathrm{arctanh}\left(\sqrt{4-x}\right)$	18
trager	$-\ln\left(2\sqrt{4-x}+5-x\right)$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x+(4-x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(x-3)-2*arctanh((4-x)^(1/2))
```

Maxima [A]
time = 0.28, size = 12, normalized size = 0.86

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

Fricas [A]
time = 0.37, size = 12, normalized size = 0.86

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.
time = 1.83, size = 32, normalized size = 2.29

$$\log\left(2\sqrt{4-x}\right)-\log\left(2\sqrt{4-x}+2\right)-\log\left(x-\sqrt{4-x}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)**(1/2)),x)
```

```
[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(x - sqrt(4 - x) - 4)
```

Giac [A]
time = 1.74, size = 12, normalized size = 0.86

$$-2\,\log\left(\sqrt{-x+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(-x + 4) + 1)
```

Mupad [B]
time = 0.19, size = 12, normalized size = 0.86

$$-2 \ln \left(\sqrt{4-x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4 - x)^(1/2) - x + 4),x)
```

```
[Out] -2*log((4 - x)^(1/2) + 1)
```

3.696 $\int \frac{1}{1+x-\sqrt{2+x}} \, dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(2+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(2+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {646, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+2}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+2}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[2 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x-\sqrt{2+x}} \, dx &= 2\text{Subst}\left(\int \frac{x}{-1-x+x^2} \, dx, x, \sqrt{2+x}\right) \\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x} \, dx, x, \sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \right. \\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{2+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right.
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 56, normalized size = 0.92

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{2+x}\right)-\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{2+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - Sqrt[2 + x])^(-1),x]
```

```
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]] - (-5 + Sqrt[5])*Log[-1 + S
qrt[5] + 2*Sqrt[2 + x]])/5
```

Maple [A]
time = 0.14, size = 91, normalized size = 1.49

method	result
derivativedivides	$\ln\left(1+x-\sqrt{x+2}\right)-\frac{2\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}-1\right)\sqrt{5}}{5}\right)}{5}$
default	$-\frac{\operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln(x^2+x-1)}{2}-\frac{\ln\left(1+x+\sqrt{x+2}\right)}{2}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(2\sqrt{x+2}+1\right)\sqrt{5}}{5}\right.}{5}$
trager	$-\ln\left(-1-x+\sqrt{x+2}\right)\operatorname{RootOf}\left(5\_Z^2-10\_Z+4\right)+\ln\left(150\operatorname{RootOf}\left(5\_Z^2-10\_Z\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*arctanh(1/5*(2*x+1)*5^(1/2))*5^(1/2)+1/2*ln(x^2+x-1)-1/2*ln(1+x+(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)+1)*5^(1/2))+1/2*ln(1+x-(x+2)^
(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))
```

Maxima [A]
time = 0.48, size = 46, normalized size = 0.75

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+2}+1}{\sqrt{5}+2\sqrt{x+2}-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1
)) + log(x - sqrt(x + 2) + 1)
```

Fricas [A]
time = 0.35, size = 63, normalized size = 1.03

$$\frac{1}{5}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}\,(x+3)-\left(\sqrt{5}\,(2\,x+1)-5\right)\sqrt{x+2}+7\,x+3}{x^2+x-1}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x +
 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)
```

Sympy [A]
time = 1.00, size = 94, normalized size = 1.54

$$4\left(\begin{cases}-\frac{\sqrt{5}\,\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2>\frac{5}{4}\\-\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+2}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x +
2) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (
sqrt(x + 2) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 2) + 1)
```

Giac [A]
time = 2.26, size = 50, normalized size = 0.82

$$\frac{1}{5}\,\sqrt{5}\,\log\left(\frac{\left|-\sqrt{5}+2\,\sqrt{x+2}-1\right|}{\left|\sqrt{5}+2\,\sqrt{x+2}-1\right|}\right)+\log\left(\left|x-\sqrt{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x +
2) - 1)) + log(abs(x - sqrt(x + 2) + 1))
```

Mupad [B]
time = 3.24, size = 71, normalized size = 1.16

$$\ln\left(2\,\sqrt{x+2}-\left(\frac{\sqrt{5}}{5}+1\right)\,\left(2\,\sqrt{x+2}-1\right)\right)\,\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\,\sqrt{x+2}+\left(\frac{\sqrt{5}}{5}-1\right)\,\left(2\,\sqrt{x+2}-1\right)\right)\,\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2) + 1),x)
```

```
[Out] log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

3.697 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}}+\log\left(4+x+\sqrt{1+x}\right)$$

```
[Out] ln(4+x+(1+x)^(1/2))-2/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {648, 632, 210, 642}

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\text{ArcTan}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4+x+\sqrt{1+x}}\,dx &= 2\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\ &= \log\left(4+x+\sqrt{1+x}\right) + 2\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 37, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}} + \log\left(4+x+\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.
time = 0.15, size = 93, normalized size = 2.51

method	result
derivativedivides	$\ln\left(4+x+\sqrt{1+x}\right) - \frac{2\arctan\left(\frac{\left(1+2\sqrt{1+x}\right)\sqrt{11}}{11}\right)\sqrt{11}}{11}$
default	$\frac{\ln\left(4+x+\sqrt{1+x}\right)}{2} - \frac{\arctan\left(\frac{\left(1+2\sqrt{1+x}\right)\sqrt{11}}{11}\right)\sqrt{11}}{11} - \frac{\ln\left(x+4-\sqrt{1+x}\right)}{2} - \frac{\sqrt{11}\,\arctan\left(\right.}{}$

trager	$\mathrm{RootOf}\left(11\_Z^2-22\_Z+12\right)\ln\left(4+x+\sqrt{1+x}\right)-\ln\left(-847\,\mathrm{RootOf}\left(11\_Z^2-22\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(4+x+(1+x)^(1/2))-1/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11^(1/
2)-1/2*ln(x+4-(1+x)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(1+x)^(1/2)-1)*11^(
1/2))+1/11*11^(1/2)*arctan(1/11*(2*x+7)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

Maxima [A]
time = 0.49, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.86

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

Sympy [A]
time = 0.98, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

Giac [A]
time = 1.89, size = 30, normalized size = 0.81

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Mupad [B]
time = 0.07, size = 32, normalized size = 0.86

$$\ln\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\,\mathrm{atan}\left(\frac{\sqrt{11}}{11}+\frac{2\sqrt{11}\,\sqrt{x+1}}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2) + 4),x)
```

```
[Out] log(x + (x + 1)^(1/2) + 4) - (2*11^(1/2)*atan(11^(1/2)/11 + (2*11^(1/2)*(x
+ 1)^(1/2))/11))/11
```

3.698 $\int \frac{1}{x-\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)-2*(1+x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)-2*(1+x)^(1/2))
*(5+5^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {646, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{x+1}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{x+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] - 2*Sqrt[1 + x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x}}\,dx &= 2\text{Subst}\left(\int \frac{x}{-1-x+x^2}\,dx,x,\sqrt{1+x}\right)\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int -\right.\\ &= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.92

$$\frac{1}{5}\left(\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+x}\right)-\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2\sqrt{1+x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x])^(-1),x]
```

```
[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]] - (-5 + Sqrt[5])*Log[-1 + S
qrt[5] + 2*Sqrt[1 + x]])/5
```

Maple [A]
time = 0.14, size = 91, normalized size = 1.49

method	result
derivativedivides	$\ln\left(x-\sqrt{1+x}\right)-\frac{2\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(2\sqrt{1+x}-1\right)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln\left(x^2-x-1\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln\left(x+\sqrt{1+x}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(1+2\sqrt{1+x}\right)\sqrt{5}}{5}\right)}{5}$
trager	$\mathrm{RootOf}\left(5\_Z^2-10\_Z+4\right)\ln\left(x-\sqrt{1+x}\right)-\ln\left(5\,\mathrm{RootOf}\left(5\_Z^2-10\_Z+4\right)^2x-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/2*ln(x+(1+x)^(1/
2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(1+x)^(1/2))*5^(1/2))+1/2*ln(x-(1+x)^(1/2)
)-1/5*5^(1/2)*arctanh(1/5*(2*(1+x)^(1/2)-1)*5^(1/2))
```

Maxima [A]
time = 0.51, size = 45, normalized size = 0.74

$$\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{x+1}+1}{\sqrt{5}+2\sqrt{x+1}-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1
)) + log(x - sqrt(x + 1))
```

Fricas [A]
time = 0.37, size = 64, normalized size = 1.05

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}+3x-2}{x^2-x-1}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x +
 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))
```

Sympy [A]
time = 0.96, size = 92, normalized size = 1.51

$$4\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{x+1}-\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)**(1/2)),x)
```

```
[Out] 4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (sqrt(x +
1) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 1) - 1/2)/5)/10, (
sqrt(x + 1) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 1))
```

Giac [A]
time = 1.92, size = 49, normalized size = 0.80

$$\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{x+1}-1\right|}{\left|\sqrt{5}+2\sqrt{x+1}-1\right|}\right)+\log\left(\left|x-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x +
1) - 1)) + log(abs(x - sqrt(x + 1)))
```

Mupad [B]
time = 0.12, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+1}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{x+1}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{x+1}-1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 1)^(1/2)),x)
```

```
[Out] log(2*(x + 1)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5 + 1
) - log(2*(x + 1)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 1)^(1/2) - 1))*(5^(1/2)/5
 - 1)
```

3.699 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

```
[Out] 4/3*ln(2-(2+x)^(1/2))+2/3*ln(1+(2+x)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {646, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{2+x}}\,dx &= 2\text{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx, x, \sqrt{2+x}\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{2+x}\right)+\frac{4}{3}\text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \sqrt{2+x}\right)\\
&= \frac{4}{3}\log\left(2-\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.94

$$\frac{4}{3}\log\left(-2+\sqrt{2+x}\right)+\frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]
[Out] (4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(23) = 46.
time = 0.08, size = 54, normalized size = 1.74

method	result
derivativedivides	$\frac{2\ln\left(1+\sqrt{x+2}\right)}{3}+\frac{4\ln\left(\sqrt{x+2}-2\right)}{3}$
trager	$\frac{\ln\left(6\sqrt{x+2}\,x^{2}-x^{3}+16\sqrt{x+2}\,x-15x^{2}+8\sqrt{x+2}-24x-12\right)}{3}$
default	$\frac{\ln(1+x)}{3}+\frac{2\ln(x-2)}{3}+\frac{\ln\left(1+\sqrt{x+2}\right)}{3}-\frac{2\ln\left(\sqrt{x+2}+2\right)}{3}+\frac{2\ln\left(\sqrt{x+2}-2\right)}{3}-\frac{\ln\left(-1+\sqrt{x+\right.}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x+2)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*ln(1+x)+2/3*ln(x-2)+1/3*ln(1+(x+2)^(1/2))-2/3*ln((x+2)^(1/2)+2)+2/3*ln(
(x+2)^(1/2)-2)-1/3*ln(-1+(x+2)^(1/2))
```

Maxima [A]
time = 0.26, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Fricas [A]
time = 0.34, size = 21, normalized size = 0.68

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [**A**]
time = 1.07, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

Giac [**A**]
time = 4.56, size = 22, normalized size = 0.71

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

Mupad [**B**]
time = 3.08, size = 25, normalized size = 0.81

$$\frac{2\ln\left(\frac{2\sqrt{x+2}}{3}+\frac{2}{3}\right)}{3}+\frac{4\ln\left(\frac{4}{3}-\frac{2\sqrt{x+2}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x + 2)^(1/2)),x)
[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3
```

3.700 $\int \frac{1}{-\sqrt{1-x}+x} \, dx$

Optimal. Leaf size=65

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)$$

```
[Out] 1/5*ln(1-5^(1/2)+2*(1-x)^(1/2))*(5-5^(1/2))+1/5*ln(1+5^(1/2)+2*(1-x)^(1/2))
*(5+5^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {646, 31}

$$\frac{1}{5}\left(5-\sqrt{5}\right)\log\left(2\sqrt{1-x}-\sqrt{5}+1\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(2\sqrt{1-x}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 +
 Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\sqrt{1-x}+x} \, dx &= 2\text{Subst}\left(\int \frac{x}{-1+x+x^2} \, dx, x, \sqrt{1-x}\right)\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x} \, dx, x, \sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\ldots}\right.\\
&= \frac{1}{5}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2\sqrt{1-x}\right)+\frac{1}{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.92

$$\frac{1}{5}\left(-\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2\sqrt{1-x}\right)\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2\sqrt{1-x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 - x] + x)^(-1),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1
+ Sqrt[5] + 2*Sqrt[1 - x]])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(49) = 98$.
time = 0.14, size = 101, normalized size = 1.55

method	result
derivativedivides	$\ln\left(-x+\sqrt{1-x}\right)+\frac{2\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(2\sqrt{1-x}+1\right)\sqrt{5}}{5}\right)}{5}$
default	$\frac{\ln(x^2+x-1)}{2}+\frac{\mathrm{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln\left(-x+\sqrt{1-x}\right)}{2}+\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\left(2\sqrt{1-x}+1\right)\sqrt{5}}{5}\right.}{5}$
trager	$\mathrm{RootOf}\left(5\_Z^2-10\_Z+4\right)\ln\left(x-\sqrt{1-x}\right)-\ln\left(5\,\mathrm{RootOf}\left(5\_Z^2-10\_Z+4\right)^2 x\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1-x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*arctanh(1/5*(2*x+1)*5^(1/2))*5^(1/2)+1/2*ln(-x+(1-x)^(1
/2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)+1)*5^(1/2))-1/2*ln(-x-(1-x)^(1/
2))+1/5*5^(1/2)*arctanh(1/5*(2*(1-x)^(1/2)-1)*5^(1/2))
```

Maxima [A]
time = 0.48, size = 51, normalized size = 0.78

$$-\frac{1}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-x+1}-1}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1)
+ 1)) + log(-x + sqrt(-x + 1))
```

Fricas [A]
time = 0.38, size = 65, normalized size = 1.00

$$\frac{1}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(x-2)-\left(\sqrt{5}(2x+1)+5\right)\sqrt{-x+1}-3x-2}{x^2+x-1}\right)+\log\left(-x+\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x
+ 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))
```

Sympy [A]
time = 0.93, size = 92, normalized size = 1.42

$$-4\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{1-x}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{1-x}+\frac{1}{2}\right)^2<\frac{5}{4}\end{cases}\right)+\log\left(x-\sqrt{1-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)**(1/2)),x)
```

```
[Out] -4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10, (sqrt(1 -
 x) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(1 - x) + 1/2)/5)/10,
(sqrt(1 - x) + 1/2)**2 < 5/4)) + log(x - sqrt(1 - x))
```

Giac [A]
time = 2.44, size = 54, normalized size = 0.83

$$-\frac{1}{5}\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\sqrt{-x+1}+1\right|}{\sqrt{5}+2\sqrt{-x+1}+1}\right)+\log\left(\left|-x+\sqrt{-x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1-x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x +
1) + 1)) + log(abs(-x + sqrt(-x + 1)))
```

Mupad [B]
time = 3.16, size = 79, normalized size = 1.22

$$\ln\left(2\sqrt{1-x}-\left(\frac{\sqrt{5}}{5}+1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}+1\right)-\ln\left(2\sqrt{1-x}+\left(\frac{\sqrt{5}}{5}-1\right)\left(2\sqrt{1-x}+1\right)\right)\left(\frac{\sqrt{5}}{5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x)^(1/2)),x)
```

```
[Out] log(2*(1 - x)^(1/2) - (5^(1/2)/5 + 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5 + 1
) - log(2*(1 - x)^(1/2) + (5^(1/2)/5 - 1)*(2*(1 - x)^(1/2) + 1))*(5^(1/2)/5
 - 1)
```

3.701 $\int \sqrt{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{4}(1+2\sqrt{x})\sqrt{1+\sqrt{x}+x}+\frac{2}{3}(1+\sqrt{x}+x)^{3/2}-\frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*x^(1/2))*3^(1/2))+2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^
(1/2))*(1+x+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1355, 654, 626, 633, 221}

$$\frac{2}{3}(x+\sqrt{x}+1)^{3/2}-\frac{1}{4}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}+1}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x]) + (2*(1 + Sqrt[x] + x)^(3/2))/
3 - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\sqrt{x}+x}\,dx &= 2\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx,x,\sqrt{x}\right)\\ &= \frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\right.\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{1}{8}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\right.\\ &= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1+\sqrt{x}+x}+\frac{2}{3}\left(1+\sqrt{x}+x\right)^{3/2}-\frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.92

$$\frac{1}{12}\sqrt{1+\sqrt{x}+x}\left(5+2\sqrt{x}+8x\right)+\frac{3}{8}\log\left(-1-2\sqrt{x}+2\sqrt{1+\sqrt{x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 + (3*Log[-1 - 2*Sqrt[x] +
2*Sqrt[1 + Sqrt[x] + x]])/8
```

Maple [A]
time = 0.02, size = 42, normalized size = 0.68

method	result	size

derivativedivides	$\frac{2\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42
default	$\frac{2\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{x}\right)\sqrt{1+x+\sqrt{x}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(1+x+x^(1/2))^(1/2)-3/8*arcsinh(2
/3*3^(1/2)*(x^(1/2)+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x) + 1), x)
```

Fricas [A]
time = 0.59, size = 51, normalized size = 0.82

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x}+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{16}\log\left(4\,\sqrt{x+\sqrt{x}+1}\,\left(2\,\sqrt{x}+1\right)-8\,x-8\,\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt
(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{x}+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + x + 1), x)
```

Giac [**A**]
time = 1.74, size = 45, normalized size = 0.73

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)+5\right)\sqrt{x+\sqrt{x}+1}+\frac{3}{8}\log\left(2\sqrt{x+\sqrt{x}+1}-2\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+x^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) + 5)*sqrt(x + sqrt(x) + 1) + 3/8*log(2*sqrt
(x + sqrt(x) + 1) - 2*sqrt(x) - 1)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/2) + 1)^(1/2),x)
[Out] int((x + x^(1/2) + 1)^(1/2), x)
```

3.702 $\int \sqrt{1+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=75

$$\frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2}-\frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right)+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}}\right)$$

```
[Out] 1/4*arctanh((1+x)^(1/2)/(1+x+(1+x)^(1/2))^(1/2))+2/3*(1+x+(1+x)^(1/2))^(3/2
)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1976, 654, 626, 634, 212}

$$\frac{2}{3}\left(x+\sqrt{x+1}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x+1}+1\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{x+\sqrt{x+1}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]
```

```
[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[
1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+x+\sqrt{1+x}}\,dx &= 2\text{Subst}\left(\int x\sqrt{x(1+x)}\,dx, x, \sqrt{1+x}\right)\\
&= 2\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{8}\text{Subst}\Bigg(\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\text{Subst}\Bigg(\\
&= \frac{2}{3}\left(1+x+\sqrt{1+x}\right)^{3/2} - \frac{1}{4}\sqrt{1+x+\sqrt{1+x}}\left(1+2\sqrt{1+x}\right) + \frac{1}{4}\tanh^{-1}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 0.79

$$\frac{1}{12}\left(\sqrt{1+x+\sqrt{1+x}}\left(5+8x+2\sqrt{1+x}\right)+3\tanh^{-1}\left(\frac{\sqrt{1+x+\sqrt{1+x}}}{\sqrt{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]],x]
[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x]) + 3*ArcTanh[Sqrt[1 + x
 + Sqrt[1 + x]]/Sqrt[1 + x]])/12
```

Maple [A]
time = 0.02, size = 55, normalized size = 0.73

method	result
derivativedivides	$\frac{2\left(1+x+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{1+x}\right)\sqrt{1+x+\sqrt{1+x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{1+x+\sqrt{1}}\right.}{8}$
default	$\frac{2\left(1+x+\sqrt{1+x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{1+x}\right)\sqrt{1+x+\sqrt{1+x}}}{4}+\frac{\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{1+x+\sqrt{1}}\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x+(1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x+(1+x)^(1/2))^(3/2)-1/4*(1+2*(1+x)^(1/2))*(1+x+(1+x)^(1/2))^(1/2)+1
/8*ln(1/2+(1+x)^(1/2)+(1+x+(1+x)^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)
```

Fricas [A]
time = 0.56, size = 61, normalized size = 0.81

$$\frac{1}{12}\left(8x+2\sqrt{x+1}+5\right)\sqrt{x+\sqrt{x+1}+1}+\frac{1}{16}\log\left(-4\sqrt{x+\sqrt{x+1}+1}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt
(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)
```

Giac [A]
time = 2.10, size = 55, normalized size = 0.73

$$\frac{1}{12}\left(2\sqrt{x+1}\left(4\sqrt{x+1}+1\right)-3\right)\sqrt{x+\sqrt{x+1}+1}-\frac{1}{8}\log\left(-2\sqrt{x+\sqrt{x+1}+1}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1/
8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x + 1)^(1/2) + 1)^(1/2),x)
[Out] int((x + (x + 1)^(1/2) + 1)^(1/2), x)
```

3.703 $\int \sqrt{\sqrt{-1+x}+x}\, dx$

Optimal. Leaf size=68

$$-\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x}+\frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2}-\frac{3}{8}\sinh^{-1}\left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -3/8*arcsinh(1/3*(1+2*(-1+x)^(1/2))*3^(1/2))+2/3*(x+(-1+x)^(1/2))^(3/2)-1/4
*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {654, 626, 633, 221}

$$\frac{2}{3}\left(x+\sqrt{x-1}\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x-1}+1\right)\sqrt{x+\sqrt{x-1}}-\frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[-1 + x] + x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x]) + (2*(Sqrt[-1 + x] + x)^
(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{\sqrt{-1+x}+x}\,dx = 2\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\text{Subst}\Bigg(\int -$$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{1}{8}\sqrt{3}\,\text{Subst}\Bigg($$
$$= -\frac{1}{4}\left(1+2\sqrt{-1+x}\right)\sqrt{\sqrt{-1+x}+x} + \frac{2}{3}\left(\sqrt{-1+x}+x\right)^{3/2} - \frac{3}{8}\sinh^{-1}\Bigg(\frac{1+}{}$$

Mathematica [A]
time = 0.08, size = 65, normalized size = 0.96

$$\frac{1}{12}\left(5+2\sqrt{-1+x}+8(-1+x)\right)\sqrt{\sqrt{-1+x}+x} + \frac{3}{8}\log\left(-1-2\sqrt{-1+x}+2\sqrt{\sqrt{-1+x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[-1 + x] + x],x]
[Out] ((5 + 2*Sqrt[-1 + x] + 8*(-1 + x))*Sqrt[Sqrt[-1 + x] + x])/12 + (3*Log[-1 -
 2*Sqrt[-1 + x] + 2*Sqrt[Sqrt[-1 + x] + x]])/8
```

Maple [A]
time = 0.02, size = 48, normalized size = 0.71

method	result
derivativedivides	$\frac{2\left(x+\sqrt{-1+x}\right)^{\frac{3}{2}}}{3} - \frac{\left(1+2\sqrt{-1+x}\right)\sqrt{x+\sqrt{-1+x}}}{4} - \frac{3\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{-1+x}+\frac{1}{2}\right)}{3}\right)}{8}$

default	$\frac{2\left(x+\sqrt{-1+x}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{-1+x}\right)\sqrt{x+\sqrt{-1+x}}}{4}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{-1+x}+\frac{1}{2}\right)}{3}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(x+(-1+x)^(1/2))^(3/2)-1/4*(1+2*(-1+x)^(1/2))*(x+(-1+x)^(1/2))^(1/2)-3/
8*arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x + sqrt(x - 1)), x)
```

Fricas [A]
time = 0.59, size = 59, normalized size = 0.87

$$\frac{1}{12}\left(8\,x+2\,\sqrt{x-1}-3\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{16}\,\log\left(-4\,\sqrt{x+\sqrt{x-1}}\,\left(2\,\sqrt{x-1}+1\right)+8\,x+8\,\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x +
 sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)**(1/2))**(1/2),x)
[Out] Integral(sqrt(x + sqrt(x - 1)), x)
```

Giac [A]
time = 1.51, size = 53, normalized size = 0.78

$$\frac{1}{12}\left(2\,\sqrt{x-1}\,\left(4\,\sqrt{x-1}+1\right)+5\right)\sqrt{x+\sqrt{x-1}}+\frac{3}{8}\,\log\left(2\,\sqrt{x+\sqrt{x-1}}-2\,\sqrt{x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x - 1)*(4*sqrt(x - 1) + 1) + 5)*sqrt(x + sqrt(x - 1)) + 3/8*lo
g(2*sqrt(x + sqrt(x - 1)) - 2*sqrt(x - 1) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (x - 1)^(1/2))^(1/2),x)
```

```
[Out] int((x + (x - 1)^(1/2))^(1/2), x)
```

3.704 $\int \sqrt{2x+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=80

$$\frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2}-\frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right)-\frac{3}{16}\sinh^{-1}\left(\frac{1+2\sqrt{-1+2x}}{\sqrt{3}}\right)$$

```
[Out] -3/16*arcsinh(1/3*(1+2*(-1+2*x)^(1/2))*3^(1/2))+1/3*(2*x+(-1+2*x)^(1/2))^(3
/2)-1/8*(1+2*(-1+2*x)^(1/2))*(2*x+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {654, 626, 633, 221}

$$\frac{1}{3}\left(2x+\sqrt{2x-1}\right)^{3/2}-\frac{1}{8}\left(2\sqrt{2x-1}+1\right)\sqrt{2x+\sqrt{2x-1}}-\frac{3}{16}\sinh^{-1}\left(\frac{2\sqrt{2x-1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
```

```
[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1
 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{2x+\sqrt{-1+2x}}\,dx = \text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{2}\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\text{Subs}$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{1}{16}\sqrt{3}\,\text{S}$$
$$= \frac{1}{3}\left(2x+\sqrt{-1+2x}\right)^{3/2} - \frac{1}{8}\sqrt{2x+\sqrt{-1+2x}}\left(1+2\sqrt{-1+2x}\right) - \frac{3}{16}\sinh$$

Mathematica [A]
time = 0.08, size = 75, normalized size = 0.94

$$\frac{1}{48}\left(2\sqrt{2x+\sqrt{-1+2x}}\left(-3+16x+2\sqrt{-1+2x}\right)+9\log\left(-1-2\sqrt{-1+2x}+2\sqrt{2x+\sqrt{-1+2x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]],x]
[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) + 9*Log[-1 - 2
*Sqrt[-1 + 2*x] + 2*Sqrt[2*x + Sqrt[-1 + 2*x]]])/48
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.75

method	result
derivativedivides	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3} - \frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8} - \frac{3\,\text{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$

default	$\frac{\left(2x+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-\frac{\left(1+2\sqrt{2x-1}\right)\sqrt{2x+\sqrt{2x-1}}}{8}-\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(2*x+(2*x-1)^(1/2))^(3/2)-1/8*(1+2*(2*x-1)^(1/2))*(2*x+(2*x-1)^(1/2))^(
1/2)-3/16*arcsinh(2/3*3^(1/2)*((2*x-1)^(1/2)+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x + sqrt(2*x - 1)), x)
```

Fricas [A]
time = 0.59, size = 73, normalized size = 0.91

$$\frac{1}{24}\left(16\,x+2\,\sqrt{2\,x-1}-3\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{32}\,\log\left(-4\,\sqrt{2\,x+\sqrt{2\,x-1}}\,\left(2\,\sqrt{2\,x-1}+1\right)+16\,x+8\,\sqrt{2\,x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*s
qrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2x+\sqrt{2x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)
[Out] Integral(sqrt(2*x + sqrt(2*x - 1)), x)
```

Giac [A]
time = 5.28, size = 67, normalized size = 0.84

$$\frac{1}{24}\left(2\,\sqrt{2\,x-1}\,\left(4\,\sqrt{2\,x-1}+1\right)+5\right)\sqrt{2\,x+\sqrt{2\,x-1}}+\frac{3}{16}\,\log\left(2\,\sqrt{2\,x+\sqrt{2\,x-1}}-2\,\sqrt{2\,x-1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*sqrt(2*x - 1)*(4*sqrt(2*x - 1) + 1) + 5)*sqrt(2*x + sqrt(2*x - 1))
+ 3/16*log(2*sqrt(2*x + sqrt(2*x - 1)) - 2*sqrt(2*x - 1) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2\,x+\sqrt{2\,x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + (2*x - 1)^(1/2))^(1/2),x)
```

```
[Out] int((2*x + (2*x - 1)^(1/2))^(1/2), x)
```

3.705 $\int \sqrt{3x+\sqrt{-7+8x}}\,dx$

Optimal. Leaf size=109

$$-\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}}+\frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{4+\cdots}{\cdots}\right)}{36\cdots}$$

```
[Out] -47/216*arcsinh(1/47*(4+3*(-7+8*x)^(1/2))*47^(1/2))*6^(1/2)+1/144*(24*x+8*(
-7+8*x)^(1/2))^(3/2)*2^(1/2)-1/36*(4+3*(-7+8*x)^(1/2))*(6*x+2*(-7+8*x)^(1/2
))^(1/2)*2^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {654, 626, 633, 221}

$$\frac{\left(-3(7-8x)+8\sqrt{8x-7}+21\right)^{3/2}}{72\sqrt{2}}-\frac{\left(3\sqrt{8x-7}+4\right)\sqrt{-3(7-8x)+8\sqrt{8x-7}+21}}{36\sqrt{2}}-\frac{47\sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
```

```
[Out] -1/36*((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/Sq
rt[2] + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcS
inh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \sqrt{3x+\sqrt{-7+8x}}\,dx = \frac{1}{4}\text{Subst}\left(\int x\sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-7+8x}\right)$$

$$= \frac{\left(21-3(7-8x)+8\sqrt{-7+8x}\right)^{3/2}}{72\sqrt{2}} - \frac{1}{3}\text{Subst}\left(\int \sqrt{\frac{21}{8}+x+\frac{3x^2}{8}}\,dx, x, \sqrt{-\right.$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{(21-3(7-8x)+8}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{(21-3(7-8x)+8}{72\sqrt{2}}$$

$$= -\frac{\left(4+3\sqrt{-7+8x}\right)\sqrt{21-3(7-8x)+8\sqrt{-7+8x}}}{36\sqrt{2}} + \frac{(21-3(7-8x)+8}{72\sqrt{2}}$$

Mathematica [**A**]
time = 0.20, size = 83, normalized size = 0.76

$$\frac{1}{18}\sqrt{3x+\sqrt{-7+8x}}\left(-4+12x+\sqrt{-7+8x}\right)+\frac{47\log\left(-4-3\sqrt{-7+8x}+2\sqrt{6}\sqrt{3x+\sqrt{-7+8x}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]],x]
[Out] (Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]))/18 + (47*Log[-4 -
 3*Sqrt[-7 + 8*x] + 2*Sqrt[6]*Sqrt[3*x + Sqrt[-7 + 8*x]]])/(36*Sqrt[6])
```

Maple [**A**]
time = 0.02, size = 67, normalized size = 0.61

method	result

derivativedivides	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\ \mathrm{arcsinh}\left(\frac{3\sqrt{4}}{}\right.}{}$
default	$\frac{\left(48x+16\sqrt{-7+8x}\right)^{\frac{3}{2}}}{288}-\frac{\left(12\sqrt{-7+8x}+16\right)\sqrt{48x+16\sqrt{-7+8x}}}{288}-\frac{47\sqrt{6}\ \mathrm{arcsinh}\left(\frac{3\sqrt{4}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*
(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+
4/3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)
```

Fricas [A]
time = 0.93, size = 101, normalized size = 0.93

$$\frac{1}{18}\left(12x+\sqrt{8x-7}-4\right)\sqrt{3x+\sqrt{8x-7}}+\frac{47}{864}\sqrt{6}\log\left(-41472x^2-192\left(144x-47\right)\sqrt{8x-7}+8\left(3\sqrt{6}\left(144x+17\right)\sqrt{8x-7}+4\sqrt{6}\left(432x-299\right)\right)\sqrt{3x+\sqrt{8x-7}}-9792x+30047\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*
log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)
*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*
x + 30047)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{3x+\sqrt{8x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)
```

Giac [A]
time = 3.31, size = 88, normalized size = 0.81

$$\frac{1}{216}\sqrt{2}\left(3\sqrt{2}\left(\sqrt{8x-7}\left(3\sqrt{8x-7}+2\right)+13\right)\sqrt{3x+\sqrt{8x-7}}+47\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8x-7}-2\sqrt{2}\sqrt{3x+\sqrt{8x-7}}\right)-4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3*
x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqr
t(2)*sqrt(3*x + sqrt(8*x - 7))) - 4))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{3\,x+\sqrt{8\,x-7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + (8*x - 7)^(1/2))^(1/2),x)
```

```
[Out] int((3*x + (8*x - 7)^(1/2))^(1/2), x)
```

3.706 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=47

$$2\sqrt{x+\sqrt{1+x}}-\tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

```
[Out] -arctanh(1/2*(1+2*(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+2*(x+(1+x)^(1/2))^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {654, 635, 212}

$$2\sqrt{x+\sqrt{x+1}}-\tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]
```

```
[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x+\sqrt{1+x}}}\,dx &= 2\text{Subst}\left(\int \frac{x}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\ &= 2\sqrt{x+\sqrt{1+x}} - \text{Subst}\left(\int \frac{1}{\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\ &= 2\sqrt{x+\sqrt{1+x}} - 2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\\ &= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1}\left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 0.91

$$2\sqrt{x+\sqrt{1+x}} + \log\left(-1-2\sqrt{1+x}+2\sqrt{x+\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + Sqrt[1 + x]],x]
[Out] 2*Sqrt[x + Sqrt[1 + x]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]]
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.68

method	result	size
derivativedivides	$2\sqrt{x+\sqrt{1+x}} - \ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)$	32
default	$2\sqrt{x+\sqrt{1+x}} - \ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(x+(1+x)^(1/2))^(1/2)-ln(1/2+(1+x)^(1/2)+(x+(1+x)^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)
```

Fricas [A]
time = 0.53, size = 47, normalized size = 1.00

$$2\sqrt{x+\sqrt{x+1}}+\frac{1}{2}\log\left(4\sqrt{x+\sqrt{x+1}}\left(2\sqrt{x+1}+1\right)-8x-8\sqrt{x+1}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) +
1) - 8*x - 8*sqrt(x + 1) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)
```

Giac [A]
time = 1.76, size = 33, normalized size = 0.70

$$2\sqrt{x+\sqrt{x+1}}+\log\left(-2\sqrt{x+\sqrt{x+1}}+2\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x + 1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x + (x + 1)^(1/2))^(1/2), x)
```

3.707 $\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=67

$$x-2\sqrt{3}\,\sqrt{-3+2x}+4\sqrt{6}\,\tan^{-1}\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{3}\,\sqrt{-3+2x}\right)$$

```
[Out] x+3*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))+4*arctan(1/12*(3+(-9+6*x)^(1/2))*6^(1/2)
)*6^(1/2)-2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1642, 648, 632, 210, 642}

$$4\sqrt{6}\,\text{ArcTan}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)+x-2\sqrt{3}\,\sqrt{2x-3}+3\log\left(x+\sqrt{3}\,\sqrt{2x-3}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqr
t[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{4+x+\sqrt{-9+6x}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(15+x^2)}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{18(11+x)}{33+6x+x^2}\right)dx, x, \sqrt{-9+6x}\right)\\ &= x-2\sqrt{3}\sqrt{-3+2x}+6\text{Subst}\left(\int \frac{11+x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)\\ &= x-2\sqrt{3}\sqrt{-3+2x}+3\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx, x, \sqrt{-9+6x}\right)+48\text{Subst}\\ &= x-2\sqrt{3}\sqrt{-3+2x}+3\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)-96\text{Subst}\left(\int \frac{1}{-96-}\right.\\ &= x-2\sqrt{3}\sqrt{-3+2x}+4\sqrt{6}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+3\log\left(4+x+\sqrt{}\right.\end{aligned}$$

Mathematica [A]
time = 0.08, size = 63, normalized size = 0.94

$$-\frac{3}{2}+x-2\sqrt{-9+6x}+4\sqrt{6}\tan^{-1}\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+3\log\left(4+x+\sqrt{-9+6x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -3/2 + x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])/(
2*Sqrt[2])] + 3*Log[4 + x + Sqrt[-9 + 6*x]]
```

Maple [A]
time = 0.19, size = 52, normalized size = 0.78

method	result

derivativedivides	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\,\arctan\left(\frac{\left(2\sqrt{-9+6x}\right.}{24}\right.$
default	$-\frac{3}{2}+x-2\sqrt{-9+6x}+3\ln\left(24+6x+6\sqrt{-9+6x}\right)+4\sqrt{6}\,\arctan\left(\frac{\left(2\sqrt{-9+6x}\right.}{24}\right.$
trager	$x-2\sqrt{-9+6x}+\mathrm{RootOf}\left(\_Z^2-6\_Z+33\right)\ln\left(4+x+\sqrt{-9+6x}\right)-\ln\left(161\,\mathrm{Roo}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2+x-2*(-9+6*x)^(1/2)+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24
*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 0.73

$$4\sqrt{6}\,\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)+x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) +
3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2
```

Fricas [A]
time = 0.34, size = 48, normalized size = 0.72

$$4\sqrt{6}\,\arctan\left(\frac{1}{12}\sqrt{6}\sqrt{6x-9}+\frac{1}{4}\sqrt{6}\right)+x-2\sqrt{6x-9}+3\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x
 - 9) + 3*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A]
time = 18.19, size = 58, normalized size = 0.87

$$x-2\sqrt{6x-9}+3\log\left(6x+6\sqrt{6x-9}+24\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sq
rt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2
```

Giac [A]
time = 2.42, size = 60, normalized size = 0.90

$$4\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right)-2\sqrt{3}\sqrt{2x-3}+x+3\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right)-\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 4*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) - 2*sqrt(3)
*sqrt(2*x - 3) + x + 3*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) - 3/2
```

Mupad [B]
time = 3.09, size = 102, normalized size = 1.52

$$x+3\ln\left(\left(6\sqrt{6x-9}+\left(-3+\sqrt{6}\,2\mathrm{i}\right)\,\left(2\sqrt{6x-9}+6\right)+66\right)\,\left(6\sqrt{6x-9}-\left(3+\sqrt{6}\,2\mathrm{i}\right)\,\left(2\sqrt{6x-9}+6\right)+66\right)\right)+4\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\sqrt{6x-9}}{12}+\frac{\sqrt{6}}{4}\right)-2\sqrt{6x-9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) + 6
6)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) + 4
*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1/2)
```

3.708 $\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx$

Optimal. Leaf size=71

$$-x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{-9+6x}}{2\sqrt{6}}\right)+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)$$

```
[Out] -x+10*ln(4+x+(-3+2*x)^(1/2)*3^(1/2))-21/2*arctan(1/12*(3+(-9+6*x)^(1/2))*6^
(1/2))*6^(1/2)+2*(-3+2*x)^(1/2)*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1642, 648, 632, 210, 642}

$$-21\sqrt{\frac{3}{2}}\,\text{ArcTan}\left(\frac{\sqrt{6x-9}+3}{2\sqrt{6}}\right)-x+2\sqrt{3}\sqrt{2x-3}+10\log\left(x+\sqrt{3}\sqrt{2x-3}+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2
*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{12-x}{4+x+\sqrt{-9+6x}}\,dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{x(-63+x^2)}{33+6x+x^2}\,dx,x,\sqrt{-9+6x}\right)\right)\\ &= -\left(\frac{1}{3}\text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right)\,dx,x,\sqrt{-9+6x}\right)\right)\\ &= -x+2\sqrt{3}\sqrt{-3+2x}-2\text{Subst}\left(\int \frac{33-10x}{33+6x+x^2}\,dx,x,\sqrt{-9+6x}\right)\\ &= -x+2\sqrt{3}\sqrt{-3+2x}+10\text{Subst}\left(\int \frac{6+2x}{33+6x+x^2}\,dx,x,\sqrt{-9+6x}\right)-126\text{S}\\ &= -x+2\sqrt{3}\sqrt{-3+2x}+10\log\left(4+x+\sqrt{3}\sqrt{-3+2x}\right)+252\text{Subst}\left(\int \frac{}{-96}\right.\\ &= -x+2\sqrt{3}\sqrt{-3+2x}-21\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right)+10\log\left(4+x\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 67, normalized size = 0.94

$$\frac{1}{2}\left(3-2x+4\sqrt{-9+6x}-21\sqrt{6}\tan^{-1}\left(\frac{\sqrt{3}+\sqrt{-3+2x}}{2\sqrt{2}}\right)+20\log\left(4+x+\sqrt{-9+6x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]
```

```
[Out] (3 - 2*x + 4*Sqrt[-9 + 6*x] - 21*Sqrt[6]*ArcTan[(Sqrt[3] + Sqrt[-3 + 2*x])/
(2*Sqrt[2])] + 20*Log[4 + x + Sqrt[-9 + 6*x]])/2
```

Maple [A]
time = 0.18, size = 54, normalized size = 0.76

method	result

derivativedivides	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\arctan\left(\frac{\left(2\sqrt{-9+6x}+6\right)}{24}\right.}{2}$
default	$\frac{3}{2}-x+2\sqrt{-9+6x}+10\ln\left(24+6x+6\sqrt{-9+6x}\right)-\frac{21\sqrt{6}\arctan\left(\frac{\left(2\sqrt{-9+6x}+6\right)}{24}\right.}{2}$
trager	$-x+2\sqrt{-9+6x}+\mathrm{RootOf}\left(8\_Z^2-160\_Z+2123\right)\ln\left(4+x+\sqrt{-9+6x}\right)-\ln\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12-x)/(4+x+(-9+6*x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 3/2-x+2*(-9+6*x)^(1/2)+10*ln(24+6*x+6*(-9+6*x)^(1/2))-21/2*6^(1/2)*arctan(1
/24*(2*(-9+6*x)^(1/2)+6)*6^(1/2))
```

Maxima [A]
time = 0.50, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}\left(\sqrt{6x-9}+3\right)\right)-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")
[Out] -21/2*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) - x + 2*sqrt(6*x - 9
) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) + 3/2
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.83

$$-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9}+\frac{1}{4}\sqrt{3}\sqrt{2}\right)-x+2\sqrt{6x-9}+10\log\left(x+\sqrt{6x-9}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(
3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)
```

Sympy [A]
time = 33.98, size = 60, normalized size = 0.85

$$-x+2\sqrt{6x-9}+10\log\left(6x+6\sqrt{6x-9}+24\right)-\frac{21\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}\left(\sqrt{6x-9}+3\right)}{12}\right)}{2}+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)
```

```
[Out] -x + 2*sqrt(6*x - 9) + 10*log(6*x + 6*sqrt(6*x - 9) + 24) - 21*sqrt(6)*atan
(sqrt(6)*(sqrt(6*x - 9) + 3)/12)/2 + 3/2
```

Giac [A]
time = 2.13, size = 62, normalized size = 0.87

$$-\frac{21}{2}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\sqrt{3}+\sqrt{2x-3}\right)\right)+2\sqrt{3}\sqrt{2x-3}-x+10\log\left(2\sqrt{3}\sqrt{2x-3}+2x+8\right)+\frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/4*sqrt(2)*(sqrt(3) + sqrt(2*x - 3))) + 2*sqr
t(3)*sqrt(2*x - 3) - x + 10*log(2*sqrt(3)*sqrt(2*x - 3) + 2*x + 8) + 3/2
```

Mupad [B]
time = 0.03, size = 118, normalized size = 1.66

$$2\sqrt{6x-9}+10\ln\left(\left(\left(2\sqrt{6x-9}+6\right)\left(-10+\frac{\sqrt{2}\sqrt{3}\,21\mathrm{i}}{4}\right)+20\sqrt{6x-9}-66\right)\left(\left(2\sqrt{6x-9}+6\right)\left(10+\frac{\sqrt{2}\sqrt{3}\,21\mathrm{i}}{4}\right)-20\sqrt{6x-9}+66\right)\right)-x-\frac{21\sqrt{2}\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3}}{4}+\frac{\sqrt{2}\sqrt{3}\sqrt{6x-9}}{12}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 12)/(x + (6*x - 9)^(1/2) + 4),x)
```

```
[Out] 10*log(((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 - 10) + 20*(6*x -
9)^(1/2) - 66)*((2*(6*x - 9)^(1/2) + 6)*((2^(1/2)*3^(1/2)*21i)/4 + 10) - 20
*(6*x - 9)^(1/2) + 66)) - x + 2*(6*x - 9)^(1/2) - (21*2^(1/2)*3^(1/2)*atan(
(2^(1/2)*3^(1/2))/4 + (2^(1/2)*3^(1/2)*(6*x - 9)^(1/2))/12))/2
```

3.709 $\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx$

Optimal. Leaf size=52

$$\frac{2x^{3/2}}{3}+\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)-\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)$$

```
[Out] 2/3*x^(3/2)-arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-arctan(1+2^(1/2)*x^(1/2))*2^
(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1816, 841, 1176, 631, 210}

$$\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{x}\right)-\sqrt{2}\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{x}+1\right)+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
```

```
[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sq
rt[2]*Sqrt[x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 841

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^3}{\sqrt{x}\,(1+x^2)}\,dx &= \int \left(\sqrt{x}-\frac{1+x}{\sqrt{x}\,(1+x^2)}\right)dx \\
&= \frac{2x^{3/2}}{3}-\int \frac{1+x}{\sqrt{x}\,(1+x^2)}\,dx \\
&= \frac{2x^{3/2}}{3}-2\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{x}\right) \\
&= \frac{2x^{3/2}}{3}-\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right)-\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right. \\
&= \frac{2x^{3/2}}{3}-\sqrt{2}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\,\sqrt{x}\right)+\sqrt{2}\,\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right. \\
&= \frac{2x^{3/2}}{3}+\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,\sqrt{x}\right)-\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.62

$$\frac{2x^{3/2}}{3}-\sqrt{2}\,\tan^{-1}\left(\frac{-1+x}{\sqrt{2}\,\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]
[Out] (2*x^(3/2))/3 - Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(36) = 72.
time = 0.36, size = 117, normalized size = 2.25

method	result
trager	$\frac{2x^{\frac{3}{2}}}{3}+\frac{\mathrm{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+2\right)x^{2}+4x^{\frac{3}{2}}-4\,\mathrm{RootOf}\left(\_Z^{2}+2\right)x-4\sqrt{x}+\mathrm{RootOf}\left(\_Z^{2}+2\right)}{x^{2}+1}\right)}{2}$
risch	$\frac{2x^{\frac{3}{2}}}{3}-\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}-\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\sqrt{2}-\frac{\sqrt{2}\,\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}}{x-\sqrt{2}\,\sqrt{x}}\right)}{4}$
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{\sqrt{2}\,\left(\ln\left(\right.\right.}{}$
default	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\,\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{\sqrt{2}\,\left(\ln\left(\right.\right.}{}$
meijerg	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)-1/4*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+2*
arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-1/4*2^(1/2)*(ln((x-
2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arc
tan(-1+2^(1/2)*x^(1/2)))
```

Maxima [A]
time = 0.50, size = 46, normalized size = 0.88

$$\frac{2}{3}\,x^{\frac{3}{2}}-\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\sqrt{2}\,\arctan\left(-\frac{1}{2}\,\sqrt{2}\,\left(\sqrt{2}-2\,\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Fricas [A]
time = 0.37, size = 23, normalized size = 0.44

$$\frac{2}{3}\,x^{\frac{3}{2}}-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x-1)}{2\,\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))
```

Sympy [A]
time = 0.21, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right) - \sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(3/2)/3 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*atan(sqrt(2)*sqr
t(x) + 1)
```

Giac [A]
time = 2.91, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2} + 2\sqrt{x}\right)\right) - \sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2} - 2\sqrt{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x)))
```

Mupad [B]
time = 3.17, size = 43, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3} - \frac{\sqrt{2}\,\left(2\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2} + \frac{\sqrt{2}\,x^{3/2}}{2}\right) + 2\operatorname{atan}\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 - 1)/(x^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2*x^(3/2))/3 - (2^(1/2)*(2*atan((2^(1/2)*x^(1/2))/2 + (2^(1/2)*x^(3/2))/2)
 + 2*atan((2^(1/2)*x^(1/2))/2)))/2
```

3.710 $\int \frac{1}{2\sqrt{-1+x}\,\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {12, 633, 221}

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}$$
$$= -\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

Mathematica [A]
time = 0.06, size = 31, normalized size = 1.55

$$-\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

Maple [A]
time = 0.35, size = 14, normalized size = 0.70

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{-1+x}-\frac{1}{2}\right)}{3}\right)$	14
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{-1+x}-\frac{1}{2}\right)}{3}\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)-1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.
time = 0.55, size = 37, normalized size = 1.85

$$\frac{1}{2}\log\left(4\sqrt{x-\sqrt{x-1}}\,\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) -
 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2
```

Giac [A]
time = 2.54, size = 25, normalized size = 1.25

$$-\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{2\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.711 $\int \frac{1+x^{7/2}}{1-x^2} dx$

Optimal. Leaf size=43

$$-2\sqrt{x}-\frac{2x^{5/2}}{5}+\tan^{-1}\left(\sqrt{x}\right)-\log\left(1-\sqrt{x}\right)+\frac{1}{2}\log(1+x)$$

```
[Out] -2/5*x^(5/2)+arctan(x^(1/2))+1/2*ln(1+x)-ln(1-x^(1/2))-2*x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1847, 281, 212, 308, 218, 209}

$$\text{ArcTan}\left(\sqrt{x}\right)-\frac{2x^{5/2}}{5}-2\sqrt{x}+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x
]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
```

```
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1847

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)/c^j)*Sum[Coeff[Pq, x,
 j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0,
 n/2 - 1}], x]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0
] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^{7/2}}{1-x^2}\,dx &= 2\text{Subst}\left(\int \frac{x(1+x^7)}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x}{1-x^4}+\frac{x^8}{1-x^4}\right)\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sqrt{x}\right)+2\text{Subst}\left(\int \frac{x^8}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(-1-x^4+\frac{1}{1-x^4}\right)\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, x\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tanh^{-1}(x)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= -2\sqrt{x}-\frac{2x^{5/2}}{5}+\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)+\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.91

$$
-\frac{2}{5}\sqrt{x}\left(5+x^2\right)+\tan^{-1}\left(\sqrt{x}\right)-\log\left(-1+\sqrt{x}\right)+\frac{1}{2}\log(1+x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(7/2))/(1 - x^2),x]
```

```
[Out] (-2*Sqrt[x]*(5 + x^2))/5 + ArcTan[Sqrt[x]] - Log[-1 + Sqrt[x]] + Log[1 + x]
/2
```

Maple [A]
time = 0.39, size = 34, normalized size = 0.79

method	result
derivativedivides	$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}+\frac{\ln(1+x)}{2}+\arctan\left(\sqrt{x}\right)-\ln\left(-1+\sqrt{x}\right)$
default	$-\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}-\frac{\ln\left(-1+\sqrt{x}\right)}{2}+\frac{\ln\left(1+\sqrt{x}\right)}{2}+\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(x\right)$
meijerg	$\operatorname{arctanh}\left(x\right)-\frac{(-1)^{\frac{3}{4}}\left(-\frac{4\sqrt{x}\,(-1)^{\frac{1}{4}}\left(9x^{2}+45\right)}{45}-\frac{\sqrt{x}\,(-1)^{\frac{1}{4}}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)-2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{\left(x^{2}\right)^{\frac{1}{4}}}\right.}{2}$
trager	$\left(-\frac{2x^{2}}{5}-2\right)\sqrt{x}-2\ln\left(-\frac{24\,\mathrm{RootOf}\left(8\_Z^{2}-4\_Z+1\right)^{2}x-48\,\mathrm{RootOf}\left(8\_Z^{2}-4\_Z+1\right)^{2}+16\,\mathrm{RootOf}\left(8\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(7/2))/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*x^(5/2)-2*x^(1/2)-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))+arctan(x^(1/2))
+arctanh(x)
```

Maxima [A]
time = 0.49, size = 29, normalized size = 0.67

$$-\frac{2}{5}\,x^{\frac{5}{2}}-2\,\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) -
 1)
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.67

$$-\frac{2}{5}\left(x^{2}+5\right)\sqrt{x}+\arctan\left(\sqrt{x}\right)+\frac{1}{2}\,\log\left(x+1\right)-\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="fricas")
```

```
[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1
)
```

Sympy [A]
time = 0.54, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log\left(\sqrt{x} - 1\right) + \frac{\log\left(x+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(7/2))/(-x**2+1),x)
[Out] -2*x**(5/2)/5 - 2*sqrt(x) - log(sqrt(x) - 1) + log(x + 1)/2 + atan(sqrt(x))
```

Giac [A]
time = 2.57, size = 30, normalized size = 0.70

$$-\frac{2}{5}\,x^{\frac{5}{2}} - 2\,\sqrt{x} + \arctan\left(\sqrt{x}\right) + \frac{1}{2}\,\log\left(x+1\right) - \log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")
[Out] -2/5*x^(5/2) - 2*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(abs(sqrt(
x) - 1))
```

Mupad [B]
time = 3.11, size = 53, normalized size = 1.23

$$-\ln\left(10\,\sqrt{x} - 10\right) - 2\,\sqrt{x} - \frac{2\,x^{5/2}}{5} + \ln\left(1 + \sqrt{x}\,\left(-3 - \mathrm{i}\right) - 3\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{1}{2}\mathrm{i}\right) + \ln\left(1 + \sqrt{x}\,\left(-3 + 1\mathrm{i}\right) + 3\mathrm{i}\right)\,\left(\frac{1}{2} - \frac{1}{2}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(7/2) + 1)/(x^2 - 1),x)
[Out] log((1 - 3i) - x^(1/2)*(3 + 1i))*(1/2 + 1i/2) - log(10*x^(1/2) - 10) + log(
(1 + 3i) - x^(1/2)*(3 - 1i))*(1/2 - 1i/2) - 2*x^(1/2) - (2*x^(5/2))/5
```

3.712 $\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx$

Optimal. Leaf size=116

$$-x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\frac{3}{5}(-1+2x)^{5/6}+\frac{3}{7}(-1+2x)^{7/6}-\frac{3}{8}(-1+2x)^{4}$$

```
[Out] -x+18*(-1+2*x)^(1/6)-9*(-1+2*x)^(1/3)-3/4*(-1+2*x)^(2/3)+3/5*(-1+2*x)^(5/6)
+3/7*(-1+2*x)^(7/6)-3/8*(-1+2*x)^(4/3)+1/3*(-1+2*x)^(3/2)-18*ln(1+(-1+2*x)^
(1/6))+6*(-1+2*x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1634}

$$\frac{1}{3}(2x-1)^{3/2}-\frac{3}{8}(2x-1)^{4/3}+\frac{3}{7}(2x-1)^{7/6}+\frac{3}{5}(2x-1)^{5/6}-\frac{3}{4}(2x-1)^{2/3}+6\sqrt{2x-1}-9\sqrt[3]{2x-1}+18\sqrt[6]{2x-1}-x-18\log\left(\sqrt[6]{2x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 +
 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 +
 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{4+2x}{\sqrt[3]{-1+2x}+\sqrt{-1+2x}}\,dx &= 3\text{Subst}\left(\int \frac{x^3(5+x^6)}{1+x}\,dx, x, \sqrt[6]{-1+2x}\right)\\ &= 3\text{Subst}\left(\int \left(6-6x+6x^2-x^3+x^4-x^5+x^6-x^7+x^8-\frac{6}{1+x}\right)dx,\right.\\ &= -x+18\sqrt[6]{-1+2x}-9\sqrt[3]{-1+2x}+6\sqrt{-1+2x}-\frac{3}{4}(-1+2x)^{2/3}+\end{aligned}$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 1.12

$$2\left(\frac{1}{4}+\frac{123}{14}\sqrt[6]{-1+2x}-\frac{69}{16}\sqrt[3]{-1+2x}+\frac{17}{6}\sqrt{-1+2x}-\frac{3}{8}(-1+2x)^{2/3}+\frac{3}{10}(-1+2x)^{5/6}+x\left(-\frac{1}{2}+\frac{3}{7}\sqrt[6]{-1+2x}-\frac{3}{8}\sqrt[3]{-1+2x}+\frac{1}{3}\sqrt{-1+2x}\right)-9\log\left(1+\sqrt[6]{-1+2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]
```

```
[Out] 2*(1/4 + (123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1
 + 2*x])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (
3*(-1 + 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[
1 + (-1 + 2*x)^(1/6)])
```

Maple [A]
time = 0.01, size = 90, normalized size = 0.78

method	result
derivativedivides	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$
default	$\frac{(2x-1)^{\frac{3}{2}}}{3}-\frac{3(2x-1)^{\frac{4}{3}}}{8}+\frac{3(2x-1)^{\frac{7}{6}}}{7}-x+\frac{1}{2}+\frac{3(2x-1)^{\frac{5}{6}}}{5}-\frac{3(2x-1)^{\frac{2}{3}}}{4}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(2*x-1)^(3/2)-3/8*(2*x-1)^(4/3)+3/7*(2*x-1)^(7/6)-x+1/2+3/5*(2*x-1)^(5/
6)-3/4*(2*x-1)^(2/3)+6*(2*x-1)^(1/2)-9*(2*x-1)^(1/3)+18*(2*x-1)^(1/6)-18*ln
(1+(2*x-1)^(1/6))
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.77

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

Fricas [A]
time = 0.34, size = 76, normalized size = 0.66

$$\frac{1}{3}(2x+17)\sqrt{2x-1}-\frac{3}{8}(2x+23)(2x-1)^{\frac{1}{3}}+\frac{3}{7}(2x+41)(2x-1)^{\frac{1}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 17)*sqrt(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^(1/3) + 3/7*(2*x +
41)*(2*x - 1)^(1/6) - x + 3/5*(2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) - 18*lo
g((2*x - 1)^(1/6) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$2\left(\int \frac{x}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx+\int \frac{2}{\sqrt[3]{2x-1}+\sqrt{2x-1}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2/((2*x - 1
)**(1/3) + sqrt(2*x - 1)), x))
```

Giac [A]
time = 2.40, size = 89, normalized size = 0.77

$$\frac{1}{3}(2x-1)^{\frac{3}{2}}-\frac{3}{8}(2x-1)^{\frac{4}{3}}+\frac{3}{7}(2x-1)^{\frac{7}{6}}-x+\frac{3}{5}(2x-1)^{\frac{5}{6}}-\frac{3}{4}(2x-1)^{\frac{2}{3}}+6\sqrt{2x-1}-9(2x-1)^{\frac{1}{3}}+18(2x-1)^{\frac{1}{6}}-18\log\left((2x-1)^{\frac{1}{6}}+1\right)+\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*(2*x - 1)^(3/2) - 3/8*(2*x - 1)^(4/3) + 3/7*(2*x - 1)^(7/6) - x + 3/5*(
2*x - 1)^(5/6) - 3/4*(2*x - 1)^(2/3) + 6*sqrt(2*x - 1) - 9*(2*x - 1)^(1/3)
+ 18*(2*x - 1)^(1/6) - 18*log((2*x - 1)^(1/6) + 1) + 1/2
```

Mupad [B]
time = 0.13, size = 88, normalized size = 0.76

$$6\sqrt{2x-1}-18\ln\left((2x-1)^{1/6}+1\right)-x-9(2x-1)^{1/3}-\frac{3(2x-1)^{2/3}}{4}+\frac{(2x-1)^{3/2}}{3}+18(2x-1)^{1/6}-\frac{3(2x-1)^{4/3}}{8}+\frac{3(2x-1)^{5/6}}{5}+\frac{3(2x-1)^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)
```

```
[Out] 6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) - (
3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x - 1
)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

3.713 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$-48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx &= 2\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)\\ &= 4\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)\\ &= 4\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}}+11\sqrt{2+x}-6(2+x)^{3/2}+(2+x)^{5/2}\right)\,dx, x, \sqrt{1+\right.\\ &= -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}}+3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A]
time = 0.16, size = 54, normalized size = 0.65

method	result
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

Giac [A]
time = 2.84, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2}-8\sqrt{x}-7\right)\mathrm{sgn}\left(4\,x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.714 $\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx$

Optimal. Leaf size=64

$$\frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2}-\frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2}+\frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

```
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(4
+x^(1/2))^(1/2))^(9/2)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {378, 1412, 786}

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{9/2}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{7/2}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
```

```
[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7
 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{2+\sqrt{4+\sqrt{x}}}\,dx = 2\text{Subst}\left(\int x\sqrt{2+\sqrt{4+x}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \sqrt{2+\sqrt{x}}\,(-4+x)\,dx, x, 4+\sqrt{x}\right)$$
$$= 4\text{Subst}\left(\int x\sqrt{2+x}\,(-4+x^2)\,dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= 4\text{Subst}\left(\int \left(8(2+x)^{3/2}-6(2+x)^{5/2}+(2+x)^{7/2}\right)dx, x, \sqrt{4+\sqrt{x}}\right)$$
$$= \frac{64}{5}\left(2+\sqrt{4+\sqrt{x}}\right)^{5/2} - \frac{48}{7}\left(2+\sqrt{4+\sqrt{x}}\right)^{7/2} + \frac{8}{9}\left(2+\sqrt{4+\sqrt{x}}\right)^{9/2}$$

Mathematica [**A**]
time = 0.05, size = 62, normalized size = 0.97

$$\frac{8}{315}\sqrt{2+\sqrt{4+\sqrt{x}}}\left(-64\left(2+\sqrt{4+\sqrt{x}}\right)+2\left(2+5\sqrt{4+\sqrt{x}}\right)\sqrt{x}+35x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]],x]
[Out] (8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-64*(2 + Sqrt[4 + Sqrt[x]]) + 2*(2 + 5*Sqrt
[4 + Sqrt[x]])*Sqrt[x] + 35*x))/315
```

Maple [**A**]
time = 0.22, size = 41, normalized size = 0.64

method	result	size
meijerg	$2x\,\text{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{4},2\right],\left[\frac{1}{2},3\right],-\frac{\sqrt{x}}{4}\right)$	17
derivativedivides	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41
default	$\frac{64\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{5}{2}}}{5}-\frac{48\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{7}{2}}}{7}+\frac{8\left(2+\sqrt{4+\sqrt{x}}\right)^{\frac{9}{2}}}{9}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(4+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 64/5*(2+(4+x^(1/2))^(1/2))^(5/2)-48/7*(2+(4+x^(1/2))^(1/2))^(7/2)+8/9*(2+(4
+x^(1/2))^(1/2))^(9/2)
```

Maxima [A]
time = 0.29, size = 40, normalized size = 0.62

$$\frac{8}{9}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-\frac{48}{7}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+\frac{64}{5}\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/9*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 48/7*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 64
/5*(sqrt(sqrt(x) + 4) + 2)^(5/2)
```

Fricas [A]
time = 0.35, size = 39, normalized size = 0.61

$$\frac{8}{315}\left(2\left(5\sqrt{x}-32\right)\sqrt{\sqrt{x}+4}+35\,x+4\sqrt{x}-128\right)\sqrt{\sqrt{\sqrt{x}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(
sqrt(sqrt(x) + 4) + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(54) = 108$.
time = 1.46, size = 216, normalized size = 3.38

$$-\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi}-\frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}-\frac{\sqrt{2}\,x\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi}+\frac{64\sqrt{2}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}+\frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\,\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] -2*sqrt(2)*sqrt(x)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/4
)*gamma(1/4)/(63*pi) - 4*sqrt(2)*sqrt(x)*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(
-1/4)*gamma(1/4)/(315*pi) - sqrt(2)*x*sqrt(sqrt(sqrt(x) + 4) + 2)*gamma(-1/
4)*gamma(1/4)/(9*pi) + 64*sqrt(2)*sqrt(sqrt(x) + 4)*sqrt(sqrt(sqrt(x) + 4)
+ 2)*gamma(-1/4)*gamma(1/4)/(315*pi) + 128*sqrt(2)*sqrt(sqrt(sqrt(x) + 4) +
 2)*gamma(-1/4)*gamma(1/4)/(315*pi)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(40) = 80$.
time = 4.55, size = 268, normalized size = 4.19

$$\frac{8}{315}\left(\left(35\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{9}{2}}-360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}+1512\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}-3360\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}+5040\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)+18\left(5\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{7}{2}}-42\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}+140\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}-280\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)-84\left(3\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{5}{2}}-20\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}+60\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)-840\left(\left(\sqrt{\sqrt{x}+4}+2\right)^{\frac{3}{2}}-6\sqrt{\sqrt{\sqrt{x}+4}+2}\right)\mathrm{sgn}\left(4\left(\sqrt{x}+4\right)^{2}-32\sqrt{x}-79\right)\right)\mathrm{sgn}(4x-15)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/2
) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/2)
 + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 7
9) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn
(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)^(5/2)
 - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4
*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^(3/2) -
6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79))*sg
n(4*x - 15)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{\sqrt{x}+4}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)
```

3.715 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal. Leaf size=82

$$\frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2}-\frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2}+\frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{9/2}$$

```
[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))
^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {378, 1412, 786}

$$\frac{8}{45}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{9/2}-\frac{48}{35}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{7/2}+\frac{64}{25}\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 +
5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{2}{5}\text{Subst}\left(\int x\sqrt{2-\sqrt{4+x}}\,dx, x, \sqrt{-9+5x}\right)$$
$$= \frac{2}{5}\text{Subst}\left(\int \sqrt{2-\sqrt{x}}\,(-4+x)\,dx, x, 4+\sqrt{-9+5x}\right)$$
$$= \frac{4}{5}\text{Subst}\left(\int \sqrt{2-x}\,x(-4+x^2)\,dx, x, \sqrt{4+\sqrt{-9+5x}}\right)$$
$$= \frac{4}{5}\text{Subst}\left(\int \left(-8(2-x)^{3/2}+6(2-x)^{5/2}-(2-x)^{7/2}\right)dx, x, \sqrt{4+\sqrt{-9}}\right.$$
$$= \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2} - \frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2} + \frac{8}{45}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 1.05

$$-\frac{8\sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\left(443-175x-4\sqrt{-9+5x}-64\sqrt{4+\sqrt{-9+5x}}+10\sqrt{-9+5x}\sqrt{4+\sqrt{-9+5x}}\right)}{1575}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

```
[Out] (-8*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]]*(443 - 175*x - 4*Sqrt[-9 + 5*x] - 64
*Sqrt[4 + Sqrt[-9 + 5*x]] + 10*Sqrt[-9 + 5*x]*Sqrt[4 + Sqrt[-9 + 5*x]]))/15
75
```

Maple [A]
time = 0.18, size = 59, normalized size = 0.72

method	result	size
derivativedivides	$\frac{64\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{9}{2}}}{45}$	59
default	$\frac{64\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{5x-9}}\right)^{\frac{9}{2}}}{45}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-(4+(5*x-9)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 64/25*(2-(4+(5*x-9)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(5*x-9)^(1/2))^(1/2))^(
7/2)+8/45*(2-(4+(5*x-9)^(1/2))^(1/2))^(9/2)
```

Maxima [A]
time = 0.29, size = 58, normalized size = 0.71

$$\frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}}-\frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}}+\frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4)
 + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

Fricas [A]
time = 0.34, size = 57, normalized size = 0.70

$$-\frac{8}{1575}\left(2\left(5\sqrt{5\,x-9}-32\right)\sqrt{\sqrt{5\,x-9}+4}-175\,x-4\sqrt{5\,x-9}+443\right)\sqrt{-\sqrt{\sqrt{5\,x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(
5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{2-\sqrt{\sqrt{5x-9}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(2 - sqrt(sqrt(5*x - 9) + 4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. 2(58) = 116.
time = 3.22, size = 474, normalized size = 5.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) +
```

```
2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2
) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*x -
9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 18*(5
*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 42*(s
qrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 140*(-sq
rt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*
sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqrt(sqrt(5*
x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt(sqrt(5*x
- 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(
5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x - 9) + 4) +
 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4
)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
[Out] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

3.716 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=83

$$-48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/2}+\frac{8}{7}\left(2+\sqrt{1+\sqrt{x}}\right)^{7/2}$$

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.176$, Rules used = {378, 1412, 786}

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{7/2}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{5/2}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{3/2}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (4
8*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 378

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}}\,dx = 2\text{Subst}\left(\int \frac{x}{\sqrt{2+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \frac{-1+x}{\sqrt{2+\sqrt{x}}}\,dx, x, 1+\sqrt{x}\right)$$
$$= 4\text{Subst}\left(\int \frac{x(-1+x^2)}{\sqrt{2+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$
$$= 4\text{Subst}\left(\int \left(-\frac{6}{\sqrt{2+x}}+11\sqrt{2+x}-6(2+x)^{3/2}+(2+x)^{5/2}\right)dx, x, \sqrt{1+\right.$$
$$= -48\sqrt{2+\sqrt{1+\sqrt{x}}}+\frac{88}{3}\left(2+\sqrt{1+\sqrt{x}}\right)^{3/2}-\frac{48}{5}\left(2+\sqrt{1+\sqrt{x}}\right)^{5/}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 0.70

$$\frac{8}{105}\sqrt{2+\sqrt{1+\sqrt{x}}}\left(-280+76\sqrt{1+\sqrt{x}}+3\left(-12+5\sqrt{1+\sqrt{x}}\right)\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sq
rt[1 + Sqrt[x]])*Sqrt[x]))/105
```

Maple [A]
time = 0.12, size = 54, normalized size = 0.65

method	result
derivativedivides	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$
default	$\frac{88\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{3}{2}}}{3}-\frac{48\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{5}{2}}}{5}+\frac{8\left(2+\sqrt{1+\sqrt{x}}\right)^{\frac{7}{2}}}{7}-48\sqrt{2+\sqrt{1+\sqrt{x}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+(1+x^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 88/3*(2+(1+x^(1/2))^(1/2))^(3/2)-48/5*(2+(1+x^(1/2))^(1/2))^(5/2)+8/7*(2+(1
+x^(1/2))^(1/2))^(7/2)-48*(2+(1+x^(1/2))^(1/2))^(1/2)
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.64

$$\frac{8}{7}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-\frac{48}{5}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+\frac{88}{3}\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-48\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
[Out] 8/7*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 48/5*(sqrt(sqrt(x) + 1) + 2)^(5/2) + 88
/3*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 48*sqrt(sqrt(sqrt(x) + 1) + 2)
```

Fricas [A]
time = 0.39, size = 35, normalized size = 0.42

$$\frac{8}{105}\left(\left(15\sqrt{x}+76\right)\sqrt{\sqrt{x}+1}-36\sqrt{x}-280\right)\sqrt{\sqrt{\sqrt{x}+1}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sq
rt(x) + 1) + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)
[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)
```

Giac [A]
time = 2.52, size = 82, normalized size = 0.99

$$\frac{8\left(15\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{7}{2}}-126\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{5}{2}}+385\left(\sqrt{\sqrt{x}+1}+2\right)^{\frac{3}{2}}-630\sqrt{\sqrt{\sqrt{x}+1}+2}\right)}{105\,\mathrm{sgn}\left(4\left(\sqrt{x}+1\right)^{2}-8\sqrt{x}-7\right)\mathrm{sgn}\left(4x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/105*(15*(sqrt(sqrt(x) + 1) + 2)^(7/2) - 126*(sqrt(sqrt(x) + 1) + 2)^(5/2)
 + 385*(sqrt(sqrt(x) + 1) + 2)^(3/2) - 630*sqrt(sqrt(sqrt(x) + 1) + 2))/(sg
n(4*(sqrt(x) + 1)^2 - 8*sqrt(x) - 7)*sgn(4*x - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x}+1}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)
```

```
[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)
```

3.717 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal. Leaf size=190

$$-\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2}+\frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{9/2}-\frac{32}{1}$$

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Rubi [A]
time = 0.25, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1632, 1634}

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{9/2}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{7/2}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1
+ Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (
320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/
15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17
```

Rule 1632

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = 2\text{Subst}\left(\int x\sqrt{1+\sqrt{1+\sqrt{1+x}}}\,dx, x, \sqrt{x}\right)$$

$$= 4\text{Subst}\left(\int x(-1+x^2)\sqrt{1+\sqrt{1+x}}\,dx, x, \sqrt{1+\sqrt{x}}\right)$$

$$= 8\text{Subst}\left(\int x^3\sqrt{1+x}\,(-2+x^2)\,(-1+x^2)\,dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$

$$= 8\text{Subst}\left(\int x^3(1+x)^{3/2}\,(2-2x-x^2+x^3)\,dx, x, \sqrt{1+\sqrt{1+\sqrt{x}}}\right)$$

$$= 8\text{Subst}\left(\int \left(-2(1+x)^{3/2}+3(1+x)^{5/2}+7(1+x)^{7/2}-20(1+x)^{9/2}+\right.\right.$$

$$= -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2}+\frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2} -$$

Mathematica [A]

time = 0.16, size = 168, normalized size = 0.88

$$\frac{16\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\left(-8\left(3519-1094\sqrt{1+\sqrt{1+\sqrt{x}}}+163\sqrt{1+\sqrt{x}}+584\sqrt{1+\sqrt{1+\sqrt{x}}}\sqrt{1+\sqrt{x}}\right)+7\left(659-504\sqrt{1+\sqrt{1+\sqrt{x}}}+33\sqrt{1+\sqrt{x}}+429\sqrt{1+\sqrt{1+\sqrt{x}}}\sqrt{1+\sqrt{x}}\right)\sqrt{x}+45045x\right)}{765765}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

```
[Out] (16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-8*(3519 - 1094*Sqrt[1 + Sqrt[1
+ Sqrt[x]]] + 163*Sqrt[1 + Sqrt[x]] + 584*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[
1 + Sqrt[x]]) + 7*(659 - 504*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 33*Sqrt[1 + Sqrt
[x]] + 429*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 45045*x
))/765765
```

Maple [A]

time = 0.36, size = 121, normalized size = 0.64

method	result
derivativedivides	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right.}{9}$

default	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^(
1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x^(
1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)-11
2/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(17/2)
```

Maxima [A]
time = 0.27, size = 120, normalized size = 0.63

$$\frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}-\frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}+\frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}-\frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}+\frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}+\frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}-\frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
+ 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) - 3
20/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32/5*(
sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

Fricas [A]
time = 0.38, size = 76, normalized size = 0.40

$$\frac{16}{765765}\left((231\sqrt{x}-1304)\sqrt{\sqrt{x}+1}+\left((3003\sqrt{x}-4672)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+1}+45045\,x+4613\sqrt{x}-28152\right)\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)*
sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 4504
5*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 7916 vs. $2(120) = 240$.
time = 35.37, size = 7916, normalized size = 41.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 16/765765*(7*(6435*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 58344*(sqrt(s
qrt(sqrt(x) + 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1
)^(13/2) - 556920*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 850850*(sqrt(s
qrt(sqrt(x) + 1) + 1) + 1)^(9/2) - 875160*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
^(7/2) + 612612*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 291720*(sqrt(sqrt
(sqrt(x) + 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1
))*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*(sqrt(s
qrt(x) + 1) + 1)^91 + 291326600895528960*(sqrt(sqrt(x) + 1) + 1)^90 - 85455
80292935516160*(sqrt(sqrt(x) + 1) + 1)^89 + 183728762437276532736*(sqrt(sqr
t(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1)^87 + 421
79809308639429132288*(sqrt(sqrt(x) + 1) + 1)^86 - 481978846822841400164352*
(sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sqrt(x) + 1) +
 1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83 + 2931836397
16003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 1916656336440269370174734336
*(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*(sqrt(sqrt(x) +
 1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) + 1) + 1)^79 + 27
3479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 - 11609566078829931
55309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469532994953426239488
*(sqrt(sqrt(x) + 1) + 1)^76 - 15624039803063454614788052615168*(sqrt(sqrt(x
) + 1) + 1)^75 + 49728771914087708805425247813632*(sqrt(sqrt(x) + 1) + 1)^7
4 - 144204022361387642459669217148928*(sqrt(sqrt(x) + 1) + 1)^73 + 38109938
4933784007520636056371200*(sqrt(sqrt(x) + 1) + 1)^72 - 9174887252144158139
57123995336704*(sqrt(sqrt(x) + 1) + 1)^71 + 200952113081899810499009723970355
52*(sqrt(sqrt(x) + 1) + 1)^70 - 3994471142582563999654557691936768*(sqrt(sq
rt(x) + 1) + 1)^69 + 7177812996901911023337169833951232*(sqrt(sqrt(x) + 1)
+ 1)^68 - 11588332903437268712897290291904512*(sqrt(sqrt(x) + 1) + 1)^67 +
16646690083818302450699356048719872*(sqrt(sqrt(x) + 1) + 1)^66 - 2093668615
18988043128935803571404 80*(sqrt(sqrt(x) + 1) + 1)^65 + 22382788038883899099
152454346866688*(sqrt(sqrt(x) + 1) + 1)^64 - 1905635422748711967745134244651
92000*(sqrt(sqrt(x) + 1) + 1)^63 + 1044679223910917373907117564913254 4*(sqr
t(sqrt(x) + 1) + 1)^62 + 1511753796217450360680303785148416*(sqrt(sqrt(x) +
```

```
 1) + 1)^61 - 12615704090193713988088537190236160*(sqrt(sqrt(x) + 1) + 1)^6
0 + 18210769010276524054435333169741824*(sqrt(sqrt(x) + 1) + 1)^59 - 158886
18239925478635050328221810688*(sqrt(sqrt(x) + 1) + 1)^58 + 7264980298352403
064896955164393472*(sqrt(sqrt(x) + 1) + 1)^57 + 271715923563468262470123743
9758336*(sqrt(sqrt(x) + 1) + 1)^56 - 8806173737385529153533018462224384*(sq
rt(sqrt(x) + 1) + 1)^55 + 8704589509518681571761496954765312*(sqrt(sqrt(x)
+ 1) + 1)^54 - 4141051044270206270604188407824384*(sqrt(sqrt(x) + 1) + 1)^5
3 - 944047265435153343329682904317952*(sqrt(sqrt(x) + 1) + 1)^52 + 34414217
59241742702311709805117440*(sqrt(sqrt(x) + 1) + 1)^51 - 2875820730830681791
678590352359424*(sqrt(sqrt(x) + 1) + 1)^50 + 881068299799276284483428560142
336*(sqrt(sqrt(x) + 1) + 1)^49 + 656876670010853235917344051560448*(sqrt(sq
rt(x) + 1) + 1)^48 - 985314730141923394087336160526336*(sqrt(sqrt(x) + 1) +
 1)^47 + 512961170622589184570169885720576*(sqrt(sqrt(x) + 1) + 1)^46 + 178
39996318603553048412869885952*(sqrt(sqrt(x) + 1) + 1)^45 - 2210745729060236
19230346738925568*(sqrt(sqrt(x) + 1) + 1)^44 + 1533206437006286733306258668
91264*(sqrt(sqrt(x) + 1) + 1)^43 - 26652891419311593866038343630848*(sqrt(s
qrt(x) + 1) + 1)^42 - 34964525177019636722858108911616*(sqrt(sqrt(x) + 1) +
 1)^41 + 31682113944794289518835974275072*(sqrt(sqrt(x) + 1) + 1)^40 - 9233
374080713604069270669492224*(sqrt(sqrt(x) + 1) + 1)^39 - 383353858045854843
1139339501568*(sqrt(sqrt(x) + 1) + 1)^38 + 5085184419428714337736452997120*
(sqrt(sqrt(x) + 1) + 1)^37 - 1982823679057600833030660816896*(sqrt(sqrt(x)
+ 1) + 1)^36 - 262480534359793423136287883264*(sqrt(sqrt(x) + 1) + 1)^35 +
711320861924448823343914680320*(sqrt(sqrt(x) + 1) + 1)^34 - 328697875402249
596865599242240*(sqrt(sqrt(x) + 1) + 1)^33 - 16461430004162620889537183744*
(sqrt(sqrt(x) + 1) + 1)^32 + 95428601176521400977360683008*(sqrt(sqrt(x) +
1) + 1)^31 - 41349359848761873167586164736*(sqrt(sqrt(x) + 1) + 1)^30 - 486
3977456557543561269084160*(sqrt(sqrt(x) + 1) + 1)^29 + 11544647980057943904
629669888*(sqrt(sqrt(x) + 1) + 1)^28 - 3333524342970261558455762944*(sqrt(s
qrt(x) + 1) + 1)^27 - 1191572683417725493401812992*(sqrt(sqrt(x) + 1) + 1)^
26 + 1026232209353398029110476800*(sqrt(sqrt(x) + 1) + 1)^25 - 934813837556
79278573023232*(sqrt(sqrt(x) + 1) + 1)^24 - 146062154264152631122427904*(sq
rt(sqrt(x) + 1) + 1)^23 + 51788232298428869952700416*(sqrt(sqrt(x) + 1) + 1
)^22 + 9053298579516313975259136*(sqrt(sqrt(x) + 1) + 1)^21 - 8934958448492
427163846656*(sqrt(sqrt(x) + 1) + 1)^20 + 641389659530470477504512*(sqrt(sq
rt(x) + 1) + 1)^19 + 915449581849135293882368*(sqrt(sqrt(x) + 1) + 1)^18 -
220733028492743248314368*(sqrt(sqrt(x) + 1) + 1...
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.718 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal. Leaf size=233

$$-\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2}-\frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right.$$

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Rubi [A]

time = 0.26, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1634\}$

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{9/2}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{7/2}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{5/2}-\frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7
/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqr
t[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 +
 (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = 2\text{Subst}\left(\int x\sqrt{2+\sqrt{3+\sqrt{-1+2x}}}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int x(1+x^2)\sqrt{2+\sqrt{3+x}}\,dx, x, \sqrt{-1+2\sqrt{x}}\right)$$
$$= 2\text{Subst}\left(\int x\sqrt{2+x}\,(-3+x^2)\left(1+(-3+x^2)^2\right)dx, x, \sqrt{3+\sqrt{-$$
$$= 2\text{Subst}\left(\int \left(-4\sqrt{2+x}+34(2+x)^{3/2}-120(2+x)^{5/2}+228(2+x$$
$$= -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2}+\frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right.$$

Mathematica [A]
time = 0.16, size = 186, normalized size = 0.80

$$\frac{8\sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\left(8\left(-15510-7428\sqrt{3+\sqrt{-1+2\sqrt{x}}}+211\sqrt{-1+2\sqrt{x}}+1700\sqrt{3+\sqrt{-1+2\sqrt{x}}}\sqrt{-1+2\sqrt{x}}\right)+7\left(-549-672\sqrt{3+\sqrt{-1+2\sqrt{x}}}-121\sqrt{-1+2\sqrt{x}}+286\sqrt{3+\sqrt{-1+2\sqrt{x}}}\sqrt{-1+2\sqrt{x}}\right)\sqrt{x}+30030x\right)}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

```
[Out] (8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(8*(-15510 - 7428*Sqrt[3 + Sqrt
[-1 + 2*Sqrt[x]]] + 211*Sqrt[-1 + 2*Sqrt[x]] + 1700*Sqrt[3 + Sqrt[-1 + 2*Sq
rt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-549 - 672*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]
] - 121*Sqrt[-1 + 2*Sqrt[x]] + 286*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[-1 +
 2*Sqrt[x]])*Sqrt[x] + 30030*x))/255255
```

Maple [A]
time = 0.42, size = 154, normalized size = 0.66

method	result
derivativedivides	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-\right.}}}{7}$
default	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-\right.}}}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^(
1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(2+
(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))^(1
/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(3+(-
1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(
17/2)
```

Maxima [A]
time = 0.28, size = 153, normalized size = 0.66

$$\frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+\frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-\frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}+\frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}-\frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}+\frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}-\frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(x
) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/2)
 - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(2*s
qrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(7/
2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt(2*sq
rt(x) - 1) + 3) + 2)^(3/2)
```

Fricas [A]
time = 0.37, size = 85, normalized size = 0.36

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\sqrt{\sqrt{2\sqrt{x}-1}+3}-30030x+3843\sqrt{x}+124080\right)\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 68
00)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1) +
3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)
```

Giac [A]
time = 6.11, size = 271, normalized size = 1.16

$$\frac{4}{255255}\left(15015\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}-238238\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}+1472625\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}-4408950\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}+6466460\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}-4375800\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}+1735734\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}-340340\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}\right)\mathrm{sgn}\left(8192x^{23}+376832x^{22}+\cdots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(s
qrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) + 3
) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646646
0*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt(x)
- 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) -
340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(8192*x^23 + 376832*x^
22 + 8224768*x^21 + 113971200*x^20 + 1130782720*x^19 + 8582063104*x^18 + 51
933387264*x^17 + 257575619584*x^16 + 1066188686592*x^15 + 3723204389632*x^1
4 + 11019822890016*x^13 + 27631512444352*x^12 + 58424530490176*x^11 + 10333
6828749760*x^10 + 151203890043312*x^9 + 180411181747936*x^8 + 1722871992929
60*x^7 + 128457231939048*x^6 + 72257964298210*x^5 + 29175203228012*x^4 + 78
30371130072*x^3 + 1228114804752*x^2 + 87490886400*x + 933120000)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)
```

```
[Out] int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)
```

3.719 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x \, dx$

Optimal. Leaf size=160

$$\frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2}-\frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2}-\frac{160}{11}\left(1+\sqrt{\right.$$

```
[Out] 16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2)
+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11/2
)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/2))
^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

Rubi [A]
time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1632, 1634}

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{9/2}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{7/2}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt
[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*
(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(
17/2))/17
```

Rule 1632

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}\, x\, dx = 2\text{Subst}\left(\int x(1+x^2)\sqrt{1+\sqrt{1+x}}\, dx, x, \sqrt{-1+x}\right)$$
$$= 4\text{Subst}\left(\int x\sqrt{1+x}\,(-1+x^2)\left(1+\left(-1+x^2\right)^2\right) dx, x, \sqrt{1+\sqrt{-1+$$
$$= 4\text{Subst}\left(\int x(1+x)^{3/2}\left(-2+2x+2x^2-2x^3-x^4+x^5\right) dx, x, \sqrt{1+$$
$$= 4\text{Subst}\left(\int \left(2(1+x)^{3/2}-3(1+x)^{5/2}+9(1+x)^{7/2}-20(1+x)^{9/2}+18$$
$$= \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2} - \frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2} + 8\left(1$$

Mathematica [A]
time = 0.11, size = 94, normalized size = 0.59

$$\frac{8\sqrt{1+\sqrt{1+\sqrt{-1+x}}}\left(-8872+1109\sqrt{-1+x}+28231(-1+x)+77(-1+x)^{3/2}+15015(-1+x)^2+\sqrt{1+\sqrt{-1+x}}\left(-7696+4544\sqrt{-1+x}+7\left(-168+143\sqrt{-1+x}\right)x\right)\right)}{255255}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

```
[Out] (8*Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*(-8872 + 1109*Sqrt[-1 + x] + 28231*(-1
+ x) + 77*(-1 + x)^(3/2) + 15015*(-1 + x)^2 + Sqrt[1 + Sqrt[-1 + x]]*(-7696
 + 4544*Sqrt[-1 + x] + 7*(-168 + 143*Sqrt[-1 + x])*x)))/255255
```

Maple [A]
time = 0.28, size = 107, normalized size = 0.67

method	result
derivativedivides	$\frac{16\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{9}{2}}-$
default	$\frac{16\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{\frac{9}{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2)
+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11/2
```

```
)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/2))
^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

Maxima [A]
time = 0.27, size = 106, normalized size = 0.66

$$\frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}-\frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}+\frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}-\frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}+8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}}-\frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}+\frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1)
^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x -
 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(sqr
t(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

Fricas [A]
time = 0.40, size = 62, normalized size = 0.39

$$\frac{8}{255255}\left(15015\,x^{2}+(77\,x+1032)\sqrt{x-1}+\left((1001\,x+4544)\sqrt{x-1}-1176\,x-7696\right)\sqrt{\sqrt{x-1}+1}-1799\,x-22088\right)\sqrt{\sqrt{\sqrt{x-1}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x -
 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sqrt
(x - 1) + 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. 2(106) = 212.
time = 5.69, size = 859, normalized size = 5.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x -
 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920*(
sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2
) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1) + 1
) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(
sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 119
*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) + 1) + 1)
^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sqrt(x - 1
) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 27027*(sqrt(s
qrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 6435
*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1)
 - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqrt(sqrt(x - 1
) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 8580*(sqrt(sq
rt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) - 6006*(
sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sg
n(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sqrt(sqrt(x - 1) +
 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) + 990*(sqrt(sqrt(x
- 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 1155*(sqrt(
sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(s
qrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(sqrt(sqrt(x - 1) + 1) +
1)^(9/2) - 180*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 378*(sqrt(sqrt(x - 1) +
1) + 1)^(5/2) - 420*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 315*sqrt(sqrt(sqrt(
x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 87516*(5
*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 21*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) +
 35*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 35*sqrt(sqrt(sqrt(x - 1) + 1) + 1))
*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 102102*(3*(sqrt(sqrt(x -
1) + 1) + 1)^(5/2) - 10*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 15*sqrt(sqrt(sq
rt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 51051
0*((sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 3*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*s
gn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7))*sgn(4*x - 7)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

3.720 $\int \frac{1}{\sqrt{-1+x}\,\sqrt{-\sqrt{-1+x}+x}}\,dx$

Optimal. Leaf size=20

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

```
[Out] -2*arcsinh(1/3*(1-2*(-1+x)^(1/2))*3^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {633, 221}

$$-2\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
```

```
[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}}\,dx = 2\text{Subst}\left(\int \frac{1}{\sqrt{1-x+x^2}}\,dx, x, \sqrt{-1+x}\right)$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, -1+2\sqrt{-1+x}\right)}{\sqrt{3}}$$

$$= -2\sinh^{-1}\left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}}\right)$$

Mathematica [A]
time = 0.00, size = 31, normalized size = 1.55

$$-2\log\left(1-2\sqrt{-1+x}+2\sqrt{-\sqrt{-1+x}+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]
[Out] -2*Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]
```

Maple [A]
time = 0.37, size = 16, normalized size = 0.80

method	result	size
derivativedivides	$2\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{-1+x}-\frac{1}{2}\right)}{3}\right)$	16
default	$2\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,\left(\sqrt{-1+x}-\frac{1}{2}\right)}{3}\right)$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*arcsinh(2/3*3^(1/2)*((-1+x)^(1/2)-1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.
time = 0.57, size = 35, normalized size = 1.75

$$\log\left(4\sqrt{x-\sqrt{x-1}}\,\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")
[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\,\sqrt{x-\sqrt{x-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)
```

Giac [A]
time = 1.42, size = 25, normalized size = 1.25

$$-2\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")
[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}}\,\sqrt{x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)),x)
[Out] int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)
```

3.721 $\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=44

$$2\sqrt{1+x+\sqrt{-1+2x}}-\sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

```
[Out] -arcsinh(1/2*(1+(-1+2*x)^(1/2))*2^(1/2))*2^(1/2)+2*(1+x+(-1+2*x)^(1/2))^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {654, 633, 221}

$$\sqrt{2}\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-1}+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2
*x])/Sqrt[2]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{\frac{3}{2}+x+\frac{x^2}{2}}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{2}}}\,dx, x, 1+\sqrt{-1+2x}\right)$$
$$= \sqrt{2}\sqrt{2+2x+2\sqrt{-1+2x}} - \sqrt{2}\sinh^{-1}\left(\frac{1+\sqrt{-1+2x}}{\sqrt{2}}\right)$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 1.34

$$2\sqrt{1+x+\sqrt{-1+2x}} + \sqrt{2}\log\left(-1-\sqrt{-1+2x}+\sqrt{2+2x+2\sqrt{-1+2x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]
```

```
[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] + Sqrt[2]*Log[-1 - Sqrt[-1 + 2*x] + Sqrt[2 +
 2*x + 2*Sqrt[-1 + 2*x]]]
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.86

method	result	size
derivativedivides	$\sqrt{4x+4+4\sqrt{2x-1}} - \operatorname{arcsinh}\left(\frac{\left(1+\sqrt{2x-1}\right)\sqrt{2}}{2}\right)\sqrt{2}$	38
default	$\sqrt{4x+4+4\sqrt{2x-1}} - \operatorname{arcsinh}\left(\frac{\left(1+\sqrt{2x-1}\right)\sqrt{2}}{2}\right)\sqrt{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.
time = 0.88, size = 85, normalized size = 1.93

$$\frac{1}{4}\sqrt{2}\log\left(-8x^2-8(2x+1)\sqrt{2x-1}+2\left(\sqrt{2}(2x+3)\sqrt{2x-1}+\sqrt{2}(6x-1)\right)\sqrt{x+\sqrt{2x-1}+1}-24x+7\right)+2\sqrt{x+\sqrt{2x-1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*s
qrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) +
 2*sqrt(x + sqrt(2*x - 1) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x+\sqrt{2x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)
```

Giac [A]
time = 3.25, size = 49, normalized size = 1.11

$$\sqrt{2}\left(\sqrt{2x+2\sqrt{2x-1}+2}+\log\left(\sqrt{2x+2\sqrt{2x-1}+2}-\sqrt{2x-1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1) +
 2) - sqrt(2*x - 1) - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x+\sqrt{2\,x-1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)
```

```
[Out] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)
```

3.722 $\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx$

Optimal. Leaf size=54

$$\frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2(bp-f^2p-aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

[Out] p*x/a-2*(-f^2*p-a*q+b*p)*ln(f+(a*x+b)^(1/2))/a^2-2*f*p*(a*x+b)^(1/2)/a^2

Rubi [A]
time = 0.26, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {711}

$$-\frac{2(-aq+bp+f^2(-p))\log\left(\sqrt{ax+b}+f\right)}{a^2}-\frac{2fp\sqrt{ax+b}}{a^2}+\frac{px}{a}$$

Antiderivative was successfully verified.

[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]

[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b + a*x]])/a^2

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{q+px}{\sqrt{b+ax}\left(f+\sqrt{b+ax}\right)}\,dx &= \frac{2\text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x}\,dx,x,\sqrt{b+ax}\right)}{a^2}\\ &= \frac{2\text{Subst}\left(\int \left(-fp+px+\frac{-bp+f^2p+aq}{f+x}\right)\,dx,x,\sqrt{b+ax}\right)}{a^2}\\ &= \frac{px}{a}-\frac{2fp\sqrt{b+ax}}{a^2}-\frac{2(bp-f^2p-aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.94

$$\frac{p\left(b+ax-2f\sqrt{b+ax}\right)+2(-bp+f^2p+aq)\log\left(f+\sqrt{b+ax}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]
```

```
[Out] (p*(b + a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b
+ a*x]])/a^2
```

Maple [A]
time = 0.36, size = 50, normalized size = 0.93

method	result	size
derivativedivides	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2(f^2p+aq-pb)\ln\left(f+\sqrt{ax+b}\right)}{a^2}$	50
default	$\frac{-2fp\sqrt{ax+b}+p(ax+b)+2(f^2p+aq-pb)\ln\left(f+\sqrt{ax+b}\right)}{a^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/a^2*(-f*p*(a*x+b)^(1/2)+1/2*p*(a*x+b)+(f^2*p+a*q-b*p)*ln(f+(a*x+b)^(1/2))
)
```

Maxima [A]
time = 0.28, size = 58, normalized size = 1.07

$$\frac{\frac{2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a}-\frac{2\sqrt{ax+b}\,fp-(ax+b)p}{a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")
```

```
[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))/a - (2*sqrt(a*x + b)*f*p - (a
*x + b)*p)/a)/a
```

Fricas [A]
time = 0.34, size = 45, normalized size = 0.83

$$\frac{apx-2\sqrt{ax+b}\,fp+2\left((f^2-b)p+aq\right)\log\left(f+\sqrt{ax+b}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b))
)/a^2
```

Sympy [A]
time = 16.59, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^2}-\frac{2f(-aq+bp-f^2p)\left(\begin{cases}\frac{1}{\sqrt{ax+b}} & \text{for } f=0\\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise}\end{cases}\right)}{a^2}+\frac{p(ax+b)}{a^2}+\frac{2(-aq+bp-f^2p)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)
```

```
[Out] -2*f*p*sqrt(a*x + b)/a**2 - 2*f*(-a*q + b*p - f**2*p)*Piecewise((1/sqrt(a*x
 + b), Eq(f, 0)), (log(f/sqrt(a*x + b) + 1)/f, True))/a**2 + p*(a*x + b)/a*
*2 + 2*(-a*q + b*p - f**2*p)*log(1/sqrt(a*x + b))/a**2
```

Giac [A]
time = 2.44, size = 61, normalized size = 1.13

$$\frac{2(f^2p-bp+aq)\log\left(\left|f+\sqrt{ax+b}\right|\right)}{a^2}-\frac{2\sqrt{ax+b}\,a^2fp-(ax+b)a^2p}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a^
2*f*p - (a*x + b)*a^2*p)/a^4
```

Mupad [B]
time = 3.10, size = 50, normalized size = 0.93

$$\frac{\ln\left(f+\sqrt{b+a\,x}\right)\,\left(2\,p\,f^2+2\,a\,q-2\,b\,p\right)}{a^2}+\frac{p\,x}{a}-\frac{2\,f\,p\,\sqrt{b+a\,x}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)
```

```
[Out] (log(f + (b + a*x)^(1/2))*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*p
*(b + a*x)^(1/2))/a^2
```

3.723 $\int \sqrt{1-\sqrt{x}-x}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{4}(1+2\sqrt{x})\sqrt{1-\sqrt{x}-x}-\frac{2}{3}(1-\sqrt{x}-x)^{3/2}-\frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

```
[Out] -5/8*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))-2/3*(1-x-x^(1/2))^(3/2)-1/4*(1+2*x^(
1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1355, 654, 626, 633, 222}

$$-\frac{5}{8}\text{ArcSin}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)-\frac{2}{3}\left(-x-\sqrt{x}+1\right)^{3/2}-\frac{1}{4}\left(2\sqrt{x}+1\right)\sqrt{-x-\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x],x]
```

```
[Out] -1/4*((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x]) - (2*(1 - Sqrt[x] - x)^(3/2))/
3 - (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\,dx &= 2\text{Subst}\left(\int x\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \text{Subst}\left(\int \sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right) \\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-x-x^2}} \right. \\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} + \frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^{}}{5}}} \right. \\
&= -\frac{1}{4}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x} - \frac{2}{3}\left(1-\sqrt{x}-x\right)^{3/2} - \frac{5}{8}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.91

$$\frac{1}{12}\sqrt{1-\sqrt{x}-x}\left(-11+2\sqrt{x}+8x\right) - \frac{5}{4}\tan^{-1}\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x],x]
[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 - (5*ArcTan[Sqrt[x]/(-1
+ Sqrt[1 - Sqrt[x] - x])])/4
```

Maple [A]
time = 0.02, size = 50, normalized size = 0.71

method	result	size
derivativedivides	$-\frac{2\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50
default	$-\frac{2\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{3}+\frac{\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{4}-\frac{5\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(
2/5*5^(1/2)*(x^(1/2)+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A]
time = 0.83, size = 84, normalized size = 1.20

$$\frac{1}{12}\left(8x+2\sqrt{x}-11\right)\sqrt{-x-\sqrt{x}+1}+\frac{5}{16}\arctan\left(-\frac{\left(8x^2-\left(16x^2-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^3-13x^2+7x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="fricas")
[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^
2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 -
 13*x^2 + 7*x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) - x + 1), x)
```

Giac [A]
time = 2.07, size = 44, normalized size = 0.63

$$\frac{1}{12}\left(2\sqrt{x}\left(4\sqrt{x}+1\right)-11\right)\sqrt{-x-\sqrt{x}+1}-\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1
/5*sqrt(5)*(2*sqrt(x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] int((1 - x^(1/2) - x)^(1/2), x)
```

3.724 $\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx$

Optimal. Leaf size=19

$$4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 1411, 785}

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Antiderivative was successfully verified.

[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]

[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1411

```
Int[((a_.) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g
 - 1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x
]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x}\,dx &= \int \frac{\left(3+\sqrt{x}\right)^2}{4\sqrt{x}+x}\,dx \\
&= 2\text{Subst}\left(\int \frac{x(3+x)^2}{4x+x^2}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(2+x+\frac{1}{4+x}\right)\,dx, x, \sqrt{x}\right) \\
&= 4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$4\sqrt{x}+x+2\log\left(4+\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]
```

```
[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]
```

Maple [A]
time = 0.41, size = 16, normalized size = 0.84

method	result	size
derivativedivides	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
default	$x+2\ln\left(4+\sqrt{x}\right)+4\sqrt{x}$	16
trager	$-1+x+4\sqrt{x}+\ln\left(8\sqrt{x}+16+x\right)$	18
meijerg	$2\ln\left(1+\frac{\sqrt{x}}{4}\right)-\frac{4\sqrt{x}\left(-\frac{3\sqrt{x}}{4}+6\right)}{3}+12\sqrt{x}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+x+6*x^(1/2))/(x+4*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x+2*ln(4+x^(1/2))+4*x^(1/2)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.79

$$x+4\sqrt{x}+2\log\left(\sqrt{x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Sympy [A]
time = 0.06, size = 17, normalized size = 0.89

$$4\sqrt{x} + x + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

Giac [A]
time = 2.20, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log\left(\sqrt{x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 2*log(sqrt(x) + 4)
```

Mupad [B]
time = 3.04, size = 15, normalized size = 0.79

$$x + 2\ln\left(\sqrt{x} + 4\right) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)),x)
```

```
[Out] x + 2*log(x^(1/2) + 4) + 4*x^(1/2)
```

3.725 $\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx$

Optimal. Leaf size=77

$$-\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

```
[Out] 125000/4782969*x+50000/1594323*x^(3/2)+2500/59049*x^2+400/6561*x^(5/2)+200/
2187*x^3+80/567*x^(7/2)+2/9*x^4-280728140/387420489*ln(5-9*x^(1/2))-5614562
8/43046721*x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1907, 196, 45, 272}

$$\frac{80x^{7/2}}{567}+\frac{400x^{5/2}}{6561}+\frac{50000x^{3/2}}{1594323}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\log\left(5-9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323
 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/56
7 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1907

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{6-8x^{7/2}}{5-9\sqrt{x}}\,dx &= \int \left(-\frac{6}{-5+9\sqrt{x}}+\frac{8x^{7/2}}{-5+9\sqrt{x}}\right)dx\\
&= -\left(6\int \frac{1}{-5+9\sqrt{x}}\,dx\right)+8\int \frac{x^{7/2}}{-5+9\sqrt{x}}\,dx\\
&= -\left(12\text{Subst}\left(\int \frac{x}{-5+9x}\,dx,x,\sqrt{x}\right)\right)+16\text{Subst}\left(\int \frac{x^8}{-5+9x}\,dx,x,\sqrt{x}\right)\\
&= -\left(12\text{Subst}\left(\int \left(\frac{1}{9}+\frac{5}{9(-5+9x)}\right)dx,x,\sqrt{x}\right)\right)+16\text{Subst}\left(\int \left(\frac{78125}{43046721}+\frac{15625x}{4782969}\right.\right.\\
&= -\frac{56145628\sqrt{x}}{43046721}+\frac{125000x}{4782969}+\frac{50000x^{3/2}}{1594323}+\frac{2500x^2}{59049}+\frac{400x^{5/2}}{6561}+\frac{200x^3}{2187}+\frac{80x^{7/2}}{567}+\frac{2x^4}{9}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.87

$$\frac{2\sqrt{x}\left(-196509698+3937500\sqrt{x}+4725000x+6378750x^{3/2}+9185400x^2+13778100x^{5/2}+21257640x^3+33480783x^{7/2}\right)}{301327047}-\frac{280728140\log\left(-5+9\sqrt{x}\right)}{387420489}$$

Antiderivative was successfully verified.

```
[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]),x]
```

```
[Out] (2*Sqrt[x]*(-196509698 + 3937500*Sqrt[x] + 4725000*x + 6378750*x^(3/2) + 91
85400*x^2 + 13778100*x^(5/2) + 21257640*x^3 + 33480783*x^(7/2)))/301327047
- (280728140*Log[-5 + 9*Sqrt[x]])/387420489
```

Maple [A]
time = 0.40, size = 50, normalized size = 0.65

method	result
derivativedivides	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln\left(-5+9\right.}{387420489}$
default	$\frac{2x^4}{9}+\frac{80x^{\frac{7}{2}}}{567}+\frac{200x^3}{2187}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{2500x^2}{59049}+\frac{50000x^{\frac{3}{2}}}{1594323}+\frac{125000x}{4782969}-\frac{56145628\sqrt{x}}{43046721}-\frac{280728140\ln\left(-5+9\right.}{387420489}$
trager	$\frac{2(531441x^3+750141x^2+851391x+913891)(-1+x)}{4782969}+2\left(\frac{40}{567}x^3+\frac{200}{6561}x^2+\frac{25000}{1594323}x-\frac{28072814}{43046721}\right)\sqrt{x}-$

meijerg	$-\frac{4\sqrt{x}}{3}-\frac{280728140\ln\left(1-\frac{9\sqrt{x}}{5}\right)}{387420489}+\frac{31250\sqrt{x}\left(\frac{301327047x^{\frac{7}{2}}}{15625}+\frac{38263752x^{3}}{3125}+\frac{4960116x^{\frac{5}{2}}}{625}+\frac{3306744x^{2}}{625}+\frac{91854x}{25}\right.}{2711943423}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1
594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/3874204
89*ln(-5+9*x^(1/2))
```

Maxima [A]
time = 0.30, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4+\frac{80}{567}x^{\frac{7}{2}}+\frac{200}{2187}x^3+\frac{400}{6561}x^{\frac{5}{2}}+\frac{2500}{59049}x^2+\frac{50000}{1594323}x^{\frac{3}{2}}+\frac{125000}{4782969}x-\frac{56145628}{43046721}\sqrt{x}-\frac{280728140}{387420489}\log\left(9\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(9*sqrt(x) - 5)
```

Fricas [A]
time = 0.37, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4+\frac{200}{2187}x^3+\frac{2500}{59049}x^2+\frac{4}{301327047}\left(10628820x^3+4592700x^2+2362500x-98254849\right)\sqrt{x}+\frac{125000}{4782969}x-\frac{280728140}{387420489}\log\left(9\sqrt{x}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 45927
00*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/38742
0489*log(9*sqrt(x) - 5)
```

Sympy [A]
time = 0.39, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567}+\frac{400x^{\frac{5}{2}}}{6561}+\frac{50000x^{\frac{3}{2}}}{1594323}-\frac{56145628\sqrt{x}}{43046721}+\frac{2x^4}{9}+\frac{200x^3}{2187}+\frac{2500x^2}{59049}+\frac{125000x}{4782969}-\frac{280728140\log\left(9\sqrt{x}-5\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)
```

```
[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqr
t(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/47829
69 - 280728140*log(9*sqrt(x) - 5)/387420489
```

Giac [A]
time = 2.18, size = 50, normalized size = 0.65

$$\frac{2}{9}\,x^{4}+\frac{80}{567}\,x^{\frac{7}{2}}+\frac{200}{2187}\,x^{3}+\frac{400}{6561}\,x^{\frac{5}{2}}+\frac{2500}{59049}\,x^{2}+\frac{50000}{1594323}\,x^{\frac{3}{2}}+\frac{125000}{4782969}\,x-\frac{56145628}{43046721}\,\sqrt{x}-\frac{280728140}{387420489}\,\log\left(|9\,\sqrt{x}-5|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2
 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 2
80728140/387420489*log(abs(9*sqrt(x) - 5))
```

Mupad [B]
time = 0.05, size = 47, normalized size = 0.61

$$\frac{125000\,x}{4782969}-\frac{280728140\,\ln\left(\sqrt{x}-\frac{5}{9}\right)}{387420489}+\frac{2500\,x^2}{59049}-\frac{56145628\,\sqrt{x}}{43046721}+\frac{200\,x^3}{2187}+\frac{2\,x^4}{9}+\frac{50000\,x^{3/2}}{1594323}+\frac{400\,x^{5/2}}{6561}+\frac{80\,x^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)
```

```
[Out] (125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)/
59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000*x
^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567
```

3.726 $\int \frac{\sqrt{1+x}\,(1+x^3)}{1+x^2}\,dx$

Optimal. Leaf size=80

$$-2\sqrt{1+x}-\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}+(1-i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1+i}}\right)$$

```
[Out] -2/3*(1+x)^(3/2)+2/5*(1+x)^(5/2)+(1-I)^(3/2)*arctanh((1+x)^(1/2)/(1-I)^(1/2
))+(1+I)^(3/2)*arctanh((1+x)^(1/2)/(1+I)^(1/2))-2*(1+x)^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 224 vs. 2(80) = 160. time = 0.21, antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1639, 1643, 839, 12, 722, 1108, 648, 632, 210, 642}

$$-\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{x+1}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\sqrt{1+\sqrt{2}}\,\text{ArcTan}\left(\frac{2\sqrt{x+1}+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}-2\sqrt{x+1}-\frac{\log\left(x-\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}+\frac{\log\left(x+\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+1}+\sqrt{2}+1\right)}{2\sqrt{1+\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[
2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]]
+ Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(
-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/
(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1
+ x]]/(2*Sqrt[1 + Sqrt[2]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 722

```
Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*
e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],
x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 839

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[(d + e*x)^(m -
 1)*(Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]/(a + c*x^2)), x], x] /; Fre
eQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m
, 0]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1639

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p,
x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemaind
er[Pq, d + e*x, x], 0]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sqrt{1+x}\,(1+x^3)}{1+x^2}\,dx = \int \frac{(1+x)^{3/2}\,(1-x+x^2)}{1+x^2}\,dx$$

$$= \int \left((1+x)^{3/2} - \frac{x(1+x)^{3/2}}{1+x^2}\right)\,dx$$

$$= \frac{2}{5}(1+x)^{5/2} - \int \frac{x(1+x)^{3/2}}{1+x^2}\,dx$$

$$= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int \frac{(-1+x)\sqrt{1+x}}{1+x^2}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int -\frac{2}{\sqrt{1+x}\,(1+x^2)}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 2\int \frac{1}{\sqrt{1+x}\,(1+x^2)}\,dx$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 4\text{Subst}\left(\int \frac{1}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right.$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{\sqrt{2\left(1+\sqrt{2}\right)}-x}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,d\right.}{\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-\sqrt{2\left(1+\sqrt{2}\right)}\,x+x^2}\,d\right.}{\sqrt{2}}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\log\left(1+\sqrt{2}+x-\sqrt{2\left(1+\sqrt{2}\right)}\right.}{2\sqrt{1+\sqrt{2}}}$$

$$= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2\sqrt{1+x}}{\sqrt{2\left(-1+\sqrt{2}\right)}}\right)}{\sqrt{-1+\sqrt{2}}}$$

Mathematica [A]

time = 0.11, size = 76, normalized size = 0.95

$$\frac{2}{15}\sqrt{1+x}\,(-17+x+3x^2)+\sqrt{2+2i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}-\frac{i}{2}}\,\sqrt{1+x}\right)+\sqrt{2-2i}\,\tan^{-1}\left(\sqrt{-\frac{1}{2}+\frac{i}{2}}\,\sqrt{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2),x]
```

```
[Out] (2*Sqrt[1 + x]*(-17 + x + 3*x^2))/15 + Sqrt[2 + 2*I]*ArcTan[Sqrt[-1/2 - I/2
]*Sqrt[1 + x]] + Sqrt[2 - 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(58) = 116$.
time = 0.64, size = 289, normalized size = 3.61

method	result
derivativedivides	$\frac{2(1+x)^{\frac{5}{2}}}{5}-\frac{2(1+x)^{\frac{3}{2}}}{3}-2\sqrt{1+x}+\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+x+\sqrt{2}+\sqrt{1+\right.}{4}$
default	$\frac{2(1+x)^{\frac{5}{2}}}{5}-\frac{2(1+x)^{\frac{3}{2}}}{3}-2\sqrt{1+x}+\frac{\left(-\sqrt{2+2\sqrt{2}}\,\sqrt{2}+2\sqrt{2+2\sqrt{2}}\right)\ln\left(1+x+\sqrt{2}+\sqrt{1+\right.}{4}$
trager	$\left(\frac{2}{5}x^2+\frac{2}{15}x-\frac{34}{15}\right)\sqrt{1+x}+\mathrm{RootOf}\left(\_Z^2+16\,\mathrm{RootOf}\left(512\_Z^4+32\_Z^2+1\right)^2+1\right)\mathrm{l}$
risch	$\frac{2(3x^2+x-17)\sqrt{1+x}}{15}+\frac{\ln\left(1+x+\sqrt{2}-\sqrt{1+x}\,\sqrt{2+2\sqrt{2}}\right)\sqrt{2+2\sqrt{2}}\,\sqrt{2}}{4}-\frac{\ln\left(1+x+\sqrt{\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*(1+x)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(1+x)^(5/2)-2/3*(1+x)^(3/2)-2*(1+x)^(1/2)+1/4*(-(2+2*2^(1/2))^(1/2)*2^(
1/2)+2*(2+2*2^(1/2))^(1/2))*ln(1+x+2^(1/2)+(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))
+(2*2^(1/2)-1/2*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2
^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)+(2+2*2^(1/2))^(1/
2))/(-2+2*2^(1/2))^(1/2))-1/4*(-(2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))
^(1/2))*ln(1+x+2^(1/2)-(1+x)^(1/2)*(2+2*2^(1/2))^(1/2))-(-2*2^(1/2)+1/2*(-(
2+2*2^(1/2))^(1/2)*2^(1/2)+2*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2))/(-2+
2*2^(1/2))^(1/2)*arctan((2*(1+x)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^
(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(50) = 100.
time = 0.36, size = 302, normalized size = 3.78

$$-\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)+\frac{1}{8}\cdot 8^{\frac{1}{4}}\sqrt{2\sqrt{2}+4}\left(\sqrt{2}-2\right)\log\left(-2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4\right)-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{16}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4}\sqrt{2\sqrt{2}+4}-\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{x+1}\sqrt{2\sqrt{2}+4}-\sqrt{2}-1\right)-\frac{1}{2}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2\sqrt{2}+4}\arctan\left(\frac{1}{16}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{-2\cdot 8^{\frac{1}{4}}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+4x+4\sqrt{2}+4}\sqrt{2\sqrt{2}+4}-\frac{1}{8}\cdot 8^{\frac{3}{4}}\sqrt{2}\sqrt{x+1}\sqrt{2\sqrt{2}+4}+\sqrt{2}+1\right)+\frac{2}{15}\left(3x^{2}+x-17\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*8^(1/4)*sqrt(x + 1)*sq
rt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 2)*log(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt
(2) + 4) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt
(2)*sqrt(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*s
qrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) -
sqrt(2) - 1) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*
sqrt(2)*sqrt(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) +
 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) +
4) + sqrt(2) + 1) + 2/15*(3*x^2 + x - 17)*sqrt(x + 1)
```

Sympy [A]
time = 5.72, size = 56, normalized size = 0.70

$$\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}+4\operatorname{RootSum}\left(512t^{4}+32t^{2}+1,\left(t\mapsto t\log\left(-128t^{3}+\sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)
```

```
[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootSum(512*_t*
*4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(x + 1))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(50) = 100.
time = 2.18, size = 171, normalized size = 2.14

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}+\sqrt{\sqrt{2}+1}\arctan\left(\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}+2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\sqrt{\sqrt{2}+1}\arctan\left(-\frac{2^{\frac{3}{4}}\left(2^{\frac{1}{4}}\sqrt{\sqrt{2}+2}-2\sqrt{x+1}\right)}{2\sqrt{-\sqrt{2}+2}}\right)+\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)-\frac{1}{2}\sqrt{\sqrt{2}-1}\log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{\sqrt{2}+2}+x+\sqrt{2}+1\right)-2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4
)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sq
rt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/
sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sq
rt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x +
 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)
```

Mupad [B]
time = 0.10, size = 255, normalized size = 3.19

$$\frac{2\,{\left(x+1\right)}^{5/2}}{5}-\frac{2\,{\left(x+1\right)}^{3/2}}{3}-2\,\sqrt{x+1}-\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}-\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}-64}\right)\,\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}+\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}+\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{x+1}\,64{}\mathrm{i}}{256\,\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}+64}\right)\,\left(\sqrt{-\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}-\sqrt{\frac{\sqrt{2}}{4}-\frac{1}{4}}\,2{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)
```

```
[Out] (2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2)
*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*
(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^
(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((
- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1/2)*(
- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(-
 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1
/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64))*((-
2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)
```

3.727 $\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx$

Optimal. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)-\tanh^{-1}\left(\frac{1+3\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right)$$

```
[Out] arctan(1/2*(3-x^(1/2))/(-1+x-x^(1/2))^(1/2))-2*arctanh(1/2*(1-2*x^(1/2))/(-
1+x-x^(1/2))^(1/2))-arctanh(1/2*(1+3*x^(1/2))/(-1+x-x^(1/2))^(1/2))
```

Rubi [A]
time = 0.19, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1004, 635, 212, 1047, 738, 210}

$$\text{ArcTan}\left(\frac{3-\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x-\sqrt{x}-1}}\right)-\tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x-\sqrt{x}-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x]
)/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x
] + x])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{\sqrt{-1-x+x^2}}{-1+x^2}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \frac{1}{\sqrt{-1-x+x^2}}\,dx, x, \sqrt{x}\right) - 2\text{Subst}\left(\int \frac{x}{(-1+x^2)\sqrt{-1-x+x^2}}\right. \\
&= 4\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2\sqrt{x}}{\sqrt{-1-\sqrt{x}+x}}\right) - \text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1-x+}}\right. \\
&= -2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) + 2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3+\sqrt{x}}{\sqrt{-1-\sqrt{x}+}}\right. \\
&= \tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - 2\tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{-1-\sqrt{x}+x}}\right) - \tanh^{-1}\left(\right.
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 81, normalized size = 0.91

$$-2\tan^{-1}\left(1-\sqrt{x}+\sqrt{-1-\sqrt{x}+x}\right)-2\tanh^{-1}\left(1+\sqrt{x}-\sqrt{-1-\sqrt{x}+x}\right)-2\log\left(1-2\sqrt{x}+2\sqrt{-1-\sqrt{x}+x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]),x]

[Out] -2*ArcTan[1 - Sqrt[x] + Sqrt[-1 - Sqrt[x] + x]] - 2*ArcTanh[1 + Sqrt[x] - Sqrt[-1 - Sqrt[x] + x]] - 2*Log[1 - 2*Sqrt[x] + 2*Sqrt[-1 - Sqrt[x] + x]]

Maple [A]
time = 0.42, size = 130, normalized size = 1.46

method	result
derivativedivides	$-\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}\right)}{2}+\operatorname{arctanh}$
default	$-\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}+\frac{3\ln\left(-\frac{1}{2}+\sqrt{x}+\sqrt{\left(1+\sqrt{x}\right)^{2}-3\sqrt{x}-2}\right)}{2}+\operatorname{arctanh}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x,method=_RETURNVERBOSE)

[Out] -((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2))+arctanh(1/2*(-1-3*x^(1/2))/((1+x^(1/2))^2-3*x^(1/2)-2)^(1/2))+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2)+1/2*ln(-1/2+x^(1/2)+((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))-arctan(1/2*(-3+x^(1/2))/((-1+x^(1/2))^2+x^(1/2)-2)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)

Fricas [A]
time = 2.04, size = 87, normalized size = 0.98

$$-\arctan\left(\frac{\left((x-4)\sqrt{x}-2x+3\right)\sqrt{x-\sqrt{x}-1}}{2\left(x^{2}-3x+1\right)}\right)+\log\left(-\frac{8x^{2}+2\left((4x-5)\sqrt{x}+2x-1\right)\sqrt{x-\sqrt{x}-1}-17x-2\sqrt{x}+11}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")

```
[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x +
1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) -
 17*x - 2*sqrt(x) + 11)/(x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{x} + x - 1}}{\sqrt{x}\,(x - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)
```

```
[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)
```

Giac [A]
time = 2.59, size = 81, normalized size = 0.91

$$-2\arctan\left(\sqrt{x-\sqrt{x}-1}-\sqrt{x}+1\right)-\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}+2\right)+\log\left(-\sqrt{x-\sqrt{x}-1}+\sqrt{x}\right)-2\log\left(\left|2\sqrt{x-\sqrt{x}-1}-2\sqrt{x}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1)
 + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqrt(
x - sqrt(x) - 1) - 2*sqrt(x) + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x-\sqrt{x}-1}}{\sqrt{x}\,\left(x-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)
```

```
[Out] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)
```

3.728 $\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=61

$$-\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)+3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

```
[Out] -arctan(1/2*(3+(1+x)^(1/2))/(x+(1+x)^(1/2))^(1/2))+3*arctanh(1/2*(1-3*(1+x)
^(1/2))/(x+(1+x)^(1/2))^(1/2))
```

Rubi [A]
time = 0.36, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1047, 738, 212, 210}

$$3\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)-\text{ArcTan}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
```

```
[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqr
t[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}}\,dx = 2\text{Subst}\left(\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)$$
$$= 3\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right) + \text{Subst}\left(\int \frac{}{(1+x}\right.$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)\right) - 6\text{Subst}\left(\int \frac{1}{4-x^2}\right.$$
$$= -\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right) + 3\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)$$

Mathematica [A]
time = 0.12, size = 55, normalized size = 0.90

$$-2\tan^{-1}\left(1+\sqrt{1+x}-\sqrt{x+\sqrt{1+x}}\right) - 6\tanh^{-1}\left(1-\sqrt{1+x}+\sqrt{x+\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]),x]
[Out] -2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 6*ArcTanh[1 - Sqrt[1 +
 x] + Sqrt[x + Sqrt[1 + x]]]
```

Maple [A]
time = 0.39, size = 68, normalized size = 1.11

method	result
derivativedivides	$-3\,\text{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(\sqrt{1+x}-1\right)^2+3\sqrt{1+x}-2}}\right)+\text{arctan}\left(\frac{-3-\sqrt{1+x}}{2\sqrt{\left(\sqrt{1+x}+1\right)^2-}}\right.$

default	$-3\,\mathrm{arctanh}\left(\frac{-1+3\sqrt{1+x}}{2\sqrt{\left(\sqrt{1+x}-1\right)^{2}+3\sqrt{1+x}-2}}\right)+\arctan\left(\frac{-3-\sqrt{1+}}{2\sqrt{\left(\sqrt{1+x}+1\right)^{2}-}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -3*arctanh(1/2*(-1+3*(1+x)^(1/2))/(((1+x)^(1/2)-1)^2+3*(1+x)^(1/2)-2)^(1/2)
)+arctan(1/2*(-3-(1+x)^(1/2))/(((1+x)^(1/2)+1)^2-(1+x)^(1/2)-2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)
```

Fricas [A]
time = 1.56, size = 62, normalized size = 1.02

$$\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+3\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3x-2\sqrt{x+1}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="fricas")
```

```
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x
 + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)
```

Giac [A]
time = 2.07, size = 65, normalized size = 1.07

$$2\arctan\left(\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}-1\right)-3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}+2\right|\right)+3\log\left(\left|\sqrt{x+\sqrt{x+1}}-\sqrt{x+1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorith
m="giac")
```

```
[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqrt
(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1
)))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}\,\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)
```

3.729 $\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx$

Optimal. Leaf size=8

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {56, 221}

$$2\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 2.00

$$2\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

```
[Out] 2*ArcTanh[Sqrt[x]/Sqrt[1 + x]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.
time = 0.38, size = 28, normalized size = 3.50

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{x\left(1+x\right)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)}{\sqrt{x}\,\sqrt{1+x}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.
time = 0.28, size = 27, normalized size = 3.38

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)-\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(6) = 12$.
time = 0.33, size = 18, normalized size = 2.25

$$-\log\left(2\sqrt{x+1}\,\sqrt{x}-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.46, size = 26, normalized size = 3.25

$$\begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for}\ |x+1|>1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 1.57, size = 14, normalized size = 1.75

$$-2 \log \left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(x + 1) - sqrt(x))
```

Mupad [B]
time = 0.16, size = 14, normalized size = 1.75

$$4 \operatorname{atanh}\left(\frac{\sqrt{x+1}-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] 4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

3.730 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

Optimal. Leaf size=8

$$2 \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arcsinh(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1978, 56, 221}

$$2 \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcSinh[Sqrt[x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx &= \int \frac{1}{\sqrt{x} \sqrt{1+x}} \, dx \\
&= 2\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \sqrt{x}\right) \\
&= 2 \sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 14, normalized size = 1.75

$$2\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)]/x,x]
```

```
[Out] 2*ArcTanh[Sqrt[x/(1 + x)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(6) = 12$.
time = 0.06, size = 32, normalized size = 4.00

method	result	size
default	$\frac{\sqrt{\frac{x}{1+x}}\,(1+x)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x\,(1+x)}}$	32
trager	$-\ln\left(2\sqrt{\frac{x}{1+x}}\,x+2\sqrt{\frac{x}{1+x}}-2x-1\right)$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (x/(1+x))^(1/2)*(1+x)/(x*(1+x))^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.
time = 0.28, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.
time = 0.35, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.
time = 2.34, size = 22, normalized size = 2.75

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

Mupad [B]
time = 0.06, size = 12, normalized size = 1.50

$$2\,\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2)/x,x)
```

```
[Out] 2*atanh((x/(x + 1))^(1/2))
```

3.731 $\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=22

$$\sqrt{x}\,\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {52, 56, 221}

$$\sqrt{x}\,\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx &= \sqrt{x}\,\sqrt{1+x}-\frac{1}{2}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx\\
&= \sqrt{x}\,\sqrt{1+x}-\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\,\sqrt{1+x}-\sinh^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.
time = 0.04, size = 48, normalized size = 2.18

$$
\frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}\,(1+x)-\sqrt{1+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcTanh[Sqrt[x/(1 + x)]]))/
Sqrt[x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(16) = 32.
time = 0.38, size = 39, normalized size = 1.77

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	27
default	$\sqrt{x}\,\sqrt{1+x}-\frac{\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{1+x}}$	39
risch	$\sqrt{x}\,\sqrt{1+x}-\frac{\sqrt{x\,(1+x)}\,\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{1+x}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^(1/2)*(1+x)^(1/2)-1/2*(x*(1+x))^(1/2)/x^(1/2)/(1+x)^(1/2)*ln(x+1/2+(x^2+x
)^(1/2))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

time = 0.28, size = 49, normalized size = 2.23

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)}-\frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A]
time = 0.34, size = 28, normalized size = 1.27

$$\sqrt{x+1}\,\sqrt{x}+\frac{1}{2}\log\left(2\sqrt{x+1}\,\sqrt{x}-2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.83, size = 63, normalized size = 2.86

$$\begin{cases}\sqrt{x}\,\sqrt{x+1}-\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1\\ i\operatorname{asin}\left(\sqrt{x+1}\right)-\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{-x}}+\frac{i\sqrt{x+1}}{\sqrt{-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x)*sqrt(x + 1) - acosh(sqrt(x + 1)), Abs(x + 1) > 1), (I*as
in(sqrt(x + 1)) - I*(x + 1)**(3/2)/sqrt(-x) + I*sqrt(x + 1)/sqrt(-x), True)
)
```

Giac [A]
time = 1.49, size = 22, normalized size = 1.00

$$\sqrt{x+1}\,\sqrt{x}+\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

Mupad [B]
time = 3.72, size = 26, normalized size = 1.18

$$\sqrt{x}\ \sqrt{x+1}\ -2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}\ -1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + 1)^(1/2),x)
[Out] x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

3.732 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{x}\,\sqrt{1+x}\ -\sinh^{-1}\left(\sqrt{x}\,\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1978, 52, 56, 221}

$$\sqrt{x}\,\sqrt{x+1}\ -\sinh^{-1}\left(\sqrt{x}\,\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{x}{1+x}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= \sqrt{x}\,\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\,\sqrt{1+x}}\,dx\\ &= \sqrt{x}\,\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{x}\,\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.
time = 0.00, size = 48, normalized size = 2.18

$$\frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}\,(1+x) - \sqrt{1+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcTanh[Sqrt[x/(1 + x)]]))/
Sqrt[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.
time = 0.06, size = 45, normalized size = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{1+x}}\,(1+x)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{x\,(1+x)}}$	45
risch	$(1+x)\,\sqrt{\frac{x}{1+x}} - \frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x}{1+x}}\,\sqrt{x\,(1+x)}}{2x}$	47
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{\frac{x}{1+x}} + \frac{\ln\left(2\sqrt{\frac{x}{1+x}}\,x+2\sqrt{\frac{x}{1+x}}-2x-1\right)}{2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/(x*(1+x
))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.
time = 0.27, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.
time = 0.33, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 2.19, size = 35, normalized size = 1.59

$$\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

Mupad [B]
time = 3.12, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
```

```
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.733 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {96, 94, 209}

$$\text{ArcTan}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right)$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

Mathematica [A]
time = 0.11, size = 66, normalized size = 1.83

$$-\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}\,(1+x)+2x\sqrt{1+x}\,\tan^{-1}\left(x-\sqrt{-1+x}\,\sqrt{1+x}\right)\right)}{\sqrt{-1+x}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
- Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Maple [A]
time = 0.40, size = 43, normalized size = 1.19

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{-1+x}\,\sqrt{1+x}}{x\sqrt{x^2-1}}$	43
risch	$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(1+x)(-1+x)}}{\sqrt{-1+x}\,\sqrt{1+x}}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/x^2/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(-1+x)^(1/2)*(1+x)^(1/2)/x/(x^2-
1)^(1/2)
```

Maxima [A]
time = 0.50, size = 20, normalized size = 0.56

$$-\frac{\sqrt{x^2-1}}{x}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

Fricas [A]
time = 0.36, size = 39, normalized size = 1.08

$$\frac{2\,x\arctan\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)
[Out] Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

Giac [A]
time = 1.85, size = 42, normalized size = 1.17

$$-\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4}-2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x
 - 1))^2)
```

Mupad [B]
time = 5.09, size = 138, normalized size = 3.83

$$-\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\,1\mathrm{i}-\frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)}-\frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
```

```
[Out] log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i)
^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2) -
 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x - 1)^
(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1)^(1
/2) - 1))
```

3.734 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$

Optimal. Leaf size=36

$$-\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

```
[Out] arctan((-1+x)^(1/2)*(1+x)^(1/2))-(-1+x)^(1/2)*(1+x)^(1/2)/x
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1978, 96, 94, 209}

$$\text{ArcTan}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx &= \int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x}\,\sqrt{1+x}\right)\\ &= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 66, normalized size = 1.83

$$-\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}\,(1+x)+2x\sqrt{1+x}\,\tan^{-1}\left(x-\sqrt{-1+x}\,\sqrt{1+x}\right)\right)}{\sqrt{-1+x}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
[Out] -((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
- Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(28) = 56.
time = 0.12, size = 59, normalized size = 1.64

method	result
risch	$-\frac{(1+x)\sqrt{\frac{-1+x}{1+x}}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{\frac{-1+x}{1+x}}\,\sqrt{(1+x)(-1+x)}}{-1+x}$
default	$\frac{\sqrt{\frac{-1+x}{1+x}}\,(1+x)\left((x^2-1)^{\frac{3}{2}}-x^2\sqrt{x^2-1}-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x\right)}{\sqrt{(1+x)(-1+x)}\,x}$

trager	$-\frac{(1+x)\sqrt{-\frac{1-x}{1+x}}}{x}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1-x}{1+x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1-x}{1+x}}\,-1}{x}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)/(1+x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-1+x)/(1+x))^(1/2)*(1+x)*((x^2-1)^(3/2)-x^2*(x^2-1)^(1/2)-arctan(1/(x^2-1
)^(1/2))*x)/((1+x)*(-1+x))^(1/2)/x
```

Maxima [A]
time = 0.50, size = 41, normalized size = 1.14

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}+2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x +
 1)))
```

Fricas [A]
time = 0.33, size = 36, normalized size = 1.00

$$\frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)-(x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

Giac [A]
time = 2.50, size = 51, normalized size = 1.42

$$-\frac{1}{2}(\pi-2)\mathrm{sgn}(x+1)+2\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sgn}(x+1)-\frac{2\,\mathrm{sgn}(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn(
x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

Mupad [B]
time = 0.06, size = 41, normalized size = 1.14

$$2\,\mathrm{atan}\left(\sqrt{\frac{x-1}{x+1}}\right)-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

```
[Out] 2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x +
 1) + 1)
```

3.735 $\int \frac{\sqrt{-1+x}\, x^3}{\sqrt{1+x}}\, dx$

Optimal. Leaf size=69

$$-\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {102, 152, 52, 54}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx\\ &= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx\\ &= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\\ &= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\end{aligned}$$

Mathematica [A]
time = 0.11, size = 74, normalized size = 1.07

$$\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}\,(-16-7x+x^2-2x^3+6x^4)+18\sqrt{1+x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\right)}{24\sqrt{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) + 1
8*Sqrt[1 + x]*ArcTanh[1/Sqrt[(-1 + x)/(1 + x)]]))/(24*Sqrt[-1 + x])
```

Maple [A]
time = 0.41, size = 76, normalized size = 1.10

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)\sqrt{1+x}\,\sqrt{-1+x}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(1+x)\,(-1+x)}}{8\sqrt{-1+x}\,\sqrt{1+x}}$	60
default	$\frac{\sqrt{-1+x}\,\sqrt{1+x}\,\left(6x^3\sqrt{x^2-1}-8x^2\sqrt{x^2-1}+9x\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(-1+x)^(1/2)*(1+x)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*x^2*(x^2-1)^(1/2)+9*x*
(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(x^2-1)^(1/2)
```

Maxima [A]
time = 0.30, size = 55, normalized size = 0.80

$$\frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x-\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{x^2-1}\,x-\sqrt{x^2-1}+\frac{3}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^
2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A]
time = 0.34, size = 46, normalized size = 0.67

$$\frac{1}{24}\left(6\,x^3-8\,x^2+9\,x-16\right)\sqrt{x+1}\,\sqrt{x-1}-\frac{3}{8}\log\left(\sqrt{x+1}\,\sqrt{x-1}-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
1)*sqrt(x - 1) - x)
```

Sympy [A]
time = 5.16, size = 83, normalized size = 1.20

$$\frac{(x-1)^{\frac{7}{2}}\sqrt{x+1}}{4}+\frac{5(x-1)^{\frac{5}{2}}\sqrt{x+1}}{12}+\frac{11(x-1)^{\frac{3}{2}}\sqrt{x+1}}{24}-\frac{3\sqrt{x-1}\,\sqrt{x+1}}{8}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 + 11*(x - 1)
**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*asinh(sqrt(2)*sqrt
(x - 1)/2)/4
```

Giac [A]
time = 1.82, size = 47, normalized size = 0.68

$$\frac{1}{24}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{x-1}-\frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3
/4*log(sqrt(x + 1) - sqrt(x - 1))
```

Mupad [B]
time = 12.86, size = 473, normalized size = 6.86

$$\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)}{2}+\frac{\frac{23\left(\sqrt{x-1}-\mathrm{i}\right)^3}{2\left(\sqrt{x+1}-1\right)^3}-\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^4 64\mathrm{i}}{\left(\sqrt{x+1}-1\right)^4}+\frac{333\left(\sqrt{x-1}-\mathrm{i}\right)^5}{2\left(\sqrt{x+1}-1\right)^5}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^6 256\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^6}+\frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^7}{2\left(\sqrt{x+1}-1\right)^7}-\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^8 128\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^8}+\frac{671\left(\sqrt{x-1}-\mathrm{i}\right)^9}{2\left(\sqrt{x+1}-1\right)^9}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^{10} 256\mathrm{i}}{3\left(\sqrt{x+1}-1\right)^{10}}+\frac{333\left(\sqrt{x-1}-\mathrm{i}\right)^{11}}{2\left(\sqrt{x+1}-1\right)^{11}}-\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^{12} 64\mathrm{i}}{\left(\sqrt{x+1}-1\right)^{12}}+\frac{23\left(\sqrt{x-1}-\mathrm{i}\right)^{13}}{2\left(\sqrt{x+1}-1\right)^{13}}-\frac{3\left(\sqrt{x-1}-\mathrm{i}\right)^{15}}{2\left(\sqrt{x+1}-1\right)^{15}}-\frac{3\left(\sqrt{x-1}-\mathrm{i}\right)}{2\left(\sqrt{x+1}-1\right)}}{1+\frac{28\left(\sqrt{x-1}-\mathrm{i}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^6}{\left(\sqrt{x+1}-1\right)^6}+\frac{70\left(\sqrt{x-1}-\mathrm{i}\right)^8}{\left(\sqrt{x+1}-1\right)^8}-\frac{56\left(\sqrt{x-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{x+1}-1\right)^{10}}+\frac{28\left(\sqrt{x-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{x+1}-1\right)^{12}}-\frac{8\left(\sqrt{x-1}-\mathrm{i}\right)^{14}}{\left(\sqrt{x+1}-1\right)^{14}}+\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^{16}}{\left(\sqrt{x+1}-1\right)^{16}}-\frac{8\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

```
[Out] (3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2)
 - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x + 1)
^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) + ((
(x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(1/2)
 - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3*((x
+ 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) - 1)^9)
 + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*((x - 1
)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i)^12*64i
)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1)^(1/2) -
 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15) - (3*((x -
 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) - 1i)^4)/((x
+ 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (56*
((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(1/2) - 1i)^8)
/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1)^(1/2) - 1)^1
0 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8*((x - 1)^(1/2)
 - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16/((x + 1)^(1/2)
- 1)^16 + 1)
```

3.736 $\int x^3 \sqrt{\frac{-1+x}{1+x}}\, dx$

Optimal. Leaf size=69

$$-\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3}{8}\cosh^{-1}(x)$$

```
[Out] 3/8*arccosh(x)+1/24*(7-2*x)*(-1+x)^(3/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(
1+x)^(1/2)-3/8*(-1+x)^(1/2)*(1+x)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1978, 102, 152, 52, 54}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}\,x^2+\frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{8}\sqrt{x-1}\,\sqrt{x+1}+\frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24
 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m +
 1)*((c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Dist[(a^2*d
^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m
+ n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n +
 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)),
Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}
, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx &= \int \frac{\sqrt{-1+x}\,x^3}{\sqrt{1+x}}\,dx \\
&= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{1}{4}\int \frac{(2-x)\sqrt{-1+x}\,x}{\sqrt{1+x}}\,dx \\
&= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}-\frac{3}{8}\int \frac{\sqrt{-1+x}}{\sqrt{1+x}}\,dx \\
&= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+ \\
&= -\frac{3}{8}\sqrt{-1+x}\,\sqrt{1+x}+\frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 74, normalized size = 1.07

$$
\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}\,(-16-7x+x^2-2x^3+6x^4)+18\sqrt{1+x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-1+x}{1+x}}}\right)\right)}{24\sqrt{-1+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

```
[Out] (Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) + 1
8*Sqrt[1 + x]*ArcTanh[1/Sqrt[(-1 + x)/(1 + x)]]))/(24*Sqrt[-1 + x])
```

Maple [A]
time = 0.10, size = 79, normalized size = 1.14

method	result	size
risch	$\frac{\left(6 x^{3}-8 x^{2}+9 x -16\right) \left(1+x \right) \sqrt{\frac{-1+x}{1+x}}}{24}+\frac{3 \ln \left(x +\sqrt{x^{2}-1}\right) \sqrt{\frac{-1+x}{1+x}}\, \sqrt{\left(1+x \right) \left(-1+x \right)}}{8 \left(-1+x \right)}$	70
trager	$\frac{\left(1+x \right) \left(6 x^{3}-8 x^{2}+9 x -16\right) \sqrt{-\frac{1-x}{1+x}}}{24}+\frac{3 \ln \left(\sqrt{-\frac{1-x}{1+x}}\, x +\sqrt{-\frac{1-x}{1+x}}+x \right)}{8}$	71
default	$-\frac{\sqrt{\frac{-1+x}{1+x}}\, \left(1+x \right) \left(-6 x \left(x^{2}-1\right)^{\frac{3}{2}}+8 \left(\left(1+x \right) \left(-1+x \right)\right)^{\frac{3}{2}}-15 x \sqrt{x^{2}-1}+24 \sqrt{x^{2}-1}-9 \ln \left(x +\sqrt{x^{2}-1}\right)\right)}{24 \sqrt{\left(1+x \right) \left(-1+x \right)}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((-1+x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*((-1+x)/(1+x))^(1/2)*(1+x)*(-6*x*(x^2-1)^(3/2)+8*((1+x)*(-1+x))^(3/2)
-15*x*(x^2-1)^(1/2)+24*(x^2-1)^(1/2)-9*ln(x+(x^2-1)^(1/2)))/((1+x)*(-1+x))^
(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(49) = 98$.
time = 0.27, size = 138, normalized size = 2.00

$$-\frac{39 \left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} - 31 \left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} + 49 \left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 9 \sqrt{\frac{x-1}{x+1}}}{12 \left(\frac{4 (x-1)}{x+1} - \frac{6 (x-1)^2}{(x+1)^2} + \frac{4 (x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} + \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1
)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^
2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sq
rt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.93

$$\frac{1}{24} \left(6 x^4 - 2 x^3 + x^2 - 7 x - 16\right) \sqrt{\frac{x-1}{x+1}} + \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="fricas")

[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)

[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)

Giac [A]
time = 2.88, size = 62, normalized size = 0.90

$$-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}(x+1)+\frac{1}{24}\left(\left(2\left(3x\operatorname{sgn}(x+1)-4\operatorname{sgn}(x+1)\right)x+9\operatorname{sgn}(x+1)\right)x-16\operatorname{sgn}(x+1)\right)\sqrt{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")

[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)

Mupad [B]
time = 0.05, size = 119, normalized size = 1.72

$$\frac{3\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}-\frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4}-\frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12}-\frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((x - 1)/(x + 1))^(1/2),x)

[Out] (3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (49*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x - 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)

3.737 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$

Optimal. Leaf size=15

$$2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)$$

```
[Out] 2*arctan((-x/(1+x))^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1980, 210}

$$2\text{ArcTan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))]/x,x]
```

```
[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1980

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Dist[q*e*(b*c - a*d), Subst[Int[x^(q*(
p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*x
)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p] &
& IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx &= -\left(2\text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)\\ &= 2\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(15) = 30.
time = 0.01, size = 37, normalized size = 2.47

$$\frac{2\sqrt{-\frac{x}{1+x}}\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)}{\sqrt{\frac{x}{1+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]
[Out] (2*Sqrt[-(x/(1 + x))]*ArcTanh[Sqrt[x/(1 + x)]])/Sqrt[x/(1 + x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.
time = 0.11, size = 33, normalized size = 2.20

method	result
default	$\frac{\sqrt{-\frac{x}{1+x}}\,(1+x)\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{\sqrt{x(1+x)}}$
trager	$-\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-2x\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{-\frac{x}{1+x}}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)+2\sqrt{-\frac{x}{1+x}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(1+x))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] (-x/(1+x))^(1/2)*(1+x)/(x*(1+x))^(1/2)*ln(x+1/2+(x^2+x)^(1/2))
```

Maxima [A]
time = 0.48, size = 13, normalized size = 0.87

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.87

$$2\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-x/(x + 1)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-x/(x + 1))/x, x)
```

Giac [**A**]
time = 3.94, size = 20, normalized size = 1.33

$$-\frac{1}{2}\,\pi\mathrm{sgn}(x+1) - \arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

Mupad [**B**]
time = 0.18, size = 13, normalized size = 0.87

$$2\,\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2)/x,x)
```

```
[Out] 2*atan((-x/(x + 1))^(1/2))
```

3.738 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} \, dx$

Optimal. Leaf size=18

$$2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

[Out] 2*arctan(((1-x)/(1+x))^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1983, 210}

$$2\text{ArcTan}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]

[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\left(4\text{Subst}\left(\int \frac{1}{-2-2x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.
time = 0.04, size = 51, normalized size = 2.83

$$-\frac{2\sqrt{\frac{1-x}{1+x}}\sqrt{1-x^2}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{1+x}}\right)}{-1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
[Out] (-2*Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/(-
1 + x)
```

Maple [A]
time = 0.36, size = 30, normalized size = 1.67

method	result
default	$-\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\arcsin(x)}{\sqrt{-(1+x)(-1+x)}}$
trager	$\text{RootOf}(\_Z^2+1)\ln\left(-\text{RootOf}(\_Z^2+1)\sqrt{-\frac{-1+x}{1+x}}\,x-\text{RootOf}(\_Z^2+1)\sqrt{-\frac{-1+x}{1+x}}+x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2)/(-1+x),x,method=_RETURNVERBOSE)
[Out] -(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(1+x)*(-1+x))^(1/2)*arcsin(x)
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.83

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="maxima")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Fricas [A]
time = 0.35, size = 15, normalized size = 0.83

$$2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="fricas")
[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

Giac [A]
time = 4.09, size = 16, normalized size = 0.89

$$-\frac{1}{2}\,\pi\mathrm{sgn}(x+1)-\arcsin\left(x\right)\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x),x, algorithm="giac")
[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

Mupad [B]
time = 3.14, size = 15, normalized size = 0.83

$$2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
[Out] 2*atan((-(x - 1)/(x + 1))^(1/2))
```

3.739 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx$

Optimal. Leaf size=24

$$\frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

```
[Out] 2*arctan(((b*x+a)/(-b*x+c))^(1/2))/b
```

Rubi [A]
time = 0.04, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1983, 12, 209}

$$\frac{2\text{ArcTan}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = (2b(a+c))\text{Subst}\left(\int \frac{1}{b^2(a+c)\,(1+x^2)}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)$$
$$= \frac{2\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$
$$= \frac{2\tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.
time = 0.07, size = 63, normalized size = 2.62

$$-\frac{2\sqrt{c-bx}\,\sqrt{\frac{a+bx}{c-bx}}\,\tan^{-1}\left(\frac{\sqrt{c-bx}}{\sqrt{a+bx}}\right)}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

```
[Out] (-2*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcTan[Sqrt[c - b*x]/Sqrt[a + b
*x]])/(b*Sqrt[a + b*x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(22) = 44.
time = 0.36, size = 85, normalized size = 3.54

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2b\sqrt{-(bx+a)(bx-c)}}\right)(bx-c)\sqrt{-\frac{bx+a}{bx-c}}}{\sqrt{b^2}\,\sqrt{-(bx+a)(bx-c)}}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(
b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

Maxima [A]
time = 0.50, size = 24, normalized size = 1.00

$$\frac{2 \arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Fricas [A]
time = 0.35, size = 24, normalized size = 1.00

$$\frac{2 \arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

Giac [A]
time = 5.86, size = 41, normalized size = 1.71

$$-\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\mathrm{sgn}(-ab-bc)\,\mathrm{sgn}(bx-c)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

Mupad [B]
time = 0.18, size = 36, normalized size = 1.50

$$-\frac{2\sqrt{-b}\,\operatorname{atanh}\left(\frac{\sqrt{-b}\,\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
```

```
[Out] -(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^(
3/2)
```

3.740 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

```
[Out] 2*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1983, 12, 214}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1983

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n),
Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(
b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1/
n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
```

```
gerQ[r]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = (2(bc-ad))\text{Subst}\left(\int \frac{1}{(bc-ad)\,(b-dx^2)}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{b-dx^2}\,dx, x, \sqrt{\frac{a+bx}{c+dx}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Mathematica [**A**]
time = 0.07, size = 77, normalized size = 1.88

$$\frac{2\sqrt{\frac{a+bx}{c+dx}}\,\sqrt{c+dx}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{\sqrt{b}\,\sqrt{d}\,\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

```
[Out] (2*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(31) = 62$.
time = 0.40, size = 80, normalized size = 1.95

method	result	size
default	$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))
*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

Maxima [A]
time = 0.48, size = 59, normalized size = 1.44

$$-\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c))
 + sqrt(b*d)))/sqrt(b*d)
```

Fricas [A]
time = 0.34, size = 105, normalized size = 2.56

$$\left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(
d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c))/(b*d*x + a*d))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

```
[Out] Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.
time = 4.22, size = 74, normalized size = 1.80

$$-\frac{\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)\operatorname{sgn}(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b
*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sgn(d*x + c)/(b*d)
```

Mupad [B]
time = 0.20, size = 31, normalized size = 0.76

$$\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
```

```
[Out] (2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

3.741 $\int \sqrt{-\frac{x}{1+x}}\, dx$

Optimal. Leaf size=32

$$\sqrt{-\frac{x}{1+x}}(1+x)-\tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)$$

```
[Out] -arctan((-x/(1+x))^(1/2))+(1+x)*(-x/(1+x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1979, 294, 210}

$$\sqrt{-\frac{x}{x+1}}(x+1)-\text{ArcTan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(x/(1 + x))],x]
```

```
[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\left(2\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)\right)$$
$$= \sqrt{-\frac{x}{1+x}}\,(1+x) + \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{x}{1+x}}\right)$$
$$= \sqrt{-\frac{x}{1+x}}\,(1+x) - \tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right)$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.53

$$\frac{\sqrt{-\frac{x}{1+x}}\left(\sqrt{x}\,(1+x) - \sqrt{1+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(x/(1 + x))],x]
[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcTanh[Sqrt[x/(1 + x)]]
))/Sqrt[x]
```

Maple [A]
time = 0.11, size = 46, normalized size = 1.44

method	result	s
risch	$(1+x)\sqrt{-\frac{x}{1+x}} - \frac{\arcsin(2x+1)\sqrt{-\frac{x}{1+x}}\,\sqrt{-x(1+x)}}{2x}$	4
default	$\frac{\sqrt{-\frac{x}{1+x}}\,(1+x)\left(2\sqrt{x^2+x} - \ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{x(1+x)}}$	4
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{-\frac{x}{1+x}} - \frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(2x\,\text{RootOf}\left(\_Z^2+1\right)+2\sqrt{-\frac{x}{1+x}}\,x+\text{RootOf}\left(\_Z^2+1\right)+2\sqrt{-\frac{x}{1+x}}\right)}{2}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(1+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/(x*(1+
x))^(1/2)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.16

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

Fricas [A]
time = 0.37, size = 28, normalized size = 0.88

$$(x+1)\sqrt{-\frac{x}{x+1}}-\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))**(1/2),x)
[Out] Integral(sqrt(-x/(x + 1)), x)
```

Giac [A]
time = 4.59, size = 36, normalized size = 1.12

$$\frac{1}{4}\,\pi\mathrm{sgn}(x+1)+\frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1)+\sqrt{-x^2-x}\,\mathrm{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x +
 1)
```

Mupad [B]
time = 3.13, size = 37, normalized size = 1.16

$$-\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x/(x + 1))^(1/2),x)
[Out] - atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.742 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{1+x}}\,(1+x)-2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

```
[Out] -2*arctan(((1-x)/(1+x))^(1/2))+(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 210}

$$\sqrt{\frac{1-x}{x+1}}\,(x+1)-2\text{ArcTan}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\left(4\text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= \sqrt{\frac{1-x}{1+x}}\,(1+x) + 2\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)$$
$$= \sqrt{\frac{1-x}{1+x}}\,(1+x) - 2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 1.74

$$\frac{\sqrt{\frac{1-x}{1+x}}\left(\sqrt{1-x}\,(1+x)+2\sqrt{1+x}\,\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
[Out] (Sqrt[(1 - x)/(1 + x)]*(Sqrt[1 - x]*(1 + x) + 2*Sqrt[1 + x]*ArcTan[Sqrt[1 +
 x]/Sqrt[1 - x]]))/Sqrt[1 - x]
```

Maple [A]
time = 0.11, size = 39, normalized size = 1.03

method	result
default	$\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(1+x)(-1+x)}}$
risch	$\sqrt{-\frac{-1+x}{1+x}}\,(1+x)-\frac{\arcsin(x)\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(1+x)(-1+x)}}{-1+x}$
trager	$\sqrt{-\frac{-1+x}{1+x}}\,(1+x)+\text{RootOf}\left(\_Z^2+1\right)\ln\left(\text{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{-1+x}{1+x}}\,x+\text{RootOf}\left(\_Z^2+1\right)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(1+x)*(-1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x)
)
```

Maxima [A]
time = 0.49, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Giac [A]
time = 4.97, size = 29, normalized size = 0.76

$$\frac{1}{2}\pi\operatorname{sgn}(x+1)+\arcsin\left(x\right)\operatorname{sgn}(x+1)+\sqrt{-x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B]
time = 0.03, size = 43, normalized size = 1.13

$$-2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/(x + 1))^(1/2),x)
[Out] - 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)/
(x + 1) - 1)
```

3.743 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$-\left((a-x)\sqrt{\frac{a+x}{a-x}}\right)+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

```
[Out] 2*a*arctan(((a+x)/(a-x))^(1/2))-(a-x)*((a+x)/(a-x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 294, 209}

$$2a\text{ArcTan}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\text{Subst}\left(\int \frac{x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\
&= -(a-x)\sqrt{\frac{a+x}{a-x}} + (2a)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right) \\
&= -(a-x)\sqrt{\frac{a+x}{a-x}} + 2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 67, normalized size = 1.60

$$
\frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x}+2a\sqrt{a-x}\tan^{-1}\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
[Out] (Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt[
a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

Maple [A]
time = 0.06, size = 61, normalized size = 1.45

method	result	size
default	$\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)}{\sqrt{(a-x)(a+x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a-x)(a+x)}}{a+x}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+x)/(a-x))^(1/2)*(a-x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))/((a
-x)*(a+x))^(1/2)
```

Maxima [A]
time = 0.50, size = 49, normalized size = 1.17

$$
-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Fricas [A]
time = 0.33, size = 38, normalized size = 0.90

$$2a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Giac [A]
time = 6.29, size = 36, normalized size = 0.86

$$a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a)-\sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

Mupad [B]
time = 3.17, size = 49, normalized size = 1.17

$$2a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{2a\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + x)/(a - x))^(1/2),x)
```

```
[Out] 2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)/
(a - x) + 1)
```

3.744 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] -2*a*arctanh(((-a+x)/(a+x))^(1/2))+(a+x)*((-a+x)/(a+x))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 294, 212}

$$\sqrt{-\frac{a-x}{a+x}}(a+x)-2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + x)/(a + x)],x]
```

```
[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = (4a)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= \sqrt{-\frac{a-x}{a+x}}\,(a+x) - (2a)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= \sqrt{-\frac{a-x}{a+x}}\,(a+x) - 2a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Mathematica [A]
time = 0.08, size = 67, normalized size = 1.63

$$\frac{\sqrt{\frac{-a+x}{a+x}}\left(\sqrt{-a+x}\,(a+x) - 2a\sqrt{a+x}\,\tanh^{-1}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)\right)}{\sqrt{-a+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + x)/(a + x)],x]
[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a*Sqrt[a + x]*ArcTanh[Sqr
t[a + x]/Sqrt[-a + x]]))/Sqrt[-a + x]
```

Maple [A]
time = 0.07, size = 60, normalized size = 1.46

method	result	size
default	$\frac{\sqrt{-\frac{a-x}{a+x}}\,(a+x)\left(\sqrt{-a^2+x^2}-a\ln\left(x+\sqrt{-a^2+x^2}\right)\right)}{\sqrt{-(a-x)(a+x)}}$	60
risch	$\frac{(a+x)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{\sqrt{(-a+x)(a+x)}}+\frac{a\ln\left(x+\sqrt{-a^2+x^2}\right)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a-x)(a+x)}}{a-x}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-a+x)/(a+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-(a-x)/(a+x))^(1/2)*(a+x)/(-(a-x)*(a+x))^(1/2)*((-a^2+x^2)^(1/2)-a*ln(x+(-
a^2+x^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 1.71

$$a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1} - \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

Fricas [A]
time = 0.34, size = 58, normalized size = 1.41

$$-a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a
 + x)*sqrt(-(a - x)/(a + x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+x}{a+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)/(a + x)), x)
```

Giac [A]
time = 6.90, size = 40, normalized size = 0.98

$$a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\operatorname{sgn}(a+x)+\sqrt{-a^2+x^2}\operatorname{sgn}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

Mupad [B]
time = 0.05, size = 51, normalized size = 1.24

$$\frac{2\,a\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - x)/(a + x))^(1/2),x)
```

```
[Out] (2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)/
(a + x))^(1/2))
```

3.745 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

```
[Out] -(-a*d+b*c)*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/d^(3/2)/b^(1/2
)+(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 294, 214}

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqr
t[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*d^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1979

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c - a*d)/n), Subst[Int[x
^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rubi steps

$$\begin{aligned}\int\sqrt{\frac{a+bx}{c+dx}}\,dx&=(2(bc-ad))\text{Subst}\left(\int\frac{x^2}{(b-dx^2)^2}\,dx,x,\sqrt{\frac{a+bx}{c+dx}}\right)\\&=\frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d}-\frac{(bc-ad)\text{Subst}\left(\int\frac{1}{b-dx^2}\,dx,x,\sqrt{\frac{a+bx}{c+dx}}\right)}{d}\\&=\frac{\sqrt{\frac{a+bx}{c+dx}}\,(c+dx)}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\,d^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.24, size = 114, normalized size = 1.50

$$\frac{\sqrt{\frac{a+bx}{c+dx}}\left(\sqrt{\frac{b}{d}}\,d\sqrt{a+bx}\,(c+dx)+(bc-ad)\sqrt{c+dx}\,\log\left(\sqrt{a+bx}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)\right)}{\sqrt{\frac{b}{d}}\,d^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

```
[Out] (Sqrt[(a + b*x)/(c + d*x)]*(Sqrt[b/d]*d*Sqrt[a + b*x]*(c + d*x) + (b*c - a*
d)*Sqrt[c + d*x]*Log[Sqrt[a + b*x] - Sqrt[b/d]*Sqrt[c + d*x]]))/(Sqrt[b/d]*
d^2*Sqrt[a + b*x])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(64) = 128$.
time = 0.05, size = 152, normalized size = 2.00

method	result

default	$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(\ln\left(\frac{2bdx+2\sqrt{(bx+a)\,(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ad-\ln\left(\frac{2bdx+2\sqrt{(bx+a)\,(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right.\right.}{2\sqrt{(bx+a)\,(dx+c)}\,d\sqrt{bd}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1
/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c
))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)*(b
*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

Maxima [A]
time = 0.49, size = 118, normalized size = 1.55

$$\frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}}+\frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\sqrt{bd}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/2
*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a
)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

Fricas [A]
time = 0.33, size = 180, normalized size = 2.37

$$\left[-\frac{(bc-ad)\sqrt{bd}\,\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\,(bd^2x+bcd)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2},\ \frac{(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{\sqrt{-bd}\,(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)+(bd^2x+bcd)\sqrt{\frac{bx+a}{dx+c}}}{bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c
)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)
))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x +
```

```
a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c))
)/(b*d^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))**(1/2),x)
[Out] Integral(sqrt((a + b*x)/(c + d*x)), x)
```

Giac [A]
time = 5.28, size = 119, normalized size = 1.57

$$\frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sgn}(dx+c)}{d}+\frac{(bc\mathrm{sgn}(dx+c)-ad\mathrm{sgn}(dx+c))\sqrt{bd}\,\log\left(\left|-2\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)bd-\sqrt{bd}\,bc-\sqrt{bd}\,ad\right|\right)}{2\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d + 1/2*(b*c*sgn(d*x + c)
- a*d*sgn(d*x + c))*sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*
x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2)
```

Mupad [B]
time = 0.27, size = 90, normalized size = 1.18

$$\frac{\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)\,(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}+\frac{(a\,d-b\,c)\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\,\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/(c + d*x))^(1/2),x)
[Out] (atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2)
*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))/(
b*(c + d*x)) - 1))
```

3.746 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{-1+x}\right)}{3\sqrt{3}}$$

```
[Out] -8/9*arcsinh(1/4*6^(1/2)*(-1+x)^(1/2))*3^(1/2)+1/3*(-1+x)^(1/2)*(5+3*x)^(1/
2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1978, 52, 56, 221}

$$\frac{1}{3}\sqrt{x-1}\,\sqrt{3x+5}-\frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

```
[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3
*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{4}{3}\int \frac{1}{\sqrt{-1+x}\,\sqrt{5+3x}}\,dx\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{8}{3}\text{Subst}\left(\int \frac{1}{\sqrt{8+3x^2}}\,dx, x, \sqrt{-1+x}\right)\\ &= \frac{1}{3}\sqrt{-1+x}\,\sqrt{5+3x} - \frac{8\sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\,\sqrt{-1+x}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.47

$$\frac{\sqrt{\frac{-1+x}{5+3x}}\left(3\sqrt{-1+x}\,(5+3x) - 8\sqrt{15+9x}\,\tanh^{-1}\left(\frac{\sqrt{5+3x}}{\sqrt{-3+3x}}\right)\right)}{9\sqrt{-1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
[Out] (Sqrt[(-1 + x)/(5 + 3*x)]*(3*Sqrt[-1 + x]*(5 + 3*x) - 8*Sqrt[15 + 9*x]*ArcT
anh[Sqrt[5 + 3*x]/Sqrt[-3 + 3*x]]))/(9*Sqrt[-1 + x])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.
time = 0.12, size = 76, normalized size = 1.55

method	result
default	$-\frac{\sqrt{\frac{-1+x}{5+3x}}\,(5+3x)\left(4\ln\left(x\sqrt{3}+\frac{\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)\sqrt{3}-3\sqrt{3x^2+2x-5}\right)}{9\sqrt{(5+3x)\,(-1+x)}}$

risch	$\frac{(5+3x)\sqrt{\frac{-1+x}{5+3x}}}{3}-\frac{4\ln\left(\frac{(1+3x)\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)\sqrt{3}\,\sqrt{\frac{-1+x}{5+3x}}\,\sqrt{(5+3x)(-1+x)}}{9(-1+x)}$
trager	$5\left(\frac{1}{3}+\frac{x}{5}\right)\sqrt{-\frac{1-x}{5+3x}}+\frac{4\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-3\,\mathrm{RootOf}\left(\_Z^2-3\right)x+9\sqrt{-\frac{1-x}{5+3x}}\,x-\mathrm{RootOf}\left(\_Z^2-3\right)+15\sqrt{-\frac{1-x}{5+3x}}\right.}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)/(5+3*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*((-1+x)/(5+3*x))^(1/2)*(5+3*x)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(3*x^2+2*x-
5)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((5+3*x)*(-1+x))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.
time = 0.50, size = 80, normalized size = 1.63

$$\frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3x+5}}}\right)-\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3(x-1)}{3x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((x
 - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1)
```

Fricas [A]
time = 0.34, size = 54, normalized size = 1.10

$$\frac{1}{3}(3x+5)\sqrt{\frac{x-1}{3x+5}}+\frac{4}{9}\sqrt{3}\log\left(\sqrt{3}(3x+5)\sqrt{\frac{x-1}{3x+5}}-3x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*s
qrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{3x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/(3*x + 5)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.
time = 5.10, size = 74, normalized size = 1.51

$$-\frac{8}{9}\sqrt{3}\log(2)\operatorname{sgn}(3x+5)+\frac{4}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2x-5}\right)-1\right|\right)\operatorname{sgn}(3x+5)+\frac{1}{3}\sqrt{3x^2+2x-5}\operatorname{sgn}(3x+5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/(5+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] -8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
- sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*sgn
(3*x + 5)
```

Mupad [B]
time = 3.16, size = 57, normalized size = 1.16

$$-\frac{8\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\sqrt{\frac{x-1}{3x+5}}\right)}{9}-\frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3x-3}{3x+5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/(3*x + 5))^(1/2),x)
```

```
[Out] - (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*x
 + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

3.747 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx$

Optimal. Leaf size=46

$$-\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}-12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

```
[Out] -12*arctan((1+7*x)^(1/2)/(-1+5*x)^(1/2))-(-1+5*x)^(1/2)*(1+7*x)^(1/2)/x
```

Rubi [A]

time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1978, 96, 95, 210}

$$-12\text{ArcTan}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)-\frac{\sqrt{5x-1}\,\sqrt{7x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x
]]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1
)/((m + 1)*(b*e - a*f))), x] - Dist[n*((d*e - c*f)/((m + 1)*(b*e - a*f))),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx &= \int \frac{\sqrt{-1+5x}}{x^2\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}+6\int \frac{1}{x\sqrt{-1+5x}\,\sqrt{1+7x}}\,dx\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}+12\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\\ &= -\frac{\sqrt{-1+5x}\,\sqrt{1+7x}}{x}-12\tan^{-1}\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)\end{aligned}$$

Mathematica [A]
time = 0.17, size = 88, normalized size = 1.91

$$-\frac{\sqrt{\frac{-1+5x}{1+7x}}\left(\sqrt{-1+5x}\,(1+7x)+12x\sqrt{1+7x}\,\tan^{-1}\left(\sqrt{35}\,x-\sqrt{-1+5x}\,\sqrt{1+7x}\right)\right)}{x\sqrt{-1+5x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[(-1 + 5*x)/(1 + 7*x)]*(Sqrt[-1 + 5*x]*(1 + 7*x) + 12*x*Sqrt[1 + 7*x
]*ArcTan[Sqrt[35]*x - Sqrt[-1 + 5*x]*Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(38) = 76$.
time = 0.13, size = 106, normalized size = 2.30

method	result
risch	$-\frac{(1+7x)\sqrt{\frac{5x-1}{1+7x}}}{x}+\frac{6\arctan\left(\frac{-2x-2}{2\sqrt{35x^2-2x-1}}\right)\sqrt{\frac{5x-1}{1+7x}}\,\sqrt{(5x-1)(1+7x)}}{5x-1}$

trager	$-\frac{(1+7x)\sqrt{-\frac{1-5x}{1+7x}}}{x}+6\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{x\,\mathrm{RootOf}\left(\_Z^2+1\right)+7\sqrt{-\frac{1-5x}{1+7x}}\,x+\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-\frac{1-}{1+}}}{x}\right.$
default	$-\frac{\sqrt{\frac{5x-1}{1+7x}}\,(1+7x)\left(-(35x^2-2x-1)^{\frac{3}{2}}+35\sqrt{35x^2-2x-1}\,x^2+6\arctan\left(\frac{1+x}{\sqrt{35x^2-2x-1}}\right)x-2\sqrt{35x^2-2x-}\right.}{\sqrt{(5x-1)(1+7x)}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x-1)/(1+7*x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((5*x-1)/(1+7*x))^(1/2)*(1+7*x)*(-(35*x^2-2*x-1)^(3/2)+35*(35*x^2-2*x-1)^(
1/2)*x^2+6*arctan((1+x)/(35*x^2-2*x-1)^(1/2))*x-2*(35*x^2-2*x-1)^(1/2)*x)/(
(5*x-1)*(1+7*x))^(1/2)/x
```

Maxima [A]
time = 0.47, size = 53, normalized size = 1.15

$$-\frac{12\sqrt{\frac{5\,x-1}{7\,x+1}}}{\frac{5\,x-1}{7\,x+1}+1}+12\,\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((5
*x - 1)/(7*x + 1)))
```

Fricas [A]
time = 0.34, size = 46, normalized size = 1.00

$$\frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right)-(7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1
)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(38) = 76.
time = 5.30, size = 114, normalized size = 2.48

$$\left(\sqrt{35} - 12 \arctan\left(\frac{1}{7} \sqrt{35}\right)\right) \mathrm{sgn}(7 x + 1) + 12 \arctan\left(-\sqrt{35} x + \sqrt{35 x^2 - 2 x - 1}\right) \mathrm{sgn}(7 x + 1) - \frac{2 \left(\left(\sqrt{35} x - \sqrt{35 x^2 - 2 x - 1}\right) \mathrm{sgn}(7 x + 1) + \sqrt{35} \mathrm{sgn}(7 x + 1)\right)}{\left(\sqrt{35} x - \sqrt{35 x^2 - 2 x - 1}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x +
 sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*x
- 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))^2 + 1)
```

Mupad [B]
time = 3.24, size = 74, normalized size = 1.61

$$12 \operatorname{atan}\left(\frac{\sqrt{5} \sqrt{7} \sqrt{35} \sqrt{\frac{5 x - 1}{7 x + 1}}}{35}\right) - \frac{12 \sqrt{5} \sqrt{7} \sqrt{35} \sqrt{\frac{5 x - 1}{7 x + 1}}}{25 \left(\frac{7 x - \frac{7}{5}}{7 x + 1} + \frac{7}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

```
[Out] 12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5^
(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x +
 1) + 7/5))
```

3.748 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx$

Optimal. Leaf size=20

$$-\sqrt{\frac{1-x}{1+x}}\,(1+x)$$

```
[Out] -(1+x)*((1-x)/(1+x))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1984, 12, 391}

$$-\sqrt{\frac{1-x}{x+1}}\,(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

```
[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1984

```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[q*e*((b*c
- a*d)/n), Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^
((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)^
(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q
)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] &
& IntegerQ[1/n] && IntegersQ[m, r]
```

Rubi steps

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}\,(1+x)}\,dx = -\left(4\text{Subst}\left(\int \frac{1-x^2}{2\,(1+x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= -\left(2\text{Subst}\left(\int \frac{1-x^2}{(1+x^2)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)$$
$$= -\sqrt{\frac{1-x}{1+x}}\,(1+x)$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 0.95

$$\frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [A]
time = 0.44, size = 36, normalized size = 1.80

method	result	size
gosper	$\frac{-1+x}{\sqrt{-\frac{-1+x}{1+x}}}$	17
risch	$\frac{-1+x}{\sqrt{-\frac{-1+x}{1+x}}}$	17
trager	$(-1-x)\sqrt{-\frac{-1+x}{1+x}}$	19
default	$\frac{(-1+x)\sqrt{-x^2+1}}{\sqrt{-\frac{-1+x}{1+x}}\,\sqrt{-(1+x)(-1+x)}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/((1-x)/(1+x))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1+x)*(-x^2+1)^(1/2)/(-(-1+x)/(1+x))^(1/2)/(-(1+x)*(-1+x))^(1/2)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 1.35

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Fricas [A]
time = 0.32, size = 17, normalized size = 0.85

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A]
time = 6.11, size = 17, normalized size = 0.85

$$-\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.85

$$-\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
[Out] -(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

3.749 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal. Leaf size=18

$$-\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)$$

```
[Out] -(1+x)*(-1+2/(1+x))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {526, 528, 382, 75}

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] -((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 526

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIn
tegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x], x
, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m]
 && NeQ[v, x]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx &= \text{Subst}\left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}\,x}\,dx,x,1+x\right)\\ &= \text{Subst}\left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}}\,dx,x,1+x\right)\\ &= -\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{-1+2x}}\,dx,x,\frac{1}{1+x}\right)\\ &= -(1+x)\sqrt{-1+\frac{2}{1+x}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.06

$$\frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

```
[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.
time = 0.41, size = 37, normalized size = 2.06

method	result	size
gosper	$\frac{-1+x}{\sqrt{-\frac{-1+x}{1+x}}}$	17

risch	$\frac{-1+x}{\sqrt{-\frac{-1+x}{1+x}}}$	17
trager	$(-1-x)\sqrt{-\frac{-1+x}{1+x}}$	19
default	$-\frac{\sqrt{-\frac{-1+x}{1+x}}\,(1+x)\sqrt{-x^{2}+1}}{\sqrt{-(1+x)(-1+x)}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(-1+2/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-1+x)/(1+x))^(1/2)*(1+x)/(-(1+x)*(-1+x))^(1/2)*(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.89

$$\frac{\sqrt{x+1}\,(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.94

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Giac [A]
time = 4.41, size = 17, normalized size = 0.94

$$-\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
[Out] int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

3.750 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

Optimal. Leaf size=54

$$\sqrt{2+x}\,\sqrt{3+x}-\sinh^{-1}\left(\sqrt{2+x}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2+x}}{\sqrt{3+x}}\right)$$

```
[Out] -arcsinh((2+x)^(1/2))+2*arctanh(2^(1/2)*(2+x)^(1/2)/(3+x)^(1/2))*2^(1/2)+(2
+x)^(1/2)*(3+x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1978, 159, 163, 56, 221, 95, 213}

$$\sqrt{x+2}\,\sqrt{x+3}-\sinh^{-1}\left(\sqrt{x+2}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]
*Sqrt[2 + x])/Sqrt[3 + x]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx &= \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}}\,dx\\
&= \sqrt{2+x}\,\sqrt{3+x} + \int \frac{-\frac{5}{2}-\frac{x}{2}}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx\\
&= \sqrt{2+x}\,\sqrt{3+x} - \frac{1}{2}\int \frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\,dx - 2\int \frac{1}{(1+x)\sqrt{2+x}\,\sqrt{3+x}}\,dx\\
&= \sqrt{2+x}\,\sqrt{3+x} - 4\mathrm{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \frac{\sqrt{2+x}}{\sqrt{3+x}}\right) - \mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\right.\\
&= \sqrt{2+x}\,\sqrt{3+x} - \sinh^{-1}\left(\sqrt{2+x}\right) + 2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{2+x}}{\sqrt{3+x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 66, normalized size = 1.22

$$\sqrt{2+x}\sqrt{3+x}-\tanh^{-1}\left(\frac{1}{\sqrt{\frac{2+x}{3+x}}}\right)+2\sqrt{2}\tanh^{-1}\left(\frac{-1-x+\sqrt{2+x}\sqrt{3+x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

```
[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcTanh[1/Sqrt[(2 + x)/(3 + x)]] + 2*Sqrt[2]*ArcT
anh[(-1 - x + Sqrt[2 + x]*Sqrt[3 + x])/Sqrt[2]]
```

Maple [A]
time = 0.44, size = 81, normalized size = 1.50

method	result
default	$\frac{(x+2)\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)+2\sqrt{x^2+5x+6}-\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)\right)}{2\sqrt{\frac{x+2}{3+x}}\sqrt{(3+x)(x+2)}}$
risch	$\frac{x+2}{\sqrt{\frac{x+2}{3+x}}}+\frac{\left(-\frac{\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{(1+x)^2+5+3x}}\right)\right)\sqrt{(3+x)(x+2)}}{\sqrt{\frac{x+2}{3+x}}\,(3+x)}$
trager	$3\left(1+\frac{x}{3}\right)\sqrt{-\frac{-x-2}{3+x}}-\frac{\ln\left(2\sqrt{-\frac{-x-2}{3+x}}\,x+6\sqrt{-\frac{-x-2}{3+x}}+2x+5\right)}{2}-\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{4\sqrt{-\frac{-x-2}{3+x}}\,x-}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/((x+2)/(3+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x+2)*(2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+2*(x^2+
5*x+6)^(1/2)-ln(5/2+x+(x^2+5*x+6)^(1/2)))/((x+2)/(3+x))^(1/2)/((3+x)*(x+2))
^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(40) = 80.
time = 0.51, size = 103, normalized size = 1.91

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)
/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x
 + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(40) = 80$.
time = 0.39, size = 83, normalized size = 1.54

$$(x+3)\sqrt{\frac{x+2}{x+3}} + \sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}} + 3x + 7}{x+1}\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)
/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*lo
g(sqrt((x + 2)/(x + 3)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

```
[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(40) = 80$.
time = 3.74, size = 129, normalized size = 2.39

$$\sqrt{2}\log\left(-\frac{\sqrt{2}-2}{\sqrt{2}+2}\right)\operatorname{sgn}(x+3) - \frac{\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+5x+6}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+5x+6}-2\right|}\right)}{\operatorname{sgn}(x+3)} + \frac{\log\left(\left|-2x+2\sqrt{x^2+5x+6}-5\right|\right)}{2\operatorname{sgn}(x+3)} + \frac{\sqrt{x^2+5x+6}}{\operatorname{sgn}(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(-(sqrt(2) - 2)/(sqrt(2) + 2))*sgn(x + 3) - sqrt(2)*log(abs(-2*x
 - 2*sqrt(2) + 2*sqrt(x^2 + 5*x + 6) - 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2
```

```
 + 5*x + 6) - 2))/sgn(x + 3) + 1/2*log(abs(-2*x + 2*sqrt(x^2 + 5*x + 6) - 5
))/sgn(x + 3) + sqrt(x^2 + 5*x + 6)/sgn(x + 3)
```

Mupad [B]
time = 0.09, size = 62, normalized size = 1.15

$$2\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sqrt{\frac{x+2}{x+3}}\right)-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1}-\mathrm{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
```

```
[Out] 2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)/
((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

3.751 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx$

Optimal. Leaf size=11

$$\frac{2}{\sqrt{1+\frac{1}{x}}}$$

[Out] 2/(1+1/x)^(1/2)

Rubi [A]

time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {25, 267}

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx = \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2} dx$$
$$= \frac{2}{\sqrt{1+\frac{1}{x}}}$$

Mathematica [A]
time = 0.02, size = 19, normalized size = 1.73

$$\frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

```
[Out] (2*x*Sqrt[(1 + x)/x])/(1 + x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(9) = 18.
time = 0.39, size = 32, normalized size = 2.91

method	result	size
gosper	$\frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$	18
risch	$\frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$	18
trager	$\frac{2x\sqrt{-\frac{-1-x}{x}}}{1+x}$	21
default	$\frac{2\sqrt{x^2+x}\, x\sqrt{\frac{1+x}{x}}}{(1+x)\sqrt{x(1+x)}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(1+x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^2+x)^(1/2)*x*((1+x)/x)^(1/2)/(1+x)/(x*(1+x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

Fricas [A]
time = 0.36, size = 17, normalized size = 1.55

$$\frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 2*x*sqrt((x + 1)/x)/(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\left(x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

```
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.
time = 4.82, size = 23, normalized size = 2.09

$$\frac{2\,\mathrm{sgn}(x)}{x-\sqrt{x^2+x}+1}-2\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

Mupad [B]
time = 3.13, size = 15, normalized size = 1.36

$$\frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(x + 1)^2,x)
[Out] (2*x*(1/x + 1)^(1/2))/(x + 1)
```

3.752 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{1+\frac{1}{x}}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{1+x}}$$

```
[Out] arcsin(-1+2*x)*(1+1/x)^(1/2)*x^(1/2)/(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1462, 26, 55, 633, 222}

$$-\frac{\sqrt{\frac{1}{x}+1}\,\sqrt{x}\,\text{ArcSin}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1462

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q]), Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a + c
*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
!IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{\sqrt{1+x}}{\sqrt{x}\,\sqrt{1-x^2}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx}{\sqrt{1+x}}\\
&= \frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\int \frac{1}{\sqrt{x-x^2}}\,dx}{\sqrt{1+x}}\\
&= -\frac{\left(\sqrt{1+\frac{1}{x}}\,\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)}{\sqrt{1+x}}\\
&= -\frac{\sqrt{1+\frac{1}{x}}\,\sqrt{x}\,\sin^{-1}(1-2x)}{\sqrt{1+x}}
\end{aligned}$$

Mathematica [A]
time = 1.57, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{1+x}{x}}\,(-1+2x)\sqrt{1-x^2}}{2\,(-1+x^2)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

```
[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

Maple [A]
time = 0.48, size = 40, normalized size = 1.38

method	result	size
default	$\frac{\sqrt{\frac{1+x}{x}}\, x\sqrt{-x^{2}+1}\, \arcsin(2x-1)}{(1+x)\sqrt{-x(-1+x)}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] ((1+x)/x)^(1/2)*x*(-x^2+1)^(1/2)/(1+x)/(-x*(-1+x))^(1/2)*arcsin(2*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Fricas [A]
time = 0.41, size = 34, normalized size = 1.17

$$-\arctan\left(\frac{2\sqrt{-x^{2}+1}\,x\sqrt{\frac{x+1}{x}}}{2\,x^{2}+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.753 $\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx$

Optimal. Leaf size=180

$$\tan^{-1}\left(\frac{\sqrt{3}-\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(-\frac{3-x-\sqrt{3}\sqrt{3-2x-x^2}}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(1+\sqrt{3}-\right.$$

```
[Out] arctan((3^(1/2)-(-x^2-2*x+3)^(1/2))/x)-1/2*ln((-3+x+3^(1/2)*(-x^2-2*x+3)^(1
/2))/x^2)+1/14*ln(1+3^(1/2)+7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)
*(7-7^(1/2))+1/14*ln(1+3^(1/2)-7^(1/2)-3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))
/x)*(7+7^(1/2))
```

Rubi [A]

time = 0.14, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1088, 646, 31, 649, 209, 266}

$$\text{ArcTan}\left(\frac{\sqrt{3}-\sqrt{-x^2-2x+3}}{x}\right)-\frac{1}{2}\log\left(-\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-x+3}{x^2}\right)+\frac{1}{14}\left(7+\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{7}+\sqrt{3}+1\right)+\frac{1}{14}\left(7-\sqrt{7}\right)\log\left(-\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}+\sqrt{7}+\sqrt{3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 -
 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*
(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] +
Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1088

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{3-2x-x^2}}\,dx = 2\text{Subst}\left(\int \frac{\sqrt{3}-2x-\sqrt{3}\,x^2}{(1+x^2)\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-}}{x}\right.$$

$$= \frac{1}{16}\text{Subst}\left(\int \frac{-6+2\sqrt{3}\left(2-\sqrt{3}\right)-4\left(1+\sqrt{3}\right)-\left(-2\sqrt{3}+2\left(2-\sqrt{3}\right)+\right.}{1+x^2}\right.$$

$$= -\left(\frac{1}{2}\left(\sqrt{\frac{3}{7}}\left(1-\sqrt{7}\right)\right)\text{Subst}\left(\int \frac{1}{1+\sqrt{3}+\sqrt{7}+\sqrt{3}\,x}\,dx, x, \frac{-\sqrt{3}+\sqrt{3}}{x}\right.\right.$$

$$= -\tan^{-1}\left(\frac{-\sqrt{3}+\sqrt{3-2x-x^2}}{x}\right)-\frac{1}{2}\log\left(\frac{-3+x+\sqrt{3}\,\sqrt{3-2x-x^2}}{x^2}\right)+$$

Mathematica [A]
time = 0.28, size = 111, normalized size = 0.62

$$\frac{1}{14}\left(-14\tan^{-1}\left(\frac{\sqrt{3-2x-x^2}}{3+x}\right)-7\log(-1+x)-\left(-7+\sqrt{7}\right)\log\left(-2+\sqrt{7}\,(-1+x)+2x-\sqrt{3-2x-x^2}\right)+\left(7+\sqrt{7}\right)\log\left(2+\sqrt{7}\,(-1+x)-2x+\sqrt{3-2x-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1),x]
```

```
[Out] (-14*ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)] - 7*Log[-1 + x] - (-7 + Sqrt[7])*L
og[-2 + Sqrt[7]*(-1 + x) + 2*x - Sqrt[3 - 2*x - x^2]] + (7 + Sqrt[7])*Log[2
 + Sqrt[7]*(-1 + x) - 2*x + Sqrt[3 - 2*x - x^2]])/14
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 358 vs. $2(138) = 276$.
time = 0.58, size = 359, normalized size = 1.99

method	result
default	$\sqrt{7}\left(\frac{\sqrt{-4\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)^{2}+4\left(-1+\sqrt{7}\right)\left(x+\frac{1}{2}+\frac{\sqrt{7}}{2}\right)+8+2\sqrt{7}}}{4}+\frac{\left(-1+\sqrt{7}\right)\arcsin\left(\ldots\right)}{\ldots}\right.$
trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2
))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1
/2))^2)^(1/2)*(1+x))-1/2*(2+1/2*7^(1/2))/(1/2*7^(1/2)+1/2)*arctanh((4+7^(1/
2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2*7^(1/2)+1/2)/(-4*(x+1/2+1/2*7^(1/
2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)))-1/7*7^(1/2)*(
1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2
))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(1/2)
*(1+x))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/
2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1
-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))+1/4*ln(2*x^2+2*x-3)+1/14
*7^(1/2)*arctanh(1/14*(2+4*x)*7^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 2*x + 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(136) = 272$.
time = 0.39, size = 372, normalized size = 2.07

$$\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2+2\sqrt{7}(3x^4+x^3-45x^2+45x)-\left(14x^3-84x^2+\sqrt{7}(8x^3-30x^2+27x-27)+126x\right)\sqrt{-x^2-2x+3}+180x-135}{4x^4+8x^3-8x^2-12x+9}\right)+\frac{1}{56}\sqrt{7}\log\left(\frac{24x^4+62x^3-153x^2-2\sqrt{7}(3x^4+x^3-45x^2+45x)+\left(14x^3-84x^2-\sqrt{7}(8x^3-30x^2+27x-27)+126x\right)\sqrt{-x^2-2x+3}+180x-135}{4x^4+8x^3-8x^2-12x+9}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{2x^2+\sqrt{7}(2x+1)+2x+4}{2x^2+2x-3}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-2x+3}(x+1)}{x^2+2x-3}\right)+\frac{1}{4}\log\left(2x^2+2x-3\right)-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-2x+3}x+2x-3}{x^2}\right)+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-2x+3}x-2x+3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x
^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*
x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))
+ 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45
*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 12
6*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
) + 1/28*sqrt(7)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3
)) - 1/2*arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)) + 1/4*log(2*x
^2 + 2*x - 3) - 1/8*log((2*sqrt(-x^2 - 2*x + 3)*x + 2*x - 3)/x^2) + 1/8*log
(-(2*sqrt(-x^2 - 2*x + 3)*x - 2*x + 3)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(136) = 272$.
time = 6.02, size = 287, normalized size = 1.59

$$-\frac{1}{28}\sqrt{7}\log\left(\frac{|4x-2\sqrt{7}+2|}{|4x+2\sqrt{7}+2|}\right)+\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6(\sqrt{-x^2-2x+3}-2)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6(\sqrt{-x^2-2x+3}-2)}{x+1}+4\right|}\right)-\frac{1}{28}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2(\sqrt{-x^2-2x+3}-2)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2(\sqrt{-x^2-2x+3}-2)}{x+1}-4\right|}\right)+\frac{1}{2}\arcsin\left(\frac{1}{2}x+\frac{1}{2}\right)+\frac{1}{4}\log\left(|2x^2+2x-3|\right)+\frac{1}{4}\log\left(\left|\frac{4(\sqrt{-x^2-2x+3}-2)}{x+1}+\frac{3(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2}-1\right|\right)-\frac{1}{4}\log\left(\left|-\frac{4(\sqrt{-x^2-2x+3}-2)}{x+1}+\frac{(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/28*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 1/28
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
```

```
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 1/28*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) + 1/2*arcsin(1/2*x + 1/2) + 1/4*
log(abs(2*x^2 + 2*x - 3)) + 1/4*log(abs(4*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1
) + 3*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 1)) - 1/4*log(abs(-4*(sqrt(-
x^2 - 2*x + 3) - 2)/(x + 1) + (sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x+\sqrt{-x^2-2\,x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

3.754 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx$

Optimal. Leaf size=172

$$\frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)}+\frac{8\tanh^{-1}\left(\frac{3-x-\sqrt{3}\,x-\sqrt{3}}{\sqrt{}}\right.}{7\sqrt{7}}$$

```
[Out] 8/49*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^(1
/2)+2/7*(4-3^(1/2)+3*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2
))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^
2)
```

Rubi [A]
time = 0.11, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1674, 12, 632, 212}

$$\frac{2\left(\frac{3\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+4\right)}{7\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{8\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}\,x-x+3}{\sqrt{7}\,x}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
```

```
[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqr
t[3 - 2*x - x^2])/(Sqrt[7]*x)])/(7*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^2}\,dx = 2\text{Subst}\left(\int \frac{-\sqrt{3}+2x+\sqrt{3}\,x^2}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)^2}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x}}{x}\right.$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right.}{x^2}\right.}$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right.}{x^2}\right.}$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right.}{x^2}\right.}$$

$$= \frac{2\left(4-\sqrt{3}+\frac{3\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{7\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right.}{x^2}\right.}$$

Mathematica [A]
time = 0.38, size = 94, normalized size = 0.55

$$\frac{3+6\sqrt{3-2x-x^2}-2x\left(4+\sqrt{3-2x-x^2}\right)}{14\left(-3+2x+2x^2\right)}+\frac{8\tanh^{-1}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}\,(-1+x)}\right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2),x]
[Out] (3 + 6*Sqrt[3 - 2*x - x^2] - 2*x*(4 + Sqrt[3 - 2*x - x^2]))/(14*(-3 + 2*x +
 2*x^2)) + (8*ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))])/
(7*Sqrt[7])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(139) = 278$.

time = 0.17, size = 1066, normalized size = 6.20

method	result
trager	$\frac{(x-3)x}{14x^2+14x-21} - \frac{(x-3)\sqrt{-x^2-2x+3}}{7(2x^2+2x-3)} + \frac{4\,\mathrm{RootOf}\left(\_Z^2-7\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-7\right)x+7\sqrt{-x^2-2x+3}-3\,\mathrm{Root}}{\mathrm{RootOf}\left(\_Z^2-7\right)x+x-3}\right.}{49}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/28*(2+4*x)/(2*x^2+2*x-3)+4/49*7^(1/2)*arctanh(1/14*(2+4*x)*7^(1/2))+1/14
*(-2*x+6)/(2*x^2+2*x-3)+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7
^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*arcsin(1/(2
+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))-1/2*(2+1/2*7^(1/2))/(1/2*7^(1
/2)+1/2)*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2*7^(1/2)+
1/2)/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/
2))^(1/2)))-2*(-1/14-1/14*7^(1/2))*(-1/4/(2+1/2*7^(1/2))/(x+1/2+1/2*7^(1/2)
)*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(
3/2)+1/8*(-1+7^(1/2))/(2+1/2*7^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+
7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2))*arcsin(1/(
2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))-(2+1/2*7^(1/2))/(1/2*7^(1/2)
+1/2)*arctanh((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2*7^(1/2)+1/2
)/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))
^(1/2)))-1/2/(2+1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(
1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2)-(-1+7^(1/2
))^2)*arcsin(1/(2+1/2*7^(1/2)+1/4*(-1+7^(1/2))^2)^(1/2)*(1+x))))-1/49*7^(1/
2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^
(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2)^(
1/2)*(1+x))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7
^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4
*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-2*(-1/14+1/14*7^(1/2
))*(-1/4/(2-1/2*7^(1/2))/(x+1/2-1/2*7^(1/2))*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(
1/2))*(1/2*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2
*7^(1/2))^(1/2)+1/2*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2)+1/4*(-1-7^(1/2))^2
)^(1/2)*(1+x))-(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^
(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2))/(-4*(x+1/2-1/2*7^(1/2))^2+4*
(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)))-1/2/(2-1/2*7^(1/2))*(
-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/
2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1/2)-(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2)+
1/4*(-1-7^(1/2))^2)^(1/2)*(1+x))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)
```

Fricas [A]
time = 0.38, size = 171, normalized size = 0.99

$$\frac{2\sqrt{7}\,(2x^2+2x-3)\log\left(\frac{x^4+44x^3-\sqrt{7}\,(3x^3+x^2-45x+45)\sqrt{-x^2-2x+3}+26x^2-276x+207}{4x^4+8x^3-8x^2-12x+9}\right)+4\sqrt{7}\,(2x^2+2x-3)\log\left(\frac{2x^2+\sqrt{7}\,(2x+1)+2x+4}{2x^2+2x-3}\right)-14\sqrt{-x^2-2x+3}\,(x-3)-56x+21}{98\,(2x^2+2x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/98*(2*sqrt(7)*(2*x^2 + 2*x - 3)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2
- 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 -
8*x^2 - 12*x + 9)) + 4*sqrt(7)*(2*x^2 + 2*x - 3)*log((2*x^2 + sqrt(7)*(2*x
+ 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*sqrt(-x^2 - 2*x + 3)*(x - 3) - 56*x
 + 21)/(2*x^2 + 2*x - 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-2x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(132) = 264.
time = 2.38, size = 350, normalized size = 2.03

$$-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|4x-2\sqrt{7}+2\right|}{\left|4x+2\sqrt{7}+2\right|}\right)+\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}{\left|2\sqrt{7}+\frac{6\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+4\right|}\right)-\frac{2}{49}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}{\left|2\sqrt{7}+\frac{2\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}-4\right|}\right)-\frac{8x-3}{14\,(2x^2+2x-3)}-\frac{8\left(\frac{5\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{11\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-6\right)}{21\left(\frac{8\left(\sqrt{-x^2-2x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2x+3}-2\right)^4}{(x+1)^4}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -2/49*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 2/49
*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs
(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 2/49*sqrt(7)*log(
abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) +
2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x -
3) - 8/21*(5*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3)
- 2)^2/(x + 1)^2 + 11*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 6)/(8*(sqrt(
-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 +
8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(
x + 1)^4 - 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-2\,x+3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)
```

3.755 $\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx$

Optimal. Leaf size=307

$$-\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)^2}{x^2}\right)^2}+\frac{2\left(18-\right.}{147\left(2-\sqrt{3}-\frac{2\left(1+\right.}{}\right.}$$

```
[Out] 12/343*arctanh(1/7*(3-x-x*3^(1/2)-3^(1/2)*(-x^2-2*x+3)^(1/2))/x*7^(1/2))*7^
(1/2)-4/21*(9-5*3^(1/2)+(21+5*3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x)/(2-3
^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^(1/2)*(3^(1/2)-(-x^2-
2*x+3)^(1/2))^2/x^2)^2+2/147*(18-43*3^(1/2)-(18+49*3^(1/2))*(3^(1/2)-(-x^2-
2*x+3)^(1/2))/x)/(2-3^(1/2)-2*(1+3^(1/2))*(3^(1/2)-(-x^2-2*x+3)^(1/2))/x+3^
(1/2)*(3^(1/2)-(-x^2-2*x+3)^(1/2))^2/x^2)
```

Rubi [A]

time = 0.19, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1674, 12, 632, 212}

$$-\frac{4\left(\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-5\sqrt{3}+9\right)}{21\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)^2}+\frac{2\left(-\frac{\left(18+49\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-43\sqrt{3}+18\right)}{147\left(\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)^2}{x^2}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{-x^2-2x+3}\right)}{x}-\sqrt{3}+2\right)}+\frac{12\tanh^{-1}\left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3}-\sqrt{3}x-x+3}{\sqrt{7}x}\right)}{49\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))
/(21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (
Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] -
((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(147*(2 - Sqrt[3] -
 (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] -
Sqrt[3 - 2*x - x^2])^2)/x^2)) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sq
rt[3 - 2*x - x^2])/(Sqrt[7]*x)])/(49*Sqrt[7])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{3-2x-x^2}\right)^3}\,dx = 2\text{Subst}\left(\int \frac{\sqrt{3}-2x-2x^3-\sqrt{3}\,x^4}{\left(2-\sqrt{3}+2\left(1+\sqrt{3}\right)x+\sqrt{3}\,x^2\right)^3}\,dx, x, \frac{-\sqrt{3}+\sqrt{3-2x}}{x}\right.$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-}\right.}{x^2}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-}\right.}{x^2}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-}\right.}{x^2}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-}\right.}{x^2}\right.}$$

$$= -\frac{4\left(9-5\sqrt{3}+\frac{\left(21+5\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}\right)}{21\left(2-\sqrt{3}-\frac{2\left(1+\sqrt{3}\right)\left(\sqrt{3}-\sqrt{3-2x-x^2}\right)}{x}+\frac{\sqrt{3}\left(\sqrt{3}-\sqrt{3-2x-}\right.}{x^2}\right.}$$

Mathematica [A]
time = 0.46, size = 115, normalized size = 0.37

$$\frac{\frac{7\left(-279+300x+26x^2-48x^3\right)}{\left(-3+2x+2x^2\right)^2}+\frac{14\sqrt{3-2x-x^2}\left(15+83x-58x^2-34x^3\right)}{\left(-3+2x+2x^2\right)^2}+48\sqrt{7}\tanh^{-1}\left(\frac{2-2x+\sqrt{3-2x-x^2}}{\sqrt{7}\,(-1+x)}\right)}{1372}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3),x]
```

```
[Out] ((7*(-279 + 300*x + 26*x^2 - 48*x^3))/(-3 + 2*x + 2*x^2)^2 + (14*Sqrt[3 - 2
*x - x^2]*(15 + 83*x - 58*x^2 - 34*x^3))/(-3 + 2*x + 2*x^2)^2 + 48*Sqrt[7]*
ArcTanh[(2 - 2*x + Sqrt[3 - 2*x - x^2])/(Sqrt[7]*(-1 + x))])/1372
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5983 vs. $2(248) = 496$.
time = 0.18, size = 5984, normalized size = 19.49

method	result
trager	$\frac{(62x^3+100x^2-111x-36)x}{98(2x^2+2x-3)^2} - \frac{(34x^3+58x^2-83x-15)\sqrt{-x^2-2x+3}}{98(2x^2+2x-3)^2} - \frac{6\,\mathrm{RootOf}(\_Z^2-7)\ln\left(-\frac{-\mathrm{RootOf}(\_Z^2-7)x+}{1}\right.}{34}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)
```

Fricas [A]
time = 0.35, size = 223, normalized size = 0.73

$$-\frac{336\,x^3 - 6\,\sqrt{7}\,(4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9)\log\left(\frac{x^4 + 44\,x^3 - \sqrt{7}\,(3\,x^3 + x^2 - 45\,x + 45)\sqrt{-x^2 - 2\,x + 3} + 26\,x^2 - 276\,x + 207}{4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9}\right) - 12\,\sqrt{7}\,(4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9)\log\left(\frac{2\,x^2 + \sqrt{7}\,(2\,x + 1) + 2\,x + 4}{2\,x^2 + 2\,x - 3}\right) - 182\,x^2 + 14\,(34\,x^3 + 58\,x^2 - 83\,x - 15)\sqrt{-x^2 - 2\,x + 3} - 2100\,x + 1953}{1372\,(4\,x^4 + 8\,x^3 - 8\,x^2 - 12\,x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 +
44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 -
276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^
3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*
```

```
x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) -
 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.94, size = 452, normalized size = 1.47

$$-\frac{3}{343}\,\sqrt{7}\log\left(\frac{\left|4\,x-2\,\sqrt{7}+2\right|}{\left|4\,x+2\,\sqrt{7}+2\right|}\right)+\frac{3}{343}\,\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}{\left|2\,\sqrt{7}+\frac{6\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+4\right|}\right)-\frac{3}{343}\,\sqrt{7}\log\left(\frac{\left|-2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}{\left|2\,\sqrt{7}+\frac{2\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}-4\right|}\right)-\frac{48\,x^3-26\,x^2-300\,x+279}{196\,(2\,x^2+2\,x-3)^2}+\frac{4\left(\frac{231\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{3286\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}-\frac{4441\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3}-\frac{18906\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4}-\frac{12487\left(\sqrt{-x^2-2\,x+3}-2\right)^5}{(x+1)^5}+\frac{946\left(\sqrt{-x^2-2\,x+3}-2\right)^6}{(x+1)^6}+\frac{1977\left(\sqrt{-x^2-2\,x+3}-2\right)^7}{(x+1)^7}-414\right)}{441\left(\frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)}{x+1}+\frac{26\left(\sqrt{-x^2-2\,x+3}-2\right)^2}{(x+1)^2}+\frac{8\left(\sqrt{-x^2-2\,x+3}-2\right)^3}{(x+1)^3}-\frac{3\left(\sqrt{-x^2-2\,x+3}-2\right)^4}{(x+1)^4}-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/3
43*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/a
bs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*l
og(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7)
 + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 30
0*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x +
 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x +
 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487
*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/
(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^
2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(
sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x +
 1)^4 - 3)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(x+\sqrt{-x^2-2\,x+3}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)
```

3.756 $\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2}{1-x-\sqrt{-3-2x+x^2}}+2\log\left(1-x-\sqrt{-3-2x+x^2}\right)-\frac{3}{2}\log\left(x+\sqrt{-3-2x+x^2}\right)$$

```
[Out] 2*ln(1-x-(x^2-2*x-3)^(1/2))-3/2*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^
(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2141, 907}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+2\log\left(-\sqrt{x^2-2x-3}-x+1\right)-\frac{3}{2}\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (
3*Log[x + Sqrt[-3 - 2*x + x^2]])/2
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x+\sqrt{-3-2x+x^2}}\,dx = 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}-\frac{3}{4x}\right)dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}} + 2\log\left(1-x-\sqrt{-3-2x+x^2}\right) - \frac{3}{2}\log\left(x+\right.$$

Mathematica [A]
time = 0.19, size = 78, normalized size = 1.20

$$\frac{1}{2}\left(x-\sqrt{-3-2x+x^2}-\log\left(-1-x+\sqrt{-3-2x+x^2}\right)+4\log\left(1+x+\sqrt{-3-2x+x^2}\right)-3\log\left(3+3x+\sqrt{-3-2x+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1),x]
```

```
[Out] (x - Sqrt[-3 - 2*x + x^2] - Log[-1 - x + Sqrt[-3 - 2*x + x^2]] + 4*Log[1 +
x + Sqrt[-3 - 2*x + x^2]] - 3*Log[3 + 3*x + Sqrt[-3 - 2*x + x^2]])/2
```

Maple [A]
time = 0.10, size = 71, normalized size = 1.09

method	result
default	$-\frac{\sqrt{4\left(\frac{3}{2}+x\right)^2-20x-21}}{4}+\frac{5\ln\left(-1+x+\sqrt{\left(\frac{3}{2}+x\right)^2-5x-\frac{21}{4}}\right)}{4}+\frac{3\operatorname{arctanh}\left(\frac{-2-\frac{10x}{3}}{\sqrt{4\left(\frac{3}{2}+x\right)^2-20x}}\right.}{4}$
trager	$\frac{x}{2}-\frac{\sqrt{x^2-2x-3}}{2}+\frac{\ln\left(\frac{\sqrt{x^2-2x-3}\,x^3+x^4+3\sqrt{x^2-2x-3}\,x^2+2x^3+\sqrt{x^2-2x-3}\,x-4x^2-3\sqrt{x^2-}}{(3+2x)^3}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(4*(3/2+x)^2-20*x-21)^(1/2)+5/4*ln(-1+x+((3/2+x)^2-5*x-21/4)^(1/2))+3/
4*arctanh(2/3*(-3-5*x)/(4*(3/2+x)^2-20*x-21)^(1/2))+1/2*x-3/4*ln(3+2*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)
```

Fricas [A]
time = 0.33, size = 77, normalized size = 1.18

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2 - 2x - 3} - \frac{3}{4}\log(2x + 3) - \frac{5}{4}\log\left(-x + \sqrt{x^2 - 2x - 3} + 1\right) + \frac{3}{4}\log\left(-x + \sqrt{x^2 - 2x - 3}\right) - \frac{3}{4}\log\left(-x + \sqrt{x^2 - 2x - 3} - 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(2*x + 3) - 5/4*log(-x + sqrt(x^2
- 2*x - 3) + 1) + 3/4*log(-x + sqrt(x^2 - 2*x - 3)) - 3/4*log(-x + sqrt(x^2
 - 2*x - 3) - 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{x^2 - 2x - 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)
```

Giac [A]
time = 1.71, size = 81, normalized size = 1.25

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2 - 2x - 3} - \frac{3}{4}\log(|2x + 3|) - \frac{5}{4}\log\left(\left|-x + \sqrt{x^2 - 2x - 3} + 1\right|\right) + \frac{3}{4}\log\left(\left|-x + \sqrt{x^2 - 2x - 3}\right|\right) - \frac{3}{4}\log\left(\left|-x + \sqrt{x^2 - 2x - 3} - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*log(abs(2*x + 3)) - 5/4*log(abs(-x +
sqrt(x^2 - 2*x - 3) + 1)) + 3/4*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3/4*lo
g(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\frac{x}{2} - \frac{3\ln\left(x + \frac{3}{2}\right)}{4} - \int \frac{\sqrt{x^2 - 2x - 3}}{2x + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2)),x)
```

```
[Out] x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)
```

3.757 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx$

Optimal. Leaf size=83

$$-\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}+4\log\left(1-x-\sqrt{-3-2x+x^2}\right)-4\log\left(x+\sqrt{-3\right.$$

```
[Out] 4*ln(1-x-(x^2-2*x-3)^(1/2))-4*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/2/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2141, 907}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{3}{2\left(\sqrt{x^2-2x-3}+x\right)}+4\log\left(-\sqrt{x^2-2x-3}-x+1\right)-4\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Lo
g[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^2}\,dx = 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x^2(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$
$$= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}-\frac{3}{4x^2}-\frac{2}{x}\right)dx, x, x+\sqrt{-3-2x+\right.$$
$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{2\left(x+\sqrt{-3-2x+x^2}\right)}+4\log\left(1-x-\right.$$

Mathematica [A]
time = 0.31, size = 69, normalized size = 0.83

$$\frac{-9+6x+4x^2-4(3+x)\sqrt{-3-2x+x^2}-32(3+2x)\tanh^{-1}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right)}{4(3+2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2),x]
[Out] (-9 + 6*x + 4*x^2 - 4*(3 + x)*Sqrt[-3 - 2*x + x^2] - 32*(3 + 2*x)*ArcTanh[(
1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])/(4*(3 + 2*x))
```

Maple [A]
time = 0.08, size = 118, normalized size = 1.42

method	result
trager	$\frac{(3+x)x}{3+2x}-\frac{(3+x)\sqrt{x^2-2x-3}}{3+2x}+4\ln\left(-\frac{\sqrt{x^2-2x-3}+3+x}{3+2x}\right)$
default	$-2\ln(3+2x)+\frac{x}{2}-\frac{9}{4(3+2x)}-\frac{\left(\left(\frac{3}{2}+x\right)^2-5x-\frac{21}{4}\right)^{\frac{3}{2}}}{3\left(\frac{3}{2}+x\right)}-\frac{2\sqrt{4\left(\frac{3}{2}+x\right)^2-20x-21}}{3}+2\ln\left(-1+x+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
[Out] -2*ln(3+2*x)+1/2*x-9/4/(3+2*x)-1/3/(3/2+x)*((3/2+x)^2-5*x-21/4)^(3/2)-2/3*(
4*(3/2+x)^2-20*x-21)^(1/2)+2*ln(-1+x+((3/2+x)^2-5*x-21/4)^(1/2))+2*arctanh(
2/3*(-3-5*x)/(4*(3/2+x)^2-20*x-21)^(1/2))+1/6*(-2+2*x)*((3/2+x)^2-5*x-21/4)
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)
```

Fricas [A]
time = 0.33, size = 97, normalized size = 1.17

$$\frac{4\,x^2-8\,(2\,x+3)\log\left(x^2-\sqrt{x^2-2\,x-3}\,(x+1)-3\right)-8\,(2\,x+3)\log\left(2\,x+3\right)+8\,(2\,x+3)\log\left(-x+\sqrt{x^2-2\,x-3}\right)-4\,\sqrt{x^2-2\,x-3}\,(x+3)+2\,x-15}{4\,(2\,x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*
x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^
2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{x^2-2x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.
time = 1.91, size = 143, normalized size = 1.72

$$\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2-2\,x-3}-\frac{3\left(5\,x-5\,\sqrt{x^2-2\,x-3}+3\right)}{4\left(\left(x-\sqrt{x^2-2\,x-3}\right)^2+3\,x-3\,\sqrt{x^2-2\,x-3}\right)}-\frac{9}{4\,(2\,x+3)}-2\,\log\left(|2\,x+3|\right)-2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}+1\right|\right)+2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}\right|\right)-2\,\log\left(\left|-x+\sqrt{x^2-2\,x-3}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x
 - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) -
2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(
-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)
```

3.758 $\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3} dx$

Optimal. Leaf size=101

$$-\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-2x+x^2}}+6\log\left(1-x-\sqrt{-3-2x+\right.$$

```
[Out] 6*ln(1-x-(x^2-2*x-3)^(1/2))-6*ln(x+(x^2-2*x-3)^(1/2))-2/(1-x-(x^2-2*x-3)^(1
/2))+3/4/(x+(x^2-2*x-3)^(1/2))^2+4/(x+(x^2-2*x-3)^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2141, 907}

$$-\frac{2}{-\sqrt{x^2-2x-3}-x+1}+\frac{4}{\sqrt{x^2-2x-3}+x}+\frac{3}{4\left(\sqrt{x^2-2x-3}+x\right)^2}+6\log\left(-\sqrt{x^2-2x-3}-x+1\right)-6\log\left(\sqrt{x^2-2x-3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/
(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x
+ Sqrt[-3 - 2*x + x^2]]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2141

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*((d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)
^2), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-2x+x^2}\right)^3}\,dx = 2\text{Subst}\left(\int \frac{-3-2x+x^2}{x^3(-2+2x)^2}\,dx, x, x+\sqrt{-3-2x+x^2}\right)$$

$$= 2\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}+\frac{3}{-1+x}-\frac{3}{4x^3}-\frac{2}{x^2}-\frac{3}{x}\right)dx, x, x+\sqrt{-3-\right.$$

$$= -\frac{2}{1-x-\sqrt{-3-2x+x^2}}+\frac{3}{4\left(x+\sqrt{-3-2x+x^2}\right)^2}+\frac{4}{x+\sqrt{-3-}}$$

Mathematica [A]
time = 0.34, size = 83, normalized size = 0.82

$$-\frac{189+108x-48x^2-16x^3+4\sqrt{-3-2x+x^2}\,(33+31x+4x^2)+96(3+2x)^2\tanh^{-1}\left(\frac{1+x}{2+2x+\sqrt{-3-2x+x^2}}\right)}{8(3+2x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3),x]
```

```
[Out] -1/8*(189 + 108*x - 48*x^2 - 16*x^3 + 4*Sqrt[-3 - 2*x + x^2]*(33 + 31*x + 4
*x^2) + 96*(3 + 2*x)^2*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])])/(
3 + 2*x)^2
```

Maple [A]
time = 0.07, size = 146, normalized size = 1.45

method	result
trager	$\frac{(4x^2+33x+36)x}{2(3+2x)^2}-\frac{(4x^2+31x+33)\sqrt{x^2-2x-3}}{2(3+2x)^2}-6\ln\left(3+x-\sqrt{x^2-2x-3}\right)$
default	$-\frac{9}{3+2x}-3\ln(3+2x)+\frac{x}{2}+\frac{27}{8(3+2x)^2}-\frac{\left(\left(\frac{3}{2}+x\right)^2-5x-\frac{21}{4}\right)^{\frac{3}{2}}}{2\left(\frac{3}{2}+x\right)}-\sqrt{4\left(\frac{3}{2}+x\right)^2-20x-21}+3\,\text{arctanh}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -9/(3+2*x)-3*ln(3+2*x)+1/2*x+27/8/(3+2*x)^2-1/2/(3/2+x)*((3/2+x)^2-5*x-21/4
)^(3/2)-(4*(3/2+x)^2-20*x-21)^(1/2)+3*arctanh(2/3*(-3-5*x)/(4*(3/2+x)^2-20*
x-21)^(1/2))+1/4*(-2+2*x)*((3/2+x)^2-5*x-21/4)^(1/2)+3*ln(-1+x+((3/2+x)^2-5
*x-21/4)^(1/2))+1/4/(3/2+x)^2*((3/2+x)^2-5*x-21/4)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)
```

Fricas [A]
time = 0.37, size = 129, normalized size = 1.28

$$\frac{8 x^3 - 10 x^2 - 12 (4 x^2 + 12 x + 9) \log \left(x^2 - \sqrt{x^2 - 2 x - 3} (x + 1) - 3\right) - 12 (4 x^2 + 12 x + 9) \log (2 x + 3) + 12 (4 x^2 + 12 x + 9) \log \left(-x + \sqrt{x^2 - 2 x - 3}\right) - 2 (4 x^2 + 31 x + 33) \sqrt{x^2 - 2 x - 3} - 156 x - 171}{4 (4 x^2 + 12 x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3)*(
x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*lo
g(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 1
56*x - 171)/(4*x^2 + 12*x + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x + \sqrt{x^2 - 2x - 3}\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)
```

```
[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(85) = 170.
time = 2.00, size = 184, normalized size = 1.82

$$\frac{1}{2} x - \frac{1}{2} \sqrt{x^2 - 2 x - 3} - \frac{104 \left(x - \sqrt{x^2 - 2 x - 3}\right)^3 + 315 \left(x - \sqrt{x^2 - 2 x - 3}\right)^2 + 162 x - 162 \sqrt{x^2 - 2 x - 3} + 27}{8 \left(\left(x - \sqrt{x^2 - 2 x - 3}\right)^2 + 3 x - 3 \sqrt{x^2 - 2 x - 3}\right)^2} - \frac{9 (16 x + 21)}{8 (2 x + 3)^2} - 3 \log (|2 x + 3|) - 3 \log \left(\left|-x + \sqrt{x^2 - 2 x - 3} + 1\right|\right) + 3 \log \left(\left|-x + \sqrt{x^2 - 2 x - 3}\right|\right) - 3 \log \left(\left|-x + \sqrt{x^2 - 2 x - 3} - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 31
5*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x -
```

```
 sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/
(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)
) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x -
3) - 3))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{x^2-2\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)
```

```
[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)
```

3.759 $\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx$

Optimal. Leaf size=108

$$-\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)-\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)+\frac{1}{2}\log(3+x)+\frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}\,x+x}{(3+x)^{3/2}}\right.$$

```
[Out] -arctan((-1-x)^(1/2)/(3+x)^(1/2))+1/2*ln(3+x)+1/2*ln((3*(-1-x)^(1/2)+x*(-1-
x)^(1/2)+x*(3+x)^(1/2))/(3+x)^(3/2))-arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/
2))*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {12, 1037, 648, 632, 210, 642, 649, 209, 266}

$$-\text{ArcTan}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)-\sqrt{2}\,\text{ArcTan}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)+\frac{1}{2}\log(x+3)+\frac{1}{2}\log\left(\frac{\sqrt{-x-1}\,x+\sqrt{x+3}\,x+3\sqrt{-x-1}}{(x+3)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
```

```
[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sq
rt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x +
 x*Sqrt[3 + x])/(3 + x)^(3/2)]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x+\sqrt{-3-4x-x^2}}\,dx &= 2\mathrm{Subst}\left(\int \frac{2x}{(1+x^2)\,(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= 4\mathrm{Subst}\left(\int \frac{x}{(1+x^2)\,(1-2x+3x^2)}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{-2-2x}{1+x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-}}{\sqrt{}}\right.\\
&= \frac{1}{2}\mathrm{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + 2\mathrm{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x\right.\\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\log\left(\frac{3\sqrt{-1-x}+\sqrt{-1-x}\,x+}{(3+x)^{3/2}}\right.\\
&= -\tan^{-1}\left(\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right) + \frac{1}{2}\log(3+x) + \frac{1}{2}\mathrm{lo}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 81, normalized size = 0.75

$$
\frac{1}{2}\left(-2\tan^{-1}\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right) + \log\left(x+\sqrt{-3-4x-x^2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1),x]
[Out] (-2*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - 2*Sqrt[2]*ArcTan[(Sqrt[2]*(1 + x
))/(1 + x + Sqrt[-3 - 4*x - x^2])] + Log[x + Sqrt[-3 - 4*x - x^2]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(85) = 170$.
time = 0.54, size = 370, normalized size = 3.43

method	result
default	$\frac{\arcsin(x+2)}{2} - \frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right) - \mathrm{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}}}\right.\right.}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$

trager	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3
/2-x))+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2
-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2
)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x
^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/
(-3/2-x))+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(85) = 170$.
time = 0.40, size = 187, normalized size = 1.73

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right)+\frac{1}{4}\log\left(2x^2+4x+3\right)-\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)+\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x +
3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 -
4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*
sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x
- 4*x - 3)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x+\sqrt{-x^2-4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(85) = 170.
time = 2.17, size = 197, normalized size = 1.82

$$-\frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,(x+1)\right)+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)+\frac{1}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\,\arcsin\left(x+2\right)+\frac{1}{4}\,\log\left(2\,x^2+4\,x+3\right)+\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right)-\frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2
+ 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x +
2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x+\sqrt{-x^2-4\,x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)
```

3.760 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}}+\frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+(1-(-1-x)^(1
/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 652, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1-\frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/
Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^2}\,dx &= 2\text{Subst}\left(\int -\frac{2x}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= -\left(4\text{Subst}\left(\int \frac{x}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+\frac{6\sqrt{-1-x}}{\sqrt{3+x}}\right)\\
&= \frac{1-\frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 84, normalized size = 0.97

$$\frac{3+x+(3+2x)\sqrt{-3-4x-x^2}+\sqrt{2}\,(3+4x+2x^2)\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)}{2\,(3+4x+2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2),x]
```

```
[Out] (3 + x + (3 + 2*x)*Sqrt[-3 - 4*x - x^2] + Sqrt[2]*(3 + 4*x + 2*x^2)*ArcTan[
(Sqrt[2]*(1 + x))/(1 + x + Sqrt[-3 - 4*x - x^2])])/(2*(3 + 4*x + 2*x^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2406 vs. $2(73) = 146$.
time = 0.20, size = 2407, normalized size = 27.67

method	result
trager	$-\frac{(3+2x)x}{2(2x^2+4x+3)}+\frac{(3+2x)\sqrt{-x^2-4x-3}}{4x^2+8x+6}-\frac{\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+2\right)x+3\,\mathrm{RootOf}\left(\_Z^2+2\right)+2\sqrt{-x^2}}{\mathrm{RootOf}\left(\_Z^2+2\right)x-2x-3}\right.}{4}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*(4+4*x)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))-1/2*(-6-
4*x)/(2*x^2+4*x+3)+1/36*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(7*2^(1
/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/
(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(
1+x/(-3/2-x))+1/72*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(arctan(1/6*
(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^2/(-3/2-x)^2-8*arctanh(3*x/(
-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2+2*2^(1/2)*arctan(1/6*(3
*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-16*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^
2-12)^(1/2))-6*(3*x^2/(-3/2-x)^2-12)^(1/2))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-
x))^2)^(1/2)/(1+x/(-3/2-x))/(x^2/(-3/2-x)^2+2)-2/9*3^(1/2)*4^(1/2)*(3*x^2/(
-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)
)+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1
+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))-2/9*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-
12)^(1/2)*(3*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^6/(-
3/2-x)^6+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^6/(-3/2-x)^6
+2*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-
x)^2-4))*x^6/(-3/2-x)^6-2*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-
3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6+(3*x^2/(-3/2-x)^2-12)^(1/2)*
x^5/(-3/2-x)^5-(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2
-12)^(1/2)*x^4/(-3/2-x)^4-36*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)
)*2^(1/2)*x^2/(-3/2-x)^2-2*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3/(-3/2-x)^3-48*ar
ctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2-24*ln(((3*x^
2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^2
/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^2/(-3/2-x)^2+24*ln(((3*x^2/(-3/
2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2
-x)^2-48*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-8*(3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-64*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12
```

```
)^(1/2))-32*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x
^2/(-3/2-x)^2-4))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)+32*ln(((3*x^2/(-3/2-x)^2-1
2)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x)^2
-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))/(x^2/(-3/2-x)^2+2)/((3*x^2/(-3/2
-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/((3*x^2/(-3/2-x)^2-12)^(1/2)*x
/(-3/2-x)-x^2/(-3/2-x)^2+4)+1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2
)*(11*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2^(1/2)*x^6/(-3/2-x)^
6+24*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^6/(-3/2-x)^6+8*ln(
((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4
))*x^6/(-3/2-x)^6-8*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)
^2-4)/(x^2/(-3/2-x)^2-4))*x^6/(-3/2-x)^6+4*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^5/
(-3/2-x)^5-(3*x^2/(-3/2-x)^2-12)^(3/2)*x^2/(-3/2-x)^2+(3*x^2/(-3/2-x)^2-12)
^(1/2)*x^4/(-3/2-x)^4-132*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))*2
^(1/2)*x^2/(-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^3/(-3/2-x)^3-288*arct
anh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2))*x^2/(-3/2-x)^2-96*ln(((3*x^2/
(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/(x^2/(-3/2-x)^2-4))*x^2/(
-3/2-x)^2-8*(3*x^2/(-3/2-x)^2-12)^(1/2)*x^2/(-3/2-x)^2+96*ln(((3*x^2/(-3/2-
x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4))*x^2/(-3/2-x
)^2-176*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-32*(3*x^2/(
-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-384*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-1
2)^(1/2))-128*ln(((3*x^2/(-3/2-x)^2-12)^(1/2)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)/
(x^2/(-3/2-x)^2-4))+16*(3*x^2/(-3/2-x)^2-12)^(1/2)+128*ln(((3*x^2/(-3/2-x)^
2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/(x^2/(-3/2-x)^2-4)))/((x^2/(-3/2-x
)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))/(x^2/(-3/2-x)^2+2)/((3*x^2/(-
3/2-x)^2-12)^(1/2)*x/(-3/2-x)+x^2/(-3/2-x)^2-4)/((3*x^2/(-3/2-x)^2-12)^(1/2
)*x/(-3/2-x)-x^2/(-3/2-x)^2+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)
```

Fricas [A]
time = 0.35, size = 121, normalized size = 1.39

$$\frac{2\sqrt{2}\,(2x^2+4x+3)\arctan\left(\sqrt{2}\,(x+1)\right)-\sqrt{2}\,(2x^2+4x+3)\arctan\left(\frac{\sqrt{2}\,(6x^2+20x+15)\sqrt{-x^2-4x-3}}{4\,(2x^3+11x^2+18x+9)}\right)+4\sqrt{-x^2-4x-3}\,(2x+3)+4x+12}{8\,(2x^2+4x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 +
 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^
3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x
^2 + 4*x + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x+\sqrt{-x^2-4x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)
```

```
[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(72) = 144.
time = 2.22, size = 263, normalized size = 3.02

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)+\frac{x+3}{2\left(2x^2+4x+3\right)}-\frac{\frac{10\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{7\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}-\frac{2\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+3}{3\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3
*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x
 - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x
^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 +
 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)
```

3.761 $\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3} dx$

Optimal. Leaf size=149

$$-\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)}-\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2}-\frac{3\tan^{-1}\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/18*(-13+2
7*(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))-2/
9*(2-(-1-x)^(1/2)/(3+x)^(1/2))/(1-3*(1+x)/(3+x)-2*(-1-x)^(1/2)/(3+x)^(1/2))
^2
```

Rubi [A]
time = 0.07, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 1674, 652, 632, 210}

$$-\frac{3\text{ArcTan}\left(\frac{1-\frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{13-\frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)}-\frac{2\left(2-\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3}-\frac{2\sqrt{-x-1}}{\sqrt{x+3}}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -1/18*(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sq
rt[-1 - x])/Sqrt[3 + x]) - (2*(2 - Sqrt[-1 - x]/Sqrt[3 + x]))/(9*(1 - (3*(1
 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[
-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(x+\sqrt{-3-4x-x^2}\right)^3}\,dx = 2\text{Subst}\left(\int \frac{2x(1+x^2)}{(1-2x+3x^2)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= 4\text{Subst}\left(\int \frac{x(1+x^2)}{(1-2x+3x^2)^3}\,dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)$$

$$= -\frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{1}{4}\text{Subst}\left(\int \frac{\frac{56}{9}+\frac{16x}{3}}{(1-2x+3x^2)^2}\,dx, x, \frac{\sqrt{-}}{\sqrt{3}}\right.$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} + \frac{3}{2}\text{Sul}$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - 3\text{Sub}$$

$$= -\frac{13-\frac{27\sqrt{-1-x}}{\sqrt{3+x}}}{18\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)} - \frac{2\left(2-\frac{\sqrt{-1-x}}{\sqrt{3+x}}\right)}{9\left(1-\frac{3(1+x)}{3+x}-\frac{2\sqrt{-1-x}}{\sqrt{3+x}}\right)^2} - \frac{3\tan}{}$$

Mathematica [A]
time = 0.40, size = 109, normalized size = 0.73

$$-\frac{9+15x+16x^2+6x^3+\sqrt{-3-4x-x^2}\,(15+26x+22x^2+8x^3)+3\sqrt{2}\,(3+4x+2x^2)^2\tan^{-1}\left(\frac{\sqrt{2}\,(1+x)}{1+x+\sqrt{-3-4x-x^2}}\right)}{4\,(3+4x+2x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3),x]
```

```
[Out] -1/4*(9 + 15*x + 16*x^2 + 6*x^3 + Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2
+ 8*x^3) + 3*Sqrt[2]*(3 + 4*x + 2*x^2)^2*ArcTan[(Sqrt[2]*(1 + x))/(1 + x +
Sqrt[-3 - 4*x - x^2])])/(3 + 4*x + 2*x^2)^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 14529 vs. 2(120) = 240.
time = 0.36, size = 14530, normalized size = 97.52

method	result
trager	$\frac{(4x^3+10x^2+12x+9)x}{4(2x^2+4x+3)^2} - \frac{(8x^3+22x^2+26x+15)\sqrt{-x^2-4x-3}}{4(2x^2+4x+3)^2} - \frac{3\,\mathrm{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{-2\,\mathrm{RootOf}\left(\_Z^2+2\right)x+2\sqrt{-}}{\mathrm{RootOf}\left(\right.}\right.}{8}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)
```

Fricas [A]
time = 0.35, size = 171, normalized size = 1.15

$$-\frac{24\,x^{3}+6\,\sqrt{2}\,(4\,x^{4}+16\,x^{3}+28\,x^{2}+24\,x+9)\arctan\left(\sqrt{2}\,(x+1)\right)-3\,\sqrt{2}\,(4\,x^{4}+16\,x^{3}+28\,x^{2}+24\,x+9)\arctan\left(\frac{\sqrt{2}\,(6\,x^{2}+20\,x+15)\sqrt{-x^{2}-4\,x-3}}{4\,(2\,x^{3}+11\,x^{2}+18\,x+9)}\right)+64\,x^{2}+4\,(8\,x^{3}+22\,x^{2}+26\,x+15)\sqrt{-x^{2}-4\,x-3}+60\,x+36}{16\,(4\,x^{4}+16\,x^{3}+28\,x^{2}+24\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(
2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqr
t(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9))
+ 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)
/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(119) = 238.
time = 2.02, size = 367, normalized size = 2.46

$$-\frac{3}{8}\sqrt{2}\arctan\left(\sqrt{2}(x+1)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)-\frac{6x^3+16x^2+15x+9}{4(2x^2+4x+3)^2}+\frac{\frac{618\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{1547\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{2362\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{2223\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+\frac{1174\left(\sqrt{-x^2-4x-3}-1\right)^5}{(x+2)^5}+\frac{377\left(\sqrt{-x^2-4x-3}-1\right)^6}{(x+2)^6}+\frac{6\left(\sqrt{-x^2-4x-3}-1\right)^7}{(x+2)^7}+117}{18\left(\frac{8\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{14\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+\frac{8\left(\sqrt{-x^2-4x-3}-1\right)^3}{(x+2)^3}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^4}{(x+2)^4}+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2
*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x
+ 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*
x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(s
qrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(
x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3
) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(x+\sqrt{-x^2-4\,x-3}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)
```

```
[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)
```

3.762 $\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A]
time = 0.14, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1694, 12, 1261, 706, 643}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4}\,dx = \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x$$
$$= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx, x, \frac{1}{2}+x$$
$$= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx, x, \left(\frac{1}{2}+x\right)$$
$$= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2} + \frac{1}{40}\text{Subst}\left(\int (-1+$$
$$= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2} - \frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-\right.$$

Mathematica [A]
time = 0.06, size = 42, normalized size = 1.00

$$\frac{1}{15}\left(-2-3x^2-6x^3-3x^4\right)\left(1-x^2-2x^3-x^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x)^3*(1 + 2*x)*Sqrt[1 - x^2 - 2*x^3 - x^4],x]
[Out] ((-2 - 3*x^2 - 6*x^3 - 3*x^4)*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(38) = 76$.
time = 0.73, size = 191, normalized size = 4.55

method	result

gosper	$\frac{(x^2+x+1)(x^2+x-1)(3x^4+6x^3+3x^2+2)\sqrt{-x^4-2x^3-x^2+1}}{15}$
trager	$\left(\frac{1}{5}x^8+\frac{4}{5}x^7+\frac{6}{5}x^6+\frac{4}{5}x^5+\frac{2}{15}x^4-\frac{2}{15}x^3-\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{-x^4-2x^3-x^2+1}$
risch	$-\frac{(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)(x^4+2x^3+x^2-1)}{15\sqrt{-x^4-2x^3-x^2+1}}$
default	$-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-}}{}$
elliptic	$-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(1+x)^3*(2*x+1)*(-x^4-2*x^3-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*x^2*(-x^4-2*x^3-x^2+1)^(1/2)-2/15*(-x^4-2*x^3-x^2+1)^(1/2)+1/5*x^8*(-
x^4-2*x^3-x^2+1)^(1/2)+4/5*x^7*(-x^4-2*x^3-x^2+1)^(1/2)+6/5*x^6*(-x^4-2*x^3
-x^2+1)^(1/2)+4/5*x^5*(-x^4-2*x^3-x^2+1)^(1/2)+2/15*x^4*(-x^4-2*x^3-x^2+1)^
(1/2)-2/15*x^3*(-x^4-2*x^3-x^2+1)^(1/2)
```

Maxima [A]
time = 0.31, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\,\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A]
time = 0.34, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(36) = 72$.
time = 0.25, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Giac [A]
time = 1.43, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.18, size = 38, normalized size = 0.90

$$-\frac{\left(3\,x^4+6\,x^3+3\,x^2+2\right)\,{\left(-x^4-2\,x^3-x^2+1\right)}^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2),x)
```

```
[Out] -((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15
```

3.763 $\int (1+2x)\left(x+x^2\right)^3 \sqrt{1-\left(x+x^2\right)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{1}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}\left(2+3x^2+6x^3+3x^4\right)$$

```
[Out] -1/15*(-x^4-2*x^3-x^2+1)^(3/2)*(3*x^4+6*x^3+3*x^2+2)
```

Rubi [A]
time = 0.16, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1607, 1694, 12, 1261, 706, 643}

$$-\frac{1}{5}x^2\left(-x^4-2x^3-x^2+1\right)^{3/2}(x+1)^2-\frac{2}{15}\left(-x^4-2x^3-x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x
^4)^(3/2))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 706

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*d*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(b*(m + 2*p
 + 1))), x] + Dist[d^2*(m - 1)*((b^2 - 4*a*c)/(b^2*(m + 2*p + 1))), Int[(d
+ e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ
[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && Rational
Q[p]) || OddQ[m])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int (1+2x)\left(x+x^2\right)^3\sqrt{1-\left(x+x^2\right)^2}\,dx &= \int x^3(1+x)^3(1+2x)\sqrt{1-\left(x+x^2\right)^2}\,dx\\
&= \text{Subst}\left(\int \frac{1}{128}x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx,x,\frac{1}{2}+x\right)\\
&= \frac{1}{128}\text{Subst}\left(\int x\left(-1+4x^2\right)^3\sqrt{15+8x^2-16x^4}\,dx,x,\frac{1}{2}+x\right)\\
&= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2}\,dx,x,\left(\frac{1}{2}+x\right)^2\right)\\
&= -\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3/2}+\frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{\right.\\
&= -\frac{2}{15}\left(1-x^2-2x^3-x^4\right)^{3/2}-\frac{1}{5}x^2(1+x)^2\left(1-x^2-2x^3-x^4\right)^{3
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 30, normalized size = 0.71

$$\frac{1}{15}\left(-2-3\left(x+x^2\right)^2\right)\left(1-\left(x+x^2\right)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2],x]
```

```
[Out] ((-2 - 3*(x + x^2)^2)*(1 - (x + x^2)^2)^(3/2))/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(38) = 76$.
time = 0.52, size = 191, normalized size = 4.55

method	result
gosper	$\frac{(x^2+x+1)(x^2+x-1)(3x^4+6x^3+3x^2+2)\sqrt{-x^4-2x^3-x^2+1}}{15}$
trager	$\left(\frac{1}{5}x^8+\frac{4}{5}x^7+\frac{6}{5}x^6+\frac{4}{5}x^5+\frac{2}{15}x^4-\frac{2}{15}x^3-\frac{1}{15}x^2-\frac{2}{15}\right)\sqrt{-x^4-2x^3-x^2+1}$
risch	$-\frac{(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)(x^4+2x^3+x^2-1)}{15\sqrt{-x^4-2x^3-x^2+1}}$
default	$-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-}}{}$
elliptic	$-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}+\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*x^2*(-x^4-2*x^3-x^2+1)^(1/2)-2/15*(-x^4-2*x^3-x^2+1)^(1/2)+1/5*x^8*(-
x^4-2*x^3-x^2+1)^(1/2)+4/5*x^7*(-x^4-2*x^3-x^2+1)^(1/2)+6/5*x^6*(-x^4-2*x^3
-x^2+1)^(1/2)+4/5*x^5*(-x^4-2*x^3-x^2+1)^(1/2)+2/15*x^4*(-x^4-2*x^3-x^2+1)^
(1/2)-2/15*x^3*(-x^4-2*x^3-x^2+1)^(1/2)
```

Maxima [A]
time = 0.31, size = 59, normalized size = 1.40

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{x^2+x+1}\,\sqrt{-x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2
+ x + 1)*sqrt(-x^2 - x + 1)
```

Fricas [A]
time = 0.34, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3\,x^8+12\,x^7+18\,x^6+12\,x^5+2\,x^4-2\,x^3-x^2-2\right)\sqrt{-x^4-2\,x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(-x^4
 - 2*x^3 - x^2 + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(36) = 72$.
time = 1.00, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5}+\frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15}-\frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)
```

```
[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2
+ 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x
**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt
(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 -
2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15
```

Giac [A]
time = 2.62, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2\,x^3+x^2-1\right)^2\sqrt{-x^4-2\,x^3-x^2+1}-\frac{1}{3}\left(-x^4-2\,x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^4 + 2*x^3 + x^2 - 1)^2*sqrt(-x^4 - 2*x^3 - x^2 + 1) - 1/3*(-x^4 - 2*
x^3 - x^2 + 1)^(3/2)
```

Mupad [B]
time = 3.42, size = 51, normalized size = 1.21

$$\sqrt{1-\left(x^2+x\right)^2}\,\left(\frac{x^8}{5}+\frac{4\,x^7}{5}+\frac{6\,x^6}{5}+\frac{4\,x^5}{5}+\frac{2\,x^4}{15}-\frac{2\,x^3}{15}-\frac{x^2}{15}-\frac{2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)
```

```
[Out] (1 - (x + x^2)^2)^(1/2)*((2*x^4)/15 - (2*x^3)/15 - x^2/15 + (4*x^5)/5 + (6*
x^6)/5 + (4*x^7)/5 + x^8/5 - 2/15)
```

3.764 $\int \left(8x-8x^2+4x^3-x^4\right)^{3/2} dx$

Optimal. Leaf size=102

$$\frac{2}{35}\left(13-3(-1+x)^2\right)\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1120, 1105, 1190, 1194, 538, 435, 430}

$$-\frac{176}{35}\sqrt{3}\,F\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}\,E\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \left(8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$
$$= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{3}\right.$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$
$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 20.45, size = 278, normalized size = 2.73

$$\frac{896-1056x+352x^2+848x^3-1420x^4+1152x^5-602x^6+206x^7-45x^8+5x^9+\frac{112i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}-304i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x^2\sqrt{\frac{4-2x+x^2}{x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{35\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (896 - 1056*x + 352*x^2 + 848*x^3 - 1420*x^4 + 1152*x^5 - 602*x^6 + 206*x^7
 - 45*x^8 + 5*x^9 + ((112*I)*Sqrt[2]*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2]*E
llipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])
/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] - (304*I)*Sqrt[2
]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*El
lipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/
(-I + Sqrt[3])])/(35*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1049 vs. 2(86) = 172.
time = 0.64, size = 1050, normalized size = 10.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-
66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+8*x)^(1/2
)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
```

```
7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3^(1/2))*((-1+I*3^(1
/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x
-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(
x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/
(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-
I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))
/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(
1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/
2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1
/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1
/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-
1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*
3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Fricas [A]
time = 0.12, size = 58, normalized size = 0.57

$$-\frac{(5x^6-30x^5+91x^4-164x^3+130x^2-12x-132)\sqrt{-x^4+4x^3-8x^2+8x}}{35(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
[Out] -1/35*(5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130*x^2 - 12*x - 132)*sqrt(-x^4
+ 4*x^3 - 8*x^2 + 8*x)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
```

```
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.765 $\int \sqrt{8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1120, 1105, 1194, 538, 435, 430}

$$-\frac{4F\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}\,(x-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{8x-8x^2+4x^3-x^4}\,dx &= \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\,d\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\,\sqrt{6+}}\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x,\right.\\ &= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F(}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.41, size = 256, normalized size = 4.13

$$-\frac{-16+24x-24x^2+14x^3-5x^4+x^5-\frac{2i\sqrt{2}(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}+8i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x^2\sqrt{\frac{4-2x+x^2}{x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{3\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -1/3*(-16 + 24*x - 24*x^2 + 14*x^3 - 5*x^4 + x^5 - ((2*I)*Sqrt[2]*(-2 + x)*
x*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[((-I)*(-2 + x))/((-I +
Sqrt[3])*x)] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*S
qrt[(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^
3))]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(54) = 108$.
time = 0.47, size = 946, normalized size = 15.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-1
-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^
(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/
2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Elliptic
F(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2)
)/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x
/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(
1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1
/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I
*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*
3^(1/2)))^(1/2)))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*(
(-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3
^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*
I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/
2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1
+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(
1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2)
)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))
^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [A]
time = 0.09, size = 36, normalized size = 0.58

$$\frac{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,(x^2-2\,x+3)}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x + 3)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
[Out] Integral(sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int((8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.766 $\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1120, 1109, 430}

$$-\frac{F\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)$$
$$= -\frac{F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.17, size = 156, normalized size = 9.18

$$\frac{\sqrt{-i+\sqrt{3}+\frac{4i}{x}}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x\left(-4+x-i\sqrt{3}\,x\right)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{2}\sqrt{i+\sqrt{3}-\frac{4i}{x}}\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[-I + Sqrt[3] + (4*I)/x]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x*(-
4 + x - I*Sqrt[3]*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(Sqrt[2]*Sqrt[I + Sqrt[3] - (4*I)/x
]*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(15) = 30.
time = 0.52, size = 200, normalized size = 11.76

method	result
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\text{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\right.}{\left(-1+i\sqrt{3}\right)\sqrt{-x\left(x-2\right)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\text{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\right.}{\left(-1+i\sqrt{3}\right)\sqrt{-x\left(x-2\right)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 16, normalized size = 0.94

$$-\frac{1}{2}\sqrt{2}\,\text{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-x^4+4x^3-8x^2+8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.767 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1120, 1106, 1194, 538, 435, 430}

$$-\frac{F\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$
$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx, x, \right.$$
$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\right.$$
$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} - \frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x, -1\right.$$
$$= \frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}} + \frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}} - \frac{F\left(\sin^{-1}(1\right.}{4\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 20.62, size = 261, normalized size = 3.58

$$\frac{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}\left(\frac{\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{\frac{4-2x+x^2}{x^2}}}-\frac{2+x^2-4i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,x^2\sqrt{\frac{4-2x+x^2}{x^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{4-2x+x^2}\right)}{24(-2+x)x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*((Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-
2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(S
qrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/Sqrt[(4 - 2*x + x^2)/x^2] -
(2 + x^2 - (4*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*x^2*Sqrt[
(4 - 2*x + x^2)/x^2]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*
3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])/(4 - 2*x + x^2)))/(24*(-2 + x)*x)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(61) = 122.
time = 0.49, size = 963, normalized size = 13.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(1/24+1/192*x^2)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/32*(-x^3+4*x^2-8*x+8)/(
x*(-x^3+4*x^2-8*x+8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/
2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-
I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/
2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))
^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1
/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2
))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2
*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2)
)*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-1/24*(x*(x-1+I*3^(1/2)
)*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(
1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(
1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-
1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/
2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2
))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I
*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^
```

```
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.07, size = 119, normalized size = 1.63

$$-\frac{5\sqrt{2}\,(x^4-4\,x^3+8\,x^2-8\,x)\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)-6\sqrt{2}\,(x^4-4\,x^3+8\,x^2-8\,x)\mathrm{weierstrassZeta}\left(-\frac{2}{3},\frac{7}{54},\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)\right)+3\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}\,(x^2+2)}{72\,(x^4-4\,x^3+8\,x^2-8\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="fricas")
[Out] -1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/54
, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta(-
2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^4 +
 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^4+4x^3-8x^2+8x\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(3/2),x)
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.768 $\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}}+\frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}-\frac{11F\left(\sin\right.}{}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1120, 1106, 1192, 1194, 538, 435, 430}

$$-\frac{11F\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{(7(x-1)^2+26)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)(x-1)}{72\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{(8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{(3-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{(3-2x^2-x^4)^{3/2}}\,d\right.$$

$$= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}$$

$$= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}$$

$$= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}$$

$$= -\frac{(26+7(1-x)^2)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{(5+(-1+x)^2)(-1+x)}{72\,(3-2(-1+x)^2-(-1+x)^4)^{3/2}}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 20.75, size = 298, normalized size = 2.73

$$\frac{\frac{7i\sqrt{2}(-2+x)x^2\sqrt{\frac{4-2x+x^2}{x^2}}E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}}+\frac{36-232x+274x^2-226x^3+115x^4-37x^5+7x^6-19i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}x^3\sqrt{\frac{4-2x+x^2}{x^2}}(-8+8x-4x^2+x^3)F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)}{-8+8x-4x^2+x^3}}{432x\sqrt{-x(-8+8x-4x^2+x^3)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] (((7*I)*Sqrt[2]*(-2 + x)*x^2*Sqrt[(4 - 2*x + x^2)/x^2]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])]])/S
qrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)] + (36 - 232*x + 274*x^2 - 226*x^3 +
 115*x^4 - 37*x^5 + 7*x^6 - (19*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt
[3])*x)]*x^3*Sqrt[(4 - 2*x + x^2)/x^2]*(-8 + 8*x - 4*x^2 + x^3)*EllipticF[A
rcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqr
t[3])])/(-8 + 8*x - 4*x^2 + x^3))/(432*x*Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))
])
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(93) = 186.
time = 0.50, size = 1039, normalized size = 9.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*
x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/768*(-x^4+4
*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1
/2)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)
^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2
))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(
1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((
-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^
(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/
2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^
(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(
1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x
-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I
*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2
*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+
I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))
^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I
*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))
/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(
1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1
/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2
))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 195, normalized size = 1.79

$$-\frac{43\sqrt{2}\left(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2\right)\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}\left(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2\right)\mathrm{weierstrassZeta}\left(-\frac{2}{3},\frac{7}{54},\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)\right)+6\left(7x^6-37x^5+115x^4-226x^3+274x^2-232x+36\right)\sqrt{-x^4+4x^3-8x^2+8x}}{2592\left(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64
*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 - 8
```

```
*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/3,
7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*x^5
+ 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x
))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-x^4+4x^3-8x^2+8x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+8*x)**(5/2),x)
[Out] Integral((-x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+8*x)^(5/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + 8*x)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x^4+4\,x^3-8\,x^2+8\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.769 $\int ((2-x)x(4-2x+x^2))^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{2}{35}\left(13-3(-1+x)^2\right)\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+$$

```
[Out] 1/7*(3-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/5*EllipticE(-1+x,1/3*I*3^(1/2))
*3^(1/2)+176/35*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+2/35*(13-3*(-1+x)^2)*
(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1120, 1105, 1190, 1194, 538, 435, 430}

$$-\frac{176}{35}\sqrt{3}\,F\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)+\frac{16}{5}\sqrt{3}\,E\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)+\frac{1}{7}(x-1)\left(-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}\left(13-3(x-1)^2\right)(x-1)\sqrt{-(x-1)^4-2(x-1)^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (2*(13 - 3*(-1 + x)^2)*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + (
(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*Sqrt[3]*EllipticE[A
rcSin[1 - x], -1/3])/5 - (176*Sqrt[3]*EllipticF[ArcSin[1 - x], -1/3])/35
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}\,dx = \text{Subst}\left(\int \left(3-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \frac{1}{7}\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int \left(6-2x^2\right)\sqrt{\right.$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$

$$= -\frac{2}{35}\left(13-3(1-x)^2\right)\sqrt{3-2(1-x)^2-(1-x)^4}\,(1-x)+\frac{1}{7}(3-2(-1$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.68, size = 278, normalized size = 2.73

$$\frac{\sqrt{-x(-8+8x-4x^2+x^3)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}\left(-224+152x+44x^2-228x^3+230x^4-116x^5+35x^6-5x^7\right)+112\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+304i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)\right)}{35(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(3/2),x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-224 + 152
*x + 44*x^2 - 228*x^3 + 230*x^4 - 116*x^5 + 35*x^6 - 5*x^7) + 112*Sqrt[2]*(
-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticE[ArcSin[Sqr
t[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])] +
(304*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3])*x)]*EllipticF[ArcSin[S
qrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-I + Sqrt[3])])
)/(35*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1049 vs. 2(86) = 172.
time = 0.56, size = 1050, normalized size = 10.29 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-
66/35*x^3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+8*x)^(1/2
)-32/35*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-4/7*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+32/
7*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
```

```
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+64/5*(-1-I*3^(1/2))*((-1+I*3^(1
/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x
-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(
x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/
(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-
I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2
),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))
/(1-I*3^(1/2)))^(1/2)))-16/5*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(
1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))
/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/
2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/
(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1
/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1
/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-
1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*
3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^
(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Fricas [**A**]
time = 0.10, size = 58, normalized size = 0.57

$$-\frac{(5\,x^6-30\,x^5+91\,x^4-164\,x^3+130\,x^2-12\,x-132)\sqrt{-x^4+4\,x^3-8\,x^2+8\,x}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
[Out] -1/35*(5*x^6 - 30*x^5 + 91*x^4 - 164*x^3 + 130*x^2 - 12*x - 132)*sqrt(-x^4
+ 4*x^3 - 8*x^2 + 8*x)/(x - 1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x(2-x)\left(x^2-2x+4\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
```

```
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.770 $\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx$

Optimal. Leaf size=62

$$\frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] -2/3*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+4/3*EllipticF(-1+x,1/3*I*3^(1/2)
)*3^(1/2)+1/3*(-1+x)*(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1120, 1105, 1194, 538, 435, 430}

$$-\frac{4F\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{2E\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{\sqrt{3}}+\frac{1}{3}\sqrt{-(x-1)^4-2(x-1)^2+3}\,(x-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/3 + (2*EllipticE[ArcSin[1 -
x], -1/3])/Sqrt[3] - (4*EllipticF[ArcSin[1 - x], -1/3])/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{(2-x)x\left(4-2x+x^2\right)}\,dx &= \text{Subst}\left(\int \sqrt{3-2x^2-x^4}\,dx, x, -1+x\right)\\
&= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{1}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{3-2x^2-x^4}}\right.\\
&= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2}{3}\text{Subst}\left(\int \frac{6-2x^2}{\sqrt{2-2x^2}\,\sqrt{6+}}\right.\\
&= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)-\frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx, x\right.\\
&= \frac{1}{3}\sqrt{3-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{4F}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.80, size = 256, normalized size = 4.13

$$\frac{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}\left(\sqrt{\frac{4-2x+x^2}{x^2}}\left(-4+4x-3x^2+x^3\right)+2\sqrt{2}\left(-i+\sqrt{3}\right)\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,E\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)+8i\sqrt{2}\sqrt{-\frac{i(-2+x)}{\left(-i+\sqrt{3}\right)x}}\,F\left(\sin^{-1}\left(\frac{\sqrt{i+\sqrt{3}-\frac{4i}{x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{-i+\sqrt{3}}\right)\right)}{3(-2+x)x\sqrt{\frac{4-2x+x^2}{x^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] (Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))]*(Sqrt[(4 - 2*x + x^2)/x^2]*(-4 + 4*x -
 3*x^2 + x^3) + 2*Sqrt[2]*(-I + Sqrt[3])*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticE[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])] + (8*I)*Sqrt[2]*Sqrt[((-I)*(-2 + x))/((-I + Sqrt[3
])*x)]*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (4*I)/x]/(Sqrt[2]*3^(1/4))], (2*
Sqrt[3])/(-I + Sqrt[3])]))/(3*(-2 + x)*x*Sqrt[(4 - 2*x + x^2)/x^2])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(54) = 108$.
time = 0.50, size = 946, normalized size = 15.26 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+8*x)^(1/2)+8/3*(-1
-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^
(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/
2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Elliptic
F(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2)
)/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+8/3*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x
/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(
1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(
x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3
^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1
/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I
*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*
3^(1/2)))^(1/2)))-2/3*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*(
(-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3
^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*
I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^
(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/
2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1
+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(
1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2)
)/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))
^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [A]
time = 0.11, size = 36, normalized size = 0.58

$$\frac{\sqrt{-x^4 + 4x^3 - 8x^2 + 8x}\,(x^2 - 2x + 3)}{3(x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x)*(x^2 - 2*x + 3)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x(2 - x)(x^2 - 2x + 4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x**2-2*x+4))**(1/2),x)
[Out] Integral(sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-x\,(x - 2)\,(x^2 - 2x + 4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
[Out] int((-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.771 $\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx$

Optimal. Leaf size=17

$$-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1120, 1109, 430}

$$-\frac{F\left(\text{ArcSin}(1-x)\,\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(EllipticF[ArcSin[1 - x], -1/3]/Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 1109

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[1/(Sqrt[b + q + 2*c*x^2]*Sqrt[-b + q
- 2*c*x^2]), x], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c
, 0]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{(2-x)x(4-2x+x^2)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx, x, -1+x\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\,dx, x, -1+x\right)$$
$$= -\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.92, size = 100, normalized size = 5.88

$$-\frac{\sqrt[3]{-1}(-2+x)^2\sqrt{\frac{x\left(-1+i\sqrt{3}+x\right)}{(-2+x)^2}}\sqrt{\frac{-2+x-\sqrt[3]{-1}\,x}{-2+x}}\,F\left(\sin^{-1}\left(\sqrt{-\frac{(-1)^{2/3}x}{-2+x}}\right)\middle|(-1)^{2/3}\right)}{\sqrt{-x\left(-8+8x-4x^2+x^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - x)*x*(4 - 2*x + x^2)],x]
```

```
[Out] -(((-1)^(1/3)*(-2 + x)^2*Sqrt[(x*(-1 + I*Sqrt[3] + x))/(-2 + x)^2]*Sqrt[(-2
 + x - (-1)^(1/3)*x)/(-2 + x)]*EllipticF[ArcSin[Sqrt[-(((-1)^(2/3)*x)/(-2 +
 x))]], (-1)^(2/3)])/Sqrt[-(x*(-8 + 8*x - 4*x^2 + x^3))])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(15) = 30.
time = 0.51, size = 200, normalized size = 11.76

method	result
default	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\text{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\right.}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$
elliptic	$\frac{2\left(-1-i\sqrt{3}\right)\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}}\,(x-2)^2\sqrt{\frac{x-1+i\sqrt{3}}{\left(1-i\sqrt{3}\right)(x-2)}}\sqrt{\frac{x-1-i\sqrt{3}}{\left(1+i\sqrt{3}\right)(x-2)}}\,\text{EllipticF}\left(\sqrt{\frac{\left(-1+i\sqrt{3}\right)x}{\left(1+i\sqrt{3}\right)(x-2)}},\right.}{\left(-1+i\sqrt{3}\right)\sqrt{-x(x-2)\left(x-1+i\sqrt{3}\right)\left(x-1-i\sqrt{3}\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1
+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2)
)^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*Ell
ipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 16, normalized size = 0.94

$$-\frac{1}{2}\sqrt{2}\,\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(2-x)\,(x^2-2x+4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(1/2),x)
[Out] Integral(1/sqrt(x*(2 - x)*(x**2 - 2*x + 4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(-(x^2 - 2*x + 4)*(x - 2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(1/2), x)
```

3.772 $\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(5+(-1+x)^2)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}-\frac{F\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}$$

```
[Out] -1/24*EllipticE(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/12*EllipticF(-1+x,1/3*I*3^(1/
2))*3^(1/2)+1/24*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1120, 1106, 1194, 538, 435, 430}

$$-\frac{F\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)}{4\sqrt{3}}+\frac{E\left(\text{ArcSin}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}+\frac{((x-1)^2+5)(x-1)}{24\sqrt{-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(24*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + Elli
pticE[ArcSin[1 - x], -1/3]/(8*Sqrt[3]) - EllipticF[ArcSin[1 - x], -1/3]/(4*
Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
```

```
SqrtQ[-b/a, -d/c]))))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{3/2}}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{3/2}}\,dx,x,-1+x\right)\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}-\frac{1}{48}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{3-2x^2-x^4}}\,dx,x\right.\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}-\frac{1}{24}\text{Subst}\left(\int \frac{-6+2x^2}{\sqrt{2-2x^2}\sqrt{6+2x^2}}\right.\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}-\frac{1}{24}\text{Subst}\left(\int \frac{\sqrt{6+2x^2}}{\sqrt{2-2x^2}}\,dx,x,-1\right.\\
&= \frac{\left(5+(-1+x)^2\right)(-1+x)}{24\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{E\left(\sin^{-1}(1-x)\middle|-\frac{1}{3}\right)}{8\sqrt{3}}-\frac{F\left(\sin^{-1}(1\right.}{4\sqrt{}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.

time = 17.89, size = 298, normalized size = 4.08

$$\frac{(-2+x)^2x(4-2x+x^2)\left(2(-1+x)x-3(4-2x+x^2)-\frac{3x(4-2x+x^2)}{-2+x}-4(2-x)\sqrt{\frac{4-2x+x^2}{(-2+x)^2}}\left(x\sqrt{\frac{4-2x+x^2}{(-2+x)^2}}-\sqrt{2}\left(i+\sqrt{3}\right)\sqrt{\frac{ix}{\left(i+\sqrt{3}\right)(-2+x)}}E\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)+4i\sqrt{2}\sqrt{\frac{ix}{\left(i+\sqrt{3}\right)(-2+x)}}F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}{\sqrt{2}\sqrt[4]{3}}\right)|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)\right)\right)}{96\left(-x\left(-8+8x-4x^2+x^3\right)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-3/2),x]
```

```
[Out] ((-2 + x)^2*x*(4 - 2*x + x^2)*(2*(-1 + x)*x - 3*(4 - 2*x + x^2) - (3*x*(4 -
 2*x + x^2))/(-2 + x) - 4*(2 - x)*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*(x*Sqrt[
(4 - 2*x + x^2)/(-2 + x)^2] - Sqrt[2]*(I + Sqrt[3])*Sqrt[(I*x)/((I + Sqrt[3
])*(-2 + x))]*EllipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]
*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])] + (4*I)*Sqrt[2]*Sqrt[(I*x)/((I + Sqr
t[3])*(-2 + x))]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt
[2]*3^(1/4))], (2*Sqrt[3])/(I + Sqrt[3])])))/(96*(-(x*(-8 + 8*x - 4*x^2 + x
^3)))^(3/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 962 vs. 2(61) = 122.
time = 0.50, size = 963, normalized size = 13.19 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-x)*x*(x^2-2*x+4))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*x*(1/24+1/192*x^2)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/32*(-x^3+4*x^2-8*x+8)/(
x*(-x^3+4*x^2-8*x+8))^(1/2)+1/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/
2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-
I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/
2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))
^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1
/6*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-
1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2
))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2
*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-
I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2)
)*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*
(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-1/24*(x*(x-1+I*3^(1/2)
)*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(
1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(
1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-
1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1
+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/
2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2
))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I
*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^
(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-
1-I*3^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 119, normalized size = 1.63

$$-\frac{5\sqrt{2}\left(x^{4}-4x^{3}+8x^{2}-8x\right)\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-6\sqrt{2}\left(x^{4}-4x^{3}+8x^{2}-8x\right)\mathrm{weierstrassZeta}\left(-\frac{2}{3},\frac{7}{54},\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)\right)+3\sqrt{-x^{4}+4x^{3}-8x^{2}+8x}\left(x^{2}+2\right)}{72\left(x^{4}-4x^{3}+8x^{2}-8x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/72*(5*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassPInverse(-2/3, 7/54
, -1/3*(x - 3)/x) - 6*sqrt(2)*(x^4 - 4*x^3 + 8*x^2 - 8*x)*weierstrassZeta(-
2/3, 7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 3*sqrt(-x^4 +
 4*x^3 - 8*x^2 + 8*x)*(x^2 + 2))/(x^4 - 4*x^3 + 8*x^2 - 8*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(2-x\right)\left(x^{2}-2x+4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(3/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2),x)
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(3/2), x)
```

3.773 $\int \frac{1}{((2-x)x(4-2x+x^2))^{5/2}}\,dx$

Optimal. Leaf size=109

$$\frac{(5+(-1+x)^2)(-1+x)}{72(3-2(-1+x)^2-(-1+x)^4)^{3/2}}+\frac{(26+7(-1+x)^2)(-1+x)}{432\sqrt{3-2(-1+x)^2-(-1+x)^4}}+\frac{7E\left(\sin^{-1}(1-x)|-\frac{1}{3}\right)}{144\sqrt{3}}-\frac{11F\left(\sin^{-1}\ldots\right.}{1\ldots}$$

```
[Out] 1/72*(5+(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(3/2)-7/432*EllipticE(-1+x
,1/3*I*3^(1/2))*3^(1/2)+11/432*EllipticF(-1+x,1/3*I*3^(1/2))*3^(1/2)+1/432*
(26+7*(-1+x)^2)*(-1+x)/(3-2*(-1+x)^2-(-1+x)^4)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1120, 1106, 1192, 1194, 538, 435, 430}

$$-\frac{11F\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{7E\left(\text{ArcSin}(1-x)\,|-\frac{1}{3}\right)}{144\sqrt{3}}+\frac{(7(x-1)^2+26)(x-1)}{432\sqrt{-(x-1)^4-2(x-1)^2+3}}+\frac{((x-1)^2+5)(x-1)}{72(-(x-1)^4-2(x-1)^2+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((5 + (-1 + x)^2)*(-1 + x))/(72*(3 - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + ((
26 + 7*(-1 + x)^2)*(-1 + x))/(432*Sqrt[3 - 2*(-1 + x)^2 - (-1 + x)^4]) + (7
*EllipticE[ArcSin[1 - x], -1/3])/(144*Sqrt[3]) - (11*EllipticF[ArcSin[1 - x
], -1/3])/(144*Sqrt[3])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
```

```
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c])))))
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1194

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*Sqrt[-c], Int[(d + e*x^2)/(Sqrt
[b + q + 2*c*x^2]*Sqrt[-b + q - 2*c*x^2]), x], x]] /; FreeQ[{a, b, c, d, e}
, x] && GtQ[b^2 - 4*a*c, 0] && LtQ[c, 0]
```

Rubi steps

$$\int \frac{1}{\left((2-x)x\left(4-2x+x^2\right)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(3-2x^2-x^4\right)^{5/2}}\,dx, x, -1+x\right)$$
$$= \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/2}} - \frac{1}{144}\text{Subst}\left(\int \frac{-38-6x^2}{\left(3-2x^2-x^4\right)^{3/2}}\right.$$
$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}}$$
$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}}$$
$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}}$$
$$= -\frac{\left(26+7(1-x)^2\right)(1-x)}{432\sqrt{3-2(1-x)^2-(1-x)^4}} + \frac{\left(5+(-1+x)^2\right)(-1+x)}{72\left(3-2(-1+x)^2-(-1+x)^4\right)^{3/}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.39, size = 327, normalized size = 3.00

$$\frac{(-2+x)^3x^2\left(4-2x+x^2\right)^2\left(-\frac{7x(4-2x+x^2)}{-2+x}+\frac{36+216x-622x^2+670x^3-445x^4+187x^5-49x^6+7x^7}{(-2+x)^2x(4-2x+x^2)}+\frac{7i\sqrt{2}\,x\sqrt{\frac{4-2x+x^2}{(-2+x)^2}}\,E\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)}{\sqrt{\frac{ix}{\left(i+\sqrt{3}\right)(-2+x)}}}-19i\sqrt{2}(-2+x)\sqrt{\frac{ix}{\left(i+\sqrt{3}\right)(-2+x)}}\sqrt{\frac{4-2x+x^2}{(-2+x)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt{-i+\sqrt{3}-\frac{4i}{-2+x}}}{\sqrt{2}\sqrt[4]{3}}\right)\middle|\frac{2\sqrt{3}}{i+\sqrt{3}}\right)\right)}{432\left(-x\left(-8+8x-4x^2+x^3\right)\right)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 - x)*x*(4 - 2*x + x^2))^(-5/2),x]
```

```
[Out] ((-2 + x)^3*x^2*(4 - 2*x + x^2)^2*((-7*x*(4 - 2*x + x^2))/(-2 + x) + (36 +
216*x - 622*x^2 + 670*x^3 - 445*x^4 + 187*x^5 - 49*x^6 + 7*x^7)/((-2 + x)^2
*x*(4 - 2*x + x^2)) + ((7*I)*Sqrt[2]*x*Sqrt[(4 - 2*x + x^2)/(-2 + x)^2]*Ell
ipticE[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4))], (2*Sq
rt[3])/(I + Sqrt[3])])/Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))] - (19*I)*Sqrt[2
]*(-2 + x)*Sqrt[(I*x)/((I + Sqrt[3])*(-2 + x))]*Sqrt[(4 - 2*x + x^2)/(-2 +
x)^2]*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (4*I)/(-2 + x)]/(Sqrt[2]*3^(1/4)
)], (2*Sqrt[3])/(I + Sqrt[3])]))/(432*(-(x*(-8 + 8*x - 4*x^2 + x^3)))^(5/2)
)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(93) = 186.
time = 0.51, size = 1039, normalized size = 9.53 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2-x)*x*(x^2-2*x+4))^(5/2),x,method=_RETURNVERBOSE)

[Out] (1/36+1/288*x^2-1/96*x)*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/(x^3-4*x^2+8*x-8)^2+2*x*(53/3456+5/1728*x^2-19/4608*x)/(-x*(x^3-4*x^2+8*x-8))^(1/2)-1/768*(-x^4+4*x^3-8*x^2+8*x)^(1/2)/x^2-1/96*(-x^3+4*x^2-8*x+8)/(x*(-x^3+4*x^2-8*x+8))^(1/2)+5/216*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+7/108*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)/(-1+I*3^(1/2))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)*(2*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-2*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(1+I*3^(1/2))/(-1+I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2)))-7/432*(x*(x-1+I*3^(1/2))*(x-1-I*3^(1/2))+2*(-1-I*3^(1/2))*((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2)*(x-2)^2*((x-1+I*3^(1/2))/(1-I*3^(1/2))/(x-2))^(1/2)*((x-1-I*3^(1/2))/(1+I*3^(1/2))/(x-2))^(1/2)*(1/2*(6+2*I*3^(1/2))/(-1+I*3^(1/2))*EllipticF(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))+1/2*(-1+I*3^(1/2))*EllipticE(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))-4/(-1+I*3^(1/2))*EllipticPi(((-1+I*3^(1/2))*x/(1+I*3^(1/2))/(x-2))^(1/2),(-1-I*3^(1/2))/(1-I*3^(1/2)),((1+I*3^(1/2))*(-1-I*3^(1/2))/(-1+I*3^(1/2))/(1-I*3^(1/2)))^(1/2))))/(-x*(x-2)*(x-1+I*3^(1/2))*(x-1-I*3^(1/2)))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="maxima")

[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 195, normalized size = 1.79

$$-\frac{43\sqrt{2}\,(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\,\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)-84\sqrt{2}\,(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)\,\mathrm{weierstrassZeta}\left(-\frac{2}{3},\frac{7}{54},\mathrm{weierstrassPInverse}\left(-\frac{2}{3},\frac{7}{54},-\frac{x-3}{3x}\right)\right)+6\,(7x^6-37x^5+115x^4-226x^3+274x^2-232x+36)\sqrt{-x^4+4x^3-8x^2+8x}}{2592\,(x^8-8x^7+32x^6-80x^5+128x^4-128x^3+64x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="fricas")

```
[Out] -1/2592*(43*sqrt(2)*(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64
*x^2)*weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x) - 84*sqrt(2)*(x^8 - 8
*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)*weierstrassZeta(-2/3,
7/54, weierstrassPInverse(-2/3, 7/54, -1/3*(x - 3)/x)) + 6*(7*x^6 - 37*x^5
+ 115*x^4 - 226*x^3 + 274*x^2 - 232*x + 36)*sqrt(-x^4 + 4*x^3 - 8*x^2 + 8*x
))/(x^8 - 8*x^7 + 32*x^6 - 80*x^5 + 128*x^4 - 128*x^3 + 64*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x\left(2-x\right)\left(x^2-2x+4\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x**2-2*x+4))**(5/2),x)
```

```
[Out] Integral((x*(2 - x)*(x**2 - 2*x + 4))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-x)*x*(x^2-2*x+4))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-(x^2 - 2*x + 4)*(x - 2)*x)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(-x\,\left(x-2\right)\,\left(x^2-2\,x+4\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2),x)
```

```
[Out] int(1/(-x*(x - 2)*(x^2 - 2*x + 4))^(5/2), x)
```

3.774 $\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=730

$$\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,\Big(7c^3+20ad^2-3cd^2}{35d^2}$$

```
[Out] 1/7*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2)+2/35*c*(c/d+x)*(7*c^3
+20*a*d^2-3*c*d^2*(c/d+x)^2)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d^2-
16/35*c^3*(8*a*d^2+c^3)*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/d
^2/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))/(4*a*d^2+c^3)^(1/2)+16/35*c^
(13/4)*(4*a*d^2+c^3)^(3/4)*(8*a*d^2+c^3)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)
))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^
(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2
))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d
+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)
^(1/2)+8/35*c^(7/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*
d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))
)*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(
3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2)
)*(-c^(3/2)*(8*a*d^2+c^3)+(5*a*d^2+c^3)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/d^5/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1120, 1105, 1190, 1211, 1117, 1209}

$$\frac{8c^{7/4}(4ad^2+c^3)^{3/4}\left(\sqrt{4ad^2+c^3}\,(5ad^2+c^3)-c^{3/2}(8ad^2+c^3)\right)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2(\frac{c}{d}+x)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2(\frac{c}{d}+x)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\,\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{16c^{13/4}(4ad^2+c^3)^{3/4}(8ad^2+c^3)\sqrt{\frac{d^2(4ac+4c^2x^2+4cdx^3+d^2x^4)}{(4ad^2+c^3)\left(\frac{d^2(\frac{c}{d}+x)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2(\frac{c}{d}+x)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\,\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{35d^5\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}-\frac{16c^3(8ad^2+c^3)\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{35d^2\sqrt{4ad^2+c^3}\left(\frac{d^2(\frac{c}{d}+x)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}+\frac{2c\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\left(20ad^2+7c^3-3cd^2\left(\frac{c}{d}+x\right)^2\right)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] ((c/d + x)*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2))/7 + (2*c*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]*(7*c^3 + 20*a*d^2 - 3*c*d
^2*(c/d + x)^2))/(35*d^2) - (16*c^3*(c^3 + 8*a*d^2)*(c/d + x)*Sqrt[4*a*c +
4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(35*d^2*Sqrt[c^3 + 4*a*d^2]*(Sqrt[c] + (d
^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (16*c^(13/4)*(c^3 + 4*a*d^2)^(3/4)*
(c^3 + 8*a*d^2)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3
```

```
+ 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/
4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*S
qrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (8*c^(7/4)*(c^3 + 4*a*d^2)^
(3/4)*(Sqrt[c^3 + 4*a*d^2]*(c^3 + 5*a*d^2) - c^(3/2)*(c^3 + 8*a*d^2))*Sqrt[
(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] +
 (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sq
rt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1
/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(35*d^5*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
```

```
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}\,dx = \text{Subst}\left(\int \left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{3}{7}\text{Subst}\left(\int \left(2c\left(4\right.\right.\right.$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+}}{}$$

$$= \frac{1}{7}\left(\frac{c}{d}+x\right)\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}+\frac{2c(c+dx)\sqrt{4ac+}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.14, size = 10468, normalized size = 14.34

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5228 vs. $2(772) = 1544$.
time = 0.35, size = 5229, normalized size = 7.16

method	result	size
default	Expression too large to display	5229
elliptic	Expression too large to display	5229
risch	Expression too large to display	6018

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.775 $\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx$

Optimal. Leaf size=622

$$\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{c^3+4ad^2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}+\frac{2c^{9/4}\left(c^3+4ad^2\right)^{3/4}}{\cdots}$$

```
[Out] 1/3*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)-2/3*c^2*(c/d+x)*(d^2*
x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(
1/2))/(4*a*d^2+c^3)^(1/2)+2/3*c^(9/4)*(4*a*d^2+c^3)^(3/4)*(cos(2*arctan((d*
x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4
*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1
/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(
4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/
(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3
+4*c^2*x^2+4*a*c)^(1/2)+1/3*c^(3/4)*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+
c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a
*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4
))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*
a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4
*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3
)^(1/2))^2)^(1/2)/d^3/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 622, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1120, 1105, 1211, 1117, 1209}

$$\frac{c^{3/4}\sqrt[4]{4ad^2+c^3}\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{2c^{9/4}\left(4ad^2+c^3\right)^{3/4}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{3d^3\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}-\frac{2c^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{3\sqrt{4ad^2+c^3}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/3 - (2*c^2*(c/d +
 x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(3*Sqrt[c^3 + 4*a*d^2]*(
Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (2*c^(9/4)*(c^3 + 4*a*d
^2)^(3/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*
d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*
(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^
3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a
*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) + (c^(3/4)*(c^3 + 4*a*d^2)^(1/4)*(c^
```

```
3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4
*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3
 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*Elliptic
F[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^
3 + 4*a*d^2])/2])/(3*d^3*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}\,dx = \text{Subst}\left(\int \sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} + \frac{1}{3}\text{Subst}\left(\int \frac{2c\left(4a \right.}{\sqrt{c\left(4a+\frac{c}{d}\right.}}\right.$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} + \frac{\left(2c^{5/2}\sqrt{c^3+4ad^2}\right)\text{Sub}}{}$$

$$= \frac{1}{3}\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4} - \frac{2c^2(c+dx)\sqrt{4ac+4c^2x}}{3d\sqrt{c^3+4ad^2}\left(\sqrt{c}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.09, size = 5218, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4889 vs. 2(668) = 1336.
time = 0.08, size = 4890, normalized size = 7.86

method	result	size
risch	Expression too large to display	4865
default	Expression too large to display	4890
elliptic	Expression too large to display	4890

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*x*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/3*c/d*(d^2*x^4+4*c*d*x^3+
4*c^2*x^2+4*a*c)^(1/2)+16/3*a*c*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d))^(1/2)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(
x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((
-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/
2))-8/3*c^3/d*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^
2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2
)*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d
)*(x+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d))^(1/2)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2
)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d)/(d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(
1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-
a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d*Ellip
ticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(
1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(
```

```
1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(-2*d*(-a*c)^(1/2)
+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+
c^2)^(1/2))/d))^(1/2))+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^
(1/2)+c^2)^(1/2))/d)*EllipticPi(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(
2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(
c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+
(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/
2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)))-2/3*c^2*((x-(-c+(2*d*(-a*
c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)+((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(
-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+
(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))
/d)^2*((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)
^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
[Out] Integral(sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
[Out] int((4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.776 $\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx$

Optimal. Leaf size=227

$$\frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(c^3+4ad^2\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)F\left(2\tan^{-1}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)\right|}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

```
[Out] 1/2*(4*a*d^2+c^3)^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))
^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(
2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^
3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+
4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^
3)^(1/2))^2)^(1/2)/c^(1/4)/d/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1120, 1117}

$$\frac{\sqrt[4]{4ad^2+c^3}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] ((c^3 + 4*a*d^2)^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))
/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sq
rt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)
/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(2
*c^(1/4)*d*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\sqrt[4]{c^3+4ad^2}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(c^3+4ad^2\right)\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)^2}}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}\right.}{\sqrt{c^3+}}\right.}{2\sqrt[4]{c}\,d\sqrt{4ac+4c^2x}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.41, size = 822, normalized size = 3.62

$$\frac{2\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}+dx\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)}}\sqrt{-\frac{\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\left(c+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}+dx\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)}}\right)\middle|\frac{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)^2}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)^2}\right)}{d\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}\sqrt{\frac{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+dx\right)}{\left(\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}+\sqrt{c^2+2i\sqrt{a}\sqrt{c}d}\right)\left(-c+\sqrt{c^2-2i\sqrt{a}\sqrt{c}d}-dx\right)}}\sqrt{4ac+x^2(2c+dx)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4],x]
```

```
[Out] (2*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)*(c + Sqrt[c^2 - (2*I)*S
qrt[a]*Sqrt[c]*d] + d*x)*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c - S
qrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[
c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - d*x)))]*Sqrt[-((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*(c + Sqrt
[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*
d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] - d*x)))]*EllipticF[ArcSin[Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]
*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sq
rt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*S
qrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x))]], (Sq
rt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])^2/
(Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])
^2])/(d*Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d]*Sqrt[((Sqrt[c^2 - (2*I)*Sqrt[a]
*Sqrt[c]*d] - Sqrt[c^2 + (2*I)*Sqrt[a]*Sqrt[c]*d])*(c + Sqrt[c^2 - (2*I)*Sq
rt[a]*Sqrt[c]*d] + d*x))/((Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] + Sqrt[c^2 +
```

```
(2*I)*Sqrt[a]*Sqrt[c]*d])*(-c + Sqrt[c^2 - (2*I)*Sqrt[a]*Sqrt[c]*d] - d*x)
)]*Sqrt[4*a*c + x^2*(2*c + d*x)^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. $2(259) = 518$.

time = 0.06, size = 1056, normalized size = 4.65

method	result
default	$\frac{2\left(\frac{-c+\sqrt{2d\sqrt{-ac}+c^2}}{d}+\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}\right)\sqrt{\frac{\left(-\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}+\frac{c+\sqrt{2d\sqrt{-ac}+c^2}}{d}\right.}{\left(-\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}-\frac{-c+\sqrt{2d\sqrt{-ac}+c^2}}{d}\right.}}}{}$
elliptic	$\frac{2\left(\frac{-c+\sqrt{2d\sqrt{-ac}+c^2}}{d}+\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}\right)\sqrt{\frac{\left(-\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}+\frac{c+\sqrt{2d\sqrt{-ac}+c^2}}{d}\right.}{\left(-\frac{c+\sqrt{-2d\sqrt{-ac}+c^2}}{d}-\frac{-c+\sqrt{2d\sqrt{-ac}+c^2}}{d}\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)
*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d))^(1/2)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)^2*((-(c+(2*d*(-a*c)^(1/2
)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(-2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/((-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1
/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)*((-(c+(2*d
*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*
d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2)
/(-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/
(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(
d^2*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(2*d*(-a*c)^(1/2)+c^2)^(1
/2))/d)*(x-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x+(c+(-2*d*(-a*c)^(1/2)+c
^2)^(1/2))/d))^(1/2)*EllipticF(((-(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d+(c+(2
*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*(x-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c
+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d-(-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(x+(
```

```
c+(2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2),((-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)*((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2
))/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/((-c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d-(-c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d)/(-(c+(2*d*(-a*c)^(1/2)+c^2)^(1/2)
)/d+(c+(-2*d*(-a*c)^(1/2)+c^2)^(1/2))/d))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="fricas"
)
[Out] integral(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4ac + 4c^2x^2 + 4cdx^3 + d^2x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(1/2),x)
[Out] Integral(1/sqrt(4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2),x)
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(1/2), x)
```

3.777 $\int \frac{1}{(4ac+4c^2x^2+4cdx^3+d^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=674

$$-\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(c^3+4ad^2\right)^{3/2}\left(\sqrt{c}+\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{c^3+4ad^2}}\right)}+\frac{\sqrt[4]{c}\sqrt{\frac{d^2(4}{(c^3+4}}}{}$$

[Out] -1/8*(c/d+x)*(c^3-4*a*d^2-c*d^2*(c/d+x)^2)/a/c/(4*a*d^2+c^3)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)-1/8*d^2*(c/d+x)*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)/a/(4*a*d^2+c^3)^(3/2)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))+1/8*c^(1/4)*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticE(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/a/d/(4*a*d^2+c^3)^(1/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)+1/16*(cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))^2)^(1/2)/cos(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4)))*EllipticF(sin(2*arctan((d*x+c)/c^(1/4)/(4*a*d^2+c^3)^(1/4))),1/2*(2+2*c^(3/2)/(4*a*d^2+c^3)^(1/2))^(1/2))*(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))*(c^3+4*a*d^2-c^(3/2)*(4*a*d^2+c^3)^(1/2))*(d^2*(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)/(4*a*d^2+c^3)/(c^(1/2)+d^2*(c/d+x)^2/(4*a*d^2+c^3)^(1/2))^2)^(1/2)/a/c^(5/4)/d/(4*a*d^2+c^3)^(3/4)/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(1/2)

Rubi [A]
time = 0.53, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1120, 1106, 1211, 1117, 1209}

$$\frac{\left(-c^{3/2}\sqrt{4ad^2+c^3}+4ad^2+c^3\right)\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)F\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{16ac^{5/4}d\left(4ad^2+c^3\right)^{3/4}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}+\frac{\sqrt[4]{c}\sqrt{\frac{d^2\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)}{\left(4ad^2+c^3\right)\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)^2}}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)E\left(2\text{ArcTan}\left(\frac{c+dx}{\sqrt[4]{c}\sqrt[4]{c^3+4ad^2}}\right)|\frac{1}{2}\left(\frac{c^{3/2}}{\sqrt{c^3+4ad^2}}+1\right)\right)}{8ad\sqrt[4]{4ad^2+c^3}\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}-\frac{d^2\left(\frac{c}{d}+x\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}{8a\left(4ad^2+c^3\right)^{3/2}\left(\frac{d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{4ad^2+c^3}}+\sqrt{c}\right)}-\frac{\left(\frac{c}{d}+x\right)\left(-4ad^2+c^3-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(4ad^2+c^3\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}}$$

Antiderivative was successfully verified.

[In] Int[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]

[Out] -1/8*((c/d + x)*(c^3 - 4*a*d^2 - c*d^2*(c/d + x)^2))/(a*c*(c^3 + 4*a*d^2)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4]) - (d^2*(c/d + x)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])/(8*a*(c^3 + 4*a*d^2)^(3/2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])) + (c^(1/4)*Sqrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])*El

```
lipticE[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2)^(1/4))], (1 + c^(3/2)/S
qrt[c^3 + 4*a*d^2])/2])/(8*a*d*(c^3 + 4*a*d^2)^(1/4)*Sqrt[4*a*c + 4*c^2*x^2
 + 4*c*d*x^3 + d^2*x^4]) + ((c^3 + 4*a*d^2 - c^(3/2)*Sqrt[c^3 + 4*a*d^2])*S
qrt[(d^2*(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4))/((c^3 + 4*a*d^2)*(Sqrt[
c] + (d^2*(c/d + x)^2)/Sqrt[c^3 + 4*a*d^2])^2)]*(Sqrt[c] + (d^2*(c/d + x)^2
)/Sqrt[c^3 + 4*a*d^2])*EllipticF[2*ArcTan[(c + d*x)/(c^(1/4)*(c^3 + 4*a*d^2
)^(1/4))], (1 + c^(3/2)/Sqrt[c^3 + 4*a*d^2])/2])/(16*a*c^(5/4)*d*(c^3 + 4*a
*d^2)^(3/4)*Sqrt[4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

c/a]

Rubi steps

$$\int \frac{1}{\left(4ac+4c^2x^2+4cdx^3+d^2x^4\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(c\left(4a+\frac{c^3}{d^2}\right)-2c^2x^2+d^2x^4\right)^{3/2}}\,dx, x, \frac{c}{d}+x\right)$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{c}}\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} + \frac{\sqrt{c}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(\frac{c}{d}+x\right)\left(c^3-4ad^2-cd^2\left(\frac{c}{d}+x\right)^2\right)}{8ac\left(c^3+4ad^2\right)\sqrt{4ac+4c^2x^2+4cdx^3+d^2x^4}} - \frac{d(c+dx)\sqrt{4a}}{8a\left(c^3+4ad^2\right)}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.11, size = 5276, normalized size = 7.83

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(4*a*c + 4*c^2*x^2 + 4*c*d*x^3 + d^2*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5023 vs. 2(720) = 1440.
time = 0.08, size = 5024, normalized size = 7.45

method	result	size

default	Expression too large to display	5024
elliptic	Expression too large to display	5024

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)/(d^4*x^8 + 8*c*d^3*x
^7 + 24*c^2*d^2*x^6 + 32*c^3*d*x^5 + 32*a*c^2*d*x^3 + 32*a*c^3*x^2 + 8*(2*c
^4 + a*c*d^2)*x^4 + 16*a^2*c^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4ac + 4c^2x^2 + 4cdx^3 + d^2x^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d**2*x**4+4*c*d*x**3+4*c**2*x**2+4*a*c)**(3/2),x)
```

```
[Out] Integral((4*a*c + 4*c**2*x**2 + 4*c*d*x**3 + d**2*x**4)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d^2*x^4+4*c*d*x^3+4*c^2*x^2+4*a*c)^(3/2),x, algorithm="giac")
[Out] integrate((d^2*x^4 + 4*c*d*x^3 + 4*c^2*x^2 + 4*a*c)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(4\,c^2\,x^2+4\,c\,d\,x^3+4\,a\,c+d^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2),x)
[Out] int(1/(4*a*c + 4*c^2*x^2 + d^2*x^4 + 4*c*d*x^3)^(3/2), x)
```

3.778 $\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$

Optimal. Leaf size=663

$$d^2(5d^4 + 256ae$$

$$\frac{1}{3}\left(\frac{d}{4e} + x\right)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4} - \frac{2d^2\left(\frac{d}{4e} + x\right)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}{\sqrt{5d^4 + 256ae^3}\left(1 + \frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4 + 256ae^3}}\right)} + \text{———}$$

```
[Out] 1/3*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)-2*d^2*(1/4*d/e+
x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(1+16*e^2*(1/4*d/e+x)^2/(256
*a*e^3+5*d^4)^(1/2))/(256*a*e^3+5*d^4)^(1/2)+1/16*d^2*(256*a*e^3+5*d^4)^(3/
4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan(
(4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*a
*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*(1+16*e^2*
(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*
e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^
(1/2)/e^2*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+1/96*(256*a*e
^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/
cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*arctan((4*
e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2
))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^4+256*a*e^3-3*d^2*
(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^
3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e^2*2^(1
/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)
```

Rubi [A]
time = 0.56, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1120, 1105, 1211, 1117, 1209}

$$\frac{\sqrt[4]{256ae^3+5d^4}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\text{ArcTan}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{48\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{d^2\left(256ae^3+5d^4\right)^{3/4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\text{ArcTan}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{8\sqrt{2}e^2\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{2d^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\sqrt{256ae^3+5d^4}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/3 - (2*d^2*
(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])/(Sqrt[5*d^4
+ 256*a*e^3]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) + (d^2
*(5*d^4 + 256*a*e^3)^(3/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticE[
```

```
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(8*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4]) + ((5*d^4 + 256*a*e^3)^(1/4)*(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 2
56*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 2
56*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (
16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*
e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/
(48*Sqrt[2]*e^2*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

`c/a]`

Rubi steps

$$\int \sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}\,dx = \text{Subst}\left(\int \sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}\,dx, x, \frac{d}{4e}+x\right)$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{1}{3}\text{Subst}\left(\int \frac{}{\sqrt{\frac{1}{32}\left(\right.}}\right.$$

$$\left(d^2\sqrt{5d^4+256ae^3}\right.$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}+\frac{}{}$$

$$= \frac{1}{3}\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}-\frac{d^2(d+4ex)\sqrt{8ae^2-}}{2e\sqrt{5d^4+256ae^3}\left(\right.}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7543 vs. 2(663) = 1326.
time = 14.87, size = 7543, normalized size = 11.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7886 vs. 2(715) = 1430.
time = 0.34, size = 7887, normalized size = 11.90

method	result	size

default	Expression too large to display	7887
elliptic	Expression too large to display	7887
risch	Expression too large to display	9561

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(8*x^4*e^3 - d^3*x + 8*(d*x^3 + a)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
[Out] int((8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.779 $\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx$

Optimal. Leaf size=235

$$\frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)F\left(2\tan^{-1}\left(\frac{d+4\ldots}{\sqrt[4]{5d^4+\ldots}}\right.\right.}{\sqrt{2}\,e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

```
[Out] 1/2*(256*a*e^3+5*d^4)^(1/4)*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)
))^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(
2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)
^(1/2))^(1/2))*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x
^4+8*d*e^2*x^3-d^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(25
6*a*e^3+5*d^4)^(1/2))^2)^(1/2)/e*2^(1/2)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e
^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1120, 1117}

$$\frac{\sqrt[4]{256ae^3+5d^4}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\text{ArcTan}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\sqrt{2}\,e\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] ((5*d^4 + 256*a*e^3)^(1/4)*Sqrt[(e*(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x
^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*
e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])*EllipticF[
2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (1 + (3*d^2)/Sqrt[5*d^4 +
256*a*e^3])/2])/(Sqrt[2]*e*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{1}{32}\left(\frac{5d^4}{e}+256ae^2\right)-3d^2ex^2+8e^3x^4}}\,dx, x, \frac{d}{4e}+x\right.$$

$$= \frac{\sqrt[4]{5d^4+256ae^3}\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(5d^4+256ae^3\right)\left(1+\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{5d^4+256ae^3}}\right)^2}}\left(1+\cdots\right.}{\sqrt{2}\,e\sqrt{8ae^2-\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1065 vs. $2(235) = 470$.
time = 11.55, size = 1065, normalized size = 4.53

$$-\frac{\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)\left(d-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+4ex\right)\sqrt{-\frac{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\left(d+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+4ex\right)}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)}}\sqrt{\frac{3d^2-2\sqrt{d^4-64ae^3}-\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\sqrt{3d^2+2\sqrt{d^4-64ae^3}}+d\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)+4e\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)x}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)}}\,F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}+4ex\right)}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)}}\right)\middle|\frac{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)^2}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)^2}\right)}{2e\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\sqrt{\frac{\sqrt{3d^2-2\sqrt{d^4-64ae^3}}\left(-d+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}-4ex\right)}{\left(\sqrt{3d^2-2\sqrt{d^4-64ae^3}}+\sqrt{3d^2+2\sqrt{d^4-64ae^3}}\right)\left(-d+\sqrt{3d^2-2\sqrt{d^4-64ae^3}}-4ex\right)}}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4],x]
```

```
[Out] -1/2*((-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)*(d - Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x)*Sqrt[-((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]]*(d + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]] + 4*e*x))/((Sqrt[3*d^2 - 2*Sq
rt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^
2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x)))]*Sqrt[(3*d^2 - 2*Sqrt[d^4 - 64*a*e^3
] - Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3
]] + d*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64
*a*e^3]]) + 4*e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt
[d^4 - 64*a*e^3]])*x)/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 +
 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*
x))]*EllipticF[ArcSin[Sqrt[((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*
d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] +
4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 -
```

```
64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - 4*e*x))]], (Sqrt[3
*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])^2/(S
qrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])
^2])/(e*(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] - Sqrt[3*d^2 + 2*Sqrt[d^4 - 6
4*a*e^3]])*Sqrt[(Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]]*(-d + Sqrt[3*d^2 + 2*
Sqrt[d^4 - 64*a*e^3]] - 4*e*x))/((Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^3]] + Sq
rt[3*d^2 + 2*Sqrt[d^4 - 64*a*e^3]])*(-d + Sqrt[3*d^2 - 2*Sqrt[d^4 - 64*a*e^
3]] - 4*e*x))]*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. $2(273) = 546$.
time = 0.05, size = 1704, normalized size = 7.25

method	result	size
default	Expression too large to display	1704
elliptic	Expression too large to display	1704

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+
(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*((-1/4*(d*e+(3*d^2*e^2-
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e
^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d*
e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*(x+1/4*(d*e+(3
*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)^2*((-1/4*(d*e+(3*d^2*e^2+
2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d
^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*
e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/
e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(x+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*((-1/4*(d*e+(
3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-
64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2
)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/
(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)/(-1/
4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e
^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-64*a*
e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)
*e^2)^(1/2))/e^2)*2^(1/2)/(e^3*(x-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1
/2)*e^2)^(1/2))/e^2)*(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1
/2))/e^2)*(x-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*
(x+1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2)*Elli
```

```
pticF(((-1/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d
*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(x-1/4*(-d*e+(3*d^2*
e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2-2*(-64*a
*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2
)*e^2)^(1/2))/e^2)/(x+1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2
))/e^2))^(1/2),((-1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e
^2-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)*(1/4*(-d*e
+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1/4*(d*e+(3*d^2*e^2-2*(
-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2)/(1/4*(-d*e+(3*d^2*e^2+2*(-64*a*e^3+d^
4)^(1/2)*e^2)^(1/2))/e^2-1/4*(-d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^
(1/2))/e^2)/(-1/4*(d*e+(3*d^2*e^2+2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2+1
/4*(d*e+(3*d^2*e^2-2*(-64*a*e^3+d^4)^(1/2)*e^2)^(1/2))/e^2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/sqrt(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(1/sqrt(8*x^4*e^3 - d^3*x + 8*(d*x^3 + a)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/sqrt(8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2),x)
```

```
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(1/2), x)
```

3.780 $\int \frac{1}{\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=748

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4-256ae^3-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(5d^8-64ad^4e^3-16384a^2e^6\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+}}{\left(d^4-64ae^3\right)\left(5d^4+256ae^3\right)^{3/2}\left(1+\frac{16e^2(}{\sqrt{5d^4+}}\right.}$$

[Out] 4*e*(1/4*d/e+x)*(13*d^4-256*a*e^3-48*d^2*e^2*(1/4*d/e+x)^2)/(-16384*a^2*e^6
-64*a*d^4*e^3+5*d^8)/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)+384*d^2*e^
2*(1/4*d/e+x)*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(1/2)/(-64*a*e^3+d^4)/(
256*a*e^3+5*d^4)^(3/2)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))-12*
d^2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))^2)^(1/2)/cos(2*arctan
((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticE(sin(2*arctan((4*e*x+d)/(256*
a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(1/2))^(1/2))*2^(1/2)*(
1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d
^3*x+8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(
1/2))^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(1/4)/(8*e^3*x^4+8*d*e^2*x
^3-d^3*x+8*a*e^2)^(1/2)-2*(cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))
^2)^(1/2)/cos(2*arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4)))*EllipticF(sin(2*
arctan((4*e*x+d)/(256*a*e^3+5*d^4)^(1/4))),1/2*(2+6*d^2/(256*a*e^3+5*d^4)^(
1/2))^(1/2))*2^(1/2)*(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))*(5*d^
4+256*a*e^3-3*d^2*(256*a*e^3+5*d^4)^(1/2))*(e*(8*e^3*x^4+8*d*e^2*x^3-d^3*x+
8*a*e^2)/(256*a*e^3+5*d^4)/(1+16*e^2*(1/4*d/e+x)^2/(256*a*e^3+5*d^4)^(1/2))
^2)^(1/2)/(-64*a*e^3+d^4)/(256*a*e^3+5*d^4)^(3/4)/(8*e^3*x^4+8*d*e^2*x^3-d^
3*x+8*a*e^2)^(1/2)

Rubi [A]
time = 0.55, antiderivative size = 748, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1120, 1106, 1211, 1117, 1209}

$$\frac{4e\left(\frac{d}{4e}+x\right)\left(-256ae^3+13d^4-48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{\left(-16384a^2e^6-64ad^4e^3+5d^8\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}-\frac{2\sqrt{2}\left(-3d^2\sqrt{256ae^3+5d^4}+256ae^3+5d^4\right)\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)F\left(2\text{ArcTan}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}-\frac{12\sqrt{2}d^2\sqrt{\frac{e\left(8ae^2-d^3x+8de^2x^3+8e^3x^4\right)}{\left(256ae^3+5d^4\right)\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)^2}}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)E\left(2\text{ArcTan}\left(\frac{d+4ex}{\sqrt[4]{5d^4+256ae^3}}\right)|\frac{1}{2}\left(\frac{3d^2}{\sqrt{5d^4+256ae^3}}+1\right)\right)}{\left(d^4-64ae^3\right)\sqrt[4]{256ae^3+5d^4}\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}+\frac{384d^2e^2\left(\frac{d}{4e}+x\right)\sqrt{8ae^2-d^3x+8de^2x^3+8e^3x^4}}{\left(d^4-64ae^3\right)\left(256ae^3+5d^4\right)^{3/2}\left(\frac{16e^2\left(\frac{d}{4e}+x\right)^2}{\sqrt{256ae^3+5d^4}}+1\right)}$$

Antiderivative was successfully verified.

[In] Int[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]

[Out] (4*e*(d/(4*e) + x)*(13*d^4 - 256*a*e^3 - 48*d^2*e^2*(d/(4*e) + x)^2))/((5*d
^8 - 64*a*d^4*e^3 - 16384*a^2*e^6)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e
^3*x^4]) + (384*d^2*e^2*(d/(4*e) + x)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 +

```
8*e^3*x^4])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e^3)^(3/2)*(1 + (16*e^2*(d/(4*
e) + x)^2)/Sqrt[5*d^4 + 256*a*e^3])) - (12*Sqrt[2]*d^2*Sqrt[(e*(8*a*e^2 - d
^3*x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e)
 + x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*
d^4 + 256*a*e^3])*EllipticE[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)]
, (1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*
a*e^3)^(1/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4]) - (2*Sqrt[2]*
(5*d^4 + 256*a*e^3 - 3*d^2*Sqrt[5*d^4 + 256*a*e^3])*Sqrt[(e*(8*a*e^2 - d^3*
x + 8*d*e^2*x^3 + 8*e^3*x^4))/((5*d^4 + 256*a*e^3)*(1 + (16*e^2*(d/(4*e) +
x)^2)/Sqrt[5*d^4 + 256*a*e^3])^2)]*(1 + (16*e^2*(d/(4*e) + x)^2)/Sqrt[5*d^4
 + 256*a*e^3])*EllipticF[2*ArcTan[(d + 4*e*x)/(5*d^4 + 256*a*e^3)^(1/4)], (
1 + (3*d^2)/Sqrt[5*d^4 + 256*a*e^3])/2])/((d^4 - 64*a*e^3)*(5*d^4 + 256*a*e
^3)^(3/4)*Sqrt[8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4])
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{1}{(8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{1}{32}\left(\frac{5d^4}{e} + 256ae^2\right) - 3d^2ex^2 + 8e^3x^4\right)^{3/2}}\,dx, x, \frac{d}{4e} + x\right.$$

8

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} - $$

(

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} - $$

$$= \frac{4e\left(\frac{d}{4e}+x\right)\left(13d^4 - 256ae^3 - 48d^2e^2\left(\frac{d}{4e}+x\right)^2\right)}{(5d^8 - 64ad^4e^3 - 16384a^2e^6)\sqrt{8ae^2 - d^3x + 8de^2x^3 + 8e^3x^4}} + $$

(

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7629 vs. $2(748) = 1496$.
time = 16.11, size = 7629, normalized size = 10.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8*a*e^2 - d^3*x + 8*d*e^2*x^3 + 8*e^3*x^4)^(-3/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8102 vs. $2(804) = 1608$.
time = 0.09, size = 8103, normalized size = 10.83

method	result	size
default	Expression too large to display	8103
elliptic	Expression too large to display	8103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(8*x^4*e^3 - d^3*x + 8*(d*x^3 + a)*e^2)/(64*x^8*e^6 + d^6*x^2
+ 64*(d^2*x^6 + 2*a*d*x^3 + a^2)*e^4 - 16*(d^3*x^5 - 8*(d*x^7 + a*x^4)*e^2)
*e^3 - 16*(d^4*x^4 + a*d^3*x)*e^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8ae^{2} - d^{3}x + 8de^{2}x^{3} + 8e^{3}x^{4}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e**3*x**4+8*d*e**2*x**3-d**3*x+8*a*e**2)**(3/2),x)
```

```
[Out] Integral((8*a*e**2 - d**3*x + 8*d*e**2*x**3 + 8*e**3*x**4)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*e^3*x^4+8*d*e^2*x^3-d^3*x+8*a*e^2)^(3/2),x, algorithm="giac"
)
[Out] integrate((8*x^4*e^3 + 8*d*x^3*e^2 - d^3*x + 8*a*e^2)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(-d^3\,x+8\,d\,e^2\,x^3+8\,e^3\,x^4+8\,a\,e^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2),x)
[Out] int(1/(8*a*e^2 - d^3*x + 8*e^3*x^4 + 8*d*e^2*x^3)^(3/2), x)
```

3.781 $\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2} dx$

Optimal. Leaf size=452

$$-\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{35\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{2}{35}\left(13+5a-3(-1+x)^2\right)\sqrt{3+a-2(-1+x)$$

```
[Out] 1/7*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/35*(13+
5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/35*(3+a)*(16+5*a)*
(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^
(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)
^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(
1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1120, 1105, 1190, 1216, 545, 429, 506, 422}

$$\frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (-16*(7 + 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x
))/(35*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)
^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1
+ x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/7 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt
```

```
[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[
1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 +
 x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2
*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)$$

$$=\frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x)+\frac{3}{7}\text{Subst}\left(\int (2(3+$$

$$=-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+$$

$$=-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+$$

$$=-\frac{2}{35}\left(13+5a-3(1-x)^2\right)\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1-x)+$$

$$=\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}(13+5a-$$

$$=\frac{16(7+2a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{35\sqrt{3+a-2(1-x)^2-(1-x)^4}}-\frac{2}{35}(13+5a-$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6287 vs. $2(452)=904$.
time = 16.11, size = 6287, normalized size = 13.91

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2654 vs. $2(506) = 1012$.
time = 0.15, size = 2655, normalized size = 5.87

method	result	size
default	Expression too large to display	2655
elliptic	Expression too large to display	2655
risch	Expression too large to display	3593

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+5/7*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)-66/35*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+14/5*x^2*(-x^4+4*x^3-8*x^2+a+8
*x)^(1/2)+(3/7*a-32/35)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-3/7*a-4/7)*(-x^4
+4*x^3-8*x^2+a+8*x)^(1/2)-(a^2-(3/7*a-32/35)*a+12/7*a+16/7)*((-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))
^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1
+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1
/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+
(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(64/35*a+32/5)*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(
4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2)
)^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*
```

```
(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1
/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*E
llipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-
2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a
)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(
1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(
4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1
-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.782 $\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=397

$$-\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x)+\frac{2\left(1-\sqrt{4+\right.}}{}$$

```
[Out] -2/3*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-
1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF
((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^
(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^
(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+
x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+
(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(
-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(
-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x
)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2
))))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1120, 1105, 1216, 545, 429, 506, 422}

$$\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-2*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)
^4]*(-1 + x))/3 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^
2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*
Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))
/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4
]) + (2*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*El
lipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[
4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + S
qrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
```

```
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int \sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{1}{3}\text{Subst}\left(\int \frac{2(3+a}{\sqrt{3+a-}}\right.$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+}}}\right.}{}$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - \frac{\left(2\sqrt{1-\frac{2(-1+x)}{-2-2\sqrt{4+}}}\right.}{}$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-}$$

$$= \frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{3\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1}{3}\sqrt{3+a-2(-1+x)^2-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3470 vs. $2(397) = 794$.

time = 16.06, size = 3470, normalized size = 8.74

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (-1/3 + x/3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4] + (2*((4*(-Sqrt[-1 - Sqrt[
4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[
((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4
 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], (
(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]
])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4
+ a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x
^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sq
rt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(S
qrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Ellipt
icF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[
-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
 Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 +
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1
- Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1
+ Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1
 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[A
```

```
rcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1
 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1
 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt
[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-
1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x
))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4
 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3
 - x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]]
+ x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt
[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] -
 x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
+ x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]]
 + x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 -
Sqrt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - S
qrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(455) = 910$.

time = 0.05, size = 2519, normalized size = 6.35

method	result	size
default	Expression too large to display	2519
elliptic	Expression too large to display	2519
risch	Expression too large to display	3022

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/
3*a+4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
```

/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1
+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(
1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(
4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-4/3*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2)
)^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(
1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1
+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(
1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(
-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+
a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^
(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+
(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(

```
1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))
^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.783 $\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=144

$$\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{-1+x}{\sqrt{1+\sqrt{4+a}}}\right)|-\frac{2\sqrt{4+a}}{1-\sqrt{4+a}}\right)}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] (1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*E
llipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-
2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)
^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1120, 1118, 429}

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[
(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt
[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqr
t[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\frac{}{-}}}\right.}{\sqrt{3+a-2(-1+x)^2-(}}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)\middle|-\right.}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 540 vs. 2(144) = 288.
time = 11.01, size = 540, normalized size = 3.75

$$\frac{2\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(1+\sqrt{-1+\sqrt{4+a}}-x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)\sqrt{\frac{\sqrt{-1-\sqrt{4+a}}\left(-1+\sqrt{-1+\sqrt{4+a}}+x\right)}{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}F\left(\sin^{-1}\left(\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\right)\middle|\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)^2}{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)^2}\right)}{\sqrt{-1-\sqrt{4+a}}\sqrt{\frac{\left(\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(-1+\sqrt{-1-\sqrt{4+a}}+x\right)}{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(1+\sqrt{-1-\sqrt{4+a}}-x\right)}}\sqrt{a-x\left(-8+8x-4x^2+x^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Ellip
ticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + S
qrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(Sq
rt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]
)*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S
qrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8 + 8*x - 4*x^
2 + x^3)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(182) = 364$.
time = 0.04, size = 530, normalized size = 3.68

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\cdots\right.}{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(x-1+\sqrt{-1+\cdots\right.}}}{\cdots}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\cdots\right.}{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(x-1+\sqrt{-1+\cdots\right.}}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+
a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(
x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
```

```
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^4 - 4*x^3 + 8*x^2 - a - 8
*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(1/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.784 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=437

$$\frac{(5+a+(-1+x)^2)\,(-1+x)}{2\,(12+7a+a^2)\,\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}-\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{2(3+a)(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/2*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*
(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1
+x)^2-(-1+x)^4)^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^
2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2
/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2
/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/
2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1
/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ell
ipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*
(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(
1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.30, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1120, 1106, 1216, 545, 429, 506, 422}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)
^2 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(
-1 + x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
 + a])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sq
rt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1
```

```
 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 +
a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]
))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{}{\sqrt{3+}}\right.}{}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-}{-2-}}\right.}{}$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\left(\sqrt{1-\frac{2(-}{-2-}}\right.}{}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(12+7a+a^2)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a}{2(12+7a+a^2)\sqrt{}}$$

$$= \frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{2(12+7a+a^2)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a}{2(12+7a+a^2)\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3526 vs. 2(437) = 874.
time = 16.07, size = 3526, normalized size = 8.07

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-3/2),x]
```

```
[Out] ((6 + a - 8*x - a*x + 3*x^2 - x^3)*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4])/(2*
(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)) + ((4*(-Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 +
 a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4
+ a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-
1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-
Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 +
a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2
 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt
[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqr
t[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Elliptic
F[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[
-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt
[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + S
qrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^
2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 -
Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 +
Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 -
 Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[Arc
Sin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 -
 Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 +
```

```
Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1
- Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])
/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))
/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 +
 a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1
- Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 -
 x^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] +
x)*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt
[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] +
 x))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sq
rt[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]
] - Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]]))*EllipticF[...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2600 vs. $2(495) = 990$.
time = 0.04, size = 2601, normalized size = 5.95

method	result	size
default	Expression too large to display	2601
elliptic	Expression too large to display	2601

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3-3/4/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x-1/4*(
6+a)/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-((a+5)/(a^2+7*a+12)-1/2*(
a+8)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
```

(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))
^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a
)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(
((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/(a^2+7
a+12)((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/
2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(
1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))
^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-1/2/(a^2+7*a+12
)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)
^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-
(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(
1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(

```
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^8 - 8*x^7 + 32*x^6 - 2*(a
- 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(-x^4+4\,x^3-8\,x^2+8\,x+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.785 $\int \frac{1}{\left(a+8x-8x^2+4x^3-x^4\right)^{5/2}}\,dx$

Optimal. Leaf size=517

$$\frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}}+\frac{(104+47a+5a^2+4(7+2a)(-1+x)^2)\,(-1+x)}{12(3+a)^2(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)}}$$

```
[Out] 1/6*(5+a+(-1+x)^2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/12
*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1
+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF(
(-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(
1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(
1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1
/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+
a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2
))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1120, 1106, 1192, 1216, 545, 429, 506, 422}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] ((5 + a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 +
x))/(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7
+ 2*a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3
 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1
- Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ell
ipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4
```

```
 + a])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1
+ (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
 ((16 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellip
ticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 +
 a])])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (
-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rubi steps

$$\int \frac{1}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{\text{Subst}\left(\int \frac{4+}{}\right.}{}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{}{6\,(12+7a+}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{}{6\,(12+7a+}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{}{6\,(12+7a+}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1\right.}{3\,(12+7a+}$$

$$= -\frac{(104+47a+5a^2+4(7+2a)(1-x)^2)\,(1-x)}{12\,(12+7a+a^2)^2\,\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(7+2a)\left(1\right.}{3\,(12+7a+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6386 vs. $2(517) = 1034$.

time = 16.12, size = 6386, normalized size = 12.35

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2756 vs. $2(571) = 1142$.
time = 0.06, size = 2757, normalized size = 5.33

method	result	size
default	Expression too large to display	2757
elliptic	Expression too large to display	2757

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3-1/2/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x-1/6*(6+
a)/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2
*(1/6*(7+2*a)/(a^2+7*a+12)^2*x^3-1/2*(7+2*a)/(a^2+7*a+12)^2*x^2+1/24*(5*a^2
+71*a+188)/(a^2+7*a+12)^2*x-1/24*(5*a^2+55*a+132)/(a^2+7*a+12)^2)/(-x^4+4*x
^3-8*x^2+a+8*x)^(1/2)-(1/6*(5*a^2+47*a+104)/(a^2+7*a+12)^2-1/12*(5*a^2+71*a
+188)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*Ellipti
cF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-2/3*(
7+2*a)/(a^2+7*a+12)^2*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
```

```
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-
1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+
a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+
(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/
2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*
(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1
+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/
2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2
))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/
2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-
(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x
-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(
((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^
(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a
)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticP
i(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)/(x^12 - 12*x^11 + 72*x^10 -
3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a -
 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 - 2
4*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral((a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(-5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(-5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(1/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.786 $\int x(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal. Leaf size=558

$$\frac{3}{16}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}+\frac{1}{8}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-\right.$$

```
[Out] 1/8*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/7*(3+a-2*(-1+x)^2-(-1+x)
^4)^(3/2)*(-1+x)+3/16*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)
^(1/2))-16/35*(7+2*a)*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(
3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/16*(4+a)*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+
x)^4)^(1/2)+2/35*(13+5*a-3*(-1+x)^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+4/35*(3+a)*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+
(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(
4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(
4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+16/35*(7+2*a)*(1/
(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Elli
pticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(
4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1
/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-
(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1694, 1687, 1105, 1190, 1216, 545, 429, 506, 422, 1121, 626, 635, 210}

$$\frac{3}{16}(a+4)^2\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{4(a+3)(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{16(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{35\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{3}{16}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{8}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{7}(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{35}(x-1)\left(5a-3(x-1)^2+13\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{16(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{35\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/16 + (
(1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/8 - (16*(7 + 2*
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(35*Sqrt[
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(13 + 5*a - 3*(-1 + x)^2)*Sqrt[3 +
 a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/35 + ((3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)*(-1 + x))/7 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4]])/16 + (16*(7 + 2*a)*(1 - Sqrt[4 + a])*Sqrt[
```

```
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1
 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (4*(3 + a)*(16 + 5*a)*Sqrt[1 + Sqrt[4
+ a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 +
 Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(35*Sqrt[(1 + (-1 + x)
^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-
1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
```

```
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx = \text{Subst}\left(\int (1+x)\left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \left(3+a-2x^2-x^4\right)^{3/2}\,dx, x, -1+x\right) + \text{Subst}\left(\int x(3+a$$

$$= \frac{1}{7}\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+x) + \frac{3}{7}\text{Subst}\left(\int (2(3$$

$$= \frac{1}{8}\left(3+a-2(1-x)^2-(1-x)^4\right)^{3/2}\left(1+(-1+x)^2\right) - \frac{2}{35}(13+5a-$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}(3+$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}(3+$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}(3+$$

$$= \frac{3}{16}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right) + \frac{1}{8}(3+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7235 vs. $2(558) = 1116$.

time = 16.12, size = 7235, normalized size = 12.97

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2693 vs. 2(600) = 1200.
time = 0.05, size = 2694, normalized size = 4.83

method	result	size
default	Expression too large to display	2694
elliptic	Expression too large to display	2694
risch	Expression too large to display	3609

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+17/28*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^
(1/2)-43/28*x^4*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+74/35*x^3*(-x^4+4*x^3-8*x^2+
a+8*x)^(1/2)+(5/16*a-9/20)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-11/56*a-29/
70)*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(11/56*a+13/14)*(-x^4+4*x^3-8*x^2+a+8*
x)^(1/2)-(-(-11/56*a-29/70)*a-11/14*a-26/7)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+
(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)
^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1
/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))
^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*
(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2)))^(1/2))-(a^2-2*(5/16*a-9/20)*a+55/14*a+62/5)*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2
```

(-1+(4+a)^(1/2))^(1/2)(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)
^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(
1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4
+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(
1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+
(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(-32/35*a-16/5)*((x-1-(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2)
)^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2)
)^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-
1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(
4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1
+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/
2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)
-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1
/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(-(x^5 - 4*x^4 + 8*x^3 - a*x - 8*x^2)*sqrt(-x^4 + 4*x^3 - 8*x^2 + a
 + 8*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a-x^{4}+4x^{3}-8x^{2}+8x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.787 $\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=466

$$\frac{1}{4}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}-\frac{2\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{3\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{1}{3}\sqrt{3}$$

```
[Out] 1/4*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))-2/3*(-1+x)*(
1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)
+1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/3*(-1+x)*(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)+2/3*(3+a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/(
(1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+2/3*(1/(1+
(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*Ellipti
cE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a
)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2)
)*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+
a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1694, 1687, 1105, 1216, 545, 429, 506, 422, 1121, 626, 635, 210}

$$\frac{1}{4}(a+4)\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{2(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{4}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{3}(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}-\frac{2\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/4 - (2*(1 - Sqrt
[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*Sqrt[3 + a - 2*(-1
 + x)^2 - (-1 + x)^4]) + (Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))
/3 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]])/4 + (2*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 +
a])/(1 - Sqrt[4 + a])])/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1
 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(
3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
```

```
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(3*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a
]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int(1+x)\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int x\sqrt{3+a-\right.$$

$$= \frac{1}{3}\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) + \frac{1}{3}\text{Subst}\left(\int\frac{2(3+}{\sqrt{3+a}}\right.$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1+(-1+x)^2) + \frac{1}{3}\sqrt{3+a-2(-}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1+(-1+x)^2) + \frac{1}{3}\sqrt{3+a-2(-}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1+(-1+x)^2) + \frac{2\left(1-\sqrt{4+a}\right)}{3\sqrt{3+a-}}$$

$$= \frac{1}{4}\sqrt{3+a-2(1-x)^2-(1-x)^4}\,(1+(-1+x)^2) + \frac{2\left(1-\sqrt{4+a}\right)}{3\sqrt{3+a-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4389 vs. 2(466) = 932.

time = 13.08, size = 4389, normalized size = 9.42

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]

[Out] (1/6 - x/6 + x^2/4)*Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)] + (Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)]*((-8*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (40*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)

```
)]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
+ a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqr
t[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + S
qrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (6*a*(-Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt
[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqr
t[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a
]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sq
rt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sq
rt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
 - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + S...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2550 vs. $2(516) = 1032$.

time = 0.05, size = 2551, normalized size = 5.47

method	result	size
default	Expression too large to display	2551
elliptic	Expression too large to display	2551
risch	Expression too large to display	3034

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/6*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)
+1/6*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(1/6*a-2/3)*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1
-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(
x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(
4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+
```

```
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2)
)^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/
2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*
(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1
/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))-(1/2*a+10/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1
/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(
-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-
1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(
1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2
)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2
)))^(1/2)))-2/3*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*
(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(
x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^
(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-
(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2
)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*E
llipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(
```

```
1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a
)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))
-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1
/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int(x*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.788 $\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=179

$$\frac{1}{2}\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)+\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{-1+x}{\sqrt{1+\sqrt{4}}}\right.\right.}{\sqrt{\frac{1+\frac{(-1+x)^2}{1-\sqrt{4+a}}}{1+\frac{(-1+x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(-1+x)^2-}}$$

```
[Out] 1/2*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(1/(1+(-1+x)^2/(1+
(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1
+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(
4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a
-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4
+a)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1694, 1687, 1118, 429, 1121, 635, 210}

$$\frac{1}{2}\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]]/2 + (Sqrt[
1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 +
x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 +
a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1118

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(b + q
))]/Sqrt[a + b*x^2 + c*x^4]), Int[1/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqrt[1 + 2
*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] &
& NegQ[c/a]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{1+x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{x}{\sqrt{3+a-}}\right.$$

$$\left(\sqrt{1-\frac{2(-1}{-2-2}}\right.$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+a-2x-x^2}}\,dx, x, (-1+x)^2\right) + \frac{\quad}{\quad}$$

$$= -\frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)F\left(\tan^{-1}\left(\frac{1-x}{\sqrt{1+\sqrt{4+a}}}\right)|-\right.}{\sqrt{\frac{1+\frac{(1-x)^2}{1-\sqrt{4+a}}}{1+\frac{(1-x)^2}{1+\sqrt{4+a}}}}\sqrt{3+a-2(1-x)^2-(1-x)^4}}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right) - \frac{\sqrt{1+\sqrt{4+a}}\left(1\right.}{\sqrt{\frac{1}{1}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 813 vs. 2(179) = 358.

time = 11.79, size = 813, normalized size = 4.54

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] (2*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1 + Sqrt[
-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*Sqrt[(
Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((-Sqrt[-1 - Sqrt
[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*((1 +
 Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
```

```
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqr
t[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])^2] - 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
 Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[a - x*(-8
+ 8*x - 4*x^2 + x^3)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(213) = 426$.
time = 0.04, size = 788, normalized size = 4.40

method	result
default	$-\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\right.}{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(x-1+\sqrt{-1+\right.}}}{}$
elliptic	$-\frac{\left(\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\sqrt{\frac{\left(-\sqrt{-1-\sqrt{4+a}}+\sqrt{-1+\sqrt{4+a}}\right)\left(x-1-\sqrt{-1+\right.}{\left(-\sqrt{-1-\sqrt{4+a}}-\sqrt{-1+\sqrt{4+a}}\right)\left(x-1+\sqrt{-1+\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+
a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/
((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(
x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/
2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
```

```
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(
1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),
((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^4 - 4*x^3 + 8*x^2 - a -
 8*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.789 $\int \frac{x}{\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=474

$$\frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{\left(5+a+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}-\frac{(}{2(3+}$$

```
[Out] 1/2*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2
)+1/2*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(
1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1
/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1
+(4+a)^(1/2))^(1/2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(
4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/2*(1/(1+(-1+x)^2/(1+(4+a
)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+
a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)
^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2)
)^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^
(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.33, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1694, 1687, 1106, 1216, 545, 429, 506, 422, 1121, 627}

$$\frac{(x-1)\left(a+(x-1)^2+5\right)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 +
 x))/(2*(3 + a)*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(2*(3 + a)*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
```

```
 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x
)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(2*(4 + a)*S
qrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*
Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{}{(3+a-}\right.$$

$$= \frac{(5+a+(-1+x)^2)(-1+x)}{2(12+7a+a^2)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{1}{2}\text{Subst}\left(\int \frac{}{(3}\right.$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1}{2(12+7a+a^2)\sqrt{3+a-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(5+a+(-1}{2(12+7a+a^2)\sqrt{3+a-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\right.}{2(12+7a+a^2)\sqrt{3+a-}}$$

$$= \frac{1+(-1+x)^2}{2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\right.}{2(12+7a+a^2)\sqrt{3+a-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3593 vs. $2(474) = 948$.

time = 13.76, size = 3593, normalized size = 7.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 2*x + a*x - a*x^2 - x^3)*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^2)/(2*(3 +
a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 + 8*x - 4*x^2 + x^3)
)^(3/2)) + ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((4*(-Sqrt[-1 - Sqrt[4 +
a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-S
qrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a
]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 -
 Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 +
a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1
- Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4
+ a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sq
rt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sq
rt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]
]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 +
 4*x^3 - x^4]) + (2*a*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-
1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
+ Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]]
 + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-
1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]
] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[
-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[
ArcSin[Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[
-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*
(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
 Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1
 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-
1 + Sqrt[4 + a]]))])/(Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqr
t[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + (4*(-Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*
Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqr
t[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqr
t[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sq
rt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sq
```

```
rt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - S
qrt[-1 - Sqrt[4 + a]] + x))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSi
n[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - S
qrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + S
qrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 -
Sqrt[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(
-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a
]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 -
Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x
^4]) - ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)
*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*(((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sq
rt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[
4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[
-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt
[4 + a]]) + ((-((-1 - Sqrt[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]]
- Sqrt[-1 + Sqrt[4 + a]])) + (-1 + Sqrt[-1 - Sq...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2615 vs. $2(528) = 1056$.

time = 0.04, size = 2616, normalized size = 5.52

method	result	size
default	Expression too large to display	2616
elliptic	Expression too large to display	2616

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(a^2+7*a+12)*x^3+1/4*a/(a^2+7*a+12)*x^2-1/4*(a-2)/(a^2+7*a+12)*x+1/4
*a/(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)+1/2*(a-2)/(
a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
```

```
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2
))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(
-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-((1+a)/(a^2+7*
a+12)-a/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1
-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(
-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(
4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1
+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(
1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2)
)^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))-
1/2/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))
*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*
(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)
)/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(
4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)
^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1
```

```
-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^
2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1
/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))
/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(
1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2),((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^8 - 8*x^7 + 32*x^6 - 2*(
a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.790 $\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=591

$$\frac{1+(-1+x)^2}{6(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{1+(-1+x)^2}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{(}{6\left(12+7a+\right.}$$

```
[Out] 1/6*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(5+a+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/3*(1+(-1+x)^2)/(4+a)^2
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(104+47*a+5*a^2+4*(7+2*a)*(-1+x)^2)*(
-1+x)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/3*(7+2*a)*(-1+x)*(1+
(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(a^2+7*a+12)^2/(3+a-2*(-1+x)^2-(-
1+x)^4)^(1/2)+1/12*(16+5*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)
^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)
^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(4+a)^2/(3+a-2*(-1+x)^2-(-1
+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1
/2)+1/3*(7+2*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)
^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^
(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^
(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)^2/(3+a-2*(-1+x)^
2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))
))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1694, 1687, 1106, 1192, 1216, 545, 429, 506, 422, 1121, 628, 627}

$$\frac{(x-1)\left(5a^2+4(2a+7)(x-1)^2+47a+104\right)}{12(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)\left(a+(x-1)^2+5\right)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(5a+16)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(2a+7)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{3(a+3)^2(a+4)^2\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{6(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}-\frac{(2a+7)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{3(a+3)^2(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(6*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (1
 + (-1 + x)^2)/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((5
+ a + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)^2 - (-1
 + x)^4)^(3/2)) + ((104 + 47*a + 5*a^2 + 4*(7 + 2*a)*(-1 + x)^2)*(-1 + x))/
(12*(3 + a)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((7 + 2*
```

```
a)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(3*(3 + a
)^2*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 2*a)*(1 - Sq
rt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*Ellipti
cE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a
])])/(3*(3 + a)^2*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-
1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1
6 + 5*a)*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF
[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])
])/(12*(3 + a)*(4 + a)^2*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 +
 x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1+x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{}{(3+a-}\right.$$

$$= \frac{(5+a+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} + \frac{1}{2}\text{Subst}\left(\int \frac{}{(}\right.$$

$$= \frac{1+(-1+x)^2}{6(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^{3/2}} - \frac{(104+47a+5a^2+4(}{12\,(12+7a+a^2)^2\,\sqrt{3+}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{1+(-1+}{3(4+a)^2\sqrt{3+a-2(1-}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{1+(-1+}{3(4+a)^2\sqrt{3+a-2(1-}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{1+(-1+}{3(4+a)^2\sqrt{3+a-2(1-}}$$

$$= \frac{1+(-1+x)^2}{6(4+a)\,(3+a-2(1-x)^2-(1-x)^4)^{3/2}} + \frac{1+(-1+}{3(4+a)^2\sqrt{3+a-2(1-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6452 vs. $2(591) = 1182$.
time = 16.11, size = 6452, normalized size = 10.92

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2776 vs. $2(637) = 1274$.
time = 0.06, size = 2777, normalized size = 4.70

method	result	size
default	Expression too large to display	2777
elliptic	Expression too large to display	2777

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/6/(a^2+7*a+12)*x^3+1/6*a/(a^2+7*a+12)*x^2-1/6*(a-2)/(a^2+7*a+12)*x+1/6*a
/(a^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(
1/6*(7+2*a)/(a^2+7*a+12)^2*x^3+1/6*(a^2-12)/(a^2+7*a+12)^2*x^2-1/24*(3*a^2-
23*a-116)/(a^2+7*a+12)^2*x+1/24*(3*a^2-7*a-60)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(1/6*(a^2+23*a+68)/(a^2+7*a+12)^2+1/12*(3*a^2-23*a-116)/
(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(2/3*(a^2+2
*a-5)/(a^2+7*a+12)^2-2/3*(a^2-12)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x
-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
```

```
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1
+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-
1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/
2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2)
)*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^
(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)
*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/
(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-
(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2)))^(1/2)))-1/3*(7+2*a)/(a^2+7*a+12)^2*((x-1-(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2)
)^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)
*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))
^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*((
1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(1
/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1/
2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2
))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(
1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1
/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x/(x^12 - 12*x^11 + 72*x^10
- 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5*a
 - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3 -
 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
```

```
[Out] Integral(x/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.791 $\int x^2(a+8x-8x^2+4x^3-x^4)^{3/2}\,dx$

Optimal. Leaf size=585

$$\frac{3}{8}(4+a)\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}+\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-\right.$$

```
[Out] 1/4*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/63*(15+7*(-1+x)^2)*(3+a-
2*(-1+x)^2-(-1+x)^4)^(3/2)*(-1+x)+3/8*(4+a)^2*arctan((1+(-1+x)^2)/(3+a-2*(-
1+x)^2-(-1+x)^4)^(1/2))+4/315*(21*a^2+111*a+140)*(-1+x)*(1+(-1+x)^2/(1-(4+a
)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+3/8*(4+a)*(1+(-1+
x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+2/315*(160+54*a+3*(20+7*a)*(-1+x)^2)*
(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+4/315*(3+a)*(100+33*a)*(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-4/315*(21*a^2+111*a+140)*(1/(1+(-1+x)^2/(1+(4+a)^(1
/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(
1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/
2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1
/2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)
^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1694, 1687, 1190, 1216, 545, 429, 506, 422, 12, 1121, 626, 635, 210}

$$-\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{4(21a^2+111a+140)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{315\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{3}{8}(a+4)^2\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{4(a+3)(33a+100)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{315\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{3}{8}(a+4)\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{4}\left((x-1)^2+1\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{1}{63}\left(7(x-1)^2+15\right)(x-1)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}+\frac{2}{315}(x-1)\left(3(7a+20)(x-1)^2+2(27a+80)\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (3*(4 + a)*(1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/8 + ((
1 + (-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2))/4 + (4*(140 + 11
1*a + 21*a^2)*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x)
)/(315*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (2*(2*(80 + 27*a) + 3*(20
 + 7*a)*(-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]*(-1 + x))/315 +
 ((15 + 7*(-1 + x)^2)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)*(-1 + x))/6
```

```
3 + (3*(4 + a)^2*ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 + x)^2 - (-1 +
x)^4]])/8 - (4*(140 + 111*a + 21*a^2)*(1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a
]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sq
rt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2
/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]) + (4*(3 + a)*(100 + 33*a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
(-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a
]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(315*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+8x-8x^2+4x^3-x^4\right)^{3/2}\,dx=\text{Subst}\left(\int(1+x)^2\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)$$

$$=\text{Subst}\left(\int 2x\left(3+a-2x^2-x^4\right)^{3/2}\,dx,x,-1+x\right)+\text{Subst}\left(\int (1$$

$$=\frac{1}{63}\left(15+7(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}(-1+$$

$$=-\frac{2}{315}\left(2(80+27a)+3(20+7a)(1-x)^2\right)\sqrt{3+a-2(1-x)^2-($$

$$=\frac{1}{4}\left(1+(-1+x)^2\right)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}-\frac{2}{315}(2($$

$$=\frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}(1+$$

$$=\frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}(1+$$

$$=\frac{3}{8}(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}\left(1+(-1+x)^2\right)+\frac{1}{4}(1+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 8500 vs. $2(585)=1170$.

time = 16.15, size = 8500, normalized size = 14.53

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2732 vs. 2(624) = 1248.
time = 0.06, size = 2733, normalized size = 4.67

method	result	size
default	Expression too large to display	2733
elliptic	Expression too large to display	2733
risch	Expression too large to display	3625

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/9*x^7*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+19/36*x^6*(-x^4+4*x^3-8*x^2+a+8*x)^
(1/2)-163/126*x^5*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+71/42*x^4*(-x^4+4*x^3-8*x^
2+a+8*x)^(1/2)+(11/45*a-16/63)*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(-13/120*
a-5/18)*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+(9/140*a+23/63)*x*(-x^4+4*x^3-8*
x^2+a+8*x)^(1/2)+(107/252*a+101/63)*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(-(9/140
*a+23/63)*a-107/63*a-404/63)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(
1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(
x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2
))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*
EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*(
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1
+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)
)-(-2*(-13/120*a-5/18)*a+827/315*a+76/9)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
```

```
+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1
/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)
*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/
2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-
1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1
/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*Elliptic
Pi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^
(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2
))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)
^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)))^(1/2)))+(a^2-3*(11/45*a-16/63)*a+68/105*a+16/9)*((x-1-(-1+(4+a)^
(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(
1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/
2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2)
)^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)
^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/
2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^
(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^
(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-
1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a
)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
[Out] integral(-(x^6 - 4*x^5 + 8*x^4 - a*x^2 - 8*x^3)*sqrt(-x^4 + 4*x^3 - 8*x^2 +
 a + 8*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a - x^{4} + 4x^{3} - 8x^{2} + 8x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
[Out] Integral(x**2*(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
[Out] integrate((-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.792 $\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx$

Optimal. Leaf size=485

$$\frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}+\frac{2(8+3a)\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+}{15\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}$$

```
[Out] 1/2*(4+a)*arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+2/15*(8+3*a)
*(-1+x)*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)
^4)^(1/2)+1/2*(1+(-1+x)^2)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/15*(7+3*(-1+x)
^2)*(-1+x)*(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+8/15*(3+a)*(1/(1+(-1+x)^2/(1+(4+
a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+x)/(1+(4
+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a
)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*
(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)
^(1/2))))^(1/2)-2/15*(8+3*a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+
x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x
)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x
)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-
(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))
^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {1694, 1687, 1190, 1216, 545, 429, 506, 422, 12, 1121, 626, 635, 210}

$$\frac{1}{2}(a+4)\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{8(a+3)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{2(3a+8)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{15\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{1}{2}\left((x-1)^2+1\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{1}{15}\left(3(x-1)^2+7\right)(x-1)\sqrt{a-(x-1)^4-2(x-1)^2+3}+\frac{2(3a+8)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{15\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 + (-1 + x)^2)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])/2 + (2*(8 + 3*a)
*(1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(15*Sqrt[3
+ a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((7 + 3*(-1 + x)^2)*Sqrt[3 + a - 2*(-1
+ x)^2 - (-1 + x)^4]*(-1 + x))/15 + ((4 + a)*ArcTan[(1 + (-1 + x)^2)/Sqrt[3
 + a - 2*(-1 + x)^2 - (-1 + x)^4]])/2 - (2*(8 + 3*a)*(1 - Sqrt[4 + a])*Sqrt
[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 +
 x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(
1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[
```

```
3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (8*(3 + a)*Sqrt[1 + Sqrt[4 + a]]*(1 +
 (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 +
a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(15*Sqrt[(1 + (-1 + x)^2/(1 - Sq
rt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 -
 (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
```

```
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int x^2\sqrt{a+8x-8x^2+4x^3-x^4}\,dx = \text{Subst}\left(\int (1+x)^2\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int 2x\sqrt{3+a-2x^2-x^4}\,dx, x, -1+x\right) + \text{Subst}\left(\int (1+x^2)\right.$$

$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - \frac{}{1}$$

$$= \frac{1}{15}\left(7+3(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}\,(-1+x) - $$

$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} + \frac{1}{15}\left(7+3(-1\right.$$

$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1\right.}{15\sqrt{}}$$

$$= \frac{1}{2}\left(1+(-1+x)^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4} - \frac{2(8+3a)\left(1\right.}{15\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5647 vs. $2(485) = 970$.
time = 13.20, size = 5647, normalized size = 11.64

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2581 vs. $2(535) = 1070$.
time = 0.05, size = 2582, normalized size = 5.32

method	result	size
default	Expression too large to display	2582
elliptic	Expression too large to display	2582
risch	Expression too large to display	3044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-1/10*x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1
/2)+1/15*x*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)+1/3*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2
)-(-1/15*a-4/3)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(
4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/
2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)
^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+
a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(
1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-
1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))
/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))
^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/5*a+28/1
5)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2
)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(
1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(
4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2)
```

)/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/
2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(
-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(
1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2)
)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(
4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))
*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/
2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)
^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2)))+(2/5*a+16/15)*((x-1
-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))
^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+
(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2)
)^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-
(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(
1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1
+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/
2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(
-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1

```
/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a-x^4+4x^3-8x^2+8x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
[Out] Integral(x**2*sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
[Out] int(x^2*(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.793 $\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx$

Optimal. Leaf size=388

$$\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(-1+x)^2}{1-\sqrt{4+a}}\right)(-1+x)}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\tan^{-1}\left(\frac{1+(-1+x)^2}{\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}\right)-\frac{\left(1-\sqrt{4+a}\right.}{}$$

```
[Out] arctan((1+(-1+x)^2)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2))+(-1+x)*(1+(-1+x)^2/(1-
(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+(1/(1+(-1+x)^
2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*EllipticF((-1+
x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)
/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1+(4+a)^(1/2))^(1/2)
/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/
(1+(4+a)^(1/2))))^(1/2)-(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/
(1+(4+a)^(1/2)))^(1/2)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(
1+(4+a)^(1/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a-2*(-1+x)^2-(-1+x
)^4)^(1/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2
)
```

Rubi [A]
time = 0.28, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {1694, 1687, 1216, 545, 429, 506, 422, 12, 1121, 635, 210}

$$\text{ArcTan}\left(\frac{(x-1)^2+1}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}\right)+\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/Sqrt[3 + a
- 2*(-1 + x)^2 - (-1 + x)^4] + ArcTan[(1 + (-1 + x)^2)/Sqrt[3 + a - 2*(-1 +
 x)^2 - (-1 + x)^4]] - ((1 - Sqrt[4 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 +
x)^2/(1 - Sqrt[4 + a]))*EllipticE[ArcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (
-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a])
)/(1 + (-1 + x)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^
4]) + (Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
```

```
/(Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4 + a])
)]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+8x-8x^2+4x^3-x^4}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1+}{\sqrt{3+a-}}\right.$$

$$\left(\sqrt{1-\frac{2(-1+x}{-2-2\sqrt{4}}}\right.$$

$$= 2\text{Subst}\left(\int \frac{x}{\sqrt{3+a-2x^2-x^4}}\,dx, x, -1+x\right) + \frac{}{}$$

$$= \frac{\left(\sqrt{1-\frac{2(-1+x)^2}{-2-2\sqrt{4+a}}}\sqrt{1-\frac{2(-1+x)^2}{-2+2\sqrt{4+a}}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-\frac{}{-}}}\right.}{\sqrt{3+a-2(-1+x)^2-(}}$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} - \frac{\sqrt{1+\sqrt{4+a}}\left(1+\frac{}{1}\right.}{\sqrt{\frac{1+\frac{}{1}}{1+\frac{}{1}}}}$$

$$= -\frac{\left(1-\sqrt{4+a}\right)\left(1+\frac{(1-x)^2}{1-\sqrt{4+a}}\right)(1-x)}{\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \tan^{-1}\left(\frac{1+(-}{\sqrt{3+a-2(1}}\right.$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1145 vs. 2(388) = 776.
time = 13.98, size = 1145, normalized size = 2.95

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4],x]
```

```
[Out] ((-1 + Sqrt[-1 - Sqrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 +
Sqrt[-1 + Sqrt[4 + a]] + x) + (2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)^2*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(1
+ Sqrt[-1 + Sqrt[4 + a]] - x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4
+ a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - S
qrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[-
((Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x)))]*((1
 + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]
] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - S
qrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] - (1 + 2*Sqrt[-1 - Sqrt[
4 + a]] + Sqrt[-1 - Sqrt[4 + a]]*Sqrt[-1 + Sqrt[4 + a]])*EllipticF[ArcSin[S
qrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt
[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt
[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 4*Sqrt[-1 - Sqr
t[4 + a]]*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sq
rt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]]
- x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqr
t[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2]))/(1 + Sqrt[4 + a] + Sqrt[-1 - Sqrt[
4 + a]]*Sqrt[-1 + Sqrt[4 + a]]))/Sqrt[a - x*(-8 + 8*x - 4*x^2 + x^3)]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(454) = 908$.
time = 0.04, size = 1147, normalized size = 2.96 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(
1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(
1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a
)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*
(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2)
)/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1
/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1
-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/
```

```
2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1
/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(
-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2
)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^
(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a
)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1
/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-1-(4+a)^
(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2
))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^
(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))))/(-(x-1-(-1
+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/
2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^4 - 4*x^3 + 8*x^2 - a
 - 8*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a - x^4 + 4x^3 - 8x^2 + 8x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a - x**4 + 4*x**3 - 8*x**2 + 8*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{-x^4+4\,x^3-8\,x^2+8\,x+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2),x)
```

```
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(1/2), x)
```

3.794 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx$

Optimal. Leaf size=311

$$\frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}-\frac{\left(1-\cdots\right.}{2(3+\cdots}$$

```
[Out] (1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(4+a)*(2+(-1+x)^2)*(
-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/2*(-1+x)*(1+(-1+x)^2/(
1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/2*(
1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)*El
lipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2),(-2
*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^
(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((1+(-1+
x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1694, 1687, 1192, 12, 1154, 506, 422, 1121, 627}

$$\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{2\left(a^2+7a+12\right)\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{2(a+3)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{2(a+3)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] (1 + (-1 + x)^2)/((4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((4 +
a)*(2 + (-1 + x)^2)*(-1 + x))/(2*(12 + 7*a + a^2)*Sqrt[3 + a - 2*(-1 + x)^2
 - (-1 + x)^4]) - ((1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1
 + x))/(2*(3 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((1 - Sqrt[4 +
 a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[Arc
Tan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(
2*(3 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt
[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1154

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt[1 + 2*c*(x^2/(
b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[x^2/(Sqrt[1 + 2*c*(x^2/(b - q))]*Sqr
t[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && NegQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
```

```
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{3/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^{3/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1}{(3+a-\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + 2\text{Subst}\left(\int \frac{}{(3}\right.$$

$$= \frac{(4+a)\left(2+(-1+x)^2\right)(-1+x)}{2\left(12+7a+a^2\right)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} - \frac{\text{Subst}\left(\int \frac{}{\sqrt{3}}\right.}{}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{(4+a)\left(2+(\right.}{2\left(12+7a+a^2\right)\sqrt{3+}}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\right.}{2(3+a)\sqrt{3+a-2(1}}$$

$$= \frac{1+(-1+x)^2}{(4+a)\sqrt{3+a-2(-1+x)^2-(-1+x)^4}} + \frac{\left(1-\sqrt{4+a}\right)\left(1+\right.}{2(3+a)\sqrt{3+a-2(1}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2941 vs. $2(311) = 622$.
time = 13.78, size = 2941, normalized size = 9.46

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x]
```

```
[Out] ((-a - 8*x - a*x + 6*x^2 + a*x^2 - 4*x^3 - a*x^3)*(a + 8*x - 8*x^2 + 4*x^3
- x^4)^2)/(2*(3 + a)*(4 + a)*(-a - 8*x + 8*x^2 - 4*x^3 + x^4)*(a - x*(-8 +
8*x - 4*x^2 + x^3))^(3/2)) - ((a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2)*((2*(-S
qrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]
] + x)^2*Sqrt[((-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqr
t[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]
)*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sq
rt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]
])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + S
qrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a
]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*EllipticF[ArcSin[Sqrt[((-Sqrt[-1 -
Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/(
(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 +
a]] + x))]], ((-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1
 + Sqrt[4 + a]])*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]))])/(Sqr
t[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*Sqrt
[a + 8*x - 8*x^2 + 4*x^3 - x^4]) - (4*(-Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))
]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1
 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x)
)]*Sqrt[(Sqrt[-1 - Sqrt[4 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-
1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x
))]*((-1 - Sqrt[-1 - Sqrt[4 + a]])*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[
-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x
))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4
 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2] + 2*Sqrt[-1 - Sqrt[4 + a]]*EllipticPi[(
Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])/(-Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] +
 Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sq
rt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 +
Sqrt[4 + a]])^2]))/(Sqrt[-1 - Sqrt[4 + a]]*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
1 + Sqrt[4 + a]])*Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]) + ((-1 + Sqrt[-1 - S
qrt[4 + a]] + x)*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x)*(-1 + Sqrt[-1 + Sqrt[4 +
 a]] + x) + 2*(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(-1 - Sqrt[
-1 - Sqrt[4 + a]] + x)^2*Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 +
 a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]*Sqrt[(Sqrt[-1 - Sqrt[4
+ a]]*(-1 - Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1
 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*Sqrt[(Sqrt[-1 - Sqrt[4
 + a]]*(-1 + Sqrt[-1 + Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-
```

```
1 + Sqrt[4 + a]])*(-1 - Sqrt[-1 - Sqrt[4 + a]] + x))]*(((Sqrt[-1 - Sqrt[4 +
 a]] - Sqrt[-1 + Sqrt[4 + a]])*EllipticE[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 -
 Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]]
, (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a
]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]) + ((-((-1 - Sqr
t[-1 - Sqrt[4 + a]])*(-2 - Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])
) + (-1 + Sqrt[-1 - Sqrt[4 + a]])*(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[
4 + a]]))*EllipticF[ArcSin[Sqrt[((Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]] + x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[
-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqrt[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 +
 a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqrt[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4
 + a]])^2])/(2*Sqrt[-1 - Sqrt[4 + a]]*(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 +
Sqrt[4 + a]])) + (4*EllipticPi[(Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 +
 a]])/(-Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]]), ArcSin[Sqrt[((Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])*(-1 + Sqrt[-1 - Sqrt[4 + a]]
+ x))/((Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])*(1 + Sqrt[-1 - Sqr
t[4 + a]] - x))]], (Sqrt[-1 - Sqrt[4 + a]] + Sqrt[-1 + Sqrt[4 + a]])^2/(Sqr
t[-1 - Sqrt[4 + a]] - Sqrt[-1 + Sqrt[4 + a]])^2])/(-Sqrt[-1 - Sqrt[4 + a]]
+ Sqrt[-1 + Sqrt[4 + a]])))/Sqrt[a + 8*x - 8*x^2 + 4*x^3 - x^4]))/(2*(3 + a
)*(a - x*(-8 + 8*x - 4*x^2 + x^3))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2606 vs. $2(331) = 662$.
time = 0.04, size = 2607, normalized size = 8.38

method	result	size
default	Expression too large to display	2607
elliptic	Expression too large to display	2607

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1/4/(3+a)*x^3-1/4*(6+a)/(a^2+7*a+12)*x^2+1/4*(a+8)/(a^2+7*a+12)*x+1/4*a/
(a^2+7*a+12))/(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)-(2/(a^2+7*a+12)-1/2*(a+8)/(a^2
+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-
1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2)
)^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/
2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(
-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)
))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^
(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-
(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4
```

```
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(
1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1
/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)
)/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(-2/(a^2+7*a+12)+
(6+a)/(a^2+7*a+12))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(
-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))
^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(
4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1
+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2)
)^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1
/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+
a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))
*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*((1-(-1+(
4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*EllipticPi(((-(-1-(4+a)^(1/2)
)^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1
/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/
2/(3+a)*((x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1
-(4+a)^(1/2))^(1/2))+((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2))
)^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-
(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-
1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2
))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(
1/2))^(1/2)))^(1/2)*(-1/2*((1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(
1/2))-(1-(-1-(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1
/2))^(1/2))*(1-(-1+(4+a)^(1/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-
(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF
(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1
/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1
/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(
-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))
```

```
^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/
2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+
a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-
(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/2
))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(
1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-
1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^8 - 8*x^7 + 32*x^6 - 2
*(a - 64)*x^4 - 80*x^5 + 8*(a - 16)*x^3 - 16*(a - 4)*x^2 + a^2 + 16*a*x), x
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a - x^4 + 4x^3 - 8x^2 + 8x\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2),x)
```

```
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(3/2), x)
```

3.795 $\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx$

Optimal. Leaf size=582

$$\frac{1+(-1+x)^2}{3(4+a)\left(3+a-2(-1+x)^2-(-1+x)^4\right)^{3/2}}+\frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(-1+x)^2-(-1+x)^4}}+\frac{(4}{6\,(12+7a+}$$

```
[Out] 1/3*(1+(-1+x)^2)/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+1/6*(4+a)*(2+(-1+x)^
2)*(-1+x)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(3/2)+2/3*(1+(-1+x)^2)/(4+
a)^2/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)+1/12*(29+7*a+(13+3*a)*(-1+x)^2)*(-1+x)
/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)-1/12*(13+3*a)*(-1+x)*(1+(-1+
x)^2/(1-(4+a)^(1/2)))*(1-(4+a)^(1/2))/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^
4)^(1/2)+1/12*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(
1/2)))^(1/2)*EllipticF((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1
/2)))^(1/2),(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1
/2)))*(1+(4+a)^(1/2))^(1/2)/(a^2+7*a+12)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1/2)/((
1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)+1/12*(13+3*
a)*(1/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)*(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2
)*EllipticE((-1+x)/(1+(4+a)^(1/2))^(1/2)/(1+(-1+x)^2/(1+(4+a)^(1/2)))^(1/2)
,(-2*(4+a)^(1/2)/(1-(4+a)^(1/2)))^(1/2))*(1+(-1+x)^2/(1-(4+a)^(1/2)))*(1-(4
+a)^(1/2))*(1+(4+a)^(1/2))^(1/2)/(3+a)^2/(4+a)/(3+a-2*(-1+x)^2-(-1+x)^4)^(1
/2)/((1+(-1+x)^2/(1-(4+a)^(1/2)))/(1+(-1+x)^2/(1+(4+a)^(1/2))))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1694, 1687, 1192, 1216, 545, 429, 506, 422, 12, 1121, 628, 627}

$$\frac{\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)F\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12\left(a^2+7a+12\right)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(a+4)\left((x-1)^2+2\right)(x-1)}{6\left(a^2+7a+12\right)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(3a+13)\left(1-\sqrt{a+4}\right)\sqrt{\sqrt{a+4}+1}\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)E\left(\text{ArcTan}\left(\frac{x-1}{\sqrt{\sqrt{a+4}+1}}\right)|-\frac{2\sqrt{a+4}}{1-\sqrt{a+4}}\right)}{12(a+3)^2(a+4)\sqrt{\frac{\frac{(x-1)^2}{1-\sqrt{a+4}}+1}{\frac{(x-1)^2}{\sqrt{a+4}+1}+1}}\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{2\left((x-1)^2+1\right)}{3(a+4)^2\sqrt{a-(x-1)^4-2(x-1)^2+3}}+\frac{(x-1)^2+1}{3(a+4)\left(a-(x-1)^4-2(x-1)^2+3\right)^{3/2}}+\frac{(x-1)\left((3a+13)(x-1)^2+7a+29\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}-\frac{(3a+13)\left(1-\sqrt{a+4}\right)(x-1)\left(\frac{(x-1)^2}{1-\sqrt{a+4}}+1\right)}{12(a+3)^2(a+4)\sqrt{a-(x-1)^4-2(x-1)^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
```

```
[Out] (1 + (-1 + x)^2)/(3*(4 + a)*(3 + a - 2*(-1 + x)^2 - (-1 + x)^4)^(3/2)) + (2
*(1 + (-1 + x)^2))/(3*(4 + a)^2*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) +
((4 + a)*(2 + (-1 + x)^2)*(-1 + x))/(6*(12 + 7*a + a^2)*(3 + a - 2*(-1 + x)
^2 - (-1 + x)^4)^(3/2)) + ((29 + 7*a + (13 + 3*a)*(-1 + x)^2)*(-1 + x))/(12
*(3 + a)^2*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) - ((13 + 3*a)*(
1 - Sqrt[4 + a])*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*(-1 + x))/(12*(3 + a)^2
```

```
*(4 + a)*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + ((13 + 3*a)*(1 - Sqrt[4
 + a])*Sqrt[1 + Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticE[A
rcTan[(-1 + x)/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])
/(12*(3 + a)^2*(4 + a)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x
)^2/(1 + Sqrt[4 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4]) + (Sqrt[1
+ Sqrt[4 + a]]*(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))*EllipticF[ArcTan[(-1 + x)
/Sqrt[1 + Sqrt[4 + a]]], (-2*Sqrt[4 + a])/(1 - Sqrt[4 + a])])/(12*(12 + 7*a
 + a^2)*Sqrt[(1 + (-1 + x)^2/(1 - Sqrt[4 + a]))/(1 + (-1 + x)^2/(1 + Sqrt[4
 + a]))]*Sqrt[3 + a - 2*(-1 + x)^2 - (-1 + x)^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1216

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[1 + 2*c*(x^2/(b - q))]*(Sqrt
[1 + 2*c*(x^2/(b + q))]/Sqrt[a + b*x^2 + c*x^4]), Int[(d + e*x^2)/(Sqrt[1 +
 2*c*(x^2/(b - q))]*Sqrt[1 + 2*c*(x^2/(b + q))]), x], x]] /; FreeQ[{a, b, c
, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[c/a]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
```

```
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+8x-8x^2+4x^3-x^4)^{5/2}}\,dx = \text{Subst}\left(\int \frac{(1+x)^2}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right)$$

$$= \text{Subst}\left(\int \frac{2x}{(3+a-2x^2-x^4)^{5/2}}\,dx, x, -1+x\right) + \text{Subst}\left(\int \frac{1}{(3+a-}\right.$$

$$= \frac{(4+a)\,(2+(-1+x)^2)\,(-1+x)}{6\,(12+7a+a^2)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} + 2\text{Subst}\left(\int \frac{}{(3}\right.$$

$$= -\frac{(29+7a+(13+3a)(1-x)^2)\,(1-x)}{12(3+a)^2(4+a)\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{(4+}{6\,(12+7a+a^2}$$

$$= \frac{1+(-1+x)^2}{3(4+a)\,(3+a-2(-1+x)^2-(-1+x)^4)^{3/2}} - \frac{(29+7a+(13}{12(3+a)^2(4+a)\sqrt{3}}$$

$$= \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1}{3(4+a)\,(3+a-2(-1+}$$

$$= \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1}{3(4+a)\,(3+a-2(-1+}$$

$$= \frac{2(1+(-1+x)^2)}{3(4+a)^2\sqrt{3+a-2(1-x)^2-(1-x)^4}} + \frac{1+(-1}{3(4+a)\,(3+a-2(-1+}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5812 vs. $2(582) = 1164$.
time = 16.12, size = 5812, normalized size = 9.99

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2779 vs. $2(628) = 1256$.
time = 0.07, size = 2780, normalized size = 4.78

method	result	size
default	Expression too large to display	2780
elliptic	Expression too large to display	2780

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] (1/6/(3+a)*x^3-1/6*(6+a)/(a^2+7*a+12)*x^2+1/6*(a+8)/(a^2+7*a+12)*x+1/6*a/(a
^2+7*a+12))*(-x^4+4*x^3-8*x^2+a+8*x)^(1/2)/(x^4-4*x^3+8*x^2-a-8*x)^2+2*(1/2
4*(13+3*a)/(3+a)/(a^2+7*a+12)*x^3-1/24*(a^2+27*a+84)/(a^2+7*a+12)^2*x^2+1/6
*(9*a+32)/(a^2+7*a+12)^2*x+1/12*(3*a^2+7*a-12)/(a^2+7*a+12)^2)/(-x^4+4*x^3-
8*x^2+a+8*x)^(1/2)-(-1/6*(a^2-9*a-44)/(a^2+7*a+12)^2-1/3*(9*a+32)/(a^2+7*a+
12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^
(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(
-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(
1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))
^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-
1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^
(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/
2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+
a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2)))^(1/2)*EllipticF(((-(-1-(4+a)
^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)
+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(
(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-(1/3*(a^2-a-16)/(a^2
+7*a+12)^2+1/6*(a^2+27*a+84)/(a^2+7*a+12)^2)*((-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-
1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1
```

```
+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2))^(1/2))^2*(-2*(-1+(4+a
)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+
a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-2*(-1+(4+a)^(1/2))^(
1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))
^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4
+a)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)/(-(x-1-(-1+(4+a)^(1/2))^(1/2))*(x-
1+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2)
)^(1/2)))^(1/2)*((1-(-1+(4+a)^(1/2))^(1/2))*EllipticF(((-(-1-(4+a)^(1/2))^(
1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-
(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))+2*(-1+(4+a)^(1/2))^(1/2)*Elli
pticPi(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^
(1/2))^(1/2)))^(1/2),(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(-(-1
-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2)),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(
4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4
+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^
(1/2))^(1/2)))^(1/2))-1/12*(13+3*a)/(3+a)/(a^2+7*a+12)*((x-1-(-1+(4+a)^(1/
2))^(1/2))*(x-1-(-1-(4+a)^(1/2))^(1/2))*(x-1+(-1-(4+a)^(1/2))^(1/2))+((-1-(
4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))*((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+
a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+
(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(x-1+(-1+(4+a)^(1/2
))^(1/2))^2*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1-(-1-(4+a)^(1/2))^(1/2))/((-1-(4
+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2
)*(-2*(-1+(4+a)^(1/2))^(1/2)*(x-1+(-1-(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2)
)^(1/2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2)*(-1/2*(
(1-(-1+(4+a)^(1/2))^(1/2))*(1+(-1+(4+a)^(1/2))^(1/2))-(1-(-1-(4+a)^(1/2))^(
1/2))*(1+(-1+(4+a)^(1/2))^(1/2))+(1-(-1-(4+a)^(1/2))^(1/2))*(1-(-1+(4+a)^(1
/2))^(1/2))+(1-(-1+(4+a)^(1/2))^(1/2))^2)/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a
)^(1/2))^(1/2))/(-1+(4+a)^(1/2))^(1/2)*EllipticF(((-(-1-(4+a)^(1/2))^(1/2)+
(-1+(4+a)^(1/2))^(1/2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/
2)-(-1+(4+a)^(1/2))^(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)
^(1/2))^(1/2)-(-1+(4+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/
2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2
))^(1/2)-(-1+(4+a)^(1/2))^(1/2)))^(1/2))-1/2*(-(-1-(4+a)^(1/2))^(1/2)+(-1+(
4+a)^(1/2))^(1/2))*EllipticE(((-(-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/
2))*(x-1-(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(1/2))^
(1/2))/(x-1+(-1+(4+a)^(1/2))^(1/2)))^(1/2),((-(-1-(4+a)^(1/2))^(1/2)-(-1+(4
+a)^(1/2))^(1/2))*((-1-(4+a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/(-(-1-(4+
a)^(1/2))^(1/2)+(-1+(4+a)^(1/2))^(1/2))/((-1-(4+a)^(1/2))^(1/2)-(-1+(4+a)^(
1/2))^(1/2)))^(1/2))/(-1+(4+a)^(1/2))^(1/2)-4/(-(-1-(4+a)^(1/2))^(1/2)+(-1+
(4+a)^(1/2))^(1/2))*EllipticPi(((-(-1-(4+a)^(1/...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="maxima")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)*x^2/(x^12 - 12*x^11 + 72*x^1
0 - 3*(a - 256)*x^8 - 280*x^9 + 24*(a - 64)*x^7 - 32*(3*a - 70)*x^6 + 48*(5
*a - 48)*x^5 + 3*(a^2 - 128*a + 512)*x^4 - 4*(3*a^2 - 96*a + 128)*x^3 - a^3
 - 24*a^2*x + 24*(a^2 - 8*a)*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a - x^4 + 4x^3 - 8x^2 + 8x)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**4+4*x**3-8*x**2+a+8*x)**(5/2),x)
[Out] Integral(x**2/(a - x**4 + 4*x**3 - 8*x**2 + 8*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^4+4*x^3-8*x^2+a+8*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^2/(-x^4 + 4*x^3 - 8*x^2 + a + 8*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(-x^4+4\,x^3-8\,x^2+8\,x+a\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2),x)
[Out] int(x^2/(a + 8*x - 8*x^2 + 4*x^3 - x^4)^(5/2), x)
```

3.796 $\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}\,(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}\,(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\,\sqrt[4]{29}\,x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\,\sqrt[4]{29}\,\sqrt{8+8x-x^3+8x^4}}$$

```
[Out] -1/696*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arc
tan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(2*arctan(1/87*(4+x)*29^(3
/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((
261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(3/4)*3^(1
/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2094, 12, 6851, 1117}

$$-\frac{x^2\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}\,(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}\,(x+4)^2}{x^2}+87\right)F\left(2\text{ArcTan}\left(\frac{x+4}{\sqrt{3}\,\sqrt[4]{29}\,x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{8\sqrt{3}\,\sqrt[4]{29}\,\sqrt{8x^4-x^3+8x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] -1/8*(x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^
2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt
[3]*29^(1/4)*x)], (29 + Sqrt[29])/58])/(Sqrt[3]*29^(1/4)*Sqrt[8 + 8*x - x^3
 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+8x-x^3+8x^4}}\,dx = -\left(1024\text{Subst}\left(\int \frac{1}{2\sqrt{2}\,(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx, x\right.\right.$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\sqrt{\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}}}\,dx,\right.\right.$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{\sqrt{10}}\right.}{\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{x^2\sqrt{\frac{261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4}{\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)^2}}\left(87+\frac{\sqrt{29}(4+x)^2}{x^2}\right)F\left(2\tan^{-1}\left(\frac{4+x}{\sqrt{3}\sqrt[4]{29}}\right.\right.}{8\sqrt{3}\sqrt[4]{29}\sqrt{8+8x-x^3+8x^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 10.58, size = 927, normalized size = 7.19

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[8 + 8*x - x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], ((Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0
])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2,
 0])^2*Sqrt[((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x -
Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 4, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1
- #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^
2*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])^2)])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 965, normalized size = 7.48

method	result	size
default	Expression too large to display	965
elliptic	Expression too large to display	965

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))
*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(
x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-R
ootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^
(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,
index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,i
ndex=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*
```

```
_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_
Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^
3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^
4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*
_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*El
lipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8
,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)
))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8x^4 - x^3 + 8x + 8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(8*x**4 - x**3 + 8*x + 8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(8*x^4 - x^3 + 8*x + 8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{8\,x^4-x^3+8\,x+8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2),x)
```

```
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(1/2), x)
```

3.797 $\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}} \, dx$

Optimal. Leaf size=431

$$-\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}}+\frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}}+\frac{7\left(261-6\left(1+\frac{4}{x}\right)^2+\left(1+\frac{4}{x}\right)^4\right)\left(1+\frac{4}{x}\right)x^2}{432\sqrt{29}\sqrt{8+8x-x^3+8x^4}\left(87+\frac{\sqrt{29}(4+}{x^2}\right.}$$

```
[Out] -1/1008*(66-(1+4/x)^2)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+1/12528*(216-7*(1+4/x)^2
)*(1+4/x)*x^2/(8*x^4-x^3+8*x+8)^(1/2)+7/12528*(261-6*(1+4/x)^2+(1+4/x)^4)*(
1+4/x)*x^2*29^(1/2)/(87+(4+x)^2*29^(1/2)/x^2)/(8*x^4-x^3+8*x+8)^(1/2)-7/125
28*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))^2)^(1/2)/cos(2*arctan(
1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticE(sin(2*arctan(1/87*(4+x)*29^(3/4)/
x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-
6*(1+4/x)^2+(1+4/x)^4)/(87+(4+x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/
(8*x^4-x^3+8*x+8)^(1/2)+1/50112*x^2*(cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(
1/2)))^2)^(1/2)/cos(2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2)))*EllipticF(sin(
2*arctan(1/87*(4+x)*29^(3/4)/x*3^(1/2))),1/58*(1682+58*29^(1/2))^(1/2))*(14
-5*29^(1/2))*(87+(4+x)^2*29^(1/2)/x^2)*((261-6*(1+4/x)^2+(1+4/x)^4)/(87+(4+
x)^2*29^(1/2)/x^2)^2)^(1/2)*29^(1/4)*3^(1/2)/(8*x^4-x^3+8*x+8)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\frac{\left(14-5\sqrt{29}\right)\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2F\left(2\text{ArcTan}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{576\sqrt{3}\,29^{3/4}\sqrt{8x^4-x^3+8x+8}}-\frac{7\sqrt{\frac{\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261}{\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)^2}}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)x^2E\left(2\text{ArcTan}\left(\frac{x+4}{\sqrt{3}\sqrt[4]{29}x}\right)|\frac{1}{58}\left(29+\sqrt{29}\right)\right)}{144\sqrt{3}\,29^{3/4}\sqrt{8x^4-x^3+8x+8}}-\frac{\left(66-\left(\frac{4}{x}+1\right)^2\right)x^2}{1008\sqrt{8x^4-x^3+8x+8}}+\frac{\left(216-7\left(\frac{4}{x}+1\right)^2\right)\left(\frac{4}{x}+1\right)x^2}{12528\sqrt{8x^4-x^3+8x+8}}+\frac{7\left(\left(\frac{4}{x}+1\right)^4-6\left(\frac{4}{x}+1\right)^2+261\right)\left(\frac{4}{x}+1\right)x^2}{432\sqrt{29}\sqrt{8x^4-x^3+8x+8}\left(\frac{\sqrt{29}(x+4)^2}{x^2}+87\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -1/1008*((66 - (1 + 4/x)^2)*x^2)/Sqrt[8 + 8*x - x^3 + 8*x^4] + ((216 - 7*(1
 + 4/x)^2)*(1 + 4/x)*x^2)/(12528*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (7*(261 - 6
*(1 + 4/x)^2 + (1 + 4/x)^4)*(1 + 4/x)*x^2)/(432*Sqrt[29]*Sqrt[8 + 8*x - x^3
 + 8*x^4]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)) - (7*x^2*Sqrt[(261 - 6*(1 + 4/x)
^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/x^2)^2]*(87 + (Sqrt[29]*(4 + x
)^2)/x^2)*EllipticE[2*ArcTan[(4 + x)/(Sqrt[3]*29^(1/4)*x)], (29 + Sqrt[29])
/58])/(144*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^3 + 8*x^4]) + ((14 - 5*Sqrt[29
])*x^2*Sqrt[(261 - 6*(1 + 4/x)^2 + (1 + 4/x)^4)/(87 + (Sqrt[29]*(4 + x)^2)/
x^2)^2]*(87 + (Sqrt[29]*(4 + x)^2)/x^2)*EllipticF[2*ArcTan[(4 + x)/(Sqrt[3]
```

```
*29^(1/4)*x)], (29 + Sqrt[29])/58])/(576*Sqrt[3]*29^(3/4)*Sqrt[8 + 8*x - x^
3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{(8+8x-x^3+8x^4)^{3/2}}\,dx = -\left(1024\text{Subst}\left(\int \frac{1}{16\sqrt{2}\,(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}\right.\right.$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(8-32x)^2\left(\frac{1069056-393216x^2+1048576x^4}{(8-32x)^4}\right)^{3/2}}\,dx, x, \frac{1}{4}+\right.\right.$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(1}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= -\frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{1}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{}{(1}\right.}{8\sqrt{8+8x-x^3+8x^4}}$$

$$= \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(\sqrt{1069056-393216\left(\frac{1}{4}+\frac{1}{x}\right)^2+1048\right.}{37}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} - \frac{\left(7\sqrt{1069\right.}{}$$

$$= -\frac{\left(66-\left(1+\frac{4}{x}\right)^2\right)x^2}{1008\sqrt{8+8x-x^3+8x^4}} + \frac{\left(216-7\left(1+\frac{4}{x}\right)^2\right)\left(1+\frac{4}{x}\right)x^2}{12528\sqrt{8+8x-x^3+8x^4}} + \frac{7\left(261\right.}{432\sqrt{29}\,\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 16.04, size = 4865, normalized size = 11.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 8*x - x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] (544 + 1539*x - 1146*x^2 + 784*x^3)/(21924*Sqrt[8 + 8*x - x^3 + 8*x^4]) + (
(28*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*(-(EllipticF[ArcSin[Sqr
t[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((-Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]
)*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4
& , 4, 0])))]*Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]) + EllipticPi[(-Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0
])/(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^
4 & , 4, 0]), ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(R
oot[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]], -(((Roo
t[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1
^4 & , 4, 0]))/((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 -
 #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0])))]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1
, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0]))*Sqrt[((-Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[
8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 +
 8*#1^4 & , 3, 0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
1, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*
Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0
] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4
]*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4
 & , 2, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 4, 0])) + (842*EllipticF[ArcSin[Sqrt[((x - Root[8 + 8*#1 - #1
^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 +
 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2
, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 4, 0]))]], ((Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] - Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 8*#1 - #1^3 + 8
```

```
*#1^4 & , 2, 0] - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*(x - Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3
 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root
[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3,
0]))]*(Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] - Root[8 + 8*#1 - #1^3 + 8*#
1^4 & , 4, 0])*Sqrt[((-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8
*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]
))/((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 +
8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))]*Sqrt[((x - Ro
ot[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & ,
2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 8*#1 - #1^
3 + 8*#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 4, 0]))])/(Sqrt[8 + 8*x - x^3 + 8*x^4]*(-Root[8 +
8*#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(
-Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 &
, 4, 0])) - (224*((x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 1, 0])*(x - Root[8
 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0])*(x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 4
, 0]) + (x - Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((-Root[8 + 8*
#1 - #1^3 + 8*#1^4 & , 1, 0] + Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 2, 0])*(x
- Root[8 + 8*#1 - #1^3 + 8*#1^4 & , 3, 0]))/((x - Root[8 + 8*#1 - #1^3 + 8*
#1^4 & , 2, 0])*(-Root[8 + 8*#1 - #1^3 + 8*#1^4...
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 4426, normalized size = 10.27

method	result	size
default	Expression too large to display	4426
risch	Expression too large to display	4426
elliptic	Expression too large to display	4426

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-x^3+8*x+8)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -16*(-17/10962-57/12992*x+191/58464*x^2-7/3132*x^3)/(8*x^4-x^3+8*x+8)^(1/2)
+421/12528*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8
*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-
_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_
Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3
```

+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+7/6264*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2)*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)*EllipticF(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))+(-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)+RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*EllipticPi(((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2),(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)),((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))*(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)))^(1/2))-7/7

```
83*((x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,ind
ex=3))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))+(RootOf(8*_Z^4-_Z^3+8*_Z+8,in
dex=1)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4))*((RootOf(8*_Z^4-_Z^3+8*_Z+8,inde
x=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index
=1))/(RootOf(8*_Z^4-_Z^3+8*_Z+8,index=4)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1)
)/(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=2))^2*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=3))/(RootOf(8*_Z^4-_Z^3+8*_Z
+8,index=3)-RootOf(8*_Z^4-_Z^3+8*_Z+8,index=1))/(x-RootOf(8*_Z^4-_Z^3+8*_Z+
8,index=2)))^(1/2)*((RootOf(8*_Z^4-_Z^3+8*_Z+8,index=2)-RootOf(8*_Z^4-_Z^3+
8*_Z+8,index=1))*(x-RootOf(8*_Z^4-_Z^3+8*_Z+8,i...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(8*x^4 - x^3 + 8*x + 8)/(64*x^8 - 16*x^7 + x^6 + 128*x^5 + 112
*x^4 - 16*x^3 + 64*x^2 + 128*x + 64), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(8x^{4} - x^{3} + 8x + 8\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-x**3+8*x+8)**(3/2),x)
[Out] Integral((8*x**4 - x**3 + 8*x + 8)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-x^3+8*x+8)^(3/2),x, algorithm="giac")
[Out] integrate((8*x^4 - x^3 + 8*x + 8)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(8\,x^4-x^3+8\,x+8)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2),x)
[Out] int(1/(8*x - x^3 + 8*x^4 + 8)^(3/2), x)
```

3.798 $\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

```
[Out] -1/10*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+
1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(
1/2))^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1
/2))^2)^(1/2)*5^(3/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2094, 6851, 1117}

$$-\frac{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\text{ArcTan}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\sqrt[4]{5}\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] -1/2*((Sqrt[5] + (1 + x^(-1))^2)*Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^
4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4
)], (5 + Sqrt[5])/10])/(5^(1/4)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
```

```
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+4x+4x^2+4x^4}}\,dx = -\left(16\text{Subst}\left(\int \frac{1}{(4-4x)^2\sqrt{\frac{1280-512x^2+256x^4}{(4-4x)^4}}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{1280-512x^2}}\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{\frac{5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4}{\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\right.}{2\sqrt[4]{5}\sqrt{1+4x+4x^2+4x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.39, size = 249, normalized size = 2.31

$$\frac{(2-i)\sqrt{-\frac{1}{10}+\frac{i}{5}}\sqrt{\frac{\left(2i+\sqrt{-1-2i}-\sqrt{-1+2i}\right)\left(-i+\sqrt{-1-2i}-2x\right)}{\left(-2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(i+\sqrt{-1-2i}+2x\right)}}\left(1+2x+2ix^2\right)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{\left(2i+\sqrt{-1-2i}+\sqrt{-1+2i}\right)\left(-i+\sqrt{-1+2i}+2x\right)}{\sqrt{-1+2i}\left(i+\sqrt{-1-2i}+2x\right)}}}{\sqrt{2}}\right)|\frac{1}{2}\left(5-\sqrt{5}\right)\right)}{\sqrt{\frac{(1+2i)\left((-1+i)+\sqrt{-1-2i}\right)\left(1+2x+2ix^2\right)}{\left(i+\sqrt{-1-2i}+2x\right)^2}}\sqrt{1+4x+4x^2+4x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[1 + 4*x + 4*x^2 + 4*x^4],x]
```

```
[Out] ((2 - I)*Sqrt[-1/10 + I/5]*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1 + 2*I])*(-
I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(I + S
```

```
qrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)*EllipticF[ArcSin[Sqrt[((2*I +
Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*
I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/(Sqrt[((1 + 2*I
)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] +
2*x)^2]*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 961, normalized size = 8.90

method	result	size
default	Expression too large to display	961
elliptic	Expression too large to display	961

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)
)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1
/2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1
,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z
^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))
/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1
))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{4x^4 + 4x^2 + 4x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**4 + 4*x**2 + 4*x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(4*x^4 + 4*x^2 + 4*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{4\,x^4 + 4\,x^2 + 4\,x + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2),x)
```

```
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(1/2), x)
```

3.799 $\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx$

Optimal. Leaf size=367

$$-\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}}+\frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}}+\frac{9\left(5-2\left(1+\frac{1}{x}\right)^2+\left(1+\frac{1}{x}\right)^4\right)\left(1+\frac{1}{x}\right)x^2}{10\left(\sqrt{5}+\left(1+\frac{1}{x}\right)^2\right)\sqrt{1+4x+4x^2+4x^4}}-\frac{9(}{}$$

```
[Out] -(3-(1+1/x)^2)*x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+1/10*(13-9*(1+1/x)^2)*(1+1/x)*
x^2/(4*x^4+4*x^2+4*x+1)^(1/2)+9/10*(5-2*(1+1/x)^2+(1+1/x)^4)*(1+1/x)*x^2/((
1+1/x)^2+5^(1/2))/(4*x^4+4*x^2+4*x+1)^(1/2)-9/10*x^2*(cos(2*arctan(1/5*(1+1
/x)*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticE(sin(2*a
rctan(1/5*(1+1/x)*5^(3/4))),1/10*(50+10*5^(1/2))^(1/2))*((1+1/x)^2+5^(1/2))
*((5-2*(1+1/x)^2+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x
^2+4*x+1)^(1/2)+3/20*x^2*(cos(2*arctan(1/5*(1+1/x)*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*(1+1/x)*5^(3/4)))*EllipticF(sin(2*arctan(1/5*(1+1/x)*5^(3/4))),
1/10*(50+10*5^(1/2))^(1/2))*(3-5^(1/2))*((1+1/x)^2+5^(1/2))*((5-2*(1+1/x)^2
+(1+1/x)^4)/((1+1/x)^2+5^(1/2))^2)^(1/2)*5^(1/4)/(4*x^4+4*x^2+4*x+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2094, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\frac{3\left(3-\sqrt{5}\right)\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2F\left(2\text{ArcTan}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{4\ 5^{3/4}\sqrt{4x^4+4x^2+4x+1}}-\frac{9\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{\frac{\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5}{\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)^2}}\,x^2E\left(2\text{ArcTan}\left(\frac{1+\frac{1}{x}}{\sqrt[4]{5}}\right)|\frac{1}{10}\left(5+\sqrt{5}\right)\right)}{2\ 5^{3/4}\sqrt{4x^4+4x^2+4x+1}}-\frac{\left(3-\left(\frac{1}{x}+1\right)^2\right)x^2}{\sqrt{4x^4+4x^2+4x+1}}+\frac{\left(13-9\left(\frac{1}{x}+1\right)^2\right)\left(\frac{1}{x}+1\right)x^2}{10\sqrt{4x^4+4x^2+4x+1}}+\frac{9\left(\left(\frac{1}{x}+1\right)^4-2\left(\frac{1}{x}+1\right)^2+5\right)\left(\frac{1}{x}+1\right)x^2}{10\left(\left(\frac{1}{x}+1\right)^2+\sqrt{5}\right)\sqrt{4x^4+4x^2+4x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] -(((3 - (1 + x^(-1))^2)*x^2)/Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + ((13 - 9*(1 +
 x^(-1))^2)*(1 + x^(-1))*x^2)/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) + (9*(5 -
2*(1 + x^(-1))^2 + (1 + x^(-1))^4)*(1 + x^(-1))*x^2)/(10*(Sqrt[5] + (1 + x^
(-1))^2)*Sqrt[1 + 4*x + 4*x^2 + 4*x^4]) - (9*(Sqrt[5] + (1 + x^(-1))^2)*Sqr
t[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*x^2
*EllipticE[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(2*5^(3/4)*Sq
rt[1 + 4*x + 4*x^2 + 4*x^4]) + (3*(3 - Sqrt[5])*(Sqrt[5] + (1 + x^(-1))^2)*
Sqrt[(5 - 2*(1 + x^(-1))^2 + (1 + x^(-1))^4)/(Sqrt[5] + (1 + x^(-1))^2)^2]*
x^2*EllipticF[2*ArcTan[(1 + x^(-1))/5^(1/4)], (5 + Sqrt[5])/10])/(4*5^(3/4)
*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
```

```
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
```

```
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{(1+4x+4x^2+4x^4)^{3/2}}\,dx = -\left(16\text{Subst}\left(\int \frac{1}{(4-4x)^2\left(\frac{1280-512x^2+256x^4}{(4-4x)^4}\right)^{3/2}}\,dx, x, 1+\frac{1}{x}\right)\right)$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{(4-4x)^4}{(1280-512x^2+256x\ldots}\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= -\frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)^4}\,x^2\right)\text{Subst}\left(\int \frac{x(-1024-1024x\ldots}{(1280-512x^2+256x\ldots}\right.}{\sqrt{1+4x+4x^2+4x^4}}$$

$$= \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(\sqrt{1280-512\left(1+\frac{1}{x}\right)^2+256\left(1+\frac{1}{x}\right)\ldots}\right.}{134217728\ldots}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} - \frac{\left(9\sqrt{1280-51\ldots}\right.}{\ldots}$$

$$= -\frac{\left(3-\left(1+\frac{1}{x}\right)^2\right)x^2}{\sqrt{1+4x+4x^2+4x^4}} + \frac{\left(13-9\left(1+\frac{1}{x}\right)^2\right)\left(1+\frac{1}{x}\right)x^2}{10\sqrt{1+4x+4x^2+4x^4}} + \frac{9\left(5-2(1+\ldots\right.}{10\left(\sqrt{5}+(1+\ldots\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.95, size = 602, normalized size = 1.64

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + 4*x + 4*x^2 + 4*x^4)^(-3/2),x]
```

```
[Out] (19 + 42*x - 16*x^2 + 36*x^3 + (9*(-I + Sqrt[-1 - 2*I] - 2*x)*(-I - Sqrt[-1
 + 2*I] + 2*x)*(-I + Sqrt[-1 + 2*I] + 2*x))/2 - ((9*I)*Sqrt[-2/5 + (4*I)/5]
*(-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(2*I + Sqrt[-1 - 2*I] + Sqrt[-1 +
 2*I])*((I + Sqrt[-1 - 2*I])/2 + x)^2*Sqrt[((2*I + Sqrt[-1 - 2*I] - Sqrt[-1
 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2
```

```
*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I
])*(1 + 2*x + (2*I)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2]*EllipticE[ArcSin[Sq
rt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[-1 + 2*I] + 2*x))/(S
qrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5 - Sqrt[5])/2])/((-1
 + I) + Sqrt[-1 - 2*I]) + ((6 - 3*I)*Sqrt[-2/5 + (4*I)/5]*Sqrt[((2*I + Sqrt
[-1 - 2*I] - Sqrt[-1 + 2*I])*(-I + Sqrt[-1 - 2*I] - 2*x))/((-2*I + Sqrt[-1
- 2*I] + Sqrt[-1 + 2*I])*(I + Sqrt[-1 - 2*I] + 2*x))]*(1 + 2*x + (2*I)*x^2)
*EllipticF[ArcSin[Sqrt[((2*I + Sqrt[-1 - 2*I] + Sqrt[-1 + 2*I])*(-I + Sqrt[
-1 + 2*I] + 2*x))/(Sqrt[-1 + 2*I]*(I + Sqrt[-1 - 2*I] + 2*x))]/Sqrt[2]], (5
 - Sqrt[5])/2])/Sqrt[((1 + 2*I)*((-1 + I) + Sqrt[-1 - 2*I])*(1 + 2*x + (2*I
)*x^2))/(I + Sqrt[-1 - 2*I] + 2*x)^2])/(10*Sqrt[1 + 4*x + 4*x^2 + 4*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 2564, normalized size = 6.99

method	result	size
default	Expression too large to display	2564
risch	Expression too large to display	2564
elliptic	Expression too large to display	2564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^4+4*x^2+4*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-9/20*x^3+1/5*x^2-21/40*x-19/80)/(4*x^4+4*x^2+4*x+1)^(1/2)+3/5*(-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*((Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-R
ootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
2)))^(1/2)*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x
-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=2)))^(1/2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_
Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2)*EllipticF(((RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*
_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-RootOf(
4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(-RootO
```

```
f(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(Root
Of(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/
2))-9/5*((x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4
*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4))+(-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*((RootOf(4*_Z^4+
4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z
^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_
Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)
*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))^2*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+
1,index=3))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)-RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2)*((RootOf(4*_Z^
4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*
_Z^4+4*_Z^2+4*_Z+1,index=4))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/
2)*((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=1)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=
4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)*RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)^2)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,inde
x=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,ind
ex=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))*EllipticF(((RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*
_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4
*_Z^2+4*_Z+1,index=1))/(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((Ro
otOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-
RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/
(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1)
)/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4
)))^(1/2))+(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z
+1,index=1))*EllipticE(((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=2))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4
*_Z^4+4*_Z^2+4*_Z+1,index=4)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(x-RootO
f(4*_Z^4+4*_Z^2+4*_Z+1,index=2)))^(1/2),((RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index
=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,ind
ex=4)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,i
ndex=3)+RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))/(RootOf(4*_Z^4+4*_Z^2+4*_Z+1,
index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(1/2))/(RootOf(4*_Z^4+4*_Z^
2+4*_Z+1,index=2)-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=1))))/((x-RootOf(4*_Z^4
+4*_Z^2+4*_Z+1,index=1))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=2))*(x-RootOf
(4*_Z^4+4*_Z^2+4*_Z+1,index=3))*(x-RootOf(4*_Z^4+4*_Z^2+4*_Z+1,index=4)))^(
1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(4*x^4 + 4*x^2 + 4*x + 1)/(16*x^8 + 32*x^6 + 32*x^5 + 24*x^4 +
 32*x^3 + 24*x^2 + 8*x + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(4x^{4} + 4x^{2} + 4x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**4+4*x**2+4*x+1)**(3/2),x)
```

```
[Out] Integral((4*x**4 + 4*x**2 + 4*x + 1)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^4+4*x^2+4*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^4 + 4*x^2 + 4*x + 1)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(4\,x^4+4\,x^2+4\,x+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2),x)
```

```
[Out] int(1/(4*x + 4*x^2 + 4*x^4 + 1)^(3/2), x)
```

3.800 $\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2F\left(2\tan^{-1}\left(\frac{4+3x}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\,\sqrt{8+24x+8x^2-15x^3+8x^4}}$$

```
[Out] -1/4136*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan
(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)
/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*
(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(3/4)/(8*x^4-15*x^3
+8*x^2+24*x+8)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2094, 12, 6851, 1117}

$$-\frac{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2F\left(2\text{ArcTan}\left(\frac{3x+4}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{8\sqrt[4]{517}\,\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] -1/8*((Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(517^(1/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8
*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{8+24x+8x^2-15x^3+8x^4}}\,dx = -\left(1024\text{Subst}\left(\int \frac{1}{2\sqrt{2}\,(24-32x)^2\sqrt{\frac{2117632-2490368x^2+10\ldots}{(24-32x)^4}}}\right.\right.$$

$$= -\left(\left(256\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\sqrt{\frac{2117632-2490368x^2+10\ldots}{(24-32x)^4}}}\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{S}\ldots}{\sqrt{8+24x+8x^2-\ldots}}$$

$$= -\frac{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)\sqrt{\frac{517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4}{\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right)^2}}\,x^2F\left(2\,\text{ta}\ldots\right.}{8\sqrt[4]{517}\,\sqrt{8+24x+8x^2-15x^3+8x\ldots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 10.67, size = 1148, normalized size = 9.11

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
& , 1, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 +
 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]], ((Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^
2 - 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4
 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))/((Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 3, 0])*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
, 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0]))]*(x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])^2*Sqrt[((Root[8 + 24*#1 +
 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
8*#1^4 & , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 3, 0]
))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24
*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1
^3 + 8*#1^4 & , 3, 0]))]*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*Sqrt[((x - Root
[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0])*(Root[8 + 24*#1 + 8*#1^2
- 15*#1^3 + 8*#1^4 & , 1, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 &
 , 2, 0])*(x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])*(Root[
8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 -
 15*#1^3 + 8*#1^4 & , 4, 0]))/((x - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#
1^4 & , 2, 0])^2*(Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1, 0] - Ro
ot[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 4, 0])^2)])/(Sqrt[8 + 24*x + 8
*x^2 - 15*x^3 + 8*x^4]*(-Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 1,
0] + Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0])*(Root[8 + 24*#1
+ 8*#1^2 - 15*#1^3 + 8*#1^4 & , 2, 0] - Root[8 + 24*#1 + 8*#1^2 - 15*#1^3 +
 8*#1^4 & , 4, 0]))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 1180, normalized size = 9.37

method	result	size
default	Expression too large to display	1180
elliptic	Expression too large to display	1180

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*
_Z^2+24*_Z+8,index=4))*((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-Root
```

```
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+
24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_
Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=2)))^(1/2)*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))^2*((Ro
otOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))/(RootOf(8
*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)*((RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+
8,index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)))^(1/2)/(RootOf(8*_Z
^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,ind
ex=2))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3
+8*_Z^2+24*_Z+8,index=1))*2^(1/2)/((x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=1))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,
index=4)))^(1/2)*EllipticF(((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-
RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2))*(x-RootOf(8*_Z^4-15*_Z^3+8*_
Z^2+24*_Z+8,index=1))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4)-RootOf
(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=1))/(x-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=2)))^(1/2),((RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-Root
Of(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=3))*(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24
*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=4))/(RootOf(8*_Z^
4-15*_Z^3+8*_Z^2+24*_Z+8,index=1)-RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,inde
x=3))/(RootOf(8*_Z^4-15*_Z^3+8*_Z^2+24*_Z+8,index=2)-RootOf(8*_Z^4-15*_Z^3+
8*_Z^2+24*_Z+8,index=4)))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{8x^4 - 15x^3 + 8x^2 + 24x + 8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(1/2),x)
[Out] Integral(1/sqrt(8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{8\,x^4 - 15\,x^3 + 8\,x^2 + 24\,x + 8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(1/2), x)
```

3.801 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=434

$$-\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}}+\frac{2455\left(517-38\left(3+\frac{4}{x}\right.\right.}{322608\left(\sqrt{517}+\left(3+\frac{4}{x}\right)^2\right.}$$

```
[Out] -1/208*(172-7*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/322608*(50
896-2455*(3+4/x)^2)*(3+4/x)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+2455/3226
08*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+517^(1/2))/(8*x^4-15
*x^3+8*x^2+24*x+8)^(1/2)-2455/322608*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3
/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticE(sin(2*a
rctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*((
3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^
(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/1290432*x^2*(cos(2*arct
an(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517^(3/4
)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(534578+196
46*517^(1/2))^(1/2))*(4910-203*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3
+4/x)^2+(3+4/x)^4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8
*x^2+24*x+8)^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 434, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1261, 650}

$$\frac{\left(4910-203\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2F\left(2\text{ArcTan}\left(\frac{3x+4}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{2496\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}-\frac{2455\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}\,x^2E\left(2\text{ArcTan}\left(\frac{3x+4}{\sqrt[4]{517}\,x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{624\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}-\frac{\left(172-7\left(\frac{4}{x}+3\right)^2\right)x^2}{208\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(50896-2455\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{322608\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{2455\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{322608\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] -1/208*((172 - 7*(3 + 4/x)^2)*x^2)/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]
+ ((50896 - 2455*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(322608*Sqrt[8 + 24*x + 8*x^2
- 15*x^3 + 8*x^4]) + (2455*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x
^2)/(322608*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^
4]) - (2455*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x
)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/
4)*x)], (517 + 19*Sqrt[517])/1034])/(624*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 -
15*x^3 + 8*x^4]) + ((4910 - 203*Sqrt[517])*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(
517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]*x^2*Ellipt
```

```
icF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034])/(2496*51
7^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{3/2}}\,dx = -\left(1024\text{Subst}\left(\int \frac{1}{16\sqrt{2}\,(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3}}\right.\right.$$

$$= -\left(\left(32\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{3/2}}\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{S}}{8\sqrt{8+24x+8x^2-15x}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)\text{S}}{8\sqrt{8+24x+8x^2-15x}}$$

$$= \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{322608\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{\left(\sqrt{2117632-2490368}\right.}{}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)}{322608\sqrt{8+24x+8x^2-}}$$

$$= -\frac{\left(172-7\left(3+\frac{4}{x}\right)^2\right)x^2}{208\sqrt{8+24x+8x^2-15x^3+8x^4}} + \frac{\left(50896-2455\left(3+\frac{4}{x}\right)^2\right)}{322608\sqrt{8+24x+8x^2-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 16.04, size = 6019, normalized size = 13.87

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-3/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 5421, normalized size = 12.49

method	result	size
default	Expression too large to display	5421
risch	Expression too large to display	5421
elliptic	Expression too large to display	5421

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="maxima")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(64*x^8 - 240*x^7 + 353*x^
6 + 144*x^5 - 528*x^4 + 144*x^3 + 704*x^2 + 384*x + 64), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(3/2),x)
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(3/2),x, algorithm="giac")
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2),x)
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(3/2), x)
```

3.802 $\int \frac{1}{\left(8+24x+8x^2-15x^3+8x^4\right)^{5/2}} dx$

Optimal. Leaf size=577

$$-\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}}-\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+8x}$$

```
[Out] -1/97344*(124415-6308*(3+4/x)^2)*x^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-1/62
4*(64489-1399*(3+4/x)^2)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x
^2+24*x+8)^(1/2)+1/78056941248*(18932921731-1086525994*(3+4/x)^2)*(3+4/x)*x
^2/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+1/483912*(11921698-359497*(3+4/x)^2)*(
3+4/x)*x^2/(517-38*(3+4/x)^2+(3+4/x)^4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)+5
43262997/39028470624*(517-38*(3+4/x)^2+(3+4/x)^4)*(3+4/x)*x^2/((3+4/x)^2+51
7^(1/2))/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)-543262997/39028470624*x^2*(cos(2
*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/517*(4+3*x)*517
^(3/4)/x))*EllipticE(sin(2*arctan(1/517*(4+3*x)*517^(3/4)/x)),1/1034*(53457
8+19646*517^(1/2))^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^
4)/((3+4/x)^2+517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/
2)+1/624455529984*x^2*(cos(2*arctan(1/517*(4+3*x)*517^(3/4)/x))^2)^(1/2)/co
s(2*arctan(1/517*(4+3*x)*517^(3/4)/x))*EllipticF(sin(2*arctan(1/517*(4+3*x)
*517^(3/4)/x)),1/1034*(534578+19646*517^(1/2))^(1/2))*(4346103976-175318963
*517^(1/2))*((3+4/x)^2+517^(1/2))*((517-38*(3+4/x)^2+(3+4/x)^4)/((3+4/x)^2+
517^(1/2))^2)^(1/2)*517^(1/4)/(8*x^4-15*x^3+8*x^2+24*x+8)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2094, 12, 6851, 1687, 1692, 1211, 1117, 1209, 1677, 1674, 650}

$$\frac{\left(4346103976-175318963\sqrt{517}\right)\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2F\left(2\text{ArcTan}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{1207844352\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}-\frac{543262997\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{\frac{\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517}{\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)^2}}x^2E\left(2\text{ArcTan}\left(\frac{3x+4}{\sqrt[4]{517}x}\right)|\frac{517+19\sqrt{517}}{1034}\right)}{75490272\ 517^{3/4}\sqrt{8x^4-15x^3+8x^2+24x+8}}-\frac{\left(124415-6308\left(\frac{4}{x}+3\right)^2\right)x^2}{97344\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(18932921731-1086525994\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{78056941248\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{543262997\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\left(\frac{4}{x}+3\right)x^2}{39028470624\left(\left(\frac{4}{x}+3\right)^2+\sqrt{517}\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}+\frac{\left(11921698-359497\left(\frac{4}{x}+3\right)^2\right)\left(\frac{4}{x}+3\right)x^2}{483912\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}-\frac{\left(64489-1399\left(\frac{4}{x}+3\right)^2\right)x^2}{624\left(\left(\frac{4}{x}+3\right)^4-38\left(\frac{4}{x}+3\right)^2+517\right)\sqrt{8x^4-15x^3+8x^2+24x+8}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] -1/97344*((124415 - 6308*(3 + 4/x)^2)*x^2)/Sqrt[8 + 24*x + 8*x^2 - 15*x^3 +
 8*x^4] - ((64489 - 1399*(3 + 4/x)^2)*x^2)/(624*(517 - 38*(3 + 4/x)^2 + (3
+ 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) + ((18932921731 - 108652
5994*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(78056941248*Sqrt[8 + 24*x + 8*x^2 - 15*x^
3 + 8*x^4]) + ((11921698 - 359497*(3 + 4/x)^2)*(3 + 4/x)*x^2)/(483912*(517
- 38*(3 + 4/x)^2 + (3 + 4/x)^4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) +
(543262997*(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)*(3 + 4/x)*x^2)/(39028470624
```

```
*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4]) - (5432
62997*(Sqrt[517] + (3 + 4/x)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(
Sqrt[517] + (3 + 4/x)^2)^2]*x^2*EllipticE[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)]
, (517 + 19*Sqrt[517])/1034])/(75490272*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 1
5*x^3 + 8*x^4]) + ((4346103976 - 175318963*Sqrt[517])*(Sqrt[517] + (3 + 4/x
)^2)*Sqrt[(517 - 38*(3 + 4/x)^2 + (3 + 4/x)^4)/(Sqrt[517] + (3 + 4/x)^2)^2]
*x^2*EllipticF[2*ArcTan[(4 + 3*x)/(517^(1/4)*x)], (517 + 19*Sqrt[517])/1034
])/(1207844352*517^(3/4)*Sqrt[8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1677

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{(8+24x+8x^2-15x^3+8x^4)^{5/2}}\,dx = -\left(1024\text{Subst}\left(\int \frac{1}{128\sqrt{2}\,(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right.}\right.\right.$$

$$= -\left(\left(4\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{(24-32x)^2\left(\frac{2117632-2490368x^2+1048576x^4}{(24-32x)^4}\right)^{5/2}}\right.\right.$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)}{64\sqrt{8+24x+8x^2-1}}$$

$$= -\frac{\left(\sqrt{2117632-2490368\left(\frac{3}{4}+\frac{1}{x}\right)^2+1048576\left(\frac{3}{4}+\frac{1}{x}\right)^4}\,x^2\right)}{64\sqrt{8-}}$$

$$= \frac{\left(11921698-359497\left(3+\frac{4}{x}\right)^2\right)\left(3+\frac{4}{x}\right)x^2}{483912\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3}}$$

$$= -\frac{\left(64489-1399\left(3+\frac{4}{x}\right)^2\right)x^2}{624\left(517-38\left(3+\frac{4}{x}\right)^2+\left(3+\frac{4}{x}\right)^4\right)\sqrt{8+24x+8x^2-15x^3+}}$$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{(6}{624\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

$$= -\frac{\left(124415-6308\left(3+\frac{4}{x}\right)^2\right)x^2}{97344\sqrt{8+24x+8x^2-15x^3+8x^4}} - \frac{(6}{624\left(517-38\left(3+\frac{4}{x}\right)^2\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 16.07, size = 6084, normalized size = 10.54

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(8 + 24*x + 8*x^2 - 15*x^3 + 8*x^4)^(-5/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 5477, normalized size = 9.49

method	result	size
risch	Expression too large to display	5441
default	Expression too large to display	5477
elliptic	Expression too large to display	5477

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)/(512*x^12 - 2880*x^11 + 69
36*x^10 - 4527*x^9 - 8808*x^8 + 16776*x^7 + 5528*x^6 - 17856*x^5 - 384*x^4
+ 20160*x^3 + 15360*x^2 + 4608*x + 512), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(8x^4 - 15x^3 + 8x^2 + 24x + 8)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x**4-15*x**3+8*x**2+24*x+8)**(5/2),x)
```

```
[Out] Integral((8*x**4 - 15*x**3 + 8*x**2 + 24*x + 8)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(8*x^4-15*x^3+8*x^2+24*x+8)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((8*x^4 - 15*x^3 + 8*x^2 + 24*x + 8)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(8\,x^4-15\,x^3+8\,x^2+24\,x+8\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2),x)
```

```
[Out] int(1/(24*x + 8*x^2 - 15*x^3 + 8*x^4 + 8)^(5/2), x)
```

3.803 $\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\tan^{-1}\left(\frac{6-x}{\sqrt[4]{613}\,x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3x^4}}$$

```
[Out] -1/7356*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1
/613*(6-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1
/1226*(751538+111566*613^(1/2))^(1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1
-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(3/4)/(3*x^4+15*x^
3-44*x^2-6*x+9)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2094, 12, 6851, 1110}

$$-\frac{\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\text{ArcTan}\left(\frac{6-x}{\sqrt[4]{613}\,x}\right)\middle|\frac{613+91\sqrt{613}}{1226}\right)}{12\sqrt[4]{613}\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] -1/12*(Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x
^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4
)*x)], (613 + 91*Sqrt[613])/1226])/(613^(1/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^
3 + 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
```

```
b/a, 0]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] &&  !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{9-6x-44x^2+15x^3+3x^4}}\,dx = -\left(1296\text{Subst}\left(\int \frac{1}{3(-6-36x)^2\sqrt{\frac{794448-8491392x^2+167961}{(-6-36x)^4}}}\right.\right.$$

$$= -\left(432\text{Subst}\left(\int \frac{1}{(-6-36x)^2\sqrt{\frac{794448-8491392x^2+1679616x}{(-6-36x)^4}}}\right.\right.$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right.}{\sqrt{9-6x-44x^2}}$$

$$= -\frac{\sqrt{\frac{613-182\left(1-\frac{6}{x}\right)^2+\left(-1+\frac{6}{x}\right)^4}{\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)^2}}\left(\sqrt{613}+\frac{(6-x)^2}{x^2}\right)x^2F\left(2\,\text{ta}\right.}{12\sqrt[4]{613}\sqrt{9-6x-44x^2+15x^3+3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

time = 10.10, size = 826, normalized size = 6.35

$$-\frac{2F\left(\sin^{-1}\left(\sqrt{\frac{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}{(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}}\right)\middle|\frac{(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,3])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}{(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,3])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}\right)\sqrt{\frac{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]}{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]}}(x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2])^2\sqrt{\frac{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,3]}{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]}}\sqrt{\frac{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4]}{x-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]}}}{\sqrt{(9-6x-44x^2+15x^3+3x^4)(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,1]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,3])(\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,2]-\text{Root}[3\#1^4+15\#1^3-44\#1^2-6\#1+9\&,4])}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4],x]
```

```
[Out] (-2*EllipticF[ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[
9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 +
 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
, 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x -
 Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 -
 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*
Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(x - Root[9
 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]])/Sqrt[(9 - 6*x - 44*x^2 +
15*x^3 + 3*x^4)*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0])]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 1182, normalized size = 9.09

method	result	size
default	Expression too large to display	1182
elliptic	Expression too large to display	1182

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-4
4*_Z^2-6*_Z+9,index=1))*((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/
(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z
^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*
_Z+9,index=2)))^(1/2)*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))^2*(
```

```
-(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))/(-RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^
2-6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)*(-(
x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4))/(-RootOf(3*_Z^4+15*_Z^3-44
*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-Roo
tOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1)))^(1/2)/(Root
Of(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_
Z+9,index=2))/(RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)-RootOf(3*_Z^4+
15*_Z^3-44*_Z^2-6*_Z+9,index=1))*3^(1/2)/((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2))*(x-RootO
f(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=4)))^(1/2)*EllipticF(((x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,
index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*
_Z^3-44*_Z^2-6*_Z+9,index=1))/(x-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index
=2))*(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-
44*_Z^2-6*_Z+9,index=2)))^(1/2),((RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,inde
x=2)-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3))*(-RootOf(3*_Z^4+15*_Z^3
-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=1))/(-R
ootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=3)+RootOf(3*_Z^4+15*_Z^3-44*_Z^2-
6*_Z+9,index=1))/(-RootOf(3*_Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=4)+RootOf(3*_
Z^4+15*_Z^3-44*_Z^2-6*_Z+9,index=2)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{3x^4 + 15x^3 - 44x^2 - 6x + 9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{3\,x^4+15\,x^3-44\,x^2-6\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2),x)
```

```
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(1/2), x)
```

3.804 $\int \frac{1}{\left(9-6x-44x^2+15x^3+3x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=444

$$-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}}+\frac{3722\left(613-182\left(1\right.\right.}{31728267\left(\sqrt{613}+\frac{(6-}{}\right.}$$

```
[Out] -1/51759*(176-23*(1-6/x)^2)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+1/3172826
7*(45401-3722*(1-6/x)^2)*(1-6/x)*x^2/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722
/31728267*(613-182*(1-6/x)^2+(-1+6/x)^4)*(1-6/x)*x^2/((6-x)^2/x^2+613^(1/2)
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)+3722/31728267*x^2*(cos(2*arctan(1/613*(
6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6-x)*613^(3/4)/x))*Elliptic
E(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(751538+111566*613^(1/2))^(
1/2))*((6-x)^2/x^2+613^(1/2))*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+
613^(1/2))^2)^(1/2)*613^(1/4)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)-1/126913068
*x^2*(cos(2*arctan(1/613*(6-x)*613^(3/4)/x))^2)^(1/2)/cos(2*arctan(1/613*(6
-x)*613^(3/4)/x))*EllipticF(sin(2*arctan(1/613*(6-x)*613^(3/4)/x)),1/1226*(
751538+111566*613^(1/2))^(1/2))*(7444-145*613^(1/2))*((6-x)^2/x^2+613^(1/2)
)*((613-182*(1-6/x)^2+(-1+6/x)^4)/((6-x)^2/x^2+613^(1/2))^2)^(1/2)*613^(1/4
)/(3*x^4+15*x^3-44*x^2-6*x+9)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {2094, 12, 6851, 1687, 1692, 1197, 1110, 1196, 1261, 650}

$$-\frac{\left(7444-145\sqrt{613}\right)\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2F\left(2\text{ArcTan}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{207036\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\sqrt{\frac{\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613}{\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)^2}}\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)x^2E\left(2\text{ArcTan}\left(\frac{6-x}{\sqrt[4]{613}x}\right)|\frac{613+91\sqrt{613}}{1226}\right)}{51759\ 613^{3/4}\sqrt{3x^4+15x^3-44x^2-6x+9}}-\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{3x^4+15x^3-44x^2-6x+9}}+\frac{3722\left(\left(\frac{6}{x}-1\right)^4-182\left(1-\frac{6}{x}\right)^2+613\right)\left(1-\frac{6}{x}\right)x^2}{31728267\left(\frac{(6-x)^2}{x^2}+\sqrt{613}\right)\sqrt{3x^4+15x^3-44x^2-6x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] -1/51759*((176 - 23*(1 - 6/x)^2)*x^2)/Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^
4] + ((45401 - 3722*(1 - 6/x)^2)*(1 - 6/x)*x^2)/(31728267*Sqrt[9 - 6*x - 44
*x^2 + 15*x^3 + 3*x^4]) + (3722*(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)*(1 -
 6/x)*x^2)/(31728267*(Sqrt[613] + (6 - x)^2/x^2)*Sqrt[9 - 6*x - 44*x^2 + 15
*x^3 + 3*x^4]) + (3722*Sqrt[(613 - 182*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[61
3] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6 - x)^2/x^2)*x^2*EllipticE[2*ArcTan[(
6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[613])/1226])/(51759*613^(3/4)*Sqrt[9
- 6*x - 44*x^2 + 15*x^3 + 3*x^4]) - ((7444 - 145*Sqrt[613])*Sqrt[(613 - 182
*(1 - 6/x)^2 + (-1 + 6/x)^4)/(Sqrt[613] + (6 - x)^2/x^2)^2]*(Sqrt[613] + (6
```

```
 - x)^2/x^2)*x^2*EllipticF[2*ArcTan[(6 - x)/(613^(1/4)*x)], (613 + 91*Sqrt[
613])/1226])/(207036*613^(3/4)*Sqrt[9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1110

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c/a, 0] && LtQ[
b/a, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2
- 4*a*c, 0] && GtQ[c/a, 0] && LtQ[b/a, 0]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0] && GtQ[c
/a, 0] && LtQ[b/a, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1687

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1692

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[x*(a + b*x^2 + c*x^
4)^(p + 1)*((a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)/(2*a*(p + 1)*(b
^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 2094

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Dist[-16*
a^2, Subst[Int[(1/(b - 4*a*x)^2)*(a*((-3*b^4 + 16*a*b^2*c - 64*a^2*b*d + 25
6*a^3*e - 32*a^2*(3*b^2 - 8*a*c)*x^2 + 256*a^4*x^4)/(b - 4*a*x)^4))^p, x],
x, b/(4*a) + 1/x], x] /; NeQ[a, 0] && NeQ[b, 0] && EqQ[b^3 - 4*a*b*c + 8*a^
2*d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && IntegerQ[2*p] && !IGtQ[p, 0]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\int \frac{1}{(9-6x-44x^2+15x^3+3x^4)^{3/2}}\,dx = -\left(1296\text{Subst}\left(\int \frac{1}{27(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx\right.\right.$$

$$= -\left(48\text{Subst}\left(\int \frac{1}{(-6-36x)^2\left(\frac{794448-8491392x^2+1679616x^4}{(-6-36x)^4}\right)^{3/2}}\,dx, x, \right.\right.$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right.}{9\sqrt{9-6x-44x^2+1}}$$

$$= -\frac{\left(\sqrt{794448-8491392\left(-\frac{1}{6}+\frac{1}{x}\right)^2+1679616\left(-\frac{1}{6}+\frac{1}{x}\right)^4}\,x^2\right.}{9\sqrt{9-6x-44x^2+1}}$$

$$= \frac{\left(45401-3722\left(1-\frac{6}{x}\right)^2\right)\left(1-\frac{6}{x}\right)x^2}{31728267\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(\sqrt{794448-8491392\right.}{}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right.\right.}{31728267\sqrt{9-6x-44x}}$$

$$= -\frac{\left(176-23\left(1-\frac{6}{x}\right)^2\right)x^2}{51759\sqrt{9-6x-44x^2+15x^3+3x^4}} + \frac{\left(45401-3722\left(1-\frac{6}{x}\right.\right.}{31728267\sqrt{9-6x-44x}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 15.79, size = 4974, normalized size = 11.20

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(9 - 6*x - 44*x^2 + 15*x^3 + 3*x^4)^(-3/2),x]
```

```
[Out] (2*(106926 + 592639*x - 232005*x^2 - 44664*x^3 + 81441*EllipticF[ArcSin[Sqr
t[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#
```

```
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^
3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], ((Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & ,
 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3,
 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 1, 0])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#
1^4 & , 2, 0])]*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])^
2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])/(x - Root
[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*Sqrt[(x - Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/((x - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1,
 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1
- 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3
+ 3*#1^4 & , 4, 0]))] - (223320*(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*
#1^4 & , 2, 0])^2*(EllipticPi[(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 &
 , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/(Root[9 -
6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 4, 0]), ArcSin[Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1
^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0
] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 -
 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15
*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4
, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] + Root[9 - 6
*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
0])))]*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#
1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0]) + EllipticF[ArcSin[Sqrt[((x - Roo
t[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2
 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4
& , 4, 0]))/((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]], -(((Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(
Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#
1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3
*#1^4 & , 1, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0] - Root[9 - 6*#1 - 44*#1^
```

```
2 + 15*#1^3 + 3*#1^4 & , 4, 0])))]*Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1
^4 & , 2, 0])*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0
])/(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])]*Sqrt[(x - Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0])/((x - Root[9 - 6*#1 - 4
4*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3
*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(Ro
ot[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^
2 + 15*#1^3 + 3*#1^4 & , 4, 0]))]*Sqrt[((x - Root[9 - 6*#1 - 44*#1^2 + 15*#
1^3 + 3*#1^4 & , 1, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2,
0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/((x - Root[9 -
6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 2, 0])*(Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 1, 0] - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0]))]*(-Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0] + Root[9 - 6*
#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]))/(-Root[9 - 6*#1 - 44*#1^2 + 15*
#1^3 + 3*#1^4 & , 2, 0] + Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4,
 0]) + 44664*((x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 1, 0])*(x
 - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 3, 0])*(x - Root[9 - 6*#1
 - 44*#1^2 + 15*#1^3 + 3*#1^4 & , 4, 0]) - ((x - Root[9 - 6*#1 - 44*#1^2 +
15*#1^3 + 3*#1^4 & , 2, 0])^2*Sqrt[(x - Root[9 - 6*#1 - 44*#1^2 + 15*#1^3 +
 3*#1^4 & , 3, 0])/(x - Root[9 - 6*#1 - 44*#1^2...
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 5427, normalized size = 12.22

method	result	size
default	Expression too large to display	5427
risch	Expression too large to display	5427
elliptic	Expression too large to display	5427

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x,method=_RETURNVERBOSE)
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)/(9*x^8 + 90*x^7 - 39*x^6 -
 1356*x^5 + 1810*x^4 + 798*x^3 - 756*x^2 - 108*x + 81), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(3x^4 + 15x^3 - 44x^2 - 6x + 9)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+15*x**3-44*x**2-6*x+9)**(3/2),x)
```

```
[Out] Integral((3*x**4 + 15*x**3 - 44*x**2 - 6*x + 9)**(-3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+15*x^3-44*x^2-6*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^4 + 15*x^3 - 44*x^2 - 6*x + 9)^(-3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(3\,x^4+15\,x^3-44\,x^2-6\,x+9\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2),x)
```

```
[Out] int(1/(15*x^3 - 44*x^2 - 6*x + 3*x^4 + 9)^(3/2), x)
```

3.805 $\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx$

Optimal. Leaf size=56

$$-4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)$$

```
[Out] -4*x-12*arcsin(-1/2+1/2*x)+21*ln(x)-9*ln(1+x)-24*arctanh(3^(1/2)*(1+x)^(1/2
)/(3-x)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6874, 36, 29, 31, 132, 55, 633, 222, 12, 95, 213}

$$12\text{ArcSin}\left(\frac{1-x}{2}\right)-4x+21\log(x)-9\log(x+1)-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt
[3 - x]] + 21*Log[x] - 9*Log[1 + x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}}\right)^2}{x}\,dx &= \int \left(-4+\frac{12}{x}+\frac{9}{x(1+x)}+\frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right)dx \\
&= -4x+12\log(x)+9\int \frac{1}{x(1+x)}\,dx+12\int \frac{\sqrt{3-x}}{x\sqrt{1+x}}\,dx \\
&= -4x+12\log(x)+9\int \frac{1}{x}\,dx-9\int \frac{1}{1+x}\,dx-12\int \frac{1}{\sqrt{3-x}\,\sqrt{1+x}}\,dx \\
&= -4x+21\log(x)-9\log(1+x)-12\int \frac{1}{\sqrt{3+2x-x^2}}\,dx+72\text{Subst}\Bigg(\int \\
&= -4x-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-9\log(1+x)+3\text{Subs} \\
&= -4x+12\sin^{-1}\left(\frac{1-x}{2}\right)-24\sqrt{3}\,\tanh^{-1}\left(\frac{\sqrt{3}\,\sqrt{1+x}}{\sqrt{3-x}}\right)+21\log(x)-
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 257 vs. 2(56) = 112.
time = 0.62, size = 257, normalized size = 4.59

$$-4-4x-48\tan^{-1}\left(\frac{\sqrt{1+x}}{-2+\sqrt{3-x}}\right)-42\log\left(-2+\sqrt{3-x}\right)-30\log(1+x)+21\log\left(-\left((-3+x)\sqrt{1+x}\right)+\sqrt{3}\left(-2+\sqrt{3-x}\right)(1+x)-2\sqrt{-((-3+x)(1+x))}\right)-12\sqrt{3}\log\left(-\left((-3+x)\sqrt{1+x}\right)+\sqrt{3}\left(-2+\sqrt{3-x}\right)(1+x)-2\sqrt{-((-3+x)(1+x))}\right)+21\log\left((-3+x)\sqrt{1+x}+\sqrt{3}\left(-2+\sqrt{3-x}\right)(1+x)+2\sqrt{-((-3+x)(1+x))}\right)+12\sqrt{3}\log\left((-3+x)\sqrt{1+x}+\sqrt{3}\left(-2+\sqrt{3-x}\right)(1+x)+2\sqrt{-((-3+x)(1+x))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]
```

```
[Out] -4 - 4*x - 48*ArcTan[Sqrt[1 + x]/(-2 + Sqrt[3 - x])] - 42*Log[-2 + Sqrt[3 -
 x]] - 30*Log[1 + x] + 21*Log[-((-3 + x)*Sqrt[1 + x]) + Sqrt[3]*(-2 + Sqrt[
3 - x])*(1 + x) - 2*Sqrt[-((-3 + x)*(1 + x))]] - 12*Sqrt[3]*Log[-((-3 + x)*
Sqrt[1 + x]) + Sqrt[3]*(-2 + Sqrt[3 - x])*(1 + x) - 2*Sqrt[-((-3 + x)*(1 +
x))]] + 21*Log[(-3 + x)*Sqrt[1 + x] + Sqrt[3]*(-2 + Sqrt[3 - x])*(1 + x) +
2*Sqrt[-((-3 + x)*(1 + x))]] + 12*Sqrt[3]*Log[(-3 + x)*Sqrt[1 + x] + Sqrt[3
]*(-2 + Sqrt[3 - x])*(1 + x) + 2*Sqrt[-((-3 + x)*(1 + x))]]
```

Maple [A]
time = 0.47, size = 76, normalized size = 1.36

method	result

default	$21\ln(x)-9\ln(1+x)+\frac{12\sqrt{-x+3}\,\sqrt{1+x}\,\left(-\arcsin\left(-\frac{1}{2}+\frac{x}{2}\right)-\sqrt{3}\,\operatorname{arctanh}\left(\frac{(3+x)\sqrt{3}}{3\sqrt{-x^2+2x+3}}\right)\right)}{\sqrt{-x^2+2x+3}}$ –

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*(-x+3)^(1/2)+3/(1+x)^(1/2))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 21*ln(x)-9*ln(1+x)+12*(-x+3)^(1/2)*(1+x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-arcsin(
-1/2+1/2*x)-3^(1/2)*arctanh(1/3*(3+x)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-4*x
```

Maxima [A]
time = 0.50, size = 57, normalized size = 1.02

$$-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|}+\frac{6}{|x|}+2\right)-4x+12\arcsin\left(-\frac{1}{2}x+\frac{1}{2}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")
```

```
[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x
 + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)
```

Fricas [A]
time = 0.36, size = 81, normalized size = 1.45

$$6\sqrt{3}\log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right)-4x+12\arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right)-9\log(x+1)+21\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")
```

```
[Out] 6*sqrt(3)*log(-(sqrt(3)*(x + 3)*sqrt(x + 1)*sqrt(-x + 3) + x^2 - 6*x - 9)/x
^2) - 4*x + 12*arctan(sqrt(x + 1)*(x - 1)*sqrt(-x + 3)/(x^2 - 2*x - 3)) - 9
*log(x + 1) + 21*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(2\sqrt{3-x}\,\sqrt{x+1}+3\right)^2}{x(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)
```

```
[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-8,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [5.38357630698]Warning, choosing root of [1,0,-8,0,%%%
{4,[2]%%%}+%%%{-
```

Mupad [**B**]
time = 7.91, size = 158, normalized size = 2.82

$$48\,\mathrm{atan}\left(\frac{\sqrt{3-x}-4\,\sqrt{3}+3\,\sqrt{3}\,\sqrt{x+1}}{\sqrt{x+1}-3\,\sqrt{3}\,\sqrt{3-x}+8}\right)-9\,\ln\left(x+1\right)-4\,x+21\,\ln\left(x\right)+12\,\sqrt{3}\,\ln\left(\frac{6\,x-12\,\sqrt{x+1}+4\,\sqrt{3}\,\sqrt{3-x}+2\,\sqrt{3}\,\sqrt{x+1}\,\sqrt{3-x}-6}{3\,x+6\,\sqrt{3}\,\sqrt{3-x}-18}\right)-12\,\sqrt{3}\,\ln\left(\frac{\sqrt{x+1}-1}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)
```

```
[Out] 48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/2
) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3^(
1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(x +
 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12*3^(
1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))
```

3.806 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=65

$$-\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)$$

```
[Out] -1/x-x-1/2*arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x+1/2
*x*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6874, 283, 221, 1605, 196, 45, 201}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSi
nh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}+\frac{x^2}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)+\int \left(-1+\sqrt{1+x^2}\right)dx-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x}}{x^2}\right.\\
&= -\frac{1}{x}-x+\int \sqrt{1+x^2}\,dx-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}-x+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\Big(\\
&= -\frac{1}{x}-x+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}+\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}$$

Mathematica [A]

time = 0.18, size = 49, normalized size = 0.75

$$-\frac{1}{x}-x+\left(1+\frac{1}{x}+\frac{x}{2}\right)\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) - x + (1 + x^(-1) + x/2)*Sqrt[1 + x^2] - ArcSinh[x]/2 - Log[1 + Sqr
t[1 + x^2]]
```

Maple [A]
time = 0.11, size = 56, normalized size = 0.86

method	result
default	$-x-\frac{1}{x}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x\sqrt{x^2+1}}{2}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}$
meijerg	$-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},1\right],\left[\frac{3}{2},2\right],-x^2\right)}{2}+\frac{x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1,\frac{3}{2}\right],\left[2,\frac{5}{2}\right],-x^2\right)}{6}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\sqrt{\ }\right.}{4\sqrt{\pi}}$
trager	$-\frac{(-1+x)^2}{x}+\frac{(x^2+2x+2)\sqrt{x^2+1}}{2x}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -x-1/x-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(
1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
+ 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

Fricas [A]
time = 0.34, size = 84, normalized size = 1.29

$$-\frac{2\,x^2+2\,x\log(x)+2\,x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}\right)-2\,x\log\left(-x+\sqrt{x^2+1}-1\right)-(x^2+2\,x+2)\sqrt{x^2+1}-2\,x+2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sqr
t(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2 +
1) - 2*x + 2)/x
```

Sympy [A]
time = 2.89, size = 63, normalized size = 0.97

$$\frac{x\sqrt{x^2+1}}{2} - x + \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1}+1\right) - \frac{\operatorname{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
+ 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

Giac [A]
time = 3.27, size = 89, normalized size = 1.37

$$\frac{1}{2}\sqrt{x^2+1}\,(x+2) - x - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right) - \log\left(|x|\right) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*l
og(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + l
og(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B]
time = 0.04, size = 55, normalized size = 0.85

$$\left(\frac{x}{2}+1\right)\sqrt{x^2+1} - \frac{\operatorname{asinh}(x)}{2} - \ln(x) - x + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \operatorname{atan}\left(\sqrt{x^2+1}\;1\mathrm{i}\right)\;1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1)
^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

3.807 $\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{12}\left(6x^2+2x^3+\left(4-3x-2x^2\right)\sqrt{1+x^2}-3\sinh^{-1}(x)-6\log\left(1+\sqrt{1+x^2}\right)\right)$$

```
[Out] 1/2*x^2+1/6*x^3-1/4*arcsinh(x)-1/2*ln(1+(x^2+1)^(1/2))+1/12*(-2*x^2-3*x+4)*
(x^2+1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6874, 2142, 907, 201, 221, 267}

$$\frac{x^3}{6}+\frac{x^2}{2}-\frac{1}{4}\sqrt{x^2+1}\,x-\frac{1}{6}\left(x^2+1\right)^{3/2}+\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-\log\left(\sqrt{x^2+1}+x+1\right)+\frac{x}{2}-\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + S
qrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt
[1 + x^2]]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 2142

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[(g + h*x^n)^p*((d^2 + a*f^
2 - 2*d*x + x^2)/(d - x)^2), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+x+\sqrt{1+x^2}}+\frac{x}{1+x+\sqrt{1+x^2}}+\frac{x^2}{1+x+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+x+\sqrt{1+x^2}}\,dx+\int \frac{x^2}{1+x+\sqrt{1+x^2}}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{2-2x+x^2}{(1-x)^2x}\,dx,x,1+x+\sqrt{1+x^2}\right)\right)+\int \left(\frac{1}{2}+\frac{x}{2}-\frac{\sqrt{1+x^2}}{2}\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{2}\int \sqrt{1+x^2}\,dx-\frac{1}{2}\int x\sqrt{1+x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\right.\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{2}\log\left(x+\right.\\
&= \frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{6}-\frac{1}{4}x\sqrt{1+x^2}-\frac{1}{6}\left(1+x^2\right)^{3/2}+\frac{1}{2\left(x+\sqrt{1+x^2}\right)}-\frac{1}{4}\sinh^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 1.00

$$\frac{1}{12}\left(6x^2+2x^3+\left(4-3x-2x^2\right)\sqrt{1+x^2}-3\sinh^{-1}(x)-6\log\left(1+\sqrt{1+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]),x]
```

```
[Out] (6*x^2 + 2*x^3 + (4 - 3*x - 2*x^2)*Sqrt[1 + x^2] - 3*ArcSinh[x] - 6*Log[1 +
 Sqrt[1 + x^2]])/12
```

Maple [A]
time = 0.07, size = 58, normalized size = 1.09

method	result	s
default	$\frac{x^2}{2}-\frac{\ln(x)}{2}+\frac{x^3}{6}-\frac{x\sqrt{x^2+1}}{4}-\frac{\operatorname{arcsinh}(x)}{4}-\frac{(x^2+1)^{\frac{3}{2}}}{6}+\frac{\sqrt{x^2+1}}{2}-\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2}$	5
trager	$\frac{(x^2+4x+4)(-1+x)}{6}+\frac{\left(-\frac{1}{3}x^2-\frac{1}{2}x+\frac{2}{3}\right)\sqrt{x^2+1}}{2}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{4}$	8

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*x*(x^2+1)^(1/2)-1/4*arcsinh(x)-1/6*(x^2+1)^(3
/2)+1/2*(x^2+1)^(1/2)-1/2*arctanh(1/(x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x
^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^
2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)
```

Fricas [A]
time = 0.36, size = 78, normalized size = 1.47

$$\frac{1}{6}x^3+\frac{1}{2}x^2-\frac{1}{12}\left(2x^2+3x-4\right)\sqrt{x^2+1}-\frac{1}{2}\log(x)-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}+1\right)+\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x - 1}{x + \sqrt{x^2 + 1} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)
```

Giac [A]
time = 4.21, size = 80, normalized size = 1.51

$$\frac{1}{6}\,x^3 + \frac{1}{2}\,x^2 - \frac{1}{12}\,((2\,x + 3)x - 4)\sqrt{x^2 + 1} + \frac{1}{4}\,\log\left(-x + \sqrt{x^2 + 1}\right) - \frac{1}{2}\,\log\left(|x|\right) - \frac{1}{2}\,\log\left(\left|-x + \sqrt{x^2 + 1} + 1\right|\right) + \frac{1}{2}\,\log\left(\left|-x + \sqrt{x^2 + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqr
t(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*
log(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B]
time = 0.04, size = 52, normalized size = 0.98

$$\frac{x^2}{2} - \frac{\ln(x)}{2} - \sqrt{x^2 + 1}\,\left(\frac{x^2}{6} + \frac{x}{4} - \frac{1}{3}\right) - \frac{\mathrm{asinh}(x)}{4} + \frac{x^3}{6} + \frac{\mathrm{atan}\left(\sqrt{x^2 + 1}\;1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)
```

```
[Out] (atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*(
x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6
```

3.808 $\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx$

Optimal. Leaf size=14

$$2\sqrt{-1+x}+2\log(x)$$

[Out] 2*ln(x)+2*(-1+x)^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6820}

$$2\sqrt{x-1}+2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}\,x}\,dx = \int \left(\frac{1}{\sqrt{-1+x}}+\frac{2}{x}\right)dx$$
$$= 2\sqrt{-1+x}+2\log(x)$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 1.00

$$2\sqrt{-1+x}+2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x),x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

Maple [A]
time = 0.51, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$2\ln(x)+2\sqrt{-1+x}$	13
default	$2\ln(x)+2\sqrt{-1+x}$	13
trager	$2\sqrt{-1+x}-2\ln\left(\frac{1}{x}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*ln(x)+2*(-1+x)^(1/2)
```

Maxima [A]
time = 0.51, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Fricas [A]
time = 0.38, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Sympy [A]
time = 0.05, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Giac [A]
time = 2.79, size = 12, normalized size = 0.86

$$2\sqrt{x-1}+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x - 1) + 2*log(x)
```

Mupad [B]
time = 3.39, size = 12, normalized size = 0.86

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)
```

```
[Out] 2*log(x) + 2*(x - 1)^(1/2)
```

3.809 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx$

Optimal. Leaf size=61

$$a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}$$

```
[Out] a^2*x+4/3*a*c*x^(3/2)+6/5*a*b*x^(5/3)+1/2*c^2*x^2+12/13*b*c*x^(13/6)+3/7*b^
2*x^(7/3)
```

Rubi [A]
time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6873, 6874}

$$a^2x+\frac{6}{5}abx^{5/3}+\frac{4}{3}acx^{3/2}+\frac{3}{7}b^2x^{7/3}+\frac{12}{13}bcx^{13/6}+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13
/6))/13 + (3*b^2*x^(7/3))/7
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+c\sqrt{x}+bx^{2/3}\right)^2\,dx &= 6\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^2\,dx,x,\sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^2\,dx,x,\sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(a^2x^5+2acx^8+2abx^9+c^2x^{11}+2bcx^{12}+b^2x^{13}\right)\,dx,x,\sqrt[6]{x}\right)\\
&= a^2x+\frac{4}{3}acx^{3/2}+\frac{6}{5}abx^{5/3}+\frac{c^2x^2}{2}+\frac{12}{13}bcx^{13/6}+\frac{3}{7}b^2x^{7/3}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.92

$$\frac{2730a^2x + 3640acx^{3/2} + 3276abx^{5/3} + 1365c^2x^2 + 2520bcx^{13/6} + 1170b^2x^{7/3}}{2730}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]
```

```
[Out] (2730*a^2*x + 3640*a*c*x^(3/2) + 3276*a*b*x^(5/3) + 1365*c^2*x^2 + 2520*b*c
*x^(13/6) + 1170*b^2*x^(7/3))/2730
```

Maple [A]
time = 0.05, size = 46, normalized size = 0.75

method	result	size
derivativedivides	$a^{2} x +\frac{4 a c \,x^{\frac{3}{2}}}{3}+\frac{6 a b \,x^{\frac{5}{3}}}{5}+\frac{c^{2} x^{2}}{2}+\frac{12 b c \,x^{\frac{13}{6}}}{13}+\frac{3 b^{2} x^{\frac{7}{3}}}{7}$	44
default	$\frac{c^{2} x^{2}}{2}+2 c \left(\frac{6 b \,x^{\frac{13}{6}}}{13}+\frac{2 a \,x^{\frac{3}{2}}}{3}\right)+a^{2} x +\frac{3 b^{2} x^{\frac{7}{3}}}{7}+\frac{6 a b \,x^{\frac{5}{3}}}{5}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+a^2*x+3/7*b^2*x^(7/3)+6/5*a
*b*x^(5/3)
```

Maxima [A]
time = 0.29, size = 45, normalized size = 0.74

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + a^2x + \frac{2}{15}\left(9\,bx^{\frac{5}{3}} + 10\,cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5
/3) + 10*c*x^(3/2))*a
```

Fricas [A]
time = 0.33, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

Sympy [A]
time = 1.23, size = 60, normalized size = 0.98

$$a^2x + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{3b^2x^{\frac{7}{3}}}{7} + \frac{12bcx^{\frac{13}{6}}}{13} + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)
```

```
[Out] a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x
**(13/6)/13 + c**2*x**2/2
```

Giac [A]
time = 1.80, size = 43, normalized size = 0.70

$$\frac{3}{7}\,b^2x^{\frac{7}{3}} + \frac{12}{13}\,bcx^{\frac{13}{6}} + \frac{1}{2}\,c^2x^2 + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{4}{3}\,acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*
a*c*x^(3/2) + a^2*x
```

Mupad [B]
time = 3.36, size = 43, normalized size = 0.70

$$a^2\,x + \frac{3\,b^2\,x^{7/3}}{7} + \frac{c^2\,x^2}{2} + \frac{6\,a\,b\,x^{5/3}}{5} + \frac{4\,a\,c\,x^{3/2}}{3} + \frac{12\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^2,x)
```

```
[Out] a^2*x + (3*b^2*x^(7/3))/7 + (c^2*x^2)/2 + (6*a*b*x^(5/3))/5 + (4*a*c*x^(3/2
))/3 + (12*b*c*x^(13/6))/13
```

3.810 $\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx$

Optimal. Leaf size=114

$$a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}bc^2x^{8/3}+\frac{18}{17}b^2cx^{17/6}+\frac{b^3x^3}{3}$$

```
[Out] a^3*x+2*a^2*c*x^(3/2)+9/5*a^2*b*x^(5/3)+3/2*a*c^2*x^2+36/13*a*b*c*x^(13/6)+
9/7*a*b^2*x^(7/3)+2/5*c^3*x^(5/2)+9/8*b*c^2*x^(8/3)+18/17*b^2*c*x^(17/6)+1/
3*b^3*x^3
```

Rubi [A]
time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6873, 6874}

$$a^3x+\frac{9}{5}a^2bx^{5/3}+2a^2cx^{3/2}+\frac{9}{7}ab^2x^{7/3}+\frac{36}{13}abcx^{13/6}+\frac{3}{2}ac^2x^2+\frac{b^3x^3}{3}+\frac{18}{17}b^2cx^{17/6}+\frac{9}{8}bc^2x^{8/3}+\frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c
*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))
/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3
```

Rule 6873

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \left(a+c\sqrt{x}+bx^{2/3}\right)^3 \, dx &= 6\text{Subst}\left(\int x^5\left(a+x^3(c+bx)\right)^3 \, dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int x^5\left(a+cx^3+bx^4\right)^3 \, dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(a^3x^5+3a^2cx^8+3a^2bx^9+3ac^2x^{11}+6abcx^{12}+3ab^2x^{13}+c^3x^{14}+\right.\right.\\
&= a^3x+2a^2cx^{3/2}+\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}ac^2x^2+\frac{36}{13}abcx^{13/6}+\frac{9}{7}ab^2x^{7/3}+\frac{2}{5}c^3x^{5/2}+\frac{9}{8}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 0.90

$$\frac{185640a^3x+371280a^2cx^{3/2}+334152a^2bx^{5/3}+278460ac^2x^2+514080abcx^{13/6}+238680ab^2x^{7/3}+74256c^3x^{5/2}+208845bc^2x^{8/3}+196560b^2cx^{17/6}+61880b^3x^3}{185640}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]
```

```
[Out] (185640*a^3*x + 371280*a^2*c*x^(3/2) + 334152*a^2*b*x^(5/3) + 278460*a*c^2*
x^2 + 514080*a*b*c*x^(13/6) + 238680*a*b^2*x^(7/3) + 74256*c^3*x^(5/2) + 20
8845*b*c^2*x^(8/3) + 196560*b^2*c*x^(17/6) + 61880*b^3*x^3)/185640
```

Maple [A]
time = 0.08, size = 86, normalized size = 0.75

method	result
derivativedivides	$a^3x+2a^2c\,x^{\frac{3}{2}}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+\frac{3a\,c^2x^2}{2}+\frac{36abc\,x^{\frac{13}{6}}}{13}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}+\frac{2c^3x^{\frac{5}{2}}}{5}+\frac{9b\,c^2x^{\frac{8}{3}}}{8}+\frac{18b^2c\,x^{\frac{17}{6}}}{17}+\frac{b^3x^3}{3}$
default	$\frac{2c^3x^{\frac{5}{2}}}{5}+3c^2\left(\frac{3b\,x^{\frac{8}{3}}}{8}+\frac{a\,x^2}{2}\right)+3c\left(\frac{6b^2x^{\frac{17}{6}}}{17}+\frac{12ab\,x^{\frac{13}{6}}}{13}+\frac{2a^2x^{\frac{3}{2}}}{3}\right)+a^3x+\frac{b^3x^3}{3}+\frac{9a^2b\,x^{\frac{5}{3}}}{5}+\frac{9a\,b^2x^{\frac{7}{3}}}{7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*b*x^(8/3)+1/2*a*x^2)+3*c*(6/17*b^2*x^(17/6)+12/1
3*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a^2*b*x^(5/3)+9/7*a*b
^2*x^(7/3)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.75

$$\frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{8}\,bc^2x^{\frac{8}{3}}+\frac{2}{5}\,c^3x^{\frac{5}{2}}+a^3x+\frac{1}{5}\left(9\,bx^{\frac{5}{3}}+10\,cx^{\frac{3}{2}}\right)a^2+\frac{3}{182}\left(78\,b^2x^{\frac{7}{3}}+168\,bcx^{\frac{13}{6}}+91\,c^2x^2\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*
b*c*x^(13/6) + 91*c^2*x^2)*a
```

Fricas [A]
time = 0.35, size = 91, normalized size = 0.80

$$\frac{1}{3}\,b^3x^3+\frac{18}{17}\,b^2cx^{\frac{17}{6}}+\frac{9}{7}\,ab^2x^{\frac{7}{3}}+\frac{36}{13}\,abcx^{\frac{13}{6}}+\frac{3}{2}\,ac^2x^2+a^3x+\frac{9}{40}\left(5\,bc^2x^2+8\,a^2bx\right)x^{\frac{2}{3}}+\frac{2}{5}\left(c^3x^2+5\,a^2cx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/
6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(
c^3*x^2 + 5*a^2*c*x)*sqrt(x)
```

Sympy [A]
time = 1.85, size = 116, normalized size = 1.02

$$a^3x + \frac{9a^2bx^{\frac{5}{3}}}{5} + 2a^2cx^{\frac{3}{2}} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{36abcx^{\frac{13}{6}}}{13} + \frac{3ac^2x^2}{2} + \frac{b^3x^3}{3} + \frac{18b^2cx^{\frac{17}{6}}}{17} + \frac{9bc^2x^{\frac{8}{3}}}{8} + \frac{2c^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)
```

```
[Out] a**3*x + 9*a**2*b*x**(5/3)/5 + 2*a**2*c*x**(3/2) + 9*a*b**2*x**(7/3)/7 + 36
*a*b*c*x**(13/6)/13 + 3*a*c**2*x**2/2 + b**3*x**3/3 + 18*b**2*c*x**(17/6)/1
7 + 9*b*c**2*x**(8/3)/8 + 2*c**3*x**(5/2)/5
```

Giac [A]
time = 1.70, size = 84, normalized size = 0.74

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{8}bc^2x^{\frac{8}{3}} + \frac{2}{5}c^3x^{\frac{5}{2}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{36}{13}abcx^{\frac{13}{6}} + \frac{3}{2}ac^2x^2 + \frac{9}{5}a^2bx^{\frac{5}{3}} + 2a^2cx^{\frac{3}{2}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) +
9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3
) + 2*a^2*c*x^(3/2) + a^3*x
```

Mupad [B]
time = 0.06, size = 84, normalized size = 0.74

$$a^3\,x+\frac{b^3\,x^3}{3}+\frac{2\,c^3\,x^{5/2}}{5}+\frac{9\,a^2\,b\,x^{5/3}}{5}+\frac{9\,a\,b^2\,x^{7/3}}{7}+\frac{3\,a\,c^2\,x^2}{2}+2\,a^2\,c\,x^{3/2}+\frac{9\,b\,c^2\,x^{8/3}}{8}+\frac{18\,b^2\,c\,x^{17/6}}{17}+\frac{36\,a\,b\,c\,x^{13/6}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(2/3) + c*x^(1/2))^3,x)
```

```
[Out] a^3*x + (b^3*x^3)/3 + (2*c^3*x^(5/2))/5 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(7/3))/7 + (3*a*c^2*x^2)/2 + 2*a^2*c*x^(3/2) + (9*b*c^2*x^(8/3))/8 + (18*b^
2*c*x^(17/6))/17 + (36*a*b*c*x^(13/6))/13
```

3.811 $\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\, x^3}\, dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A]
time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {528, 457, 81, 65, 214}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rubi steps

$$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx = \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx$$
$$= -\left(\frac{1}{2}\mathrm{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)$$
$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\mathrm{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b}$$
$$= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Mathematica [A]
time = 0.11, size = 103, normalized size = 1.78

$$\frac{\sqrt{a-b}\,(b+ax^2-bx^2)-bx\sqrt{b+ax^2-bx^2}\,\log\left(-\sqrt{a-b}\,x+\sqrt{b+(a-b)x^2}\right)}{\sqrt{a-b}\,b\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) - b*x*Sqrt[b + a*x^2 - b*x^2]*Log[-(Sqrt[a
 - b]*x) + Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))]
*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.
time = 0.05, size = 102, normalized size = 1.76

method	result	size

default	$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)bx+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x^{2}\sqrt{a-b}\,b}$	102
risch	$\frac{a\,x^{2}-b\,x^{2}+b}{b\,x^{2}\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}}+\frac{\ln\left(x\sqrt{a-b}+\sqrt{x^{2}\left(a-b\right)+b}\right)\sqrt{a\,x^{2}-b\,x^{2}+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.38, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\,(a-b)x^{2}-2\,\sqrt{a-b}\,x^{2}\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{2\,(ab-b^{2})},\ \frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\,x^{2}\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{(a-b)x^{2}+b}\right)+(a-b)\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{ab-b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A]
time = 1.58, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.
time = 2.57, size = 92, normalized size = 1.59

$$-\frac{\log\left(\left(\sqrt{a-b}\,x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}\,x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
- 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B]
time = 4.08, size = 46, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)),x)
```

```
[Out] atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)^
(1/2)/b
```

3.812 $\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^3}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

```
[Out] arctanh((a-b*(1-1/x^2))^(1/2)/(a-b)^(1/2))/(a-b)^(1/2)+(a-b*(1-1/x^2))^(1/2
)/b
```

Rubi [A]
time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2003, 528, 457, 81, 65, 214}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b}+\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/
Sqrt[a - b]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
```

```
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 2003

```
Int[(Pq_)*(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*Pq*Expand
ToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x]
 &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^3}\,dx &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}\,x^3}\,dx \\
&= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}\,x}\,dx \\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right)\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}}\,dx, x, \frac{1}{x^2}\right) \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
&= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 103, normalized size = 1.78

$$\frac{\sqrt{a-b}\,(b+ax^2-bx^2)-bx\sqrt{b+ax^2-bx^2}\,\log\left(-\sqrt{a-b}\,x+\sqrt{b+(a-b)x^2}\right)}{\sqrt{a-b}\,b\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\,x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))]*x^3),x]
```

```
[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) - b*x*Sqrt[b + a*x^2 - b*x^2]*Log[-(Sqrt[a
 - b]*x) + Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))]
*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(50) = 100.
time = 0.05, size = 102, normalized size = 1.76

method	result	size
default	$\frac{\sqrt{a\,x^{2}-b\,x^{2}+b}\,\left(\ln\left(x\sqrt{a-b}+\sqrt{a\,x^{2}-b\,x^{2}+b}\right)bx+\sqrt{a\,x^{2}-b\,x^{2}+b}\,\sqrt{a-b}\right)}{\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x^{2}\sqrt{a-b}\,b}$	102
risch	$\frac{a\,x^{2}-b\,x^{2}+b}{b\,x^{2}\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}}+\frac{\ln\left(x\sqrt{a-b}+\sqrt{x^{2}\left(a-b\right)+b}\right)\sqrt{a\,x^{2}-b\,x^{2}+b}}{\sqrt{a-b}\,\sqrt{\frac{a\,x^{2}-b\,x^{2}+b}{x^{2}}}\,x}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (a*x^2-b*x^2+b)^(1/2)*(ln(x*(a-b)^(1/2)+(a*x^2-b*x^2+b)^(1/2))*b*x+(a*x^2-b
*x^2+b)^(1/2)*(a-b)^(1/2))/((a*x^2-b*x^2+b)/x^2)^(1/2)/x^2/(a-b)^(1/2)/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.41, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}\,b\log\left(-2\,(a-b)x^{2}-2\,\sqrt{a-b}\,x^{2}\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}-b\right)+2\,(a-b)\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{2\,(ab-b^{2})},\ \frac{\sqrt{-a+b}\,b\arctan\left(-\frac{\sqrt{-a+b}\,x^{2}\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{(a-b)x^{2}+b}\right)+(a-b)\sqrt{\frac{(a-b)x^{2}+b}{x^{2}}}}{ab-b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^
2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqr
t(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2)/((a - b)*x
^2 + b)) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]
```

Sympy [A]
time = 3.65, size = 70, normalized size = 1.21

$$-\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b=0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)
```

```
[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True)
)/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a
- b))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(44) = 88.
time = 2.56, size = 92, normalized size = 1.59

$$-\frac{\log\left(\left(\sqrt{a-b}\,x-\sqrt{ax^2-bx^2+b}\right)^2\right)}{2\sqrt{a-b}\operatorname{sgn}(x)} - \frac{2\sqrt{a-b}}{\left(\left(\sqrt{a-b}\,x-\sqrt{ax^2-bx^2+b}\right)^2-b\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2)/(sqrt(a - b)*sgn(x))
- 2*sqrt(a - b)/(((sqrt(a - b)*x - sqrt(a*x^2 - b*x^2 + b))^2 - b)*sgn(x))
```

Mupad [B]
time = 4.04, size = 62, normalized size = 1.07

$$\frac{\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}{b} + \frac{\ln\left(x^2\left(2\,a-2\,b+2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}+\frac{b}{x^2}\right)\right)}{2\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)
```

```
[Out] (a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*(
1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))
```

3.813 $\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1024, 385, 209, 455, 65, 213}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{5}\,x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/S
qrt[5]]/Sqrt[5]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}}\,dx &= \int \frac{1}{(4+x^2)\sqrt{9+x^2}}\,dx + \int \frac{x}{(4+x^2)\sqrt{9+x^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{9+x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} + \text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{9+x^2}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}\,x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.34, size = 55, normalized size = 1.04

$$-\frac{\tan^{-1}\left(\frac{4+x^2-x\sqrt{9+x^2}}{2\sqrt{5}}\right) + 2\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]),x]
```

```
[Out] -1/2*(ArcTan[(4 + x^2 - x*Sqrt[9 + x^2])/(2*Sqrt[5])] + 2*ArcTanh[Sqrt[9 +
x^2]/Sqrt[5]])/Sqrt[5]
```

Maple [A]
time = 0.72, size = 39, normalized size = 0.74

method	result
default	$\frac{\arctan\left(\frac{x\sqrt{5}}{2\sqrt{x^2+9}}\right)\sqrt{5}}{10}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+9}\,\sqrt{5}}{5}\right)\sqrt{5}}{5}$
trager	$16\ln\left(\frac{6400\,\mathrm{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)^5 x-1120\,\mathrm{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)^3 x+1440\,\mathrm{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)}{80\,\mathrm{RootOf}\left(1280\_Z^4-96\_Z^2+5\right)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+4)/(x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1
/2)*5^(1/2))*5^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(38) = 76.
time = 0.35, size = 182, normalized size = 3.43

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x+\sqrt{5}\right)+\sqrt{5}x+9}+\frac{1}{2}x+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}\left(x-\sqrt{5}\right)-\sqrt{5}x+9}+\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(50x^2-50\sqrt{x^2+9}\left(x+\sqrt{5}\right)+50\sqrt{5}x+450\right)-\frac{1}{10}\sqrt{5}\log\left(50x^2-50\sqrt{x^2+9}\left(x-\sqrt{5}\right)-50\sqrt{5}x+450\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x + sqrt(5)) + sqr
t(5)*x + 9) + 1/2*x + 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/5*sqrt(5)*arctan
(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9)*(x - sqrt(5)) - sqrt(5)*x + 9) + 1/2*
x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^
2 + 9)*(x + sqrt(5)) + 50*sqrt(5)*x + 450) - 1/10*sqrt(5)*log(50*x^2 - 50*s
qrt(x^2 + 9)*(x - sqrt(5)) - 50*sqrt(5)*x + 450)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x^2+4)\sqrt{x^2+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(38) = 76.
time = 4.11, size = 123, normalized size = 2.32

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2+2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)-\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2-2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(5
)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x -
sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log((x
 - sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9)
```

Mupad [B]
time = 3.59, size = 67, normalized size = 1.26

$$\sqrt{5}\left(\ln(x-2\mathrm{i})-\ln\left(\sqrt{5}\sqrt{x^2+9}+9+x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}-\frac{1}{20}\mathrm{i}\right)+\sqrt{5}\left(\ln(x+2\mathrm{i})-\ln\left(\sqrt{5}\sqrt{x^2+9}+9-x\,2\mathrm{i}\right)\right)\left(\frac{1}{10}+\frac{1}{20}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)
```

```
[Out] 5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i/
20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/10
 + 1i/20)
```

3.814 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {14, 267}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x^2]),x]
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+\sqrt{1-x^2}\right)\,dx&=\int\left(x+x\sqrt{1-x^2}\right)\,dx\\&=\frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\&=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.04

$$\frac{1}{6}\left(-1+x^2\right)\left(3+2\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

```
[Out] ((-1 + x^2)*(3 + 2*Sqrt[1 - x^2]))/6
```

Maple [A]
time = 0.47, size = 18, normalized size = 0.78

method	result	size
default	$\frac{x^2}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	18
derivativedivides	$\frac{x^2}{2}-\frac{1}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	19
trager	$\frac{x^2}{2}+\left(\frac{x^2}{3}-\frac{1}{3}\right)\sqrt{-x^2+1}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Maxima [A]
time = 0.30, size = 17, normalized size = 0.74

$$\frac{1}{2}x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.34, size = 22, normalized size = 0.96

$$\frac{1}{2}x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

Sympy [A]
time = 0.07, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{1-x^2}}{3}+\frac{x^2}{2}-\frac{\sqrt{1-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x**2+1)**(1/2)),x)
```

```
[Out] x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

Giac [A]
time = 3.77, size = 18, normalized size = 0.78

$$\frac{1}{2}\,x^2-\frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

Mupad [B]
time = 0.03, size = 23, normalized size = 1.00

$$\frac{x^2}{2}+\sqrt{1-x^2}\,\left(\frac{x^2}{3}-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x^2)^(1/2) + 1),x)
```

```
[Out] x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

3.815 $\int x\left(1+\sqrt{1-x}\,\sqrt{1+x}\right)\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

```
[Out] 1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {14, 267}

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] x^2/2 - (1 - x^2)^(3/2)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(1+\sqrt{1-x}\,\sqrt{1+x}\right)\,dx &= \int\left(x+x\sqrt{1-x^2}\right)\,dx\\
&= \frac{x^2}{2}+\int x\sqrt{1-x^2}\,dx\\
&= \frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 24, normalized size = 1.04

$$\frac{1}{6}\left(-1+x^2\right)\left(3+2\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]
[Out] ((-1 + x^2)*(3 + 2*Sqrt[1 - x^2]))/6
```

Maple [A]
time = 0.50, size = 26, normalized size = 1.13

method	result	size
default	$\frac{\sqrt{1-x}\,\sqrt{1+x}\,(x^2-1)}{3}+\frac{x^2}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/3*(1-x)^(1/2)*(1+x)^(1/2)*(x^2-1)+1/2*x^2
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.74

$$\frac{1}{2}\,x^2-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Fricas [A]
time = 0.33, size = 25, normalized size = 1.09

$$\frac{1}{2}\,x^2+\frac{1}{3}\left(x^2-1\right)\sqrt{x+1}\,\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 1/3*(x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)
```

Sympy [A]
time = 53.32, size = 139, normalized size = 6.04

$$-x+\frac{(x+1)^2}{2}-2\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)+2\left(\begin{cases}\frac{x\sqrt{1-x}\,\sqrt{x+1}}{4}-\frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)}{2} & \text{for}\ \sqrt{x+1}>-\sqrt{2}\wedge\sqrt{x+1}<\sqrt{2}\end{cases}\right)-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] -x + (x + 1)**2/2 - 2*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)
*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -sqrt(2)))) + 2
*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 +
 asin(sqrt(2)*sqrt(x + 1)/2)/2, (sqrt(x + 1) < sqrt(2)) & (sqrt(x + 1) > -s
qrt(2)))) - 1
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(17) = 34$.
time = 2.56, size = 54, normalized size = 2.35

$$\frac{1}{2}\,{\left(x+1\right)}^{2}+\frac{1}{6}\,{\left({\left(2\,x-5\right)}{\left(x+1\right)}+9\right)}\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\,\sqrt{x+1}\,{\left(x-2\right)}\sqrt{-x+1}-x-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)^(1/2)*(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*
sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - x - 1
```

Mupad [B]
time = 3.69, size = 35, normalized size = 1.52

$$\frac{x^2}{2}-\frac{\sqrt{1-x}\,\left(-\frac{x^3}{3}-\frac{x^2}{3}+\frac{x}{3}+\frac{1}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((1 - x)^(1/2)*(x + 1)^(1/2) + 1),x)
```

```
[Out] x^2/2 - ((1 - x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x + 1)^(1/2)
```

3.816 $\int x\left(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}\right)\,dx$

Optimal. Leaf size=33

$$\frac{x^2}{2}+\sqrt{2+x}\,\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)$$

[Out] 1/2*x^2-5*arcsinh((2+x)^(1/2))+(2+x)^(1/2)*(3+x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {14, 81, 56, 221}

$$\frac{x^2}{2}+\sqrt{x+2}\,\sqrt{x+3}-5\sinh^{-1}\left(\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]

[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x\left(1+\frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\right)dx &= \int\left(x+\frac{x}{\sqrt{2+x}\,\sqrt{3+x}}\right)dx\\
&= \frac{x^2}{2}+\int\frac{x}{\sqrt{2+x}\,\sqrt{3+x}}\,dx\\
&= \frac{x^2}{2}+\sqrt{2+x}\,\sqrt{3+x}-\frac{5}{2}\int\frac{1}{\sqrt{2+x}\,\sqrt{3+x}}\,dx\\
&= \frac{x^2}{2}+\sqrt{2+x}\,\sqrt{3+x}-5\text{Subst}\left(\int\frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{2+x}\right)\\
&= \frac{x^2}{2}+\sqrt{2+x}\,\sqrt{3+x}-5\sinh^{-1}\left(\sqrt{2+x}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.05, size = 54, normalized size = 1.64

$$\frac{x^2}{2}+\frac{\sqrt{3+x}}{\sqrt{2+x}\,\left(-1+\frac{3+x}{2+x}\right)}-5\tanh^{-1}\left(\frac{\sqrt{3+x}}{\sqrt{2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]
```

```
[Out] x^2/2 + Sqrt[3 + x]/(Sqrt[2 + x]*(-1 + (3 + x)/(2 + x))) - 5*ArcTanh[Sqrt[3
 + x]/Sqrt[2 + x]]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.
time = 0.50, size = 58, normalized size = 1.76

method	result	size
default	$-\frac{\sqrt{x+2}\,\sqrt{3+x}\,\left(-2\sqrt{x^2+5x+6}+5\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)\right)}{2\sqrt{x^2+5x+6}}+\frac{x^2}{2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+1/(x+2)^(1/2)/(3+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x+2)^(1/2)*(3+x)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(5/2+x+(x^2+5*x+6)^(
1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2
```

Maxima [**A**]
time = 0.28, size = 36, normalized size = 1.09

$$\frac{1}{2}\,x^2+\sqrt{x^2+5\,x+6}-\frac{5}{2}\,\log\left(2\,x+2\,\sqrt{x^2+5\,x+6}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="maxima")
[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)
```

Fricas [A]
time = 0.36, size = 37, normalized size = 1.12

$$\frac{1}{2}x^2 + \sqrt{x+3}\,\sqrt{x+2} + \frac{5}{2}\log\left(2\sqrt{x+3}\,\sqrt{x+2} - 2x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x
 - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\sqrt{x+2}\,\sqrt{x+3}+1\right)}{\sqrt{x+2}\,\sqrt{x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)**(1/2)/(3+x)**(1/2)),x)
[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)
```

Giac [A]
time = 2.61, size = 39, normalized size = 1.18

$$\frac{1}{2}(x+3)^2 + \sqrt{x+3}\,\sqrt{x+2} - 3x + 5\log\left(\sqrt{x+3} - \sqrt{x+2}\right) - 9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/(2+x)^(1/2)/(3+x)^(1/2)),x, algorithm="giac")
[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x
+ 2)) - 9
```

Mupad [B]
time = 7.56, size = 180, normalized size = 5.45

$$\frac{\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)}{\sqrt{x+3}-\sqrt{3}}+\frac{10\left(\sqrt{x+2}-\sqrt{2}\right)^3}{\left(\sqrt{x+3}-\sqrt{3}\right)^3}-\frac{8\sqrt{6}\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}}{\frac{\left(\sqrt{x+2}-\sqrt{2}\right)^4}{\left(\sqrt{x+3}-\sqrt{3}\right)^4}-\frac{2\left(\sqrt{x+2}-\sqrt{2}\right)^2}{\left(\sqrt{x+3}-\sqrt{3}\right)^2}+1} - 10\,\mathrm{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)
```

```
[Out] ((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(1
/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2) -
 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/(((x + 2)^(1/2) - 2^(1/2))^4/((x
+ 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) -
3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) - 3^(1/
2))) + x^2/2
```

3.817 $\int \frac{x-\sqrt{x^6}}{x(1-x^4)}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A]

time = 0.11, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^6}\,\text{ArcTan}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A]
time = 0.76, size = 57, normalized size = 1.27

$$\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A]
time = 0.46, size = 35, normalized size = 0.78

method	result	size
default	$\frac{\sqrt{x^{6}}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^{3}}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-(x^6)^(1/2))/x/(-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctan
h(x)
```

Maxima [A]
time = 0.49, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")
[Out] arctan(x)
```

Fricas [A]
time = 0.34, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [A]
time = 0.03, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)
[Out] atan(x)
```

Giac [A]
time = 2.83, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan\left(x\right)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x-\sqrt{x^6}}{x\left(x^4-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)
```

```
[Out] int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)
```

3.818 $\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^6}\,\text{ArcTan}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4}\,dx &= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.27

$$\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A]
time = 0.49, size = 35, normalized size = 0.78

method	result	size
default	$\frac{\sqrt{x^{6}}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^{3}}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(x^6)^(1/2)/x)/(-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctan
h(x)
```

Maxima [A]
time = 0.51, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")
```

```
[Out] arctan(x)
```

Fricas [A]
time = 0.38, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")
```

```
[Out] arctan(x)
```

Sympy [A]
time = 0.03, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)
```

```
[Out] atan(x)
```

Giac [A]
time = 2.40, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\mathrm{sgn}(x)+1\right)\arctan\left(x\right)-\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\mathrm{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\frac{\sqrt{x^6}}{x}-1}{x^4-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)
```

```
[Out] int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)
```

3.819 $\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$

Optimal. Leaf size=45

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\,\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A]

time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {1607, 6857, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^6}\,\text{ArcTan}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}-\frac{\sqrt{x^6}\,\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[x^6])/(x - x^5),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx &= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx\\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)dx\\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx\\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3}\\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 57, normalized size = 1.27

$$
\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^4}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[x^6])/(x - x^5),x]

[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 - x]/4 + Log[1 + x]/4

Maple [A]
time = 0.48, size = 35, normalized size = 0.78

method	result	size
default	$\frac{\sqrt{x^{6}}\,(\ln(-1+x)-\ln(1+x)+2\arctan(x))}{4x^{3}}+\frac{\arctan(x)}{2}+\frac{\operatorname{arctanh}(x)}{2}$	35
meijerg	$-\frac{x\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)-2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{1}{4}}}+\frac{\sqrt{x^{6}}\left(\ln\left(1-(x^{4})^{\frac{1}{4}}\right)-\ln\left(1+(x^{4})^{\frac{1}{4}}\right)+2\arctan\left((x^{4})^{\frac{1}{4}}\right)\right)}{4(x^{4})^{\frac{3}{4}}}$	80

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^6)^(1/2))/(-x^5+x),x,method=_RETURNVERBOSE)

[Out] 1/4*(x^6)^(1/2)*(ln(-1+x)-ln(1+x)+2*arctan(x))/x^3+1/2*arctan(x)+1/2*arctan h(x)

Maxima [A]
time = 0.53, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")

[Out] arctan(x)

Fricas [A]
time = 0.34, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="fricas")

[Out] arctan(x)

Sympy [A]
time = 0.03, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)
```

```
[Out] atan(x)
```

Giac [A]
time = 2.07, size = 31, normalized size = 0.69

$$\frac{1}{2}\left(\operatorname{sgn}(x)+1\right)\arctan\left(x\right)-\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x+1|\right)+\frac{1}{4}\left(\operatorname{sgn}(x)-1\right)\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")
```

```
[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x)
 - 1)*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x-\sqrt{x^6}}{x-x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (x^6)^(1/2))/(x - x^5),x)
```

```
[Out] int((x - (x^6)^(1/2))/(x - x^5), x)
```

3.820 $\int \frac{x}{x+\sqrt{x^6}} \, dx$

Optimal. Leaf size=45

$$\frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}$$

```
[Out] 1/2*arctan(x)+1/2*arctanh(x)+1/2*arctan(x)*(x^6)^(1/2)/x^3-1/2*arctanh(x)*(
x^6)^(1/2)/x^3
```

Rubi [A]
time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6861, 1598, 6857, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^6}\,\text{ArcTan}(x)}{2x^3}+\frac{\text{ArcTan}(x)}{2}-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*Arc
Tanh[x])/(2*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{x+\sqrt{x^6}}\,dx &= \int \frac{x\left(x-\sqrt{x^6}\right)}{x^2-x^6}\,dx \\
&= \int \frac{x-\sqrt{x^6}}{x\left(1-x^4\right)}\,dx \\
&= \int \left(\frac{1}{1-x^4}+\frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\right)\,dx \\
&= \int \frac{1}{1-x^4}\,dx+\int \frac{\sqrt{x^6}}{x\left(-1+x^4\right)}\,dx \\
&= \frac{1}{2}\int \frac{1}{1-x^2}\,dx+\frac{1}{2}\int \frac{1}{1+x^2}\,dx+\frac{\sqrt{x^6}\int \frac{x^2}{-1+x^4}\,dx}{x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\int \frac{1}{1-x^2}\,dx}{2x^3}+\frac{\sqrt{x^6}\int \frac{1}{1+x^2}\,dx}{2x^3} \\
&= \frac{1}{2}\tan^{-1}(x)+\frac{\sqrt{x^6}\tan^{-1}(x)}{2x^3}+\frac{1}{2}\tanh^{-1}(x)-\frac{\sqrt{x^6}\tanh^{-1}(x)}{2x^3}
\end{aligned}
$$

Mathematica [A]
time = 3.89, size = 57, normalized size = 1.27

$$
\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{x^6}}{x^2}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^6}}{x^2}\right)-\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[x^6]),x]
```

```
[Out] ArcTan[x]/2 + ArcTan[Sqrt[x^6]/x^2]/2 - ArcTanh[Sqrt[x^6]/x^2]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

Maple [A]
time = 0.56, size = 27, normalized size = 0.60

method	result	size
meijerg	$\frac{x^{\frac{3}{2}}\arctan\left(\frac{\left(x^6\right)^{\frac{1}{4}}}{\sqrt{x}}\right)}{\left(x^6\right)^{\frac{1}{4}}}$	20

default	$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^6}}{x^3}}\, x\right)}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(x^6)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)
```

Maxima [A]
time = 0.50, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")
[Out] arctan(x)
```

Fricas [A]
time = 0.33, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")
[Out] arctan(x)
```

Sympy [A]
time = 0.03, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x**6)**(1/2)),x)
[Out] atan(x)
```

Giac [A]
time = 2.50, size = 12, normalized size = 0.27

$$\frac{\arctan\left(x\sqrt{\operatorname{sgn}(x)}\right)}{\sqrt{\operatorname{sgn}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")
```

```
[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{x+\sqrt{x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + (x^6)^(1/2)),x)
```

```
[Out] int(x/(x + (x^6)^(1/2)), x)
```

3.821 $\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} \, dx$

Optimal. Leaf size=52

$$\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {1607, 6857, 335, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^3}\,\text{ArcTan}\left(\sqrt{x}\right)}{x^{3/2}}+\text{ArcTan}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx = \int \frac{\sqrt{x}-\sqrt{x^3}}{x(1-x^2)}\,dx$$
$$= \int \left(-\frac{1}{\sqrt{x}\,(-1+x^2)}+\frac{\sqrt{x^3}}{x(-1+x^2)}\right)dx$$
$$= -\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx + \int \frac{\sqrt{x^3}}{x(-1+x^2)}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{x}\right)\right) + \frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{}{1}\right.$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\right.}{x^{3/2}}$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x^3}\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}} + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Mathematica [A]
time = 1.63, size = 39, normalized size = 0.75

$$\tan^{-1}\left(\sqrt{x}\right) + \tan^{-1}\left(\frac{\sqrt{x^3}}{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) - \tanh^{-1}\left(\frac{\sqrt{x^3}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

Maple [A]
time = 0.49, size = 41, normalized size = 0.79

method	result
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\left(\ln\left(-1+\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)+2\arctan\left(\sqrt{x}\right)\right)}{2x^{\frac{3}{2}}}$
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)-2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x^3}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)+2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{3}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+1/2*(x^3)^(1/2)*(ln(-1+x^(1/2))-ln(1+x^(1/
2))+2*arctan(x^(1/2)))/x^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x
) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*l
og(sqrt(x) - 1)
```

Fricas [A]
time = 0.34, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x}}{x^3 - x}\,dx - \int \left(-\frac{\sqrt{x^3}}{x^3 - x}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)
```

```
[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)
```

Giac [A]
time = 2.10, size = 6, normalized size = 0.12

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{\sqrt{x^3}-\sqrt{x}}{x-x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)
[Out] -int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)
```

3.822 $\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx$

Optimal. Leaf size=52

$$\tan^{-1}\left(\sqrt{x}\right)+\frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))+arctan(x^(1/2))*(x^3)^(1/2)/x^(3/2)-arctan
h(x^(1/2))*(x^3)^(1/2)/x^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {6861, 1607, 6857, 335, 218, 212, 209, 15, 304}

$$\frac{\sqrt{x^3}\,\text{ArcTan}\left(\sqrt{x}\right)}{x^{3/2}}+\text{ArcTan}\left(\sqrt{x}\right)-\frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] -
(Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx = \int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx$$
$$= \int \frac{\sqrt{x}-\sqrt{x^3}}{x(1-x^2)}\,dx$$
$$= \int \left(-\frac{1}{\sqrt{x}\,(-1+x^2)}+\frac{\sqrt{x^3}}{x(-1+x^2)}\right)dx$$
$$= -\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx + \int \frac{\sqrt{x^3}}{x(-1+x^2)}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{x}\right)\right) + \frac{\sqrt{x^3}\int \frac{\sqrt{x}}{-1+x^2}\,dx}{x^{3/2}}$$
$$= \frac{\left(2\sqrt{x^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+\cdots}\right.$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)}{x^{3/2}} + \frac{\sqrt{x^3}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\cdots\right.}{x^{3/2}}$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \frac{\sqrt{x^3}\,\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}} + \tanh^{-1}\left(\sqrt{x}\right) - \frac{\sqrt{x^3}\,\tanh^{-1}\left(\sqrt{x}\right)}{x^{3/2}}$$

Mathematica [A]
time = 1.61, size = 39, normalized size = 0.75

$$\tan^{-1}\left(\sqrt{x}\right) + \tan^{-1}\left(\frac{\sqrt{x^3}}{x}\right) + \tanh^{-1}\left(\sqrt{x}\right) - \tanh^{-1}\left(\frac{\sqrt{x^3}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3
]/x]
```

Maple [A]
time = 0.50, size = 30, normalized size = 0.58

method	result	size
meijerg	$\frac{2x^{\frac{3}{4}}\arctan\left(\frac{\left(x^3\right)^{\frac{1}{4}}}{x^{\frac{1}{4}}}\right)}{\left(x^3\right)^{\frac{1}{4}}}$	21

default	$\frac{2\arctan\left(\sqrt{\frac{\sqrt{x^{3}}}{x^{\frac{3}{2}}}}\,\sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^{3}}}{x^{\frac{3}{2}}}}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(x^3)^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(((x^3)^(1/2)/x^(3/2))^(1/2)*x^(1/2))
```

Maxima [A]
time = 0.52, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

Fricas [A]
time = 0.34, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}+\sqrt{x^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```

Giac [A]
time = 2.30, size = 6, normalized size = 0.12

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(x^3)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3)^(1/2) + x^(1/2)),x)
```

```
[Out] int(1/((x^3)^(1/2) + x^(1/2)), x)
```

3.823 $\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx$

Optimal. Leaf size=68

$$\tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\tanh^{-1}\left(\sqrt{-1+\right.}}{(-1+x)^{3/2}}$$

```
[Out] arctan((-1+x)^(1/2))+arctanh((-1+x)^(1/2))+arctan((-1+x)^(1/2))*((-1+x)^3)^
(1/2)/(-1+x)^(3/2)-arctanh((-1+x)^(1/2))*((-1+x)^3)^(1/2)/(-1+x)^(3/2)
```

Rubi [A]
time = 0.10, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6861, 1607, 6857, 335, 218, 212, 209, 15, 304}

$$\frac{\sqrt{(x-1)^3}\,\text{ArcTan}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\text{ArcTan}\left(\sqrt{x-1}\right)-\frac{\sqrt{(x-1)^3}\tanh^{-1}\left(\sqrt{x-1}\right)}{(x-1)^{3/2}}+\tanh^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/
2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 +
 x)^(3/2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6861

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_)]), x_Symbol] :> In
t[u*((a*x^m - b*Sqrt[c*x^n])/(a^2*x^(2*m) - b^2*c*x^n)), x] /; FreeQ[{a, b,
 c, m, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{x}+\sqrt{x^3}}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \frac{\sqrt{x}-\sqrt{x^3}}{x(1-x^2)}\,dx, x, -1+x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{x}\,(-1+x^2)}+\frac{\sqrt{x^3}}{x(-1+x^2)}\right)dx, x, -1+x\right)$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx, x, -1+x\right)+\text{Subst}\left(\int \frac{\sqrt{x^3}}{x(-1+x^2)}\,dx\right.$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)\right)+\frac{\sqrt{(-1+x)^3}\,\text{Subst}\left(\int \right.}{(-1+}$$
$$= \frac{\left(2\sqrt{(-1+x)^3}\right)\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\text{Subst}\left(\int \frac{1}{1-x^2}\right.$$
$$= \tan^{-1}\left(\sqrt{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)-\frac{\sqrt{(-1+x)^3}\,\text{Subst}\left(\int \frac{1}{1-x^2}\right.}{(-1+x)^{3}}$$
$$= \tan^{-1}\left(\sqrt{-1+x}\right)+\frac{\sqrt{(-1+x)^3}\,\tan^{-1}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}+\tanh^{-1}\left(\sqrt{-1\right.$$

Mathematica [A]
time = 1.76, size = 67, normalized size = 0.99

$$\tan^{-1}\left(\sqrt{-1+x}\right)+\tan^{-1}\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right)+\tanh^{-1}\left(\sqrt{-1+x}\right)-\tanh^{-1}\left(\frac{\sqrt{-1+3x-3x^2+x^3}}{-1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1),x]
```

```
[Out] ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)] + ArcT
anh[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)]
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.59

method	result	size

default	$\frac{2\arctan\left(\sqrt{\frac{\sqrt{(-1+x)^{3}}}{(-1+x)^{\frac{3}{2}}}}\,\sqrt{-1+x}\right)}{\sqrt{\frac{\sqrt{(-1+x)^{3}}}{(-1+x)^{\frac{3}{2}}}}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(((-1+x)^3)^(1/2)/(-1+x)^(3/2))^(1/2)*arctan((((-1+x)^3)^(1/2)/(-1+x)^(3/
2))^(1/2)*(-1+x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)
```

Fricas [A]
time = 0.33, size = 8, normalized size = 0.12

$$2\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x-1}+\sqrt{(x-1)^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)
```

Giac [A]
time = 3.01, size = 8, normalized size = 0.12

$$2 \arctan \left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x-1} - \sqrt{(x-1)^3}}{(x-1)^3 - x + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)),x)
[Out] int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)
```

3.824 $\int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx$

Optimal. Leaf size=31

$$\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = $\{817\}$

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 817

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*
g, 0]
```

Rubi steps

$$\begin{aligned} \int \left(-\frac{3}{(4+5x)^2}-\frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\right) dx &= \frac{3}{5(4+5x)}-\int \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}}\, dx \\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x} \end{aligned}$$

Mathematica [A]
time = 0.27, size = 39, normalized size = 1.26

$$\frac{3+\frac{5}{\sqrt{1-x^2}}-\frac{5x^2}{\sqrt{1-x^2}}}{20+25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] (3 + 5/Sqrt[1 - x^2] - (5*x^2)/Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A]
time = 0.57, size = 32, normalized size = 1.03

method	result	size
trager	$-\frac{3x}{4(5x+4)}+\frac{\sqrt{-x^2+1}}{5x+4}$	29
default	$\frac{\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}}{5x+4}+\frac{3}{5(5x+4)}$	32
risch	$\frac{3}{25\left(x+\frac{4}{5}\right)}-\frac{x^2-1}{(5x+4)\sqrt{-x^2+1}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/(5*x+4)^2+(-5-4*x)/(5*x+4)^2/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(5*x+4)
```

Maxima [A]
time = 0.54, size = 27, normalized size = 0.87

$$\frac{\sqrt{-x^2+1}}{5\,x+4}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)
```

Fricas [A]
time = 0.34, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int \frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 2.34, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5\,x+4}+\frac{9}{(5\,x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)}+\frac{3}{5\,(5\,x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5\,x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac
")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

Mupad [B]
time = 3.33, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(- 3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.825 $\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=31

$$\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A]
time = 0.19, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {6874, 745, 739, 212, 821}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
```

```
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}}\,dx &= \int \left(-\frac{3}{(4+5x)^2}-\frac{5}{(4+5x)^2\sqrt{1-x^2}}-\frac{4x}{(4+5x)^2\sqrt{1-x^2}}\right)dx\\ &= \frac{3}{5(4+5x)}-4\int \frac{x}{(4+5x)^2\sqrt{1-x^2}}\,dx-5\int \frac{1}{(4+5x)^2\sqrt{1-x^2}}\,dx\\ &= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 23, normalized size = 0.74

$$\frac{3+5\sqrt{1-x^2}}{20+25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A]
time = 0.53, size = 32, normalized size = 1.03

method	result	size
trager	$-\frac{3x}{4(5x+4)}+\frac{\sqrt{-x^2+1}}{5x+4}$	29
default	$\frac{\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}}{5x+4}+\frac{3}{5(5x+4)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(5*x+4)^2/(-x^2+1)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(5*x+4)
```

Maxima [A]
time = 0.32, size = 25, normalized size = 0.81

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
maxima")
```

```
[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)
```

Fricas [A]
time = 0.37, size = 25, normalized size = 0.81

$$\frac{25x+20\sqrt{-x^2+1}+32}{20(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{4x}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx-\int\frac{5}{25x^2\sqrt{1-x^2}+40x\sqrt{1-x^2}+16\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x
**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1
 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40
*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.08, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5x+4}+\frac{9}{(5x+4)^2}-1}}{5\,\mathrm{sgn}\left(\frac{1}{5x+4}\right)}+\frac{3}{5(5x+4)}-\frac{1}{5}i\,\mathrm{sgn}\left(\frac{1}{5x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="
giac")
```

```
[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4)
- 1/5*I*sgn(1/(5*x + 4))
```

Mupad [B]
time = 0.04, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.826 $\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A]
time = 0.10, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6874, 679, 222, 747, 858, 739, 212, 749}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)}\,dx &= \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20\sqrt{1}}{9(4}\right.\\
&= \frac{3}{5(4+5x)}+\frac{1}{18}\int \frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int \frac{\sqrt{1-}}{4+}\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int \frac{5+4x}{(4+5x)\sqrt{1-}}\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}\int\\
&= \frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 23, normalized size = 0.74

$$\frac{3+5\sqrt{1-x^2}}{20+25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(27) = 54$.
time = 0.04, size = 81, normalized size = 2.61

method	result
trager	$-\frac{3x}{4(5x+4)}+\frac{\sqrt{-x^2+1}}{5x+4}$
default	$\frac{3}{5(5x+4)}+\frac{5\left(-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x+\frac{4}{5}\right)}+\frac{5x\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}-\frac{\sqrt{-\left(1+x\right)^2+2+2x}}{2}+\frac{\sqrt{-(-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5/(5*x+4)+5/9/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(3/2)+5/9*x*(-(x+4/5)^2+8/
5*x+41/25)^(1/2)-1/2*(-(1+x)^2+2+2*x)^(1/2)+1/18*(-(-1+x)^2+2-2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)
```

Fricas [A]
time = 0.32, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 2.59, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B]
time = 0.07, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.827 $\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

```
[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)
```

Rubi [A]
time = 0.09, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6874, 679, 222, 747, 858, 739, 212, 749}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 679

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] - Dist[2*c*d*(p/(e^
2*(m + 2*p + 1))), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}}\,dx = \int \left(-\frac{3}{(4+5x)^2}+\frac{\sqrt{1-x^2}}{18(-1+x)}-\frac{\sqrt{1-x^2}}{2(1+x)}-\frac{5\sqrt{1-x^2}}{(4+5x)^2}+\frac{20}{9}\right.$$

$$=\frac{3}{5(4+5x)}+\frac{1}{18}\int\frac{\sqrt{1-x^2}}{-1+x}\,dx-\frac{1}{2}\int\frac{\sqrt{1-x^2}}{1+x}\,dx+\frac{20}{9}\int$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{1}{18}\int\frac{1}{\sqrt{1-x^2}}\,dx+\frac{4}{9}\int\frac{5+4}{(4+5x)\sqrt{}}$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}-\frac{5}{9}\sin^{-1}(x)+\frac{1}{5}\int\frac{1}{\sqrt{1-x^2}}\,dx+\frac{16}{45}$$

$$=\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A]
time = 0.28, size = 23, normalized size = 0.74

$$\frac{3+5\sqrt{1-x^2}}{20+25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1),x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(27) = 54.
time = 0.04, size = 81, normalized size = 2.61

method	result
trager	$-\frac{3x}{4(5x+4)}+\frac{\sqrt{-x^2+1}}{5x+4}$
default	$\frac{3}{5(5x+4)}+\frac{5\left(-(x+\frac{4}{5})^2+\frac{8x}{5}+\frac{41}{25}\right)^{\frac{3}{2}}}{9(x+\frac{4}{5})}+\frac{5x\sqrt{-\left(x+\frac{4}{5}\right)^2+\frac{8x}{5}+\frac{41}{25}}}{9}-\frac{\sqrt{-\left(1+x\right)^2+2+2x}}{2}+\frac{\sqrt{-(-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE
)
```

```
[Out] 3/5/(5*x+4)+5/9/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(3/2)+5/9*x*(-(x+4/5)^2+8/
5*x+41/25)^(1/2)-1/2*(-(1+x)^2+2+2*x)^(1/2)+1/18*(-(-1+x)^2+2-2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="max
ima")
```

```
[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)
```

Fricas [A]
time = 0.33, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^2+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fri
cas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2)-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(27) = 54.
time = 3.04, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - 4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 2\right)} + \frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="gia
c")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B]
time = 0.05, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(4*x*(1 - x^2)^(1/2) + 3*x^2 + 5*(1 - x^2)^(1/2) - 3),x)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.828 $\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx$

Optimal. Leaf size=31

$$\frac{3}{5(4+5x)}+\frac{\sqrt{1-x^2}}{4+5x}$$

[Out] 3/5/(4+5*x)+(-x^2+1)^(1/2)/(4+5*x)

Rubi [A]
time = 0.45, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.302, Rules used = {6874, 283, 222, 272, 52, 65, 212, 747, 858, 739, 749, 270, 745}

$$\frac{\sqrt{1-x^2}}{5x+4}+\frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}\left(2+x-2\sqrt{1-x^2}\right)^2}\,dx = \int \left(\frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2} - \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x^2}\right)\right.$$

$$= \int \frac{1}{\left(-2-x+2\sqrt{1-x^2}\right)^2}\,dx - \int \frac{1}{\sqrt{1-x^2}\left(-2-x+2\sqrt{1-x\right.}$$

$$= -\int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-}\right.$$

$$= \frac{3}{5(4+5x)} - \frac{1}{2}\int \frac{1}{x^2\sqrt{1-x^2}}\,dx + \frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}\,dx - \frac{9}{2}\int \frac{}{(4+}$$

$$= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx + \frac{1}{2}\text{Subst}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2}\sin^{-1}(x) + \frac{5}{3}\tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) -$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1}\left(\frac{5+4x}{3\sqrt{1-x^2}}\right) - \tanh^{-1}\left(\sqrt{1-x\right.}$$

$$= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}$$

Mathematica [A]
time = 0.28, size = 23, normalized size = 0.74

$$\frac{3+5\sqrt{1-x^2}}{20+25x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),
x]
```

```
[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)
```

Maple [A]
time = 0.50, size = 32, normalized size = 1.03

method	result	size
trager	$-\frac{3x}{4(5x+4)} + \frac{\sqrt{-x^2+1}}{5x+4}$	29

default	$\frac{\sqrt{-\left(x+\frac{4}{5}\right)^{2}+\frac{8x}{5}+\frac{41}{25}}}{5x+4}+\frac{3}{5(5x+4)}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-x^2+1)^(1/2)-1)/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x,method=_RE
TURNVERBOSE)
```

```
[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(5*x+4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="maxima")
```

```
[Out] -1/56*sqrt(7)*log((3*x - 2*sqrt(7) - 2)/(3*x + 2*sqrt(7) - 2)) - integrate(
-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2
022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 1
01*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*sqrt(x + 1)*sq
rt(-x + 1) - 4096*x - 2048), x) - 1/24*log(x + 2) + 1/16*log(x + 1) - 1/48*
log(x - 1)
```

Fricas [A]
time = 0.33, size = 25, normalized size = 0.81

$$\frac{25\,x+20\,\sqrt{-x^{2}+1}+32}{20\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="fricas")
```

```
[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2
),x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.
time = 5.72, size = 68, normalized size = 2.19

$$\frac{\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-4}{4\left(\frac{5\left(\sqrt{-x^2+1}-1\right)}{x}-\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-2\right)}+\frac{3}{5\,(5\,x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2),x, al
gorithm="giac")
```

```
[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2
 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)
```

Mupad [B]
time = 3.32, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2}+\frac{3}{5}}{5\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1 - x^2)^(1/2) - 1)/((1 - x^2)^(1/2)*(x - 2*(1 - x^2)^(1/2) + 2)^2),x
)
```

```
[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)
```

3.829 $\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx$

Optimal. Leaf size=43

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}$$

```
[Out] b*ln(x)/d-(-a*d+b*c)*ln(d+c*x^(1-n))/c/d/(1-n)
```

Rubi [A]
time = 0.05, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1607, 528, 457, 78}

$$\frac{b\log(x)}{d}-\frac{(bc-ad)\log\left(cx^{1-n}+d\right)}{cd(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 528

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^{-1+n}}{cx+dx^n}\,dx &= \int \frac{x^{-n}(a+bx^{-1+n})}{d+cx^{1-n}}\,dx \\
&= \int \frac{b+ax^{1-n}}{x\,(d+cx^{1-n})}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)}\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{dx}+\frac{-bc+ad}{d(d+cx)}\right)\,dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{b\log(x)}{d} - \frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{cd(1-n)}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 38, normalized size = 0.88

$$\frac{b\log(x)+\frac{(bc-ad)\log\left(d+cx^{1-n}\right)}{c(-1+n)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n),x]
[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d
```

Maple [A]
time = 0.55, size = 58, normalized size = 1.35

method	result	size
norman	$\frac{(adn-bc)\ln(x)}{cd(-1+n)} - \frac{(ad-bc)\ln\left(cx+d\,\mathrm{e}^{n\ln(x)}\right)}{cd(-1+n)}$	58
risch	$\frac{b\ln(x)}{d} + \frac{n\ln(x)a}{c(-1+n)} - \frac{n\ln(x)b}{d(-1+n)} - \frac{\ln\left(x^n+\frac{xc}{d}\right)a}{c(-1+n)} + \frac{\ln\left(x^n+\frac{xc}{d}\right)b}{d(-1+n)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(-1+n))/(c*x+d*x^n),x,method=_RETURNVERBOSE)
[Out] (a*d*n-b*c)/c/d/(-1+n)*ln(x)-(a*d-b*c)/c/d/(-1+n)*ln(c*x+d*exp(n*ln(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(40) = 80$.
time = 0.29, size = 85, normalized size = 1.98

$$b\left(\frac{\log(x)}{d} - \frac{n\log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)}\right) + a\left(\frac{n\log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="maxima")
```

```
[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*
(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))
```

Fricas [A]
time = 0.35, size = 44, normalized size = 1.02

$$\frac{(bc-ad)\log(cx+dx^n) + (adn-bc)\log(x)}{cdn-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(29) = 58$.
time = 1.95, size = 216, normalized size = 5.02

$$\begin{cases} \tilde{\infty}(a+b)\log(x) & \text{for } c=0 \wedge d=0 \wedge n=1 \\ \frac{-\frac{anx}{n^2x^n-nx^n} - \frac{bnx^n\log\left(x^{-n}\right)}{n^2x^n-nx^n} - \frac{bnx^n}{n^2x^n-nx^n} + \frac{bx^n\log\left(x^{-n}\right)}{n^2x^n-nx^n}}{d} & \text{for } c=0 \\ \frac{\frac{anx\log(x)}{nx-x} - \frac{ax\log(x)}{nx-x} + \frac{bx^n}{nx-x}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=1 \\ \frac{adn\log(x)}{cdn-cd} - \frac{ad\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} - \frac{bc\log(x)}{cdn-cd} + \frac{bc\log\left(x+\frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n),x)
```

```
[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*n*x/(n
**2*x**n - n*x**n) - b*n*x**n*log(x**(-n))/(n**2*x**n - n*x**n) - b*n*x**n/
(n**2*x**n - n*x**n) + b*x**n*log(x**(-n))/(n**2*x**n - n*x**n))/d, Eq(c, 0
)), ((a*n*x*log(x)/(n*x - x) - a*x*log(x)/(n*x - x) + b*x**n/(n*x - x))/c,
```

```
Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d*n - c*d)
- a*d*log(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(
x + d*x**n/c)/(c*d*n - c*d), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n),x, algorithm="giac")
[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,x^{n-1}}{d\,x^n+c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^(n - 1))/(d*x^n + c*x),x)
[Out] int((a + b*x^(n - 1))/(d*x^n + c*x), x)
```

3.830 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}} \, dx$

Optimal. Leaf size=42

$$-\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -1/2/x+x-1/2*arcsinh(x*2^(1/2))*2^(1/2)+1/2*(2*x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6872, 6874, 283, 221}

$$\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

```
[Out] -1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 6872

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx &= \int \left(1+\frac{1}{-1-\sqrt{1+2x^2}}\right)dx\\
&= x+\int \frac{1}{-1-\sqrt{1+2x^2}}\,dx\\
&= x+\int \left(\frac{1}{2x^2}-\frac{\sqrt{1+2x^2}}{2x^2}\right)dx\\
&= -\frac{1}{2x}+x-\frac{1}{2}\int \frac{\sqrt{1+2x^2}}{x^2}\,dx\\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\int \frac{1}{\sqrt{1+2x^2}}\,dx\\
&= -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\sinh^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 53, normalized size = 1.26

$$\frac{-1+2x^2+\sqrt{1+2x^2}+\sqrt{2}\,x\log\left(-\sqrt{2}\,x+\sqrt{1+2x^2}\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
[Out] (-1 + 2*x^2 + Sqrt[1 + 2*x^2] + Sqrt[2]*x*Log[-(Sqrt[2]*x) + Sqrt[1 + 2*x^2
]])/(2*x)
```

Maple [A]
time = 0.49, size = 45, normalized size = 1.07

method	result	size
default	$x-\frac{1}{2x}+\frac{(2x^2+1)^{\frac{3}{2}}}{2x}-x\sqrt{2x^2+1}-\frac{\operatorname{arcsinh}\left(\sqrt{2}\,x\right)\sqrt{2}}{2}$	45
trager	$\frac{(-1+x)(2x+1)}{2x}+\frac{\sqrt{2x^2+1}}{2x}+\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2-2\right)x+\sqrt{2x^2+1}\right)}{2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(2^(1/2)*x)*2^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.05

$$\frac{\sqrt{2}\,x\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) -
 1)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

Giac [A]
time = 2.35, size = 57, normalized size = 1.36

$$\frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - s
qrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

Mupad [B]
time = 3.39, size = 31, normalized size = 0.74

$$x-\frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2}+\frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
[Out] x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

3.831 $\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx$

Optimal. Leaf size=65

$$\frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\sqrt{3}\,\sqrt{-1+4x^2}\right)}{3\sqrt{3}}$$

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)+1/9*arctanh(3^(1/2)*(4*x^2-1)^(1/2))*3
^(1/2)-1/3*(4*x^2-1)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {6874, 455, 52, 65, 213, 396}

$$-\frac{1}{3}\sqrt{4x^2-1}+\frac{\tanh^{-1}\left(\sqrt{3}\,\sqrt{4x^2-1}\right)}{3\sqrt{3}}+\frac{4x}{3}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqr
t[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}}\,dx &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2}+\frac{-1+4x^2}{-1+3x^2}\right) dx \\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2}\,dx+\int \frac{-1+4x^2}{-1+3x^2}\,dx \\
&= \frac{4x}{3}+\frac{1}{3}\int \frac{1}{-1+3x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+4x}}{-1+3x}\,dx,x,x^2\right) \\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}}\,dx,x, \right. \\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}-\frac{1}{12}\text{Subst}\left(\int \frac{1}{-\frac{1}{4}+\frac{3x^2}{4}}\,dx,x,\sqrt{-1+4x^2}\right. \\
&= \frac{4x}{3}-\frac{1}{3}\sqrt{-1+4x^2}-\frac{\tanh^{-1}\left(\sqrt{3}\,x\right)}{3\sqrt{3}}+\frac{\tanh^{-1}\left(\sqrt{3}\,\sqrt{-1+4x^2}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A]

time = 0.12, size = 50, normalized size = 0.77

$$\frac{1}{9}\left(12x-3\sqrt{-1+4x^2}+2\sqrt{3}\tanh^{-1}\left(\frac{-2x+\sqrt{-1+4x^2}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1 + 4*x^2] + 2*Sqrt[3]*ArcTanh[(-2*x + Sqrt[-1 + 4*x^2])/Sq
rt[3]])/9
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(45) = 90$.
time = 0.51, size = 262, normalized size = 4.03

method	result
trager	$\frac{4x}{3}-\frac{\sqrt{4x^2-1}}{3}-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-3\right)-3\sqrt{4x^2-1}}{\mathrm{RootOf}\left(\_Z^2-3\right)x-1}\right)}{9}$
default	$\frac{4x}{3}-\frac{\mathrm{arctanh}\left(x\sqrt{3}\right)\sqrt{3}}{9}-\frac{\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^2-24\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)+3}}{18}+\frac{\sqrt{3}\ln\left(\sqrt{4}\,x+\sqrt{4\left(\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/3*x-1/9*arctanh(x*3^(1/2))*3^(1/2)-1/18*(36*(x+1/3*3^(1/2))^2-24*3^(1/2)*
(x+1/3*3^(1/2))+3)^(1/2)+1/18*3^(1/2)*ln(4^(1/2)*x+(4*(x+1/3*3^(1/2))^2-8/3
*3^(1/2)*(x+1/3*3^(1/2))+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3-
8/3*3^(1/2)*(x+1/3*3^(1/2)))*3^(1/2)/(36*(x+1/3*3^(1/2))^2-24*3^(1/2)*(x+1/
3*3^(1/2))+3)^(1/2))-1/18*(36*(x-1/3*3^(1/2))^2+24*3^(1/2)*(x-1/3*3^(1/2))+
3)^(1/2)-1/18*3^(1/2)*ln(4^(1/2)*x+(4*(x-1/3*3^(1/2))^2+8/3*3^(1/2)*(x-1/3*
3^(1/2))+1/3)^(1/2))*4^(1/2)+1/18*3^(1/2)*arctanh(3/2*(2/3+8/3*3^(1/2)*(x-1
/3*3^(1/2)))*3^(1/2)/(36*(x-1/3*3^(1/2))^2+24*3^(1/2)*(x-1/3*3^(1/2))+3)^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)
```

Fricas [A]
time = 0.34, size = 80, normalized size = 1.23

$$\frac{1}{18}\sqrt{3}\log\left(\frac{6x^2+\sqrt{3}\sqrt{4x^2-1}-1}{3x^2-1}\right)+\frac{1}{18}\sqrt{3}\log\left(\frac{3x^2-2\sqrt{3}x+1}{3x^2-1}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*log((6*x^2 + sqrt(3)*sqrt(4*x^2 - 1) - 1)/(3*x^2 - 1)) + 1/18*
sqrt(3)*log((3*x^2 - 2*sqrt(3)*x + 1)/(3*x^2 - 1)) + 4/3*x - 1/3*sqrt(4*x^2
 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{(2x-1)(2x+1)}}{x+\sqrt{4x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)
```

```
[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(45) = 90.
time = 2.67, size = 133, normalized size = 2.05

$$\frac{1}{18}\sqrt{3}\log\left(\frac{\left|6x-2\sqrt{3}\right|}{\left|6x+2\sqrt{3}\right|}\right)-\frac{1}{18}\sqrt{3}\log\left(-\frac{\left|-12x-4\sqrt{3}+6\sqrt{4x^2-1}+\frac{6}{2x-\sqrt{4x^2-1}}\right|}{2\left(6x-2\sqrt{3}-3\sqrt{4x^2-1}-\frac{3}{2x-\sqrt{4x^2-1}}\right)}\right)+\frac{4}{3}x-\frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(3)*log(abs(6*x - 2*sqrt(3))/abs(6*x + 2*sqrt(3))) - 1/18*sqrt(3)*
log(-1/2*abs(-12*x - 4*sqrt(3) + 6*sqrt(4*x^2 - 1) + 6/(2*x - sqrt(4*x^2 -
1)))/(6*x - 2*sqrt(3) - 3*sqrt(4*x^2 - 1) - 3/(2*x - sqrt(4*x^2 - 1)))) + 4
/3*x - 1/3*sqrt(4*x^2 - 1)
```

Mupad [B]
time = 3.44, size = 60, normalized size = 0.92

$$\frac{4\,x}{3}+\frac{\sqrt{3}\,\ln\left(x-\frac{\sqrt{3}}{3}\right)}{18}-\frac{\sqrt{3}\,\ln\left(x+\frac{\sqrt{3}}{3}\right)}{18}+\frac{\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{4\,x^2-1}\right)}{9}-\frac{\sqrt{4\,x^2-1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)
```

```
[Out] (4*x)/3 + (3^(1/2)*log(x - 3^(1/2)/3))/18 - (3^(1/2)*log(x + 3^(1/2)/3))/18
 + (3^(1/2)*atanh(3^(1/2)*(4*x^2 - 1)^(1/2)))/9 - (4*x^2 - 1)^(1/2)/3
```

3.832 $\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\,(d^2-e^2)\,(d+ex)^2}+\frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-1+x^2}}{2e\,(d^2-e^2)^2\,(d+ex)}-\frac{(3bde-a(2d^2+e^2)-$$

```
[Out] -1/2*(3*b*d*e-a*(2*d^2+e^2)-c*(d^2+2*e^2))*arctanh((d*x+e)/(d^2-e^2)^(1/2)/
(x^2-1)^(1/2))/(d^2-e^2)^(5/2)-1/2*(a*e^2-b*d*e+c*d^2)*(x^2-1)^(1/2)/e/(d^2
-e^2)/(e*x+d)^2+1/2*(c*(d^3-4*d*e^2)-e*(3*a*d*e-b*(d^2+2*e^2)))*(x^2-1)^(1/
2)/e/(d^2-e^2)^2/(e*x+d)
```

Rubi [A]
time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1665, 821, 739, 212}

$$-\frac{\sqrt{x^2-1}\,(ae^2-bde+cd^2)}{2e\,(d^2-e^2)\,(d+ex)^2}-\frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\,\sqrt{d^2-e^2}}\right)(-a(2d^2+e^2)+3bde-c(d^2+2e^2))}{2\,(d^2-e^2)^{5/2}}+\frac{\sqrt{x^2-1}\,(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))}{2e\,(d^2-e^2)^2\,(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] -1/2*((c*d^2 - b*d*e + a*e^2)*Sqrt[-1 + x^2])/(e*(d^2 - e^2)*(d + e*x)^2) +
 ((c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*Sqrt[-1 + x^2])/(2*e*
(d^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*A
rcTanh[(e + d*x)/(Sqrt[d^2 - e^2]*Sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}}\,dx = -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\,(d^2-e^2)\,(d+ex)^2} - \frac{\int \frac{-2(ad+cd-be)-\left(bd+\frac{cd^2}{e}-ae-2ce\right)x}{(d+ex)^2\sqrt{-1+x^2}}\,dx}{2\,(d^2-e^2)}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\,(d^2-e^2)\,(d+ex)^2} + \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-}}{2e\,(d^2-e^2)^2\,(d+ex)}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\,(d^2-e^2)\,(d+ex)^2} + \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-}}{2e\,(d^2-e^2)^2\,(d+ex)}$$

$$= -\frac{(cd^2-bde+ae^2)\sqrt{-1+x^2}}{2e\,(d^2-e^2)\,(d+ex)^2} + \frac{(c(d^3-4de^2)-e(3ade-b(d^2+2e^2)))\sqrt{-}}{2e\,(d^2-e^2)^2\,(d+ex)}$$

Mathematica [A]
time = 1.02, size = 179, normalized size = 0.92

$$\frac{\frac{(d-e)(d+e)\sqrt{-1+x^2}\,(ae(-4d^2+e^2-3dex)+cd(-3de+d^2x-4e^2x)+b(2d^3+de^2+d^2ex+2e^3x))}{(d+ex)^2}+2\sqrt{-d^2+e^2}\,(-3bde+a(2d^2+e^2)+c(d^2+2e^2))\tan^{-1}\left(\frac{d+e\left(x-\sqrt{-1+x^2}\right)}{\sqrt{-d^2+e^2}}\right)}{2(d-e)^3(d+e)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]),x]
```

```
[Out] (((d - e)*(d + e)*Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*
e + d^2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/(d + e*x)^2
```

```
+ 2*Sqrt[-d^2 + e^2]*(-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*ArcTan[
(d + e*(x - Sqrt[-1 + x^2]))/Sqrt[-d^2 + e^2]])/(2*(d - e)^3*(d + e)^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 728 vs. $2(179) = 358$.

time = 0.52, size = 729, normalized size = 3.74

method	result
default	$-\frac{c \ln\left(\frac{\frac{2d^2-2e^2}{e^2}-\frac{2d\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{d^2-e^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d\left(x+\frac{d}{e}\right)}{e}+\frac{d^2-e^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^3\sqrt{\frac{d^2-e^2}{e^2}}}+\frac{(eb-2cd)\left(-\frac{e^2\sqrt{\left(x+\frac{d}{e}\right)^2-\frac{2d(x+}{e}}}{(d^2-e^2)(x+\frac{d}{e}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -c/e^3/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)
/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))+(b*e-2*
c*d)/e^4*(-1/(d^2-e^2)*e^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^
(1/2)-d*e/(d^2-e^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)
+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+
d/e)))+(a*e^2-b*d*e+c*d^2)/e^5*(-1/2/(d^2-e^2)*e^2/(x+d/e)^2*((x+d/e)^2-2*d
/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)+3/2*d*e/(d^2-e^2)*(-1/(d^2-e^2)*e^2/(x+d/e)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)-d*e/(d^2-e^2)/((d^2-e^2)/e^2
)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d/e)^
2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e)))+1/2/(d^2-e^2)*e^2/((d^2-e^2
)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+
d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e)))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(d-%e>0)', see `assume?` for more de
tails)I
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(175) = 350$.
time = 0.40, size = 1122, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(c*d^7 - 2*b*x^2*e^7 + ((2*a + c)*d^4*e^2 + (a + 2*c)*x^2*e^6 - (3*b*d
*x^2 - 2*(a + 2*c)*d*x)*e^5 + ((2*a + c)*d^2*x^2 - 6*b*d^2*x + (a + 2*c)*d^
2)*e^4 + (2*(2*a + c)*d^3*x - 3*b*d^3)*e^3)*sqrt(d^2 - e^2)*log((d^2*x + d*
e + sqrt(d^2 - e^2)*(d*x + e) + (d^2 + sqrt(d^2 - e^2)*d - e^2)*sqrt(x^2 -
1))/(x*e + d)) + ((3*a + 4*c)*d*x^2 - 4*b*d*x)*e^6 + (b*d^2*x^2 + 2*(3*a +
4*c)*d^2*x - 2*b*d^2)*e^5 - ((3*a + 5*c)*d^3*x^2 - 2*b*d^3*x - (3*a + 4*c)*
d^3)*e^4 + (b*d^4*x^2 - 2*(3*a + 5*c)*d^4*x + b*d^4)*e^3 + (c*d^5*x^2 + 2*b
*d^5*x - (3*a + 5*c)*d^5)*e^2 + (2*c*d^6*x + b*d^6)*e - sqrt(x^2 - 1)*((2*b
*x + a)*e^7 - ((3*a + 4*c)*d*x - b*d)*e^6 - (b*d^2*x + (5*a + 3*c)*d^2)*e^5
 + ((3*a + 5*c)*d^3*x + b*d^3)*e^4 - (b*d^4*x - (4*a + 3*c)*d^4)*e^3 - (c*d
^5*x + 2*b*d^5)*e^2))/(2*d^7*x*e^3 + d^8*e^2 - 6*d^5*x*e^5 + 6*d^3*x*e^7 -
x^2*e^10 - 2*d*x*e^9 + (3*d^2*x^2 - d^2)*e^8 - 3*(d^4*x^2 - d^4)*e^6 + (d^6
*x^2 - 3*d^6)*e^4), 1/2*(c*d^7 - 2*b*x^2*e^7 - 2*((2*a + c)*d^4*e^2 + (a +
2*c)*x^2*e^6 - (3*b*d*x^2 - 2*(a + 2*c)*d*x)*e^5 + ((2*a + c)*d^2*x^2 - 6*b
*d^2*x + (a + 2*c)*d^2)*e^4 + (2*(2*a + c)*d^3*x - 3*b*d^3)*e^3)*sqrt(-d^2
+ e^2)*arctan(sqrt(-d^2 + e^2)*(x*e - sqrt(x^2 - 1)*e + d)/(d^2 - e^2)) + (
(3*a + 4*c)*d*x^2 - 4*b*d*x)*e^6 + (b*d^2*x^2 + 2*(3*a + 4*c)*d^2*x - 2*b*d
^2)*e^5 - ((3*a + 5*c)*d^3*x^2 - 2*b*d^3*x - (3*a + 4*c)*d^3)*e^4 + (b*d^4*
x^2 - 2*(3*a + 5*c)*d^4*x + b*d^4)*e^3 + (c*d^5*x^2 + 2*b*d^5*x - (3*a + 5*
c)*d^5)*e^2 + (2*c*d^6*x + b*d^6)*e - sqrt(x^2 - 1)*((2*b*x + a)*e^7 - ((3*
a + 4*c)*d*x - b*d)*e^6 - (b*d^2*x + (5*a + 3*c)*d^2)*e^5 + ((3*a + 5*c)*d^
3*x + b*d^3)*e^4 - (b*d^4*x - (4*a + 3*c)*d^4)*e^3 - (c*d^5*x + 2*b*d^5)*e^
2))/(2*d^7*x*e^3 + d^8*e^2 - 6*d^5*x*e^5 + 6*d^3*x*e^7 - x^2*e^10 - 2*d*x*e
^9 + (3*d^2*x^2 - d^2)*e^8 - 3*(d^4*x^2 - d^4)*e^6 + (d^6*x^2 - 3*d^6)*e^4)
]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx+cx^2}{\sqrt{(x-1)(x+1)}\,(d+ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(175) = 350.
time = 1.74, size = 536, normalized size = 2.75

$$\frac{(2ad^2+cd^2-3bde+ae^2+2ce^2)\arctan\left(-\frac{(x-\sqrt{x^2-1})e+d}{\sqrt{-d^2+e^2}}\right)}{(d^4-2d^2e^2+e^4)\sqrt{-d^2+e^2}}+\frac{2cd^4\left(x-\sqrt{x^2-1}\right)^3e+2cd^5\left(x-\sqrt{x^2-1}\right)^2+2bd^4\left(x-\sqrt{x^2-1}\right)^2e-2ad^2\left(x-\sqrt{x^2-1}\right)^3e^3-5cd^2\left(x-\sqrt{x^2-1}\right)^3e^3-6ad^3\left(x-\sqrt{x^2-1}\right)^2e^2-7cd^3\left(x-\sqrt{x^2-1}\right)^2e^2+2cd^4\left(x-\sqrt{x^2-1}\right)e+3bd\left(x-\sqrt{x^2-1}\right)^3e^4+5bd^2\left(x-\sqrt{x^2-1}\right)^2e^3+4bd^3\left(x-\sqrt{x^2-1}\right)e^2-a\left(x-\sqrt{x^2-1}\right)^3e^5-3ad\left(x-\sqrt{x^2-1}\right)^2e^4-4cd\left(x-\sqrt{x^2-1}\right)^2e^4-10ad^2\left(x-\sqrt{x^2-1}\right)e^3-11cd^2\left(x-\sqrt{x^2-1}\right)e^3+cd^3e^2+2b\left(x-\sqrt{x^2-1}\right)^2e^5+5bd\left(x-\sqrt{x^2-1}\right)e^4+bd^2e^3+a\left(x-\sqrt{x^2-1}\right)e^5-3ade^4-4cde^4+2be^5}{(d^4e^2-2d^2e^4+e^6)\left(\left(x-\sqrt{x^2-1}\right)^2e+2d\left(x-\sqrt{x^2-1}\right)+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(x^2 - 1))*
e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*
d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*(x -
sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(
x^2 - 1))^3*e^3 - 6*a*d^3*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2
 - 1))^2*e^2 + 2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*
e^4 + 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*e^2 -
 a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e^4 - 4*c*d*(x -
 sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1))*e^3 - 11*c*d^2*(x - sq
rt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x - sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - s
qrt(x^2 - 1))*e^4 + b*d^2*e^3 + a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c
*d*e^4 + 2*b*e^5)/((d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2
*d*(x - sqrt(x^2 - 1)) + e)^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c\,x^2+b\,x+a}{\sqrt{x^2-1}\,{\left(d+e\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

3.833 $\int \frac{1+2x^8}{x(1+x^8)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {457, 79, 65, 213}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x^8}{x\left(1+x^8\right)^{3/2}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right)\\ &= -\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A]
time = 0.53, size = 27, normalized size = 0.96

method	result	size
trager	$-\frac{1}{4\sqrt{x^8+1}}+\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}$	29

meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{x^8+1}}-\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{x^8+1}}{2}\right)+\frac{(2-2\ln(2)+8\ln(x))\sqrt{\pi}}{2}}{4\sqrt{\pi}}+\frac{\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{x^8+1}}}{2\sqrt{\pi}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/x^4)
```

Maxima [A]
time = 0.51, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.
time = 0.34, size = 52, normalized size = 1.86

$$-\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8(x^8+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [A]
time = 10.75, size = 37, normalized size = 1.32

$$\frac{\log\left(\sqrt{x^8+1}-1\right)}{8}-\frac{\log\left(\sqrt{x^8+1}+1\right)}{8}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)
[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1)
)
```

Giac [A]
time = 2.80, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Mupad [B]
time = 3.85, size = 20, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4}-\frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)
[Out] - atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))
```

3.834 $\int \frac{\sqrt{1+x^8}\,(1+2x^8)}{x+2x^9+x^{17}}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{1+x^8}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

```
[Out] -1/4*arctanh((x^8+1)^(1/2))-1/4/(x^8+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1600, 1607, 457, 79, 65, 213}

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{4}\tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
```

```
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+x^8}\,(1+2x^8)}{x+2x^9+x^{17}}\,dx &= \int \frac{1+2x^8}{\sqrt{1+x^8}\,(x+x^9)}\,dx \\
&= \int \frac{1+2x^8}{x\,(1+x^8)^{3/2}}\,dx \\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{x(1+x)^{3/2}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^8\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^8}\right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4}\tanh^{-1}\left(\sqrt{1+x^8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17),x]
[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4
```

Maple [A]
time = 0.51, size = 27, normalized size = 0.96

method	result	size
trager	$-\frac{1}{4\sqrt{x^8+1}}-\frac{\ln\left(\frac{\sqrt{x^8+1}+1}{x^4}\right)}{4}$	27
risch	$-\frac{1}{4\sqrt{x^8+1}}+\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^8+1)^(1/2)-1/4*ln(((x^8+1)^(1/2)+1)/x^4)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")
[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.
time = 0.35, size = 52, normalized size = 1.86

$$-\frac{(x^8+1)\log\left(\sqrt{x^8+1}+1\right)-(x^8+1)\log\left(\sqrt{x^8+1}-1\right)+2\sqrt{x^8+1}}{8\,(x^8+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="fricas")
[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) +
 2*sqrt(x^8 + 1))/(x^8 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{2x^8+1}{x\,(x^8+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)
```

```
[Out] Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)
```

Giac [A]
time = 2.44, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}}-\frac{1}{8}\log\left(\sqrt{x^8+1}+1\right)+\frac{1}{8}\log\left(\sqrt{x^8+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")
```

```
[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^8+1}\,\left(2\,x^8+1\right)}{x^{17}+2\,x^9+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)
```

```
[Out] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)
```

3.835 $\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx$

Optimal. Leaf size=22

$$x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {267}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)\,dx &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A]
time = 0.51, size = 19, normalized size = 0.86

method	result	size
default	$x-3x^{3}-\frac{\sqrt{-9x^{2}+1}}{9}$	19
trager	$-\left(3x^{2}-1\right)x-\frac{\sqrt{-9x^{2}+1}}{9}$	23
risch	$-3x^{3}+x+\frac{9x^{2}-1}{9\sqrt{-9x^{2}+1}}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.82

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.77

$$-3x^{3}+x-\frac{\sqrt{1-9x^{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Giac [A]
time = 2.97, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Mupad [B]
time = 0.04, size = 18, normalized size = 0.82

$$x - 3x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)
```

```
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.836 $\int \frac{x+\left(1-9x^2\right)^{3/2}}{\sqrt{1-9x^2}}\,dx$

Optimal. Leaf size=22

$$x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6874, 267}

$$-3x^3-\frac{1}{9}\sqrt{1-9x^2}+x$$

Antiderivative was successfully verified.

```
[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}}\,dx &= \int \left(1-9x^2+\frac{x}{\sqrt{1-9x^2}}\right)dx\\ &= x-3x^3+\int \frac{x}{\sqrt{1-9x^2}}\,dx\\ &= x-3x^3-\frac{1}{9}\sqrt{1-9x^2}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 22, normalized size = 1.00

$$x-3x^3-\frac{1}{9}\sqrt{1-9x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2],x]
```

```
[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9
```

Maple [A]
time = 0.56, size = 19, normalized size = 0.86

method	result	size
default	$x-3x^{3}-\frac{\sqrt{-9x^{2}+1}}{9}$	19
trager	$-\left(3x^{2}-1\right)x-\frac{\sqrt{-9x^{2}+1}}{9}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.82

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Fricas [A]
time = 0.35, size = 18, normalized size = 0.82

$$-3\,x^{3}+x-\frac{1}{9}\,\sqrt{-9\,x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Sympy [A]
time = 0.25, size = 17, normalized size = 0.77

$$-3x^{3}+x-\frac{\sqrt{1-9x^{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2),x)
```

```
[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9
```

Giac [A]
time = 2.32, size = 18, normalized size = 0.82

$$-3\,x^3+x-\frac{1}{9}\,\sqrt{-9\,x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)
```

Mupad [B]
time = 0.03, size = 18, normalized size = 0.82

$$x-3\,x^3-\frac{\sqrt{\frac{1}{9}-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2),x)
```

```
[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3
```

3.837 $\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A]
time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2059, 643}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(-3+2\sqrt{x}\right)\left(-3\sqrt{x}+x\right)^{2/3}}{\sqrt{x}}\,dx &= 2\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)\\ &= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x],x]
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [A]
time = 0.54, size = 12, normalized size = 0.71

method	result
derivativedivides	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$
default	$\frac{6\left(x-3\sqrt{x}\right)^{\frac{5}{3}}}{5}$
meijerg	$-\frac{18\,3^{\frac{2}{3}}\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{5}{6}}\operatorname{hypergeom}\left(\left[-\frac{2}{3},\frac{5}{3}\right],\left[\frac{8}{3}\right],\frac{\sqrt{x}}{3}\right)}{5\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{2}{3}} x^{\frac{4}{3}}\operatorname{hypergeom}\left(\left[\right.\right.}{2\left(-\operatorname{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="maxima")
[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)
```

Fricas [A]
time = 0.41, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.
time = 0.27, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}+\frac{6x\left(-3\sqrt{x}+x\right)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)
```

```
[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5
```

Giac [A]
time = 3.26, size = 11, normalized size = 0.65

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Mupad [B]
time = 3.70, size = 11, normalized size = 0.65

$$\frac{6\left(x-3\sqrt{x}\right)^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2),x)
```

```
[Out] (6*(x - 3*x^(1/2))^(5/3))/5
```

3.838 $\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx$

Optimal. Leaf size=17

$$\frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

```
[Out] 6/5*(x-3*x^(1/2))^(5/3)
```

Rubi [A]
time = 0.03, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2068, 1645, 643}

$$\frac{6}{5}\left(x-3\sqrt{x}\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```

```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 2068

```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{d =
 Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x ->
 x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x]] /; FreeQ[{a, b, j
, n, p}, x] && PolyQ[Pq, x^n] &&  !IntegerQ[p] && NeQ[n, j] && RationalQ[j,
 n] && IntegerQ[j/n] && LtQ[-1, n, 1]
```

Rubi steps

$$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}}\,dx = 2\text{Subst}\left(\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx, x, \sqrt{x}\right)$$
$$= \frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 1.00

$$\frac{6}{5}\left(-3\sqrt{x}+x\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3),x]
```
```
[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 2.
time = 0.56, size = 125, normalized size = 7.35

method	result
meijerg	$\frac{18\,3^{\frac{2}{3}}\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{5}{6}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{5}{3}\right],\left[\frac{8}{3}\right],\frac{\sqrt{x}}{3}\right)}{5\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}+\frac{4\,3^{\frac{2}{3}}\left(-\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)\right)^{\frac{1}{3}} x^{\frac{11}{6}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\cdots\right.\right.}{11\,\mathrm{signum}\left(-1+\frac{\sqrt{x}}{3}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x,method=_RETURNVERBOSE)
```
```
[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x
^(5/6)*hypergeom([1/3,5/3],[8/3],1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^
(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3,11/3],
[14/3],1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/
3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3,8/3],[11/3],1/3*x^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="maxima")
[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)
```

Fricas [A]
time = 0.40, size = 11, normalized size = 0.65

$$\frac{6}{5} \left(x - 3 \sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="fricas")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9\sqrt{x} + 2x + 9}{\sqrt[3]{-3\sqrt{x} + x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3),x)
[Out] Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)
```

Giac [A]
time = 1.65, size = 11, normalized size = 0.65

$$\frac{6}{5} \left(x - 3 \sqrt{x}\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3),x, algorithm="giac")
[Out] 6/5*(x - 3*sqrt(x))^(5/3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2\,x - 9\,\sqrt{x} + 9}{{\left(x - 3\,\sqrt{x}\right)}^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3),x)
[Out] int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)
```

3.839 $\int \frac{1}{\sqrt{4-9x^2}} \, dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4-9x^2}} \, dx = \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 24, normalized size = 2.40

$$\frac{1}{3}i\log\left(-3ix+\sqrt{4-9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 - 9*x^2],x]
```

```
[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]
```

Maple [A]
time = 0.53, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-9*x^2+4)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*arcsin(3/2*x)
```

Maxima [A]
time = 0.50, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.
time = 0.34, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A]
time = 0.05, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x**2+4)**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.
time = 2.67, size = 19, normalized size = 1.90

$$\frac{1}{2}\sqrt{-9x^2+4}\,x+\frac{2}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.60

$$\frac{\mathrm{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4 - 9*x^2)^(1/2),x)
[Out] asin((3*x)/2)/3
```

3.840 $\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.00, size = 24, normalized size = 2.40

$$\frac{1}{3}i\log\left(-3ix+\sqrt{4-9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(6) = 12$.
time = 0.52, size = 34, normalized size = 3.40

method	result	size
default	$\frac{\sqrt{(2-3x)(2+3x)}\,\arcsin\left(\frac{3x}{2}\right)}{3\sqrt{2-3x}\,\sqrt{2+3x}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((2-3*x)*(2+3*x))^(1/2)/(2-3*x)^(1/2)/(2+3*x)^(1/2)*arcsin(3/2*x)
```

Maxima [A]
time = 0.55, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(6) = 12$.
time = 0.35, size = 25, normalized size = 2.50

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{3\,x+2}\,\sqrt{-3\,x+2}-2}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*arctan(1/3*(sqrt(3*x + 2)*sqrt(-3*x + 2) - 2)/x)
```

Sympy [C] Result contains complex when optimal does not.

time = 0.55, size = 49, normalized size = 4.90

$$\begin{cases} -\frac{2i\operatorname{acosh}\left(\frac{\sqrt{3}\,\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{for } \left|x+\frac{2}{3}\right| > \frac{4}{3} \\ \frac{2\operatorname{asin}\left(\frac{\sqrt{3}\,\sqrt{x+\frac{2}{3}}}{2}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, Abs(x + 2/3) > 4/3), (2*a
sin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

Giac [A]
time = 3.25, size = 12, normalized size = 1.20

$$\frac{2}{3}\arcsin\left(\frac{1}{2}\sqrt{3\,x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arcsin(1/2*sqrt(3*x + 2))
```

Mupad [B]
time = 0.15, size = 32, normalized size = 3.20

$$-\frac{4\operatorname{atan}\left(\frac{\sqrt{2}-\sqrt{2-3\,x}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

3.841 $\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)$$

```
[Out] 1/3*arcsin(3/2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1976, 222}

$$\frac{1}{3}\text{ArcSin}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
```

```
[Out] ArcSin[(3*x)/2]/3
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(2-3x)(2+3x)}}\,dx &= \int \frac{1}{\sqrt{4-9x^2}}\,dx\\ &= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.00, size = 24, normalized size = 2.40

$$\frac{1}{3}i\log\left(-3ix+\sqrt{4-9x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]
[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]
```

Maple [A]
time = 0.51, size = 7, normalized size = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
meijerg	$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$	7
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-9x^2+4}+3x\right)}{3}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2-3*x)*(2+3*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*arcsin(3/2*x)
```

Maxima [A]
time = 0.49, size = 6, normalized size = 0.60

$$\frac{1}{3}\arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")
[Out] 1/3*arcsin(3/2*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.
time = 0.39, size = 19, normalized size = 1.90

$$-\frac{2}{3}\arctan\left(\frac{\sqrt{-9\,x^2+4}-2}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")
[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)
```

Sympy [A]
time = 0.65, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)
```

```
[Out] asin(3*x/2)/3
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.
time = 3.58, size = 19, normalized size = 1.90

$$\frac{1}{2}\sqrt{-9\,x^2+4}\,x+\frac{2}{3}\,\arcsin\left(\frac{3}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-9*x^2 + 4)*x + 2/3*arcsin(3/2*x)
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3\,x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)
```

```
[Out] asin((3*x)/2)/3
```

3.842 $\int \frac{1}{\sqrt{15-2x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A]
time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$-\text{ArcSin}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{15-2x-x^2}}\,dx &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 23, normalized size = 1.92

$$-2\tan^{-1}\left(\frac{\sqrt{15-2x-x^2}}{5+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[15 - 2*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]
```

Maple [A]
time = 0.60, size = 7, normalized size = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)-\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-2x+15}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+15)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A]
time = 0.49, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.
time = 0.35, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2\,x+15}\,(x+1)}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 2x + 15}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 3.12, size = 26, normalized size = 2.17

$$\frac{1}{2}\sqrt{-x^2 - 2\,x + 15}\,(x+1) + 8\,\arcsin\left(\frac{1}{4}\,x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+15)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

Mupad [B]
time = 3.12, size = 6, normalized size = 0.50

$$\mathrm{asin}\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15 - x^2 - 2*x)^(1/2),x)
```

```
[Out] asin(x/4 + 1/4)
```

3.843 $\int \frac{1}{\sqrt{3-x}\,\sqrt{5+x}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A]

time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$-\text{ArcSin}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x}\,\sqrt{5+x}}\,dx = \int \frac{1}{\sqrt{15-2x-x^2}}\,dx$$
$$= -\left(\frac{1}{8}\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)$$
$$= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. $2(12) = 24$.
time = 0.03, size = 44, normalized size = 3.67

$$\frac{2\sqrt{-3+x}\,\sqrt{5+x}\,\tanh^{-1}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3 +
 x)*(5 + x))]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.
time = 0.52, size = 31, normalized size = 2.58

method	result	size
default	$\frac{\sqrt{(-x+3)(5+x)}\,\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)}{\sqrt{-x+3}\,\sqrt{5+x}}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-x+3)*(5+x))^(1/2)/(-x+3)^(1/2)/(5+x)^(1/2)*arcsin(1/4+1/4*x)
```

Maxima [A]
time = 0.51, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.
time = 0.34, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+1)\sqrt{-x+3}}{x^2+2\,x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 39, normalized size = 3.25

$$\begin{cases} -2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } |x+5|>8 \\ 2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5) > 8), (2*asin(sqrt
(2)*sqrt(x + 5)/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. $2(6) = 12$.
time = 2.70, size = 13, normalized size = 1.08

$$2\arcsin\left(\frac{1}{4}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))
```

Mupad [B]
time = 3.43, size = 30, normalized size = 2.50

$$4\operatorname{atan}\left(\frac{\sqrt{3}-\sqrt{3-x}}{\sqrt{x+5}-\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
```

```
[Out] 4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.844 $\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

```
[Out] arcsin(1/4+1/4*x)
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1976, 633, 222}

$$-\text{ArcSin}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] -ArcSin[(-1 - x)/4]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(3-x)(5+x)}}\,dx = \int \frac{1}{\sqrt{15-2x-x^2}}\,dx$$
$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}}\,dx, x, -2-2x\right)\right)$$
$$= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.
time = 0.00, size = 44, normalized size = 3.67

$$\frac{2\sqrt{-3+x}\,\sqrt{5+x}\,\tanh^{-1}\left(\frac{\sqrt{5+x}}{\sqrt{-3+x}}\right)}{\sqrt{-((-3+x)(5+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(3 - x)*(5 + x)],x]
```

```
[Out] (2*Sqrt[-3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[5 + x]/Sqrt[-3 + x]])/Sqrt[-((-3 + 
 x)*(5 + x))]
```

Maple [A]
time = 0.57, size = 7, normalized size = 0.58

method	result	size
default	$\arcsin\left(\frac{1}{4}+\frac{x}{4}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)-\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-2x+15}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-x+3)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(1/4+1/4*x)
```

Maxima [A]
time = 0.51, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}\,x-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/4*x - 1/4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.
time = 0.37, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2-2x+15}\,(x+1)}{x^2+2x-15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(3-x)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((3 - x)*(x + 5)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.
time = 3.17, size = 26, normalized size = 2.17

$$\frac{1}{2}\sqrt{-x^2-2x+15}\,(x+1)+8\arcsin\left(\frac{1}{4}x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 2*x + 15)*(x + 1) + 8*arcsin(1/4*x + 1/4)
```

Mupad [B]
time = 3.37, size = 6, normalized size = 0.50

$$\operatorname{asin}\left(\frac{x}{4}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x - 3)*(x + 5))^(1/2),x)
```

```
[Out] asin(x/4 + 1/4)
```

3.845 $\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A]
time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {633, 222}

$$\text{ArcSin}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-15-8x-x^2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\ &= \sin^{-1}(4+x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(4) = 8.
time = 0.05, size = 23, normalized size = 5.75

$$-2\tan^{-1}\left(\frac{\sqrt{-15-8x-x^2}}{5+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-15 - 8*x - x^2],x]
```

```
[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]
```

Maple [A]
time = 0.56, size = 5, normalized size = 1.25

method	result	size
default	$\arcsin(x+4)$	5
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)-4\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-8x-15}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-8*x-15)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(x+4)
```

Maxima [A]
time = 0.52, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.
time = 0.34, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8\,x-15}\,(x+4)}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-x^2-8x-15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-8*x-15)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.
time = 2.49, size = 24, normalized size = 6.00

$$\frac{1}{2}\sqrt{-x^2 - 8x - 15}(x + 4) + \frac{1}{2}\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

Mupad [B]
time = 3.18, size = 4, normalized size = 1.00

$$\mathrm{asin}(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(- 8*x - x^2 - 15)^(1/2),x)
```

```
[Out] asin(x + 4)
```

3.846 $\int \frac{1}{\sqrt{-3-x}\,\sqrt{5+x}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {55, 633, 222}

$$\text{ArcSin}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
[Out] ArcSin[4 + x]
```

Rule 55

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-x}\,\sqrt{5+x}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\
&= \sin^{-1}(4+x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(4) = 8.
time = 0.03, size = 44, normalized size = 11.00

$$\frac{2\sqrt{3+x}\sqrt{5+x}\tanh^{-1}\left(\frac{\sqrt{3+x}}{\sqrt{5+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]
```

```
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[3 + x]/Sqrt[5 + x]])/Sqrt[-((3 + x)
*(5 + x))]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(4) = 8.
time = 0.62, size = 29, normalized size = 7.25

method	result	size
default	$\frac{\sqrt{(-3-x)(5+x)}\,\arcsin(x+4)}{\sqrt{-3-x}\,\sqrt{5+x}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3-x)^(1/2)/(5+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-3-x)*(5+x))^(1/2)/(-3-x)^(1/2)/(5+x)^(1/2)*arcsin(x+4)
```

Maxima [A]
time = 0.49, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.
time = 0.35, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{x+5}\,(x+4)\sqrt{-x-3}}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.49, size = 39, normalized size = 9.75

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } |x+5| > 2 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5) > 2), (2*asin(sqrt
(2)*sqrt(x + 5)/2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(4) = 8.
time = 2.86, size = 13, normalized size = 3.25

$$2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))
```

Mupad [B]
time = 0.08, size = 33, normalized size = 8.25

$$4 \operatorname{atan}\left(\frac{-\sqrt{-x-3} + \sqrt{3}\ 1\mathrm{i}}{\sqrt{x+5} - \sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- x - 3)^(1/2)*(x + 5)^(1/2)),x)
```

```
[Out] 4*atan((3^(1/2)*1i - (- x - 3)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

3.847 $\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx$

Optimal. Leaf size=4

$$\sin^{-1}(4+x)$$

```
[Out] arcsin(4+x)
```

Rubi [A]

time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1976, 633, 222}

$$\text{ArcSin}(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] ArcSin[4 + x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(-3-x)(5+x)}}\,dx &= \int \frac{1}{\sqrt{-15-8x-x^2}}\,dx\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -8-2x\right)\right)\\
&= \sin^{-1}(4+x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 44 vs. 2(4) = 8.
time = 0.00, size = 44, normalized size = 11.00

$$\frac{2\sqrt{3+x}\,\sqrt{5+x}\,\tanh^{-1}\left(\frac{\sqrt{3+x}}{\sqrt{5+x}}\right)}{\sqrt{-((3+x)(5+x))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)],x]
```

```
[Out] (2*Sqrt[3 + x]*Sqrt[5 + x]*ArcTanh[Sqrt[3 + x]/Sqrt[5 + x]])/Sqrt[-((3 + x)
*(5 + x))]
```

Maple [A]
time = 0.58, size = 5, normalized size = 1.25

method	result	size
default	$\arcsin(x+4)$	5
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\mathrm{RootOf}\left(\_Z^2+1\right)-4\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-x^2-8x-15}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-3-x)*(5+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(x+4)
```

Maxima [A]
time = 0.50, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x - 4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.
time = 0.33, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2-8\,x-15}\,(x+4)}{x^2+8\,x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-x-3)(x+5)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))**(1/2),x)
[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.
time = 2.37, size = 24, normalized size = 6.00

$$\frac{1}{2}\sqrt{-x^2-8x-15}\,(x+4)+\frac{1}{2}\arcsin(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 - 8*x - 15)*(x + 4) + 1/2*arcsin(x + 4)
```

Mupad [B]
time = 3.36, size = 4, normalized size = 1.00

$$\mathrm{asin}(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 3)*(x + 5))^(1/2),x)
[Out] asin(x + 4)
```

3.848 $\int \left(1-\sqrt{x}\right) \, dx$

Optimal. Leaf size=11

$$x-\frac{2 x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x-\frac{2 x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\int \left(1-\sqrt{x}\right) \, dx = x-\frac{2 x^{3/2}}{3}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2 x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.73

method	result	size

derivativedivides	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
default	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
risch	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
trager	$-1 + x - \frac{2x^{\frac{3}{2}}}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A]
time = 0.34, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A]
time = 2.41, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

Mupad [B]
time = 0.03, size = 7, normalized size = 0.64

$$x-\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
```

```
[Out] x - (2*x^(3/2))/3
```

3.849 $\int \frac{1-x}{1+\sqrt{x}} \, dx$

Optimal. Leaf size=11

$$x - \frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1412, 26, 45}

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x}{1+\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{x(1-x^2)}{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int (1-x)x\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(x-x^2\right)\,dx, x, \sqrt{x}\right)\\ &= x-\frac{2x^{3/2}}{3}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)/(1 + Sqrt[x]),x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A]
time = 0.52, size = 8, normalized size = 0.73

method	result	size
derivativedivides	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
default	$x-\frac{2x^{\frac{3}{2}}}{3}$	8
trager	$-1+x-\frac{2x^{\frac{3}{2}}}{3}$	9
meijerg	$2\sqrt{x}-\frac{\sqrt{x}\left(4x-6\sqrt{x}+12\right)}{6}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A]
time = 0.37, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A]
time = 2.50, size = 7, normalized size = 0.64

$$-\frac{2}{3}\,x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

Mupad [B]
time = 0.03, size = 7, normalized size = 0.64

$$x-\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^(1/2) + 1),x)
```

```
[Out] x - (2*x^(3/2))/3
```

3.850 $\int \sqrt{\frac{1}{1-x^2}}\,dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}}\,\sqrt{1-x^2}\,\sin^{-1}(x)$$

```
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1972, 222}

$$\sqrt{\frac{1}{1-x^2}}\,\sqrt{1-x^2}\,\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x^2)^(-1)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1}{1-x^2}}\,dx &= \left(\sqrt{\frac{1}{1-x^2}}\,\sqrt{1-x^2}\right)\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{\frac{1}{1-x^2}}\,\sqrt{1-x^2}\,\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.44

$$\sqrt{\frac{1}{1-x^2}}\,\sqrt{1-x^2}\,\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x^2)^(-1)],x]
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[x/Sqrt[1 - x^2]]
```

Maple [A]
time = 0.52, size = 30, normalized size = 1.11

method	result	s
meijerg	$\arcsin(x)\sqrt{\frac{1}{-x^2+1}}\,\sqrt{-x^2+1}$	2
default	$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	3
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}\,x^2+\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}+x\right)$	5

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(-x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-1/(x^2 - 1)), x)
```

Fricas [A]
time = 0.43, size = 26, normalized size = 0.96

$$2\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [A]
time = 0.86, size = 7, normalized size = 0.26

$$\begin{cases} \operatorname{asin}(x) & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x**2+1))**(1/2),x)
```

```
[Out] Piecewise((asin(x), (x > -1) & (x < 1)))
```

Giac [A]
time = 2.17, size = 10, normalized size = 0.37

$$-\arcsin(x)\operatorname{sgn}(x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(x)*sgn(x^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-\frac{1}{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/(x^2 - 1))^(1/2),x)
```

```
[Out] int((-1/(x^2 - 1))^(1/2), x)
```

3.851 $\int \sqrt{\frac{1+x^2}{1-x^4}} \, dx$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

```
[Out] arcsin(x)*(1/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6820, 1972, 222}

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \, \text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(1 - x^4)],x]
```

```
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1+x^2}{1-x^4}} \, dx &= \int \sqrt{\frac{1}{1-x^2}} \, dx \\
&= \left(\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2}\right) \int \frac{1}{\sqrt{1-x^2}} \, dx \\
&= \sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.44

$$\sqrt{\frac{1}{1-x^2}}\sqrt{1-x^2}\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)],x]
[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcTan[x/Sqrt[1 - x^2]]
```

Maple [A]
time = 0.08, size = 30, normalized size = 1.11

method	result
default	$\sqrt{-\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}\,x^2+\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{1}{x^2-1}}+x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(-x^4+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)
```

Fricas [A]
time = 0.37, size = 26, normalized size = 0.96

$$2\arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}}+1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^2+1}{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

Giac [A]
time = 2.77, size = 10, normalized size = 0.37

$$-\arcsin(x)\,\mathrm{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")
[Out] -arcsin(x)*sgn(x^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int((-(x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.852 $\int \sqrt{\frac{1}{-1+x^2}}\,dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\,\sin^{-1}(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1973, 222}

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\,\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^(-1)],x]
```

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1}{-1+x^2}}\,dx &= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx\\
&= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\
&= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.
time = 0.01, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^(-1)],x]
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A]
time = 0.58, size = 28, normalized size = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$\ln\left(\sqrt{\frac{1}{x^2-1}}\,x^2-\sqrt{\frac{1}{x^2-1}}+x\right)$	28
meijerg	$\frac{\sqrt{\frac{1}{x^2-1}}\,\sqrt{x^2-1}\,\sqrt{-\mathrm{signum}\left(x^2-1\right)}\,\arcsin(x)}{\sqrt{\mathrm{signum}\left(x^2-1\right)}}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2-1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [A]
time = 0.29, size = 14, normalized size = 0.56

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A]
time = 0.33, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [**A**]
time = 0.76, size = 15, normalized size = 0.60

$$\begin{cases} \log\left(x + \sqrt{x^2 - 1}\right) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x**2-1))**(1/2),x)
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

Giac [**A**]
time = 3.44, size = 26, normalized size = 1.04

$$\frac{1}{2}\sqrt{x^2 - 1}\,x + \frac{1}{2}\log\left(\left|-x + \sqrt{x^2 - 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^2 - 1)*x + 1/2*log(abs(-x + sqrt(x^2 - 1)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{1}{x^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2 - 1))^(1/2),x)
[Out] int((1/(x^2 - 1))^(1/2), x)
```

3.853 $\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx$

Optimal. Leaf size=25

$$\sqrt{1-x^2}\sqrt{\frac{1}{-1+x^2}}\,\sin^{-1}(x)$$

```
[Out] arcsin(x)*(-x^2+1)^(1/2)*(1/(x^2-1))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6820, 1973, 222}

$$\sqrt{1-x^2}\sqrt{\frac{1}{x^2-1}}\,\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] Sqrt[1 - x^2]*Sqrt[(-1 + x^2)^(-1)]*ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\int \sqrt{\frac{1+x^2}{-1+x^4}}\,dx = \int \sqrt{\frac{1}{-1+x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\int \frac{1}{\sqrt{-1+x^2}}\,dx$$
$$= \left(\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)$$
$$= \sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.
time = 0.00, size = 56, normalized size = 2.24

$$\frac{1}{2}\sqrt{\frac{1}{-1+x^2}}\sqrt{-1+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-1+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-1+x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)],x]
```

```
[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 +
 x/Sqrt[-1 + x^2]]))/2
```

Maple [A]
time = 0.08, size = 28, normalized size = 1.12

method	result	size
default	$\sqrt{\frac{1}{x^2-1}}\sqrt{x^2-1}\ \ln\left(x+\sqrt{x^2-1}\right)$	28
trager	$-\ln\left(-\sqrt{\frac{1}{x^2-1}}\,x^2+\sqrt{\frac{1}{x^2-1}}+x\right)$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+1)/(x^4-1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)
```

Fricas [A]
time = 0.35, size = 14, normalized size = 0.56

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="fricas")
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(x**4-1))**(1/2),x)
[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)
```

Giac [A]
time = 2.96, size = 21, normalized size = 0.84

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+1)/(x^4-1))^(1/2),x, algorithm="giac")
[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\sqrt{\frac{x^2+1}{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)/(x^4 - 1))^(1/2),x)
[Out] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)
```

3.854 $\int \frac{1}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

[Out] -2*(1-x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - x],x]

[Out] -2*Sqrt[1 - x]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - x],x]

[Out] -2*Sqrt[1 - x]

Maple [A]
time = 0.55, size = 10, normalized size = 0.91

method	result	size
gosper	$-2\sqrt{1-x}$	10
derivativedivides	$-2\sqrt{1-x}$	10
default	$-2\sqrt{1-x}$	10
trager	$-2\sqrt{1-x}$	10
risch	$\frac{-2+2x}{\sqrt{1-x}}$	13
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-x}}{\sqrt{\pi}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1-x)^(1/2)
```

Maxima [A]
time = 0.29, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x + 1)
```

Fricas [A]
time = 0.33, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x + 1)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2),x)
```

```
[Out] -2*sqrt(1 - x)
```

Giac [A]
time = 3.25, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x + 1)
```

Mupad [B]
time = 0.19, size = 9, normalized size = 0.82

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x)^(1/2),x)
```

```
[Out] -2*(1 - x)^(1/2)
```

3.855 $\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

```
[Out] -2*(1-x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {26, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] -2*Sqrt[1 - x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1-x}}\,dx\\ &= -2\sqrt{1-x} \end{aligned}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.82

$$-\frac{2\sqrt{1-x^2}}{\sqrt{1+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2],x]
```

```
[Out] (-2*Sqrt[1 - x^2])/Sqrt[1 + x]
```

Maple [A]
time = 0.56, size = 17, normalized size = 1.55

method	result	size
default	$-\frac{2\sqrt{-x^{2}+1}}{\sqrt{1+x}}$	17
gosper	$\frac{2(-1+x)\sqrt{1+x}}{\sqrt{-x^{2}+1}}$	20
risch	$\frac{2\sqrt{\frac{-x^{2}+1}{1+x}}\,\sqrt{1+x}\,(-1+x)}{\sqrt{-x^{2}+1}\,\sqrt{1-x}}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-x^2+1)^(1/2)/(1+x)^(1/2)
```

Maxima [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.30, size = 12, normalized size = 1.09

$$\frac{2\,(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)/sqrt(-x + 1)
```

Fricas [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.33, size = 16, normalized size = 1.45

$$-\frac{2\,\sqrt{-x^{2}+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A]
time = 2.60, size = 15, normalized size = 1.36

$$2\sqrt{2}-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(2) - 2*sqrt(-x + 1)
```

Mupad [B]
time = 3.56, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.856 $\int \frac{1}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=9

$$2\sqrt{1+x}$$

```
[Out] 2*(1+x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{1+x}$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 1.00

$$2\sqrt{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Maple [A]
time = 0.54, size = 8, normalized size = 0.89

method	result	size
gosper	$2\sqrt{1+x}$	8
derivativedivides	$2\sqrt{1+x}$	8
default	$2\sqrt{1+x}$	8
trager	$2\sqrt{1+x}$	8
risch	$2\sqrt{1+x}$	8
meijerg	$\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+x}}{\sqrt{\pi}}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [A]
time = 0.32, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1)
```

Sympy [A]
time = 0.01, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2),x)
```

```
[Out] 2*sqrt(x + 1)
```

Giac [A]
time = 3.85, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(x + 1)
```

Mupad [B]
time = 0.09, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1)^(1/2),x)
[Out] 2*(x + 1)^(1/2)
```

3.857 $\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=9

$$2\sqrt{1+x}$$

[Out] 2*(1+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] 2*Sqrt[1 + x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{\sqrt{1-x^2}}\,dx &= \int \frac{1}{\sqrt{1+x}}\,dx\\ &= 2\sqrt{1+x} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 0.03, size = 22, normalized size = 2.44

$$\frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2],x]
```

```
[Out] (2*Sqrt[1 - x^2])/Sqrt[1 - x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.
time = 0.56, size = 24, normalized size = 2.67

method	result	size
gosper	$\frac{2(1+x)\sqrt{1-x}}{\sqrt{-x^2+1}}$	22
default	$-\frac{2\sqrt{1-x}\,\sqrt{-x^2+1}}{-1+x}$	24
risch	$-\frac{2\sqrt{\frac{(1-x)(-x^2+1)}{(-1+x)^2}}\,(-1+x)\sqrt{1+x}}{\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1-x)^(1/2)*(-x^2+1)^(1/2)/(-1+x)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1)
```

Fricas [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.35, size = 23, normalized size = 2.56

$$-\frac{2\sqrt{-x^2+1}\,\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [A]
time = 2.64, size = 13, normalized size = 1.44

$$-2\sqrt{2} + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(2) + 2*sqrt(x + 1)
```

Mupad [B]
time = 3.64, size = 18, normalized size = 2.00

$$\frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] (2*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.858 $\int \sqrt{1-x}\,dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Maple [A]
time = 0.53, size = 10, normalized size = 0.77

method	result	size
gosper	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
derivativedivides	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
default	$-\frac{2(1-x)^{\frac{3}{2}}}{3}$	10
trager	$\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x}$	14
risch	$-\frac{2(-1+x)^{2}}{3\sqrt{1-x}}$	15
meijerg	$\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2-2x)\sqrt{1-x}}{3}}{2\sqrt{\pi}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(1-x)^(3/2)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.69

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="maxima")
[Out] -2/3*(-x + 1)^(3/2)
```

Fricas [A]
time = 0.33, size = 12, normalized size = 0.92

$$\frac{2}{3}\,(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="fricas")
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Sympy [A]
time = 0.01, size = 10, normalized size = 0.77

$$-\frac{2(1-x)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2),x)
```

```
[Out] -2*(1 - x)**(3/2)/3
```

Giac [A]
time = 4.74, size = 9, normalized size = 0.69

$$-\frac{2}{3}\,(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2)
```

Mupad [B]
time = 3.51, size = 9, normalized size = 0.69

$$-\frac{2\,{\left(1-x\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2),x)
```

```
[Out] -(2*(1 - x)^(3/2))/3
```

3.859 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} \, dx$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

```
[Out] -2/3*(1-x)^(3/2)
```

Rubi [A]

time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {26, 32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (-2*(1 - x)^(3/2))/3
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} \, dx &= \int \sqrt{1-x} \, dx \\ &= -\frac{2}{3}(1-x)^{3/2}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 22, normalized size = 1.69

$$-\frac{2(1-x^2)^{3/2}}{3(1+x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x],x]
```

```
[Out] (-2*(1 - x^2)^(3/2))/(3*(1 + x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.60, size = 20, normalized size = 1.54

method	result	size
gosper	$\frac{2(-1+x)\sqrt{-x^2+1}}{3\sqrt{1+x}}$	20
default	$\frac{2(-1+x)\sqrt{-x^2+1}}{3\sqrt{1+x}}$	20
risch	$-\frac{2\sqrt{\frac{-x^2+1}{1+x}}\,\sqrt{1+x}\,(-1+x)^2}{3\sqrt{-x^2+1}\,\sqrt{1-x}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(-1+x)*(-x^2+1)^(1/2)/(1+x)^(1/2)
```

Maxima [A]
time = 0.27, size = 12, normalized size = 0.92

$$\frac{2}{3}\,(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x - 1)*sqrt(-x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.
time = 0.34, size = 19, normalized size = 1.46

$$\frac{2\sqrt{-x^2+1}\,(x-1)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-x^2 + 1)*(x - 1)/sqrt(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)
```

Giac [A]
time = 2.30, size = 15, normalized size = 1.15

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}+\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(-x + 1)^(3/2) + 4/3*sqrt(2)
```

Mupad [B]
time = 3.52, size = 20, normalized size = 1.54

$$\frac{\left(\frac{2x}{3}-\frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] (((2*x)/3 - 2/3)*(1 - x^2)^(1/2))/(x + 1)^(1/2)
```

3.860 $\int \sqrt{1+x}\, dx$

Optimal. Leaf size=11

$$\frac{2}{3}(1+x)^{3/2}$$

[Out] 2/3*(1+x)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2))/3

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+x}\, dx = \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{2}{3}(1+x)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x],x]

[Out] (2*(1 + x)^(3/2))/3

Maple [A]
time = 0.55, size = 8, normalized size = 0.73

method	result	size
gosper	$\frac{2(1+x)^{\frac{3}{2}}}{3}$	8
derivativedivides	$\frac{2(1+x)^{\frac{3}{2}}}{3}$	8
default	$\frac{2(1+x)^{\frac{3}{2}}}{3}$	8
risch	$\frac{2(1+x)^{\frac{3}{2}}}{3}$	8
trager	$\left(\frac{2}{3}+\frac{2x}{3}\right)\sqrt{1+x}$	12
meijerg	$-\frac{\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,(2x+2)\sqrt{1+x}}{3}}{2\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(1+x)^(3/2)
```

Maxima [A]
time = 0.31, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [A]
time = 0.35, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$\frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2),x)
```

```
[Out] 2*(x + 1)**(3/2)/3
```

Giac [A]
time = 3.19, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Mupad [B]
time = 3.42, size = 7, normalized size = 0.64

$$\frac{2\,{\left(x+1\right)}^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2),x)
```

```
[Out] (2*(x + 1)^(3/2))/3
```

3.861 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=11

$$\frac{2}{3}(1+x)^{3/2}$$

[Out] 2/3*(1+x)^(3/2)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x],x]

[Out] (2*(1 + x)^(3/2))/3

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}}\,dx &= \int \sqrt{1+x}\,dx\\ &= \frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

time = 0.03, size = 24, normalized size = 2.18

$$\frac{2(1-x^2)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x],x]
```

```
[Out] (2*(1 - x^2)^(3/2))/(3*(1 - x)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(7) = 14$.
time = 0.59, size = 27, normalized size = 2.45

method	result	size
gosper	$\frac{2(1+x)\sqrt{-x^2+1}}{3\sqrt{1-x}}$	22
default	$-\frac{2\sqrt{-x^2+1}\,\sqrt{1-x}\,(1+x)}{3(-1+x)}$	27
risch	$-\frac{2\sqrt{\frac{(1-x)(-x^2+1)}{(-1+x)^2}}\,(-1+x)(1+x)^{\frac{3}{2}}}{3\sqrt{1-x}\,\sqrt{-x^2+1}}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(1-x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-x^2+1)^(1/2)*(1-x)^(1/2)/(-1+x)*(1+x)
```

Maxima [A]
time = 0.28, size = 7, normalized size = 0.64

$$\frac{2}{3}\,(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(7) = 14$.
time = 0.33, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{-x^2+1}\,(x+1)\sqrt{-x+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(-x^2 + 1)*(x + 1)*sqrt(-x + 1)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(1 - x), x)
```

Giac [A]
time = 2.37, size = 13, normalized size = 1.18

$$\frac{2}{3}\left(x+1\right)^{\frac{3}{2}}-\frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 4/3*sqrt(2)
```

Mupad [B]
time = 3.49, size = 22, normalized size = 2.00

$$\frac{\left(\frac{2\,x}{3}+\frac{2}{3}\right)\,\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x)^(1/2),x)
```

```
[Out] (((2*x)/3 + 2/3)*(1 - x^2)^(1/2))/(1 - x)^(1/2)
```

3.862 $\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=35

$$\sqrt{1+x}\,\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {52, 56, 221}

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx = \sqrt{1+x}\,\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\,\sqrt{2+3x}}\,dx$$
$$= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}}$$
$$= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

Mathematica [A]
time = 0.08, size = 43, normalized size = 1.23

$$\sqrt{1+x}\,\sqrt{2+3x} - \frac{\tanh^{-1}\left(\sqrt{\frac{2+3x}{3+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcTanh[Sqrt[(2 + 3*x)/(3 + 3*x)]]/Sqrt[3]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(27) = 54$.
time = 0.56, size = 67, normalized size = 1.91

method	result	size
default	$\sqrt{1+x}\,\sqrt{2+3x} - \frac{\sqrt{(1+x)(2+3x)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{6\sqrt{2+3x}\,\sqrt{1+x}}$	67
risch	$\sqrt{1+x}\,\sqrt{2+3x} - \frac{\sqrt{(1+x)(2+3x)}\,\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}}{6\sqrt{2+3x}\,\sqrt{1+x}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^(1/2)/(1+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (1+x)^(1/2)*(2+3*x)^(1/2)-1/6*((1+x)*(2+3*x))^(1/2)/(2+3*x)^(1/2)/(1+x)^(1/
2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.52, size = 41, normalized size = 1.17

$$-\frac{1}{6}\sqrt{3}\,\log\left(2\sqrt{3}\,\sqrt{3\,x^2+5\,x+2}+6\,x+5\right)+\sqrt{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5) + sqrt(3*x^2 +
5*x + 2)
```

Fricas [A]
time = 0.35, size = 52, normalized size = 1.49

$$\frac{1}{12}\sqrt{3}\log\left(-4\sqrt{3}(6x+5)\sqrt{3x+2}\sqrt{x+1}+72x^2+120x+49\right)+\sqrt{3x+2}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 +
120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.92, size = 100, normalized size = 2.86

$$\begin{cases} \sqrt{x+1}\sqrt{3x+2} - \frac{\sqrt{3}\operatorname{acosh}\left(\sqrt{3}\sqrt{x+1}\right)}{3} & \text{for } |x+1| > \frac{1}{3} \\ \frac{\sqrt{3}i\operatorname{asin}\left(\sqrt{3}\sqrt{x+1}\right)}{3} - \frac{3i(x+1)^{\frac{3}{2}}}{\sqrt{-3x-2}} + \frac{i\sqrt{x+1}}{\sqrt{-3x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(x + 1)*sqrt(3*x + 2) - sqrt(3)*acosh(sqrt(3)*sqrt(x + 1))/3
, Abs(x + 1) > 1/3), (sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3 - 3*I*(x + 1)**
(3/2)/sqrt(-3*x - 2) + I*sqrt(x + 1)/sqrt(-3*x - 2), True))
```

Giac [A]
time = 3.40, size = 39, normalized size = 1.11

$$\frac{1}{3}\sqrt{3}\left(\sqrt{3x+3}\sqrt{3x+2}+\log\left(\sqrt{3x+3}-\sqrt{3x+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2
)))
```

Mupad [B]
time = 6.14, size = 172, normalized size = 4.91

$$\frac{2\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{3\,\left(\sqrt{x+1}-1\right)}\right)}{3}-\frac{\frac{30\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)}{\sqrt{x+1}-1}+\frac{10\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{24\,\sqrt{2}\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}}{\frac{\left(\sqrt{2}-\sqrt{3\,x+2}\right)^4}{\left(\sqrt{x+1}-1\right)^4}-\frac{6\,\left(\sqrt{2}-\sqrt{3\,x+2}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] (2*3^(1/2)*atanh((3^(1/2)*(2^(1/2) - (3*x + 2)^(1/2)))/(3*((x + 1)^(1/2) -
1))))/3 - ((30*(2^(1/2) - (3*x + 2)^(1/2)))/((x + 1)^(1/2) - 1) + (10*(2^(1
/2) - (3*x + 2)^(1/2))^3)/((x + 1)^(1/2) - 1)^3 + (24*2^(1/2)*(2^(1/2) - (3
*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^2)/((2^(1/2) - (3*x + 2)^(1/2))^4/((x
 + 1)^(1/2) - 1)^4 - (6*(2^(1/2) - (3*x + 2)^(1/2))^2)/((x + 1)^(1/2) - 1)^
2 + 9)
```

3.863 $\int \frac{\sqrt{1-x}\,\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=35

$$\sqrt{1+x}\,\sqrt{2+3x}-\frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}$$

[Out] -1/3*arcsinh((2+3*x)^(1/2))*3^(1/2)+(1+x)^(1/2)*(2+3*x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {26, 52, 56, 221}

$$\sqrt{x+1}\,\sqrt{3x+2}-\frac{\sinh^{-1}\left(\sqrt{3x+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1-x}\,\sqrt{2+3x}}{\sqrt{1-x^2}}\,dx &= \int \frac{\sqrt{2+3x}}{\sqrt{1+x}}\,dx \\
&= \sqrt{1+x}\,\sqrt{2+3x} - \frac{1}{2}\int \frac{1}{\sqrt{1+x}\,\sqrt{2+3x}}\,dx \\
&= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+3x}\right)}{\sqrt{3}} \\
&= \sqrt{1+x}\,\sqrt{2+3x} - \frac{\sinh^{-1}\left(\sqrt{2+3x}\right)}{\sqrt{3}}
\end{aligned}
$$

Mathematica [**A**]
time = 1.74, size = 49, normalized size = 1.40

$$
\frac{3\sqrt{1+x}\,(2+3x) - \sqrt{6+9x}\,\sinh^{-1}\left(\sqrt{2+3x}\right)}{3\sqrt{2+3x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]
[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2
 + 3*x])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(27) = 54$.
time = 0.64, size = 86, normalized size = 2.46

method	result
default	$\frac{\sqrt{1-x}\,\sqrt{2+3x}\,\sqrt{-x^2+1}\,\left(\ln\left(\frac{5\sqrt{3}}{6}+x\sqrt{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}-6\sqrt{3x^2+5x+2}\right)}{6(-1+x)\sqrt{3x^2+5x+2}}$
risch	$-\frac{(1+x)\sqrt{2+3x}\,\sqrt{\frac{(1-x)(2+3x)(-x^2+1)}{(-1+x)^2}}\,(-1+x)}{\sqrt{(1+x)\,(2+3x)}\,\sqrt{1-x}\,\sqrt{-x^2+1}}+\frac{\ln\left(\frac{\left(\frac{5}{2}+3x\right)\sqrt{3}}{3}+\sqrt{3x^2+5x+2}\right)\sqrt{3}\,\sqrt{\frac{(1-x)(2+3x)(-x}{(-1+x)^2}}}{6\sqrt{1-x}\,\sqrt{2+3x}\,\sqrt{-x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(1-x)^(1/2)*(2+3*x)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+x*3^(1/2)+(3*x
^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(-1+x)/(3*x^2+5*x+2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(27) = 54$.
time = 0.42, size = 96, normalized size = 2.74

$$\frac{\sqrt{3}\,(x-1)\log\left(-\frac{72\,x^3+4\,\sqrt{3}\,\sqrt{-x^2+1}\,(6\,x+5)\sqrt{3\,x+2}\,\sqrt{-x+1}+48\,x^2-71\,x-49}{x-1}\right)-12\,\sqrt{-x^2+1}\,\sqrt{3\,x+2}\,\sqrt{-x+1}}{12\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqr
t(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*
sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\,\sqrt{3x+2}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{3\,x+2}\,\sqrt{1-x}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2),x)
```

```
[Out] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)
```

3.864 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{1+x}\right)$$

```
[Out] -arcsin(x)-arctanh((1-x)^(1/2)*(1+x)^(1/2))+4*(1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {100, 21, 132, 41, 222, 94, 212}

$$-\text{ArcSin}(x) + \frac{4\sqrt{x+1}}{\sqrt{1-x}} - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
```

```
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x}\,dx &= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-2\int \frac{-\frac{1}{2}+\frac{x}{2}}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx\\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}}+\int \frac{\sqrt{1-x}}{x\sqrt{1+x}}\,dx\\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx+\int \frac{1}{\sqrt{1-x}\,x\sqrt{1+x}}\,dx\\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x^2}}\,dx-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x}\,\sqrt{1+x}\right)\\
&= \frac{4\sqrt{1+x}}{\sqrt{1-x}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x}\,\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.13, size = 61, normalized size = 1.42

$$-\frac{4\sqrt{1-x^2}}{-1+x}-2\tan^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{1-x}}\right)+2i\tan^{-1}\left(x+i\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x),x]
```

```
[Out] (-4*Sqrt[1 - x^2])/(-1 + x) - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]] + (2*I)*Arc
Tan[x + I*Sqrt[1 - x^2]]
```

Maple [A]
time = 0.60, size = 70, normalized size = 1.63

method	result
default	$\frac{\left(-\arcsin(x)x-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)x+\arcsin(x)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-4\sqrt{-x^2+1}\right)\sqrt{1-x}\,\sqrt{1+x}}{(-1+x)\sqrt{-x^2+1}}$
risch	$\frac{4\sqrt{1+x}\,\sqrt{(1-x)(1+x)}}{\sqrt{-(1+x)(-1+x)}\,\sqrt{1-x}}-\frac{\left(\arcsin(x)+\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)\right)\sqrt{(1-x)(1+x)}}{\sqrt{1-x}\,\sqrt{1+x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)/(1-x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] (-arcsin(x)*x-arctanh(1/(-x^2+1)^(1/2))*x+arcsin(x)+arctanh(1/(-x^2+1)^(1/2
))-4*(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/(-1+x)/(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 53, normalized size = 1.23

$$\frac{4\,x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin\left(x\right)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/ab
s(x) + 2/abs(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(35) = 70.
time = 0.35, size = 74, normalized size = 1.72

$$\frac{2\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+4\,x-4\,\sqrt{x+1}\,\sqrt{-x+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}}{x(1-x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters values [5.38357
630698]Warning, choosing root of [1,0,-4,0,%%%{4,[2]%%%}] at parameters val
ues [81.11954429
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{x\,(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)),x)
```

```
[Out] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)
```

3.865 $\int \frac{(1+x)^3}{x(1-x^2)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

[Out] -arcsin(x)-arctanh((-x^2+1)^(1/2))+4*(1+x)/(-x^2+1)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1819, 858, 222, 272, 65, 212}

$$-\text{ArcSin}(x)+\frac{4(x+1)}{\sqrt{1-x^2}}-\tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]

[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^3}{x\left(1-x^2\right)^{3/2}}\,dx &= \frac{4(1+x)}{\sqrt{1-x^2}}-\int \frac{-1+x}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\int \frac{1}{\sqrt{1-x^2}}\,dx+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx,x,x^2\right)\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= \frac{4(1+x)}{\sqrt{1-x^2}}-\sin^{-1}(x)-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 59, normalized size = 1.69

$$-\frac{4\sqrt{1-x^2}}{-1+x}+2\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)-2\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)),x]
```

[Out] (-4*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - 2*ArcTanh[Sqrt[1 - x^2]/(1 + x)]

Maple [A]
time = 0.62, size = 41, normalized size = 1.17

method	result
risch	$\frac{4+4x}{\sqrt{-x^2+1}}-\arcsin(x)-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
default	$\frac{4x}{\sqrt{-x^2+1}}-\arcsin(x)+\frac{4}{\sqrt{-x^2+1}}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$
trager	$-\frac{4\sqrt{-x^2+1}}{-1+x}+\operatorname{RootOf}(\_Z^2+1)\ln\left(x\operatorname{RootOf}(\_Z^2+1)+\sqrt{-x^2+1}\right)+\ln\left(\frac{\sqrt{-x^2+1}-1}{x}\right.$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-x^2+1}}-\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{-x^2+1}}{2}\right)+\frac{(2-2\ln(2)+2\ln(x)+i\pi)\sqrt{\pi}}{2}}{\sqrt{\pi}}+\frac{3x}{\sqrt{-x^2+1}}-\frac{3\left(\sqrt{\pi}-\frac{\sqrt{}}{\sqrt{-x}}\right.}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^3/x/(-x^2+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] 4*x/(-x^2+1)^(1/2)-arcsin(x)+4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))

Maxima [A]
time = 0.48, size = 53, normalized size = 1.51

$$\frac{4x}{\sqrt{-x^2+1}}+\frac{4}{\sqrt{-x^2+1}}-\arcsin(x)-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")

[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(31) = 62.
time = 0.35, size = 63, normalized size = 1.80

$$\frac{2(x-1)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+(x-1)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+4x-4\sqrt{-x^2+1}-4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="fricas")

```
[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1
)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^3}{x\left(-(x-1)(x+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

Giac [A]
time = 3.41, size = 44, normalized size = 1.26

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x}+1} - \arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(
x))
```

Mupad [B]
time = 3.20, size = 37, normalized size = 1.06

$$\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right) - \mathrm{asin}(x) - \frac{4\sqrt{1-x^2}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x -
 1)
```

3.866 $\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)$$

```
[Out] -arcsin(a*x)-arctanh((-a*x+1)^(1/2)*(a*x+1)^(1/2))+4*(a*x+1)^(1/2)/(-a*x+1)
^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {100, 21, 132, 41, 222, 94, 214}

$$-\text{ArcSin}(ax) + \frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[
1 + a*x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)
*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}}\,dx &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-\frac{2\int \frac{-\frac{a}{2}+\frac{a^2x}{2}}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx}{a}\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}+\int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-a\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx+\int \frac{1}{x\sqrt{1-ax}\,\sqrt{1+ax}}\,dx\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx-a\text{Subst}\left(\int \frac{1}{a-ax^2}\,dx,x,\sqrt{1-ax}\,\sqrt{1+ax}\right)\\
&= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-ax}\,\sqrt{1+ax}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.16, size = 75, normalized size = 1.47

$$\frac{4\sqrt{1-a^2x^2}}{1-ax} - 2\tan^{-1}\left(\frac{\sqrt{1+ax}}{\sqrt{1-ax}}\right) + 2i\tan^{-1}\left(ax + i\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 - a^2*x^2])/(1 - a*x) - 2*ArcTan[Sqrt[1 + a*x]/Sqrt[1 - a*x]] + (
2*I)*ArcTan[a*x + I*Sqrt[1 - a^2*x^2]]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.70, size = 134, normalized size = 2.63

method	result
default	$\frac{\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)ax - \arctan\left(\frac{\operatorname{csgn}(a)ax}{\sqrt{-(ax+1)(ax-1)}}\right)ax + \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\operatorname{csgn}(a)\right.}{(ax-1)\sqrt{-a^2x^2+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)*a*x-arctan(csgn(a)*a*x/(-(a*x+1)*(a
*x-1))^(1/2))*a*x+arctanh(1/(-a^2*x^2+1)^(1/2))*csgn(a)-4*csgn(a)*(-a^2*x^2
+1)^(1/2)+arctan(csgn(a)*a*x/(-(a*x+1)*(a*x-1))^(1/2)))*csgn(a)*(-a*x+1)^(1
/2)*(a*x+1)^(1/2)/(a*x-1)/(-a^2*x^2+1)^(1/2)
```

Maxima [A]
time = 0.48, size = 65, normalized size = 1.27

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(
-a^2*x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(43) = 86.
time = 0.37, size = 93, normalized size = 1.82

$$\frac{4\,ax + 2\,(ax-1)\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right) + (ax-1)\log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right) - 4\sqrt{ax+1}\sqrt{-ax+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/(a*x)) + (a*
x - 1)*log((sqrt(a*x + 1)*sqrt(-a*x + 1) - 1)/x) - 4*sqrt(a*x + 1)*sqrt(-a*
x + 1) - 4)/(a*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{\frac{3}{2}}}{x\,(-ax+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [42,56
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[-9,-13](2*sageV
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{x\,(1-a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)
```

```
[Out] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)
```

3.867 $\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arcsin(a*x)-arctanh((-a^2*x^2+1)^(1/2))+4*(a*x+1)/(-a^2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1819, 858, 222, 272, 65, 214}

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \text{ArcSin}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

```
[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^3}{x\left(1-a^2x^2\right)^{3/2}}\,dx &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\int \frac{-1+ax}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-a\int \frac{1}{\sqrt{1-a^2x^2}}\,dx+\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx,x,\sqrt{1-a^2x^2}\right)}{a^2}\\
&= \frac{4(1+ax)}{\sqrt{1-a^2x^2}}-\sin^{-1}(ax)-\tanh^{-1}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 94 vs. 2(45) = 90.
time = 0.24, size = 94, normalized size = 2.09

$$-\frac{4\sqrt{1-a^2x^2}}{-1+ax}+2\tanh^{-1}\left(\sqrt{-a^2}\,x-\sqrt{1-a^2x^2}\right)+\frac{a\log\left(-\sqrt{-a^2}\,x+\sqrt{1-a^2x^2}\right)}{\sqrt{-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]
```

[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) + 2*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*x ^2]] + (a*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/Sqrt[-a^2]

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.
time = 0.59, size = 99, normalized size = 2.20

method	result
default	$a^3\left(\frac{x}{a^2\sqrt{-a^2x^2+1}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{a^2\sqrt{a^2}}\right)+\frac{4}{\sqrt{-a^2x^2+1}}+\frac{3ax}{\sqrt{-a^2x^2+1}}-\operatorname{arctanh}\left(\frac{}{\sqrt{}}\right.$
meijerg	$\frac{-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{-a^2x^2+1}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-a^2x^2+1}}{2}\right)+\frac{(2-2\ln(2)+2\ln(x)+\ln(-a^2))\sqrt{\pi}}{2}}{\sqrt{\pi}}-\frac{a\left(\frac{\sqrt{\pi}\,x(-a^2)^{\frac{3}{2}}}{a^2\sqrt{-a^2x^2+1}}-\right.}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] a^3*(x/a^2/(-a^2*x^2+1)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2)))+4/(-a^2*x^2+1)^(1/2)+3*a*x/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))

Maxima [A]
time = 0.49, size = 65, normalized size = 1.44

$$\frac{4\,ax}{\sqrt{-a^2x^2+1}}+\frac{4}{\sqrt{-a^2x^2+1}}-\arcsin\left(ax\right)-\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))

Fricas [A]
time = 0.40, size = 82, normalized size = 1.82

$$\frac{4\,ax+2\,(ax-1)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)+(ax-1)\log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right)-4\sqrt{-a^2x^2+1}-4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="fricas")

```
[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log
((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{x\left(-(ax-1)(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.
time = 3.83, size = 87, normalized size = 1.93

$$-\frac{a \arcsin(ax)\,\mathrm{sgn}(a)}{|a|} - \frac{a \log\left(\frac{\left|-2\sqrt{-a^2x^2+1}\,|a|-2a\right|}{2a^2|x|}\right)}{|a|} + \frac{8a}{\left(\frac{\sqrt{-a^2x^2+1}\,|a|+a}{a^2x}-1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) -
 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x)
- 1)*abs(a))
```

Mupad [B]
time = 3.49, size = 82, normalized size = 1.82

$$\frac{4\,a\,\sqrt{1-a^2\,x^2}}{\left(x\,\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\,\sqrt{-a^2}} - \frac{a\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \mathrm{atanh}\left(\sqrt{1-a^2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)
```

```
[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2))
- (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh((1 - a^2*x^2)^(1/2))
```

3.868 $\int \frac{1}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

[Out] arcsin(x)

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

$$\text{ArcSin}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcSin[x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.00, size = 14, normalized size = 7.00

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[1 - x^2]]
```

Maple [A]
time = 0.61, size = 3, normalized size = 1.50

method	result	size
default	$\arcsin(x)$	3
meijerg	$\arcsin(x)$	3
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(x)
```

Maxima [A]
time = 0.49, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(2) = 4.
time = 0.34, size = 18, normalized size = 9.00

$$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.05, size = 2, normalized size = 1.00

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)**(1/2),x)
```

```
[Out] asin(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(2) = 4.
time = 3.74, size = 17, normalized size = 8.50

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B]
time = 0.01, size = 2, normalized size = 1.00

$$\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1 - x^2)^(1/2),x)
```

```
[Out] asin(x)
```

3.869 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

[Out] arcsin(x)

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 222}

$$\text{ArcSin}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]

[Out] ArcSin[x]

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 32 vs. 2(2) = 4.
time = 0.54, size = 32, normalized size = 16.00

$$-\tan^{-1}\left(\frac{x\sqrt{1+x^2}\,\sqrt{1-x^4}}{-1+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4],x]
```

```
[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.
time = 0.57, size = 29, normalized size = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\arcsin(x)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(2) = 4$.
time = 0.36, size = 27, normalized size = 13.50

$$-\arctan\left(\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)
```

3.870 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{221\}$

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12 vs. $2(2) = 4$.
time = 0.01, size = 12, normalized size = 6.00

$$\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[1 + x^2]]
```

Maple [A]
time = 0.54, size = 3, normalized size = 1.50

method	result	size
default	$\operatorname{arcsinh}(x)$	3
meijerg	$\operatorname{arcsinh}(x)$	3
trager	$-\ln\left(x-\sqrt{x^2+1}\right)$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(x)
```

Maxima [A]
time = 0.48, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(2) = 4.
time = 0.35, size = 14, normalized size = 7.00

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A]
time = 0.05, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(2) = 4.
time = 3.68, size = 25, normalized size = 12.50

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.03, size = 2, normalized size = 1.00

$$\mathrm{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)
```

3.871 $\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] arcsinh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {26, 221}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] ArcSinh[x]
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx &= \int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \sinh^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(2) = 4.
time = 0.52, size = 42, normalized size = 21.00

$$\log\left(1-x^2\right)-\log\left(-x+x^3+\sqrt{1-x^2}\,\sqrt{1-x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4],x]
```

```
[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(2) = 4$.
time = 0.50, size = 29, normalized size = 14.50

method	result	size
default	$\frac{\sqrt{-x^4+1}\, \operatorname{arcsinh}(x)}{\sqrt{-x^2+1}\, \sqrt{x^2+1}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(2) = 4$.
time = 0.36, size = 81, normalized size = 40.50

$$-\frac{1}{2}\,\log\left(\frac{x^3+\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)+\frac{1}{2}\,\log\left(-\frac{x^3-\sqrt{-x^4+1}\,\sqrt{-x^2+1}-x}{x^3-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x
^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{\sqrt{-\left(x-1\right)\left(x+1\right)\left(x^2+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)
```

3.872 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {201, 222}

$$\frac{\text{ArcSin}(x)}{2}+\frac{1}{2}\sqrt{1-x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\,dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2}-\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]
[Out] (x*Sqrt[1 - x^2])/2 - ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.50, size = 18, normalized size = 0.78

method	result	size
default	$\frac{\arcsin(x)}{2}+\frac{x\sqrt{-x^2+1}}{2}$	18
risch	$-\frac{x(x^2-1)}{2\sqrt{-x^2+1}}+\frac{\arcsin(x)}{2}$	23
meijerg	$\frac{i\left(-2i\sqrt{\pi}\,x\sqrt{-x^2+1}-2i\sqrt{\pi}\,\arcsin(x)\right)}{4\sqrt{\pi}}$	32
trager	$\frac{x\sqrt{-x^2+1}}{2}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A]
time = 0.38, size = 31, normalized size = 1.35

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [A]
time = 3.33, size = 17, normalized size = 0.74

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.74

$$\frac{\operatorname{asin}(x)}{2}+\frac{x\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2),x)
```

```
[Out] asin(x)/2 + (x*(1 - x^2)^(1/2))/2
```

3.873 $\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {26, 201, 222}

$$\frac{\text{ArcSin}(x)}{2}+\frac{1}{2}\sqrt{1-x^2}\,x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}}\,dx &= \int \sqrt{1-x^2}\,dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.
time = 0.61, size = 50, normalized size = 2.17

$$\frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1+x^2}} + \tan^{-1}\left(\frac{x\sqrt{1+x^2}}{\sqrt{1-x^4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 + x^2] + ArcTan[(x*Sqrt[1 + x^2])/Sqrt[1 - x^4]])
/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.
time = 0.48, size = 42, normalized size = 1.83

method	result	size
default	$\frac{\sqrt{-x^4+1}\,\left(x\sqrt{-x^2+1}+\arcsin(x)\right)}{2\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	42
risch	$-\frac{x(x^2-1)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^2+1}\,\sqrt{-x^4+1}} + \frac{\arcsin(x)\sqrt{\frac{-x^4+1}{x^2+1}}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*(x*(-x^2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(17) = 34$.
time = 0.35, size = 60, normalized size = 2.61

$$\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}\,x-(x^2+1)\arctan\left(\frac{\sqrt{-x^4+1}\,\sqrt{x^2+1}}{x^3+x}\right)}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-x^4 + 1)*sqrt(x^2 + 1)*x - (x^2 + 1)*arctan(sqrt(-x^4 + 1)*sqrt(
x^2 + 1)/(x^3 + x)))/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)\,(x+1)\,(x^2+1)}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\sqrt{1-x^4}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2),x)
```

```
[Out] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.874 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {201, 221}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\,dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.33

$$\frac{1}{2}\left(x\sqrt{1+x^2}+\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
[Out] (x*Sqrt[1 + x^2] + ArcTanh[x/Sqrt[1 + x^2]])/2
```

Maple [A]
time = 0.51, size = 16, normalized size = 0.76

method	result	size
default	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^{2}+1}}{2}$	16
risch	$\frac{\operatorname{arcsinh}(x)}{2}+\frac{x\sqrt{x^{2}+1}}{2}$	16
trager	$\frac{x\sqrt{x^{2}+1}}{2}+\frac{\ln\left(x+\sqrt{x^{2}+1}\right)}{2}$	24
meijerg	$-\frac{-2\sqrt{\pi}\,x\sqrt{x^{2}+1}-2\sqrt{\pi}\,\operatorname{arcsinh}(x)}{4\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Maxima [A]
time = 0.50, size = 15, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^{2}+1}\,x+\frac{1}{2}\,\operatorname{arsinh}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A]
time = 0.38, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^{2}+1}\,x-\frac{1}{2}\,\log\left(-x+\sqrt{x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2}+\frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Giac [A]
time = 2.74, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.03, size = 15, normalized size = 0.71

$$\frac{\operatorname{asinh}(x)}{2}+\frac{x\sqrt{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)^(1/2),x)
```

```
[Out] asinh(x)/2 + (x*(x^2 + 1)^(1/2))/2
```

3.875 $\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {26, 201, 221}

$$\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx = \int \sqrt{1+x^2}\,dx$$
$$= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}(x)$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 70 vs. 2(21) = 42.
time = 0.62, size = 70, normalized size = 3.33

$$\frac{1}{2}\left(\frac{x\sqrt{1-x^4}}{\sqrt{1-x^2}} + \log\left(1-x^2\right) - \log\left(-x+x^3+\sqrt{1-x^2}\sqrt{1-x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]
```

```
[Out] ((x*Sqrt[1 - x^4])/Sqrt[1 - x^2] + Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x
^2]*Sqrt[1 - x^4]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(15) = 30.
time = 0.70, size = 47, normalized size = 2.24

method	result	size
default	$-\frac{\sqrt{-x^4+1}\,\sqrt{-x^2+1}\,\left(x\sqrt{x^2+1}+\operatorname{arcsinh}(x)\right)}{2(x^2-1)\sqrt{x^2+1}}$	47
risch	$-\frac{x\sqrt{x^2+1}\,\sqrt{\frac{(-x^2+1)(-x^4+1)}{(x^2-1)^2}}\,(x^2-1)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}} - \frac{\operatorname{arcsinh}(x)\sqrt{\frac{(-x^2+1)(-x^4+1)}{(x^2-1)^2}}\,(x^2-1)}{2\sqrt{-x^4+1}\,\sqrt{-x^2+1}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*(x*(x^2+1)^(1/2)+arcsinh(x))/(x^2-1)/(x^
2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(15) = 30$.

time = 0.33, size = 120, normalized size = 5.71

$$-\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}\,x+(x^2-1)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)-(x^2-1)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-x^4 + 1)*sqrt(-x^2 + 1)*x + (x^2 - 1)*log((x^3 + sqrt(-x^4 +
1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*log(-(x^3 - sqrt(-x^4 + 1)*sq
rt(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-(x-1)\,(x+1)\,(x^2+1)}}{\sqrt{-(x-1)\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.05

$$\int\frac{\sqrt{1-x^4}}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2),x)
```

```
[Out] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)
```

3.876 $\int \left(\frac{a+b+cx^2}{d}\right)^m dx$

Optimal. Leaf size=49

$$\frac{dx\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^{1+m} {}_2F_1\left(1,\frac{3}{2}+m;\frac{3}{2};-\frac{cx^2}{a+b}\right)}{a+b}$$

```
[Out] d*x*((a+b)/d+x^2*c/d)^(1+m)*hypergeom([1, 3/2+m],[3/2],-c*x^2/(a+b))/(a+b)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1997, 252, 251}

$$x\left(\frac{cx^2}{a+b}+1\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a +
 b))])/(1 + (c*x^2)/(a + b))^m
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx = \int \left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m dx$$
$$= \left(\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m\right)\int \left(1+\frac{cx^2}{a+b}\right)^m dx$$
$$= x\left(1+\frac{cx^2}{a+b}\right)^{-m}\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.08

$$x\left(\frac{a+b+cx^2}{d}\right)^m\left(1+\frac{cx^2}{a+b}\right)^{-m} {}_2F_1\left(\frac{1}{2},-m;\frac{3}{2};-\frac{cx^2}{a+b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b + c*x^2)/d)^m,x]
```

```
[Out] (x*((a + b + c*x^2)/d)^m*Hypergeometric2F1[1/2, -m, 3/2, -((c*x^2)/(a + b))
])/(1 + (c*x^2)/(a + b))^m
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(\frac{c\,x^2+a+b}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2+a+b)/d)^m,x)
```

```
[Out] int(((c*x^2+a+b)/d)^m,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="maxima")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="fricas")
```

```
[Out] integral(((c*x^2 + a + b)/d)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{a+b+cx^2}{d}\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x**2+a+b)/d)**m,x)
```

```
[Out] Integral(((a + b + c*x**2)/d)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((c*x^2+a+b)/d)^m,x, algorithm="giac")
```

```
[Out] integrate(((c*x^2 + a + b)/d)^m, x)
```

Mupad [B]
time = 4.67, size = 54, normalized size = 1.10

$$\frac{x\left(\frac{a+b}{d}+\frac{cx^2}{d}\right)^m {}_2F_1\left(\frac{1}{2},-m;\ \frac{3}{2};\ -\frac{cx^2}{a+b}\right)}{\left(\frac{cx^2}{a+b}+1\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b + c*x^2)/d)^m,x)
```

```
[Out] (x*((a + b)/d + (c*x^2)/d)^m*hypergeom([1/2, -m], 3/2, -(c*x^2)/(a + b)))/(
(c*x^2)/(a + b) + 1)^m
```

3.877 $\int \frac{1}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=28

$$-\frac{x^2}{2}-\frac{1}{2}x\sqrt{1+x^2}-\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2131, 30, 201, 221}

$$-\frac{x^2}{2}-\frac{1}{2}\sqrt{x^2+1}\,x-\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + x^2])^(-1),x]
```

```
[Out] -1/2*x^2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x-\sqrt{1+x^2}}\,dx &= -\int x\,dx - \int \sqrt{1+x^2}\,dx\\ &= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.18

$$\frac{1}{2}\left(-x\left(x+\sqrt{1+x^2}\right)-\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + x^2])^(-1),x]
[Out] (-(x*(x + Sqrt[1 + x^2])) - ArcTanh[x/Sqrt[1 + x^2]])/2
```

Maple [A]
time = 0.51, size = 21, normalized size = 0.75

method	result	size
default	$-\frac{x^2}{2}-\frac{\operatorname{arcsinh}(x)}{2}-\frac{x\sqrt{x^2+1}}{2}$	21
trager	$-\frac{x^2}{2}-\frac{x\sqrt{x^2+1}}{2}-\frac{\ln\left(x+\sqrt{x^2+1}\right)}{2}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*x*(x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 1.07

$$-\frac{1}{2}x^2-\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(22) = 44$.
time = 0.18, size = 58, normalized size = 2.07

$$-\frac{x\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}+\frac{x}{2x-2\sqrt{x^2+1}}+\frac{\sqrt{x^2+1}\operatorname{asinh}(x)}{2x-2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x**2+1)**(1/2)),x)
[Out] -x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x*
*2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))
```

Giac [A]
time = 3.45, size = 30, normalized size = 1.07

$$-\frac{1}{2}x^2-\frac{1}{2}\sqrt{x^2+1}\,x+\frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.71

$$-\frac{\operatorname{asinh}(x)}{2}-\frac{x\sqrt{x^2+1}}{2}-\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (x^2 + 1)^(1/2)),x)
[Out] - asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2
```

3.878 $\int \frac{1}{x-\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)$$

```
[Out] -1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))+1/4*ln(-2*x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6874, 266, 399, 222, 385, 213}

$$-\frac{\text{ArcSin}(x)}{2}+\frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1-x^2}}\,dx &= \int \left(\frac{x}{-1+2x^2}+\frac{\sqrt{1-x^2}}{-1+2x^2}\right)dx\\
&= \int \frac{x}{-1+2x^2}\,dx+\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx\\
&= \frac{1}{4}\log\left(1-2x^2\right)-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\,(-1+2x^2)}\,dx\\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{4}\log\left(1-2x^2\right)+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{1-x^2}}\right)\\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{1}{4}\log\left(1-2x^2\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 39, normalized size = 1.05

$$\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)+\frac{1}{2}\log\left(-x+\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - x^2]/(1 + x)] + Log[-x + Sqrt[1 - x^2]]/2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(29) = 58.
time = 0.15, size = 197, normalized size = 5.32

method	result

trager	$-\ln\left(-\frac{\sqrt{-x^2+1}+x}{2x^2-1}\right)\operatorname{RootOf}\left(2\_Z^2+2\_Z+1\right)-\ln\left(-\frac{\sqrt{-x^2+1}+x}{2x^2-1}\right)+\frac{\ln\left(-\frac{2\operatorname{RootOf}\left(2\_Z^2 \right.}{}\right.}{}$
default	$\frac{\ln(2x^2-1)}{4}+\frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}-\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{}{\sqrt{-4\left(\right.}}\right.}{}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(2*x^2-1)+1/2*2^(1/2)*(1/4*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^
(1/2)+2)^(1/2)-1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh((1-(x-1/2*2^(1/2))
*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2))
)-1/2*2^(1/2)*(1/4*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)
+1/4*2^(1/2)*arcsin(x)-1/4*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x - sqrt(-x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(29) = 58$.
time = 0.39, size = 84, normalized size = 2.27

$$\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(2x^2-1\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right)-\frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(
-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) +
x - 1)/x^2)
```

Sympy [A]
time = 0.08, size = 17, normalized size = 0.46

$$\frac{\log\left(x - \sqrt{1 - x^2}\right)}{2} - \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] log(x - sqrt(1 - x**2))/2 - asin(x)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(29) = 58.
time = 3.42, size = 140, normalized size = 3.78

$$-\frac{1}{4}\pi\operatorname{sgn}(x) - \frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right) + \frac{1}{4}\log\left(\left|x+\frac{1}{2}\sqrt{2}\right|\right) + \frac{1}{4}\log\left(\left|x-\frac{1}{2}\sqrt{2}\right|\right) - \frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right) + \frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-
x^2 + 1) - 1)) + 1/4*log(abs(x + 1/2*sqrt(2))) + 1/4*log(abs(x - 1/2*sqrt(2
))) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) +
1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))
```

Mupad [B]
time = 0.14, size = 105, normalized size = 2.84

$$\frac{\ln\left(x-\frac{\sqrt{2}}{2}\right)}{4} + \frac{\ln\left(x+\frac{\sqrt{2}}{2}\right)}{4} - \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4} + \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4} - \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] log(x - 2^(1/2)/2)/4 + log(x + 2^(1/2)/2)/4 - log((2^(1/2)*((2^(1/2)*x)/2 -
 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/2))/4 + log((2^(1/2)*((2^(1/2)*x)
/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/4 - asin(x)/2
```

3.879 $\int \frac{1}{x-\sqrt{1+2x^2}}\,dx$

Optimal. Leaf size=40

$$-\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 266, 399, 221, 385, 212}

$$-\frac{1}{2}\log\left(x^2+1\right)+\tanh^{-1}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-\sqrt{1+2x^2}}\,dx &= \int \left(-\frac{x}{1+x^2}-\frac{\sqrt{1+2x^2}}{1+x^2}\right)dx\\
&= -\int \frac{x}{1+x^2}\,dx-\int \frac{\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -\frac{1}{2}\log\left(1+x^2\right)-2\int \frac{1}{\sqrt{1+2x^2}}\,dx+\int \frac{1}{\left(1+x^2\right)\sqrt{1+2x^2}}\,dx\\
&= -\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)-\frac{1}{2}\log\left(1+x^2\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{1+2x^2}}\right)\\
&= -\sqrt{2}\,\sinh^{-1}\left(\sqrt{2}\,x\right)+\tanh^{-1}\left(\frac{x}{\sqrt{1+2x^2}}\right)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 68, normalized size = 1.70

$$\left(1+\sqrt{2}\right)\log\left(2\left(-1+\sqrt{2}\right)x+\left(-2+\sqrt{2}\right)\sqrt{1+2x^2}\right)-\log\left(-2+\sqrt{2}-2x^2+x\sqrt{2+4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1),x]
```

```
[Out] (1 + Sqrt[2])*Log[2*(-1 + Sqrt[2])*x + (-2 + Sqrt[2])*Sqrt[1 + 2*x^2]] - Lo
g[-2 + Sqrt[2] - 2*x^2 + x*Sqrt[2 + 4*x^2]]
```

Maple [A]
time = 0.13, size = 33, normalized size = 0.82

method	result
default	$\operatorname{arctanh}\left(\frac{x}{\sqrt{2x^2+1}}\right)-\frac{\ln\left(x^2+1\right)}{2}-\operatorname{arcsinh}\left(\sqrt{2}\,x\right)\sqrt{2}$

trager	$\text{RootOf}\left(\_Z^2-2\_Z-1\right)\ln\left(\frac{x+\sqrt{2x^2+1}}{x^2+1}\right)-\ln\left(\frac{\text{RootOf}\left(\_Z^2-2\_Z-1\right)^2x^2+3\,\text{RootOf}\left(\_Z^2-2\_Z-1\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(2^(1/2)*x)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.
time = 0.34, size = 90, normalized size = 2.25

$$\sqrt{2}\log\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\log\left(\frac{2\,x^2-\sqrt{2\,x^2+1}\,(x+1)+x+1}{x^2}\right)+\frac{1}{2}\log\left(\frac{2\,x^2+\sqrt{2\,x^2+1}\,(x-1)-x+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^
2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1
)*(x - 1) - x + 1)/x^2)
```

Sympy [A]
time = 0.07, size = 27, normalized size = 0.68

$$-\log\left(-x+\sqrt{2x^2+1}\right)-\sqrt{2}\,\operatorname{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)
[Out] -log(-x + sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(32) = 64$.
time = 3.65, size = 88, normalized size = 2.20

$$\sqrt{2}\log\left(-\sqrt{2}\,x+\sqrt{2\,x^2+1}\right)+\frac{1}{2}\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2+2\sqrt{2}+3\right)-\frac{1}{2}\log\left(\left(\sqrt{2}\,x-\sqrt{2\,x^2+1}\right)^2-2\sqrt{2}+3\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^2
 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqr
t(2) + 3) - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 3.50, size = 57, normalized size = 1.42

$$-\ln(x-\mathrm{i})-\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}+\frac{1}{2}\mathrm{i}\right)}{2}+\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}\mathrm{i}\right)}{2}-\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 + 1
/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)
```

3.880 $\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(-x^2+1)+1/4*x*(-x^2+2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}=0.235$, Rules used = {6857, 266, 272, 45, 489, 12, 385, 213}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x+\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/
4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 489

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^q/(b*(m + n*(p + q) + 1))), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2}\,dx &= \int \left(\frac{x}{-1+x^2}-\frac{x^3}{2\left(-1+x^2\right)}+\frac{x^2\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)\,dx \\
&= -\left(\frac{1}{2}\int \frac{x^3}{-1+x^2}\,dx\right)+\frac{1}{2}\int \frac{x^2\sqrt{2-x^2}}{-1+x^2}\,dx+\int \frac{x}{-1+x^2}\,dx \\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\int -\frac{2}{\sqrt{2-x^2}\left(-1+x^2\right)}\,dx-\frac{1}{4}\text{Subst}\Bigg(\int \\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{2}\log\left(1-x^2\right)-\frac{1}{4}\text{Subst}\left(\int \left(1+\frac{1}{-1+x}\right)\,dx,x,x^2\right)+\frac{1}{2}\int \\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log\left(1-x^2\right)+\frac{1}{2}\text{Subst}\Bigg(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-x^2}} \\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.11, size = 77, normalized size = 1.43

$$\frac{1}{4}\left(-x^2+x\sqrt{2-x^2}+\log(1-x)-\log(1+x)+\log\left(1-x^2\right)-\log\left(2-x+\sqrt{2-x^2}\right)+\log\left(2+x+\sqrt{2-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(42) = 84$.
time = 0.27, size = 111, normalized size = 2.06

method	result
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-x^2+2}}{4}-\frac{\ln\left(-\frac{\sqrt{-x^2+2}+x}{(1+x)(-1+x)}\right)}{2}$
default	$\frac{x\sqrt{-x^2+2}}{4}+\frac{\sqrt{-(-1+x)^2+3-2x}}{4}-\frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(-1+x)^2+3-2x}}\right)}{4}-\frac{\sqrt{-(1+x)^2}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(-1+x)^2+3-2*x)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-
(-1+x)^2+3-2*x)^(1/2))-1/4*(-(1+x)^2+3+2*x)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(
-(1+x)^2+3+2*x)^(1/2))-1/4*x^2+1/4*ln(-1+x)+1/4*ln(1+x)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(42) = 84$.
time = 0.50, size = 94, normalized size = 1.74

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)+\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|}+\frac{2}{|2x+2|}+1\right)-\frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|}+\frac{2}{|2x-2|}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) + 1/4*log(2*sqrt(-x^2 +
2)/abs(2*x + 2) + 2/abs(2*x + 2) + 1) - 1/4*log(2*sqrt(-x^2 + 2)/abs(2*x -
2) + 2/abs(2*x - 2) - 1)
```

Fricas [A]
time = 0.35, size = 67, normalized size = 1.24

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{2x}{x^2-1}\right)dx+\int\frac{x^3}{x^2-1}\,dx+\int\left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x
**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(42) = 84.
time = 3.99, size = 117, normalized size = 2.17

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

Mupad [B]
time = 3.33, size = 86, normalized size = 1.59

$$\frac{\ln\left(x-1\right)}{4}+\frac{\ln\left(x+1\right)}{4}-\frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4}+\frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4}+\frac{x\sqrt{2-x^2}}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

3.881 $\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$

Optimal. Leaf size=60

$$-\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)$$

```
[Out] -1/4*x^2-1/2*arctanh(x/(-x^2+2)^(1/2))+1/4*ln(1-x)+1/4*ln(1+x)+1/4*x*(-x^2+
2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6874, 201, 222, 711, 399, 385, 213}

$$-\frac{x^2}{4}+\frac{1}{4}\sqrt{2-x^2}\,x-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4
+ Log[1 + x]/4
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx &= \int \left(\frac{\sqrt{2-x^2}}{2}+\frac{2-x^2}{4(-1+x)}+\frac{2-x^2}{4(1+x)}+\frac{\sqrt{2-x^2}}{2\left(-1+x^2\right)}\right)dx\\
&= \frac{1}{4}\int \frac{2-x^2}{-1+x}\,dx+\frac{1}{4}\int \frac{2-x^2}{1+x}\,dx+\frac{1}{2}\int \sqrt{2-x^2}\,dx+\frac{1}{2}\int \frac{\sqrt{2-x^2}}{-1+x^2}\,dx\\
&= \frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\int \left(-1+\frac{1}{-1+x}-x\right)dx+\frac{1}{4}\int \left(1-x+\frac{1}{1+x}\right)dx+\frac{1}{2}\int \frac{}{\sqrt{2}}\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\frac{x}{\sqrt{2-}}\right.\\
&= -\frac{x^2}{4}+\frac{1}{4}x\sqrt{2-x^2}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)+\frac{1}{4}\log(1-x)+\frac{1}{4}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 77, normalized size = 1.28

$$\frac{1}{4}\left(-x^2+x\sqrt{2-x^2}+\log(1-x)-\log(1+x)+\log\left(1-x^2\right)-\log\left(2-x+\sqrt{2-x^2}\right)+\log\left(2+x+\sqrt{2-x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 -
x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(46) = 92$.
time = 0.27, size = 111, normalized size = 1.85

method	result
trager	$-\frac{x^2}{4}+\frac{x\sqrt{-x^2+2}}{4}-\frac{\ln\left(-\frac{\sqrt{-x^2+2}+x}{(1+x)(-1+x)}\right)}{2}$
default	$\frac{x\sqrt{-x^2+2}}{4}+\frac{\sqrt{-(-1+x)^2+3-2x}}{4}-\frac{\operatorname{arctanh}\left(\frac{4-2x}{2\sqrt{-(-1+x)^2+3-2x}}\right)}{4}-\frac{\sqrt{-(1+x)^2}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(-x^2+2)^(1/2)+1/4*(-(-1+x)^2+3-2*x)^(1/2)-1/4*arctanh(1/2*(4-2*x)/(-
(-1+x)^2+3-2*x)^(1/2))-1/4*(-(1+x)^2+3+2*x)^(1/2)+1/4*arctanh(1/2*(4+2*x)/(
-(1+x)^2+3+2*x)^(1/2))-1/4*x^2+1/4*ln(-1+x)+1/4*ln(1+x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)
```

Fricas [A]
time = 0.33, size = 67, normalized size = 1.12

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(x^2-1\right)-\frac{1}{8}\log\left(-\frac{\sqrt{-x^2+2}\,x+1}{x^2}\right)+\frac{1}{8}\log\left(\frac{\sqrt{-x^2+2}\,x-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 +
2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(46) = 92.
time = 3.54, size = 117, normalized size = 1.95

$$-\frac{1}{4}x^2+\frac{1}{4}\sqrt{-x^2+2}\,x+\frac{1}{4}\log\left(\left|x^2-1\right|\right)-\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}+2\right|\right)+\frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}}-\frac{\sqrt{2}-\sqrt{-x^2+2}}{x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sq
rt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(
x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))
```

Mupad [B]
time = 3.38, size = 86, normalized size = 1.43

$$\frac{\ln(x-1)}{4}+\frac{\ln(x+1)}{4}-\frac{\ln\left(\frac{-x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x-1}\right)}{4}+\frac{\ln\left(\frac{x\,1\mathrm{i}+\sqrt{2-x^2}\,1\mathrm{i}+2\mathrm{i}}{x+1}\right)}{4}+\frac{x\sqrt{2-x^2}}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)
```

```
[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))
/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/
4 - x^2/4
```

3.882 $\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=51

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6874, 699, 702, 213}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{-x+\sqrt{2x-x^2}}\,dx &= \int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\frac{x}{2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx, x, \sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 45, normalized size = 0.88

$$\frac{1}{2}\left(i\pi-x-\sqrt{-((-2+x)x)}+\log(-2+x)-2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]
```

```
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A]
time = 0.05, size = 38, normalized size = 0.75

method	result	size
trager	$-\frac{x}{2}+\frac{3}{2}-\frac{\sqrt{-x^2+2x}}{2}-\frac{\ln\left(\sqrt{-x^2+2x}-1\right)}{2}$	35
default	$-\frac{x}{2}-\frac{\ln(-1+x)}{2}-\frac{\sqrt{-(-1+x)^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x+(-x^2+2*x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x-1/2*ln(-1+x)-1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1)^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")
[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)
```

Fricas [A]
time = 0.33, size = 66, normalized size = 1.29

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x}{x-\sqrt{-x^2+2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)
[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)
```

Giac [A]
time = 3.58, size = 50, normalized size = 0.98

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x}{x-\sqrt{2\,x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/(x - (2*x - x^2)^(1/2)),x)
[Out] int(-x/(x - (2*x - x^2)^(1/2)), x)
```

3.883 $\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {6874, 45, 699, 702, 213}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx &= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 45, normalized size = 0.88

$$\frac{1}{2}\left(i\pi-x-\sqrt{-((-2+x)x)}+\log(-2+x)-2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]
```

```
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A]
time = 0.25, size = 38, normalized size = 0.75

method	result	size
trager	$-\frac{x}{2}+\frac{3}{2}-\frac{\sqrt{-x^2+2x}}{2}-\frac{\ln\left(\sqrt{-x^2+2x}-1\right)}{2}$	35

default	$-\frac{x}{2}-\frac{\ln(-1+x)}{2}-\frac{\sqrt{-(-1+x)^2+1}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(-1+x)^2+1}}\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(-x^2+2*x)^(1/2))/(2-2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x-1/2*ln(-1+x)-1/2*(-(-1+x)^2+1)^(1/2)+1/2*arctanh(1/(-(-1+x)^2+1)^(1/
2))
```

Maxima [A]
time = 0.50, size = 54, normalized size = 1.06

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A]
time = 0.33, size = 66, normalized size = 1.29

$$-\frac{1}{2}x-\frac{1}{2}\sqrt{-x^2+2x}-\frac{1}{2}\log(x-1)+\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2x}}{x}\right)-\frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2
*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{x-1}\,dx+\int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(-x**2 + 2*x)/(x - 1), x))/2
```

Giac [A]
time = 2.71, size = 50, normalized size = 0.98

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x
+ 2)) - 1/2*log(abs(x - 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x+\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)
```

```
[Out] int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)
```

3.884 $\int \frac{\sqrt{2-x}\,\sqrt{x}+x}{2-2x}\,dx$

Optimal. Leaf size=51

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((-x^2+2*x)^(1/2))-1/2*ln(1-x)-1/2*(-x^2+2*x)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6820, 2140, 6874, 45, 699, 702, 213}

$$-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{x}{2}-\frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] -1/2*x - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 699

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1))), Int[(d + e*x)^m*(a + b*x
+ c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c
, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m,
 -1] &&  !(IGtQ[(m - 1)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && Rationa
lQ[m] && IntegerQ[2*p]
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 2140

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{2-x}\,\sqrt{x}+x}{2-2x}\,dx &= \int \frac{x+\sqrt{-(-2+x)x}}{2-2x}\,dx \\
&= \int \frac{x+\sqrt{2x-x^2}}{2-2x}\,dx \\
&= \int \left(-\frac{x}{2(-1+x)}+\frac{\sqrt{2x-x^2}}{2(1-x)}\right)dx \\
&= -\left(\frac{1}{2}\int \frac{x}{-1+x}\,dx\right)+\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}\,dx \\
&= -\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\int \left(1+\frac{1}{-1+x}\right)dx+\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}\,dx \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}-\frac{1}{2}\log(1-x)-2\text{Subst}\left(\int \frac{1}{-4+4x^2}\,dx,x,\sqrt{2x-x^2}\right) \\
&= -\frac{x}{2}-\frac{1}{2}\sqrt{2x-x^2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\log(1-x)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.00, size = 45, normalized size = 0.88

$$\frac{1}{2}\left(i\pi - x - \sqrt{-((-2+x)x)} + \log(-2+x) - 2\log\left(-2+x+\sqrt{-((-2+x)x)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]
```

```
[Out] (I*Pi - x - Sqrt[-((-2 + x)*x)] + Log[-2 + x] - 2*Log[-2 + x + Sqrt[-((-2 +
 x)*x)]])/2
```

Maple [A]
time = 0.25, size = 51, normalized size = 1.00

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\,\left(\sqrt{-x\,(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x\,(x-2)}}\right)\right)}{2\sqrt{-x\,(x-2)}}-\frac{x}{2}-\frac{\ln(-1+x)}{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(-1+x)
```

Maxima [A]
time = 0.50, size = 54, normalized size = 1.06

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{-x^2+2\,x}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{2\,\sqrt{-x^2+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log(2*sqrt(-x^2 + 2*x)
/abs(x - 1) + 2/abs(x - 1))
```

Fricas [A]
time = 0.33, size = 64, normalized size = 1.25

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\,\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\,\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\,\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{x-1}\,dx + \int \frac{\sqrt{x}\,\sqrt{2-x}}{x-1}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-
```

Mupad [**B**]
time = 4.78, size = 56, normalized size = 1.10

$$\mathrm{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + x^(1/2)*(2 - x)^(1/2))/(2*x - 2),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.885 $\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)-\frac{1}{2}\log(1-x)$$

```
[Out] -1/2*x+1/2*arctanh((2-x)^(1/2)*x^(1/2))-1/2*ln(1-x)-1/2*(2-x)^(1/2)*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2130, 103, 12, 94, 212, 45}

$$-\frac{x}{2}-\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] -1/2*(Sqrt[2 - x]*Sqrt[x]) - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 -
 x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 94

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[(a + b*x)^m*(c + d*x)^n*((e + f*x)^(p + 1)/(f*
```

```
(m + n + p + 1))), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2130

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2
)*x), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^
2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && N
eQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}}\,dx &= \int \frac{\sqrt{2-x}\,\sqrt{x}}{2-2x}\,dx+\int \frac{x}{2-2x}\,dx\\ &= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}+\frac{1}{2}\int \frac{2}{(2-2x)\sqrt{2-x}\,\sqrt{x}}\,dx+\int \left(-\frac{1}{2}-\frac{1}{2(-1+x)}\right)dx\\ &= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+\int \frac{1}{(2-2x)\sqrt{2-x}\,\sqrt{x}}\,dx\\ &= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}-\frac{1}{2}\log(1-x)+2\text{Subst}\left(\int \frac{1}{4-4x^2}\,dx,x,\sqrt{2-x}\,\sqrt{x}\right)\\ &= -\frac{1}{2}\sqrt{2-x}\,\sqrt{x}-\frac{x}{2}+\frac{1}{2}\tanh^{-1}\left(\sqrt{2-x}\,\sqrt{x}\right)-\frac{1}{2}\log(1-x)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 53, normalized size = 0.98

$$\frac{1}{2}\left(1-x-\sqrt{-((-2+x)x)}+2i\tan^{-1}\left(1-x-i\sqrt{-((-2+x)x)}\right)-\log(2-2x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]),x]
```

```
[Out] (1 - x - Sqrt[-((-2 + x)*x)] + (2*I)*ArcTan[1 - x - I*Sqrt[-((-2 + x)*x)]]
- Log[2 - 2*x])/2
```

Maple [A]
time = 0.02, size = 51, normalized size = 0.94

method	result	size
default	$-\frac{\sqrt{2-x}\,\sqrt{x}\,\left(\sqrt{-x\,(x-2)}-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x\,(x-2)}}\right)\right)}{2\sqrt{-x\,(x-2)}}-\frac{x}{2}-\frac{\ln(-1+x)}{2}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2-x)^(1/2)*x^(1/2)/(-x*(x-2))^(1/2)*((-x*(x-2))^(1/2)-arctanh(1/(-x*(
x-2))^(1/2)))-1/2*x-1/2*ln(-1+x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)
```

Fricas [A]
time = 0.32, size = 64, normalized size = 1.19

$$-\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x}\,\sqrt{-x+2}-\frac{1}{2}\,\log\left(x-1\right)+\frac{1}{2}\,\log\left(\frac{x+\sqrt{x}\,\sqrt{-x+2}}{x}\right)-\frac{1}{2}\,\log\left(-\frac{x-\sqrt{x}\,\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*s
qrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{-\sqrt{x}+\sqrt{2-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)
```

```
[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2]%%%}+%%%{-8,[1]%%%}+%%%{4,[0]%%%}]
at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%%
{4,[2]%%%}+%%%{-
```

Mupad [**B**]
time = 0.06, size = 56, normalized size = 1.04

$$\operatorname{atanh}\left(\frac{\sqrt{x}\,\left(\sqrt{2}-\sqrt{2-x}\right)}{x+\sqrt{2}\,\sqrt{2-x}-2}\right)-\frac{\ln\left(x-1\right)}{2}-\frac{x}{2}-\frac{\sqrt{x}\,\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((2 - x)^(1/2) - x^(1/2)),x)
```

```
[Out] atanh((x^(1/2)*(2^(1/2) - (2 - x)^(1/2)))/(x + 2^(1/2)*(2 - x)^(1/2) - 2))
- log(x - 1)/2 - x/2 - (x^(1/2)*(2 - x)^(1/2))/2
```

3.886 $\int \frac{1}{\left((1+x)\left(-1+x^2\right)\right)^{2/3}}\,dx$

Optimal. Leaf size=27

$$-\frac{3(1-x^2)}{2\left(-\left((1+x)\left(1-x^2\right)\right)\right)^{2/3}}$$

```
[Out] -3/2*(-x^2+1)/(-(1+x)*(-x^2+1))^(2/3)
```

Rubi [A]

time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2092, 2089, 37}

$$-\frac{3(1-x)(x+1)}{2\left(x^3+x^2-x-1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)*(-1 + x^2))^(-2/3),x]
```

```
[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2089

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 2092

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - (c^2 - 3*b*d)*(x/(3*d)) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rubi steps

$$\int \frac{1}{((1+x)(-1+x^2))^{2/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(-\frac{16}{27}-\frac{4x}{3}+x^3\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right)$$
$$= \frac{\left(32\sqrt[3]{2}\,(-1-x)^{4/3}(-1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(-\frac{16}{9}-\frac{8x}{3}\right)^{4/3}\left(-\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, \frac{1}{3}+x\right.}{9\left(-1-x+x^2+x^3\right)^{2/3}}$$
$$= -\frac{3(1-x)(1+x)}{2\left(-1-x+x^2+x^3\right)^{2/3}}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.85

$$\frac{3(-1+x)(1+x)}{2\left((-1+x)(1+x)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3),x]
[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.74

method	result	size
gosper	$\frac{3(1+x)(-1+x)}{2((1+x)(x^2-1))^{\frac{2}{3}}}$	20
risch	$\frac{3(1+x)(-1+x)}{2\left((-1+x)(1+x)^2\right)^{\frac{2}{3}}}$	20
trager	$\frac{3\left(x^3+x^2-x-1\right)^{\frac{1}{3}}}{2(1+x)}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)*(x^2-1))^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(1+x)*(-1+x)/((1+x)*(x^2-1))^(2/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 0.74

$$\frac{3\,(x^3+x^2-x-1)^{\frac{1}{3}}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x+1)\,(x^2-1))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)
[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="giac")
[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)
```

Mupad [B]
time = 3.42, size = 20, normalized size = 0.74

$$\frac{3\,((x^2-1)\,\,(x+1))^{1/3}}{2\,\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)*(x + 1))^(2/3),x)
[Out] (3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))
```

3.887 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx$

Optimal. Leaf size=14

$$-\frac{2x}{\sqrt{x(1+x^2)}}$$

```
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6851, 460}

$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x*(1 + x^2)]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}}\,dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{\sqrt{x(1+x^2)}}\\ &= -\frac{2x}{\sqrt{x(1+x^2)}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A]
time = 0.26, size = 13, normalized size = 0.93

method	result	size
gosper	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
default	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
risch	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
elliptic	$-\frac{2x}{\sqrt{x\left(x^{2}+1\right)}}$	13
trager	$-\frac{2\sqrt{x^{3}+x}}{x^{2}+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,\mathrm{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^{2}\right)}{5}-2\sqrt{x}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-x^{2}\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Fricas [A]
time = 0.36, size = 16, normalized size = 1.14

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)
```

Mupad [B]
time = 3.38, size = 138, normalized size = 9.86

$$-\frac{2\,x}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)\,\sqrt{x\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x^3+x}}+\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{\frac{1}{2}+\frac{x\,1\mathrm{i}}{2}}\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{1-x\,1\mathrm{i}}\right)\middle|\frac{1}{2}\right)\,\sqrt{x\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x^3+x}}-\frac{\sqrt{1-x\,1\mathrm{i}}\,\sqrt{1+x\,1\mathrm{i}}\,\sqrt{-x\,1\mathrm{i}}\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{-x\,1\mathrm{i}}\right)\middle|-1\right)\,1\mathrm{i}}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)
```

```
[Out] ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^(1/2)*ellipticF(asin((1 - x*1i)^(1/2)),
1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*((x*1i)/2 + 1/2)^
(1/2)*ellipticE(asin((1 - x*1i)^(1/2)), 1/2)*(x*1i)^(1/2)*2i)/(x + x^3)^(1/
2) - (2*x)/(x + x^3)^(1/2) - ((1 - x*1i)^(1/2)*(x*1i + 1)^(1/2)*(-x*1i)^(1/
2)*ellipticE(asin((-x*1i)^(1/2)), -1)*1i)/(x + x^3)^(1/2)
```

3.888 $\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{x+x^3}}$$

```
[Out] -2*x/(x^3+x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2081, 460}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
```

```
[Out] (-2*x)/Sqrt[x + x^3]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\,(1+x^2)^{3/2}}\,dx}{\sqrt{x+x^3}}\\
&= -\frac{2x}{\sqrt{x+x^3}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 1.00

$$-\frac{2x}{\sqrt{x+x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]
[Out] (-2*x)/Sqrt[x + x^3]
```

Maple [A]
time = 0.24, size = 13, normalized size = 1.08

method	result	size
gosper	$-\frac{2x}{\sqrt{x^3+x}}$	11
default	$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$	13
risch	$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$	13
elliptic	$-\frac{2x}{\sqrt{x\left(x^2+1\right)}}$	13
trager	$-\frac{2\sqrt{x^3+x}}{x^2+1}$	17
meijerg	$\frac{2x^{\frac{5}{2}}\,\mathrm{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-x^2\right)}{5}-2\sqrt{x}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-x^2\right)$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2*x/(x*(x^2+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Fricas [A]
time = 0.36, size = 16, normalized size = 1.33

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")
[Out] -2*sqrt(x^3 + x)/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")
[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)
```

Mupad [B]
time = 0.05, size = 10, normalized size = 0.83

$$-\frac{2\,x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)
[Out] -(2*x)/(x + x^3)^(1/2)
```

3.889 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/x/(x^2+1))^(1/2)/(-x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6850, 460}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 6850

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.)*(z_)^(q_.))^(p_), x_Symbol] :> Dist[
a^IntPart[p]*((a*v^m*w^n*z^q)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[
p])*z^(q*FracPart[p]))), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a,
 m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !Fr
eeQ[z, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2}\,dx = \frac{\left(\sqrt{x}\,\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}\,\sqrt{1+x^2}\right)\int \frac{-1+x^2}{\sqrt{x}\,(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2}$$

Mathematica [A]
time = 0.01, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{-1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A]
time = 0.21, size = 31, normalized size = 0.86

method	result	size
default	$-\frac{2\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}\,x}{x^2-1}$	31
risch	$-\frac{2\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}\,x}{x^2-1}$	31
gosper	$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}}{(1+x)(-1+x)}$	34
trager	$-\frac{2x\sqrt{-\frac{-x^4+2x^2-1}{x^3+x}}}{x^2-1}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Fricas [A]
time = 0.32, size = 30, normalized size = 0.83

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{(x-1)^2\,(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)
```

Mupad [B]
time = 3.49, size = 48, normalized size = 1.33

$$-\frac{(2x^3+2x)\sqrt{\frac{1}{x^2+1}}\sqrt{(x^2-1)^2}\sqrt{\frac{1}{x}}}{(x^2-1)(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2
- 1)*(x^2 + 1))
```

3.890 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

```
[Out] 2*x*((-x^2+1)^2/(x^3+x))^(1/2)/(-x^2+1)
```

Rubi [A]
time = 0.12, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6851, 2081, 460}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
```

```
[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
```

```
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2}\,dx = \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\sqrt{x+x^3}\right)\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}}\,dx}{-1+x^2}$$
$$= \frac{\left(\sqrt{x}\sqrt{1+x^2}\sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right)\int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}}\,dx}{-1+x^2}$$
$$= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{-1+x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2),x]
[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)
```

Maple [A]
time = 0.22, size = 31, normalized size = 0.94

method	result	size
default	$-\frac{2\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}\,x}{x^2-1}$	31
risch	$-\frac{2\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}\,x}{x^2-1}$	31
gosper	$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x(x^2+1)}}}{(1+x)(-1+x)}$	34

trager	$-\frac{2x\sqrt{-\frac{-x^4+2x^2-1}{x^3+x}}}{x^2-1}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2*((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2-1)*x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Fricas [A]
time = 0.33, size = 30, normalized size = 0.91

$$-\frac{2\,x\sqrt{\frac{x^4-2\,x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{(x-1)^2\,(x+1)^2}{x^3+x}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)
```

Mupad [B]
time = 3.49, size = 43, normalized size = 1.30

$$-\frac{\sqrt{\frac{1}{x^3+x}}\,\left(2\,x^3+2\,x\right)\,\sqrt{{\left(x^2-1\right)}^2}}{\left(x^2-1\right)\,\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)
```

```
[Out] -((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 +
1))
```

3.891 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b+ax^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

```
[Out] arctanh(d^(1/2)*(a*x^2+b)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))*(a*x^2+b)^(1/2)/x/
a^(1/2)/d^(1/2)/(a+b/x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {446, 455, 65, 223, 212}

$$\frac{\sqrt{ax^2+b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{ax^2+b}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\,x\sqrt{a+\frac{b}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[x^(n*FracPart[q])*((c + d/x^n)^FracPart[q]/(d + c*x^n)^FracPart[
q]), Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{b+ax^2}\int \frac{x}{\sqrt{b+ax^2}\sqrt{c+dx^2}}\,dx}{\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{b+ax}\sqrt{c+dx}}\,dx,x,x^2\right)}{2\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{bd}{a}+\frac{dx^2}{a}}}\,dx,x,\sqrt{b+ax^2}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{a}}\,dx,x,\frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}}\right)}{a\sqrt{a+\frac{b}{x^2}}\,x}$$

$$= \frac{\sqrt{b+ax^2}\,\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

Mathematica [A]
time = 0.38, size = 70, normalized size = 1.00

$$\frac{\sqrt{b+ax^2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{b+ax^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2]
)])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)
```

Maple [A]
time = 0.23, size = 103, normalized size = 1.47

method	result	size
default	$\frac{(a\,x^2+b)\ln\left(\frac{2ad\,x^2+2\sqrt{(a\,x^2+b)\,(d\,x^2+c)}\,\sqrt{ad}+ac+bd}{2\sqrt{ad}}\right)\sqrt{d\,x^2+c}}{2\sqrt{\frac{a\,x^2+b}{x^2}}\,x\sqrt{ad}\,\sqrt{(a\,x^2+b)\,(d\,x^2+c)}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*((a*x^2+b)*(d*x^2
+c))^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^(1/2)/((
a*x^2+b)*(d*x^2+c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)
```

Fricas [A]
time = 0.36, size = 208, normalized size = 2.97

$$\left[\frac{\sqrt{ad}\,\log\left(8\,a^2d^2x^4+a^2c^2+6\,abcd+b^2d^2+8\,(a^2cd+abd^2)x^2+4\,(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\,\sqrt{ad}\,\sqrt{\frac{ax^2+b}{x^2}}\right)}{4\,ad}, -\frac{\sqrt{-ad}\,\arctan\left(\frac{(2\,adx^3+(ac+bd)x)\sqrt{dx^2+c}\,\sqrt{-ad}\,\sqrt{\frac{ax^2+b}{x^2}}}{2\,(a^2d^2x^4+abcd+(a^2cd+abd^2)x^2)}\right)}{2\,ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c
*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)
*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c
 + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 +
a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,\sqrt{c+dx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)
```

Giac [**A**]
time = 2.29, size = 92, normalized size = 1.31

$$\frac{a\log\left(\left|-\sqrt{ad}\,\sqrt{b}+\sqrt{a^2c}\right|\right)\operatorname{sgn}(x)}{\sqrt{ad}\,|a|}-\frac{a\log\left(\left|-\sqrt{ax^2+b}\,\sqrt{ad}+\sqrt{a^2c+(ax^2+b)ad-abd}\right|\right)}{\sqrt{ad}\,|a|\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(a*d)*sqrt(b) + sqrt(a^2*c)))*sgn(x)/(sqrt(a*d)*abs(a)) - a*
log(abs(-sqrt(a*x^2 + b)*sqrt(a*d) + sqrt(a^2*c + (a*x^2 + b)*a*d - a*b*d))
)/(sqrt(a*d)*abs(a)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{x^2}}\,\sqrt{d\,x^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.892 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=83

$$\frac{2\sqrt{-2x^2+x^4}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}-\frac{\sqrt{-2x^2+x^4}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

```
[Out] 2/3*arctan(1/2*(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)-1/3*arctan(
(x^2-2)^(1/2))*(x^4-2*x^2)^(1/2)/x/(x^2-2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2081, 585, 85, 65, 209}

$$\frac{2\sqrt{x^4-2x^2}\,\text{ArcTan}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}-\frac{\sqrt{x^4-2x^2}\,\text{ArcTan}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
```

```
[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqr
t[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] && !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx &= \frac{\sqrt{-2x^2+x^4}\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}} \\
&= \frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}} \\
&= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx,x,x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2\sqrt{-2x^2+x^4}\right)\text{Subst}}{3x\sqrt{}} \\
&= -\frac{\sqrt{-2x^2+x^4}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4\sqrt{-2x^2+x^4}\right)\text{Subst}\left(\int\right.}{3x\sqrt{-2}} \\
&= \frac{2\sqrt{-2x^2+x^4}\,\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} - \frac{\sqrt{-2x^2+x^4}\,\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.67

$$
\frac{x\sqrt{-2+x^2}\left(2\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\tan^{-1}\left(\sqrt{-2+x^2}\right)\right)}{3\sqrt{x^2(-2+x^2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)),x]
[Out] (x*Sqrt[-2 + x^2]*(2*ArcTan[Sqrt[-2 + x^2]/2] - ArcTan[Sqrt[-2 + x^2]]))/(3
*Sqrt[x^2*(-2 + x^2)])
```

Maple [A]
time = 0.32, size = 63, normalized size = 0.76

method	result
default	$-\frac{\sqrt{x^4-2x^2}\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6x\sqrt{x^2-2}}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)x^7-15\,\mathrm{RootOf}\left(\_Z^2+1\right)x^5-6\sqrt{x^4-2x^2}\,x^4+24\,\mathrm{RootOf}\left(\_Z^2+1\right)x^3+16\sqrt{x^4-2x^2}}{\left(x^2+2\right)^2x(1+x)(-1+x)}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x,method=_RETURNVERBOSE)
[Out] -1/6*(x^4-2*x^2)^(1/2)*(arctan((x-2)/(x^2-2)^(1/2))-arctan((x+2)/(x^2-2)^(1
/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)
```

Fricas [A]
time = 0.35, size = 38, normalized size = 0.46

$$-\frac{1}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{x}\right)+\frac{2}{3}\arctan\left(\frac{\sqrt{x^4-2\,x^2}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="fricas")
[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x^2\,(x^2-2)}}{(x-1)\,(x+1)\,(x^2+2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)
```

Giac [C] Result contains complex when optimal does not.
time = 3.11, size = 46, normalized size = 0.55

$$\frac{1}{3}\left(\arctan\left(i\sqrt{2}\right)-2\arctan\left(\frac{1}{2}i\sqrt{2}\right)\right)\operatorname{sgn}(x)+\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}(x)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*s
qrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4-2\,x^2}}{\left(x^2-1\right)\,\left(x^2+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)
```

```
[Out] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)
```

3.893 $\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx$

Optimal. Leaf size=47

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

```
[Out] (-x^2+1)*arctan((x^2-2)^(1/2))*(1-1/(-x^2+1)^2)^(1/2)/x/(x^2-2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6854, 6857, 2015, 1160, 21, 267, 455, 52, 65, 209}

$$\frac{(1-x^2)\sqrt{x^4-2x^2}\sqrt{1-\frac{1}{(1-x^2)^2}}\,\text{ArcTan}\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}\sqrt{(x^2-1)^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

```
[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2
]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1160

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbo
l] :> Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p])*(b + c*x^2)^FracP
art[p]), Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d,
 e, p, q}, x] &&  !IntegerQ[p]
```

Rule 2015

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

Rule 6854

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] &&  !LinearQ[v, x]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx = \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{(2-x^2)(-1+x^2)}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \left(\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}+\frac{\sqrt{-1+\left(-1+x^2\right)^2}}{-1+x^2}\right.}{\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-1+\left(-1+x^2\right)^2}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1\right.}{}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{2-x^2}\,dx}{\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^2\right)\sqrt{1-\frac{}{(-1}}\right.}{\sqrt{-1}}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{2-x^2}\,dx}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}} + \frac{\left(\left(-1+x^{\right.}}{}$$

$$= \frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{-1+x}\,dx,x,x^2\right)}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx,\right.}{2x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= -\frac{\left(\left(-1+x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-2+x}\right.}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

$$= \frac{\left(1-x^2\right)\sqrt{-2x^2+x^4}\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}\sqrt{-1+\left(-1+x^2\right)^2}}$$

Mathematica [A]
time = 5.52, size = 44, normalized size = 0.94

$$-\frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
[Out] -(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
+ x^2]))
```

Maple [A]
time = 0.23, size = 63, normalized size = 1.34

method	result	size
default	$-\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right)\right)}{2x\sqrt{x^2-2}}$	63
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x^3+2x^2\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}+3x\,\mathrm{RootOf}\left(\_Z^2+1\right)-2\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}}{x(1+x)(-1+x)}\right)}{2}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x,method=_RETURNVERBOSE)
[Out] -1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Fricas [A]
time = 0.37, size = 36, normalized size = 0.77

$$-\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

```
[Out] -arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

```
[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2
 - 2), x)
```

Giac [A]
time = 5.51, size = 18, normalized size = 0.38

$$-\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{1-\frac{1}{\left(x^2-1\right)^2}}}{x^2-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
```

```
[Out] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

3.894 $\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx$

Optimal. Leaf size=123

$$-\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}+\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

```
[Out] -2/3*(-x^2+1)*arctan(1/2*(x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/2)/x/(x
^2-2)^(1/2)+1/3*(-x^2+1)*arctan((x^2-2)^(1/2))*((x^4-2*x^2)/(-x^2+1)^2)^(1/
2)/x/(x^2-2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6851, 2081, 585, 85, 65, 209}

$$\frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\,\text{ArcTan}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}-\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\,\text{ArcTan}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
```

```
[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/
(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan
[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x),
 x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 585

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 2081

```
Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}
, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int
[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x]] /; FreeQ[p, x] &&  !IntegerQ[p] &&
SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] &&  !PolyQ[P, x, 2]
```

Rule 6851

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p]))), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rubi steps

$$\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)}\,dx}{\sqrt{-2x^2+x^4}}$$

$$= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)}\,dx}{x\sqrt{-2+x^2}}$$

$$= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)}\,dx, x, x^2\right)}{2x\sqrt{-2+x^2}}$$

$$= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-2+x}\,(-1+x)}\,dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2(-1+x^2)\right.}{}$$

$$= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right)\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4(-1+x^2)\right.\sqrt{}}{}$$

$$= -\frac{2(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{-\frac{2x^2-x^4}{(1-x^2)^2}}\tan^{-1}\left(\right.}{3x\sqrt{-2+x^2}}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 0.57

$$\frac{\sqrt{\frac{x^2(-2+x^2)}{(-1+x^2)^2}}\,(-1+x^2)\left(2\tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\tan^{-1}\left(\sqrt{-2+x^2}\right)\right)}{3x\sqrt{-2+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2),x]
[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2]
 - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])
```

Maple [A]
time = 0.23, size = 75, normalized size = 0.61

method	result

default	$-\frac{\sqrt{\frac{x^{2}\left(x^{2}-2\right)}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right)\left(\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{x+2}{\sqrt{x^{2}-2}}\right)-4\arctan\left(\frac{\sqrt{x^{2}-2}}{2}\right)\right)}{6x\sqrt{x^{2}-2}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{7}-6\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}\,x^{6}-15\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{5}+22x^{4}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}+24\operatorname{RootOf}\left(\_\right.}{x(1+x)(-1+x)\left(x^{2}+2\right)^{2}}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arc
tan((x+2)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)
```

Fricas [A]
time = 0.36, size = 74, normalized size = 0.60

$$-\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan
(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{\frac{x^2\,(x^2-2)}{x^4-2x^2+1}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2*(x**2 - 2)/(x**4 - 2*x**2 + 1))/(x**2 + 2), x)
```

Giac [A]
time = 3.27, size = 39, normalized size = 0.32

$$\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)-\frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(
x^3 - x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{-\frac{2\,x^2-x^4}{{\left(x^2-1\right)}^2}}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

```
[Out] int((-(2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

3.895 $\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx$

Optimal. Leaf size=133

$$-\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}-\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)}-\frac{(4+3x)\left(1+x^2\right)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{5\sqrt{1+$$

[Out] -4/3*(1-2*x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-1/3*(1-x)*(1+x)^3*(1+2*x/(x^2+1))^(1/2)/(x^2+1)-(4+3*x)*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+5*arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)

Rubi [A]

time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6855, 984, 753, 833, 794, 221}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)^3}{3\left(x^2+1\right)}-\frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}\,(x+1)-\frac{(3x+4)\left(x^2+1\right)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + (2*x)/(1 + x^2))^(5/2),x]

[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/3 - ((1 - x)*(1 + x)^3*Sqrt[1 + (2*x)/(1 + x^2)])/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
```

```
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \left(1+\frac{2x}{1+x^2}\right)^{5/2} dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(1+2x+x^2)^{5/2}}{(1+x^2)^{5/2}}\,dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^5}{(1+x^2)^{5/2}}\,dx}{16(2+2x)}$$

$$= -\frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(24-8x)(2+2x)^3}{(1+x^2)^{3/2}}\,dx}{48(2+2x)}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} + \frac{\left(\sqrt{1+x^2}\right.}{}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\,(1}{}$$

$$= -\frac{4}{3}(1-2x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3\sqrt{1+\frac{2x}{1+x^2}}}{3\left(1+x^2\right)} - \frac{(4+3x)\,(1}{}$$

Mathematica [A]
time = 0.28, size = 74, normalized size = 0.56

$$\frac{(1+x)\left(-17-12x-18x^2-8x^3+3x^4+15(1+x^2)^{3/2}\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)}{3\sqrt{\frac{(1+x)^2}{1+x^2}}\,\left(1+x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2),x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2)*ArcTanh[
x/Sqrt[1 + x^2]]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

Maple [A]
time = 0.09, size = 62, normalized size = 0.47

method	result	size

default	$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{5}{2}}(x^2+1)\left(15\,\mathrm{arcsinh}(x)(x^2+1)^{\frac{3}{2}}+3x^4-8x^3-18x^2-12x-17\right)}{3(1+x)^5}$	62
risch	$\frac{(3x^4-8x^3-18x^2-12x-17)\sqrt{\frac{(1+x)^2}{x^2+1}}}{3(x^2+1)(1+x)}+\frac{5\,\mathrm{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(1+x)^2}{x^2+1}}}{1+x}$	82
trager	$\frac{(3x^4-8x^3-18x^2-12x-17)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{3(x^2+1)(1+x)}+5\ln\left(\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2+x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}+x}{1+x}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(1+x)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2
)+3*x^4-8*x^3-18*x^2-12*x-17)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)
```

Fricas [A]
time = 0.35, size = 117, normalized size = 0.88

$$-\frac{8\,x^3+8\,x^2+15\,(x^3+x^2+x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(3\,x^4-8\,x^3-18\,x^2-12\,x-17)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+8\,x+8}{3\,(x^3+x^2+x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^
2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17
)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(5/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

Giac [A]
time = 4.03, size = 86, normalized size = 0.65

$$\left(\sqrt{2} + 5\,\log\left(\sqrt{2} + 1\right)\right)\mathrm{sgn}(x+1) - 5\,\log\left(-x + \sqrt{x^2+1}\right)\mathrm{sgn}(x+1) + \frac{(((3\,x\mathrm{sgn}(x+1) - 8\,\mathrm{sgn}(x+1))x - 18\,\mathrm{sgn}(x+1))x - 12\,\mathrm{sgn}(x+1))x - 17\,\mathrm{sgn}(x+1)}{3\,(x^2+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(5/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x
 + 1) + 1/3*((((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sg
n(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(\frac{2\,x}{x^2+1}+1\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(5/2),x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

3.896 $\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx$

Optimal. Leaf size=90

$$-\left((1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}}\right)-\frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{3\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\sinh^{-1}(x)}{1+x}$$

```
[Out] -(1-x)*(1+x)*(1+2*x/(x^2+1))^(1/2)-x*(x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+3*
arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6855, 984, 753, 531, 396, 221}

$$-\left((1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)\right)-\frac{x(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}+\frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/
(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x]
)/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 531

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
```

```
Q[a2, 0]))
```

Rule 753

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^
2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; Free
Q[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && I
ntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \left(1+\frac{2x}{1+x^2}\right)^{3/2} dx = \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(1+2x+x^2)^{3/2}}{(1+x^2)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$

$$= \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(2+2x)^3}{(1+x^2)^{3/2}}\,dx}{4(2+2x)}$$

$$= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} + \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{(8-8x)(2+2x)}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} + \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{16-16x^2}{\sqrt{1+x^2}}\,dx}{4(2+2x)}$$

$$= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{\left(6\sqrt{1+x^2}\,\sqrt{1+\frac{}{1}\right.}{2}$$

$$= -(1-x)(1+x)\sqrt{1+\frac{2x}{1+x^2}} - \frac{x(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x} + \frac{3\sqrt{1+x^2}\,\sqrt{1+\frac{2}{1+}}}{1+x}$$

Mathematica [A]
time = 0.15, size = 54, normalized size = 0.60

$$\frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(-1-2x+x^2+3\sqrt{1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2),x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcTanh[x/Sqrt
[1 + x^2]]))/(1 + x)
```

Maple [A]
time = 0.08, size = 49, normalized size = 0.54

method	result	size

default	$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}(x^2+1)\left(3\,\mathrm{arcsinh}(x)\sqrt{x^2+1}+x^2-2x-1\right)}{(1+x)^3}$	49
risch	$\frac{(x^2-2x-1)\sqrt{\frac{(1+x)^2}{x^2+1}}}{1+x}+\frac{3\,\mathrm{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(1+x)^2}{x^2+1}}}{1+x}$	62
trager	$\frac{(x^2-2x-1)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{1+x}-3\ln\left(-\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}-x}{1+x}\right)$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(1+x)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2
-2*x-1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)
```

Fricas [A]
time = 0.33, size = 83, normalized size = 0.92

$$-\frac{3\,(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(x^2-2\,x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+2\,x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x +
 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)
```

Giac [A]
time = 4.60, size = 67, normalized size = 0.74

$$-\left(\sqrt{2}-3\,\log\left(\sqrt{2}+1\right)\right)\mathrm{sgn}(x+1)-3\,\log\left(-x+\sqrt{x^2+1}\right)\mathrm{sgn}(x+1)+\frac{(x\mathrm{sgn}(x+1)-2\,\mathrm{sgn}(x+1))x-\mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(
x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{2\,x}{x^2+1}+1\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.897 $\int \sqrt{1+\frac{2x}{1+x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\,\sinh^{-1}(x)}{1+x}$$

```
[Out] (x^2+1)*(1+2*x/(x^2+1))^(1/2)/(1+x)+arcsinh(x)*(x^2+1)^(1/2)*(1+2*x/(x^2+1)
)^(1/2)/(1+x)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6855, 984, 655, 221}

$$\frac{\sqrt{\frac{2x}{x^2+1}+1}\,(x^2+1)}{x+1}+\frac{\sqrt{\frac{2x}{x^2+1}+1}\,\sqrt{x^2+1}\,\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*
x)/(1 + x^2)]*ArcSinh[x])/(1 + x)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\frac{2x}{1+x^2}}\,dx &= \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}}\,dx}{\sqrt{1+2x+x^2}}\\ &= \frac{\left(\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{\sqrt{1+x^2}}\,dx}{2+2x}\\ &= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\left(2\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2+2x}\\ &= \frac{(1+x^2)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}+\frac{\sqrt{1+x^2}\,\sqrt{1+\frac{2x}{1+x^2}}\,\sinh^{-1}(x)}{1+x}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 50, normalized size = 0.82

$$\frac{\sqrt{\frac{(1+x)^2}{1+x^2}}\left(1+x^2+\sqrt{1+x^2}\,\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcTanh[x/Sqrt[1 + x^2]
]))/(1 + x)
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.69

method	result	size
default	$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}}\,\sqrt{x^2+1}\,\left(\operatorname{arcsinh}(x)+\sqrt{x^2+1}\right)}{1+x}$	42

risch	$\frac{(x^2+1)\sqrt{\frac{(1+x)^2}{x^2+1}}}{1+x}+\frac{\operatorname{arcsinh}(x)\sqrt{x^2+1}\,\sqrt{\frac{(1+x)^2}{x^2+1}}}{1+x}$	58
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{1+x}-\ln\left(-\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}-x}{1+x}\right)$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(1+x)*(x^2+1)^(1/2)*(arcsinh(x)+(x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)
```

Fricas [A]
time = 0.35, size = 75, normalized size = 1.23

$$-\frac{(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")
[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1
)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{2x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2),x)
```

[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)

Giac [A]
time = 6.35, size = 49, normalized size = 0.80

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\sqrt{x^2+1}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")

[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x + 1) + sqrt(x^2 + 1)*sgn(x + 1)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{\frac{2\,x}{x^2+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*x)/(x^2 + 1) + 1)^(1/2),x)

[Out] int(((2*x)/(x^2 + 1) + 1)^(1/2), x)

3.898 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx$

Optimal. Leaf size=109

$$\frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}}-\frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{\sqrt{2}\,(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

```
[Out] (1+x)/(1+2*x/(x^2+1))^(1/2)-(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^
(1/2)-(1+x)*arctanh(1/2*(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/
(1+2*x/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6855, 984, 749, 858, 221, 739, 212}

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}}-\frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{\sqrt{2}\,(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqr
t[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1
+ x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 749

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 2*p + 1))), x] + Dist[2*(p/(e*(m +
 2*p + 1))), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x],
x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && Ne
Q[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&  !ILtQ[m + 2*p, 0] &&
 IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(2+2x)\int \frac{\sqrt{1+x^2}}{2+2x}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2+2x)\int \frac{2-2x}{(2+2x)\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2+2x)\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(2(2+2x))\int \frac{1}{(2+2x)\sqrt{1+x^2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(2(2+2x))\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{2-2x}{\sqrt{1+}}\right.}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{1+x}{\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{\sqrt{2}(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.75

$$\frac{(1+x)\left(\sqrt{1+x^2} - \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) + 2\sqrt{2}\tanh^{-1}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)\right)}{\sqrt{\frac{(1+x)^2}{1+x^2}}\sqrt{1+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)],x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcTanh[x/Sqrt[1 + x^2]] + 2*Sqrt[2]*ArcTanh[(1 +
 x - Sqrt[1 + x^2])/Sqrt[2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])
```

Maple [A]
time = 0.16, size = 79, normalized size = 0.72

method	result
risch	$\frac{1+x}{\sqrt{\frac{(1+x)^2}{x^2+1}}}+\frac{\left(-\operatorname{arcsinh}(x)-\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2x)\sqrt{2}}{4\sqrt{(1+x)^2-2x}}\right)\right)(1+x)}{\sqrt{\frac{(1+x)^2}{x^2+1}}\,\sqrt{x^2+1}}$
trager	$\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,(x^2+1)}{1+x}+\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x^2+2\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-\operatorname{RootOf}\left(\_Z^2-2\right)+}{(1+x)^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+2*x/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/((1+x)^2/(x^2+1))^(1/2)*(1+x)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(2-2*x)*2^(1/2)/((1+x)^2-2*x)^(1/2)))/((1+x)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(1+x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)

Fricas [A]
time = 0.35, size = 142, normalized size = 1.30

$$\frac{\sqrt{2}\,(x+1)\log\left(-\frac{x^2+\sqrt{2}\,(x^2-1)+\left(2\,x^2+\sqrt{2}\,(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)
```

Giac [A]
time = 5.72, size = 88, normalized size = 0.81

$$\frac{\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2\right|}\right)}{\operatorname{sgn}(x+1)}+\frac{\log\left(-x+\sqrt{x^2+1}\right)}{\operatorname{sgn}(x+1)}+\frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2
) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) +
 sqrt(x^2 + 1)/sgn(x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(1/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)
```

3.899 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}}-\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}}-\frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}-\frac{9(1+x)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

```
[Out] 3/2*(2+x)/(1+2*x/(x^2+1))^(1/2)+1/2*(-x^2-1)/(1+x)/(1+2*x/(x^2+1))^(1/2)-3*
(1+x)*arcsinh(x)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)-9/4*(1+x)*arctanh(1/2*
(1-x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)/(x^2+1)^(1/2)/(1+2*x/(x^2+1))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6855, 984, 747, 827, 858, 221, 739, 212}

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}+1}}-\frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}+1}}-\frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}-\frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (
2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1
+ x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*
Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 747

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m + 1)*((a + c*x^2)^p/(e*(m + 1))), x] - Dist[2*c*(p/(e*(m + 1))
), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e
, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1
]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e,
m, p, x]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}}\,dx = \frac{\sqrt{1+2x+x^2}\int \frac{(1+x^2)^{3/2}}{(1+2x+x^2)^{3/2}}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{(4(2+2x))\int \frac{(1+x^2)^{3/2}}{(2+2x)^3}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= -\frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(3(2+2x))\int \frac{x\sqrt{1+x^2}}{(2+2x)^2}\,dx}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}}\,dx}{8\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(3(2+2x))\int \frac{1}{\sqrt{1+x^2}}\,dx}{2\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} + \frac{(9(2+2x)}{2\sqrt{1+}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{(9(2+2x))\mathrm{Su}}{2\sqrt{1}}$$

$$= \frac{3(2+x)}{2\sqrt{1+\frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1+\frac{2x}{1+x^2}}} - \frac{3(1+x)\sinh^{-1}(x)}{\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}} - \frac{9(1+x)\tanh}{2\sqrt{2}\sqrt{1+}}$$

Mathematica [A]
time = 0.30, size = 106, normalized size = 0.74

$$\frac{(1+x)\left(\sqrt{1+x^2}\,(5+9x+2x^2)-6(1+x)^2\tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)+9\sqrt{2}\,(1+x)^2\tanh^{-1}\left(\frac{1+x-\sqrt{1+x^2}}{\sqrt{2}}\right)\right)}{2\left(\frac{(1+x)^2}{1+x^2}\right)^{3/2}(1+x^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2),x]
```

```
[Out] ((1 + x)*(Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) - 6*(1 + x)^2*ArcTanh[x/Sqrt[1 +
x^2]] + 9*Sqrt[2]*(1 + x)^2*ArcTanh[(1 + x - Sqrt[1 + x^2])/Sqrt[2]]))/(2*(
(1 + x)^2/(1 + x^2))^(3/2)*(1 + x^2)^(3/2))
```

Maple [A]

time = 0.16, size = 217, normalized size = 1.51

method	result
risch	$\frac{2x^4+9x^3+7x^2+9x+5}{2(1+x)(x^2+1)\sqrt{\frac{(1+x)^2}{x^2+1}}}+\frac{\left(-3\operatorname{arcsinh}(x)-\frac{9\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2x)\sqrt{2}}{4\sqrt{(1+x)^2-2x}}\right)}{4}\right)(1+x)}{\sqrt{\frac{(1+x)^2}{x^2+1}}\,\sqrt{x^2+1}}$
trager	$\frac{(x^2+1)(2x^2+9x+5)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{2(1+x)^3}+3\ln\left(-\frac{\sqrt{-\frac{-x^2-2x-1}{x^2+1}}\,x^2-x^2+\sqrt{-\frac{-x^2-2x-1}{x^2+1}}-x}{1+x}\right)+\frac{9\operatorname{RootOf}\left(\_Z^2-\right.}{}$
default	$-\frac{(1+x)\left(-(x^2+1)^{\frac{5}{2}}x+(x^2+1)^{\frac{3}{2}}x^3+(x^2+1)^{\frac{5}{2}}-(x^2+1)^{\frac{3}{2}}x^2-18\operatorname{arctanh}\left(\frac{(-1+x)\sqrt{2}}{2\sqrt{x^2+1}}\right)\sqrt{2}\,x^2-5x(x^2+1)^{\frac{3}{2}}+6\sqrt{x^2+1}\,x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*x/(x^2+1))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/((x^2+2*x+1)/(x^2+1))^(3/2)*(1+x)*(-(x^2+1)^(5/2)*x+(x^2+1)^(3/2)*x^3+
(x^2+1)^(5/2)-(x^2+1)^(3/2)*x^2-18*arctanh(1/2*(-1+x)*2^(1/2)/(x^2+1)^(1/2)
)*2^(1/2)*x^2-5*x*(x^2+1)^(3/2)+6*(x^2+1)^(1/2)*x^3+24*arcsinh(x)*x^2-36*ar
ctanh(1/2*(-1+x)*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)*x-3*(x^2+1)^(3/2)-6*(x^2+1)
^(1/2)*x^2+48*arcsinh(x)*x-18*2^(1/2)*arctanh(1/2*(-1+x)*2^(1/2)/(x^2+1)^(1
/2))-30*x*(x^2+1)^(1/2)+24*arcsinh(x)-18*(x^2+1)^(1/2))/(x^2+1)^(3/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)
```

Fricas [A]

time = 0.33, size = 205, normalized size = 1.42

$$\frac{10\,x^3+9\,\sqrt{2}\,(x^3+3\,x^2+3\,x+1)\log\left(-\frac{x^2+\sqrt{2}\,(x^2-1)+\left(2\,x^2+\sqrt{2}\,(x^2+1)+2\right)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}-1}{x^2+2\,x+1}\right)+30\,x^2+12\,(x^3+3\,x^2+3\,x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x}{x+1}\right)+2\,(2\,x^4+9\,x^3+7\,x^2+9\,x+5)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+30\,x+10}{4\,(x^3+3\,x^2+3\,x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 -
1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(
x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)
*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 +
 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x +
 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x**2+1))**(3/2),x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)
```

Giac [A]
time = 5.33, size = 170, normalized size = 1.18

$$\frac{9\sqrt{2}\log\left(\frac{\left|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2\right|}{\left|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2\right|}\right)}{4\,\mathrm{sgn}\left(x+1\right)}+\frac{3\log\left(-x+\sqrt{x^2+1}\right)}{\mathrm{sgn}\left(x+1\right)}+\frac{\sqrt{x^2+1}}{\mathrm{sgn}\left(x+1\right)}+\frac{7\left(x-\sqrt{x^2+1}\right)^3+5\left(x-\sqrt{x^2+1}\right)^2-13x+13\sqrt{x^2+1}+5}{\left(\left(x-\sqrt{x^2+1}\right)^2+2x-2\sqrt{x^2+1}-1\right)^2\mathrm{sgn}\left(x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="giac")
```

```
[Out] 9/4*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sq
rt(2) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + 3*log(-x + sqrt(x^2 + 1))/sgn(x
+ 1) + sqrt(x^2 + 1)/sgn(x + 1) + (7*(x - sqrt(x^2 + 1))^3 + 5*(x - sqrt(x^
2 + 1))^2 - 13*x + 13*sqrt(x^2 + 1) + 5)/(((x - sqrt(x^2 + 1))^2 + 2*x - 2*
sqrt(x^2 + 1) - 1)^2*sgn(x + 1))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(\frac{2\,x}{x^2+1}+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)
```

```
[Out] int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)
```

3.900 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx$

Optimal. Leaf size=28

$$-\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

```
[Out] -(1-x)*(1+2*x/(x^2+1))^(1/2)/(1+x)
```

Rubi [A]
time = 0.08, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6855, 984, 651}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 6855

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a +
 b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] &&  !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2}\,dx = \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{\sqrt{1+2x+x^2}}{(1+x^2)^{3/2}}\,dx}{\sqrt{1+2x+x^2}}$$
$$= \frac{\left(\sqrt{1+x^2}\sqrt{1+\frac{2x}{1+x^2}}\right)\int \frac{2+2x}{(1+x^2)^{3/2}}\,dx}{2+2x}$$
$$= -\frac{(1-x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

Mathematica [A]
time = 0.09, size = 25, normalized size = 0.89

$$\frac{(-1+x)\sqrt{1+\frac{2x}{1+x^2}}}{1+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2),x]
```

```
[Out] ((-1 + x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x)
```

Maple [A]
time = 0.22, size = 28, normalized size = 1.00

method	result	size
risch	$\frac{\sqrt{\frac{(1+x)^2}{x^2+1}}\,(-1+x)}{1+x}$	25
gosper	$\frac{(-1+x)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{1+x}$	28
default	$\frac{(-1+x)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{1+x}$	28
trager	$\frac{(-1+x)\sqrt{-\frac{-x^2-2x-1}{x^2+1}}}{1+x}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x/(x^2+1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)/(1+x)*((x^2+2*x+1)/(x^2+1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)
```

Fricas [A]
time = 0.38, size = 31, normalized size = 1.11

$$\frac{(x-1)\sqrt{\frac{x^2+2\,x+1}{x^2+1}}+x+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1),x)
[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)
```

Giac [A]
time = 5.22, size = 30, normalized size = 1.07

$$\sqrt{2}\,\mathrm{sgn}(x+1)+\frac{x\mathrm{sgn}(x+1)-\mathrm{sgn}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1),x, algorithm="giac")
[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)
```

Mupad [B]
time = 3.52, size = 23, normalized size = 0.82

$$\frac{\sqrt{\frac{2\,x}{x^2+1}+1}\,\left(x-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1),x)
[Out] ((((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)
```

3.901 $\int \sqrt{x-x^2}\,F(x)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\sqrt{x-x^2}\,F(x),x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{x-x^2}\,F(x)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{x-x^2}\,F(x)\,dx=\int \sqrt{x-x^2}\,F(x)\,dx$$

Mathematica [A]
time = 0.07, size = 0, normalized size = 0.00

$$\int \sqrt{x-x^2}\,F(x)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[x - x^2]*F[x],x]
```

```
[Out] Integrate[Sqrt[x - x^2]*F[x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int F(x)\,\sqrt{-x^2+x}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(-x^2+x)^(1/2),x)
[Out] int(F(x)*(-x^2+x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(-x^2 + x)*F(x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x\,(x-1)}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x**2+x)**(1/2),x)
[Out] Integral(sqrt(-x*(x - 1))*F(x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(-x^2+x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + x)*F(x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int F(x)\,\sqrt{x-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(x - x^2)^(1/2),x)
[Out] int(F(x)*(x - x^2)^(1/2), x)
```

3.902 $\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[x]/Sqrt[x - x^2],x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[x]/Sqrt[x - x^2],x]
[Out] Integrate[F[x]/Sqrt[x - x^2], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(-x^2+x)^(1/2),x)
```

```
[Out] int(F(x)/(-x^2+x)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-x^2 + x)*F(x)/(x^2 - x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{-x(x-1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x**2+x)**(1/2),x)
```

```
[Out] Integral(F(x)/sqrt(-x*(x - 1)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(-x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(F(x)/sqrt(-x^2 + x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(x - x^2)^(1/2),x)
```

```
[Out] int(F(x)/(x - x^2)^(1/2), x)
```

3.903 $\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\sqrt{x-x^2}\,F(x),x\right)$$

```
[Out] CannotIntegrate(F(x)*(-x^2+x)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Defer[Int][Sqrt[x - x^2]*F[x], x]
```

Rubi steps

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx = \int \sqrt{x-x^2}\,F(x)\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{1-x}\,\sqrt{x}\,F(x)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x],x]
[Out] Integrate[Sqrt[1 - x]*Sqrt[x]*F[x], x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int F(x)\,\sqrt{1-x}\,\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

```
[Out] int(F(x)*(1-x)^(1/2)*x^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{1-x}\,F(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)**(1/2)*x**(1/2),x)
[Out] Integral(sqrt(x)*sqrt(1 - x)*F(x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)*(1-x)^(1/2)*x^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)*sqrt(-x + 1)*F(x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \sqrt{x}\;F(x)\;\sqrt{1-x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*F(x)*(1 - x)^(1/2),x)
[Out] int(x^(1/2)*F(x)*(1 - x)^(1/2), x)
```

3.904 $\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{F(x)}{\sqrt{x-x^2}},x\right)$$

```
[Out] CannotIntegrate(F(x)/(-x^2+x)^(1/2),x)
```

Rubi [A]

time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Defer[Int][F[x]/Sqrt[x - x^2], x]
```

Rubi steps

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}\,dx = \int \frac{F(x)}{\sqrt{x-x^2}}\,dx$$

Mathematica [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] Integrate[F[x]/(Sqrt[1 - x]*Sqrt[x]), x]
```

Maple [A]

time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{1-x}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
[Out] int(F(x)/(1-x)^(1/2)/x^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(x)*sqrt(-x + 1)*F(x)/(x^2 - x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)**(1/2)/x**(1/2),x)
[Out] Integral(F(x)/(sqrt(x)*sqrt(1 - x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x)/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(F(x)/(sqrt(x)*sqrt(-x + 1)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{F(x)}{\sqrt{x}\,\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x)/(x^(1/2)*(1 - x)^(1/2)),x)
[Out] int(F(x)/(x^(1/2)*(1 - x)^(1/2)), x)
```

3.905 $\int F\left(\frac{a+bx}{x}\right)\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(F\left(b+\frac{a}{x}\right),x\right)$$

```
[Out] CannotIntegrate(F(b+a/x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F\left(\frac{a+bx}{x}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[(a + b*x)/x],x]
```

```
[Out] Defer[Int][F[b + a/x], x]
```

Rubi steps

$$\int F\left(\frac{a+bx}{x}\right)\,dx = \int F\left(b+\frac{a}{x}\right)\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx}{x}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[(a + b*x)/x],x]
```

```
[Out] Integrate[F[(a + b*x)/x], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{bx+a}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x+a)/x),x)
[Out] int(F((b*x+a)/x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="maxima")
[Out] integrate(F((b*x + a)/x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="fricas")
[Out] integral(F((b*x + a)/x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x)
[Out] Integral(F((a + b*x)/x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x+a)/x),x, algorithm="giac")
[Out] integrate(F((b*x + a)/x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int F\left(\frac{a+b\,x}{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((a + b*x)/x),x)
```

```
[Out] int(F((a + b*x)/x), x)
```

3.906 $\int F\left(\frac{a+bx^2}{x^2}\right) dx$

Optimal. Leaf size=11

$$\text{Int}\left(F\left(b+\frac{a}{x^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(b+a/x^2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Int[F[(a + b*x^2)/x^2],x]
[Out] Defer[Int][F[b + a/x^2], x]
```

Rubi steps

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx = \int F\left(b+\frac{a}{x^2}\right) dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx^2}{x^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[(a + b*x^2)/x^2],x]
[Out] Integrate[F[(a + b*x^2)/x^2], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{b\,x^2+a}{x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((b*x^2+a)/x^2),x)
[Out] int(F((b*x^2+a)/x^2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="maxima")
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="fricas")
[Out] integral(F((b*x^2 + a)/x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{a+bx^2}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x**2+a)/x**2),x)
[Out] Integral(F((a + b*x**2)/x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F((b*x^2+a)/x^2),x, algorithm="giac")
[Out] integrate(F((b*x^2 + a)/x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int F\left(\frac{b\,x^2+a}{x^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F((a + b*x^2)/x^2),x)
[Out] int(F((a + b*x^2)/x^2), x)
```

3.907 $\int F\left(\frac{x}{a+bx}\right)\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(F\left(\frac{x}{a+bx}\right),x\right)$$

```
[Out] CannotIntegrate(F(x/(b*x+a)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x}{a+bx}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[F[x/(a + b*x)],x]
```

```
[Out] Defer[Int][F[x/(a + b*x)], x]
```

Rubi steps

$$\int F\left(\frac{x}{a+bx}\right)\,dx=\int F\left(\frac{x}{a+bx}\right)\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{a+bx}\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[x/(a + b*x)],x]
```

```
[Out] Integrate[F[x/(a + b*x)], x]
```

Maple [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{bx+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x/(b*x+a)),x)
```

```
[Out] int(F(x/(b*x+a)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="fricas")
```

```
[Out] integral(F(x/(b*x + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x}{a+bx}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x)
```

```
[Out] Integral(F(x/(a + b*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x/(b*x+a)),x, algorithm="giac")
```

```
[Out] integrate(F(x/(b*x + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int F\left(\frac{x}{a+b\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x/(a + b*x)),x)
[Out] int(F(x/(a + b*x)), x)
```

3.908 $\int F\left(\frac{x^2}{a+bx^2}\right) dx$

Optimal. Leaf size=17

$$\text{Int}\left(F\left(\frac{x^2}{a+bx^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^2/(b*x^2+a)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Int[F[x^2/(a + b*x^2)],x]
```

```
[Out] Defer[Int][F[x^2/(a + b*x^2)], x]
```

Rubi steps

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx = \int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{a+bx^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[x^2/(a + b*x^2)],x]
```

```
[Out] Integrate[F[x^2/(a + b*x^2)], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{b\,x^2+a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(b*x^2+a)),x)
```

```
[Out] int(F(x^2/(b*x^2+a)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="maxima")
```

```
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="fricas")
```

```
[Out] integral(F(x^2/(b*x^2 + a)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{a+bx^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x**2+a)),x)
```

```
[Out] Integral(F(x**2/(a + b*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x^2+a)),x, algorithm="giac")
```

```
[Out] integrate(F(x^2/(b*x^2 + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int F\left(\frac{x^2}{b\,x^2+a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(a + b*x^2)),x)
[Out] int(F(x^2/(a + b*x^2)), x)
```

3.909 $\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$

Optimal. Leaf size=15

$$\text{Int}\left(F\left(\frac{x^2}{(a+bx)^2}\right), x\right)$$

`[Out] CannotIntegrate(F(x^2/(b*x+a)^2),x)`

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is not applicable to the result.

`[In] Int[F[x^2/(a + b*x)^2],x]`

`[Out] Defer[Int][F[x^2/(a + b*x)^2], x]`

Rubi steps

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx = \int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right) dx$$

Verification is not applicable to the result.

`[In] Integrate[F[x^2/(a + b*x)^2],x]`

`[Out] Integrate[F[x^2/(a + b*x)^2], x]`

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(bx+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(b*x+a)^2),x)
[Out] int(F(x^2/(b*x+a)^2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="maxima")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="fricas")
[Out] integral(F(x^2/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^2}{(a+bx)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**2/(b*x+a)**2),x)
[Out] Integral(F(x**2/(a + b*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^2/(b*x+a)^2),x, algorithm="giac")
[Out] integrate(F(x^2/(b*x + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int F\left(\frac{x^2}{{\left(a+b\,x\right)}^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^2/(a + b*x)^2),x)
[Out] int(F(x^2/(a + b*x)^2), x)
```

3.910 $\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(F\left(\frac{x^4}{(a+bx^2)^2}\right),x\right)$$

```
[Out] CannotIntegrate(F(x^4/(b*x^2+a)^2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Int[F[x^4/(a + b*x^2)^2],x]
```

```
[Out] Defer[Int][F[x^4/(a + b*x^2)^2], x]
```

Rubi steps

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx = \int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[F[x^4/(a + b*x^2)^2],x]
```

```
[Out] Integrate[F[x^4/(a + b*x^2)^2], x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{(b\,x^2+a)^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^4/(b*x^2+a)^2),x)
```

```
[Out] int(F(x^4/(b*x^2+a)^2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="maxima")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="fricas")
```

```
[Out] integral(F(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int F\left(\frac{x^4}{(a+bx^2)^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x**4/(b*x**2+a)**2),x)
```

```
[Out] Integral(F(x**4/(a + b*x**2)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(F(x^4/(b*x^2+a)^2),x, algorithm="giac")
```

```
[Out] integrate(F(x^4/(b*x^2 + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int F\left(\frac{x^4}{(b\,x^2+a)^2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(F(x^4/(a + b*x^2)^2),x)
[Out] int(F(x^4/(a + b*x^2)^2), x)
```

3.911 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

```
[Out] 1/2*arctanh(x*2^(1/2)*b^(1/2)/(b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1
/2)
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2157, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[
b])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1-2bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [A]
time = 0.26, size = 49, normalized size = 1.04

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{2}\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
[Out] ArcTanh[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/(Sqrt[2]*Sqrt[b]*x)]/(Sqrt[2]*Sqrt[
b])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A]
time = 1.15, size = 135, normalized size = 2.87

$$\left[\frac{\sqrt{2}\log\left(4\,b^2x^4+4\,\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^{\frac{3}{2}}x^3+\sqrt{2}\,\sqrt{b^2x^4+a}\,\sqrt{b}\,x\right)\sqrt{bx^2+\sqrt{b^2x^4+a}}+a\right)}{4\,\sqrt{b}}, -\frac{1}{2}\,\sqrt{2}\,\sqrt{-\frac{1}{b}}\arctan\left(\frac{\sqrt{2}\,\sqrt{bx^2+\sqrt{b^2x^4+a}}\,\sqrt{-\frac{1}{b}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)
*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))
 + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt
(b^2*x^4 + a))*sqrt(-1/b)/x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2\,x^4+a}+b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

```
[Out] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.912 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

[Out] 1/2*arctan(x*2^(1/2)*b^(1/2)/(-b*x^2+(b^2*x^4+a)^(1/2))^(1/2))*2^(1/2)/b^(1/2)

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2157, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2157

```
Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*
(x_)^4], x_Symbol] :> Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^
2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0
]
```

Rubi steps

$$\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx = \text{Subst}\left(\int \frac{1}{1+2bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}\,x}{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [A]
time = 0.25, size = 51, normalized size = 1.06

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{2}\sqrt{b}\,x}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]
[Out] -(ArcTan[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/(Sqrt[2]*Sqrt[b]*x)]/(Sqrt[2]*S
qrt[b]))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b\,x^2+\sqrt{b^2x^4+a}}}{\sqrt{b^2x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Fricas [A]
time = 1.25, size = 146, normalized size = 3.04

$$\left[\frac{1}{4}\,\sqrt{2}\sqrt{-\frac{1}{b}}\log\left(4\,b^2x^4-4\,\sqrt{b^2x^4+a}\,bx^2+2\left(\sqrt{2}\,b^2x^3\sqrt{-\frac{1}{b}}-\sqrt{2}\,\sqrt{b^2x^4+a}\,bx\sqrt{-\frac{1}{b}}\right)\sqrt{-bx^2+\sqrt{b^2x^4+a}}+a\right),-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-bx^2+\sqrt{b^2x^4+a}}}{2\,\sqrt{b}\,x}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt
(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*
x^2 + sqrt(b^2*x^4 + a)) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2
+ sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\sqrt{b^2\,x^4+a}-b\,x^2}}{\sqrt{b^2\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)
```

```
[Out] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)
```

3.913 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}\,d^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{2ic^2+\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}+2ix^2}}\right)}{\sqrt{2ic^2+\sqrt{3}\,d^2}}$$

```
[Out] (1/2-1/2*I)*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^
2*3^(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(1/2)-(1/2+1/2*I)*arctanh((-2*I*c*x
+d*3^(1/2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d
^2*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2158, 739, 210, 212}

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\text{ArcTan}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\,\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}\,d^2+2ic^2}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\,\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}\,d^2+2ic^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*
Sqrt[Sqrt[3] - (2*I)*x^2])])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*A
rcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3]
+ (2*I)*x^2])])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)\sqrt{\sqrt{3}+2ix^2}}$$

$$= \left(-\frac{1}{2}-\frac{i}{2}\right)\text{Subst}\left(\int \frac{1}{2ic^2+\sqrt{3}\,d^2-x^2}\,dx, x, \frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}}\right) + \left(-\frac{1}{2}+\frac{i}{2}\right.$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\,\sqrt{\sqrt{3}-2ix^2}}\right)}{\sqrt{2ic^2-\sqrt{3}\,d^2}} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)\tanh^{-1}\left(\frac{\cdots}{\sqrt{2ic\cdots}}\right.}{\sqrt{2ic^2\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 4.00, size = 531, normalized size = 3.14

$$\frac{-\sqrt{-2c^2-\sqrt{4c^4+3d^4}}\tan^{-1}\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2-\sqrt{4c^4+3d^4}}}\right)+\sqrt{-2c^2+\sqrt{4c^4+3d^4}}\tan^{-1}\left(\frac{d\sqrt{2x^2+\sqrt{3+4x^4}}}{\sqrt{-2c^2+\sqrt{4c^4+3d^4}}}\right)+2ic\sqrt{12c^4+9d^4}\,\text{RootSum}\left[12d^2+16i\sqrt{3}c^2\#1+24d^2\#1+24i\sqrt{3}c^2\#1^2+24d^2\#1^2+8i\sqrt{3}c^2\#1^3+12d^2\#1^3+3d^2\#1^4\&, \frac{\log\left(2x^2+\sqrt{3+4x^4}\right)-\log\left(i\sqrt{3}-2x^2+2x\sqrt{2x^2+\sqrt{3+4x^4}}-2x^2\#1-\sqrt{3+4x^4}(1+\#1)\right)+\log\left(2x^2+\sqrt{3+4x^4}\right)\#1-\log\left(i\sqrt{3}-2x^2+2x\sqrt{2x^2+\sqrt{3+4x^4}}-2x^2\#1-\sqrt{3+4x^4}(1+\#1)\right)\#1}{4i\sqrt{3}c^2+6d^2+12i\sqrt{3}c^2\#1+12d^2\#1+6i\sqrt{3}c^2\#1^2+9d^2\#1^2+3d^2\#1^3}\&\right]}{\sqrt{4c^4+3d^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]),x]
```

```
[Out] (-(Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4
]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]]) + Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4
]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^
4]]] + (2*I)*c*Sqrt[12*c^4 + 9*d^4]*RootSum[12*d^2 + (16*I)*Sqrt[3]*c^2*#1
+ 24*d^2*#1 + (24*I)*Sqrt[3]*c^2*#1^2 + 24*d^2*#1^2 + (8*I)*Sqrt[3]*c^2*#1^
```

```
3 + 12*d^2*#1^3 + 3*d^2*#1^4 & , (Log[2*x^2 + Sqrt[3 + 4*x^4]] - Log[I*Sqrt
[3] - 2*x^2 + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - 2*x^2*#1 - Sqrt[3 + 4*x^4
]*(1 + #1)] + Log[2*x^2 + Sqrt[3 + 4*x^4]]*#1 - Log[I*Sqrt[3] - 2*x^2 + 2*x
*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - 2*x^2*#1 - Sqrt[3 + 4*x^4]*(1 + #1)]*#1)/(
(4*I)*Sqrt[3]*c^2 + 6*d^2 + (12*I)*Sqrt[3]*c^2*#1 + 12*d^2*#1 + (6*I)*Sqrt[
3]*c^2*#1^2 + 9*d^2*#1^2 + 3*d^2*#1^3) & ])/Sqrt[4*c^4 + 3*d^4]
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)\sqrt{4x^4+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

```
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="maxima")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(c+dx)\sqrt{4x^4+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2\,x^2+\sqrt{4\,x^4+3}}}{\sqrt{4\,x^4+3}\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)
```

```
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)
```

3.914 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$

Optimal. Leaf size=268

$$\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)}+\frac{(1+i)c\tan^{-1}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{2ic^2-\sqrt{3}\,d^2}\sqrt{\sqrt{3}-2ix^2}}\right)}{\left(2ic^2-\sqrt{3}\,d^2\right)^{3/2}}+\cdots$$

```
[Out] (1+I)*c*arctan((2*I*c*x+d*3^(1/2))/(-2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2-d^2*3^
(1/2))^(1/2))/(2*I*c^2-d^2*3^(1/2))^(3/2)+(1-I)*c*arctanh((-2*I*c*x+d*3^(1/
2))/(2*I*x^2+3^(1/2))^(1/2)/(2*I*c^2+d^2*3^(1/2))^(1/2))/(2*I*c^2+d^2*3^(1/
2))^(3/2)+(1/2-1/2*I)*d*(-2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2-d^2*3^(1/
2))-(1/2+1/2*I)*d*(2*I*x^2+3^(1/2))^(1/2)/(d*x+c)/(2*I*c^2+d^2*3^(1/2))
```

Rubi [A]

time = 0.22, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2158, 745, 739, 210, 212}

$$\frac{(1+i)c\,\text{ArcTan}\left(\frac{\sqrt{3}\,d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}\,d^2+2ic^2}}\right)}{\left(-\sqrt{3}\,d^2+2ic^2\right)^{3/2}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(-\sqrt{3}\,d^2+2ic^2\right)(c+dx)}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(\sqrt{3}\,d^2+2ic^2\right)(c+dx)}+\frac{(1-i)c\tanh^{-1}\left(\frac{\sqrt{3}\,d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}\,d^2+2ic^2}}\right)}{\left(\sqrt{3}\,d^2+2ic^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
```

```
[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d
*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)
*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - S
qrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2])])/((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) +
((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*S
qrt[Sqrt[3] + (2*I)*x^2])])/((2*I)*c^2 + Sqrt[3]*d^2)^(3/2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2158

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^
4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^
m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sq
rt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ
[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx = \left(\frac{1}{2}-\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}-2ix^2}}\,dx + \left(\frac{1}{2}+\frac{i}{2}\right)\int \frac{1}{(c+dx)^2\sqrt{\sqrt{3}+2i}}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + \frac{((1+i)c)\int \frac{}{(c+dx)\sqrt{}}}{2c^2-i\sqrt{3}}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + -\frac{((1+i)c)\text{Subst}\left(\int\right.}{}$$

$$= \frac{\left(\frac{1}{2}-\frac{i}{2}\right)d\sqrt{\sqrt{3}-2ix^2}}{\left(2ic^2-\sqrt{3}\,d^2\right)(c+dx)} - \frac{\left(\frac{1}{2}+\frac{i}{2}\right)d\sqrt{\sqrt{3}+2ix^2}}{\left(2ic^2+\sqrt{3}\,d^2\right)(c+dx)} + \frac{(1+i)c\tan^{-1}\left(\frac{}{\sqrt{2i}}\right.}{\left(2ic^2\right.}$$

Mathematica [F]
time = 10.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]
[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2+\sqrt{4x^4+3}}}{(dx+c)^2\sqrt{4x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="maxima")
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)^2 \sqrt{4x^4 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2),x)
[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)**2*sqrt(4*x**4 + 3)), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algori
thm="giac")
[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^2 + \sqrt{4\,x^4 + 3}}}{\sqrt{4\,x^4 + 3}\,{\left(c + d\,x\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2),x)
[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)
```

3.915 $\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$-30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1854, 1634, 52, 65, 209}

$$30\text{ArcTan}\left(\sqrt[6]{x}\right)+\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-30\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)
]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1854

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g =
Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a +
b*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x
] && FractionQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-4+x}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= 3\text{Subst}\left(\int \frac{\sqrt{x}\left(-4+x^3\right)}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 3\text{Subst}\left(\int \left(\sqrt{x}-x^{3/2}+x^{5/2}-\frac{5\sqrt{x}}{1+x}\right)\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}-15\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+15\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -30\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+30\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.02

$$\frac{2}{35}\left(-525\sqrt[6]{x}+35\sqrt{x}-21x^{5/6}+15x^{7/6}\right)+30\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]
[Out] (2*(-525*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 30*ArcTan[x^
(1/6)]
```

Maple [A]
time = 0.22, size = 28, normalized size = 0.68

method	result	size
derivativedivides	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
default	$-30x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+30\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+30\arctan\left(x^{\frac{1}{6}}\right)-24x^{\frac{1}{6}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-4)/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -30*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+30*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A]
time = 0.52, size = 27, normalized size = 0.66

$$\frac{6}{7}\,x^{\frac{7}{6}}-\frac{6}{5}\,x^{\frac{5}{6}}+2\,\sqrt{x}-30\,x^{\frac{1}{6}}+30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.61

$$\frac{6}{7}\,(x-35)x^{\frac{1}{6}}-\frac{6}{5}\,x^{\frac{5}{6}}+2\,\sqrt{x}+30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x-4}{\sqrt{x}\,\left(\sqrt[3]{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x**(1/3))/x**(1/2),x)
[Out] Integral((x - 4)/(sqrt(x)*(x**(1/3) + 1)), x)
```

Giac [A]
time = 4.15, size = 27, normalized size = 0.66

$$\frac{6}{7}\,x^{\frac{7}{6}} - \frac{6}{5}\,x^{\frac{5}{6}} + 2\,\sqrt{x} - 30\,x^{\frac{1}{6}} + 30\,\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 30*x^(1/6) + 30*arctan(x^(1/6))
```

Mupad [B]
time = 3.37, size = 27, normalized size = 0.66

$$30\,\mathrm{atan}\left(x^{1/6}\right) + 2\,\sqrt{x} - 30\,x^{1/6} - \frac{6\,x^{5/6}}{5} + \frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 4)/(x^(1/2)*(x^(1/3) + 1)),x)
[Out] 30*atan(x^(1/6)) + 2*x^(1/2) - 30*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.916 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx$

Optimal. Leaf size=26

$$3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Rubi [A]
time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1607, 1833, 1824, 649, 209, 266}

$$6\text{ArcTan}\left(\sqrt[6]{x}\right)+3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
```

```
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1833

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/(m
 + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[
Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}}\,dx &= \int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)x^{5/6}}\,dx\\
&= 6\text{Subst}\left(\int \frac{1+x^3}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(x+\frac{1-x}{1+x^2}\right)\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)-6\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 26, normalized size = 1.00

$$3\sqrt[3]{x}+6\tan^{-1}\left(\sqrt[6]{x}\right)-3\log\left(1+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]
[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]
```

Maple [A]
time = 0.22, size = 21, normalized size = 0.81

method	result	size
derivativedivides	$3x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	21

default	$3x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	21
meijerg	$3x^{\frac{1}{3}}+6\arctan\left(x^{\frac{1}{6}}\right)-3\ln\left(1+x^{\frac{1}{3}}\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(x^(5/6)+x^(7/6)),x,method=_RETURNVERBOSE)
[Out] 3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))
```

Maxima [A]
time = 0.49, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="fricas")
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Sympy [A]
time = 1.11, size = 24, normalized size = 0.92

$$3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)+6\,\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)
[Out] 3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))
```

Giac [A]
time = 2.39, size = 20, normalized size = 0.77

$$3\,x^{\frac{1}{3}}+6\,\arctan\left(x^{\frac{1}{6}}\right)-3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="giac")
```

```
[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)
```

Mupad [B]
time = 3.36, size = 22, normalized size = 0.85

$$6\,\mathrm{atan}\left(x^{1/6}\right)-3\,\ln\left(36\,x^{1/3}+36\right)+3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)
```

```
[Out] 6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)
```

3.917 $\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx$

Optimal. Leaf size=42

$$6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/6)-3*x^(1/3)+3/2*x^(2/3)-6*arctan(x^(1/6))+3*ln(1+x^(1/3))
```

Rubi [A]
time = 0.10, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6820, 1607, 1816, 649, 209, 266}

$$-6\text{ArcTan}\left(\sqrt[6]{x}\right)+\frac{3x^{2/3}}{2}-3\sqrt[3]{x}+6\sqrt[6]{x}+3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/
3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt{x}}{\left(1+\sqrt[3]{x}\right)\sqrt{x}}\,dx &= \int \frac{1+\frac{1}{\sqrt{x}}}{1+\sqrt[3]{x}}\,dx\\
&= 6\text{Subst}\left(\int \frac{x^2+x^5}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \frac{x^2(1+x^3)}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\text{Subst}\left(\int \left(1-x+x^3-\frac{1-x}{1+x^2}\right)\,dx, x, \sqrt[6]{x}\right)\\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\text{Subst}\left(\int \frac{1-x}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)+6\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x\right.\\
&= 6\sqrt[6]{x}-3\sqrt[3]{x}+\frac{3x^{2/3}}{2}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{3}{2}\left(4-2\sqrt[6]{x}+\sqrt{x}\right)\sqrt[6]{x}-6\tan^{-1}\left(\sqrt[6]{x}\right)+3\log\left(1+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]
[Out] (3*(4 - 2*x^(1/6) + Sqrt[x])*x^(1/6))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(
1/3)]
```

Maple [A]
time = 0.22, size = 48, normalized size = 1.14

method	result	si
derivativedivides	$6x^{\frac{1}{6}}-3x^{\frac{1}{3}}+\frac{3x^{\frac{2}{3}}}{2}-6\arctan\left(x^{\frac{1}{6}}\right)+3\ln\left(1+x^{\frac{1}{3}}\right)$	31
meijerg	$6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)-\frac{x^{\frac{1}{3}}\left(-3x^{\frac{1}{3}}+6\right)}{2}+3\ln\left(1+x^{\frac{1}{3}}\right)$	33
default	$\ln\left(1+x\right)+\frac{3x^{\frac{2}{3}}}{2}+2\ln\left(1+x^{\frac{1}{3}}\right)-\ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right)-3x^{\frac{1}{3}}+6x^{\frac{1}{6}}-6\arctan\left(x^{\frac{1}{6}}\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+x)+3/2*x^(2/3)+2*ln(1+x^(1/3))-ln(x^(2/3)-x^(1/3)+1)-3*x^(1/3)+6*x^(1/
6)-6*arctan(x^(1/6))
```

Maxima [A]
time = 0.51, size = 30, normalized size = 0.71

$$\frac{3}{2}\,x^{\frac{2}{3}}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\arctan\left(x^{\frac{1}{6}}\right)+3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Fricas [A]
time = 0.37, size = 30, normalized size = 0.71

$$\frac{3}{2}\,x^{\frac{2}{3}}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\arctan\left(x^{\frac{1}{6}}\right)+3\,\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Sympy [A]
time = 39.36, size = 39, normalized size = 0.93

$$6\sqrt[6]{x}+\frac{3x^{\frac{2}{3}}}{2}-3\sqrt[3]{x}+3\log\left(\sqrt[3]{x}+1\right)-6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)
```

```
[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1
/6))
```

Giac [A]
time = 4.54, size = 30, normalized size = 0.71

$$\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\arctan\left(x^{\frac{1}{6}}\right) + 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1
)
```

Mupad [B]
time = 0.03, size = 42, normalized size = 1.00

$$\frac{3x^{2/3}}{2} + 3\ln\left(\left(-6 + x^{1/6}\,6\mathrm{i}\right)\,\left(6 + x^{1/6}\,6\mathrm{i}\right)\right) - 3x^{1/3} - 6\,\mathrm{atan}\left(x^{1/6}\right) + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)
```

```
[Out] 3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (3
*x^(2/3))/2 + 6*x^(1/6)
```

3.918 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsch(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {25, 342, 221}

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2}\,dx = \int \frac{1}{\sqrt{2+\frac{b}{x^2}}\,x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\text{csch}^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(20) = 40.
time = 0.02, size = 48, normalized size = 2.40

$$-\frac{\sqrt{2+\frac{b}{x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2),x]
```

```
[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*
x^2]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(14) = 28.
time = 0.22, size = 50, normalized size = 2.50

method	result	size
default	$-\frac{\sqrt{\frac{2x^2+b}{x^2}}\,x\ln\left(\frac{2b+2\sqrt{b}\,\sqrt{2x^2+b}}{x}\right)}{\sqrt{2x^2+b}\,\sqrt{b}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+b/x^2)^(1/2)/(2*x^2+b),x,method=_RETURNVERBOSE)
```

```
[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1
/2)+b)/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.
time = 0.35, size = 75, normalized size = 3.75

$$\left[\frac{\log\left(-\frac{x^2-\sqrt{b}\,x\sqrt{\frac{2\,x^2+b}{x^2}}+b}{x^2}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x\sqrt{\frac{2\,x^2+b}{x^2}}}{2\,x^2+b}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b
)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2}+2}}{b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)
```

```
[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.
time = 4.00, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2+b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\mathrm{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))
*sgn(x)/sqrt(-b)
```

Mupad [B]
time = 3.46, size = 17, normalized size = 0.85

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}}{2\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)
[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)
```

3.919 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

```
[Out] -arccsc(x*2^(1/2)/b^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {25, 342, 222}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[u*((a + b*x^n)^(m + p)/x^(n*p)), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2}\,dx = \int \frac{1}{\sqrt{2-\frac{b}{x^2}}\,x^2}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{\csc^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.
time = 0.02, size = 52, normalized size = 2.60

$$\frac{\sqrt{2-\frac{b}{x^2}}\,x\tan^{-1}\left(\frac{\sqrt{-b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{-b+2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2),x]
```

```
[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x
^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(14) = 28$.
time = 0.22, size = 61, normalized size = 3.05

method	result	size
default	$-\frac{\sqrt{-\frac{-2x^2+b}{x^2}}\,x\ln\left(\frac{-2b+2\sqrt{-b}\,\sqrt{2x^2-b}}{x}\right)}{\sqrt{2x^2-b}\,\sqrt{-b}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-b/x^2)^(1/2)/(2*x^2-b),x,method=_RETURNVERBOSE)
```

```
[Out] -(-(-2*x^2+b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x
^2-b)^(1/2)-b)/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(14) = 28$.
time = 0.39, size = 84, normalized size = 4.20

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{x^2-\sqrt{-b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}-b}{x^2}\right)}{2\,b}, -\frac{\arctan\left(\frac{\sqrt{b}\,x\sqrt{\frac{2\,x^2-b}{x^2}}}{2\,x^2-b}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -a
rctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{b}{x^2}+2}}{-b+2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)
```

```
[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.
time = 4.23, size = 40, normalized size = 2.00

$$\frac{\arctan\left(\frac{\sqrt{2\,x^2-b}}{\sqrt{b}}\right)\mathrm{sgn}(x)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right)\mathrm{sgn}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="giac")
```

```
[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*s
gn(x)/sqrt(b)
```

Mupad [B]
time = 3.47, size = 21, normalized size = 1.05

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{-b}}{2\,x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)
[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)
```

3.920 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d}$$

```
[Out] arctanh((a+c/x^2)^(1/2)/a^(1/2))*a^(1/2)/e-arctanh(c^(1/2)/x/(a+c/x^2)^(1/2
))*c^(1/2)/d-arctanh((a*d-c*e/x)/(a*d^2+c*e^2)^(1/2)/(a+c/x^2)^(1/2))*(a*d^
2+c*e^2)^(1/2)/d/e
```

Rubi [A]
time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {1458, 1489, 910, 272, 65, 214, 858, 223, 212, 739}

$$-\frac{\sqrt{ad^2+ce^2}\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{a+\frac{c}{x^2}}\sqrt{ad^2+ce^2}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh
[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2])])/(d*e) - (Sqrt[c]*A
rcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 910

```
Int[((a_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))),
 x_Symbol] :> Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)
/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[Simp[c*d*f + a*e*g - c*(e*
f - d*g)*x, x]*((a + c*x^2)^(p - 1)/(f + g*x)), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] &
& GtQ[p, 0]
```

Rule 1458

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sy
mbol] :> Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d,
e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] ||  !IntegerQ[p
])
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}}}{\left(e+\frac{d}{x}\right)x}\,dx$$

$$= -\text{Subst}\left(\int \frac{\sqrt{a+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{ad-cex}{(e+dx)\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, \frac{1}{x^2}\right)}{2e} + \left(\frac{ad}{e}+\frac{ce}{d}\right)\text{Subs}$$

$$= -\frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{1}{\sqrt{a+\frac{c}{x^2}}\,x}\right)}{d} - \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+\frac{c}{x^2}}\right)}{ce} + \left(-\frac{ad}{e}\right.$$

$$= \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+\frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2+ce^2}\,\tanh^{-1}\left(\frac{ad-\frac{ce}{x}}{\sqrt{ad^2+ce^2}\sqrt{a+\frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c}\,\tan}{}$$

Mathematica [A]

time = 0.26, size = 160, normalized size = 1.32

$$-\frac{\sqrt{a+\frac{c}{x^2}}\,x\left(2\sqrt{-ad^2-ce^2}\,\tan^{-1}\left(\frac{\sqrt{a}\,(d+ex)-e\sqrt{c+ax^2}}{\sqrt{-ad^2-ce^2}}\right)-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{a}\,x-\sqrt{c+ax^2}}{\sqrt{c}}\right)+\sqrt{a}\,d\log\left(-\sqrt{a}\,x+\sqrt{c+ax^2}\right)\right)}{de\sqrt{c+ax^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2]/(d + e*x),x]
```

[Out] -((Sqrt[a + c/x^2]*x*(2*Sqrt[-(a*d^2) - c*e^2]*ArcTan[(Sqrt[a]*(d + e*x) - e*Sqrt[c + a*x^2])/Sqrt[-(a*d^2) - c*e^2]] - 2*Sqrt[c]*e*ArcTanh[(Sqrt[a]*x - Sqrt[c + a*x^2])/Sqrt[c]] + Sqrt[a]*d*Log[-(Sqrt[a]*x) + Sqrt[c + a*x^2]]))/(d*e*Sqrt[c + a*x^2]))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(103) = 206.
time = 0.26, size = 247, normalized size = 2.04

method	result
default	$-\frac{\sqrt{\frac{a\,x^{2}+c}{x^{2}}}\,x\left(\sqrt{c}\,\sqrt{\frac{a\,d^{2}+c\,e^{2}}{e^{2}}}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{a\,x^{2}+c}}{x}\right)e^{2}-\sqrt{a}\,d\ln\left(\frac{\sqrt{a}\,\sqrt{a\,x^{2}+c}+ax}{\sqrt{a}}\right)e\sqrt{\frac{a\,d^{2}+c\,e^{2}}{e^{2}}}-\cdots\right.}{\sqrt{a\,x^{2}+c}\,d\,e^{2}\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)

[Out] -((a*x^2+c)/x^2)^(1/2)*x*(c^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2)*ln(2*(c^(1/2)*(a*x^2+c)^(1/2)+c)/x)*e^2-a^(1/2)*d*ln((a^(1/2)*(a*x^2+c)^(1/2)+a*x)/a^(1/2))*e*((a*d^2+c*e^2)/e^2)^(1/2)-ln(2*(((a*d^2+c*e^2)/e^2)^(1/2)*(a*x^2+c)^(1/2)*e-a*d*x+c*e)/(e*x+d))*a*d^2-ln(2*(((a*d^2+c*e^2)/e^2)^(1/2)*(a*x^2+c)^(1/2)*e-a*d*x+c*e)/(e*x+d))*c*e^2)/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(a + c/x^2)/(x*e + d), x)

Fricas [A]
time = 0.85, size = 1515, normalized size = 12.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="fricas")

[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt

```
(a*d^2 + c*e^2)*log(-(2*a^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*(a*d*x^2 -
c*x*e)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2) + (a*c*x^2 + 2*c^2)*e^2)/(
x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d, -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2
*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*s
qrt((a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log(-(2*a^2*d^2*x^2
- 2*a*c*d*x*e + a*c*d^2 - 2*(a*d*x^2 - c*x*e)*sqrt(a*d^2 + c*e^2)*sqrt((a*x
^2 + c)/x^2) + (a*c*x^2 + 2*c^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d,
 1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + s
qrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sq
rt(-a*d^2 - c*e^2)*arctan(-(a*d*x^2 - c*x*e)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x
^2 + c)/x^2)/(a^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + c^2)*e^2)))*e^(-1)/d, -1/2
*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqr
t(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt
(-a*d^2 - c*e^2)*arctan(-(a*d*x^2 - c*x*e)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2
 + c)/x^2)/(a^2*d^2*x^2 + a*c*d^2 + (a*c*x^2 + c^2)*e^2)))*e^(-1)/d, 1/2*(2
*sqrt(-c)*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c))*e + sqrt(a)*
d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*
e^2)*log(-(2*a^2*d^2*x^2 - 2*a*c*d*x*e + a*c*d^2 - 2*(a*d*x^2 - c*x*e)*sqrt
(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2) + (a*c*x^2 + 2*c^2)*e^2)/(x^2*e^2 + 2
*d*x*e + d^2)))*e^(-1)/d, -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^
2 + c)/x^2)/(a*x^2 + c)) - 2*sqrt(-c)*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^
2)/(a*x^2 + c))*e - sqrt(a*d^2 + c*e^2)*log(-(2*a^2*d^2*x^2 - 2*a*c*d*x*e +
 a*c*d^2 - 2*(a*d*x^2 - c*x*e)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2) +
(a*c*x^2 + 2*c^2)*e^2)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d, 1/2*(2*sqrt(-c
)*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c))*e + sqrt(a)*d*log(-2
*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) - 2*sqrt(-a*d^2 - c*e^2)*
arctan(-(a*d*x^2 - c*x*e)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a^2*d
^2*x^2 + a*c*d^2 + (a*c*x^2 + c^2)*e^2)))*e^(-1)/d, -(sqrt(-a)*d*arctan(sqr
t(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(-c)*arctan(sqrt(-c)*x*s
qrt((a*x^2 + c)/x^2)/(a*x^2 + c))*e + sqrt(-a*d^2 - c*e^2)*arctan(-(a*d*x^2
 - c*x*e)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a^2*d^2*x^2 + a*c*d^2
 + (a*c*x^2 + c^2)*e^2)))*e^(-1)/d]
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)
```

Giac [**F(-2)**]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c/x^2)^(1/2)/(d + e*x), x)
```

3.921 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx$

Optimal. Leaf size=181

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d}-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))*a^(1/2)/e-arctanh(1/2*(b
+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)/d-arctanh(1/2*(2*a*d-b*e+(b*d-
2*c*e)/x)/(a*d^2-e*(b*d-c*e))^(1/2)/(a+c/x^2+b/x)^(1/2))*(a*d^2-e*(b*d-c*e)
)^(1/2)/d/e
```

Rubi [A]
time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$,
Rules used = {1457, 1488, 909, 738, 212, 857, 635}

$$-\frac{\sqrt{ad^2-e(bd-ce)}\tanh^{-1}\left(\frac{2ad+\frac{bd-2ce}{x}-be}{2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{ad^2-e(bd-ce)}}\right)}{de}-\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/e - (Sqrt[
c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/d - (Sqrt[a*d^
2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x)/(2*Sqrt[a*d^2 -
e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x])])/(d*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1457

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)
^(n2_.))^(p_.), x_Symbol] :> Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/
x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -
n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex}\,dx = \int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{\left(e+\frac{d}{x}\right)x}\,dx$$

$$= -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(e+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{ad-be-cex}{(e+dx)\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e} - \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{d} + \frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+\frac{2c}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d}$$

$$= \frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{e} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad}}{}$$

Mathematica [A]
time = 0.53, size = 186, normalized size = 1.03

$$\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(-2\sqrt{-ad^2+bde-ce^2}\tan^{-1}\left(\frac{\sqrt{a}(d+ex)-e\sqrt{c+x(b+ax)}}{\sqrt{-ad^2+bde-ce^2}}\right)+2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{a}x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{a}\,d\log\left(e\left(b+2ax-2\sqrt{a}\sqrt{c+x(b+ax)}\right)\right)\right)}{de\sqrt{c+x(b+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(-2*Sqrt[-(a*d^2) + b*d*e - c*e^2]*ArcTan[(Sqrt[
a]*(d + e*x) - e*Sqrt[c + x*(b + a*x)])/Sqrt[-(a*d^2) + b*d*e - c*e^2]] + 2
*Sqrt[c]*e*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - Sqrt[a]*d
*Log[e*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]))/(d*e*Sqrt[c + x*(b
+ a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(157) = 314$.
time = 0.23, size = 397, normalized size = 2.19

method	result
default	$-\frac{\sqrt{\frac{a x^2+bx+c}{x^2}}\, x \left(\sqrt{c}\, \ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a x^2+bx+c}}{x}\right)\sqrt{a}\,\sqrt{\frac{a d^2-deb+c e^2}{e^2}}\, e^2-\ln\left(\frac{2\sqrt{a x^2+bx+c}\,\sqrt{}}{2\sqrt{a}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*x^2+b*x+c)/x^2)^(1/2)*x*(c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(
1/2))/x)*a^(1/2)*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*e^2-ln(1/2*(2*(a*x^2+b*x+c
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*a*d*e-ln(
(2*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*(a*x^2+b*x+c)^(1/2)*e-2*a*d*x+e*b*x-b*d+
2*c*e)/(e*x+d))*a^(3/2)*d^2+ln((2*((a*d^2-b*d*e+c*e^2)/e^2)^(1/2)*(a*x^2+b*
x+c)^(1/2)*e-2*a*d*x+e*b*x-b*d+2*c*e)/(e*x+d))*a^(1/2)*b*d*e-ln((2*((a*d^2-
b*d*e+c*e^2)/e^2)^(1/2)*(a*x^2+b*x+c)^(1/2)*e-2*a*d*x+e*b*x-b*d+2*c*e)/(e*x
+d))*a^(1/2)*c*e^2)/(a*x^2+b*x+c)^(1/2)/d/e^2/a^(1/2)/((a*d^2-b*d*e+c*e^2)/
e^2)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2)/(x*e + d), x)
```

Fricas [A]
time = 41.38, size = 2407, normalized size = 13.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*
sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a
*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^
```

```
2) + sqrt(a*d^2 - b*d*e + c*e^2)*log(-(8*a^2*d^2*x^2 + 8*a*b*d^2*x + (b^2 +
 4*a*c)*d^2 - 4*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sqrt(a*d^2 - b*d*e
+ c*e^2)*sqrt((a*x^2 + b*x + c)/x^2) + (8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2
)*e^2 - 2*(4*a*b*d*x^2 + 4*b*c*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*e^2 + 2*d*x
*e + d^2)))*e^(-1)/d, -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a
)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-(8*
b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 +
 b*x + c)/x^2))/x^2) - sqrt(a*d^2 - b*d*e + c*e^2)*log(-(8*a^2*d^2*x^2 + 8*
a*b*d^2*x + (b^2 + 4*a*c)*d^2 - 4*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*s
qrt(a*d^2 - b*d*e + c*e^2)*sqrt((a*x^2 + b*x + c)/x^2) + (8*b*c*x + (b^2 +
4*a*c)*x^2 + 8*c^2)*e^2 - 2*(4*a*b*d*x^2 + 4*b*c*d + (3*b^2 + 4*a*c)*d*x)*e
)/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d, 1/2*(sqrt(a)*d*log(-8*a^2*x^2 - 8*a
*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2))
 + sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*
sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*
arctan(-1/2*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sqrt(-a*d^2 + b*d*e - c
*e^2)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*d^2*x^2 + a*b*d^2*x + a*c*d^2 + (a*c
*x^2 + b*c*x + c^2)*e^2 - (a*b*d*x^2 + b^2*d*x + b*c*d)*e)))*e^(-1)/d, -1/2
*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x
^2)/(a^2*x^2 + a*b*x + a*c)) - sqrt(c)*e*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2
+ 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 2*s
qrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x
)*e)*sqrt(-a*d^2 + b*d*e - c*e^2)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*d^2*x^2
+ a*b*d^2*x + a*c*d^2 + (a*c*x^2 + b*c*x + c^2)*e^2 - (a*b*d*x^2 + b^2*d*x
+ b*c*d)*e)))*e^(-1)/d, 1/2*(2*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)
*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2))*e + sqrt(a)*d*log(-8*
a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b
*x + c)/x^2)) + sqrt(a*d^2 - b*d*e + c*e^2)*log(-(8*a^2*d^2*x^2 + 8*a*b*d^2
*x + (b^2 + 4*a*c)*d^2 - 4*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sqrt(a*d
^2 - b*d*e + c*e^2)*sqrt((a*x^2 + b*x + c)/x^2) + (8*b*c*x + (b^2 + 4*a*c)*
x^2 + 8*c^2)*e^2 - 2*(4*a*b*d*x^2 + 4*b*c*d + (3*b^2 + 4*a*c)*d*x)*e)/(x^2*
e^2 + 2*d*x*e + d^2)))*e^(-1)/d, -1/2*(2*sqrt(-a)*d*arctan(1/2*(2*a*x^2 + b
*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 2*sqrt(
-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^
2 + b*c*x + c^2))*e - sqrt(a*d^2 - b*d*e + c*e^2)*log(-(8*a^2*d^2*x^2 + 8*a
*b*d^2*x + (b^2 + 4*a*c)*d^2 - 4*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sq
rt(a*d^2 - b*d*e + c*e^2)*sqrt((a*x^2 + b*x + c)/x^2) + (8*b*c*x + (b^2 + 4
*a*c)*x^2 + 8*c^2)*e^2 - 2*(4*a*b*d*x^2 + 4*b*c*d + (3*b^2 + 4*a*c)*d*x)*e)
/(x^2*e^2 + 2*d*x*e + d^2)))*e^(-1)/d, 1/2*(2*sqrt(-c)*arctan(1/2*(b*x^2 +
2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2))*e + sq
rt(a)*d*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) - 2*sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(-1/2*(
2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sqrt(-a*d^2 + b*d*e - c*e^2)*sqrt((a
*x^2 + b*x + c)/x^2)/(a^2*d^2*x^2 + a*b*d^2*x + a*c*d^2 + (a*c*x^2 + b*c*x
+ c^2)*e^2 - (a*b*d*x^2 + b^2*d*x + b*c*d)*e)))*e^(-1)/d, -(sqrt(-a)*d*arct
```

```
an(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*
x + a*c)) - sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x
+ c)/x^2)/(a*c*x^2 + b*c*x + c^2))*e + sqrt(-a*d^2 + b*d*e - c*e^2)*arctan(
-1/2*(2*a*d*x^2 + b*d*x - (b*x^2 + 2*c*x)*e)*sqrt(-a*d^2 + b*d*e - c*e^2)*s
qrt((a*x^2 + b*x + c)/x^2)/(a^2*d^2*x^2 + a*b*d^2*x + a*c*d^2 + (a*c*x^2 +
b*c*x + c^2)*e^2 - (a*b*d*x^2 + b^2*d*x + b*c*d)*e)))*e^(-1)/d]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)
```

3.922 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx$

Optimal. Leaf size=26

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\,\sqrt[5]{x^3}$$

[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {14, 15, 30}

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt[5]{x^3}\,\sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x}}+\frac{\sqrt[5]{x^3}}{\sqrt{x}}\right)dx\\ &= \frac{3x^{2/3}}{2}+\int \frac{\sqrt[5]{x^3}}{\sqrt{x}}\,dx\\ &= \frac{3x^{2/3}}{2}+\frac{\sqrt[5]{x^3}\int \sqrt[10]{x}\,dx}{x^{3/5}}\\ &= \frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\,\sqrt[5]{x^3}\end{aligned}$$

Mathematica [A]
time = 7.08, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2}+\frac{10}{11}\sqrt{x}\,\sqrt[5]{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]
[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.65

method	result	size
default	$\frac{3x^{\frac{2}{3}}}{2}+\frac{10\left(x^3\right)^{\frac{1}{5}}\sqrt{x}}{11}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)+10/11*(x^3)^(1/5)*x^(1/2)
```

Maxima [A]
time = 0.30, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^3\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Fricas [A]
time = 0.35, size = 16, normalized size = 0.62

$$\frac{10}{11}\left(x^{3}\right)^{\frac{1}{5}}\sqrt{x}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")
[Out] 10/11*(x^3)^(1/5)*sqrt(x) + 3/2*x^(2/3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [A]
time = 5.06, size = 11, normalized size = 0.42

$$\frac{10}{11}x^{\frac{11}{10}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")
[Out] 10/11*x^(11/10) + 3/2*x^(2/3)
```

Mupad [B]
time = 3.54, size = 16, normalized size = 0.62

$$\frac{10\sqrt{x}\,\left(x^3\right)^{1/5}}{11}+\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
[Out] (10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

3.923 $\int \frac{2+x}{\sqrt{4x-x^2}} dx$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2}-4\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] 4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {654, 633, 222}

$$-4\text{ArcSin}\left(1-\frac{x}{2}\right)-\sqrt{4x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/Sqrt[4*x - x^2],x]
```

```
[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{2+x}{\sqrt{4x-x^2}}\,dx = -\sqrt{4x-x^2} + 4\int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\sqrt{4x-x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)$$
$$= -\sqrt{4x-x^2} - 4\sin^{-1}\left(1-\frac{x}{2}\right)$$

Mathematica [A]
time = 0.05, size = 43, normalized size = 1.65

$$\frac{(-4+x)x + 8\sqrt{-4+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{1}{\sqrt{\frac{-4+x}{x}}}\right)}{\sqrt{-((-4+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/Sqrt[4*x - x^2],x]
[Out] ((-4 + x)*x + 8*Sqrt[-4 + x]*Sqrt[x]*ArcTanh[1/Sqrt[(-4 + x)/x]])/Sqrt[-((-
4 + x)*x)]
```

Maple [A]
time = 0.24, size = 23, normalized size = 0.88

method	result
default	$4 \arcsin\left(-1+\frac{x}{2}\right) - \sqrt{-x^2+4x}$
risch	$\frac{x(x-4)}{\sqrt{-x\,(x-4)}} + 4\arcsin\left(-1+\frac{x}{2}\right)$
meijerg	$4\arcsin\left(\frac{\sqrt{x}}{2}\right) + \frac{4i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{-\frac{x}{4}+1}}{2} - i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$
trager	$-\sqrt{-x^2+4x} + 4\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\text{RootOf}\left(\_Z^2+1\right) + 2\,\text{RootOf}\left(\_Z^2+1\right) + \sqrt{-x^2 -}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+2)/(-x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 4*arcsin(-1+1/2*x)-(-x^2+4*x)^(1/2)
```

Maxima [A]
time = 0.52, size = 22, normalized size = 0.85

$$-\sqrt{-x^2+4\,x}-4\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 1.23

$$-\sqrt{-x^2+4\,x}-8\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+2}{\sqrt{-x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

Giac [A]
time = 4.98, size = 22, normalized size = 0.85

$$-\sqrt{-x^2+4\,x}+4\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x^2+4*x)^(1/2),x, algorithm="giac")
[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)
```

Mupad [B]
time = 3.53, size = 22, normalized size = 0.85

$$4\,\mathrm{asin}\left(\frac{x}{2}-1\right)-\sqrt{4\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 2)/(4*x - x^2)^(1/2),x)
[Out] 4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

3.924 $\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx$

Optimal. Leaf size=15

$$\frac{3}{4}\left(6x+x^2\right)^{2/3}$$

```
[Out] 3/4*(x^2+6*x)^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {643}

$$\frac{3}{4}\left(x^2+6x\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(6*x + x^2)^(2/3))/4
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} \, dx = \frac{3}{4}\left(6x+x^2\right)^{2/3}$$

Mathematica [A]
time = 10.01, size = 13, normalized size = 0.87

$$\frac{3}{4}(x(6+x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/(6*x + x^2)^(1/3),x]
```

```
[Out] (3*(x*(6 + x))^(2/3))/4
```

Maple [A]
time = 0.22, size = 12, normalized size = 0.80

method	result	size
default	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
trager	$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$	12
risch	$\frac{3x(x+6)}{4(x(x+6))^{\frac{1}{3}}}$	14
gosper	$\frac{3x(x+6)}{4(x^2+6x)^{\frac{1}{3}}}$	16
meijerg	$\frac{3\,9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{2}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],\left[\frac{5}{3}\right],-\frac{x}{6}\right)}{4}+\frac{9^{\frac{1}{3}}2^{\frac{2}{3}}x^{\frac{5}{3}}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{5}{3}\right],\left[\frac{8}{3}\right],-\frac{x}{6}\right)}{10}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(x^2+6*x)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*(x^2+6*x)^(2/3)
```

Maxima [A]
time = 0.29, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Fricas [A]
time = 0.32, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^2+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Sympy [A]
time = 0.07, size = 12, normalized size = 0.80

$$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+6*x)**(1/3),x)
```

```
[Out] 3*(x**2 + 6*x)**(2/3)/4
```

Giac [A]
time = 5.17, size = 11, normalized size = 0.73

$$\frac{3}{4}\left(x^{2}+6\,x\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*(x^2 + 6*x)^(2/3)
```

Mupad [B]
time = 3.56, size = 9, normalized size = 0.60

$$\frac{3\left(x\left(x+6\right)\right)^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 3)/(6*x + x^2)^(1/3),x)
```

```
[Out] (3*(x*(x + 6))^(2/3))/4
```

3.925 $\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

```
[Out] 1/9*(-12+7*x)/(-x^2+6*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {650}

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] -1/9*(12 - 7*x)/Sqrt[6*x - x^2]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{4+x}{(6x-x^2)^{3/2}}\,dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 0.86

$$\frac{-12+7x}{9\sqrt{-((-6+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x)/(6*x - x^2)^(3/2),x]
```

```
[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])
```

Maple [A]
time = 0.24, size = 31, normalized size = 1.41

method	result	size
risch	$\frac{-12+7x}{9\sqrt{-x\left(-6+x\right)}}$	16
gosper	$-\frac{x(-6+x)(-12+7x)}{9(-x^2+6x)^{\frac{3}{2}}}$	23
trager	$-\frac{(-12+7x)\sqrt{-x^2+6x}}{9x(-6+x)}$	27
default	$\frac{1}{\sqrt{-x^2+6x}}-\frac{7(-2x+6)}{18\sqrt{-x^2+6x}}$	31
meijerg	$-\frac{2\sqrt{2}\,\sqrt{3}\,\left(1-\frac{x}{3}\right)}{9\sqrt{x}\,\sqrt{1-\frac{x}{6}}}+\frac{\sqrt{x}\,\sqrt{6}}{18\sqrt{1-\frac{x}{6}}}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+4)/(-x^2+6*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/(-x^2+6*x)^(1/2)-7/18*(-2*x+6)/(-x^2+6*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 28, normalized size = 1.27

$$\frac{7\,x}{9\,\sqrt{-x^2+6\,x}}-\frac{4}{3\,\sqrt{-x^2+6\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")
[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)
```

Fricas [A]
time = 0.36, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6\,x}\,(7\,x-12)}{9\,(x^2-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x+4}{\left(-x\,(x-6)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)
```

```
[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)
```

Giac [A]
time = 3.64, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6\,x}\,(7\,x-12)}{9\,(x^2-6\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)
```

Mupad [B]
time = 3.50, size = 18, normalized size = 0.82

$$\frac{7\,x-12}{9\,\sqrt{6\,x-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 4)/(6*x - x^2)^(3/2),x)
```

```
[Out] (7*x - 12)/(9*(6*x - x^2)^(1/2))
```

3.926 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{2x+x^2}\right)$$

```
[Out] arctan((x^2+2*x)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {702, 209}

$$\text{ArcTan}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] ArcTan[Sqrt[2*x + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

time = 0.04, size = 41, normalized size = 3.42

$$-\frac{2\sqrt{x}\,\sqrt{2+x}\,\tan^{-1}\left(1+x-\sqrt{x}\,\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x)
]
```

Maple [A]
time = 0.24, size = 13, normalized size = 1.08

method	result	size
default	$-\arctan\left(\frac{1}{\sqrt{(1+x)^{2}-1}}\right)$	13
trager	$\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{1+x}\right)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A]
time = 0.55, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A]
time = 0.34, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^{2}+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Giac [A]
time = 3.17, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2x}\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
```

```
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.927 $\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x+x^2}\right)$$

```
[Out] arctan(2*(x^2+x)^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {702, 209}

$$\text{ArcTan}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

```
[Out] ArcTan[2*Sqrt[x + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+2x)\sqrt{x+x^2}}\,dx &= 4\text{Subst}\left(\int \frac{1}{2+8x^2}\,dx, x, \sqrt{x+x^2}\right)\\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(12) = 24.

time = 0.04, size = 43, normalized size = 3.58

$$-\frac{2\sqrt{x}\,\sqrt{1+x}\,\tan^{-1}\left(1+2x-2\sqrt{x}\,\sqrt{1+x}\right)}{\sqrt{x(1+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[1 + x]*ArcTan[1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x]])/Sqrt[x*(1
+ x)]
```

Maple [A]
time = 0.24, size = 15, normalized size = 1.25

method	result	size
default	$-\arctan\left(\frac{1}{\sqrt{4\left(x+\frac{1}{2}\right)^{2}-1}}\right)$	15
trager	$\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)+2\sqrt{x^{2}+x}}{2x+1}\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+1)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))
```

Maxima [A]
time = 0.53, size = 11, normalized size = 0.92

$$-\arcsin\left(\frac{1}{|2\,x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(2*x + 1))
```

Fricas [A]
time = 0.34, size = 17, normalized size = 1.42

$$2\,\arctan\left(-2\,x+2\,\sqrt{x^{2}+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+1)}\,(2x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```

Giac [A]
time = 2.88, size = 17, normalized size = 1.42

$$2\,\arctan\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{(2\,x+1)\,\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + 1)*(x + x^2)^(1/2)),x)
```

```
[Out] int(1/((2*x + 1)*(x + x^2)^(1/2)), x)
```

3.928 $\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=15

$$-\sqrt{2x-x^2}$$

```
[Out] -(-x^2+2*x)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {643}

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/Sqrt[2*x - x^2],x]
[Out] -Sqrt[2*x - x^2]
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{-1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2}$$

Mathematica [A]
time = 0.03, size = 12, normalized size = 0.80

$$-\sqrt{-((-2+x)x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/Sqrt[2*x - x^2],x]
[Out] -Sqrt[-((-2 + x)*x)]
```

Maple [A]
time = 0.22, size = 14, normalized size = 0.93

method	result	size
default	$-\sqrt{-x^2+2x}$	14
trager	$-\sqrt{-x^2+2x}$	14
risch	$\frac{x(x-2)}{\sqrt{-x\,(x-2)}}$	14
gosper	$\frac{x(x-2)}{\sqrt{-x^2+2x}}$	17
meijerg	$-2\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)+\frac{2i\left(\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{1-\frac{x}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)\right)}{\sqrt{\pi}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-x^2+2*x)^(1/2)
```

Maxima [A]
time = 0.30, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.67

$$-\sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] -sqrt(-x**2 + 2*x)
```

Giac [A]
time = 4.34, size = 13, normalized size = 0.87

$$-\sqrt{-x^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x)
```

Mupad [B]
time = 3.64, size = 10, normalized size = 0.67

$$-\sqrt{-x\,\left(x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/(2*x - x^2)^(1/2),x)
```

```
[Out] -(-x*(x - 2))^(1/2)
```

3.929 $\int \frac{\sqrt{x-x^2}}{1+x} \, dx$

Optimal. Leaf size=54

$$\sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\,\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\,\sqrt{x-x^2}}\right)$$

```
[Out] 3/2*arcsin(-1+2*x)+arctan(1/4*(1-3*x)*2^(1/2)/(-x^2+x)^(1/2))*2^(1/2)+(-x^2
+x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {748, 857, 633, 222, 738, 210}

$$-\frac{3}{2}\text{ArcSin}(1-2x)+\sqrt{2}\,\text{ArcTan}\left(\frac{1-3x}{2\sqrt{2}\,\sqrt{x-x^2}}\right)+\sqrt{x-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]
*Sqrt[x - x^2])]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x-x^2}}{1+x}\,dx &= \sqrt{x-x^2}-\frac{1}{2}\int \frac{1-3x}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx-2\int \frac{1}{(1+x)\sqrt{x-x^2}}\,dx\\
&= \sqrt{x-x^2}-\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)+4\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,\frac{-1+3x}{\sqrt{x-x^2}}\right.\\
&= \sqrt{x-x^2}-\frac{3}{2}\sin^{-1}(1-2x)+\sqrt{2}\,\tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\,\sqrt{x-x^2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 82, normalized size = 1.52

$$
\frac{\sqrt{-((-1+x)x)}\left(\sqrt{-1+x}\,\sqrt{x}-6\tanh^{-1}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)+2\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{\frac{-1+x}{x}}}\right)\right)}{\sqrt{-1+x}\,\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x - x^2]/(1 + x),x]
```

```
[Out] (Sqrt[-((-1 + x)*x)]*(Sqrt[-1 + x]*Sqrt[x] - 6*ArcTanh[Sqrt[-1 + x]/(-1 + S
qrt[x])] + 2*Sqrt[2]*ArcTanh[Sqrt[2]/Sqrt[(-1 + x)/x]]))/(Sqrt[-1 + x]*Sqrt
[x])
```

Maple [A]
time = 0.26, size = 54, normalized size = 1.00

method	result
default	$\sqrt{-\left(1+x\right)^{2}+1+3x}+\frac{3\arcsin\left(2x-1\right)}{2}-\sqrt{2}\,\arctan\left(\frac{\left(-1+3x\right)\sqrt{2}}{4\sqrt{-\left(1+x\right)^{2}+1+3x}}\right)$
risch	$-\frac{x\left(-1+x\right)}{\sqrt{-x\left(-1+x\right)}}+\frac{3\arcsin\left(2x-1\right)}{2}-\sqrt{2}\,\arctan\left(\frac{\left(-1+3x\right)\sqrt{2}}{4\sqrt{-\left(1+x\right)^{2}+1+3x}}\right)$
trager	$\sqrt{-x^{2}+x}+\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^{2}+2\right)x-\operatorname{RootOf}\left(\_Z^{2}+2\right)+4\sqrt{-x^{2}+x}}{1+x}\right)-\frac{3\operatorname{RootOf}\left(\_\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)^(1/2)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] (-(1+x)^2+1+3*x)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(-1+3*x)*2^(1/2
)/(-(1+x)^2+1+3*x)^(1/2))
```

Maxima [A]
time = 0.52, size = 42, normalized size = 0.78

$$-\sqrt{2}\arcsin\left(\frac{3\,x}{|x+1|}-\frac{1}{|x+1|}\right)+\sqrt{-x^{2}+x}+\frac{3}{2}\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsi
n(2*x - 1)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 0.91

$$2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^{2}+x}}{2\,x}\right)+\sqrt{-x^{2}+x}-3\arctan\left(\frac{\sqrt{-x^{2}+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(
sqrt(-x^2 + x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x\left(x-1\right)}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)**(1/2)/(1+x),x)
```

```
[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)
```

Giac [A]
time = 5.06, size = 53, normalized size = 0.98

$$2\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2x-1}-1\right)\right)+\sqrt{-x^2+x}+\frac{3}{2}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sq
rt(-x^2 + x) + 3/2*arcsin(2*x - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x-x^2}}{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)^(1/2)/(x + 1),x)
```

```
[Out] int((x - x^2)^(1/2)/(x + 1), x)
```

3.930 $\int \sqrt{\sqrt[4]{x}+x}\,dx$

Optimal. Leaf size=59

$$\frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

```
[Out] -1/3*arctanh(x^(1/2)/(x^(1/4)+x)^(1/2))+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+2/3*x
*(x^(1/4)+x)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2029, 2043, 2049, 2054, 212}

$$\frac{2}{3}\sqrt{x+\sqrt[4]{x}}\,x+\frac{1}{3}\sqrt{x+\sqrt[4]{x}}\,\sqrt[4]{x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(1/4) + x],x]
```

```
[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]
/Sqrt[x^(1/4) + x]]/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Dist[a*(n - j)*(p/(n*p + 1)), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2049

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Dist[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1))), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2054

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt[4]{x}+x}\,dx &= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\frac{1}{4}\int \frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x}+x}}\,dx\\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x}+x}+\text{Subst}\left(\int \frac{x^4}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{x+x^4}}\,dx,x,\sqrt[4]{x}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)\\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x}+x}+\frac{2}{3}x\sqrt{\sqrt[4]{x}+x}-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.34, size = 47, normalized size = 0.80

$$\frac{1}{3}\sqrt{\sqrt[4]{x}+x}\left(\sqrt[4]{x}+2x\right)-\frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(1/4) + x],x]
```

```
[Out] (Sqrt[x^(1/4) + x]*(x^(1/4) + 2*x))/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]/
3
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.53, size = 342, normalized size = 5.80

method	result
meijerg	$-\frac{2\left(-\frac{\sqrt{\pi}\, x^{\frac{3}{8}}\left(6x^{\frac{3}{4}}+3\right)\sqrt{x^{\frac{3}{4}}+1}}{6}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{3}{8}}\right)}{2}\right)}{3\sqrt{\pi}}$
derivativedivides	$\frac{2x\sqrt{x^{\frac{1}{4}}+x}}{3}+\frac{x^{\frac{1}{4}}\sqrt{x^{\frac{1}{4}}+x}}{3}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(1+x^{\frac{1}{4}}\right)}}\left(1+x^{\frac{1}{4}}\right)^{2}\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}+\frac{i\sqrt{}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(}}}{}$
default	$\frac{2x\sqrt{x^{\frac{1}{4}}+x}}{3}+\frac{x^{\frac{1}{4}}\sqrt{x^{\frac{1}{4}}+x}}{3}+\frac{\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)x^{\frac{1}{4}}}{\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(1+x^{\frac{1}{4}}\right)}}\left(1+x^{\frac{1}{4}}\right)^{2}\sqrt{-\frac{x^{\frac{1}{4}}-\frac{1}{2}+\frac{i\sqrt{}}{2}}{\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/4)+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I*3^(1/2))*
((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)*(1+x^(1
/4))^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)
*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2)/(3/2+
1/2*I*3^(1/2))/(x^(1/4)*(1+x^(1/4))*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/
2-1/2*I*3^(1/2)))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I
*3^(1/2))/(1+x^(1/4)))^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1
/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/
2))*x^(1/4)/(1/2+1/2*I*3^(1/2))/(1+x^(1/4)))^(1/2),(1/2+1/2*I*3^(1/2))/(3/2
+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1
/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + x^(1/4)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt[4]{x} + x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/4)+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**(1/4) + x), x)
```

Giac [A]
time = 5.97, size = 45, normalized size = 0.76

$$\frac{1}{3}\sqrt{x + x^{\frac{1}{4}}}\, x^{\frac{1}{4}}\left(2\,x^{\frac{3}{4}} + 1\right) - \frac{1}{6}\log\left(\sqrt{\frac{1}{x^{\frac{3}{4}}} + 1} + 1\right) + \frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^{\frac{3}{4}}} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1)
 + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))
```

Mupad [B]
time = 3.53, size = 27, normalized size = 0.46

$$\frac{8\,x\,\sqrt{x + x^{1/4}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{2};\ \frac{5}{2};\ -x^{3/4}\right)}{9\,\sqrt{x^{3/4} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(1/4))^(1/2),x)
```

```
[Out] (8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4)
 + 1)^(1/2))
```

3.931 $\int \sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=59

$$\frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}}-\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}$$

```
[Out] 32/105*(x+x^(3/2))^(3/2)/x^(3/2)-16/35*(x+x^(3/2))^(3/2)/x+4/7*(x+x^(3/2))^
(3/2)/x^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 2041, 2039}

$$\frac{4\left(x^{3/2}+x\right)^{3/2}}{7\sqrt{x}}-\frac{16\left(x^{3/2}+x\right)^{3/2}}{35x}+\frac{32\left(x^{3/2}+x\right)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(3/2)],x]
```

```
[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) +
(4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x+x^{3/2}}\,dx &= \frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}-\frac{4}{7}\int \frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx\\ &= -\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}+\frac{8}{35}\int \frac{\sqrt{x+x^{3/2}}}{x}\,dx\\ &= \frac{32\left(x+x^{3/2}\right)^{3/2}}{105x^{3/2}}-\frac{16\left(x+x^{3/2}\right)^{3/2}}{35x}+\frac{4\left(x+x^{3/2}\right)^{3/2}}{7\sqrt{x}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 39, normalized size = 0.66

$$\frac{4\sqrt{x+x^{3/2}}\left(8-4\sqrt{x}+3x+15x^{3/2}\right)}{105\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/(105*Sqrt[x])
```

Maple [A]
time = 0.22, size = 28, normalized size = 0.47

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,\left(1+\sqrt{x}\right)\left(15x-12\sqrt{x}+8\right)}{105\sqrt{x}}$	28
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,\left(1+\sqrt{x}\right)\left(15x-12\sqrt{x}+8\right)}{105\sqrt{x}}$	28
meijerg	$-\frac{\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\,\left(1+\sqrt{x}\right)^{\frac{3}{2}}\left(15x-12\sqrt{x}+8\right)}{105}}{\sqrt{\pi}}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/105*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(15*x-12*x^(1/2)+8)/x^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(3/2) + x), x)
```

Fricas [A]
time = 0.36, size = 30, normalized size = 0.51

$$\frac{4\left(15\,x^{2}+(3\,x+8)\sqrt{x}-4\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(3/2))**(1/2),x)
[Out] Integral(sqrt(x**(3/2) + x), x)
```

Giac [A]
time = 5.35, size = 33, normalized size = 0.56

$$\frac{4}{105}\left(15\left(\sqrt{x}+1\right)^{\frac{7}{2}}-42\left(\sqrt{x}+1\right)^{\frac{5}{2}}+35\left(\sqrt{x}+1\right)^{\frac{3}{2}}-8\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(3/2))^(1/2),x, algorithm="giac")
[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(
3/2) - 8)*sgn(x)
```

Mupad [B]
time = 3.54, size = 27, normalized size = 0.46

$$\frac{2\,x\,\sqrt{x+x^{3/2}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},3;\;4;\;-\sqrt{x}\right)}{3\,\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(3/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) + 1
)^(1/2))
```

3.932 $\int x\sqrt{x+x^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}$$

```
[Out] -32/99*(x+x^(3/2))^(3/2)+512/3465*(x+x^(3/2))^(3/2)/x^(3/2)-256/1155*(x+x^(
3/2))^(3/2)/x+64/231*(x+x^(3/2))^(3/2)/x^(1/2)+4/11*(x+x^(3/2))^(3/2)*x^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2041, 2027, 2039}

$$\frac{4}{11}\sqrt{x}\left(x^{3/2}+x\right)^{3/2}+\frac{64\left(x^{3/2}+x\right)^{3/2}}{231\sqrt{x}}-\frac{256\left(x^{3/2}+x\right)^{3/2}}{1155x}+\frac{512\left(x^{3/2}+x\right)^{3/2}}{3465x^{3/2}}-\frac{32}{99}\left(x^{3/2}+x\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x + x^(3/2)],x]
```

```
[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (
256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x])
+ (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{x+x^{3/2}}\,dx &= \frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{8}{11}\int\sqrt{x}\sqrt{x+x^{3/2}}\,dx\\ &= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{16}{33}\int\sqrt{x+x^{3/2}}\,dx\\ &= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}-\frac{64}{231}\int\frac{\sqrt{x+x^{3/2}}}{\sqrt{x}}\,dx\\ &= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\left(x+x^{3/2}\right)^{3/2}+\frac{12}{}\\ &= -\frac{32}{99}\left(x+x^{3/2}\right)^{3/2}+\frac{512\left(x+x^{3/2}\right)^{3/2}}{3465x^{3/2}}-\frac{256\left(x+x^{3/2}\right)^{3/2}}{1155x}+\frac{64\left(x+x^{3/2}\right)^{3/2}}{231\sqrt{x}}+\frac{4}{11}\sqrt{x}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.54

$$\frac{4\sqrt{x+x^{3/2}}\left(128-64\sqrt{x}+48x-40x^{3/2}+35x^2+315x^{5/2}\right)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x + x^(3/2)],x]
[Out] (4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x^2 + 315*x
^(5/2)))/(3465*Sqrt[x])
```

Maple [A]
time = 0.22, size = 38, normalized size = 0.40

method	result	size
derivativedivides	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,\left(1+\sqrt{x}\right)\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38
default	$\frac{4\sqrt{x+x^{\frac{3}{2}}}\,\left(1+\sqrt{x}\right)\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465\sqrt{x}}$	38

meijerg	$-\frac{\frac{512\sqrt{\pi}}{3465}-\frac{4\sqrt{\pi}\left(1+\sqrt{x}\right)^{\frac{3}{2}}\left(315x^2-280x^{\frac{3}{2}}+240x-192\sqrt{x}+128\right)}{3465}}{\sqrt{\pi}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x+x^(3/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 4/3465*(x+x^(3/2))^(1/2)*(1+x^(1/2))*(315*x^2-280*x^(3/2)+240*x-192*x^(1/2)
+128)/x^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^(3/2) + x)*x, x)
```

Fricas [A]
time = 0.41, size = 40, normalized size = 0.43

$$\frac{4\left(315\,x^3-40\,x^2+\left(35\,x^2+48\,x+128\right)\sqrt{x}-64\,x\right)\sqrt{x^{\frac{3}{2}}+x}}{3465\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="fricas")
[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/
2) + x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x**(3/2))**(1/2),x)
[Out] Integral(x*sqrt(x**(3/2) + x), x)
```

Giac [A]
time = 6.00, size = 51, normalized size = 0.54

$$\frac{4}{3465}\left(315\left(\sqrt{x}+1\right)^{\frac{11}{2}}-1540\left(\sqrt{x}+1\right)^{\frac{9}{2}}+2970\left(\sqrt{x}+1\right)^{\frac{7}{2}}-2772\left(\sqrt{x}+1\right)^{\frac{5}{2}}+1155\left(\sqrt{x}+1\right)^{\frac{3}{2}}-128\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x+x^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(315*(sqrt(x) + 1)^(11/2) - 1540*(sqrt(x) + 1)^(9/2) + 2970*(sqrt(x)
 + 1)^(7/2) - 2772*(sqrt(x) + 1)^(5/2) + 1155*(sqrt(x) + 1)^(3/2) - 128)*sg
n(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{x+x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x + x^(3/2))^(1/2),x)
```

```
[Out] int(x*(x + x^(3/2))^(1/2), x)
```

3.933 $\int (1-x^2) \sqrt{\frac{1}{2-x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

[Out] 1/2*x/(1/(-x^2+2))^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1972, 391}

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]

[Out] x/(2*Sqrt[(2 - x^2)^(-1)])

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 1972

```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)], Int[u*(a + b*x^n)^(p*q), x], x]
 /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int (1-x^2)\sqrt{\frac{1}{2-x^2}}\,dx &= \left(\sqrt{\frac{1}{2-x^2}}\sqrt{2-x^2}\right)\int \frac{1-x^2}{\sqrt{2-x^2}}\,dx\\ &= \frac{x}{2\sqrt{\frac{1}{2-x^2}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 18, normalized size = 1.00

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]
```

```
[Out] x/(2*Sqrt[(2 - x^2)^(-1)])
```

Maple [A]
time = 0.23, size = 20, normalized size = 1.11

method	result
gosper	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$
default	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$
trager	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$
risch	$-\frac{(x^2-2)x\sqrt{-\frac{1}{x^2-2}}}{2}$
meijerg	$\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\,\arcsin\left(\frac{\sqrt{2}\,x}{2}\right)-\frac{i\sqrt{\frac{1}{-x^2+2}}\,\sqrt{-x^2+2}\,\left(\frac{i\sqrt{\pi}\,x\sqrt{2}\,\sqrt{1-\frac{x^2}{2}}}{2}-i\sqrt{\pi}\,\arcsin\left(\right.\right.}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)*(1/(-x^2+2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 1.11

$$-\frac{1}{2}\left(x^3-2x\right)\sqrt{-\frac{1}{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.
time = 0.41, size = 29, normalized size = 1.61

$$-\frac{x^3\sqrt{-\frac{1}{x^2-2}}}{2}+x\sqrt{-\frac{1}{x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

```
[Out] -x**3*sqrt(-1/(x**2 - 2))/2 + x*sqrt(-1/(x**2 - 2))
```

Giac [A]
time = 4.88, size = 18, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-x^2+2}\,x\mathrm{sgn}\left(x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)
```

Mupad [B]
time = 3.50, size = 19, normalized size = 1.06

$$-\frac{x\left(x^2-2\right)\sqrt{-\frac{1}{x^2-2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2),x)
```

```
[Out] -(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2
```

3.934 $\int \sqrt{x^2+x^3-x^4}\,dx$

Optimal. Leaf size=107

$$-\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x}-\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x}-\frac{5\sqrt{x^2+x^3-x^4}\,\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{1+x-x^2}}$$

```
[Out] -1/8*(1-2*x)*(-x^4+x^3+x^2)^(1/2)/x-1/3*(-x^2+x+1)*(-x^4+x^3+x^2)^(1/2)/x-5
/16*arcsin(1/5*(1-2*x)*5^(1/2))*(-x^4+x^3+x^2)^(1/2)/x/(-x^2+x+1)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1917, 654, 626, 633, 222}

$$-\frac{5\sqrt{-x^4+x^3+x^2}\,\text{ArcSin}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2+x+1}}-\frac{\sqrt{-x^4+x^3+x^2}\,(1-2x)}{8x}-\frac{(-x^2+x+1)\sqrt{-x^4+x^3+x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3 - x^4],x]
```

```
[Out] -1/8*((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/x - ((1 + x - x^2)*Sqrt[x^2 + x^3 -
x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqr
t[1 + x - x^2])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{x^2+x^3-x^4}\,dx = \frac{\sqrt{x^2+x^3-x^4}\int x\sqrt{1+x-x^2}\,dx}{x\sqrt{1+x-x^2}}$$

$$= -\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\sqrt{x^2+x^3-x^4}\int \sqrt{1+x-x^2}\,dx}{2x\sqrt{1+x-x^2}}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\left(5\sqrt{x^2+x^3-x^4}\right)\int}{16x\sqrt{1+x}}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} - \frac{\left(\sqrt{5}\sqrt{x^2+x^3-x^4}\right)}{16}$$

$$= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4}\sin^{-1}}{16x\sqrt{1+x-x}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.77

$$\frac{\sqrt{x^2+x^3-x^4}\left(2\sqrt{-1-x+x^2}\,(-11-2x+8x^2)+15\log\left(1-2x+2\sqrt{-1-x+x^2}\right)\right)}{48x\sqrt{-1-x+x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3 - x^4],x]
[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) + 15*Log[1
 - 2*x + 2*Sqrt[-1 - x + x^2]]))/(48*x*Sqrt[-1 - x + x^2])
```

Maple [A]
time = 0.12, size = 81, normalized size = 0.76

method	result
default	$-\frac{\sqrt{-x^{4}+x^{3}+x^{2}}\,\left(16\left(-x^{2}+x+1\right)^{\frac{3}{2}}-12x\sqrt{-x^{2}+x+1}+6\sqrt{-x^{2}+x+1}-15\arcsin\left(\frac{\sqrt{5}\,(2x-1)}{5}\right)\right)}{48x\sqrt{-x^{2}+x+1}}$
risch	$\frac{(8x^{2}-2x-11)\sqrt{-x^{2}\left(x^{2}-x-1\right)}}{24x}-\frac{5\arcsin\left(\frac{2\sqrt{5}\,(x-\frac{1}{2})}{5}\right)\sqrt{-x^{2}\left(x^{2}-x-1\right)}\,\sqrt{-x^{2}+x+1}}{16x(x^{2}-x-1)}$
trager	$\frac{(8x^{2}-2x-11)\sqrt{-x^{4}+x^{3}+x^{2}}}{24x}+\frac{5\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{2\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{2}-x\operatorname{RootOf}\left(\_Z^{2}+1\right)-2\sqrt{-x^{4}+x^{3}}}{x}\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/48*(-x^4+x^3+x^2)^(1/2)*(16*(-x^2+x+1)^(3/2)-12*x*(-x^2+x+1)^(1/2)+6*(-x
^2+x+1)^(1/2)-15*arcsin(1/5*5^(1/2)*(2*x-1)))/x/(-x^2+x+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)
```

Fricas [A]
time = 0.39, size = 62, normalized size = 0.58

$$-\frac{15\,x\arctan\left(-\frac{x-\sqrt{-x^{4}+x^{3}+x^{2}}}{x^{2}}\right)-\sqrt{-x^{4}+x^{3}+x^{2}}\,(8\,x^{2}-2\,x-11)+11\,x}{24\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="fricas")
[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x
^2)*(8*x^2 - 2*x - 11) + 11*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-x^{4}+x^{3}+x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)
```

Giac [A]
time = 4.08, size = 60, normalized size = 0.56

$$\frac{1}{48}\left(15\arcsin\left(\frac{1}{5}\sqrt{5}\right)+22\right)\mathrm{sgn}(x)+\frac{5}{16}\arcsin\left(\frac{1}{5}\sqrt{5}\,(2x-1)\right)\mathrm{sgn}(x)+\frac{1}{24}\left(2\left(4x\mathrm{sgn}(x)-\mathrm{sgn}(x)\right)x-11\,\mathrm{sgn}(x)\right)\sqrt{-x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x -
1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-x^4+x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3 - x^4)^(1/2),x)
```

```
[Out] int((x^2 + x^3 - x^4)^(1/2), x)
```

3.935 $\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx$

Optimal. Leaf size=25

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

```
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1973, 197}

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a^2 + x^2)^3],x]
```

```
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1973

```
Int[(u_.)*((c_.)*((a_) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Dist[Si
mp[(c*(a + b*x^n)^q)^p/(1 + b*(x^n/a))^(p*q)], Int[u*(1 + b*(x^n/a))^(p*q),
 x], x] /; FreeQ[{a, b, c, n, p, q}, x] &&  !GeQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(a^2+x^2)^3}}\,dx &= \frac{(a^2+x^2)^{3/2}\int \frac{1}{(a^2+x^2)^{3/2}}\,dx}{\sqrt{(a^2+x^2)^3}}\\ &= \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.00

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]
[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])
```

Maple [A]
time = 0.22, size = 24, normalized size = 0.96

method	result	size
gosper	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
default	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
risch	$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$	24
trager	$\frac{x\sqrt{a^6+3a^4x^2+3a^2x^4+x^6}}{a^2(a^2+x^2)^2}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+x^2)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2+x^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="maxima")
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(23) = 46.
time = 0.42, size = 64, normalized size = 2.56

$$\frac{a^4+2\,a^2x^2+x^4+\sqrt{a^6+3\,a^4x^2+3\,a^2x^4+x^6}\,x}{a^6+2\,a^4x^2+a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 +
2*a^4*x^2 + a^2*x^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a^2+x^2\right)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+x**2)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a**2 + x**2)**3), x)
```

Giac [A]
time = 3.46, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2+x^2}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+x^2)^3)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(a^2 + x^2)*a^2)
```

Mupad [B]
time = 3.51, size = 25, normalized size = 1.00

$$\frac{x\,\sqrt{\left(a^2+x^2\right)^3}}{a^2\,\left(a^2+x^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2 + x^2)^3)^(1/2),x)
```

```
[Out] (x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)
```

3.936 $\int \frac{\sqrt{x}}{1+\sqrt{x}+x} dx$

Optimal. Leaf size=42

$$2\sqrt{x} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}} - \log\left(1+\sqrt{x}+x\right)$$

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1371, 717, 648, 632, 210, 642}

$$-\frac{2\text{ArcTan}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}} + 2\sqrt{x} - \log\left(x+\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt{x}+x}\,dx &= 2\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}+2\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,\sqrt{x}\right)-\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,\sqrt{x}\right)\\
&= 2\sqrt{x}-\log\left(1+\sqrt{x}+x\right)+2\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt{x}\right)\\
&= 2\sqrt{x}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 1.00

$$
2\sqrt{x}-\frac{2\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}-\log\left(1+\sqrt{x}+x\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x),x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] +
 x]
```

Maple [A]
time = 0.16, size = 34, normalized size = 0.81

method	result
derivativedivides	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
default	$-\ln\left(1+x+\sqrt{x}\right)-\frac{2\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}+2\sqrt{x}$
trager	$2\sqrt{x}+2\,\mathrm{RootOf}\left(3\_Z^2+3\_Z+1\right)\ln\left(1+x+\sqrt{x}\right)-2\ln\left(3\,\mathrm{RootOf}\left(3\_Z^2+3\_Z+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1+x+x^(1/2))-2/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Maxima [A]
time = 0.50, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

Fricas [A]
time = 0.37, size = 35, normalized size = 0.83

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\log\left(x+\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x
+ sqrt(x) + 1)
```

Sympy [A]
time = 0.09, size = 49, normalized size = 1.17

$$2\sqrt{x} - \log\left(4\sqrt{x} + 4x + 4\right) - \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)
[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 +
 sqrt(3)/3)/3
```

Giac [A]
time = 2.68, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + 1\right)\right) + 2\sqrt{x} - \log\left(x + \sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")
[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt
(x) + 1)
```

Mupad [B]
time = 3.45, size = 35, normalized size = 0.83

$$2\sqrt{x} - \frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - \ln\left(x + \sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x + x^(1/2) + 1),x)
[Out] 2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x +
 x^(1/2) + 1)
```

3.937 $\int \frac{x}{1+\sqrt{x}+x}\,dx$

Optimal. Leaf size=32

$$-2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1371, 715, 632, 210}

$$\frac{4\text{ArcTan}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}+x-2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[x] + x),x]
```

```
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+\sqrt{x}+x}\,dx &= 2\text{Subst}\left(\int \frac{x^3}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x+x^2}\right)\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x+2\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt{x}\right)\\ &= -2\sqrt{x}+x-4\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt{x}\right)\\ &= -2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$-2\sqrt{x}+x+\frac{4\tan^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[x] + x),x]
[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]
```

Maple [A]
time = 0.16, size = 26, normalized size = 0.81

method	result	size
derivativedivides	$x+\frac{4\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26
default	$x+\frac{4\arctan\left(\frac{\left(1+2\sqrt{x}\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}-2\sqrt{x}$	26

trager	$-1+x-2\sqrt{x}+\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\sqrt{x}+x-1}{\mathrm{RootOf}\left(\_Z^2+3\right)x-\mathrm{RootOf}\left(\_Z^2+3\right)-3x-3}\right)}{3}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] x+4/3*arctan(1/3*(1+2*x^(1/2))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Maxima [A]
time = 0.49, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Fricas [A]
time = 0.32, size = 27, normalized size = 0.84

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\sqrt{x}+\frac{1}{3}\sqrt{3}\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="fricas")
[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)
```

Sympy [A]
time = 0.08, size = 37, normalized size = 1.16

$$-2\sqrt{x}+x+\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x**(1/2)),x)
[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3
```

Giac [A]
time = 3.29, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+1\right)\right)+x-2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x+x^(1/2)),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)
```

Mupad [B]
time = 0.04, size = 27, normalized size = 0.84

$$x+\frac{4\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}+\frac{2\,\sqrt{3}\,\sqrt{x}}{3}\right)}{3}-2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x + x^(1/2) + 1),x)
```

```
[Out] x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)
```

3.938 $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx$

Optimal. Leaf size=76

$$\frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{512\left(1+2\sqrt{x}\right)}{405\sqrt{1+\sqrt{x}+x}}$$

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 628, 627}

$$\frac{512\left(2\sqrt{x}+1\right)}{405\sqrt{x+\sqrt{x}+1}}+\frac{64\left(2\sqrt{x}+1\right)}{135\left(x+\sqrt{x}+1\right)^{3/2}}+\frac{4\left(2\sqrt{x}+1\right)}{15\left(x+\sqrt{x}+1\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
```

```
[Out] (4*(1 + 2*Sqrt[x]))/(15*(1 + Sqrt[x] + x)^(5/2)) + (64*(1 + 2*Sqrt[x]))/(13
5*(1 + Sqrt[x] + x)^(3/2)) + (512*(1 + 2*Sqrt[x]))/(405*Sqrt[1 + Sqrt[x] +
x])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
```

```
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}+x\right)^{7/2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{7/2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{32}{15}\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)^{5/2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{256}{135}\text{Subst}\left(\int \frac{1}{\left(1+x+x^2\right)}\right.\\ &= \frac{4\left(1+2\sqrt{x}\right)}{15\left(1+\sqrt{x}+x\right)^{5/2}}+\frac{64\left(1+2\sqrt{x}\right)}{135\left(1+\sqrt{x}+x\right)^{3/2}}+\frac{512\left(1+2\sqrt{x}\right)}{405\sqrt{1+\sqrt{x}+x}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 49, normalized size = 0.64

$$\frac{4\left(1+2\sqrt{x}\right)\left(203+304\sqrt{x}+432x+256x^{3/2}+128x^2\right)}{405\left(1+\sqrt{x}+x\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 + Sqrt[x] + x)^(7/2)),x]
[Out] (4*(1 + 2*Sqrt[x])*(203 + 304*Sqrt[x] + 432*x + 256*x^(3/2) + 128*x^2))/(40
5*(1 + Sqrt[x] + x)^(5/2))
```

Maple [A]
time = 0.01, size = 53, normalized size = 0.70

method	result	size
derivativedivides	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53
default	$\frac{\frac{4}{15}+\frac{8\sqrt{x}}{15}}{\left(1+x+\sqrt{x}\right)^{\frac{5}{2}}}+\frac{\frac{64}{135}+\frac{128\sqrt{x}}{135}}{\left(1+x+\sqrt{x}\right)^{\frac{3}{2}}}+\frac{\frac{512}{405}+\frac{1024\sqrt{x}}{405}}{\sqrt{1+x+\sqrt{x}}}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4/15*(1+2*x^(1/2))/(1+x+x^(1/2))^(5/2)+64/135*(1+2*x^(1/2))/(1+x+x^(1/2))^(
3/2)+512/405*(1+2*x^(1/2))/(1+x+x^(1/2))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")
[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)
```

Fricas [A]
time = 0.37, size = 95, normalized size = 1.25

$$-\frac{4\left(128\,x^{5}+272\,x^{4}+455\,x^{3}+232\,x^{2}-\left(256\,x^{5}+736\,x^{4}+1366\,x^{3}+1427\,x^{2}+839\,x+101\right)\sqrt{x}-128\,x-203\right)\sqrt{x+\sqrt{x}+1}}{405\left(x^{6}+3\,x^{5}+6\,x^{4}+7\,x^{3}+6\,x^{2}+3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")
[Out] -4/405*(128*x^5 + 272*x^4 + 455*x^3 + 232*x^2 - (256*x^5 + 736*x^4 + 1366*x
^3 + 1427*x^2 + 839*x + 101)*sqrt(x) - 128*x - 203)*sqrt(x + sqrt(x) + 1)/(
x^6 + 3*x^5 + 6*x^4 + 7*x^3 + 6*x^2 + 3*x + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x}\,\left(\sqrt{x}+x+1\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)
[Out] Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)
```

Giac [A]
time = 3.28, size = 45, normalized size = 0.59

$$\frac{4\left(2\left(8\left(2\left(4\sqrt{x}\left(2\sqrt{x}+5\right)+35\right)\sqrt{x}+65\right)\sqrt{x}+355\right)\sqrt{x}+203\right)}{405\left(x+\sqrt{x}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")
```

```
[Out] 4/405*(2*(8*(2*(4*sqrt(x)*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355
)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(x+\sqrt{x}+1\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)
```

3.939 $\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)$$

```
[Out] -1/x-arcsinh(x)-ln(1+(x^2+1)^(1/2))+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {6874, 283, 221, 1605, 196, 45}

$$\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}-\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

```
[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x
^2]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
```

```
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1605

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}}+\frac{x}{1+\sqrt{1+x^2}}\right)dx\\
&= -\int \frac{1}{1+\sqrt{1+x^2}}\,dx+\int \frac{x}{1+\sqrt{1+x^2}}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x^2\right)-\int \left(-\frac{1}{x^2}+\frac{\sqrt{1+x^2}}{x^2}\right)dx\\
&= -\frac{1}{x}-\int \frac{\sqrt{1+x^2}}{x^2}\,dx+\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\frac{\sqrt{1+x^2}}{x}-\int \frac{1}{\sqrt{1+x^2}}\,dx+\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,\sqrt{1+x^2}\right)\\
&= -\frac{1}{x}+\sqrt{1+x^2}+\frac{\sqrt{1+x^2}}{x}-\sinh^{-1}(x)-\log\left(1+\sqrt{1+x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.89

$$-\frac{1}{x}+\frac{(1+x)\sqrt{1+x^2}}{x}-4\tanh^{-1}\left(1-2x+2\sqrt{1+x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
[Out] -x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]
```

Maple [A]
time = 0.06, size = 53, normalized size = 1.15

method	result	size
trager	$\frac{-1+x}{x}+\frac{(1+x)\sqrt{x^2+1}}{x}+2\ln\left(-\frac{\sqrt{x^2+1}-1-x}{x}\right)$	43
default	$-\frac{1}{x}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}-x\sqrt{x^2+1}-\operatorname{arcsinh}(x)$	53
meijerg	$-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},1\right],\left[\frac{3}{2},2\right],-x^2\right)}{2}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+1/x*(x^2+1)^(3/2)-x*(x^2+
1)^(1/2)-arcsinh(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)
```

Fricas [A]
time = 0.39, size = 64, normalized size = 1.39

$$\frac{x\log\left(2x^2-\sqrt{x^2+1}\,(2x+1)+x+1\right)-x\log(x)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}\,(x+1)+x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqr
t(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

Sympy [A]
time = 1.50, size = 48, normalized size = 1.04

$$\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\operatorname{asinh}(x)-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+(x**2+1)**(1/2)),x)
```

```
[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/
x + 1/(x*sqrt(x**2 + 1))
```

Giac [A]
time = 4.23, size = 79, normalized size = 1.72

$$\sqrt{x^2+1} - \frac{2}{\left(x - \sqrt{x^2+1}\right)^2 - 1} - \frac{1}{x} + \log\left(-x + \sqrt{x^2+1}\right) - \log\left(|x|\right) - \log\left(\left|-x + \sqrt{x^2+1} + 1\right|\right) + \log\left(\left|-x + \sqrt{x^2+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1
)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2
 + 1) - 1))
```

Mupad [B]
time = 0.04, size = 46, normalized size = 1.00

$$\sqrt{x^2+1} - \ln\left(x\right) - \mathrm{asinh}(x) + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \mathrm{atan}\left(\sqrt{x^2+1}\ 1\mathrm{i}\right)\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

```
[Out] atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
1)^(1/2)/x - 1/x
```

3.940 $\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx$

Optimal. Leaf size=20

$$\frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

```
[Out] 3/2*(x^2-1)^(1/3)/(1+x)^(2/3)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {665}

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^m*((a + c*x^2)^(p + 1)/(2*c*d*(p + 1))), x] /; FreeQ[{a, c, d,
e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2,
0]
```

Rubi steps

$$\int \frac{1}{(1+x)^{2/3}\left(-1+x^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Mathematica [A]
time = 0.24, size = 20, normalized size = 1.00

$$\frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)),x]
```

```
[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))
```

Maple [A]
time = 0.25, size = 18, normalized size = 0.90

method	result	size
gosper	$\frac{3(1+x)^{\frac{1}{3}}(-1+x)}{2(x^2-1)^{\frac{2}{3}}}$	18
risch	$\frac{3(1+x)^{\frac{1}{3}}\left(\frac{(x^2-1)^2}{1+x}\right)^{\frac{1}{3}}(-1+x)}{2(x^2-1)^{\frac{2}{3}}\left((-1+x)^2(1+x)\right)^{\frac{1}{3}}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(2/3)/(x^2-1)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/2*(1+x)^(1/3)*(-1+x)/(x^2-1)^(2/3)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

Fricas [A]
time = 0.38, size = 14, normalized size = 0.70

$$\frac{3\,(x^2-1)^{\frac{1}{3}}}{2\,(x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")
[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x-1)\,(x+1))^{\frac{2}{3}}\,(x+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)
```

```
[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)
```

Giac [A]
time = 2.85, size = 13, normalized size = 0.65

$$\frac{3}{2}\left(-\frac{2}{x+1}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")
```

```
[Out] 3/2*(-2/(x + 1) + 1)^(1/3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\left(x^2-1\right)^{2/3}\left(x+1\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)
```

```
[Out] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)
```

3.941 $\int \left(\left(1-x^6\right)^{2/3} + \frac{\left(1-x^6\right)^{2/3}}{x^6} \right) dx$

Optimal. Leaf size=35

$$-\frac{\left(1-x^6\right)^{2/3}}{5x^5}+\frac{1}{5}x\left(1-x^6\right)^{2/3}$$

```
[Out] -1/5*(-x^6+1)^(2/3)/x^5+1/5*x*(-x^6+1)^(2/3)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 2 in optimal.
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {251, 371}

$$x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)-\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -1/5*Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/x^5 + x*Hypergeometric2F1[-2/3
, 1/6, 7/6, x^6]
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\left(1-x^6\right)^{2/3}+\frac{\left(1-x^6\right)^{2/3}}{x^6}\right)dx&=\int \left(1-x^6\right)^{2/3}\,dx+\int \frac{\left(1-x^6\right)^{2/3}}{x^6}\,dx\\&=-\frac{{}_2F_1\left(-\frac{5}{6},-\frac{2}{3};\frac{1}{6};x^6\right)}{5x^5}+x\,{}_2F_1\left(-\frac{2}{3},\frac{1}{6};\frac{7}{6};x^6\right)\end{aligned}$$

Mathematica [A]
time = 0.11, size = 18, normalized size = 0.51

$$-\frac{(1-x^6)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]
```

```
[Out] -1/5*(1 - x^6)^(5/3)/x^5
```

Maple [A]
time = 0.27, size = 20, normalized size = 0.57

method	result	size
trager	$\frac{(x^6-1)\left(-x^6+1\right)^{\frac{2}{3}}}{5x^5}$	20
risch	$-\frac{x^{12}-2x^6+1}{5x^5\left(-x^6+1\right)^{\frac{1}{3}}}$	25
meijerg	$x\,\mathrm{hypergeom}\left(\left[-\frac{2}{3},\frac{1}{6}\right],\left[\frac{7}{6}\right],x^6\right)-\frac{\mathrm{hypergeom}\left(\left[-\frac{5}{6},-\frac{2}{3}\right],\left[\frac{1}{6}\right],x^6\right)}{5x^5}$	27
gosper	$\frac{\left(-x^6+1\right)^{\frac{2}{3}}(x^2-x+1)(x^2+x+1)(1+x)(-1+x)}{5x^5}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(x^6-1)/x^5*(-x^6+1)^(2/3)
```

Maxima [A]
time = 0.52, size = 38, normalized size = 1.09

$$\frac{(x^6-1)(x^2+x+1)^{\frac{2}{3}}(-x^2+x-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}(x-1)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1
)^(2/3)/x^5
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.54

$$\frac{(x^6-1)(-x^6+1)^{\frac{2}{3}}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 68, normalized size = 1.94

$$\frac{x\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|x^6e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)}+\frac{\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{2}{3}\\ \frac{1}{6}\end{matrix}\middle|x^6e^{2i\pi}\right)}{6x^5\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6,x)
```

```
[Out] x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/
6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x*
*5*gamma(1/6))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")
```

```
[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)
```

Mupad [B]
time = 3.59, size = 14, normalized size = 0.40

$$-\frac{{\left(1-x^6\right)}^{5/3}}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3),x)
```

```
[Out] -(1 - x^6)^(5/3)/(5*x^5)
```

3.942 $\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

```
[Out] x^m/(a+b*x^n)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 460}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2\left(a+bx^n\right)^{3/2}}\,dx &= \frac{1}{2}\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{\left(a+bx^n\right)^{3/2}}\,dx\\ &= \frac{x^m}{\sqrt{a+bx^n}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 15, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x^n]
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}(2am+b(2m-n)\,x^{n})}{2\,(a+b\,x^{n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

```
[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x)
```

Maxima [A]
time = 0.35, size = 13, normalized size = 0.87

$$\frac{x^{m}}{\sqrt{bx^{n}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
maxima")
```

```
[Out] x^m/sqrt(b*x^n + a)
```

Fricas [A]
time = 0.33, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^{n}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
fricas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x^n + a)
```

Sympy [C] Result contains complex when optimal does not.
time = 65.65, size = 100, normalized size = 6.67

$$\frac{mx^{m}\Gamma\left(\frac{m}{n}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{m}{n}\\ \frac{m}{n}+1\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{\sqrt{a}\,n\Gamma\left(\frac{m}{n}+1\right)}+\frac{bx^{m}x^{n}(2m-n)\,\Gamma\left(\frac{m}{n}+1\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{m}{n}+1\\ \frac{m}{n}+2\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}n\Gamma\left(\frac{m}{n}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)
```

```
[Out] m*x**m*gamma(m/n)*hyper((3/2, m/n), (m/n + 1,), b*x**n*exp_polar(I*pi)/a)/(
sqrt(a)*n*gamma(m/n + 1)) + b*x**m*x**n*(2*m - n)*gamma(m/n + 1)*hyper((3/2
, m/n + 1), (m/n + 2,), b*x**n*exp_polar(I*pi)/a)/(2*a**(3/2)*n*gamma(m/n +
 2))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)
```

Mupad [**B**]
time = 3.68, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{a+b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)),x)
```

```
[Out] x^m/(a + b*x^n)^(1/2)
```

3.943 $\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx$

Optimal. Leaf size=53

$$-\frac{4}{81}\sqrt{2+3x} - \frac{10}{81}(2+3x)^{3/2} + \frac{8}{135}(2+3x)^{5/2} - \frac{4}{567}(2+3x)^{7/2}$$

```
[Out] -10/81*(2+3*x)^(3/2)+8/135*(2+3*x)^(5/2)-4/567*(2+3*x)^(7/2)-4/81*(2+3*x)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1607, 786}

$$-\frac{4}{567}(3x+2)^{7/2} + \frac{8}{135}(3x+2)^{5/2} - \frac{10}{81}(3x+2)^{3/2} - \frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 -
 (4*(2 + 3*x)^(7/2))/567
```

Rule 786

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x-2x^3}{\sqrt{2+3x}} \, dx &= \int \frac{x(1-2x^2)}{\sqrt{2+3x}} \, dx \\
&= \int \left(-\frac{2}{27\sqrt{2+3x}} - \frac{5}{9}\sqrt{2+3x} + \frac{4}{9}(2+3x)^{3/2} - \frac{2}{27}(2+3x)^{5/2}\right) dx \\
&= -\frac{4}{81}\sqrt{2+3x} - \frac{10}{81}(2+3x)^{3/2} + \frac{8}{135}(2+3x)^{5/2} - \frac{4}{567}(2+3x)^{7/2}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 0.53

$$-\frac{2\sqrt{2+3x}\left(164-123x-216x^2+270x^3\right)}{2835}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x],x]
```

```
[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835
```

Maple [A]
time = 0.22, size = 38, normalized size = 0.72

method	result
trager	$\left(-\frac{4}{21}x^3+\frac{16}{105}x^2+\frac{82}{945}x-\frac{328}{2835}\right)\sqrt{2+3x}$
gosper	$-\frac{2\left(270x^3-216x^2-123x+164\right)\sqrt{2+3x}}{2835}$
risch	$-\frac{2\left(270x^3-216x^2-123x+164\right)\sqrt{2+3x}}{2835}$
derivativedivides	$-\frac{10(2+3x)^{\frac{3}{2}}}{81}+\frac{8(2+3x)^{\frac{5}{2}}}{135}-\frac{4(2+3x)^{\frac{7}{2}}}{567}-\frac{4\sqrt{2+3x}}{81}$
default	$-\frac{10(2+3x)^{\frac{3}{2}}}{81}+\frac{8(2+3x)^{\frac{5}{2}}}{135}-\frac{4(2+3x)^{\frac{7}{2}}}{567}-\frac{4\sqrt{2+3x}}{81}$
meijerg	$-\frac{16\sqrt{2}\left(\frac{32\sqrt{\pi}}{35}-\frac{\sqrt{\pi}\left(-135x^3+108x^2-96x+128\right)\sqrt{1+\frac{3x}{2}}}{140}\right)}{81\sqrt{\pi}}+\frac{2\sqrt{2}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-6x+8\right)\sqrt{1+\frac{3x}{2}}}{6}\right.}{9\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^3+x)/(2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -10/81*(2+3*x)^(3/2)+8/135*(2+3*x)^(5/2)-4/567*(2+3*x)^(7/2)-4/81*(2+3*x)^(
1/2)
```

Maxima [A]
time = 0.37, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

Fricas [A]
time = 0.34, size = 24, normalized size = 0.45

$$-\frac{2}{2835}\left(270\,x^3-216\,x^2-123\,x+164\right)\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)
```

Sympy [A]
time = 5.97, size = 46, normalized size = 0.87

$$-\frac{4(3x+2)^{\frac{7}{2}}}{567}+\frac{8(3x+2)^{\frac{5}{2}}}{135}-\frac{10(3x+2)^{\frac{3}{2}}}{81}-\frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)
```

```
[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 -
 4*sqrt(3*x + 2)/81
```

Giac [A]
time = 5.29, size = 37, normalized size = 0.70

$$-\frac{4}{567}\,(3\,x+2)^{\frac{7}{2}}+\frac{8}{135}\,(3\,x+2)^{\frac{5}{2}}-\frac{10}{81}\,(3\,x+2)^{\frac{3}{2}}-\frac{4}{81}\,\sqrt{3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/
81*sqrt(3*x + 2)
```

Mupad [B]
time = 0.04, size = 37, normalized size = 0.70

$$\frac{8\,{\left(3\,x+2\right)}^{5/2}}{135}-\frac{10\,{\left(3\,x+2\right)}^{3/2}}{81}-\frac{4\,\sqrt{3\,x+2}}{81}-\frac{4\,{\left(3\,x+2\right)}^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2*x^3)/(3*x + 2)^(1/2),x)
```

```
[Out] (8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81
- (4*(3*x + 2)^(7/2))/567
```

3.944 $\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=31

$$-4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)$$

```
[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2037, 1607, 272, 45}

$$2\sqrt{x+1}-4\sqrt[4]{x+1}+4\log\left(\sqrt[4]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
```

```
[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 2037

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{1+x}+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{x}+\sqrt{x}}\,dx, x, 1+x\right)\\
&= \text{Subst}\left(\int \frac{1}{\left(1+\sqrt[4]{x}\right)\sqrt[4]{x}}\,dx, x, 1+x\right)\\
&= 4\text{Subst}\left(\int \frac{x^2}{1+x}\,dx, x, \sqrt[4]{1+x}\right)\\
&= 4\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)\,dx, x, \sqrt[4]{1+x}\right)\\
&= -4\sqrt[4]{1+x}+2\sqrt{1+x}+4\log\left(1+\sqrt[4]{1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 31, normalized size = 1.00

$$2\sqrt[4]{1+x}\left(-2+\sqrt[4]{1+x}\right)+4\log\left(1+\sqrt[4]{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1),x]
[Out] 2*(1 + x)^(1/4)*(-2 + (1 + x)^(1/4)) + 4*Log[1 + (1 + x)^(1/4)]
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.84

method	result	size
derivativedivides	$-4(1+x)^{\frac{1}{4}}+4\ln\left(1+(1+x)^{\frac{1}{4}}\right)+2\sqrt{1+x}$	26
default	$-4(1+x)^{\frac{1}{4}}+4\ln\left(1+(1+x)^{\frac{1}{4}}\right)+2\sqrt{1+x}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)^(1/4)+(1+x)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -4*(1+x)^(1/4)+4*ln(1+(1+x)^(1/4))+2*(1+x)^(1/2)
```

Maxima [A]
time = 0.37, size = 25, normalized size = 0.81

$$2\sqrt{x+1}-4\,(x+1)^{\frac{1}{4}}+4\,\log\left((x+1)^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Fricas [A]
time = 0.32, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Sympy [A]
time = 0.10, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1} + 2\sqrt{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)),x)
[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)
```

Giac [A]
time = 4.10, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4\,(x+1)^{\frac{1}{4}} + 4\,\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)
```

Mupad [B]
time = 0.07, size = 25, normalized size = 0.81

$$4\,\ln\left((x+1)^{1/4} + 1\right) + 2\sqrt{x+1} - 4\,(x+1)^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x + 1)^(1/2) + (x + 1)^(1/4)),x)
[Out] 4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)
```

3.945 $\int \frac{1+2x}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=11

$$2\sqrt{x+x^2}$$

[Out] 2*(x^2+x)^(1/2)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{643\}$

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/Sqrt[x + x^2],x]

[Out] 2*Sqrt[x + x^2]

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{x+x^2}}\,dx = 2\sqrt{x+x^2}$$

Mathematica [A]
time = 0.03, size = 11, normalized size = 1.00

$$2\sqrt{x(1+x)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/Sqrt[x + x^2],x]

[Out] 2*Sqrt[x*(1 + x)]

Maple [A]
time = 0.22, size = 10, normalized size = 0.91

method	result	size
derivativedivides	$2\sqrt{x^2+x}$	10
default	$2\sqrt{x^2+x}$	10
trager	$2\sqrt{x^2+x}$	10
gosper	$\frac{2x(1+x)}{\sqrt{x^2+x}}$	14
risch	$\frac{2x(1+x)}{\sqrt{x(1+x)}}$	14
meijerg	$\frac{2\sqrt{\pi}\,\sqrt{x}\,\sqrt{1+x}-2\sqrt{\pi}\,\operatorname{arcsinh}(\sqrt{x})}{\sqrt{\pi}}+2\operatorname{arcsinh}\left(\sqrt{x}\right)$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(x^2+x)^(1/2)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x^2 + x)
```

Fricas [A]
time = 0.37, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + x)
```

Sympy [A]
time = 0.06, size = 8, normalized size = 0.73

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] 2*sqrt(x**2 + x)
```

Giac [A]
time = 5.64, size = 9, normalized size = 0.82

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x^2 + x)
```

Mupad [B]
time = 3.52, size = 9, normalized size = 0.82

$$2\sqrt{x\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/(x + x^2)^(1/2),x)
```

```
[Out] 2*(x*(x + 1))^(1/2)
```

3.946 $\int \frac{1}{2\sqrt{x}\,(1+x)}\,dx$

Optimal. Leaf size=6

$$\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 65, 209}

$$\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{2\sqrt{x}\,(1+x)}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\
&= \tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 6, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.22, size = 5, normalized size = 0.83

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)$	5
default	$\arctan\left(\sqrt{x}\right)$	5
meijerg	$\arctan\left(\sqrt{x}\right)$	5
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{x}+x-1}{1+x}\right)}{2}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/(1+x)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/2))
```

Maxima [A]
time = 0.47, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))
```

Fricas [A]
time = 0.35, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))
```

Sympy [A]
time = 0.08, size = 5, normalized size = 0.83

$$\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x**(1/2),x)
[Out] atan(sqrt(x))
```

Giac [A]
time = 4.87, size = 4, normalized size = 0.67

$$\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(x))
```

Mupad [B]
time = 0.14, size = 4, normalized size = 0.67

$$\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)*(x + 1)),x)
[Out] atan(x^(1/2))
```

3.947 $\int \frac{1}{x\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

```
[Out] -1/3*(-x^2+6*x)^(1/2)/x
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {664}

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[6*x - x^2]),x]
[Out] -1/3*Sqrt[6*x - x^2]/x
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt{6x-x^2}}\,dx = -\frac{\sqrt{6x-x^2}}{3x}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 0.85

$$\frac{-6+x}{3\sqrt{-((-6+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]
[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])
```

Maple [A]
time = 0.23, size = 17, normalized size = 0.85

method	result	size
risch	$\frac{-6+x}{3\sqrt{-x\left(-6+x\right)}}$	14
gosper	$\frac{-6+x}{3\sqrt{-x^{2}+6x}}$	17
default	$-\frac{\sqrt{-x^{2}+6x}}{3x}$	17
trager	$-\frac{\sqrt{-x^{2}+6x}}{3x}$	17
meijerg	$-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1-\frac{x}{6}}}{3\sqrt{x}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*(-x^2+6*x)^(1/2)/x
```

Maxima [A]
time = 0.49, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^{2}+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Fricas [A]
time = 0.36, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^{2}+6\,x}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(-x^2 + 6*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{-x\left(x-6\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+6*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)
```

Giac [A]
time = 4.96, size = 25, normalized size = 1.25

$$\frac{2}{3\left(\frac{\sqrt{-x^2+6\,x}-3}{x-3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")
[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)
```

Mupad [B]
time = 3.51, size = 16, normalized size = 0.80

$$-\frac{\sqrt{6\,x-x^2}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x - x^2)^(1/2)),x)
[Out] -(6*x - x^2)^(1/2)/(3*x)
```

3.948 $\int \left(1+\sqrt{x}\right) \sqrt{x} \, dx$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \left(1+\sqrt{x}\right) \sqrt{x} \, dx &= \int \left(\sqrt{x}+x\right) \, dx \\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x^{3/2}}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A]
time = 0.02, size = 12, normalized size = 0.71

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
default	$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$	12
trager	$\frac{(1+x)(-1+x)}{2}+\frac{2x^{\frac{3}{2}}}{3}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 2/3*x^(3/2)+1/2*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(11) = 22.
time = 0.27, size = 26, normalized size = 1.53

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Fricas [A]
time = 0.34, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Sympy [A]
time = 0.06, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

Giac [A]
time = 4.58, size = 11, normalized size = 0.65

$$\frac{1}{2}\,x^{2} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.65

$$\frac{x^2}{2} + \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) + 1),x)
```

```
[Out] x^2/2 + (2*x^(3/2))/3
```

3.949 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{14\}$

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{\sqrt[3]{x}} \, dx &= \int \left(\frac{1}{\sqrt[3]{x}} - \sqrt[6]{x}\right) \, dx \\ &= \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 19, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A]
time = 0.03, size = 12, normalized size = 0.63

method	result	size
derivativedivides	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12
default	$\frac{3x^{\frac{2}{3}}}{2}-\frac{6x^{\frac{7}{6}}}{7}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.58

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Fricas [A]
time = 0.38, size = 11, normalized size = 0.58

$$-\frac{6}{7}x^{\frac{7}{6}}+\frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A]
time = 0.83, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Giac [A]
time = 3.70, size = 11, normalized size = 0.58

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Mupad [B]
time = 0.02, size = 12, normalized size = 0.63

$$-\frac{3\,x^{2/3}\,\left(4\,\sqrt{x}-7\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/x^(1/3),x)
[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14
```

3.950 $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

Optimal. Leaf size=41

$$-6\sqrt[6]{x}+2\sqrt{x}-\frac{6x^{5/6}}{5}+\frac{6x^{7/6}}{7}+6\tan^{-1}\left(\sqrt[6]{x}\right)$$

```
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {348, 52, 65, 209}

$$6\text{ArcTan}\left(\sqrt[6]{x}\right)+\frac{6x^{7/6}}{7}-\frac{6x^{5/6}}{5}+2\sqrt{x}-6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x^(1/3)),x]
```

```
[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx &= 3\text{Subst}\left(\int \frac{x^{7/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{6x^{7/6}}{7} - 3\text{Subst}\left(\int \frac{x^{5/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\text{Subst}\left(\int \frac{x^{3/2}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3\text{Subst}\left(\int \frac{\sqrt{x}}{1+x}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(1+x)}\,dx, x, \sqrt[3]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[6]{x}\right)\\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 42, normalized size = 1.02

$$\frac{2}{35}\left(-105\sqrt[6]{x} + 35\sqrt{x} - 21x^{5/6} + 15x^{7/6}\right) + 6\tan^{-1}\left(\sqrt[6]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x^(1/3)),x]
[Out] (2*(-105*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 6*ArcTan[x^(
1/6)]
```

Maple [A]
time = 0.22, size = 28, normalized size = 0.68

method	result	size
derivativedivides	$-6x^{\frac{1}{6}} - \frac{6x^{\frac{5}{6}}}{5} + \frac{6x^{\frac{7}{6}}}{7} + 6\arctan\left(x^{\frac{1}{6}}\right) + 2\sqrt{x}$	28

default	$-6x^{\frac{1}{6}}-\frac{6x^{\frac{5}{6}}}{5}+\frac{6x^{\frac{7}{6}}}{7}+6\arctan\left(x^{\frac{1}{6}}\right)+2\sqrt{x}$	28
meijerg	$-\frac{2x^{\frac{1}{6}}\left(-45x+63x^{\frac{2}{3}}-105x^{\frac{1}{3}}+315\right)}{105}+6\arctan\left(x^{\frac{1}{6}}\right)$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x^(1/3)),x,method=_RETURNVERBOSE)
[Out] -6*x^(1/6)-6/5*x^(5/6)+6/7*x^(7/6)+6*arctan(x^(1/6))+2*x^(1/2)
```

Maxima [A]
time = 0.51, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-6x^{\frac{1}{6}}+6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

Fricas [A]
time = 0.35, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-7)x^{\frac{1}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}+6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="fricas")
[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))
```

Sympy [A]
time = 1.89, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7}-\frac{6x^{\frac{5}{6}}}{5}-6\sqrt[6]{x}+2\sqrt{x}+6\operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x**(1/3)),x)
[Out] 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 2*sqrt(x) + 6*atan(x**(1/6))
```

Giac [A]
time = 3.15, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}}-\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-6x^{\frac{1}{6}}+6\arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="giac")
```

```
[Out] 6/7*x^(7/6) - 6/5*x^(5/6) + 2*sqrt(x) - 6*x^(1/6) + 6*arctan(x^(1/6))
```

Mupad [B]
time = 0.03, size = 27, normalized size = 0.66

$$6\,\mathrm{atan}\left(x^{1/6}\right)+2\,\sqrt{x}-6\,x^{1/6}-\frac{6\,x^{5/6}}{5}+\frac{6\,x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/3) + 1),x)
```

```
[Out] 6*atan(x^(1/6)) + 2*x^(1/2) - 6*x^(1/6) - (6*x^(5/6))/5 + (6*x^(7/6))/7
```

3.951 $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx$

Optimal. Leaf size=67

$$6\sqrt[3]{1+\sqrt{x}}-2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right)+3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right)-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+3*ln(1-(1+x^(1/2))^(1/3))-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3
^(1/2))*3^(1/2)+6*(1+x^(1/2))^(1/3)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {272, 52, 59, 632, 210, 31}

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)+6\sqrt[3]{\sqrt{x}+1}+3\log\left(1-\sqrt[3]{\sqrt{x}+1}\right)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt{x}}}{x}\,dx &= 2\text{Subst}\left(\int \frac{\sqrt[3]{1+x}}{x}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} + 2\text{Subst}\left(\int \frac{1}{x(1+x)^{2/3}}\,dx, x, \sqrt{x}\right)\\
&= 6\sqrt[3]{1+\sqrt{x}} - \frac{\log(x)}{2} - 3\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+\sqrt{x}}\right) - 3\text{Subst}\left(\int \frac{1}{1+x+\ldots}\right.\\
&= 6\sqrt[3]{1+\sqrt{x}} + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2} + 6\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\ldots\right.\\
&= 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 3\log\left(1-\sqrt[3]{1+\sqrt{x}}\right) - \frac{\log(x)}{2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 86, normalized size = 1.28

$$6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}}\right) + 2\log\left(-1+\sqrt[3]{1+\sqrt{x}}\right) - \log\left(1+\sqrt[3]{1+\sqrt{x}} + \left(1+\sqrt{x}\right)^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x,x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3
]] + 2*Log[-1 + (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + S
qrt[x])^(2/3)]
```

Maple [A]
time = 0.23, size = 64, normalized size = 0.96

method	result
meijerg	$-\frac{2\left(-\Gamma\left(\frac{2}{3}\right)\sqrt{x}\ \text{hypergeom}\left(\left[\frac{2}{3},1,1\right],[2,2],-\sqrt{x}\right)-3\left(3+\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+\frac{\ln(x)}{2}\right)\Gamma\left(\frac{2}{3}\right)\right)}{3\Gamma\left(\frac{2}{3}\right)}$
derivativedivides	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}+2\ln\left(\left(1+\sqrt{x}\right)^{\frac{1}{3}}-1\right)-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\,\text{arcta}$
default	$6\left(1+\sqrt{x}\right)^{\frac{1}{3}}+2\ln\left(\left(1+\sqrt{x}\right)^{\frac{1}{3}}-1\right)-\ln\left(\left(1+\sqrt{x}\right)^{\frac{2}{3}}+\left(1+\sqrt{x}\right)^{\frac{1}{3}}+1\right)-2\,\text{arcta}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2))^(1/3)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 6*(1+x^(1/2))^(1/3)+2*ln((1+x^(1/2))^(1/3)-1)-ln((1+x^(1/2))^(2/3)+(1+x^(1/
2))^(1/3)+1)-2*arctan(1/3*(1+2*(1+x^(1/2))^(1/3))*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.51, size = 63, normalized size = 0.94

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log((sqrt(
x) + 1)^(1/3) - 1)
```

Fricas [A]
time = 0.37, size = 65, normalized size = 0.97

$$-2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*arctan(2/3*sqrt(3)*(sqrt(x) + 1)^(1/3) + 1/3*sqrt(3)) + 6*(sqrt(
x) + 1)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(
(sqrt(x) + 1)^(1/3) - 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.60, size = 39, normalized size = 0.58

$$-\frac{2\sqrt[6]{x}\,\Gamma\left(-\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

Giac [A]
time = 2.74, size = 64, normalized size = 0.96

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)\right)+6\left(\sqrt{x}+1\right)^{\frac{1}{3}}-\log\left(\left(\sqrt{x}+1\right)^{\frac{2}{3}}+\left(\sqrt{x}+1\right)^{\frac{1}{3}}+1\right)+2\log\left(\left|\left(\sqrt{x}+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(x) + 1)^(1/3) + 1)) + 6*(sqrt(x) + 1
)^(1/3) - log((sqrt(x) + 1)^(2/3) + (sqrt(x) + 1)^(1/3) + 1) + 2*log(abs((s
qrt(x) + 1)^(1/3) - 1))
```

Mupad [B]
time = 3.83, size = 73, normalized size = 1.09

$$2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3}+\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2) + 1)^(1/3)/x,x)
```

```
[Out] 2*log((x^(1/2) + 1)^(1/3) - 1) + 6*(x^(1/2) + 1)^(1/3) + log((x^(1/2) + 1)^
(1/3) - (3^(1/2)*1i)/2 + 1/2)*(3^(1/2)*1i - 1) - log((3^(1/2)*1i)/2 + (x^(1
/2) + 1)^(1/3) + 1/2)*(3^(1/2)*1i + 1)
```

3.952 $\int \left(1-\sqrt{x}\right) dx$

Optimal. Leaf size=11

$$x-\frac{2x^{3/2}}{3}$$

```
[Out] x-2/3*x^(3/2)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Rubi steps

$$\int \left(1-\sqrt{x}\right) dx = x-\frac{2x^{3/2}}{3}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - Sqrt[x],x]
```

```
[Out] x - (2*x^(3/2))/3
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.73

method	result	size

derivativedivides	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
default	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
risch	$x - \frac{2x^{\frac{3}{2}}}{3}$	8
trager	$-1 + x - \frac{2x^{\frac{3}{2}}}{3}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x-2/3*x^(3/2)
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*x^(3/2) + x
```

Fricas [A]
time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*x^(3/2) + x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/2),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

Giac [A]
time = 3.15, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*x^(3/2) + x
```

Mupad [B]
time = 0.00, size = 7, normalized size = 0.64

$$x - \frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/2),x)
```

```
[Out] x - (2*x^(3/2))/3
```

3.953 $\int (1 - \sqrt[4]{x})\, dx$

Optimal. Leaf size=11

$$x - \frac{4x^{5/4}}{5}$$

```
[Out] x-4/5*x^(5/4)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^(1/4),x]
[Out] x - (4*x^(5/4))/5
```

Rubi steps

$$\int (1 - \sqrt[4]{x})\, dx = x - \frac{4x^{5/4}}{5}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^(1/4),x]
[Out] x - (4*x^(5/4))/5
```

Maple [A]
time = 0.02, size = 8, normalized size = 0.73

method	result	size

derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
default	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
risch	$x-\frac{4x^{\frac{5}{4}}}{5}$	8
trager	$-1+x-\frac{4x^{\frac{5}{4}}}{5}$	9

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-x^(1/4),x,method=_RETURNVERBOSE)
```

```
[Out] x-4/5*x^(5/4)
```

Maxima [A]
time = 0.26, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="maxima")
```

```
[Out] -4/5*x^(5/4) + x
```

Fricas [A]
time = 0.33, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="fricas")
```

```
[Out] -4/5*x^(5/4) + x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x**(1/4),x)
```

```
[Out] -4*x**(5/4)/5 + x
```

Giac [A]
time = 3.87, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-x^(1/4),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

Mupad [B]
time = 0.02, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - x^(1/4),x)
[Out] x - (4*x^(5/4))/5
```

3.954 $\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx$

Optimal. Leaf size=11

$$x-\frac{4x^{5/4}}{5}$$

```
[Out] x-4/5*x^(5/4)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {26}

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]
```

```
[Out] x - (4*x^(5/4))/5
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c,
 d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && G
tQ[a, 0] && LtQ[d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} \, dx &= \int \left(1-\sqrt[4]{x}\right) \, dx \\ &= x-\frac{4x^{5/4}}{5} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$x-\frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]
[Out] x - (4*x^(5/4))/5
```

Maple [C] Result contains higher order function than in optimal. Order 3 vs. order 2.
time = 0.22, size = 46, normalized size = 4.18

method	result
derivativedivides	$x-\frac{4x^{\frac{5}{4}}}{5}$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3}-\frac{x^{\frac{1}{4}}\left(12x-15x^{\frac{3}{4}}+20\sqrt{x}-30x^{\frac{1}{4}}+60\right)}{15}$
default	$-\frac{4x^{\frac{5}{4}}}{5}+x+2\ln\left(1+x^{\frac{1}{4}}\right)-\ln\left(1-x\right)-\ln\left(-1+\sqrt{x}\right)+\ln\left(1+\sqrt{x}\right)+2\ln\left(-1+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/(1+x^(1/4)),x,method=_RETURNVERBOSE)
[Out] -4/5*x^(5/4)+x+2*ln(1+x^(1/4))-ln(1-x)-ln(-1+x^(1/2))+ln(1+x^(1/2))+2*ln(-1
+x^(1/4))
```

Maxima [A]
time = 0.27, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")
[Out] -4/5*x^(5/4) + x
```

Fricas [A]
time = 0.32, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")
[Out] -4/5*x^(5/4) + x
```

Sympy [A]
time = 2.14, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)
[Out] -4*x**(5/4)/5 + x
```

Giac [A]
time = 5.44, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")
[Out] -4/5*x^(5/4) + x
```

Mupad [B]
time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{4\,x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)
[Out] x - (4*x^(5/4))/5
```

3.955 $\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] arctanh(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(a*d+b*c)*x+b*d*x^2)^(1/
2))/b^(1/2)/d^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1976, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x +
 b*d*x^2])]/(Sqrt[b]*Sqrt[d])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}}\,dx = \int \frac{1}{\sqrt{ac+(bc+ad)x+bdx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{4bd-x^2}\,dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [**A**]
time = 0.02, size = 75, normalized size = 1.23

$$\frac{2\sqrt{a+bx}\sqrt{c+dx}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]
```

```
[Out] (2*Sqrt[a + b*x]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])
```

Maple [**A**]
time = 0.31, size = 49, normalized size = 0.80

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ad+\frac{1}{2}bc+bdx}{\sqrt{bd}}+\sqrt{ac+(ad+bc)x+bd\,x^2}\right)}{\sqrt{bd}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*
d)^(1/2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.37, size = 192, normalized size = 3.15

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,\sqrt{bdx^2+ac+(bc+ad)x}\,(2\,bdx+bc+ad)\sqrt{bd}+8\,(b^2cd+abd^2)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{\sqrt{bdx^2+ac+(bc+ad)x}\,(2\,bdx+bc+ad)\sqrt{-bd}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b
*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d
+ a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d
)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a
*b*d^2)*x))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x)*(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(49) = 98.
time = 3.91, size = 123, normalized size = 2.02

$$\frac{1}{4}\sqrt{bdx^2+bcx+adx+ac}\left(2\,x+\frac{bc+ad}{bd}\right)+\frac{(b^2c^2-2\,abcd+a^2d^2)\log\left(\left|-bc-ad-2\,\sqrt{bd}\left(\sqrt{bd}\,x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{8\,\sqrt{bd}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*(2*x + (b*c + a*d)/(b*d)) + 1/8*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x
- sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c + d*x))^(1/2), x)
```

3.956 $\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

```
[Out] -arctan(1/2*(-2*b*d*x-a*d+b*c)/b^(1/2)/d^(1/2)/(a*c+(-a*d+b*c)*x-b*d*x^2)^(
1/2))/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1976, 635, 210}

$$-\frac{\text{ArcTan}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\,\sqrt{d}\,\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x
- b*d*x^2])]/(Sqrt[b]*Sqrt[d]))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}}\,dx = \int \frac{1}{\sqrt{ac+(bc-ad)x-bdx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{-4bd-x^2}\,dx, x, \frac{bc-ad-2bdx}{\sqrt{ac+(bc-ad)x-bdx^2}}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 1.20

$$-\frac{2\sqrt{a+bx}\sqrt{c-dx}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c-dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]
```

```
[Out] (-2*Sqrt[a + b*x]*Sqrt[c - d*x]*ArcTan[(Sqrt[b]*Sqrt[c - d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)])
```

Maple [A]
time = 0.30, size = 55, normalized size = 0.85

method	result	size
default	$\frac{\arctan\left(\frac{\sqrt{bd}\left(x-\frac{-ad+bc}{2bd}\right)}{\sqrt{ac+(-ad+bc)x-bd\,x^2}}\right)}{\sqrt{bd}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b
*d*x^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 detail
```

Fricas [A]
time = 0.39, size = 202, normalized size = 3.11

$$\left[-\frac{\sqrt{-bd}\,\log\left(8\,b^2d^2x^2+b^2c^2-6\,abcd+a^2d^2-4\,\sqrt{-bdx^2+ac+(bc-ad)x}\,(2\,bdx-bc+ad)\sqrt{-bd}-8\,(b^2cd-abd^2)x\right)}{2\,bd},-\frac{\sqrt{bd}\,\arctan\left(\frac{\sqrt{-bdx^2+ac+(bc-ad)x}\,(2\,bdx-bc+ad)\sqrt{bd}}{2\,(b^2d^2x^2-abcd-(b^2cd-abd^2)x)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt
(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*
c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c -
 a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d
- a*b*d^2)*x))/(b*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)\,(c-dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x)*(c - d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(53) = 106.
time = 5.54, size = 131, normalized size = 2.02

$$\frac{1}{4}\,\sqrt{-bdx^2+bcx-adx+ac}\,\left(2\,x-\frac{bc-ad}{bd}\right)-\frac{(b^2c^2+2\,abcd+a^2d^2)\log\left(\left|bc-ad+2\,\sqrt{-bd}\,\left(\sqrt{-bd}\,x-\sqrt{-bdx^2+bcx-adx+ac}\right)\right|\right)}{8\,\sqrt{-bd}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c)*(2*x - (b*c - a*d)/(b*d)) - 1/8*(b
^2*c^2 + 2*a*b*c*d + a^2*d^2)*log(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*
x - sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c))))/(sqrt(-b*d)*b*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+b\,x)\,\left(c-d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c - d*x))^(1/2),x)
[Out] int(1/((a + b*x)*(c - d*x))^(1/2), x)
```

3.957 $\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.267$, Rules used = {335, 218, 212, 209}

$$\text{ArcTan}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(1 - x^2)),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx = 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= \tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right) + \tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A]
time = 0.23, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)-2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{1}{4}}}$	40
trager	$-\frac{\ln\left(\frac{-1-x+2\sqrt{x}}{-1+x}\right)}{2}+\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{x\,\text{RootOf}\left(\_Z^2+1\right)-\text{RootOf}\left(\_Z^2+1\right)-2\sqrt{x}}{1+x}\right)}{2}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+1)/x^(1/2),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.50, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.37, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x} + 1\right) - \frac{1}{2}\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.12, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x} - 1\right)}{2} + \frac{\log\left(\sqrt{x} + 1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+1)/x**(1/2),x)
```

```
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.
time = 4.66, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x} + 1\right) - \frac{1}{2}\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 3.34, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(x^2 - 1)),x)
```

```
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.958 $\int \frac{\sqrt{x}}{x-x^3}\,dx$

Optimal. Leaf size=13

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1598, 335, 218, 212, 209}

$$\text{ArcTan}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x - x^3),x]
```

```
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{x-x^3}\,dx &= \int \frac{1}{\sqrt{x}\,(1-x^2)}\,dx\\ &= 2\text{Subst}\left(\int \frac{1}{1-x^4}\,dx, x, \sqrt{x}\right)\\ &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{x}\right)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= \tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 13, normalized size = 1.00

$$\tan^{-1}\left(\sqrt{x}\right)+\tanh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x - x^3),x]
[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Maple [A]
time = 0.23, size = 10, normalized size = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
default	$\arctan\left(\sqrt{x}\right)+\operatorname{arctanh}\left(\sqrt{x}\right)$	10
meijerg	$-\frac{\sqrt{x}\,\left(\ln\left(1-(x^2)^{\frac{1}{4}}\right)-\ln\left(1+(x^2)^{\frac{1}{4}}\right)-2\arctan\left((x^2)^{\frac{1}{4}}\right)\right)}{2(x^2)^{\frac{1}{4}}}$	40
trager	$\frac{\ln\left(\frac{2\sqrt{x}+1+x}{-1+x}\right)}{2}-\frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{x\,\text{RootOf}\left(\_Z^2+1\right)-\text{RootOf}\left(\_Z^2+1\right)+2\sqrt{x}}{1+x}\right)}{2}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-x^3+x),x,method=_RETURNVERBOSE)
[Out] arctan(x^(1/2))+arctanh(x^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.49, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(9) = 18$.
time = 0.35, size = 21, normalized size = 1.62

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="fricas")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.17, size = 26, normalized size = 2.00

$$-\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-x**3+x),x)
[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.
time = 5.45, size = 22, normalized size = 1.69

$$\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="giac")
[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B]
time = 0.03, size = 9, normalized size = 0.69

$$\operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atanh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x - x^3),x)
[Out] atan(x^(1/2)) + atanh(x^(1/2))
```

3.959 $\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx$

Optimal. Leaf size=72

$$\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right)+\frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right)$$

```
[Out] 1/2*ln(2+x^2-3^(1/2)+x*(1+3^(1/2)))+1/23*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1
/2))^(1/2))*(299+184*3^(1/2))^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {648, 632, 212, 642}

$$\frac{1}{2}\log\left(x^2+\left(1+\sqrt{3}\right)x-\sqrt{3}+2\right)+\sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{2x+\sqrt{3}+1}{\sqrt{2\left(3\sqrt{3}-2\right)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3
])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx = \frac{1}{2}\int \frac{1+\sqrt{3}+2x}{2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2}\,dx + \frac{1}{2}\left(-1-\sqrt{3}\right)\int \frac{}{2-\sqrt{3}+}$$

$$= \frac{1}{2}\log\left(2-\sqrt{3}+\left(1+\sqrt{3}\right)x+x^2\right) + \left(1+\sqrt{3}\right)\text{Subst}\left(\int \frac{}{-2\left(2\right.}\right.$$

$$= \sqrt{\frac{1}{23}\left(13+8\sqrt{3}\right)}\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{2\left(-2+3\sqrt{3}\right)}}\right) + \frac{1}{2}\log\left(2-\sqrt{3}\right.$$

Mathematica [A]
time = 0.06, size = 72, normalized size = 1.00

$$\frac{\left(1+\sqrt{3}\right)\tanh^{-1}\left(\frac{1+\sqrt{3}+2x}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}} + \frac{1}{2}\log\left(2-\sqrt{3}+x+\sqrt{3}\,x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2),x]
```

```
[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 +
 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2
```

Maple [A]
time = 0.31, size = 58, normalized size = 0.81

method	result	size

default	$\frac{\ln\left(x\sqrt{3}+x^{2}-\sqrt{3}+x+2\right)}{2}-\frac{2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\operatorname{arctanh}\left(\frac{1+2x+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x^2-3^(1/2)+(1+3^(1/2))*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x*3^(1/2)+x^2-3^(1/2)+x+2)-2*(-1/2-1/2*3^(1/2))/(-4+6*3^(1/2))^(1/2)
*arctanh((1+2*x+3^(1/2))/(-4+6*3^(1/2))^(1/2))
```

Maxima [A]
time = 0.52, size = 77, normalized size = 1.07

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1}{2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(x^{2}+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(3) + 1)*log((2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/(2*x + sqr
t(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(x^2 + x*(sqr
t(3) + 1) - sqrt(3) + 2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(51) = 102.
time = 0.34, size = 131, normalized size = 1.82

$$\frac{1}{46}\sqrt{23}\sqrt{8\sqrt{3}+13}\log\left(\frac{23x^{4}+46x^{3}+\sqrt{23}\left(11x^{3}+24x^{2}-\sqrt{3}\left(5x^{3}+13x^{2}-6x-4\right)-4x+5\right)\sqrt{8\sqrt{3}+13}+23\sqrt{3}\left(3x^{2}+5x+4\right)-23x+138}{x^{4}+2x^{3}+2x^{2}+10x+1}\right)+\frac{1}{2}\log\left(x^{2}+\sqrt{3}(x-1)+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log((23*x^4 + 46*x^3 + sqrt(23)*(11*x^3
+ 24*x^2 - sqrt(3)*(5*x^3 + 13*x^2 - 6*x - 4) - 4*x + 5)*sqrt(8*sqrt(3) + 1
3) + 23*sqrt(3)*(3*x^2 + 5*x + 4) - 23*x + 138)/(x^4 + 2*x^3 + 2*x^2 + 10*x
 + 1)) + 1/2*log(x^2 + sqrt(3)*(x - 1) + x + 2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(58) = 116.
time = 0.75, size = 202, normalized size = 2.81

$$\left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}+\frac{1}{2}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\left(\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}+\frac{1}{2}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)+\frac{11}{5+4\sqrt{3}}\right)+\left(\frac{1}{2}-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\log\left(x-\frac{5\sqrt{3}}{5+4\sqrt{3}}+\frac{11}{5+4\sqrt{3}}+\left(\frac{1}{2}-\frac{\sqrt{5+4\sqrt{3}}}{2\cdot\left(2-3\sqrt{3}\right)}\right)\left(\frac{47}{22+13\sqrt{3}}+\frac{33\sqrt{3}}{22+13\sqrt{3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)
```

```
[Out] (sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*log(x - 5*sqrt(3)/(5 + 4*sq
rt(3)) + (sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))) + 1/2)*(47/(22 + 13*sqrt(
3)) + 33*sqrt(3)/(22 + 13*sqrt(3))) + 11/(5 + 4*sqrt(3))) + (1/2 - sqrt(5 +
 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*log(x - 5*sqrt(3)/(5 + 4*sqrt(3)) + 11/(5
+ 4*sqrt(3)) + (1/2 - sqrt(5 + 4*sqrt(3))/(2*(2 - 3*sqrt(3))))*(47/(22 + 13
*sqrt(3)) + 33*sqrt(3)/(22 + 13*sqrt(3))))
```

Giac [A]
time = 5.07, size = 80, normalized size = 1.11

$$-\frac{\left(\sqrt{3}+1\right)\log\left(\frac{\left|2x+\sqrt{3}-\sqrt{6\sqrt{3}-4}+1\right|}{\left|2x+\sqrt{3}+\sqrt{6\sqrt{3}-4}+1\right|}\right)}{2\sqrt{6\sqrt{3}-4}}+\frac{1}{2}\log\left(\left|x^2+x\left(\sqrt{3}+1\right)-\sqrt{3}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(3) + 1)*log(abs(2*x + sqrt(3) - sqrt(6*sqrt(3) - 4) + 1)/abs(2*x
 + sqrt(3) + sqrt(6*sqrt(3) - 4) + 1))/sqrt(6*sqrt(3) - 4) + 1/2*log(abs(x^
2 + x*(sqrt(3) + 1) - sqrt(3) + 2))
```

Mupad [B]
time = 4.23, size = 233, normalized size = 3.24

$$\ln\left(x-\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}+\frac{1}{2}\right)-\ln\left(x+\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)\left(2x+\sqrt{3}+1\right)\right)\left(\frac{\frac{\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}+\frac{\sqrt{3}\sqrt{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}}{2}}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+7\right)}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)
```

```
[Out] log(x - ((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(
3^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2)*(2*x + 3^(1/2)
 + 1))*((((3^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3
^(1/2) + 7))^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) + 1/2) - log(x + ((((3
^(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))
^(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)*(2*x + 3^(1/2) + 1))*((((3^
(1/2) - 1)*(3^(1/2) + 7))^(1/2)/2 + (3^(1/2)*((3^(1/2) - 1)*(3^(1/2) + 7))^
(1/2))/2)/((3^(1/2) - 1)*(3^(1/2) + 7)) - 1/2)
```

3.960 $\int \sqrt{x^2+x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{4(x^2+x^3)^{3/2}}{15x^3}+\frac{2(x^2+x^3)^{3/2}}{5x^2}$$

```
[Out] -4/15*(x^3+x^2)^(3/2)/x^3+2/5*(x^3+x^2)^(3/2)/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2027, 2039}

$$\frac{2(x^3+x^2)^{3/2}}{5x^2}-\frac{4(x^3+x^2)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2 + x^3],x]
```

```
[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^2+x^3}\,dx &= \frac{2(x^2+x^3)^{3/2}}{5x^2}-\frac{2}{5}\int \frac{\sqrt{x^2+x^3}}{x}\,dx\\ &= -\frac{4(x^2+x^3)^{3/2}}{15x^3}+\frac{2(x^2+x^3)^{3/2}}{5x^2}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 0.62

$$\frac{2(x^2(1+x))^{3/2}(-2+3x)}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2 + x^3],x]
```

```
[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)
```

Maple [A]
time = 0.23, size = 23, normalized size = 0.62

method	result	size
gosper	$\frac{2(1+x)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
default	$\frac{2(1+x)(3x-2)\sqrt{x^3+x^2}}{15x}$	23
trager	$\frac{2(3x^2+x-2)\sqrt{x^3+x^2}}{15x}$	23
risch	$\frac{2\sqrt{x^2(1+x)}\,(3x^2+x-2)}{15x}$	23
meijerg	$-\frac{-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,(1+x)^{\frac{3}{2}}(2-3x)}{15}}{2\sqrt{\pi}}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(1+x)*(3*x-2)*(x^3+x^2)^(1/2)/x
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.41

$$\frac{2}{15}\left(3\,x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Fricas [A]
time = 0.34, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}\,(3\,x^2+x-2)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

Giac [A]
time = 4.88, size = 48, normalized size = 1.30

$$\frac{2}{15}\left(3\,(x+1)^{\frac{5}{2}}-10\,(x+1)^{\frac{3}{2}}+15\,\sqrt{x+1}\right)\mathrm{sgn}(x)+\frac{2}{3}\left((x+1)^{\frac{3}{2}}-3\,\sqrt{x+1}\right)\mathrm{sgn}(x)+\frac{4}{15}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((x
 + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)
```

Mupad [B]
time = 3.51, size = 22, normalized size = 0.59

$$\frac{2\,\sqrt{x^3+x^2}\,\left(3\,x^2+x-2\right)}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + x^3)^(1/2),x)
```

```
[Out] (2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)
```

3.961 $\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{2x+x^2}\right)$$

[Out] arctan((x^2+2*x)^(1/2))

Rubi [A]
time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {702, 209}

$$\text{ArcTan}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] ArcTan[Sqrt[2*x + x^2]]

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 702

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{2x+x^2}}\,dx &= 4\text{Subst}\left(\int \frac{1}{4+4x^2}\,dx, x, \sqrt{2x+x^2}\right)\\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 41 vs. 2(12) = 24.

time = 0.00, size = 41, normalized size = 3.42

$$-\frac{2\sqrt{x}\,\sqrt{2+x}\,\tan^{-1}\left(1+x-\sqrt{x}\,\sqrt{2+x}\right)}{\sqrt{x(2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 + x]*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/Sqrt[x*(2 + x)
]
```

Maple [A]
time = 0.24, size = 13, normalized size = 1.08

method	result	size
default	$-\arctan\left(\frac{1}{\sqrt{(1+x)^{2}-1}}\right)$	13
trager	$\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)+\sqrt{x^{2}+2x}}{1+x}\right)$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctan(1/((1+x)^2-1)^(1/2))
```

Maxima [A]
time = 0.52, size = 9, normalized size = 0.75

$$-\arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/abs(x + 1))
```

Fricas [A]
time = 0.33, size = 17, normalized size = 1.42

$$2\arctan\left(-x+\sqrt{x^{2}+2\,x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(x+2)}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)
```

Giac [A]
time = 4.32, size = 17, normalized size = 1.42

$$2\,\arctan\left(-x+\sqrt{x^2+2\,x}\,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2+2\,x}\,\,(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x + x^2)^(1/2)*(x + 1)),x)
```

```
[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)
```

3.962 $\int \sqrt{1-\sqrt{x}-x}\,\sqrt{x}\,dx$

Optimal. Leaf size=95

$$\frac{9}{32}(1+2\sqrt{x})\sqrt{1-\sqrt{x}-x}+\frac{5}{12}(1-\sqrt{x}-x)^{3/2}-\frac{1}{2}(1-\sqrt{x}-x)^{3/2}\sqrt{x}+\frac{45}{64}\sin^{-1}\left(\frac{1+2\sqrt{x}}{\sqrt{5}}\right)$$

```
[Out] 45/64*arcsin(1/5*(1+2*x^(1/2))*5^(1/2))+5/12*(1-x-x^(1/2))^(3/2)-1/2*(1-x-x
^(1/2))^(3/2)*x^(1/2)+9/32*(1+2*x^(1/2))*(1-x-x^(1/2))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1371, 756, 654, 626, 633, 222}

$$\frac{45}{64}\text{ArcSin}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)-\frac{1}{2}\sqrt{x}\,(-x-\sqrt{x}+1)^{3/2}+\frac{5}{12}(-x-\sqrt{x}+1)^{3/2}+\frac{9}{32}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/
12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[
5]])/64
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-\sqrt{x}-x}\,\sqrt{x}\,dx &= 2\text{Subst}\left(\int x^2\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{5x}{2}\right)\sqrt{1-x-x^2}\,dx, x, \sqrt{x}\right.\\
&= \frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\sqrt{x}+\frac{9}{8}\text{Subst}\left(\int \sqrt{1-x-x^2}\,d\right.\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}\\
&= \frac{9}{32}\left(1+2\sqrt{x}\right)\sqrt{1-\sqrt{x}-x}+\frac{5}{12}\left(1-\sqrt{x}-x\right)^{3/2}-\frac{1}{2}\left(1-\sqrt{x}-x\right)^{3/2}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 71, normalized size = 0.75

$$\frac{1}{96}\sqrt{1-\sqrt{x}-x}\left(67-34\sqrt{x}+8x+48x^{3/2}\right)+\frac{45}{32}\tan^{-1}\left(\frac{\sqrt{x}}{-1+\sqrt{1-\sqrt{x}-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 + (45*ArcTa
n[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/32
```

Maple [A]
time = 0.01, size = 67, normalized size = 0.71

method	result
derivativedivides	$-\frac{\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{12}-\frac{9\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}\right.}{5}\right.}{64}$
default	$-\frac{\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{5\left(1-x-\sqrt{x}\right)^{\frac{3}{2}}}{12}-\frac{9\left(-2\sqrt{x}-1\right)\sqrt{1-x-\sqrt{x}}}{32}+\frac{45\arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}\right.}{5}\right.}{64}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1-x-x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(1-x-x^(1/2))^(3/2)*x^(1/2)+5/12*(1-x-x^(1/2))^(3/2)-9/32*(-2*x^(1/2)-
1)*(1-x-x^(1/2))^(1/2)+45/64*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)
```

Fricas [A]
time = 0.89, size = 89, normalized size = 0.94

$$\frac{1}{96}\left(2\left(24x-17\right)\sqrt{x}+8x+67\right)\sqrt{-x-\sqrt{x}+1}-\frac{45}{128}\arctan\left(-\frac{\left(8x^{2}-\left(16x^{2}-38x+11\right)\sqrt{x}-9x+3\right)\sqrt{-x-\sqrt{x}+1}}{4\left(4x^{3}-13x^{2}+7x-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 17)*sqrt(x) + 8*x + 67)*sqrt(-x - sqrt(x) + 1) - 45/128*arc
tan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x)
 + 1)/(4*x^3 - 13*x^2 + 7*x - 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{-\sqrt{x}-x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)
```

Giac [**A**]
time = 4.12, size = 51, normalized size = 0.54

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-17\right)\sqrt{x}+67\right)\sqrt{-x-\sqrt{x}+1}+\frac{45}{64}\arcsin\left(\frac{1}{5}\sqrt{5}\left(2\sqrt{x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 17)*sqrt(x) + 67)*sqrt(-x - sqrt(x) +
1) + 45/64*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{1-\sqrt{x}-x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)
```

```
[Out] int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)
```

3.963 $\int \sqrt[3]{1+\sqrt{-3+x}}\,dx$

Optimal. Leaf size=35

$$-\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}$$

```
[Out] -3/2*(1+(-3+x)^(1/2))^(4/3)+6/7*(1+(-3+x)^(1/2))^(7/3)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {253, 196, 45}

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{7/3}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[-3 + x])^(1/3),x]
```

```
[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \sqrt[3]{1+\sqrt{-3+x}}\,dx = \text{Subst}\left(\int \sqrt[3]{1+\sqrt{x}}\,dx, x, -3+x\right)$$
$$= 2\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt{-3+x}\right)$$
$$= 2\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt{-3+x}\right)$$
$$= -\frac{3}{2}\left(1+\sqrt{-3+x}\right)^{4/3}+\frac{6}{7}\left(1+\sqrt{-3+x}\right)^{7/3}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.80

$$\frac{3}{14}\left(1+\sqrt{-3+x}\right)^{4/3}\left(-3+4\sqrt{-3+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[-3 + x])^(1/3),x]
[Out] (3*(1 + Sqrt[-3 + x])^(4/3)*(-3 + 4*Sqrt[-3 + x]))/14
```

Maple [A]
time = 0.06, size = 24, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{3\left(1+\sqrt{x-3}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{x-3}\right)^{\frac{7}{3}}}{7}$	24
default	$-\frac{3\left(1+\sqrt{x-3}\right)^{\frac{4}{3}}}{2}+\frac{6\left(1+\sqrt{x-3}\right)^{\frac{7}{3}}}{7}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(x-3)^(1/2))^(1/3),x,method=_RETURNVERBOSE)
[Out] -3/2*(1+(x-3)^(1/2))^(4/3)+6/7*(1+(x-3)^(1/2))^(7/3)
```

Maxima [A]
time = 0.28, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="maxima")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Fricas [A]
time = 0.36, size = 21, normalized size = 0.60

$$\frac{3}{14}\left(4x+\sqrt{x-3}-15\right)\left(\sqrt{x-3}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/14*(4*x + sqrt(x - 3) - 15)*(sqrt(x - 3) + 1)^(1/3)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(29) = 58$.
time = 0.61, size = 184, normalized size = 5.26

$$\frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}-\frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{9(x-3)^{\frac{5}{2}}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{15(x-3)^{3}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}-\frac{9(x-3)^{2}\sqrt[3]{\sqrt{x-3}+1}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}+\frac{9(x-3)^{2}}{14\left(x-3\right)^{\frac{5}{2}}+14\left(x-3\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)**(1/2))**(1/3),x)
```

```
[Out] 12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)*
*2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x
- 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3
)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x -
3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x -
 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)
```

Giac [A]
time = 3.73, size = 23, normalized size = 0.66

$$\frac{6}{7}\left(\sqrt{x-3}+1\right)^{\frac{7}{3}}-\frac{3}{2}\left(\sqrt{x-3}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(-3+x)^(1/2))^(1/3),x, algorithm="giac")
```

```
[Out] 6/7*(sqrt(x - 3) + 1)^(7/3) - 3/2*(sqrt(x - 3) + 1)^(4/3)
```

Mupad [B]
time = 3.51, size = 16, normalized size = 0.46

$$(x-3)\ {}_2\mathrm{F}_1\left(-\frac{1}{3},2;\ 3;\ -\sqrt{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 3)^(1/2) + 1)^(1/3),x)
```

```
[Out] (x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))
```

3.964 $\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx$

Optimal. Leaf size=37

$$-6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}$$

```
[Out] 2/3*(3+(-1+2*x)^(1/2))^(3/2)-6*(3+(-1+2*x)^(1/2))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {253, 196, 45}

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{3/2}-6\sqrt{\sqrt{2x-1}+3}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
```

```
[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 253

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{3+\sqrt{-1+2x}}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+\sqrt{x}}}\,dx, x, -1+2x\right)$$
$$= \text{Subst}\left(\int \frac{x}{\sqrt{3+x}}\,dx, x, \sqrt{-1+2x}\right)$$
$$= \text{Subst}\left(\int \left(-\frac{3}{\sqrt{3+x}}+\sqrt{3+x}\right)\,dx, x, \sqrt{-1+2x}\right)$$
$$= -6\sqrt{3+\sqrt{-1+2x}}+\frac{2}{3}\left(3+\sqrt{-1+2x}\right)^{3/2}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.81

$$\frac{2}{3}\left(-6+\sqrt{-1+2x}\right)\sqrt{3+\sqrt{-1+2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]],x]
[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3
```

Maple [A]
time = 0.04, size = 28, normalized size = 0.76

method	result	size
derivativedivides	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28
default	$\frac{2\left(3+\sqrt{2x-1}\right)^{\frac{3}{2}}}{3}-6\sqrt{3+\sqrt{2x-1}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+(2*x-1)^(1/2))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)
```

Maxima [A]
time = 0.31, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2\,x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2\,x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Fricas [A]
time = 0.33, size = 22, normalized size = 0.59

$$\frac{2}{3}\sqrt{\sqrt{2x-1}+3}\left(\sqrt{2x-1}-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(31) = 62.
time = 0.61, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^{3}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}-\frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^{2}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}+\frac{108\left(x-\frac{1}{2}\right)^{2}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)
```

```
[Out] -6*sqrt(6)*(x - 1/2)**(5/2)*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x -
 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 36*sqrt(2)*(x - 1/2)**(5/2)/(3*sqr
t(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 4*sqrt(3)*(x - 1/2)**3*sq
rt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x -
1/2)**2) - 36*sqrt(3)*(x - 1/2)**2*sqrt(sqrt(2)*sqrt(x - 1/2) + 3)/(3*sqrt(
6)*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*sqrt(6)
*(x - 1/2)**(5/2) + 9*sqrt(3)*(x - 1/2)**2)
```

Giac [A]
time = 3.67, size = 27, normalized size = 0.73

$$\frac{2}{3}\left(\sqrt{2x-1}+3\right)^{\frac{3}{2}}-6\sqrt{\sqrt{2x-1}+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)
```

Mupad [B]
time = 3.58, size = 24, normalized size = 0.65

$$\frac{\sqrt{3}\,\left(2x-1\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ 3;\ -\frac{\sqrt{2x-1}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)
[Out] (3^(1/2)*(2*x - 1)*hypergeom([1/2, 2], 3, -(2*x - 1)^(1/2)/3))/6
```

3.965 $\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$-\left(\left(2-\sqrt{x}\right)\sqrt{1-x}\right)-\sin^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsin(x^(1/2))-(1-x)^(1/2)*(2-x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1412, 799, 794, 222}

$$-\text{ArcSin}\left(\sqrt{x}\right)-\sqrt{1-x}\left(2-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]),x]
```

```
[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{1+\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1+x}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(2-\sqrt{x}\right)\sqrt{1-x} - \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(2-\sqrt{x}\right)\sqrt{1-x} - \sin^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 40, normalized size = 1.38

$$\left(-2+\sqrt{x}\right)\sqrt{1-x} + 2\tan^{-1}\left(\frac{\sqrt{1-x}}{1+\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]),x]
[Out] (-2 + Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[1 - x]/(1 + Sqrt[x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(23) = 46.
time = 0.24, size = 48, normalized size = 1.66

method	result	size
default	$-\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-x\,(-1+x)}+\arcsin(2x-1)\right)}{2\sqrt{-x\,(-1+x)}}-2\sqrt{1-x}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1+x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(-1+x))^(1/2)+arcsin(2*x-1))/(-x*(-1+x))^(
1/2)-2*(1-x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")
[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)
```

Fricas [A]
time = 0.34, size = 33, normalized size = 1.14

$$\sqrt{x}\,\sqrt{-x+1} - 2\,\sqrt{-x+1} + \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="fricas")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.38, size = 32, normalized size = 1.10

$$i\sqrt{x}\,\sqrt{x-1} - 2i\sqrt{x-1} + i\,\mathrm{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)
[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))
```

Giac [A]
time = 3.14, size = 29, normalized size = 1.00

$$\sqrt{x}\,\sqrt{-x+1} - 2\,\sqrt{-x+1} + \arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="giac")
[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))
```

Mupad [B]
time = 3.90, size = 39, normalized size = 1.34

$$\sqrt{x}\,\sqrt{1-x} - 2\,\sqrt{1-x} - 2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x^(1/2) + 1),x)
[Out] x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) - 1
))
```

3.966 $\int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx$

Optimal. Leaf size=25

$$-\left(\left(2+\sqrt{x}\right)\sqrt{1-x}\right)+\sin^{-1}\left(\sqrt{x}\right)$$

```
[Out] arcsin(x^(1/2))-(1-x)^(1/2)*(2+x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1412, 799, 794, 222}

$$\text{ArcSin}\left(\sqrt{x}\right)-\left(\sqrt{x}+2\right)\sqrt{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]),x]
```

```
[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 799

```
Int[(x_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
Dist[d^m*e^m, Int[x*((a + c*x^2)^(m + p)/(a*e + c*d*x)^m), x], x] /; FreeQ[
{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && ILtQ[m, 0]
 && EqQ[m, -1] &&  !ILtQ[p - 1/2, 0]
```

Rule 1412

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
```

```
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{1-\sqrt{x}}\,dx &= 2\text{Subst}\left(\int \frac{x\sqrt{1-x^2}}{1-x}\,dx, x, \sqrt{x}\right)\\ &= -\left(2\text{Subst}\left(\int \frac{(-1-x)x}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\right)\\ &= -\left(2+\sqrt{x}\right)\sqrt{1-x}+\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \sqrt{x}\right)\\ &= -\left(2+\sqrt{x}\right)\sqrt{1-x}+\sin^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.09, size = 42, normalized size = 1.68

$$\left(-2-\sqrt{x}\right)\sqrt{1-x}+2\tan^{-1}\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]),x]
[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.
time = 0.25, size = 48, normalized size = 1.92

method	result	size
default	$-2\sqrt{1-x}+\frac{\sqrt{1-x}\,\sqrt{x}\,\left(-2\sqrt{-x\left(-1+x\right)}+\arcsin\left(2x-1\right)\right)}{2\sqrt{-x\left(-1+x\right)}}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^(1/2)/(1-x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -2*(1-x)^(1/2)+1/2*(1-x)^(1/2)*x^(1/2)*(-2*(-x*(-1+x))^(1/2)+arcsin(2*x-1))
/(-x*(-1+x))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")
[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)
```

Fricas [A]
time = 0.36, size = 36, normalized size = 1.44

$$-\sqrt{x}\,\sqrt{-x+1}-2\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

Sympy [A]
time = 2.18, size = 87, normalized size = 3.48

$$2\left(\begin{cases}-\sqrt{1-x}+\frac{i\operatorname{acosh}\left(\sqrt{1-x}\right)}{2}-\frac{i(1-x)^{\frac{3}{2}}}{2\sqrt{-x}}+\frac{i\sqrt{1-x}}{2\sqrt{-x}} & \text{for}\ |x-1|>1\\ \frac{\sqrt{x}\,\sqrt{1-x}}{2}-\sqrt{1-x}+\frac{\operatorname{asin}\left(\sqrt{1-x}\right)}{2} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)
[Out] 2*Piecewise((-sqrt(1 - x) + I*acosh(sqrt(1 - x))/2 - I*(1 - x)**(3/2)/(2*sq
rt(-x)) + I*sqrt(1 - x)/(2*sqrt(-x)), Abs(x - 1) > 1), (sqrt(x)*sqrt(1 - x)
/2 - sqrt(1 - x) + asin(sqrt(1 - x))/2, True))
```

Giac [A]
time = 2.84, size = 32, normalized size = 1.28

$$-\sqrt{x}\,\sqrt{-x+1}-2\,\sqrt{-x+1}-\arcsin\left(\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))
```

Mupad [B]
time = 3.65, size = 40, normalized size = 1.60

$$2\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-2\,\sqrt{1-x}-\sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)
```

```
[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1/
2)
```

3.967 $\int \frac{x}{x-\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}$$

```
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2131, 30, 267}

$$-\frac{x^3}{3}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + x^2]),x]
```

```
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2131

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> Dist[-b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1+x^2}}\,dx &= -\int x^2\,dx-\int x\sqrt{1+x^2}\,dx\\ &= -\frac{x^3}{3}-\frac{1}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 21, normalized size = 1.00

$$-\frac{x^3}{3}-\frac{1}{3}(1+x^2)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + x^2]),x]
[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3
```

Maple [A]
time = 0.22, size = 16, normalized size = 0.76

method	result	size
default	$-\frac{x^3}{3}-\frac{(x^2+1)^{\frac{3}{2}}}{3}$	16
trager	$-\frac{x^3}{3}+\left(-\frac{x^2}{3}-\frac{1}{3}\right)\sqrt{x^2+1}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(x^2 + 1)), x)
```

Fricas [A]
time = 0.32, size = 15, normalized size = 0.71

$$-\frac{1}{3}\,x^3-\frac{1}{3}\,(x^2+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(15) = 30$.
time = 0.17, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x - 3\sqrt{x^2 + 1}} - \frac{x\sqrt{x^2 + 1}}{3x - 3\sqrt{x^2 + 1}} + \frac{1}{3x - 3\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))
 + 1/(3*x - 3*sqrt(x**2 + 1))
```

Giac [A]
time = 4.88, size = 15, normalized size = 0.71

$$-\frac{1}{3}x^3 - \frac{1}{3}\left(x^2 + 1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)
```

Mupad [B]
time = 0.04, size = 22, normalized size = 1.05

$$-\sqrt{x^2 + 1}\left(\frac{x^2}{3} + \frac{1}{3}\right) - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (x^2 + 1)^(1/2)),x)
```

```
[Out] - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3
```

3.968 $\int \frac{x}{x-\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=65

$$\frac{x}{2}+\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}}-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{2\sqrt{2}}$$

```
[Out] 1/2*x-1/4*arctanh(x*2^(1/2))*2^(1/2)-1/4*arctanh(2^(1/2)*(-x^2+1)^(1/2))*2^
(1/2)+1/2*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2132, 327, 212, 455, 52, 65, 213}

$$\frac{\sqrt{1-x^2}}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{2\sqrt{2}}+\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sq
rt[1 - x^2]]/(2*Sqrt[2])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2132

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Dist[-d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-\sqrt{1-x^2}}\,dx &= -\int \frac{x^2}{1-2x^2}\,dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2}\,dx \\ &= \frac{x}{2} - \frac{1}{2}\int \frac{1}{1-2x^2}\,dx - \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{1-2x}\,dx, x, x^2\right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{(1-2x)\sqrt{1-x}}\,dx, x, x^2\right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{1-x^2}\right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\,x\right)}{2\sqrt{2}} - \frac{\tanh^{-1}\left(\sqrt{2}\,\sqrt{1-x^2}\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 49, normalized size = 0.75

$$\frac{1}{2}\left(x+\sqrt{1-x^2}+\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,x}{-1-x+\sqrt{1-x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 - x^2]),x]
```

```
[Out] (x + Sqrt[1 - x^2] + Sqrt[2]*ArcTanh[(Sqrt[2]*x)/(-1 - x + Sqrt[1 - x^2])])
/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(45) = 90$.
time = 0.14, size = 175, normalized size = 2.69

method	result
trager	$\frac{x}{2}+\frac{\sqrt{-x^2+1}}{2}-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-2\right)+2\sqrt{-x^2+1}}{\mathrm{RootOf}\left(\_Z^2-2\right)x-1}\right)}{4}$
default	$\frac{x}{2}-\frac{\mathrm{arctanh}\left(\sqrt{2}\,x\right)\sqrt{2}}{4}+\frac{\sqrt{-4\left(x+\frac{\sqrt{2}}{2}\right)^2+4\left(x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{8}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{\quad}{\sqrt{-4\left(x\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x-1/4*arctanh(2^(1/2)*x)*2^(1/2)+1/8*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(
1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1
/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*(-4*(x-1/
2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1-(x-1
/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2
)+2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.
time = 0.33, size = 97, normalized size = 1.49

$$\frac{1}{8}\sqrt{2}\log\left(\frac{6x^2-2\sqrt{2}(2x^2-3)+2\sqrt{-x^2+1}\left(3\sqrt{2}-4\right)-9}{2x^2-1}\right)+\frac{1}{8}\sqrt{2}\log\left(\frac{2x^2-2\sqrt{2}x+1}{2x^2-1}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2
) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2
 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x-\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(45) = 90.

time = 5.39, size = 105, normalized size = 1.62

$$\frac{1}{8}\sqrt{2}\log\left(\frac{\left|4x-2\sqrt{2}\right|}{\left|4x+2\sqrt{2}\right|}\right)-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}{\left|4\sqrt{2}+\frac{2\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-6\right|}\right)+\frac{1}{2}x+\frac{1}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*lo
g(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqr
t(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)
```

Mupad [B]

time = 3.50, size = 127, normalized size = 1.95

$$\frac{x}{2}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{8}-\frac{\sqrt{2}\,\ln\left(\frac{\sqrt{2}\,\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{8}+\frac{\sqrt{2}\,\ln\left(x-\frac{\sqrt{2}}{2}\right)}{8}-\frac{\sqrt{2}\,\ln\left(x+\frac{\sqrt{2}}{2}\right)}{8}+\frac{\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (1 - x^2)^(1/2)),x)
```

```
[Out] x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x
 - 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2
)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1/2)
*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

3.969 $\int \frac{x}{x-\sqrt{1+2x^2}} \, dx$

Optimal. Leaf size=31

$$-x-\sqrt{1+2x^2}+\tan^{-1}(x)+\tan^{-1}\left(\sqrt{1+2x^2}\right)$$

```
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2132, 327, 209, 455, 52, 65}

$$\text{ArcTan}\left(\sqrt{2x^2+1}\right)+\text{ArcTan}(x)-\sqrt{2x^2+1}-x$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 2132

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] :> Dist[-d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{x-\sqrt{1+2x^2}}\,dx &= -\int \frac{x^2}{1+x^2}\,dx-\int \frac{x\sqrt{1+2x^2}}{1+x^2}\,dx\\
&= -x-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1+2x}}{1+x}\,dx,x,x^2\right)+\int \frac{1}{1+x^2}\,dx\\
&= -x-\sqrt{1+2x^2}+\tan^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+2x}}\,dx,x,x^2\right)\\
&= -x-\sqrt{1+2x^2}+\tan^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^2}{2}}\,dx,x,\sqrt{1+2x^2}\right)\\
&= -x-\sqrt{1+2x^2}+\tan^{-1}(x)+\tan^{-1}\left(\sqrt{1+2x^2}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 50, normalized size = 1.61

$$-x-\sqrt{1+2x^2}+2\tan^{-1}\left(\left(2+\sqrt{2}\right)x-\left(1+\sqrt{2}\right)\sqrt{1+2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 + 2*x^2]),x]
```

```
[Out] -x - Sqrt[1 + 2*x^2] + 2*ArcTan[(2 + Sqrt[2])*x - (1 + Sqrt[2])*Sqrt[1 + 2*
x^2]]
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.90

method	result	size
default	$-x+\arctan(x)+\arctan\left(\sqrt{2x^2+1}\right)-\sqrt{2x^2+1}$	28
trager	$-x-\sqrt{2x^2+1}+\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{2x^2+1}+\mathrm{RootOf}\left(\_Z^2+1\right)}{x\,\mathrm{RootOf}\left(\_Z^2+1\right)+1}\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x-(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
[Out] -x+arctan(x)+arctan((2*x^2+1)^(1/2))-(2*x^2+1)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")
[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)
```

Fricas [A]
time = 0.34, size = 41, normalized size = 1.32

$$-x-\sqrt{2\,x^2+1}+\arctan(x)-\arctan\left(-\frac{x^2-\sqrt{2\,x^2+1}+1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")
[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{x-\sqrt{2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.
time = 3.70, size = 63, normalized size = 2.03

$$-\frac{1}{2}\pi - x - \sqrt{2x^2+1} + \arctan(x) + \arctan\left(-\frac{\left(\sqrt{2}x - \sqrt{2x^2+1}\right)^2 + 1}{2\left(\sqrt{2}x - \sqrt{2x^2+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*pi - x - sqrt(2*x^2 + 1) + arctan(x) + arctan(-1/2*((sqrt(2)*x - sqrt(
2*x^2 + 1))^2 + 1)/(sqrt(2)*x - sqrt(2*x^2 + 1)))
```

Mupad [B]
time = 0.19, size = 64, normalized size = 2.06

$$-x - \sqrt{2}\sqrt{x^2+\frac{1}{2}} - \ln(x-\mathrm{i})\,1\mathrm{i} + \frac{\ln\left(x - \frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2} + \frac{1}{2}\mathrm{i}\right)1\mathrm{i}}{2} + \frac{\ln\left(x + \frac{\sqrt{2}\sqrt{x^2+\frac{1}{2}}}{2} - \frac{1}{2}\mathrm{i}\right)1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - (2*x^2 + 1)^(1/2)),x)
```

```
[Out] (log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)*1i)/2 - log(x - 1i)*1i - x +
 (log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)*1i)/2 - 2^(1/2)*(x^2 + 1/2)
^(1/2)
```

3.970 $\int \sqrt{x} \sqrt{\sqrt{x}+x}\,dx$

Optimal. Leaf size=82

$$\frac{5}{32}(1+2\sqrt{x})\sqrt{\sqrt{x}+x}-\frac{5}{12}(\sqrt{x}+x)^{3/2}+\frac{1}{2}\sqrt{x}(\sqrt{x}+x)^{3/2}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)$$

```
[Out] -5/32*arctanh(x^(1/2)/(x+x^(1/2))^(1/2))-5/12*(x+x^(1/2))^(3/2)+1/2*x^(1/2)
*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2043, 684, 654, 626, 634, 212}

$$\frac{1}{2}\sqrt{x}\left(x+\sqrt{x}\right)^{3/2}-\frac{5}{12}\left(x+\sqrt{x}\right)^{3/2}+\frac{5}{32}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}}-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sq
rt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 684

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1))), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2043

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,\sqrt{\sqrt{x}+x}\,dx &= 2\text{Subst}\left(\int x^2\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{4}\text{Subst}\left(\int x\sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= -\frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} + \frac{5}{8}\text{Subst}\left(\int \sqrt{x+x^2}\,dx, x, \sqrt{x}\right)\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{64}\text{Subst}\Bigg(\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\text{Subst}\Bigg(\\
&= \frac{5}{32}\left(1+2\sqrt{x}\right)\sqrt{\sqrt{x}+x} - \frac{5}{12}\left(\sqrt{x}+x\right)^{3/2} + \frac{1}{2}\sqrt{x}\left(\sqrt{x}+x\right)^{3/2} - \frac{5}{32}\tanh^{-1}
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 57, normalized size = 0.70

$$\frac{1}{96}\sqrt{\sqrt{x}+x}\left(15-10\sqrt{x}+8x+48x^{3/2}\right)-\frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}+x}}{\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x],x]
```

```
[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*ArcTanh[Sq
rt[Sqrt[x] + x]/Sqrt[x]])/32
```

Maple [A]
time = 0.22, size = 54, normalized size = 0.66

method	result	siz
meijerg	$-\frac{-\frac{\sqrt{\pi}\,x^{\frac{1}{4}}\left(336x^{\frac{3}{2}}+56x-70\sqrt{x}+105\right)\sqrt{1+\sqrt{x}}}{672}+\frac{5\sqrt{\pi}\,\operatorname{arcsinh}\left(x^{\frac{1}{4}}\right)}{32}}{\sqrt{\pi}}$	46
derivativedivides	$\frac{\sqrt{x}\,\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{2}-\frac{5\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{12}+\frac{5\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{32}-\frac{5\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54
default	$\frac{\sqrt{x}\,\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{2}-\frac{5\left(x+\sqrt{x}\right)^{\frac{3}{2}}}{12}+\frac{5\left(1+2\sqrt{x}\right)\sqrt{x+\sqrt{x}}}{32}-\frac{5\ln\left(\frac{1}{2}+\sqrt{x}+\sqrt{x+\sqrt{x}}\right)}{64}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x+x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^(1/2)*(x+x^(1/2))^(3/2)-5/12*(x+x^(1/2))^(3/2)+5/32*(1+2*x^(1/2))*(x+
x^(1/2))^(1/2)-5/64*ln(1/2+x^(1/2)+(x+x^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x))*sqrt(x), x)
```

Fricas [A]
time = 0.57, size = 54, normalized size = 0.66

$$\frac{1}{96}\left(2\left(24x-5\right)\sqrt{x}+8x+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{128}\log\left(4\sqrt{x+\sqrt{x}}\left(2\sqrt{x}+1\right)-8x-8\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(2*(24*x - 5)*sqrt(x) + 8*x + 15)*sqrt(x + sqrt(x)) + 5/128*log(4*sqrt
(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{\sqrt{x}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) + x), x)
```

Giac [A]
time = 5.44, size = 50, normalized size = 0.61

$$\frac{1}{96}\left(2\left(4\sqrt{x}\left(6\sqrt{x}+1\right)-5\right)\sqrt{x}+15\right)\sqrt{x+\sqrt{x}}+\frac{5}{64}\log\left(-2\sqrt{x+\sqrt{x}}+2\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*(4*sqrt(x)*(6*sqrt(x) + 1) - 5)*sqrt(x) + 15)*sqrt(x + sqrt(x)) + 5
/64*log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{x+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x + x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)*(x + x^(1/2))^(1/2), x)
```

3.971 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=74

$$-3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\,\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

```
[Out] -3*x^(1/3)+6/5*x^(5/6)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))-2*arctan(1/3*(
1-2*x^(1/6))*3^(1/2))*3^(1/2)+2*x^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {1607, 1901, 1888, 31, 648, 632, 210, 642}

$$-2\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)+\frac{6x^{5/6}}{5}+2\sqrt{x}-3\sqrt[3]{x}-4\log\left(\sqrt[6]{x}+1\right)-\log\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}}\,dx = 6\text{Subst}\left(\int \frac{x^5+x^7}{1+x^3}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \frac{x^5(1+x^2)}{1+x^3}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\text{Subst}\left(\int \left(-x+x^2+x^4+\frac{(1-x)x}{1+x^3}\right)\,dx, x, \sqrt[6]{x}\right)$$
$$= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+6\text{Subst}\left(\int \frac{(1-x)x}{1+x^3}\,dx, x, \sqrt[6]{x}\right)$$
$$= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}+2\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-4\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[6]{\right.$$
$$= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)+3\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[6]{x}\right)-\text{Subst}\left(\right.$$
$$= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)-6\text{Subst}\left(\int \frac{1}{-3-x^2}\right.$$
$$= -3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\right.$$

Mathematica [A]
time = 0.05, size = 74, normalized size = 1.00

$$-3\sqrt[3]{x}+2\sqrt{x}+\frac{6x^{5/6}}{5}-2\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}}\right)-4\log\left(1+\sqrt[6]{x}\right)-\log\left(1-\sqrt[6]{x}+\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]),x]
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

Maple [A]
time = 0.22, size = 56, normalized size = 0.76

method	result
derivativedivides	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$
default	$\frac{6x^{\frac{5}{6}}}{5}+2\sqrt{x}-3x^{\frac{1}{3}}-4\ln\left(1+x^{\frac{1}{6}}\right)-\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)+2\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{6}}-1\right)\sqrt{3}}{3}\right)$

meijerg	$2\sqrt{x} - 2\ln\left(1+\sqrt{x}\right) - \frac{3x^{\frac{1}{3}}\left(-8\sqrt{x}+20\right)}{20} + 2x^{\frac{1}{3}}\left(-\frac{\ln\left(1+x^{\frac{1}{6}}\right)}{x^{\frac{1}{3}}} + \frac{\ln\left(1-x^{\frac{1}{6}}+x^{\frac{1}{3}}\right)}{2x^{\frac{1}{3}}} + \frac{\sqrt{3}\ \mathrm{arcta}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/3))/(1+x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-4*ln(1+x^(1/6))-ln(1-x^(1/6)+x^(1/3))+2*3^(
1/2)*arctan(1/3*(2*x^(1/6)-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Fricas [A]
time = 0.35, size = 57, normalized size = 0.77

$$2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}}-\frac{1}{3}\sqrt{3}\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(
x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.11, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)}-\frac{8\sqrt[3]{x}\,\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)}+2\sqrt{x}-2\log\left(\sqrt{x}+1\right)-\frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16\log\left(-\sqrt[6]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}-\frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[6]{x}\,e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)
```

```
[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3)
+ 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_pola
```

```
r(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi
) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_pola
r(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))
```

Giac [A]
time = 3.80, size = 55, normalized size = 0.74

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}}-1\right)\right)+\frac{6}{5}x^{\frac{5}{6}}+2\sqrt{x}-3x^{\frac{1}{3}}-\log\left(x^{\frac{1}{3}}-x^{\frac{1}{6}}+1\right)-4\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3
*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)
```

Mupad [B]
time = 3.41, size = 95, normalized size = 1.28

$$2\sqrt{x}+\ln\left(\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(27+\sqrt{3}\,9\mathrm{i}\right)+36x^{1/6}+36\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)-\ln\left(\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(-27+\sqrt{3}\,9\mathrm{i}\right)+36x^{1/6}+36\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)-4\ln\left(36x^{1/6}+36\right)-3x^{1/3}+\frac{6x^{5/6}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/2) + 1),x)
[Out] log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1)
- 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^(1
/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5
```

3.972 $\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx$

Optimal. Leaf size=115

$$12\sqrt[12]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}-2\sqrt{x}+\frac{12x^{7/12}}{7}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{13/12}}{13}+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\sqrt[12]{x}\right.$$

```
[Out] 12*x^(1/12)+4*x^(1/4)-3*x^(1/3)+12/7*x^(7/12)+4/3*x^(3/4)-6/5*x^(5/6)+12/13
*x^(13/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))+4*arctan(1/3*(1-2*x^(1
/12))*3^(1/2))*3^(1/2)-2*x^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1607, 1850, 1901, 1888, 31, 648, 632, 210, 642}

$$4\sqrt{3}\,\text{ArcTan}\left(\frac{1-2\sqrt[12]{x}}{\sqrt{3}}\right)+\frac{12x^{13/12}}{13}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}+\frac{12x^{7/12}}{7}-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}+12\sqrt[12]{x}-8\log\left(\sqrt[12]{x}+1\right)-2\log\left(\sqrt[6]{x}-\sqrt[12]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3
/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/1
2))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1850

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Wi
th[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p
 + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*
(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[Pqq*(c*x)^(m + q
 - n + 1)*((a + b*x^n)^(p + 1)/(b*c^(q - n + 1)*(m + q + n*p + 1))), x]] /;
 NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q
 + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1888

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[q*((A - B*q
+ C*q^2)/(3*a)), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1901

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}}\,dx &= 12\text{Subst}\left(\int \frac{x^{11}+x^{15}}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= 12\text{Subst}\left(\int \frac{x^{11}(1+x^4)}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= \frac{12x^{13/12}}{13} + \frac{12}{13}\text{Subst}\left(\int \frac{(13-13x)x^{11}}{1+x^3}\,dx, x, \sqrt[12]{x}\right) \\
&= \frac{12x^{13/12}}{13} + \frac{12}{13}\text{Subst}\left(\int \left(13+13x^2-13x^3-13x^5+13x^6+13x^8-13x^9-\frac{13(1+x^2)}{1+x^3}\right)\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 12\text{Subst}\left(\int \frac{1}{1\ldots}\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 4\text{Subst}\left(\int \frac{\ldots}{1-\ldots}\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\ldots\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8\log\left(1+\sqrt[12]{x}\ldots\right. \\
&= 12\sqrt[12]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} + 4\sqrt{3}\tan^{-1}\left(\frac{1}{\ldots}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 115, normalized size = 1.00

$$
\frac{16380\sqrt[12]{x}+5460\sqrt[4]{x}-4095\sqrt[3]{x}-2730\sqrt{x}+2340x^{7/12}+1820x^{3/4}-1638x^{5/6}+1260x^{13/12}}{1365}+4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[12]{x}}{\sqrt{3}}\right)-8\log\left(1+\sqrt[12]{x}\right)-2\log\left(1-\sqrt[12]{x}+\sqrt[6]{x}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)),x]
```

```
[Out] (16380*x^(1/12) + 5460*x^(1/4) - 4095*x^(1/3) - 2730*Sqrt[x] + 2340*x^(7/12
) + 1820*x^(3/4) - 1638*x^(5/6) + 1260*x^(13/12))/1365 + 4*Sqrt[3]*ArcTan[1
/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12)
 + x^(1/6)]
```

Maple [A]
time = 0.22, size = 81, normalized size = 0.70

method	result
derivativedivides	$\frac{12x^{\frac{13}{12}}}{13}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-8\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-\ldots\right.$

default	$\frac{12x^{\frac{13}{12}}}{13}-\frac{6x^{\frac{5}{6}}}{5}+\frac{4x^{\frac{3}{4}}}{3}+\frac{12x^{\frac{7}{12}}}{7}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-8\ln\left(1+x^{\frac{1}{12}}\right)-2\ln\left(1-x\right.$
meijerg	$\frac{x^{\frac{1}{4}}\left(4\sqrt{x}-6x^{\frac{1}{4}}+12\right)}{3}-4\ln\left(1+x^{\frac{1}{4}}\right)+\frac{3x^{\frac{1}{12}}\left(560x-728x^{\frac{3}{4}}+1040\sqrt{x}-1820x^{\frac{1}{4}}+7280\right)}{1820}-4x^{\frac{1}{12}}\left(\frac{\ln(1+}{x^{\frac{1}{}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/3))/(1+x^(1/4)),x,method=_RETURNVERBOSE)
```

```
[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4
*x^(1/4)+12*x^(1/12)-8*ln(1+x^(1/12))-2*ln(1-x^(1/12)+x^(1/6))-4*3^(1/2)*ar
ctan(1/3*(2*x^(1/12)-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Fricas [A]
time = 0.35, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}}-\frac{1}{3}\sqrt{3}\right)+\frac{12}{13}(x+13)x^{\frac{1}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1
/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) +
4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.43, size = 221, normalized size = 1.92

$$\frac{64x^{\frac{13}{12}}\Gamma\left(\frac{16}{3}\right)}{13\Gamma\left(\frac{19}{3}\right)}+\frac{64x^{\frac{7}{12}}\Gamma\left(\frac{16}{3}\right)}{7\Gamma\left(\frac{19}{3}\right)}+\frac{64\sqrt[12]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-\frac{32x^{\frac{5}{6}}\Gamma\left(\frac{16}{3}\right)}{5\Gamma\left(\frac{19}{3}\right)}+\frac{4x^{\frac{3}{4}}}{3}+4\sqrt[4]{x}-\frac{16\sqrt[3]{x}\,\Gamma\left(\frac{16}{3}\right)}{\Gamma\left(\frac{19}{3}\right)}-2\sqrt{x}-4\log\left(\sqrt[4]{x}+1\right)+\frac{64e^{-\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}\,e^{\frac{i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}-\frac{64\log\left(-\sqrt[12]{x}\,e^{i\pi}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}+\frac{64e^{\frac{i\pi}{3}}\log\left(-\sqrt[12]{x}\,e^{\frac{5i\pi}{3}}+1\right)\Gamma\left(\frac{16}{3}\right)}{3\Gamma\left(\frac{19}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/3))/(1+x**(1/4)),x)
```

```
[Out] 64*x**(13/12)*gamma(16/3)/(13*gamma(19/3)) + 64*x**(7/12)*gamma(16/3)/(7*ga
mma(19/3)) + 64*x**(1/12)*gamma(16/3)/gamma(19/3) - 32*x**(5/6)*gamma(16/3)
/(5*gamma(19/3)) + 4*x**(3/4)/3 + 4*x**(1/4) - 16*x**(1/3)*gamma(16/3)/gamm
a(19/3) - 2*sqrt(x) - 4*log(x**(1/4) + 1) + 64*exp(-I*pi/3)*log(-x**(1/12)*
exp_polar(I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3)) - 64*log(-x**(1/12)*exp_
polar(I*pi) + 1)*gamma(16/3)/(3*gamma(19/3)) + 64*exp(I*pi/3)*log(-x**(1/12
)*exp_polar(5*I*pi/3) + 1)*gamma(16/3)/(3*gamma(19/3))
```

Giac [A]
time = 3.18, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}}-1\right)\right)+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{4}{3}x^{\frac{3}{4}}+\frac{12}{7}x^{\frac{7}{12}}-2\sqrt{x}-3x^{\frac{1}{3}}+4x^{\frac{1}{4}}+12x^{\frac{1}{12}}-2\log\left(x^{\frac{1}{6}}-x^{\frac{1}{12}}+1\right)-8\log\left(x^{\frac{1}{12}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/3))/(1+x^(1/4)),x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12
*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)
```

Mupad [B]
time = 0.09, size = 130, normalized size = 1.13

$$4x^{1/4}+\ln\left(\left(-2+\sqrt{3}\,2\mathrm{i}\right)\left(54-36x^{1/12}+\sqrt{3}\,18\mathrm{i}\right)-144x^{1/12}+144\right)\left(-2+\sqrt{3}\,2\mathrm{i}\right)-\ln\left(\left(2+\sqrt{3}\,2\mathrm{i}\right)\left(36x^{1/12}-54+\sqrt{3}\,18\mathrm{i}\right)-144x^{1/12}+144\right)\left(2+\sqrt{3}\,2\mathrm{i}\right)-2\sqrt{x}-3x^{1/3}-8\ln\left(144x^{1/12}+144\right)+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+12x^{1/12}+\frac{12x^{7/12}}{7}+\frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/3) + 1)/(x^(1/4) + 1),x)
```

```
[Out] log((3^(1/2)*2i - 2)*(3^(1/2)*18i - 36*x^(1/12) + 54) - 144*x^(1/12) + 144)
*(3^(1/2)*2i - 2) - 8*log(144*x^(1/12) + 144) - log((3^(1/2)*2i + 2)*(3^(1/
2)*18i + 36*x^(1/12) - 54) - 144*x^(1/12) + 144)*(3^(1/2)*2i + 2) - 2*x^(1/
2) - 3*x^(1/3) + 4*x^(1/4) + (4*x^(3/4))/3 - (6*x^(5/6))/5 + 12*x^(1/12) +
(12*x^(7/12))/7 + (12*x^(13/12))/13
```

3.973 $\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=4

$$x+\sin^{-1}(x)$$

```
[Out] x+arcsin(x)
```

Rubi [A]
time = 0.03, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2187, 8, 222}

$$\text{ArcSin}(x)+x$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
[Out] x + ArcSin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2187

```
Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_
Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/
((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && EqQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}}\,dx &= -\int -1\,dx+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= x+\sin^{-1}(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 22 vs. 2(4) = 8.
time = 0.10, size = 22, normalized size = 5.50

$$x+2\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] x + 2*ArcTan[x/(-1 + Sqrt[1 - x^2])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(4) = 8$.
time = 0.08, size = 51, normalized size = 12.75

method	result
trager	$x + \mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)$
default	$x + \frac{\ln(-1+x)}{2} - \frac{\ln(1+x)}{2} + \operatorname{arctanh}(x) - \frac{\sqrt{-\left(-1+x\right)^2+2-2x}}{2} + \arcsin(x) + \frac{\sqrt{-\left(1+x\right)^2+}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/2*ln(-1+x)-1/2*ln(1+x)+arctanh(x)-1/2*(-(-1+x)^2+2-2*x)^(1/2)+arcsin(x)
+1/2*(-(1+x)^2+2+2*x)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(4) = 8$.
time = 0.33, size = 20, normalized size = 5.00

$$x - 2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2+\sqrt{1-x^2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)
```

Giac [A]
time = 2.95, size = 4, normalized size = 1.00

$$x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x + arcsin(x)
```

Mupad [B]
time = 0.03, size = 4, normalized size = 1.00

$$x + \mathrm{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)
```

```
[Out] x + asin(x)
```

3.974 $\int \sqrt{\frac{1+x}{x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{1+\frac{1}{x}}\,x+\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[Out] arctanh((1+1/x)^(1/2))+x*(1+1/x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1997, 248, 43, 65, 213}

$$\sqrt{\frac{1}{x}+1}\,x+\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x}}\,dx &= \int \sqrt{1+\frac{1}{x}}\,dx\\ &= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx,x,\frac{1}{x}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x-\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\sqrt{1+\frac{1}{x}}\right)\\ &= \sqrt{1+\frac{1}{x}}\,x+\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 1.00

$$\sqrt{1+\frac{1}{x}}\,x+\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x],x]
```

```
[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.06, size = 41, normalized size = 1.86

method	result	size

trager	$\sqrt{-\frac{-1-x}{x}}\, x+\frac{\ln\left(2\sqrt{-\frac{-1-x}{x}}\, x+2x+1\right)}{2}$	39
default	$\frac{\sqrt{\frac{1+x}{x}}\, x\left(2\sqrt{x^{2}+x}+\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\right)}{2\sqrt{x\left(1+x\right)}}$	41
risch	$x\sqrt{\frac{1+x}{x}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{\frac{1+x}{x}}\,\sqrt{x\left(1+x\right)}}{2x+2}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/(x*(1+x))^(
1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.
time = 0.28, size = 50, normalized size = 2.27

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sq
rt((x + 1)/x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.33, size = 40, normalized size = 1.82

$$x\sqrt{\frac{x+1}{x}}+\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\frac{1}{2}\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x)
- 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x+1}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/x), x)
```

Giac [A]
time = 3.31, size = 31, normalized size = 1.41

$$-\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\mathrm{sgn}(x)+\sqrt{x^2+x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)
```

Mupad [B]
time = 3.39, size = 18, normalized size = 0.82

$$\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)+x\,\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2),x)
```

```
[Out] atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)
```

3.975 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{-1+\frac{1}{x}}\,x-\tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)$$

```
[Out] -arctan((-1+1/x)^(1/2))+x*(-1+1/x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1997, 248, 43, 65, 209}

$$\sqrt{\frac{1}{x}-1}\,x-\text{ArcTan}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}\,x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\sqrt{-1+\frac{1}{x}}\,x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Maple [A]
time = 0.10, size = 40, normalized size = 1.67

method	result	size
default	$\frac{\sqrt{-\frac{-1+x}{x}}\,x\left(2\sqrt{-x^2+x}+\arcsin(2x-1)\right)}{2\sqrt{-x\,(-1+x)}}$	40

risch	$\sqrt{-\frac{-1+x}{x}}\, x-\frac{\arcsin(2x-1)\sqrt{-\frac{-1+x}{x}}\,\sqrt{-x\,(-1+x)}}{2(-1+x)}$	45
trager	$\sqrt{-\frac{-1+x}{x}}\, x-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\sqrt{-\frac{-1+x}{x}}\, x+2x\operatorname{RootOf}\left(\_Z^2+1\right)-\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(-(-1+x)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(-1+x))^(1/2)
```

Maxima [A]
time = 0.49, size = 37, normalized size = 1.54

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.08

$$x\sqrt{-\frac{x-1}{x}}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
[Out] Integral(sqrt((1 - x)/x), x)
```

Giac [**A**]
time = 4.68, size = 28, normalized size = 1.17

$$\frac{1}{4}\,\pi\mathrm{sgn}(x) + \frac{1}{2}\,\arcsin\left(2\,x - 1\right)\mathrm{sgn}(x) + \sqrt{-x^2 + x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

Mupad [**B**]
time = 0.04, size = 20, normalized size = 0.83

$$x\,\sqrt{\frac{1}{x} - 1} - \mathrm{atan}\left(\sqrt{\frac{1}{x} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x - 1)/x)^(1/2),x)
```

```
[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))
```

3.976 $\int \sqrt{\frac{-1+x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{-1+x}\,\sqrt{x}\;-\sinh^{-1}\left(\sqrt{-1+x}\right)$$

```
[Out] -arcsinh((-1+x)^(1/2))+(-1+x)^(1/2)*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1997, 248, 43, 65, 212}

$$\sqrt{\frac{x-1}{x}}\,x-\tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-1+x}{x}}\,dx &= \int \sqrt{1-\frac{1}{x}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x - \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\frac{-1+x}{x}}\right) \\
&= \sqrt{\frac{-1+x}{x}}\,x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 34, normalized size = 1.42

$$\sqrt{-1+x}\,\sqrt{x} - 2\tanh^{-1}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-1 + x)/x],x]
```

```
[Out] Sqrt[-1 + x]*Sqrt[x] - 2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.
time = 0.06, size = 45, normalized size = 1.88

method	result	size
trager	$\sqrt{-\frac{1-x}{x}}\,x-\frac{\ln\left(2\sqrt{-\frac{1-x}{x}}\,x+2x-1\right)}{2}$	39
default	$-\frac{\sqrt{\frac{-1+x}{x}}\,x\left(-2\sqrt{x^2-x}+\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)\right)}{2\sqrt{x\,(-1+x)}}$	45

risch	$x\sqrt{\frac{-1+x}{x}}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)\sqrt{\frac{-1+x}{x}}\,\sqrt{x\left(-1+x\right)}}{2(-1+x)}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((-1+x)/x)^(1/2)*x*(-2*(x^2-x)^(1/2)+ln(x-1/2+(x^2-x)^(1/2)))/(x*(-1+x
))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(18) = 36$.
time = 0.28, size = 51, normalized size = 2.12

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(s
qrt((x - 1)/x) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(18) = 36$.
time = 0.38, size = 40, normalized size = 1.67

$$x\sqrt{\frac{x-1}{x}}-\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x-1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x)
- 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x-1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((x - 1)/x), x)
```

Giac [A]
time = 5.63, size = 35, normalized size = 1.46

$$\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)\mathrm{sgn}(x)+\sqrt{x^2-x}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)/x)^(1/2),x, algorithm="giac")
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 1.00

$$x\,\sqrt{1-\frac{1}{x}}-\mathrm{atanh}\left(\sqrt{1-\frac{1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x - 1)/x)^(1/2),x)
[Out] x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))
```

3.977 $\int \frac{\sqrt{\frac{1+x}{x}}}{x}\,dx$

Optimal. Leaf size=24

$$-2\sqrt{1+\frac{1}{x}}+2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)$$

```
[Out] 2*arctanh((1+1/x)^(1/2))-2*(1+1/x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1998, 272, 52, 65, 213}

$$2\tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right)-2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x]/x,x]
```

```
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1998

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] &&  !BinomialMatchQ[u, x
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{\frac{1+x}{x}}}{x} \, dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{x} \, dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x} \, dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} \, dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - 2\text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= -2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 24, normalized size = 1.00

$$-2\sqrt{1+\frac{1}{x}} + 2\tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x]/x,x]
[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(20) = 40$.
time = 0.06, size = 60, normalized size = 2.50

method	result	size
trager	$-2\sqrt{-\frac{-1-x}{x}}-\ln\left(2\sqrt{-\frac{-1-x}{x}}\, x-2x-1\right)$	39
risch	$-2\sqrt{\frac{1+x}{x}}+\frac{\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)\sqrt{\frac{1+x}{x}}\, \sqrt{x\left(1+x\right)}}{1+x}$	46
default	$-\frac{\sqrt{\frac{1+x}{x}}\, \left(2\left(x^{2}+x\right)^{\frac{3}{2}}-2\sqrt{x^{2}+x}\, x^{2}-\ln\left(x+\frac{1}{2}+\sqrt{x^{2}+x}\right)x^{2}\right)}{x\sqrt{x\left(1+x\right)}}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -((1+x)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*(x^2+x)^(1/2)*x^2-ln(x+1/2+(x^2+x)^(1
/2))*x^2)/(x*(1+x))^(1/2)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Fricas [A]
time = 0.33, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}}+\log\left(\sqrt{\frac{x+1}{x}}+1\right)-\log\left(\sqrt{\frac{x+1}{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)
```

Sympy [A]
time = 1.77, size = 32, normalized size = 1.33

$$-2\sqrt{1+\frac{1}{x}}-\log\left(\sqrt{1+\frac{1}{x}}-1\right)+\log\left(\sqrt{1+\frac{1}{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)**(1/2)/x,x)
```

```
[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)
```

Giac [A]
time = 4.71, size = 38, normalized size = 1.58

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x)+\frac{2\operatorname{sgn}(x)}{x-\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.83

$$2\,\mathrm{atanh}\left(\sqrt{\frac{1}{x}+1}\right)-2\,\sqrt{\frac{1}{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x)^(1/2)/x,x)
```

```
[Out] 2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)
```

3.978 $\int \sqrt{\frac{x}{1+x}} \, dx$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arcsinh(x^(1/2))+x^(1/2)*(1+x)^(1/2)
```

Rubi [A]

time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1978, 52, 56, 221}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x/(1 + x)],x]
```

```
[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$$
$$= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{1+x} - \sinh^{-1}\left(\sqrt{x}\right)$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(22) = 44.
time = 0.02, size = 48, normalized size = 2.18

$$\frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}\,(1+x) - \sqrt{1+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x/(1 + x)],x]
```

```
[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcTanh[Sqrt[x/(1 + x)]]))/
Sqrt[x]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(16) = 32.
time = 0.05, size = 45, normalized size = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{1+x}}\,(1+x)\left(2\sqrt{x^2+x}-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\right)}{2\sqrt{x\,(1+x)}}$	45
risch	$(1+x)\sqrt{\frac{x}{1+x}} - \frac{\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)\sqrt{\frac{x}{1+x}}\,\sqrt{x\,(1+x)}}{2x}$	47
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{\frac{x}{1+x}} - \frac{\ln\left(2\sqrt{\frac{x}{1+x}}\,x+2\sqrt{\frac{x}{1+x}}+2x+1\right)}{2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(1+x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x/(1+x))^(1/2)*(1+x)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/(x*(1+x
))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.
time = 0.27, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(s
qrt(x/(x + 1)) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.
time = 0.35, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}}-\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
+ 1)) - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt(x/(x + 1)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.
time = 5.00, size = 35, normalized size = 1.59

$$\frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)+\sqrt{x^2+x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x +
 1)
```

Mupad [B]
time = 0.04, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x/(x + 1))^(1/2),x)
[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

3.979 $\int \frac{1}{\sqrt{\frac{-1-x}{x}}} \, dx$

Optimal. Leaf size=29

$$-x\sqrt{-\frac{1+x}{x}}+\tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)$$

[Out] arctan(((-1-x)/x)^(1/2))-x*((-1-x)/x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1997, 248, 44, 65, 210}

$$\text{ArcTan}\left(\sqrt{-\frac{x+1}{x}}\right)-x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(-1 - x)/x],x]

[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 248

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1997

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{-1-x}{x}}}\,dx &= \int \frac{1}{\sqrt{-1-\frac{1}{x}}}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x^2}\,dx, x, \frac{1}{x}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1-x}\,x}\,dx, x, \frac{1}{x}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{-\frac{1+x}{x}}\right)\\
&= -x\sqrt{-\frac{1+x}{x}} + \tan^{-1}\left(\sqrt{-\frac{1+x}{x}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.69

$$\frac{\sqrt{x}\,(1+x)-\sqrt{1+x}\,\tanh^{-1}\left(\sqrt{\frac{x}{1+x}}\right)}{\sqrt{x}\,\sqrt{-\frac{1+x}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(-1 - x)/x],x]
```

```
[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcTanh[Sqrt[x/(1 + x)]])/(Sqrt[x]*Sqrt[-((1
 + x)/x)])
```

Maple [A]
time = 0.10, size = 44, normalized size = 1.52

method	result	size
default	$\frac{(1+x)\left(2\sqrt{-x^{2}-x}+\arcsin(2x+1)\right)}{2\sqrt{-\frac{1+x}{x}}\,\sqrt{-x\,(1+x)}}$	44
risch	$\frac{1+x}{\sqrt{-\frac{1+x}{x}}}-\frac{\arcsin(2x+1)\sqrt{-x\,(1+x)}}{2\sqrt{-\frac{1+x}{x}}\,x}$	45
trager	$-\sqrt{-\frac{1+x}{x}}\,x+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(2x\,\mathrm{RootOf}\left(\_Z^{2}+1\right)+2\sqrt{-\frac{1+x}{x}}\,x+\mathrm{RootOf}\left(\_Z^{2}+1\right)\right)}{2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1-x)/x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(1+x)*(2*(-x^2-x)^(1/2)+arcsin(2*x+1))/(-(1+x)/x)^(1/2)/(-x*(1+x))^(1/2
)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 1.21

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x}-1}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-(x + 1)/x)/((x + 1)/x - 1) + arctan(sqrt(-(x + 1)/x))
```

Fricas [A]
time = 0.33, size = 25, normalized size = 0.86

$$-x\sqrt{-\frac{x+1}{x}}+\arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="fricas")
[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{-x-1}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((-x - 1)/x), x)
```

Giac [A]
time = 3.69, size = 35, normalized size = 1.21

$$\frac{1}{4}\pi\operatorname{sgn}(x) - \frac{\arcsin(2x+1)}{2\operatorname{sgn}(x)} - \frac{\sqrt{-x^2-x}}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) - 1/2*arcsin(2*x + 1)/sgn(x) - sqrt(-x^2 - x)/sgn(x)
```

Mupad [B]
time = 3.37, size = 23, normalized size = 0.79

$$\operatorname{atan}\left(\sqrt{-\frac{1}{x}-1}\right) - x\sqrt{-\frac{1}{x}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x + 1)/x)^(1/2),x)
```

```
[Out] atan((- 1/x - 1)^(1/2)) - x*(- 1/x - 1)^(1/2)
```

3.980 $\int \sqrt{(4-x)x}\, dx$

Optimal. Leaf size=33

$$-\frac{1}{2}(2-x)\sqrt{4x-x^2} - 2\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] 2*arcsin(-1+1/2*x)-1/2*(2-x)*(-x^2+4*x)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1976, 626, 633, 222}

$$-2\text{ArcSin}\left(1-\frac{x}{2}\right) - \frac{1}{2}\sqrt{4x-x^2}\,(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(4 - x)*x],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[4*x - x^2]) - 2*ArcSin[1 - x/2]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \sqrt{(4-x)x}\,dx = \int \sqrt{4x-x^2}\,dx$$
$$= -\frac{1}{2}(2-x)\sqrt{4x-x^2} + 2\int \frac{1}{\sqrt{4x-x^2}}\,dx$$
$$= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right)$$
$$= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - 2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Mathematica [A]
time = 0.08, size = 49, normalized size = 1.48

$$\frac{1}{2}\sqrt{-((-4+x)x)}\left(-2+x-\frac{16\tanh^{-1}\left(\frac{\sqrt{-4+x}}{-2+\sqrt{x}}\right)}{\sqrt{-4+x}\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(4 - x)*x],x]
[Out] (Sqrt[-((-4 + x)*x)]*(-2 + x - (16*ArcTanh[Sqrt[-4 + x]/(-2 + Sqrt[x])])/(S
qrt[-4 + x]*Sqrt[x])))/2
```

Maple [A]
time = 0.24, size = 28, normalized size = 0.85

method	result
risch	$-\frac{(x-2)x(x-4)}{2\sqrt{-x(x-4)}}+2\arcsin\left(-1+\frac{x}{2}\right)$
default	$-\frac{(4-2x)\sqrt{-x^2+4x}}{4}+2\arcsin\left(-1+\frac{x}{2}\right)$
meijerg	$-\frac{8i\left(-\frac{i\sqrt{\pi}\sqrt{x}\left(-\frac{3x}{2}+3\right)\sqrt{-\frac{x}{4}+1}}{12}+\frac{i\sqrt{\pi}\arcsin\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+4x}+2\,\text{RootOf}\left(\_Z^2+1\right)\ln\left(-x\,\text{RootOf}\left(\_Z^2+1\right)+2\,\text{RootOf}\left(\_Z^2+1\right)+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((4-x)*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(4-2*x)*(-x^2+4*x)^(1/2)+2*arcsin(-1+1/2*x)
```

Maxima [A]
time = 0.47, size = 36, normalized size = 1.09

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,x-\sqrt{-x^2+4\,x}-2\,\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(-x^2 + 4*x)*x - sqrt(-x^2 + 4*x) - 2*arcsin(-1/2*x + 1)
```

Fricas [A]
time = 0.33, size = 35, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,(x-2)-4\,\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) - 4*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x\,(4-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)**(1/2),x)
[Out] Integral(sqrt(x*(4 - x)), x)
```

Giac [A]
time = 4.58, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{-x^2+4\,x}\,(x-2)+2\,\arcsin\left(\frac{1}{2}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(-x^2 + 4*x)*(x - 2) + 2*arcsin(1/2*x - 1)
```

Mupad [B]
time = 3.47, size = 26, normalized size = 0.79

$$2\,\mathrm{asin}\left(\frac{x}{2}-1\right)+\left(\frac{x}{2}-1\right)\,\sqrt{4\,x-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 4))^(1/2),x)
[Out] 2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

3.981 $\int \frac{1}{\sqrt{(1-x)x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] arcsin(-1+2*x)
```

Rubi [A]
time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1976, 633, 222}

$$-\text{ArcSin}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(1 - x)*x],x]
```

```
[Out] -ArcSin[1 - 2*x]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{(1-x)x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right) \\
&= -\sin^{-1}(1-2x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 38 vs. $2(8) = 16$.
time = 0.03, size = 38, normalized size = 4.75

$$\frac{2\sqrt{-1+x}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{-1+x}}\right)}{\sqrt{-((-1+x)x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(1 - x)*x],x]
```

```
[Out] (2*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sqrt[x]/Sqrt[-1 + x]])/Sqrt[-((-1 + x)*x)]
```

Maple [A]
time = 0.23, size = 7, normalized size = 0.88

method	result	size
default	$\arcsin(2x-1)$	7
meijerg	$2\arcsin\left(\sqrt{x}\right)$	7
trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-2x\,\mathrm{RootOf}\left(\_Z^2+1\right)+\mathrm{RootOf}\left(\_Z^2+1\right)+2\sqrt{-x^2+x}\right)$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1-x)*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsin(2*x-1)
```

Maxima [A]
time = 0.50, size = 6, normalized size = 0.75

$$\arcsin(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.
time = 0.34, size = 16, normalized size = 2.00

$$-2\,\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\,(1-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)**(1/2),x)
[Out] Integral(1/sqrt(x*(1 - x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(6) = 12.
time = 4.27, size = 25, normalized size = 3.12

$$\frac{1}{4}\,\sqrt{-x^2+x}\,(2\,x-1)+\frac{1}{8}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)*x)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(-x^2 + x)*(2*x - 1) + 1/8*arcsin(2*x - 1)
```

Mupad [B]
time = 0.01, size = 6, normalized size = 0.75

$$\mathrm{asin}(2\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(x - 1))^(1/2),x)
[Out] asin(2*x - 1)
```

3.982 $\int \frac{x}{(x(2+x))^{3/2}}\,dx$

Optimal. Leaf size=13

$$\frac{x}{\sqrt{2x+x^2}}$$

[Out] x/(x^2+2*x)^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1976, 650}

$$\frac{x}{\sqrt{x^2+2x}}$$

Antiderivative was successfully verified.

[In] Int[x/(x*(2 + x))^(3/2),x]

[Out] x/Sqrt[2*x + x^2]

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(x(2+x))^{3/2}}\,dx &= \int \frac{x}{\left(2x+x^2\right)^{3/2}}\,dx\\ &= \frac{x}{\sqrt{2x+x^2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(2+x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x*(2 + x))^(3/2),x]
```

```
[Out] x/Sqrt[x*(2 + x)]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.22, size = 29, normalized size = 2.23

method	result	size
risch	$\frac{x}{\sqrt{x\,(x+2)}}$	10
gosper	$\frac{x^{2}(x+2)}{(x(x+2))^{\frac{3}{2}}}$	15
trager	$\frac{\sqrt{x^{2}+2x}}{x+2}$	16
meijerg	$\frac{\sqrt{2}\,\sqrt{x}}{2\sqrt{1+\frac{x}{2}}}$	16
default	$-\frac{1}{\sqrt{x^{2}+2x}}+\frac{2x+2}{2\sqrt{x^{2}+2x}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x+2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2+2*x)^(1/2)+1/2*(2*x+2)/(x^2+2*x)^(1/2)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x^2+2\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 2*x)
```

Fricas [A]
time = 0.36, size = 18, normalized size = 1.38

$$\frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")
```

```
[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x\left(x+2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))**(3/2),x)
```

```
[Out] Integral(x/(x*(x + 2))**(3/2), x)
```

Giac [A]
time = 4.30, size = 18, normalized size = 1.38

$$\frac{2}{x-\sqrt{x^2+2\,x}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")
```

```
[Out] 2/(x - sqrt(x^2 + 2*x) + 2)
```

Mupad [B]
time = 3.52, size = 13, normalized size = 1.00

$$\frac{\sqrt{x\,\left(x+2\right)}}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x*(x + 2))^(3/2),x)
```

```
[Out] (x*(x + 2))^(1/2)/(x + 2)
```

3.983 $\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$

Optimal. Leaf size=22

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)$$

```
[Out] arctanh(1/2*(1+1/x)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1460, 1483, 641, 65, 213}

$$\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 641

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
```

```
rQ[m + p]))
```

Rule 1460

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1483

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2}\,dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{\left(-1+\frac{1}{x^2}\right)x^2}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{1+x}}{-1+x^2}\,dx, x, \frac{1}{x}\right) \\
&= -\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+x}}\,dx, x, \frac{1}{x}\right) \\
&= -\left(2\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right) \\
&= \sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 1.09

$$\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{\frac{1+x}{x}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2),x]
```

```
[Out] Sqrt[2]*ArcTanh[Sqrt[(1 + x)/x]/Sqrt[2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(17) = 34$.
time = 0.23, size = 41, normalized size = 1.86

method	result	size
default	$\frac{\sqrt{\frac{1+x}{x}}\, x\sqrt{2}\, \operatorname{arctanh}\left(\frac{(1+3x)\sqrt{2}}{4\sqrt{x^2+x}}\right)}{2\sqrt{x\,(1+x)}}$	41
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right) \ln\left(\frac{4\sqrt{-\frac{-1-x}{x}}\, x-3\operatorname{RootOf}\left(\_Z^2-2\right) x-\operatorname{RootOf}\left(\_Z^2-2\right)}{-1+x}\right)}{2}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((1+x)/x)^(1/2)*x/(x*(1+x))^(1/2)*2^(1/2)*arctanh(1/4*(1+3*x)*2^(1/2)/(
x^2+x)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)
```

Fricas [A]
time = 0.33, size = 33, normalized size = 1.50

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\,x\sqrt{\frac{x+1}{x}}+3\,x+1}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{1+\frac{1}{x}}}{x^2-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)
```

```
[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(17) = 34$.
time = 4.63, size = 73, normalized size = 3.32

$$\frac{1}{2}\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\operatorname{sgn}(x)-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+x}+2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+x}+2\right|}\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-
2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 +
x) + 2))*sgn(x)
```

Mupad [B]
time = 3.58, size = 17, normalized size = 0.77

$$\sqrt{2}\,\operatorname{atanh}\left(\frac{\sqrt{2}\,\sqrt{\frac{1}{x}+1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/x + 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] 2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)
```

3.984 $\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}\,x^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)$$

```
[Out] 1/2*arctan(x*(1/2*5^(1/2)-1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6, 209}

$$\frac{1}{2}\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sqrt{5}-x^2+\sqrt{5}\,x^2}\,dx &= \int \frac{1}{1+\sqrt{5}+\left(-1+\sqrt{5}\right)x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}\,x\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.02, size = 39, normalized size = 1.62

$$\frac{1}{4}i\log\left(1+\sqrt{5}-2ix\right)-\frac{1}{4}i\log\left(1+\sqrt{5}+2ix\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1),x]
```

```
[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.
time = 0.24, size = 32, normalized size = 1.33

method	result	size
risch	$\frac{\arctan\left(\frac{2x}{\sqrt{5}+1}\right)}{4}-\frac{\arctan\left(\frac{2x}{-\sqrt{5}-1}\right)}{4}$	30
default	$\frac{4\arctan\left(\frac{4x}{2\sqrt{5}+2}\right)}{\left(\sqrt{5}-1\right)\left(2\sqrt{5}+2\right)}$	32
meijerg	$\frac{\arctan\left(\frac{x\sqrt{\sqrt{5}-1}}{\sqrt{\sqrt{5}+1}}\right)}{\sqrt{\sqrt{5}+1}\,\sqrt{\sqrt{5}-1}}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(5^(1/2)-1)/(2*5^(1/2)+2)*arctan(4*x/(2*5^(1/2)+2))
```

Maxima [A]
time = 0.81, size = 11, normalized size = 0.46

$$\frac{1}{2}\arctan\left(\frac{1}{2}\,x\left(\sqrt{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.54

$$\frac{1}{2}\arctan\left(\frac{1}{2}\sqrt{5}\,x-\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(1/2*sqrt(5)*x - 1/2*x)
```

Sympy [A]
time = 0.32, size = 15, normalized size = 0.62

$$-\frac{\operatorname{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)
```

```
[Out] -atan(x*(1/2 - sqrt(5)/2))/2
```

Giac [A]
time = 2.96, size = 13, normalized size = 0.54

$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(2*x/(sqrt(5) + 1))
```

Mupad [B]
time = 0.10, size = 45, normalized size = 1.88

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\,x\left(\sqrt{5}+1\right)}{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\sqrt{\sqrt{5}+3}}\right)\left(\sqrt{5}+1\right)}{4\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1),x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)^
(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))
```

3.985 $\int \frac{1}{\sqrt{ax+bx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax+bx^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [**A**]
time = 0.23, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [**A**]
time = 0.49, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [**A**]
time = 0.38, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 3.12, size = 61, normalized size = 2.18

$$\frac{1}{4}\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.47, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.986 $\int \frac{1}{\sqrt{x(a+bx)}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1976, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x(a+bx)}} \, dx = \int \frac{1}{\sqrt{ax+bx^2}} \, dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(a + b*x)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A]
time = 0.23, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.29, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))**(1/2),x)
[Out] Integral(1/sqrt(x*(a + b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 1.95, size = 61, normalized size = 2.18

$$\frac{1}{4}\,\sqrt{bx^2+ax}\,\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\,\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}\,-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x))^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.987 $\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\left(b+\frac{a}{x}\right)x^2}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b + a/x)*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.39, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 0.36, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b+a/x)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a/x + b)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 2.44, size = 61, normalized size = 2.18

$$\frac{1}{4}\,\sqrt{bx^2+ax}\,\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b+a/x)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.59, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a/x))^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.988 $\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\left(\frac{a}{x^2}+\frac{b}{x}\right)x^3}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/x^2+b/x)*x^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.53, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 0.34, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^3\left(\frac{a}{x^2}+\frac{b}{x}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(22) = 44$.
time = 2.44, size = 61, normalized size = 2.18

$$\frac{1}{4}\,\sqrt{bx^2+ax}\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a/x^2 + b/x))^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.989 $\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a*x^2)/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.30, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 0.32, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 2.80, size = 61, normalized size = 2.18

$$\frac{1}{4}\,\sqrt{bx^2+ax}\,\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\,\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}\,-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^2 + b*x^3)/x)^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.990 $\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x)^(1/2))/b^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx = \int \frac{1}{\sqrt{ax+bx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{ax+bx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 1.96

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[x*(a + b*x)])
```

Maple [A]
time = 0.02, size = 29, normalized size = 1.04

method	result	size
default	$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{\sqrt{b}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.31, size = 27, normalized size = 0.96

$$\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)
```

Fricas [A]
time = 0.35, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2\,bx+a+2\,\sqrt{bx^2+ax}\,\sqrt{b}\right)}{\sqrt{b}},-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^2+ax}\,\sqrt{-b}}{bx}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.
time = 4.49, size = 61, normalized size = 2.18

$$\frac{1}{4}\,\sqrt{bx^2+ax}\,\left(2\,x+\frac{a}{b}\right)+\frac{a^2\log\left(\left|-2\,\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\sqrt{b}\,-a\right|\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x^2 + a*x)*(2*x + a/b) + 1/8*a^2*log(abs(-2*(sqrt(b)*x - sqrt(b*
x^2 + a*x))*sqrt(b) - a))/b^(3/2)
```

Mupad [B]
time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+b\,x}{\sqrt{b}}+\sqrt{b\,x^2+a\,x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

3.991 $\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{acx+bcx^2}}\,dx &= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*c*x + b*c*x^2],x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [**A**]
time = 0.23, size = 37, normalized size = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^{2}+acx}\right)}{\sqrt{bc}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c*x^2+a*c*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [**A**]
time = 0.29, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^{2}+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Fricas [**A**]
time = 0.34, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^{2}+acx}\,\sqrt{bc}\right)}{bc},\,-\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^{2}+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x**2+a*c*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*c*x + b*c*x**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.
time = 3.15, size = 76, normalized size = 1.90

$$\frac{a^2c\log\left(\left|-ac-2\sqrt{bc}\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\sqrt{bc}\,b}+\frac{1}{4}\sqrt{bcx^2+acx}\left(2\,x+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c*x^2+a*c*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B]
time = 3.90, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c*x + b*c*x^2)^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.992 $\int \frac{1}{\sqrt{c(ax+bx^2)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\,(ax+bx^2)}}\,dx = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(a*x + b*x^2)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A]
time = 0.23, size = 37, normalized size = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b*x^2+a*x))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [A]
time = 0.26, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Fricas [A]
time = 0.33, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx + ac + 2\,\sqrt{bcx^2 + acx}\,\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2 + acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,(ax + bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.
time = 2.61, size = 76, normalized size = 1.90

$$\frac{a^2c\log\left(\left|-ac - 2\,\sqrt{bc}\,\left(\sqrt{bc}\,x - \sqrt{bcx^2 + acx}\right)\right|\right)}{8\,\sqrt{bc}\,b} + \frac{1}{4}\,\sqrt{bcx^2 + acx}\,\left(2\,x + \frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B]
time = 3.56, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c + 2\,\sqrt{b\,c}\,\sqrt{c\,x\,(a + b\,x)} + 2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(a*x + b*x^2))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.993 $\int \frac{1}{\sqrt{cx(a+bx)}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1976, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*x*(a + b*x)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{cx(a+bx)}}\,dx = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{x}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A]
time = 0.23, size = 37, normalized size = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [A]
time = 0.33, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Fricas [A]
time = 0.33, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc}, -\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x*(a + b*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(30) = 60$.
time = 2.21, size = 76, normalized size = 1.90

$$\frac{a^2c\log\left(\left|-ac-2\,\sqrt{bc}\,\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\,\sqrt{bc}\,b}+\frac{1}{4}\,\sqrt{bcx^2+acx}\,\left(2\,x+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [B]
time = 3.47, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x))^(1/2),x)
```

```
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.994 $\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx$

Optimal. Leaf size=40

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] 2*arctanh(x*b^(1/2)*c^(1/2)/(b*c*x^2+a*c*x)^(1/2))/b^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2004, 634, 212}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*(b + a/x)*x^2],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2004

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}}\,dx = \int \frac{1}{\sqrt{acx+bcx^2}}\,dx$$
$$= 2\text{Subst}\left(\int \frac{1}{1-bcx^2}\,dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c}\,x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\,\sqrt{c}}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.40

$$-\frac{2\sqrt{x}\,\sqrt{a+bx}\,\log\left(-\sqrt{b}\,\sqrt{x}+\sqrt{a+bx}\right)}{\sqrt{b}\,\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*(b + a/x)*x^2],x]
[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]
*Sqrt[c*x*(a + b*x)])
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.92

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}ac+bcx}{\sqrt{bc}}+\sqrt{bc\,x^2+acx}\right)}{\sqrt{bc}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*(b+a/x)*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln((1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))/(b*c)^(1/2)
```

Maxima [A]
time = 0.40, size = 36, normalized size = 0.90

$$\frac{\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)
```

Fricas [**A**]
time = 0.33, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\,\log\left(2\,bcx+ac+2\,\sqrt{bcx^2+acx}\,\sqrt{bc}\right)}{bc},-\frac{2\,\sqrt{-bc}\,\arctan\left(\frac{\sqrt{bcx^2+acx}\,\sqrt{-bc}}{bcx}\right)}{bc}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2
*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^2\left(\frac{a}{x}+b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b+a/x)*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(30) = 60.
time = 3.08, size = 76, normalized size = 1.90

$$\frac{a^2c\log\left(\left|-ac-2\,\sqrt{bc}\,\left(\sqrt{bc}\,x-\sqrt{bcx^2+acx}\right)\right|\right)}{8\,\sqrt{bc}\,b}+\frac{1}{4}\,\sqrt{bcx^2+acx}\,\left(2\,x+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*(b+a/x)*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a^2*c*log(abs(-a*c - 2*sqrt(b*c)*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x)))
)/(sqrt(b*c)*b) + 1/4*sqrt(b*c*x^2 + a*c*x)*(2*x + a/b)
```

Mupad [**B**]
time = 3.65, size = 33, normalized size = 0.82

$$\frac{\ln\left(a\,c+2\,\sqrt{b\,c}\,\sqrt{c\,x\,\left(a+b\,x\right)}+2\,b\,c\,x\right)}{\sqrt{b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2*(b + a/x))^(1/2),x)
[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)
```

3.995 $\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{1}{4}\left(3x+\sqrt{-1+x^2}\right)\sqrt{1-x^2+x\sqrt{-1+x^2}}+\frac{3\sin^{-1}\left(x-\sqrt{-1+x^2}\right)}{4\sqrt{2}}$$

```
[Out] 3/8*arcsin(x-(x^2-1)^(1/2))*2^(1/2)+1/4*(3*x+(x^2-1)^(1/2))*(1-x^2+x*(x^2-1
)^(1/2))^(1/2)
```

Rubi [F]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]
```

Rubi steps

$$\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx=\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx$$

Mathematica [A]
time = 0.53, size = 110, normalized size = 1.75

$$\frac{1}{8}\left(\frac{2(-1+x^2)\left(3x+\sqrt{-1+x^2}\right)}{\sqrt{1-x^2+x\sqrt{-1+x^2}}\left(-1+x^2+x\sqrt{-1+x^2}\right)}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-1+x^2}}{\sqrt{1-x^2+x\sqrt{-1+x^2}}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]],x]
```

```
[Out] ((2*(-1 + x^2)*(3*x + Sqrt[-1 + x^2]))/(Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]*(-
1 + x^2 + x*Sqrt[-1 + x^2])) - 3*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[-1 + x^2])/Sq
rt[1 - x^2 + x*Sqrt[-1 + x^2]]])/8
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{1-x^2+x\sqrt{x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

```
[Out] int((1-x^2+x*(x^2-1)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Fricas [A]
time = 0.89, size = 68, normalized size = 1.08

$$\frac{1}{4}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}\left(3\,x+\sqrt{x^2-1}\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+\sqrt{x^2-1}\,x+1}}{2\sqrt{x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*ar
ctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+x\sqrt{x^2-1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x\,\sqrt{x^2-1}-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)
```

```
[Out] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)
```

3.996 $\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=66

$$\frac{1}{2}\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-\frac{3\sin^{-1}\left(\sqrt{x}-\sqrt{1+x}\right)}{2\sqrt{2}}$$

```
[Out] -3/4*arcsin(x^(1/2)-(1+x)^(1/2))*2^(1/2)+1/2*(x^(1/2)+3*(1+x)^(1/2))*(-x+x^
(1/2)*(1+x)^(1/2))^(1/2)
```

Rubi [F]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 +
 x]]
```

Rubi steps

$$\int \frac{\sqrt{-x+\sqrt{x}\sqrt{1+x}}}{\sqrt{1+x}}\,dx=2\text{Subst}\left(\int \sqrt{1-x^2+x\sqrt{-1+x^2}}\,dx,x,\sqrt{1+x}\right)$$

Mathematica [A]
time = 0.64, size = 91, normalized size = 1.38

$$\frac{1}{4}\left(2\left(\sqrt{x}+3\sqrt{1+x}\right)\sqrt{-x+\sqrt{x}\sqrt{1+x}}-3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-2x+2\sqrt{x}\sqrt{1+x}}}{-\sqrt{x}+\sqrt{1+x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] (2*(Sqrt[x] + 3*Sqrt[1 + x])*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] - 3*Sqrt[2]*Arc
Tan[Sqrt[-2*x + 2*Sqrt[x]*Sqrt[1 + x]]/(-Sqrt[x] + Sqrt[1 + x])])/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x+\sqrt{x}\,\sqrt{1+x}}}{\sqrt{1+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

```
[Out] int((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(46) = 92$.
time = 190.19, size = 127, normalized size = 1.92

$$\frac{1}{2}\sqrt{\sqrt{x+1}\,\sqrt{x}-x}\left(3\sqrt{x+1}+\sqrt{x}\right)+\frac{3}{8}\sqrt{2}\arctan\left(\frac{2\left(132301464716464979417539 20\sqrt{2}\,(4x+1)\sqrt{x+1}\,\sqrt{x}+472818412040\sqrt{2}\,(111925814517792x^2+83944360888344x+621904154881)+3497681703681\left(\sqrt{2}\,(50987674019848x-621904154881)\sqrt{x+1}+\sqrt{2}\,(60938140497944x-24871932855043)\sqrt{x}\right)\sqrt{\sqrt{x+1}\,\sqrt{x}-x}\right)}{1392142581741099885962855 04x^2-678551053586366160645570576x+386764777858250836124161}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(sqrt(x + 1)*sqrt(x) - x)*(3*sqrt(x + 1) + sqrt(x)) + 3/8*sqrt(2)*a
rctan(2*(132301464716464979417539 20*sqrt(2)*(4*x + 1)*sqrt(x + 1)*sqrt(x) +
 472818412040*sqrt(2)*(111925814517792*x^2 + 83944360888344*x + 62190415488
1) + 3497681703681*(sqrt(2)*(50987674019848*x - 621904154881)*sqrt(x + 1) +
 sqrt(2)*(60938140497944*x - 24871932855043)*sqrt(x))*sqrt(sqrt(x + 1)*sqrt
(x) - x))/(139214258174109988596285504*x^2 - 678551053586366160645570576*x
+ 386764777858250836124161))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x)*sqrt(x + 1) - x)/sqrt(x + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x + 1)*sqrt(x) - x)/sqrt(x + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x}\,\sqrt{x+1}-x}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] int((x^(1/2)*(x + 1)^(1/2) - x)^(1/2)/(x + 1)^(1/2), x)
```

3.997 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=78

$$-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{-2+\sqrt{5}}\left(x+\sqrt{1+x^2}\right)\right)+\sqrt{2\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{2+\sqrt{5}}\left(x+\sqrt{1\right.\right.$$

```
[Out] arctanh((x+(x^2+1)^(1/2))*(2+5^(1/2))^(1/2))*(-2+2*5^(1/2))^(1/2)-arctan((x
+(x^2+1)^(1/2))*(-2+5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 319 vs. 2(78) = 156.
time = 0.41, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$,
Rules used = {6874, 267, 1144, 209, 213, 1265, 838, 721, 1107, 1261, 713, 1293}

$$-\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\,\text{ArcTan}\left(\sqrt{\frac{2}{\sqrt{5}-1}}\sqrt{x^2+1}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)-2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\,\text{ArcTan}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)-\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\sqrt{x^2+1}\right)+\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)-2\sqrt{\frac{2}{5\left(\sqrt{5}-1\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sq
rt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcT
an[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*Arc
Tanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1
 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*S
qrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqr
t[1 + x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 721

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx = -\int \left(\frac{x}{x+x^3+\sqrt{1+x^2}} + \frac{2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\right)dx$$

$$= -\left(2\int \frac{\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}}\,dx\right) - \int \frac{x}{x+x^3+\sqrt{1+x^2}}\,dx$$

$$= -\left(2\int \left(1+\frac{x\sqrt{1+x^2}}{-1+x^2+x^4} - \frac{x^2(1+x^2)}{-1+x^2+x^4}\right)dx\right) - \int \left(\frac{x}{\sqrt{1+x^2}} + \frac{}{-1+}\right.$$

$$= -2x - 2\int \frac{x\sqrt{1+x^2}}{-1+x^2+x^4}\,dx + 2\int \frac{x^2(1+x^2)}{-1+x^2+x^4}\,dx - \int \frac{x}{\sqrt{1+x^2}}\,dx - \int$$

$$= -\sqrt{1+x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{-1+x+x^2}\,dx, x, x^2\right) + 2\int \frac{1}{-1+x^2+x^4}\,dx + \frac{1}{1}$$

$$= -\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) + \sqrt{\frac{1}{10}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\right.$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{}{1+}}\right.$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{}{1+}}\right.$$

$$= -2\sqrt{\frac{2}{5\left(1+\sqrt{5}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{}{1+}}\right.$$

Mathematica [**A**]
time = 0.35, size = 83, normalized size = 1.06

$$-\sqrt{2\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right) - \sqrt{2\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{-2+\sqrt{5}}\left(x-\sqrt{1+x^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]) - Sq
rt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[-2 + Sqrt[5]]*(x - Sqrt[1 + x^2])]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(58) = 116$.
time = 1.13, size = 438, normalized size = 5.62

method	result
trager	$\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)\ln\left(-\frac{\sqrt{x^2+1}\,\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)^2+\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)^3+\sqrt{x^2+1}+}{\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)^3x+\mathrm{RootOf}\left(\_Z^4+\_Z^2-1\right)x+}\right.$
default	$-\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}-\frac{\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{\sqrt{2\sqrt{5}-2}}+\frac{\mathrm{arctanh}\left(\frac{}{\sqrt{2}}\right.}{\sqrt{2\sqrt{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-1/(2*5^(1/2)+2
)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh
(2*x/(2*5^(1/2)-2)^(1/2))+1/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(
1/2))-1/2*(x^2+1)^(1/2)-1/2*x+1/2/(-x+(x^2+1)^(1/2))+1/2/(2+5^(1/2))^(1/2)*
arctanh((-x+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)
*arctanh((-x+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arcta
n((-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arc
tan((-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*a
rctan((-x+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan((-
x+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))+3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh
((-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctanh((-x+
(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan((-x
+(x^2+1)^(1/2))/(2+5^(1/2))^(1/2))+2/5*(-2+5^(1/2))^(1/2)*5^(1/2)*arctanh((
-x+(x^2+1)^(1/2))/(-2+5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 1/2*arctan(x) + integrate(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 +
 2*x^2 + 2*(x^3 + x)*sqrt(x^2 + 1) + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(58) = 116$.
time = 0.40, size = 383, normalized size = 4.91

$$\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{4}\sqrt{2}\sqrt{4x^4+4x^2+\sqrt{5}(2x^2+1)-2\left(2x^3+\sqrt{5}x+x\right)\sqrt{x^2+1}+1}\left(\sqrt{2}x+\sqrt{2}\sqrt{x^2+1}\right)\sqrt{\sqrt{5}+1}-\frac{1}{2}\sqrt{2}\sqrt{x^2+1}\sqrt{\sqrt{5}+1}\right)+\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{8}\sqrt{4x^2+2\sqrt{5}+2}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{4}\left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{5}+1}\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x+\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(4x^2-4\sqrt{x^2+1}x-\left(\sqrt{5}\sqrt{2}x-\sqrt{x^2+1}\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)+\sqrt{2}x\right)\sqrt{\sqrt{5}-1}+4\right)-\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x+\sqrt{2}\sqrt{\sqrt{5}-1}\right)+\frac{1}{4}\sqrt{2}\sqrt{\sqrt{5}-1}\log\left(2x-\sqrt{2}\sqrt{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5)*(
2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqrt
(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt(
5) + 1)) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)
*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - s
qrt(2)*x)*sqrt(sqrt(5) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*
sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqr
t(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*l
og(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*
sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(s
qrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5)
 - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(58) = 116.
time = 2.45, size = 218, normalized size = 2.79

$$-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(\left|-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1))
)/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5)
+ 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) +
 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2
*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) -
1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5
) + 2) - 1/(x - sqrt(x^2 + 1))))
```

Mupad [**B**]
time = 4.38, size = 649, normalized size = 8.32

$$\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}-\frac{5}{2}\right)}{2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}+\frac{\ln\left(x-\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\ln\left(x+\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\left(\frac{\sqrt{5}}{2}+\frac{5}{2}\right)}{2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}}-\frac{\left(\ln\left(x-\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}}{2}+\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\sqrt{5}+1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}-\frac{\left(\ln\left(x+\frac{\sqrt{2}\,\sqrt{\sqrt{5}-1}}{2}\right)-\ln\left(\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{\sqrt{5}+1}}{2}-\frac{\sqrt{2}\,x\,\sqrt{\sqrt{5}-1}}{2}+1\right)\right)\left(\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{1}{2}}}+\frac{\left(\ln\left(\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{1-\sqrt{5}}}{2}-\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}}{2}+1\right)-\ln\left(x+\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}+\frac{\left(\ln\left(\frac{\sqrt{2}\,x\,\sqrt{-\sqrt{5}-1}}{2}+\frac{\sqrt{2}\,\sqrt{x^2+1}\,\sqrt{1-\sqrt{5}}}{2}+1\right)-\ln\left(x-\frac{\sqrt{2}\,\sqrt{-\sqrt{5}-1}}{2}\right)\right)\left(\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+2\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{1}{2}-\frac{\sqrt{5}}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x + 2*(x^2 + 1)^(1/2))/(x + (x^2 + 1)^(1/2) + x^3),x)
```

```
[Out] (log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2)) - (log(x - (2^(1/2)*(5^(1/2) - 1)
^(1/2))/2)*(5^(1/2)/2 - 5/2))/(2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1
/2)^(3/2)) + (log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))
/(2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - (log(x + (2^
(1/2)*(- 5^(1/2) - 1)^(1/2))/2)*(5^(1/2)/2 + 5/2))/(2*(- 5^(1/2)/2 - 1/2)^(
1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2)) - ((log(x - (2^(1/2)*(5^(1/2) - 1)^(1/2
))/2) - log((2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(5
^(1/2) + 1)^(1/2))/2 + 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(
3/2)))/((2*(5^(1/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2
+ 1/2)^(1/2)) - ((log(x + (2^(1/2)*(5^(1/2) - 1)^(1/2))/2) - log((2^(1/2)*(
x^2 + 1)^(1/2)*(5^(1/2) + 1)^(1/2))/2 - (2^(1/2)*x*(5^(1/2) - 1)^(1/2))/2 +
 1))*((5^(1/2)/2 - 1/2)^(1/2) + 2*(5^(1/2)/2 - 1/2)^(3/2)))/((2*(5^(1/2)/2
- 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 1/2)^(1/2)) + ((log(
(2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2 - (2^(1/2)*x*(- 5^(1/2) - 1
)^(1/2))/2 + 1) - log(x + (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2
 - 1/2)^(1/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2)
 + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2)) + ((log((2^(1/2)*x
*(- 5^(1/2) - 1)^(1/2))/2 + (2^(1/2)*(x^2 + 1)^(1/2)*(1 - 5^(1/2))^(1/2))/2
 + 1) - log(x - (2^(1/2)*(- 5^(1/2) - 1)^(1/2))/2))*((- 5^(1/2)/2 - 1/2)^(1
/2) + 2*(- 5^(1/2)/2 - 1/2)^(3/2)))/((2*(- 5^(1/2)/2 - 1/2)^(1/2) + 4*(- 5^
(1/2)/2 - 1/2)^(3/2))*(1/2 - 5^(1/2)/2)^(1/2))
```

3.998 $\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=126

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right)-\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}\tanh^{-1}\left(\frac{2\sqrt{5}+\left(5\right.}{\sqrt{10\left(-1+\sqrt{5}\right.}}\right.$$

```
[Out] -1/2*arctanh((x*(5-5^(1/2))+2*5^(1/2))/(x^2+2*x+2)^(1/2)/(-10+10*5^(1/2))^(
1/2))*(-2+2*5^(1/2))^(1/2)-1/2*arctan((2*5^(1/2)-x*(5+5^(1/2)))/(x^2+2*x+2)
^(1/2)/(10+10*5^(1/2))^(1/2))*(2+2*5^(1/2))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1050, 1044, 213, 209}

$$-\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{x^2+2x+2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\left(5-\sqrt{5}\right)x+2\sqrt{5}}{\sqrt{10\left(\sqrt{5}-1\right)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 +
Sqrt[5])]*Sqrt[2 + 2*x + x^2])]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5
] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5])]*Sqrt[2 + 2*x + x^2])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
```

```
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx = -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}}\,dx}{2\sqrt{5}}$$

$$= \left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{20\left(1-\sqrt{5}\right)+2x^2}\,dx, x, \frac{2\sqrt{5}+\left(5-\sqrt{5}\right)x}{\sqrt{2+2x+x^2}}\right.$$

$$= -\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{2\sqrt{5}-\left(5+\sqrt{5}\right)x}{\sqrt{10\left(1+\sqrt{5}\right)}\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}\left(-1\right.}$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 97, normalized size = 0.77

$$\text{RootSum}\left[8-8\#1+\#1^4\&, \frac{-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)-\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)\#1+\log\left(-x+\sqrt{2+2x+x^2}-\#1\right)\#1^2}{-2+\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]),x]
[Out] RootSum[8 - 8*#1 + #1^4 & , (-Log[-x + Sqrt[2 + 2*x + x^2] - #1] - Log[-x +
 Sqrt[2 + 2*x + x^2] - #1]*#1 + Log[-x + Sqrt[2 + 2*x + x^2] - #1]*#1^2)/(-
2 + #1^3) & ]
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 752 vs. 2(94) = 188.
time = 0.77, size = 753, normalized size = 5.98 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+1)/(x^2+1)/(x^2+2*x+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(10*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(x-1/2*5^(1
/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/2))^(1/2)*(3*(-22+10*5^(1/2))^(
1/2)*(-10+10*5^(1/2))^(1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))
*(2*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1
/2)*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(x-1/2*5^(1/2)+1/2)^2/(
-1/2*5^(1/2)-1/2-x)^2+4*5^(1/2)+10)*(x-1/2*5^(1/2)+1/2)/(-1/2*5^(1/2)-1/2-x
)*(5^(1/2)-5)/((x-1/2*5^(1/2)+1/2)^4/(-1/2*5^(1/2)-1/2-x)^4+3*(x-1/2*5^(1/2
)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+1))*5^(1/2)+5*(-22+10*5^(1/2))^(1/2)*(-10+1
0*5^(1/2))^(1/2)*arctan(1/80*(-22+10*5^(1/2))^(1/2)*((5-5^(1/2))*(2*(x-1/2*
5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+5^(1/2)+3))^(1/2)*(11*5^(1/2)*(x-1/2*
5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+25*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2
)-1/2-x)^2+4*5^(1/2)+10)*(x-1/2*5^(1/2)+1/2)/(-1/2*5^(1/2)-1/2-x)*(5^(1/2)-
5)/((x-1/2*5^(1/2)+1/2)^4/(-1/2*5^(1/2)-1/2-x)^4+3*(x-1/2*5^(1/2)+1/2)^2/(-
1/2*5^(1/2)-1/2-x)^2+1))+20*arctanh((10*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)
-1/2-x)^2-2*5^(1/2)*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2+10+2*5^(1/
2))^(1/2)/(-10+10*5^(1/2))^(1/2))*5^(1/2)-60*arctanh((10*(x-1/2*5^(1/2)+1/2
)^2/(-1/2*5^(1/2)-1/2-x)^2-2*5^(1/2)*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/
2-x)^2+10+2*5^(1/2))^(1/2)/(-10+10*5^(1/2))^(1/2)))/(-2*(5^(1/2)*(x-1/2*5^(
1/2)+1/2)^2/(-1/2*5^(1/2)-1/2-x)^2-5*(x-1/2*5^(1/2)+1/2)^2/(-1/2*5^(1/2)-1/
2-x)^2-5^(1/2)-5)/(1+(x-1/2*5^(1/2)+1/2)/(-1/2*5^(1/2)-1/2-x))^2)^(1/2)/(1+
(x-1/2*5^(1/2)+1/2)/(-1/2*5^(1/2)-1/2-x))/(5^(1/2)-5)/(-10+10*5^(1/2))^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(93) = 186$.
time = 0.38, size = 770, normalized size = 6.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")
```

[Out] 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^2 + 2*x + 2)*x - (2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(sqrt(5) + 5) + 10*sqrt(10)*(sqrt(5) + 3)) + 1/10*sqrt(5)*(sqrt(5)*(2*x + 1) + 5) + 1/2*sqrt(5)*x + 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2)) - sqrt(x^2 + 2*x + 2)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2)) + 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) - 1/2*sqrt(x^2 + 2*x + 2)*(sqrt(5) + 3) + 1/2*x + 1) + 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5) + 5)*arctan(1/200*sqrt(20*x^2 - 20*sqrt(x^2 + 2*x + 2)*x + (2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 20*x + 10*sqrt(5) + 30)*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2))*sqrt(sqrt(5) + 5) - 10*sqrt(10)*(sqrt(5) + 3)) - 1/10*sqrt(5)*(sqrt(5)*(2*x + 1) + 5) - 1/2*sqrt(5)*x + 1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x - sqrt(2)*(x - 2)) - sqrt(x^2 + 2*x + 2)*(5^(3/4)*(sqrt(5)*sqrt(2) - sqrt(2)) + 2*5^(3/4)*sqrt(2)) + 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) + 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 1/2*sqrt(x^2 + 2*x + 2)*(sqrt(5) + 3) - 1/2*x - 1) + 1/40*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x + 1/10*(2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3) - 1/40*5^(1/4)*(sqrt(5)*sqrt(2) - 5*sqrt(2))*sqrt(sqrt(5) + 5)*log(2*x^2 - 2*sqrt(x^2 + 2*x + 2)*x - 1/10*(2*5^(3/4)*sqrt(2)*sqrt(x^2 + 2*x + 2) - 5^(1/4)*(sqrt(5)*sqrt(2)*(2*x + 1) - 5*sqrt(2)))*sqrt(sqrt(5) + 5) + 2*x + sqrt(5) + 3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)

[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(93) = 186.
time = 2.26, size = 444, normalized size = 3.52

$$\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2x+2}\right)-2x+\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2x+2}-2\sqrt{\sqrt{5}-2}-2\right)^2+256\left(\sqrt{5}\left(x-\sqrt{x^2+2x+2}\right)-2x-\sqrt{5}+2\sqrt{x^2+2x+2}+\sqrt{\sqrt{5}-2}+2\right)^2\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(256\left(\sqrt{5}\left(x-\sqrt{x^2+2x+2}\right)-2x-\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+2\sqrt{x^2+2x+2}+2\sqrt{\sqrt{5}-2}-2\right)^2+256\left(\sqrt{5}\left(x-\sqrt{x^2+2x+2}\right)-2x-\sqrt{5}+2\sqrt{x^2+2x+2}-\sqrt{\sqrt{5}-2}+2\right)^2\right)+\frac{\left(\pi+4\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}+\sqrt{5}+4\sqrt{\sqrt{5}-2}+3\right)+\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\sqrt{5}+\frac{7}{2}\sqrt{\sqrt{5}-2}+\frac{3}{2}\right)\right)\sqrt{2\sqrt{5}-2}}{4\left(\sqrt{5}-1\right)}-\frac{\left(\pi+4\arctan\left(-\frac{1}{2}\left(x-\sqrt{x^2+2x+2}\right)\left(2\sqrt{5}\sqrt{\sqrt{5}-2}-\sqrt{5}+4\sqrt{\sqrt{5}-2}-3\right)-\frac{3}{2}\sqrt{5}\sqrt{\sqrt{5}-2}+\frac{1}{2}\sqrt{5}-\frac{7}{2}\sqrt{\sqrt{5}-2}+\frac{3}{2}\right)\right)\sqrt{2\sqrt{5}-2}}{4\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(2*sqrt(5) - 2)*log(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x + sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - 2*sqrt(sqrt(5

```
) - 2) - 2)^2 + 256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*
sqrt(x^2 + 2*x + 2) + sqrt(sqrt(5) - 2) + 2)^2) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(256*(sqrt(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5)*sqrt(sqrt(5) - 2)
 + sqrt(5) + 2*sqrt(x^2 + 2*x + 2) + 2*sqrt(sqrt(5) - 2) - 2)^2 + 256*(sqrt
(5)*(x - sqrt(x^2 + 2*x + 2)) - 2*x - sqrt(5) + 2*sqrt(x^2 + 2*x + 2) - sqr
t(sqrt(5) - 2) + 2)^2) + 1/4*(pi + 4*arctan(1/2*(x - sqrt(x^2 + 2*x + 2))*(
2*sqrt(5)*sqrt(sqrt(5) - 2) + sqrt(5) + 4*sqrt(sqrt(5) - 2) + 3) + 3/2*sqrt
(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) + 7/2*sqrt(sqrt(5) - 2) + 3/2))*sqrt(2*
sqrt(5) - 2)/(sqrt(5) - 1) - 1/4*(pi + 4*arctan(-1/2*(x - sqrt(x^2 + 2*x +
2))*(2*sqrt(5)*sqrt(sqrt(5) - 2) - sqrt(5) + 4*sqrt(sqrt(5) - 2) - 3) - 3/2
*sqrt(5)*sqrt(sqrt(5) - 2) + 1/2*sqrt(5) - 7/2*sqrt(sqrt(5) - 2) + 3/2))*sq
rt(2*sqrt(5) - 2)/(sqrt(5) - 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2\,x+1}{\left(x^2+1\right)\,\sqrt{x^2+2\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)
```

3.999 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)$$

```
[Out] arctan(x/(-x^2+(x^4+1)^(1/2))^(1/2))
```

Rubi [A]

time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2153, 209}

$$\text{ArcTan}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 96, normalized size = 4.36

$$i\tanh^{-1}\left(\sqrt{2}+\sqrt{2}\,x^4-ix^3\sqrt{-x^2+\sqrt{1+x^4}}+\frac{\sqrt{1+x^4}\left(-2x^2+i\sqrt{2}\,x\sqrt{-x^2+\sqrt{1+x^4}}\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[
1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(18) = 36$.
time = 0.77, size = 62, normalized size = 2.82

$$-\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+\left(7\,x^5-x\right)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + \sqrt{x^4 + 1}}\,(x^4 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^4 + 1} - x^2}\,(x^4 + 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)
```

3.1000 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2153, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Mathematica [**A**]
time = 0.84, size = 42, normalized size = 1.05

$$\frac{\tanh^{-1}\left(\frac{\sqrt{cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}\,x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]
[Out] ArcTanh[Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c])
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)\sqrt{c\,x^2+d\sqrt{b\,x^4+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,\sqrt{b\,x^4+a}+c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)
```

3.1001 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

```
[Out] arctan(x*c^(1/2)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2))/a/c^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2153, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{c}\,x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2153

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{-cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Mathematica [A]
time = 0.84, size = 44, normalized size = 1.07

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-cx^2+d\sqrt{a+bx^4}}}{\sqrt{c}\,x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]
[Out] -(ArcTan[Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c]))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^4+a)\sqrt{-c\,x^2+d\sqrt{b\,x^4+a}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
[Out] int(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d\sqrt{a+bx^4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,\sqrt{b\,x^4+a}-c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)
```

3.1002 $\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}$

Optimal. Leaf size=184

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\ldots}{\ldots}\right)\right.}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*arctanh(d^2*(c/d+x)^2*b^(1/2)/(a+b*d^4*(c/d+x)^4)^(1/2))/d^2/b^(1/2)-1/
2*c*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d^2/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1694, 1899, 226, 281, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2}{\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}\right)}{2\sqrt{b}\,d^2}-\frac{c\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*d^2*(c/d + x)^2)/Sqrt[a + b*d^4*(c/d + x)^4]]/(2*Sqrt[b]*d
^2) - (c*(Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(
Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x)
)/a^(1/4)], 1/2])/(2*a^(1/4)*b^(1/4)*d^2*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1694

```
Int[(Pq_)*(Q4_)^(p_), x_Symbol] :> With[{a = Coeff[Q4, x, 0], b = Coeff[Q4,
 x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Sub
st[Int[SimplifyIntegrand[(Pq /. x -> -d/(4*e) + x)*(a + d^4/(256*e^3) - b*(
d/(8*e)) + (c - 3*(d^2/(8*e)))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; E
qQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x]
 && PolyQ[Q4, x, 4] &&  !IGtQ[p, 0]
```

Rule 1899

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*(n/2)]*x^(k*(n/2)), {k, 0, 2
*((q - j)/n) + 1}]*(a + b*x^n)^p, {j, 0, n/2 - 1}], x]] /; FreeQ[{a, b, p},
 x] && PolyQ[Pq, x] && IGtQ[n/2, 0] &&  !PolyQ[Pq, x^(n/2)]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \text{Subst}\left(\int \frac{-\frac{c}{d}+x}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \text{Subst}\left(\int \left(-\frac{c}{d\sqrt{a+bd^4x^4}}+\frac{x}{\sqrt{a+bd^4x^4}}\right) dx \cdots$$

$$= -\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)}{d} + \text{Subst}\Big(\cdots$$

$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx \cdots}}}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c \cdots}}$$

$$= -\frac{c\left(\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\sqrt{\frac{a+b(c+dx)^4}{\left(\sqrt{a}+\sqrt{b}\,(c+dx \cdots}}}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d^2\sqrt{a+b(c \cdots}}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,(c+dx)^2}{\sqrt{a+b(c+dx)^4}}\right)}{2\sqrt{b}\,d^2} - \frac{c\left(\sqrt{a}+\sqrt{b}\,(c \cdots\right.}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.40, size = 330, normalized size = 1.79

$$\frac{\sqrt[4]{-1}\sqrt{2}\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}\,(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\left(i\sqrt{a}+\sqrt{b}\,(c+dx)^2\right)\left(\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}\,c\right)F\left(\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}\,(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\right)\middle|-1\right)-2\sqrt[4]{-1}\sqrt[4]{a}\,\Pi\left(-i;\sin^{-1}\left(\sqrt{-\frac{i\left(\sqrt[4]{-1}\sqrt[4]{a}+\sqrt[4]{b}\,(c+dx)\right)}{\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)}}\right)\middle|-1\right)\right)}{\sqrt[4]{a}\sqrt{b}\,d^2\sqrt{\frac{i\sqrt{a}+\sqrt{b}\,(c+dx)^2}{\left(\sqrt[4]{-1}\sqrt[4]{a}-\sqrt[4]{b}\,(c+dx)\right)^2}}\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-1)^(1/4)*Sqrt[2]*Sqrt[((-I)*((-1)^(1/4)*a^(1/4) + b^(1/4)*(c + d*x)))/((
-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]*(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)*(
((-1)^(1/4)*a^(1/4) - b^(1/4)*c)*EllipticF[ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^
(1/4) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]
 - 2*(-1)^(1/4)*a^(1/4)*EllipticPi[-I, ArcSin[Sqrt[((-I)*((-1)^(1/4)*a^(1/4
) + b^(1/4)*(c + d*x)))/((-1)^(1/4)*a^(1/4) - b^(1/4)*(c + d*x))]], -1]))/(
a^(1/4)*Sqrt[b]*d^2*Sqrt[(I*Sqrt[a] + Sqrt[b]*(c + d*x)^2)/((-1)^(1/4)*a^(1
/4) - b^(1/4)*(c + d*x))^2]*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.56, size = 1528, normalized size = 8.30

method	result	size
default	Expression too large to display	1528
elliptic	Expression too large to display	1528

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-
a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)
-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)
-c)/d))^(1/2)*((I/b*(-a*b^3)^(1/4)-c)/d*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)
/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(
1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((
I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/
d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)
-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))+((1/b*(
-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*EllipticPi((((-I/b*(-a*b^3)^(1
/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*
b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/
2),((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d),(((I/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)
^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)/((1/b*(-a
*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)-c)/d-(-I/b
*(-a*b^3)^(1/4)-c)/d))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{b\,c^4 + 4\,b\,c^3\,d\,x + 6\,b\,c^2\,d^2\,x^2 + 4\,b\,c\,d^3\,x^3 + b\,d^4\,x^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
```

```
[Out] int(x/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

3.1003 $\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x}}$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

```
[Out] 1/2*(cos(2*arctan(b^(1/4)*(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*(
d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*(d*x+c)/a^(1/4))),1/2*2^(1/
2))*(a^(1/2)+d^2*(c/d+x)^2*b^(1/2))*((a+b*d^4*(c/d+x)^4)/(a^(1/2)+d^2*(c/d+
x)^2*b^(1/2))^2)^(1/2)/a^(1/4)/b^(1/4)/d/(a+b*d^4*(c/d+x)^4)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 49, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.041, Rules used = {1120, 226}

$$\frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)^4}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{b}\,(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\frac{c}{d}+x\right)^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3 + b*d^
4*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[b]*d^2*(c/d + x)^2)*Sqrt[(a + b*d^4*(c/d + x)^4)/(Sqrt[a]
+ Sqrt[b]*d^2*(c/d + x)^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*(c + d*x))/a^(1/4
)], 1/2])/(2*a^(1/4)*b^(1/4)*d*Sqrt[a + b*d^4*(c/d + x)^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
```

```
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bc^4+4bc^3dx+6bc^2d^2x^2+4bcd^3x^3+bd^4x^4}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{a+bd^4x^4}}\,dx, x, \frac{c}{d}+x\right)$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+x\right)^2\right)\sqrt{\frac{a+bd^4\left(\frac{c}{d}+x\right)}{\left(\sqrt{a}+\sqrt{b}\,d^2\left(\frac{c}{d}+\right.\right.}}}{2\sqrt[4]{a}\sqrt[4]{b}\,d\sqrt{a+bd^4\left(\right.}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.07, size = 90, normalized size = 0.69

$$-\frac{i\sqrt{\frac{a+b(c+dx)^4}{a}}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,(c+dx)\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\,d\sqrt{a+b(c+dx)^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*c^4 + 4*b*c^3*d*x + 6*b*c^2*d^2*x^2 + 4*b*c*d^3*x^3
+ b*d^4*x^4],x]
```

```
[Out] ((-I)*Sqrt[(a + b*(c + d*x)^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt
[a]]*(c + d*x)], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[a]]*d*Sqrt[a + b*(c + d*x)^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.05, size = 1036, normalized size = 7.91 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 2*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d)*(((-I/b*(-a*b^3)^(1/
4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)*(x-(I/b*(-a*b^3)^(1/4)-c)/d)^2*(((I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(
```

```
1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)/((-1/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-
a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2)*(((I/b*(-a*b^3)^(1/4
)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b
^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2
)/((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4)
-c)/d-(1/b*(-a*b^3)^(1/4)-c)/d)/(d^4*b*(x-(1/b*(-a*b^3)^(1/4)-c)/d)*(x-(I/b
*(-a*b^3)^(1/4)-c)/d)*(x-(-1/b*(-a*b^3)^(1/4)-c)/d)*(x-(-I/b*(-a*b^3)^(1/4)
-c)/d))^(1/2)*EllipticF((((-I/b*(-a*b^3)^(1/4)-c)/d-(I/b*(-a*b^3)^(1/4)-c)/
d)*(x-(1/b*(-a*b^3)^(1/4)-c)/d)/((-I/b*(-a*b^3)^(1/4)-c)/d-(1/b*(-a*b^3)^(1
/4)-c)/d)/(x-(I/b*(-a*b^3)^(1/4)-c)/d))^(1/2),(((I/b*(-a*b^3)^(1/4)-c)/d-(-
1/b*(-a*b^3)^(1/4)-c)/d)*((1/b*(-a*b^3)^(1/4)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/
d)/((1/b*(-a*b^3)^(1/4)-c)/d-(-1/b*(-a*b^3)^(1/4)-c)/d)/((I/b*(-a*b^3)^(1/4
)-c)/d-(-I/b*(-a*b^3)^(1/4)-c)/d))^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x +
 b*c^4 + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bc^4 + 4bc^3dx + 6bc^2d^2x^2 + 4bcd^3x^3 + bd^4x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d**4*x**4+4*b*c*d**3*x**3+6*b*c**2*d**2*x**2+4*b*c**3*d*x+b*
c**4+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*c**4 + 4*b*c**3*d*x + 6*b*c**2*d**2*x**2 + 4*b*c*d**3
*x**3 + b*d**4*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*d^4*x^4+4*b*c*d^3*x^3+6*b*c^2*d^2*x^2+4*b*c^3*d*x+b*c^4+a)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,c^4+4\,b\,c^3\,d\,x+6\,b\,c^2\,d^2\,x^2+4\,b\,c\,d^3\,x^3+b\,d^4\,x^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2),x)
```

```
[Out] int(1/(a + b*c^4 + b*d^4*x^4 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + 4*b*c*d^3*x^
3)^(1/2), x)
```

3.1004 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,x}{\sqrt{d}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{bd-ae}}$$

```
[Out] arctanh(x*(-a*e+b*d)^(1/2)/d^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^(1/2)/(-a*e+b*d
)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2137, 214}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqr
t[b*d - a*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx &= a\text{Subst}\left(\int \frac{1}{ad-(abd-a^2e)\,x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}\,x}{\sqrt{d}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{bd-ae}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 10.56, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae-\sqrt{a}\sqrt{-4cd^2+ae^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{ae+\sqrt{a}\sqrt{-4cd^2+ae^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e
^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/
(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[
b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt
[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 514, normalized size = 9.52

method	result
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^4+b\,x^2+a}}{x\sqrt{(ae-bd)\,d}}\right)}{\sqrt{(ae-bd)\,d}}$
default	$-\frac{\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\text{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4d\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/4/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1
/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_
alpha^2*c*d+a*e)*(-1/(_alpha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alpha^2*
c*x^2+_alpha^2*b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+a)^(
1/2))+1/d/a*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((-b+(-4*a*c+b^2)^(1/2))/a)^(
1/2)*(2+b*x^2/a-x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a+x^2/a*(-4*a*c+b^
2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*d+(
-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/c/d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*e+a
*d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*x^2*e + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

Fricas [A]
time = 7.99, size = 318, normalized size = 5.89

$$\left[\frac{\log\left(-\frac{c^2d^2x^8+8\,bcd^2x^6+2\,(4\,b^2+ac)\,d^2x^4+a^2x^4e^2+8\,abd^2x^2+a^2d^2+4\,(cdx^5+2\,bdx^3-ax^3e+adx)\,\sqrt{cx^4+bx^2+a}\,\sqrt{bd^2-ade}-2\,(3\,acdx^6+4\,abdx^4+3\,a^2dx^2)\,e}{c^2d^2x^8+2\,acd^2x^4+a^2x^4e^2+a^2d^2+2\,(acdx^6+a^2dx^2)\,e}\right)}{4\,\sqrt{bd^2-ade}}, \frac{\sqrt{-bd^2+ade}\,\arctan\left(-\frac{2\,\sqrt{cx^4+bx^2+a}\,\sqrt{-bd^2+ade}\,x}{cdx^4+2\,bdx^2-ax^2e+ad}\right)}{2\,(bd^2-ade)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [1/4*log(-(c^2*d^2*x^8 + 8*b*c*d^2*x^6 + 2*(4*b^2 + a*c)*d^2*x^4 + a^2*x^4*
e^2 + 8*a*b*d^2*x^2 + a^2*d^2 + 4*(c*d*x^5 + 2*b*d*x^3 - a*x^3*e + a*d*x)*s
qrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 - a*d*e) - 2*(3*a*c*d*x^6 + 4*a*b*d*x^4 +
 3*a^2*d*x^2)*e)/(c^2*d^2*x^8 + 2*a*c*d^2*x^4 + a^2*x^4*e^2 + a^2*d^2 + 2*(
a*c*d*x^6 + a^2*d*x^2)*e))/sqrt(b*d^2 - a*d*e), 1/2*sqrt(-b*d^2 + a*d*e)*ar
ctan(-2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + 2*b*d*x^2
 - a*x^2*e + a*d))/(b*d^2 - a*d*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{a}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cx^4}{ad\sqrt{a+bx^2+cx^4}+aex^2\sqrt{a+bx^2+cx^4}+cdx^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*
x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(
a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*x^2*e + a*d)*sqrt(c*x^4 + b*x^2 + a)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a-c\,x^4}{\left(c\,d\,x^4+a\,e\,x^2+a\,d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1005 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\,\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{bd+ae}}$$

```
[Out] arctan(x*(a*e+b*d)^(1/2)/d^(1/2)/(c*x^4-b*x^2+a)^(1/2))/d^(1/2)/(a*e+b*d)^(
1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2137, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\,\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]
```

```
[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt
[b*d + a*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}\,(ad+aex^2+cdx^4)}\,dx &= a\text{Subst}\left(\int \frac{1}{ad-(-abd-a^2e)\,x^2}\,dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}\,x}{\sqrt{d}\,\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{bd+ae}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 10.53, size = 416, normalized size = 7.85

$$\frac{i\sqrt{2+\frac{4cx^2}{-b+\sqrt{b^2-4ac}}}\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b-\sqrt{b^2-4ac}\right)d}{-ae+\sqrt{a}\sqrt{-4cd^2+ae^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(-b+\sqrt{b^2-4ac}\right)d}{ae+\sqrt{a}\sqrt{-4cd^2+ae^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\right)}{2\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}d\sqrt{a-bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x
]
```

```
[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4
*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])] - EllipticPi[(
(b - Sqrt[b^2 - 4*a*c])*d)/(-(a*e) + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*Arc
Sinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(
b + Sqrt[b^2 - 4*a*c])] - EllipticPi[((-b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sq
rt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*
a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])]))/(Sqrt[c/(-b +
 Sqrt[b^2 - 4*a*c])]*d*Sqrt[a - b*x^2 + c*x^4])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 517, normalized size = 9.75

method	result
elliptic	$-\frac{\arctan\left(\frac{d\sqrt{c\,x^4-b\,x^2+a}}{x\sqrt{(ae+bd)\,d}}\right)}{\sqrt{(ae+bd)\,d}}$
default	$-\frac{\sqrt{2}\sqrt{4-\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}}{2}\right.}{4d\sqrt{\frac{b+\sqrt{-4ac+b^2}}{a}}\sqrt{c\,x^4-b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x,method=_RETURN
VERBOSE)
```

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_a
lpha^2*c*d+a*e)*(-1/(-_alpha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c
*x^2-_alpha^2*b-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1
/2))+1/d/a*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(2-b*x^2/a-x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(2-b*x^2/a+x^2/a*(-4*a*c+b^2)
^(1/2))^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),-1/2*(-_alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*d-(-
4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/c/d,(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2))),_alpha=RootOf(_Z^4*c*d+_Z^2*a*e+a*
d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*x^2*e + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

Fricas [A]
time = 8.15, size = 319, normalized size = 6.02

$$\left[-\frac{\sqrt{-bd^2-ade}\,\log\left(-\frac{c^2d^2x^8-8\,bcd^2x^6+2\,(4\,b^2+ac)d^2x^4+a^2x^4e^2-8\,abd^2x^2+a^2d^2+4\,(cdx^5-2\,bdx^3-ax^3e+adx)\sqrt{cx^4-bx^2+a}\,\sqrt{-bd^2-ade}-2\,(3\,acdx^6-4\,abdx^4+3\,a^2dx^2)e}{c^2d^2x^8+2\,acd^2x^4+a^2x^4e^2+a^2d^2+2\,(acdx^6+a^2dx^2)e}\right)}{4\,(bd^2+ade)},\ -\frac{\arctan\left(-\frac{2\,\sqrt{cx^4-bx^2+a}\,\sqrt{bd^2+ade}\,x}{cdx^4-2\,bdx^2-ax^2e+ad}\right)}{2\,\sqrt{bd^2+ade}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

```
[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 8*b*c*d^2*x^6 + 2*(4*b^2 + a
*c)*d^2*x^4 + a^2*x^4*e^2 - 8*a*b*d^2*x^2 + a^2*d^2 + 4*(c*d*x^5 - 2*b*d*x^
3 - a*x^3*e + a*d*x)*sqrt(c*x^4 - b*x^2 + a)*sqrt(-b*d^2 - a*d*e) - 2*(3*a*
c*d*x^6 - 4*a*b*d*x^4 + 3*a^2*d*x^2)*e)/(c^2*d^2*x^8 + 2*a*c*d^2*x^4 + a^2*
x^4*e^2 + a^2*d^2 + 2*(a*c*d*x^6 + a^2*d*x^2)*e))/(b*d^2 + a*d*e), -1/2*arc
tan(-2*sqrt(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - 2*b*d*x^2 -
 a*x^2*e + a*d))/sqrt(b*d^2 + a*d*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{a}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\right)dx-\int\frac{cx^4}{ad\sqrt{a-bx^2+cx^4}+aex^2\sqrt{a-bx^2+cx^4}+cdx^4\sqrt{a-bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*x^2*e + a*d)*sqrt(c*x^4 - b*x^2 + a)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a-c\,x^4}{\left(c\,d\,x^4+a\,e\,x^2+a\,d\right)\,\sqrt{c\,x^4-b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

3.1006 $\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5-2x+x^2}\right)$$

```
[Out] 1/12*arctanh((x^2-2*x+5)^(1/2))-1/12*arctan(1/3*(1-x)*3^(1/2)/(x^2-2*x+5)^(
1/2))*3^(1/2)-1/156*arctanh(1/13*(7-3*x)*13^(1/2)/(x^2-2*x+5)^(1/2))*13^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2099, 738, 212, 1039, 996, 209, 1038, 213}

$$-\frac{\text{ArcTan}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
```

```
[Out] -1/4*ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/Sqrt[3] - ArcTanh[(7 - 3
*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x +
x^2]]/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2099

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{5-2x+x^2}\,(8+x^3)}\,dx = \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}} + \frac{4-x}{12\,(4-2x+x^2)\,\sqrt{5-2x+x^2}}\right) dx$$

$$= \frac{1}{12}\int \frac{1}{(2+x)\sqrt{5-2x+x^2}}\,dx + \frac{1}{12}\int \frac{4-x}{(4-2x+x^2)\,\sqrt{5-2x+x^2}}\,dx$$

$$= -\left(\frac{1}{24}\int \frac{-2+2x}{(4-2x+x^2)\,\sqrt{5-2x+x^2}}\,dx\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{52-x^2}\,dx, x, \right.$$

$$= -\frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\,\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{-2+2x^2}\,dx, x, \sqrt{5-2x}\right.$$

$$= \frac{\tan^{-1}\left(\frac{-2+2x}{2\sqrt{3}\,\sqrt{5-2x+x^2}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\,\sqrt{5-2x+x^2}}\right)}{12\sqrt{13}} + \frac{1}{12}\tan$$

Mathematica [**A**]
time = 0.24, size = 90, normalized size = 1.07

$$\frac{1}{156}\left(-13\sqrt{3}\tan^{-1}\left(\frac{4-2x+x^2-(-1+x)\sqrt{5-2x+x^2}}{\sqrt{3}}\right) + 13\tanh^{-1}\left(\sqrt{5-2x+x^2}\right) + 2\sqrt{13}\tanh^{-1}\left(\frac{2+x-\sqrt{5-2x+x^2}}{\sqrt{13}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]
[Out] (-13*Sqrt[3]*ArcTan[(4 - 2*x + x^2 - (-1 + x)*Sqrt[5 - 2*x + x^2])/Sqrt[3]]
 + 13*ArcTanh[Sqrt[5 - 2*x + x^2]] + 2*Sqrt[13]*ArcTanh[(2 + x - Sqrt[5 - 2
*x + x^2])/Sqrt[13]])/156
```

Maple [**A**]
time = 0.54, size = 69, normalized size = 0.82

method	result
default	$\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\,(-2+2x)}{6\sqrt{x^2-2x+5}}\right)}{12} - \frac{\sqrt{13}\operatorname{arctanh}\left(\frac{(14-6x)\sqrt{13}}{26\sqrt{(x+2)^2-6x+1}}\right)}{156}$
trager	$\frac{\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{3\,\text{RootOf}\left(\_Z^2-13\right)x-7\,\text{RootOf}\left(\_Z^2-13\right)+13\sqrt{x^2-2x+5}}{x+2}\right)}{156} - \ln\left(\frac{5760\,\text{RootOf}\left(144\_Z^2+12\_Z\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*arctanh((x^2-2*x+5)^(1/2))+1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2-2*x+5)
^(1/2)*(-2+2*x))-1/156*13^(1/2)*arctanh(1/26*(14-6*x)*13^(1/2)/((x+2)^2-6*x
+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 8)*sqrt(x^2 - 2*x + 5)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(64) = 128.
time = 0.37, size = 154, normalized size = 1.83

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x-2)+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2-2x+5}\right)+\frac{1}{156}\sqrt{13}\log\left(-\frac{\sqrt{13}(3x-7)+\sqrt{x^2-2x+5}\left(3\sqrt{13}+13\right)+9x-21}{x+2}\right)+\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}(x-2)-3x+6\right)-\frac{1}{24}\log\left(x^2-\sqrt{x^2-2x+5}x-x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - 2) + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 - 2*x + 5)) +
1/156*sqrt(13)*log(-(sqrt(13)*(3*x - 7) + sqrt(x^2 - 2*x + 5)*(3*sqrt(13) +
 13) + 9*x - 21)/(x + 2)) + 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*(x - 2) - 3*
x + 6) - 1/24*log(x^2 - sqrt(x^2 - 2*x + 5)*x - x + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x+2)\left(x^2-2x+4\right)\sqrt{x^2-2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(64) = 128.
time = 2.14, size = 164, normalized size = 1.95

$$-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2-2x+5}-2\right)\right)+\frac{1}{156}\sqrt{13}\log\left(\frac{\left|-2x-2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}{\left|-2x+2\sqrt{13}+2\sqrt{x^2-2x+5}-4\right|}\right)+\frac{1}{24}\log\left(\left(x-\sqrt{x^2-2x+5}\right)^2-4x+4\sqrt{x^2-2x+5}+7\right)-\frac{1}{24}\log\left(\left(x-\sqrt{x^2-2x+5}\right)^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5))) + 1/12*sqrt(3)
*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 - 2*x + 5) - 2)) + 1/156*sqrt(13)*log(ab
s(-2*x - 2*sqrt(13) + 2*sqrt(x^2 - 2*x + 5) - 4)/abs(-2*x + 2*sqrt(13) + 2*
sqrt(x^2 - 2*x + 5) - 4)) + 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 - 4*x + 4*
sqrt(x^2 - 2*x + 5) + 7) - 1/24*log((x - sqrt(x^2 - 2*x + 5))^2 + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3+8)\,\sqrt{x^2-2\,x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)),x)
```

```
[Out] int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)
```

3.1007 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal. Leaf size=20

$$\frac{\sqrt{x^2}\,\sqrt{1+x^2}}{x}$$

[Out] (x^2)^(1/2)*(x^2+1)^(1/2)/x

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1978, 15, 267}

$$\frac{\sqrt{x^2}\,\sqrt{x^2+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(1 + x^2)],x]

[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^F
racPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}}\,dx$$
$$= \frac{\sqrt{x^2}\int \frac{x}{\sqrt{1+x^2}}\,dx}{x}$$
$$= \frac{\sqrt{x^2}\,\sqrt{1+x^2}}{x}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{1+x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2/(1 + x^2)],x]
[Out] x/Sqrt[x^2/(1 + x^2)]
```

Maple [A]
time = 0.02, size = 23, normalized size = 1.15

method	result	size
gosper	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
default	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
trager	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
risch	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
[Out] (x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

Maxima [A]
time = 0.48, size = 7, normalized size = 0.35

$$\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)
```

Fricas [A]
time = 0.38, size = 22, normalized size = 1.10

$$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
```

```
[Out] (x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

Sympy [A]
time = 0.16, size = 26, normalized size = 1.30

$$x\sqrt{\frac{x^2}{x^2+1}}+\frac{\sqrt{\frac{x^2}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2/(x**2+1))**(1/2),x)
```

```
[Out] x*sqrt(x**2/(x**2 + 1)) + sqrt(x**2/(x**2 + 1))/x
```

Giac [A]
time = 1.92, size = 15, normalized size = 0.75

$$\sqrt{x^2+1}\,\mathrm{sgn}(x)-\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

Mupad [B]
time = 3.41, size = 13, normalized size = 0.65

$$\frac{\sqrt{x^4+x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2/(x^2 + 1))^(1/2),x)
```

```
[Out] (x^2 + x^4)^(1/2)/x
```

3.1008 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal. Leaf size=46

$$\frac{2x\sqrt{x^n}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}$$

```
[Out] 2*x*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)*(x^n)^(1/2)/(2+n)
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1978, 15, 371}

$$\frac{2x\sqrt{x^n}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^n/(1 + x^n)],x]
```

```
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3 + 2/n)/2, -x^n])/(2 + n)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a*x^n)^FracPart[m]/x^(n*FracPart[m])), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1978

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Rubi steps

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \frac{\sqrt{x^n}}{\sqrt{1+x^n}}\,dx$$
$$= \left(x^{-n/2}\sqrt{x^n}\right)\int \frac{x^{n/2}}{\sqrt{1+x^n}}\,dx$$
$$= \frac{2x\sqrt{x^n}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right);\frac{1}{2}\left(3+\frac{2}{n}\right);-x^n\right)}{2+n}$$

Mathematica [**A**]
time = 0.01, size = 38, normalized size = 0.83

$$\frac{2x\sqrt{x^n}\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n};\frac{3}{2}+\frac{1}{n};-x^n\right)}{2+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^n/(1 + x^n)],x]
[Out] (2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(2
 + n)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n/(1+x^n))^(1/2),x)
[Out] int((x^n/(1+x^n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^n}{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**n/(1+x**n))**(1/2),x)
```

```
[Out] Integral(sqrt(x**n/(x**n + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^n/(x^n + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{x^n}{x^n + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n/(x^n + 1))^(1/2),x)
```

```
[Out] int((x^n/(x^n + 1))^(1/2), x)
```

3.1009 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tan^{-1}\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

```
[Out] e*f*arctan(1/2*(a*b+(4*a^2-2*a*c+b^2)*x+a*b*x^2)/a/(2*a-c)^(1/2)/(a*x^4+b*x
^3+c*x^2+b*x+a)^(1/2))/a/d/(2*a-c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {2109}

$$\frac{ef\,\text{ArcTan}\left(\frac{x(4a^2-2ac+b^2)+abx^2+ab}{2a\sqrt{2a-c}\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 +
 a*x^4]),x]
```

```
[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sq
rt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Rule 2109

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[a*(f/(d*Rt[a^2*(2*a - c), 2]))*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b
*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])], x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}}\,dx = \frac{ef\tan^{-1}\left(\frac{ab+(4a^2+b^2-2ac)x+abx^2}{2a\sqrt{2a-c}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

Mathematica [A]
time = 0.72, size = 83, normalized size = 0.94

$$-\frac{2ef\tan^{-1}\left(\frac{\sqrt{a}\sqrt{2a-c}\,x}{a+bx+ax^2-\sqrt{a}\sqrt{a+bx+cx^2+bx^3+ax^4}}\right)}{a\sqrt{2a-c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b
*x^3 + a*x^4]),x]
```

```
[Out] (-2*e*f*ArcTan[(Sqrt[a]*Sqrt[2*a - c]*x)/(a + b*x + a*x^2 - Sqrt[a]*Sqrt[a
+ b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.37, size = 242984, normalized size = 2761.18

method	result	size
default	Expression too large to display	242984
elliptic	Expression too large to display	254498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x,me
thod=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((f*x^2*e - f*e)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

Fricas [A]
time = 2.23, size = 326, normalized size = 3.70

$$\left[-\frac{\sqrt{-2\,a+c}\,fe\log\left(\frac{2\,ab^3x^3+2\,ab^3x-(8\,a^4-a^2b^2-4\,a^3c)x^4-8\,a^4+a^2b^2+4\,a^3c+(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2-4\,(6\,a^3+ab^2)c)x^2-4\,(a^2bx^2+a^2b+(4\,a^3+ab^2-2\,a^2c)x)\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{-2\,a+c}}{a^2x^4+2\,abx^3+2\,abx+(2\,a^2+b^2)x^2+a^2}\right)}{2\,(2\,a^2-ac)d},\ -\frac{\sqrt{2\,a-c}\,f\arctan\left(\frac{2\,\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{2\,a-c}\,a}{abx^2+ab+(4\,a^2+b^2-2\,ac)x}\right)e}{(2\,a^2-ac)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*f*e*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 -
4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a
^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 -
 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4
 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 - a*c)*d), -sqrt
(2*a - c)*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*a/
(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))*e/((2*a^2 - a*c)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\,dx+\int \left(-\frac{1}{ax^2\sqrt{ax^4+a+bx^3+bx+cx^2}+a\sqrt{ax^4+a+bx^3+bx+cx^2}+bx\sqrt{ax^4+a+bx^3+bx+cx^2}}\right)dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)
**(1/2),x)
```

```
[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sq
rt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x
 + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x*
*2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*
x**3 + b*x + c*x**2)), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(f*x^2*e - f*e)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2
+ b*d*x + a*d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,f-e\,f\,x^2}{\left(a\,d\,x^2+b\,d\,x+a\,d\right)\,\sqrt{a\,x^4+b\,x^3+c\,x^2+b\,x+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)),x)
```

```
[Out] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x
^2)^(1/2)), x)
```

3.1010 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx$

Optimal. Leaf size=88

$$\frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3-ax^4}}\right)}{a\sqrt{2a+c}\,d}$$

```
[Out] e*f*arctanh(1/2*(a*b-(4*a^2+2*a*c+b^2)*x+a*b*x^2)/a/(2*a+c)^(1/2)/(-a*x^4+b
*x^3+c*x^2+b*x-a)^(1/2))/a/d/(2*a+c)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2110}

$$\frac{ef\tanh^{-1}\left(\frac{-x(4a^2+2ac+b^2)+abx^2+ab}{2a\sqrt{2a+c}\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x
^3 - a*x^4]),x]
```

```
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*S
qrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*d)
```

Rule 2110

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(-a)*(f/(d*Rt[(-a^2)*(2*a - c), 2]))*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*
x + a*b*x^2)/(2*Rt[(-a^2)*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^
4])], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f +
 g, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}}\,dx = \frac{ef\tanh^{-1}\left(\frac{ab-(4a^2+b^2+2ac)x+abx^2}{2a\sqrt{2a+c}\sqrt{-a+bx+cx^2+bx^3}}\right.}{a\sqrt{2a+c}\,d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 16.39, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-(a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2
 + b*x^3 - a*x^4]),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.33, size = 269221, normalized size = 3059.33

method	result	size
default	Expression too large to display	269221
elliptic	Expression too large to display	281960

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x,
method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2*e - f*e)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

Fricas [A]
time = 2.28, size = 333, normalized size = 3.78

$$\left[\frac{\sqrt{2\,a+c}\,fe\log\left(\frac{2\,ab^3x^3+2\,ab^3x+(8\,a^4-a^2b^2+4\,a^3c)x^4+8\,a^4-a^2b^2+4\,a^3c-(16\,a^4+10\,a^2b^2+b^4+8\,a^2c^2+4\,(6\,a^3+ab^2)c)x^2-4\,(a^2bx^2+a^2b-(4\,a^3+ab^2+2\,a^2c)x)\sqrt{-ax^4+bx^3+cx^2+bx-a}\,\sqrt{2\,a+c}}{a^2x^4-2\,abx^3-2\,abx+(2\,a^2+b^2)x^2+a^2}\right)}{2\,(2\,a^2+ac)d}, -\frac{\sqrt{-2\,a-c}\,f\arctan\left(\frac{2\,\sqrt{-ax^4+bx^3+cx^2+bx-a}\,a\sqrt{-2\,a-c}}{abx^2+ab-(4\,a^2+b^2+2\,ac)x}\right)e}{(2\,a^2+ac)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2*a + c)*f*e*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*
a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2
*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2
*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 -
 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-
2*a - c)*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a - c)
/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))*e/((2*a^2 + a*c)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{ef\left(\int \frac{x^2}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\,dx+\int \left(-\frac{1}{ax^2\sqrt{-ax^4-a+bx^3+bx+cx^2}+a\sqrt{-ax^4-a+bx^3+bx+cx^2}-bx\sqrt{-ax^4-a+bx^3+bx+cx^2}}\right)dx\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-
a)**(1/2),x)
```

```
[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sq
rt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b
*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c
*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a
 + b*x**3 + b*x + c*x**2)), x))/d
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1
/2),x, algorithm="giac")
```

```
[Out] integrate((f*x^2*e - f*e)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2
- b*d*x + a*d)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{e\,f-e\,f\,x^2}{\left(a\,d\,x^2-b\,d\,x+a\,d\right)\,\sqrt{-a\,x^4+b\,x^3+c\,x^2+b\,x-a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)),x)
```

```
[Out] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*
x^2)^(1/2)), x)
```

3.1011 $\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2155, 221}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 107 vs. $2(46) = 92$.
time = 4.94, size = 107, normalized size = 2.33

$$-\frac{\sqrt{2}\,b\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\tan^{-1}\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a(-1+ax^2)}{b^2}}\right)}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-
1 + a*x^2))/b^2])]])/(a*x))
```

Maple [**F**]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Fricas [A]
time = 5.85, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\, b \log\left(-4\, a x^2 - 4\, b x \sqrt{\frac{a^2 x^2 - a}{b^2}} - 2\, \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}} \left(\sqrt{2}\, \sqrt{a}\, x + \frac{\sqrt{2}\, b \sqrt{\frac{a^2 x^2 - a}{b^2}}}{\sqrt{a}}\right) + 1\right)}{2\, \sqrt{a}}, -\sqrt{2}\, b \sqrt{-\frac{1}{a}} \arctan\left(\frac{\sqrt{2}\, \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}}\, \sqrt{-\frac{1}{a}}}{2\, x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x \left(a x + b \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}}\right)}}{x \sqrt{\frac{a \left(a x^2 - 1\right)}{b^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a*
*2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a\,x^2+b\,x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

3.1012 $\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.40, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2155, 222}

$$\frac{\sqrt{2}\,b\text{ArcSin}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx = -\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx, x, -ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. $2(46) = 92$.
time = 4.49, size = 114, normalized size = 2.48

$$\frac{\sqrt{2}\,b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x + b
*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1 +
 a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a\,x^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

[Out] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)

Fricas [A]
time = 5.48, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}\,b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}\,ax\sqrt{-\frac{1}{a}}-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}\,x}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-x\left(ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a\,(ax^2+1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**
2*x**2/b**2)**(1/2),x)

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}-a\,x^2}}{x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)),x)
```

```
[Out] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)
/b^2)^(1/2)), x)
```

3.1013 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsinh((a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2156, 2155, 221}

$$\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^
2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[
a]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2156

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{ax^2+bx\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=\frac{\left(\sqrt{2}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\,dx,x,ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 107 vs. 2(46) = 92.
time = 0.03, size = 107, normalized size = 2.33

$$-\frac{\sqrt{2}\,b\sqrt{x\left(-ax+b\sqrt{\frac{a\left(-1+ax^2\right)}{b^2}}\right)}\sqrt{x\left(ax+b\sqrt{\frac{a\left(-1+ax^2\right)}{b^2}}\right)}\tan^{-1}\left(\sqrt{2}\sqrt{x\left(-ax+b\sqrt{\frac{a\left(-1+ax^2\right)}{b^2}}\right)}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2)
 + (a^2*x^2)/b^2]),x]
```

```
[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-1 + a*x^2))/b^2])]*Sqrt[x*(a*x +
b*Sqrt[(a*(-1 + a*x^2))/b^2])]*ArcTan[Sqrt[2]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(-
1 + a*x^2))/b^2])]])/(a*x))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

```
[Out] int((x*(a*x+b*(-a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/
2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Fricas [A]
time = 5.76, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2}\,b\log\left(-4\,ax^2-4\,bx\sqrt{\frac{a^2x^2-a}{b^2}}-2\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\left(\sqrt{2}\,\sqrt{a}\,x+\frac{\sqrt{2}\,b\sqrt{\frac{a^2x^2-a}{b^2}}}{\sqrt{a}}\right)+1\right)}{2\,\sqrt{a}},\ -\sqrt{2}\,b\sqrt{-\frac{1}{a}}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{ax^2+bx\sqrt{\frac{a^2x^2-a}{b^2}}}\,\sqrt{-\frac{1}{a}}}{2\,x}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2
+ b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2
 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*
sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(ax+b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2-1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(-a/b**2+a**
2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2
- 1)/b**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^
2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a
/b^2)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x\left(a\,x+b\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}\right)}}{x\,\sqrt{\frac{a^2\,x^2}{b^2}-\frac{a}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a
/b^2)^(1/2)), x)
```

3.1014 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] b*arcsin((a*x-b*(a/b^2+a^2*x^2/b^2)^(1/2))/a^(1/2))*2^(1/2)/a^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2156, 2155, 222}

$$\frac{\sqrt{2}\,b\text{ArcSin}\left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x
^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 2155

```
Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c
_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[Sqrt[2]*(b/a), Subst[Int[1/Sqrt[1 +
x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]
```

Rule 2156

```
Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*S
qrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d
*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^
2*d, 0] && EqQ[b^2*c*e + a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx=\int \frac{\sqrt{-ax^2+bx\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

$$=-\frac{\left(\sqrt{2}\,b\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^2}{a}}}\,dx,x,-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}{a}$$

$$=\frac{\sqrt{2}\,b\sin^{-1}\left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 114 vs. 2(46) = 92.
time = 0.01, size = 114, normalized size = 2.48

$$\frac{\sqrt{2}\,b\sqrt{x\left(-ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{ax\left(ax+b\sqrt{\frac{a(1+ax^2)}{b^2}}\right)}}{\sqrt{a}}\right)}{a^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 +
(a^2*x^2)/b^2]),x]
```

```
[Out] (Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[a*x*(a*x + b
*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTan[(Sqrt[2]*Sqrt[a*x*(a*x + b*Sqrt[(a*(1 +
 a*x^2))/b^2])])/Sqrt[a]])/(a^(3/2)*x)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
[Out] int((x*(-a*x+b*(a/b^2+a^2/b^2*x^2)^(1/2)))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2
),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Fricas [A]
time = 5.27, size = 161, normalized size = 3.50

$$\left[\frac{1}{2}\sqrt{2}\,b\sqrt{-\frac{1}{a}}\log\left(4\,ax^2-4\,bx\sqrt{\frac{a^2x^2+a}{b^2}}+2\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}\left(\sqrt{2}\,ax\sqrt{-\frac{1}{a}}-\sqrt{2}\,b\sqrt{-\frac{1}{a}}\sqrt{\frac{a^2x^2+a}{b^2}}\right)+1\right),-\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\sqrt{-ax^2+bx\sqrt{\frac{a^2x^2+a}{b^2}}}}{2\sqrt{a}\,x}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="fricas")
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*s
qrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)
*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1), -sqrt(2)*b*arctan(1/2*sqrt(2)*
sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x \left(ax - b\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}}\right)}}{x\sqrt{\frac{a(ax^2+1)}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-a*x+b*(a/b**2+a**2*x**2/b**2)**(1/2)))**(1/2)/x/(a/b**2+a**2
*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2
 + 1)/b**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-a*x+b*(a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(a/b^2+a^2*x^2/b^2
)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 +
a/b^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x\,\left(a\,x-b\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}\right)}}{x\,\sqrt{\frac{a}{b^2}+\frac{a^2\,x^2}{b^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)),x)
```

```
[Out] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2)^(1/2)))^(1/2)/(x*(a/b^2 + (a^2*x^2
)/b^2)^(1/2)), x)
```

3.1015 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}$

Optimal. Leaf size=19

$$2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)$$

```
[Out] 2*ln(1+(x-4)^(1/2)+(-1+x)^(1/2))
```

Rubi [A]
time = 0.37, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6820, 1600, 6816}

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1
+ Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]
```

Rule 1600

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6816

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 6820

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}\,x+\sqrt{-1+x}\,x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+x^2)}\,dx &= \int \frac{\sqrt{-1+x}\left(-4+\sqrt{-4+x}\sqrt{-1+x}+\right.}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(4-5x+} \\
&= \int \frac{-4+\sqrt{-4+x}\sqrt{-1+x}+x}{\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)(-4+x)} \\
&= 2\log\left(1+\sqrt{-4+x}+\sqrt{-1+x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 27, normalized size = 1.42

$$4\tanh^{-1}\left(1-\frac{2\sqrt{-4+x}}{3}+\frac{2\sqrt{-1+x}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x
)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]
```

```
[Out] 4*ArcTanh[1 - (2*Sqrt[-4 + x])/3 + (2*Sqrt[-1 + x])/3]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(15) = 30$.
time = 0.40, size = 147, normalized size = 7.74

method	result
default	$\frac{\ln(-5+x)}{2}+\frac{\ln\left(2+\sqrt{-1+x}\right)}{2}-\frac{\ln\left(-2+\sqrt{-1+x}\right)}{2}-\frac{\ln\left(1+\sqrt{x-4}\right)}{2}+\frac{\ln\left(-1+\sqrt{x-4}\right)}{2}+\frac{7\sqrt{x-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(x-4)^(1/2)+x*(x-4)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2-5*x+4)/
(1+(x-4)^(1/2)+(-1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-5+x)+1/2*ln(2+(-1+x)^(1/2))-1/2*ln(-2+(-1+x)^(1/2))-1/2*ln(1+(x-4)^
(1/2))+1/2*ln(-1+(x-4)^(1/2))+7/4*(x-4)^(1/2)*(-1+x)^(1/2)/(x^2-5*x+4)^(1/2
)*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2))+1/4*(x-4)^(1/2)*(-1+x)^(1/2)*(2*
ln(-5/2+x+(x^2-5*x+4)^(1/2))-5*arctanh(1/4*(-17+5*x)/(x^2-5*x+4)^(1/2)))/(x
^2-5*x+4)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(15) = 30$.
time = 0.33, size = 94, normalized size = 4.95

$$\frac{1}{2}\log\left(x-1\right)+\frac{1}{2}\log\left(\frac{2x^{2}+2\left(\left(x-1\right)\sqrt{x-4}+2x-6\right)\sqrt{x-1}+2\left(2x-3\right)\sqrt{x-4}-7x+3}{2\left(\left(x-1\right)\sqrt{x-4}+2x-6\right)}\right)+\frac{1}{2}\log\left(\frac{\left(x-1\right)\sqrt{x-4}+2x-6}{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*((x - 1)*sqrt(x - 4) + 2*x - 6)*sqr
t(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x -
6)) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(15) = 30$.
time = 0.36, size = 96, normalized size = 5.05

$$-\frac{1}{2}\log\left(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23\right)+\frac{1}{2}\log\left(\sqrt{x-1}\sqrt{x-4}-x+7\right)+\frac{1}{2}\log(x-5)+\frac{1}{2}\log\left(\sqrt{x-1}+2\right)-\frac{1}{2}\log\left(\sqrt{x-1}-2\right)-\frac{1}{2}\log\left(\sqrt{x-4}+1\right)+\frac{1}{2}\log\left(\sqrt{x-4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(-(4*x - 11)*sqrt(x - 1)*sqrt(x - 4) + 4*x^2 - 21*x + 23) + 1/2*log
(sqrt(x - 1)*sqrt(x - 4) - x + 7) + 1/2*log(x - 5) + 1/2*log(sqrt(x - 1) +
2) - 1/2*log(sqrt(x - 1) - 2) - 1/2*log(sqrt(x - 4) + 1) + 1/2*log(sqrt(x -
 4) - 1)
```

Sympy [A]
time = 157.08, size = 17, normalized size = 0.89

$$2\log\left(\sqrt{x-4}+\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/
(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)
```

```
[Out] 2*log(sqrt(x - 4) + sqrt(x - 1) + 1)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(15) = 30$.
time = 3.33, size = 58, normalized size = 3.05

$$-\log\left(\sqrt{x-1}-\sqrt{x-4}+1\right)-\log\left(\sqrt{x-1}-\sqrt{x-4}\right)+\log\left(\sqrt{x-1}+2\right)+\log\left(\left|-\sqrt{x-1}+\sqrt{x-4}-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2
-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(x - 1) - sqrt(x - 4) + 1) - log(sqrt(x - 1) - sqrt(x - 4)) + log(
sqrt(x - 1) + 2) + log(abs(-sqrt(x - 1) + sqrt(x - 4) - 3))
```

Mupad [B]
time = 6.10, size = 132, normalized size = 6.95

$$\frac{\ln(x-5)}{2}+2\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right)+\frac{7\,\mathrm{atanh}\left(\frac{4\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2}-\frac{5\,\mathrm{atanh}\left(\frac{194400\left(\sqrt{x-1}-\sqrt{3}\right)}{\left(\frac{48600\left(\sqrt{x-1}-\sqrt{3}\right)^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2}-\mathrm{atanh}\left(\sqrt{x-4}\right)+\mathrm{atanh}\left(\frac{\sqrt{x-1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/(
(x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)
```

```
[Out] log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh(
(4*((x - 1)^(1/2) - 3^(1/2)))/((((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*(x
 - 4)^(1/2))))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/(((48600*((x
 - 1)^(1/2) - 3^(1/2))^2)/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((x -
4)^(1/2)) + atanh((x - 1)^(1/2)/2)
```

3.1016 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(1+x)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(1+x)-\sqrt[3]{2+(1+x)^3}\right)}{2\sqrt[3]{3}}$$

```
[Out] -1/3*arctan(1/3*(1+2*3^(1/3)*(1+x)/(2+(1+x)^3)^(1/3))*3^(1/2))*3^(1/6)-1/18
*ln(1-(1+x)^3)*3^(2/3)+1/6*ln(3^(1/3)*(1+x)-(2+(1+x)^3)^(1/3))*3^(2/3)
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {444, 442, 384}

$$-\frac{\text{ArcTan}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
- Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^(1
/3)]/(2*3^(1/3))
```

Rule 384

```
Int[1/(((a_) + (b_.)*(x_)^3)^(1/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wit
h[{q = Rt[(b*c - a*d)/c, 3]}, Simp[ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3))/S
qrt[3]]/(Sqrt[3]*c*q), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*q), x]
 + Simp[Log[c + d*x^3]/(6*c*q), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 442

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 444

```
Int[(u_)^(p_.)*(v_)^(q_.)*(x_)^(m_.), x_Symbol] :> Int[NormalizePseudoBinom
ial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
 && IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(3+3x+x^2\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx &= \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx \\
&= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right) \\
&= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}\,x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)+\frac{1}{3}\text{Subst}\Bigg(\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{1}{2}\text{Subst}\Bigg(\int \frac{1}{1+\sqrt[3]{3}\,x+3^{2/3}x^2} \\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{(2+(1+x)^3)^{2/3}}+\frac{\sqrt[3]{3}}{\sqrt[3]{2+}}\right.}{6\sqrt[3]{3}} \\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 180, normalized size = 2.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{3+3x+3x^2+x^3}}{2\sqrt[3]{3}+2\sqrt[3]{3}x+\sqrt[3]{3+3x+3x^2+x^3}}\right)}{3^{5/6}}+\frac{2\log\left(\sqrt[3]{3}+\sqrt[3]{3}x-\sqrt[3]{3+3x+3x^2+x^3}\right)-\log\left(3^{2/3}+2\,3^{2/3}x+3^{2/3}x^2+\sqrt[3]{3}(1+x)\sqrt[3]{3+3x+3x^2+x^3}+\left(3+3x+3x^2+x^3\right)^{2/3}\right)}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (
3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + (2*Log[3^(1/3) + 3^(1/3)*x - (3 +
```

```
3*x + 3*x^2 + x^3)^(1/3)] - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + 3^(1/
3)*(1 + x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3)])/
(6*3^(1/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 9.40, size = 2499, normalized size = 27.77

method	result	size
trager	Expression too large to display	2499

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*RootOf(_Z^3-9)*ln(-(15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_
Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*x-12802499577*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-404177657
1*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3-530
38926819*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10200674203*R
ootOf(_Z^3-9)*x^3-30602022609*RootOf(_Z^3-9)*x^2-30602022609*RootOf(_Z^3-9)
*x-32311070361*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-969
33211083*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-969332110
83*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-16744502937*RootO
f(_Z^3-9)+6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x+2837496903*(x^3+3*x^2+3*x+3)
^(1/3)*RootOf(_Z^3-9)^2+6809512275*(x^3+3*x^2+3*x+3)^(2/3)+5486785533*RootO
f(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2+1732
189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^
3*x^2+5486785533*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*Roo
tOf(_Z^3-9)^2*x+1732189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_
Z^2)*RootOf(_Z^3-9)^3*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_
Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2837496903*(x^3+3*x^
2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x^2+5674993806*(x^3+3*x^2+3*x+3)^(1/3)*Root
Of(_Z^3-9)^2*x-20428536825*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+
9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)-20428536825*(x^3+3*x^2+3*x+3)^(
1/3)*RootOf(_Z^3-9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^
2-40857073650*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*x+1828928511*RootOf(RootOf(_Z^3-9)^2+9*_Z*Ro
otOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+577396653*RootOf(RootOf(_Z^3-9
)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3)/x/(x^2+3*x+3))-1/9*l
n((15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(
_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*x+23573489562*RootOf(RootOf(_Z^3-9)^2+9*_
Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+4041776571*RootOf(RootOf(_Z^3-
9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3-74088110052*RootOf(RootO
f(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10778070856*RootOf(_Z^3-9)*x^3-323
34212568*RootOf(_Z^3-9)*x^2-32334212568*RootOf(_Z^3-9)*x-62862638832*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-188587916496*RootOf(Root
```

```
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-188587916496*RootOf(RootOf(_Z
^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-12702726366*RootOf(_Z^3-9)+851249070
9*(x^3+3*x^2+3*x+3)^(2/3)*x+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-
9)^2+8512490709*(x^3+3*x^2+3*x+3)^(2/3)-10102924098*RootOf(RootOf(_Z^3-9)^2
+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1732189959*RootOf(Root
Of(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-10102924098*
RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-1
732189959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-
9)^3*x+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*Roo
tOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*Ro
otOf(_Z^3-9)^2*x^2+4539674850*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2*x-25
537472127*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-
9)+81*_Z^2)*RootOf(_Z^3-9)-25537472127*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-
9)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-51074944254*(x^
3+3*x^2+3*x+3)^(1/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*R
ootOf(_Z^3-9)*x-3367641366*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_
Z^2)^2*RootOf(_Z^3-9)^2*x^3-577396653*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_
Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3)/x/(x^2+3*x+3))*RootOf(_Z^3-9)-ln((153
22002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9
)+81*_Z^2)*RootOf(_Z^3-9)^2*x+23573489562*RootOf(RootOf(_Z^3-9)^2+9*_Z*Root
Of(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2+4041776571*RootOf(RootOf(_Z^3-9)^2+9
*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3-74088110052*RootOf(RootOf(_Z^3
-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)-10778070856*RootOf(_Z^3-9)*x^3-323342125
68*RootOf(_Z^3-9)*x^2-32334212568*RootOf(_Z^3-9)*x-62862638832*RootOf(RootO
f(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^3-188587916496*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x^2-188587916496*RootOf(RootOf(_Z^3-9)^
2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*x-12702726366*RootOf(_Z^3-9)+8512490709*(x^3
+3*x^2+3*x+3)^(2/3)*x+2269837425*(x^3+3*x^2+3*x+3)^(1/3)*RootOf(_Z^3-9)^2+8
512490709*(x^3+3*x^2+3*x+3)^(2/3)-10102924098*RootOf(RootOf(_Z^3-9)^2+9*_Z*
RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^2-1732189959*RootOf(RootOf(_Z^
3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^2-10102924098*RootOf
(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-1732189
959*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x
+15322002984*(x^3+3*x^2+3*x+3)^(2/3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z
^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2+2269837425*(x^3...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(71) = 142$.
time = 5.25, size = 458, normalized size = 5.09

$$-\frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(\frac{3 \cdot 3^{\frac{2}{3}}(7x^4 + 28x^3 + 42x^2 + 30x + 9)(x^3 + 3x^2 + 3x + 3)^{\frac{2}{3}} + 3^{\frac{1}{3}}(31x^6 + 186x^5 + 465x^4 + 666x^3 + 603x^2 + 324x + 81) + 9(5x^5 + 25x^4 + 50x^3 + 54x^2 + 33x + 9)(x^3 + 3x^2 + 3x + 3)^{\frac{1}{3}}}{x^6 + 6x^5 + 15x^4 + 18x^3 + 9x^2}\right) + \frac{1}{27} \cdot 3^{\frac{2}{3}} \log\left(\frac{2 \cdot 3^{\frac{2}{3}}(x^3 + 3x^2 + 3x) - 9 \cdot 3^{\frac{1}{3}}(x^3 + 3x^2 + 3x + 3)^{\frac{1}{3}}(x^2 + 2x + 1) + 9(x^3 + 3x^2 + 3x + 3)^{\frac{2}{3}}(x + 1)}{x^3 + 3x^2 + 3x}\right) - \frac{1}{9} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{3^{\frac{1}{6}}\left(12 \cdot 3^{\frac{2}{3}}(7x^7 + 49x^6 + 147x^5 + 240x^4 + 225x^3 + 117x^2 + 27x)(x^3 + 3x^2 + 3x + 3)^{\frac{2}{3}} - 3^{\frac{1}{3}}(127x^9 + 1143x^8 + 4572x^7 + 11070x^6 + 18414x^5 + 22032x^4 + 18900x^3 + 11178x^2 + 4131x + 729) - 18(31x^8 + 248x^7 + 868x^6 + 1782x^5 + 2400x^4 + 2196x^3 + 1332x^2 + 486x + 81)(x^3 + 3x^2 + 3x + 3)^{\frac{1}{3}}\right)}{3(251x^9 + 2259x^8 + 9036x^7 + 21546x^6 + 34398x^5 + 38556x^4 + 30348x^3 + 16038x^2 + 5103x + 729)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^{2} + 3x + 3\right)\sqrt[3]{x^{3} + 3x^{2} + 3x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(x^2+3\,x+3\right)\,{\left(x^3+3\,x^2+3\,x+3\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)
[Out] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)
```

3.1017 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}}-\frac{\log\left(1+2(1-x)^3-x^3\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}(1-x)+\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(1+2*(1-x)^3-x^3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)+(-x^3+1)^(1/3))*2^(1/
3)+1/2*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)*2^(1/
3)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 425 vs. $2(103) = 206$.
time = 0.40, antiderivative size = 425, normalized size of antiderivative = 4.13, number of steps used = 42, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.517$, Rules used = {2183, 421, 251, 420, 493, 298, 31, 648, 631, 210, 642, 503, 455, 59, 494}

$$\frac{\sqrt[3]{2}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(x^3+1\right)}{3\ 2^{2/3}}+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{1}{3}\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] (2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]])/Sqrt[3]
 + ArcTan[(1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]
) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) +
 ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 +
x^3]/(3*2^(2/3)) + Log[2^(2/3) - (1 - x)/(1 - x^3)^(1/3)]/(3*2^(2/3)) - Log
[1 + (2^(2/3)*(1 - x)^2)/(1 - x^3)^(2/3) - (2^(1/3)*(1 - x))/(1 - x^3)^(1/3
)]/(3*2^(2/3)) + (2^(1/3)*Log[1 + (2^(1/3)*(1 - x))/(1 - x^3)^(1/3)])/3 - L
og[2*2^(1/3) + (1 - x)^2/(1 - x^3)^(2/3) + (2^(2/3)*(1 - x))/(1 - x^3)^(1/3
)]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3)) - Log[-(2^(1/3)
*x) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 420

```
Int[((a_) + (b_.)*(x_)^3)^(1/3)/((c_) + (d_.)*(x_)^3), x_Symbol] :> With[{q
 = Rt[b/a, 3]}, Dist[9*(a/(c*q)), Subst[Int[x/((4 - a*x^3)*(1 + 2*a*x^3)),
x], x, (1 + q*x)/(a + b*x^3)^(1/3)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b
*c - a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 421

```
Int[1/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^3)^(2/3), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^3)^(1/3)/(c + d*x^3), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0] && EqQ[b*c + a*d, 0]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 493

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 494

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[a*(e^n/b), Int[(e*x)^(m - n)*((c + d*x^n)^q/(a + b*x^n)), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 503

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)^(2/3)*((c_) + (d_.)*(x_)^3)), x_Symbol] :>
With[{q = Rt[(b*c - a*d)/c, 3]}, Simp[-ArcTan[(1 + (2*q*x)/(a + b*x^3)^(1/3
))/Sqrt[3]]/(Sqrt[3]*c*q^2), x] + (-Simp[Log[q*x - (a + b*x^3)^(1/3)]/(2*c*
q^2), x] + Simp[Log[c + d*x^3]/(6*c*q^2), x])] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 2183

```
Int[(Px_.)*((c_) + (d_.)*(x_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^3)^(p
_.), x_Symbol] :> Dist[1/c^q, Int[ExpandIntegrand[(c^3 - d^3*x^3)^q*(a + b*
x^3)^p, Px/(c - d*x)^q, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ
```

```
[Px, x] && EqQ[d^2 - c*e, 0] && ILtQ[q, 0] && RationalQ[p] && EqQ[Denominat
or[p], 3]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}}\,dx &= \int \left(-\frac{1}{(1-x^3)^{2/3}}+\frac{2-x}{(1-x+x^2)(1-x^3)^{2/3}}\right)dx\\ &= -\int \frac{1}{(1-x^3)^{2/3}}\,dx+\int \frac{2-x}{(1-x+x^2)(1-x^3)^{2/3}}\,dx\\ &= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\int \left(\frac{-1-i\sqrt{3}}{\left(-1-i\sqrt{3}+2x\right)(1-x^3)^{2/3}}+\frac{-}{\left(-1+i\sqrt{3}\right.}\right.\\ &= -x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\left(-1-i\sqrt{3}\right)\int \frac{1}{\left(-1-i\sqrt{3}+2x\right)(1-x^3)^{2/3}}\,d\end{aligned}$$

Mathematica [A]
time = 1.15, size = 181, normalized size = 1.76

$$-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}(1-x^3)^{2/3}}{2^{2/3}+2^{2/3}x+2^{2/3}x^2-(1-x^3)^{2/3}}\right)-2\log\left(2^{2/3}+2^{2/3}x+2^{2/3}x^2+2(1-x^3)^{2/3}\right)+\log\left(-\left((1+x+x^2)\left(\sqrt[3]{2}+\sqrt[3]{2}\,x^2-(2-2x^3)^{2/3}+2\sqrt[3]{1-x^3}+x\left(\sqrt[3]{2}-2\sqrt[3]{1-x^3}\right)\right)\right)\right)}{2\;2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(2/3))/(2^(2/3) + 2^(2/3)*x + 2^(
2/3)*x^2 - (1 - x^3)^(2/3))] - 2*Log[2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + 2*
(1 - x^3)^(2/3)] + Log[-((1 + x + x^2)*(2^(1/3) + 2^(1/3)*x^2 - (2 - 2*x^3)
^(2/3) + 2*(1 - x^3)^(1/3) + x*(2^(1/3) - 2*(1 - x^3)^(1/3))))]])/2^(2/3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 5.43, size = 690, normalized size = 6.70

method	result	size
trager	Expression too large to display	690

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-
2)^4*x-2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)
```

```
*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)^2*x^2-RootOf(_Z^3-2)^2*x-4*(-x^3+1)^(1/3)*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z^3-2)^2+4*(-
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2))/(x^2-x+1)
)*RootOf(_Z^3-2)-ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*
RootOf(_Z^3-2)^4*x-2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^
3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)^2*x^2-RootOf(_Z^3-2)^2*x-4*(-x
^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+RootOf(_Z
^3-2)^2+4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2
))/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)+RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+2
*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x-2*(-x^3+1)^(2/3)*RootOf(RootO
f(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x
^2+3*RootOf(_Z^3-2)^2*x-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x-4*(-x^3+1)^(1/3)*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x-RootOf(_Z^3-2)^2+2*(-
x^3+1)^(1/3)*RootOf(_Z^3-2)+4*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*R
ootOf(_Z^3-2)+4*_Z^2)-2*(-x^3+1)^(2/3))/(x^2-x+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(78) = 156$.
time = 3.24, size = 289, normalized size = 2.81

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(2\cdot 4^{\frac{2}{3}}(x^5-x^4-3x^3+3x^2+x-1)(-x^3+1)^{\frac{1}{3}}+4(x^4-4x^3+5x^2-4x+1)(-x^3+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^6-7x^5+10x^4-7x^3+10x^2-7x+1)\right)}{6(3x^6-9x^5+6x^4-x^3+6x^2-9x+3)}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}(-x^3+1)^{\frac{2}{3}}(x^2-3x+1)-4^{\frac{2}{3}}(x^4-3x^2+1)-8(-x^3+1)^{\frac{1}{3}}(x^2-x)}{x^4-2x^3+3x^2-2x+1}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(-\frac{4^{\frac{2}{3}}(-x^3+1)^{\frac{1}{3}}(x-1)-4^{\frac{1}{3}}(x^2-x+1)-2(-x^3+1)^{\frac{2}{3}}}{x^2-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x
^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-
x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))
/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(
1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3
 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(
-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(
2/3))/(x^2 - x + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\,dx-\int \left(-\frac{1}{x^2\left(1-x^3\right)^{\frac{2}{3}}-x\left(1-x^3\right)^{\frac{2}{3}}+\left(1-x^3\right)^{\frac{2}{3}}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)
```

```
[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**
(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1
 - x**3)**(2/3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2-1}{{\left(1-x^3\right)}^{2/3}\,\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)
```

```
[Out] -int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)
```

3.1018 $\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4}\tan^{-1}\left(\frac{1+x^2}{x\sqrt{-1+x^4}}\right)-\frac{1}{4}\tanh^{-1}\left(\frac{1-x^2}{x\sqrt{-1+x^4}}\right)$$

```
[Out] -1/4*arctan((x^2+1)/x/(x^4-1)^(1/2))-1/4*arctanh((-x^2+1)/x/(x^4-1)^(1/2))
```

Rubi [C] Result contains complex when optimal does not.
time = 0.08, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {504, 1225, 228, 1713, 212, 209}

$$\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)-\left(\frac{1}{8}+\frac{i}{8}\right)\text{ArcTan}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 +
 I)*x)/Sqrt[-1 + x^4]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
```

```
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1225

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d
+ e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 +
a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{-1+x^4}\,(1+x^4)}\,dx &= -\left(\frac{1}{2}\int \frac{1}{(i-x^2)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1}{(i+x^2)\sqrt{-1+x^4}}\,dx \\
&= -\left(\frac{1}{4}i\int \frac{i-x^2}{(i+x^2)\sqrt{-1+x^4}}\,dx\right)+\frac{1}{4}i\int \frac{i+x^2}{(i-x^2)\sqrt{-1+x^4}}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{i-2x^2}\,dx, x, \frac{x}{\sqrt{-1+x^4}}\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{i+2x^2}\,dx, x, \frac{x}{\sqrt{-1+x}}\right. \\
&= \left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}+\frac{i}{8}\right)\tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 53, normalized size = 1.08

$$\left(-\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right)+\left(\frac{1}{8}-\frac{i}{8}\right)\tan^{-1}\left(\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{-1+x^4}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]
```

```
[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2
+ I/2)*Sqrt[-1 + x^4])/x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(41) = 82$.
time = 0.50, size = 101, normalized size = 2.06

method	result
default	$\frac{\left(\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)\sqrt{2}}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)\sqrt{2}}{8}+\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}\right)\sqrt{2}}{2}$
elliptic	$\frac{\left(\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}+1\right)\sqrt{2}}{8}+\frac{\arctan\left(\frac{\sqrt{x^4-1}}{x}-1\right)\sqrt{2}}{8}+\frac{\sqrt{2}\,\ln\left(\frac{1+\frac{x^4-1}{2x^2}+\frac{\sqrt{x^4-1}}{x}}{1+\frac{x^4-1}{2x^2}-\frac{\sqrt{x^4-1}}{x}}\right)}{16}\right)\sqrt{2}}{2}$
trager	$-\frac{\ln\left(\frac{8\,\mathrm{RootOf}\left(32\_Z^2+8\_Z+1\right)x-\sqrt{x^4-1}+2x}{8x^2\,\mathrm{RootOf}\left(32\_Z^2+8\_Z+1\right)+x^2+1}\right)}{4}-\ln\left(\frac{8\,\mathrm{RootOf}\left(32\_Z^2+8\_Z+1\right)x-\sqrt{x^4-1}+2x}{8x^2\,\mathrm{RootOf}\left(32\_Z^2+8\_Z+1\right)+x^2+1}\right)\mathrm{RootOf}\,($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(1/8*arctan((x^4-1)^(1/2)/x+1)*2^(1/2)+1/8*arctan((x^4-1)^(1/2)/x-1)*2^
(1/2)+1/16*2^(1/2)*ln((1+1/2*(x^4-1)/x^2+(x^4-1)^(1/2)/x)/(1+1/2*(x^4-1)/x^
2-(x^4-1)^(1/2)/x)))*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Fricas [A]
time = 0.40, size = 51, normalized size = 1.04

$$\frac{1}{4}\arctan\left(\frac{\sqrt{x^4-1}\,x}{x^2+1}\right)+\frac{1}{8}\log\left(\frac{x^4+2\,x^2+2\,\sqrt{x^4-1}\,x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 -
1)*x - 1)/(x^4 + 1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)(x^2+1)}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^4-1}\,(x^4+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)
```

```
[Out] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)
```

3.1019 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{cd^2-bde+ae^2}}$$

```
[Out] arctan(x*(a*e^2-b*d*e+c*d^2)^(1/2)/d^(1/2)/e^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d
^(1/2)/e^(1/2)/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2137, 211}

$$\frac{\text{ArcTan}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2137

```
Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v,
x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Co
eff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)
*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0]] /;
FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]
```

Rubi steps

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = a\mathrm{Subst}\left(\int \frac{1}{ade-(abde-a(cd^2+ae^2))x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}\,x}{\sqrt{d}\,\sqrt{e}\,\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\,\sqrt{e}\,\sqrt{cd^2-bde+ae^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 10.56, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)d}{2ae};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-\Pi\left(\frac{\left(b+\sqrt{b^2-4ac}\right)e}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}de\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])
,x]

[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])]))/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

Maple [C] Result contains higher order function than in optimal. Order 4 vs. order 3.
time = 0.28, size = 555, normalized size = 6.94

method	result
elliptic	$-\frac{\arctan\left(\frac{de\sqrt{c\,x^4+b\,x^2+a}}{x\sqrt{(a\,e^2-deb+c\,d^2)\,de}}\right)}{\sqrt{(a\,e^2-deb+c\,d^2)\,de}}$
default	$-\frac{\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b}}{a}}}{2}\right.}{4ed\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/e/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b
*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/e/d*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1
/2))^(1/2)*(1+1/2*b*x^2/a-1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/
a+1/2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*a*e/
d,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2))+1/d/e*2^(1/2)/(-b/a+1/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a-1/
2*x^2/a*(-4*a*c+b^2)^(1/2))^(1/2)*(1+1/2*b*x^2/a+1/2*x^2/a*(-4*a*c+b^2)^(1/
2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),-2/(-b+(-4*a*c+b^2)^(1/2))*c*d/e,(-1/2*(b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(x^2*e + d)
), x)
```

Fricas [A]
time = 29.13, size = 477, normalized size = 5.96

$$\left[-\frac{\sqrt{-cd^3e+bd^2e^2-ade^3}\log\left(-\frac{c^2d^4x^4+a^2x^4e^4-4\left(cd^2x^3+ax^3e^2-\left(cdx^5+2\,bdx^3+adx\right)e\right)\sqrt{cx^4+bx^2+a}\sqrt{-cd^3e+bd^2e^2-ade^3}-2\left(3\,acdx^6+4\,abdx^4+3\,a^2dx^2\right)e^3+\left(c^2d^2x^8+8\,bcd^2x^6+4\left(2\,b^2+ac\right)d^2x^4+8\,abd^2x^2+a^2d^2\right)e^2-2\left(3\,c^2d^3x^6+4\,bcd^3x^4+3\,acd^3x^2\right)e}{c^2d^4x^4+a^2x^4e^4+2\left(acdx^6+a^2dx^2\right)e^3+\left(c^2d^2x^8+4\,acd^2x^4+a^2d^2\right)e^2+2\left(c^2d^3x^6+acd^3x^2\right)e}\right)}{4\left(cd^3e-bd^2e^2+ade^3\right)},\ -\frac{\arctan\left(\frac{2\sqrt{cx^4+bx^2+a}\sqrt{cd^3e-bd^2e^2+ade^3}\,x}{cd^2x^2+ax^2e^2-\left(cdx^4+2\,bdx^2+ad\right)e}\right)}{2\sqrt{cd^3e-bd^2e^2+ade^3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^4*x^4 + a^2*x^4*e^4
- 4*(c*d^2*x^3 + a*x^3*e^2 - (c*d*x^5 + 2*b*d*x^3 + a*d*x)*e)*sqrt(c*x^4 +
b*x^2 + a)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3) - 2*(3*a*c*d*x^6 + 4*a*b*d*
x^4 + 3*a^2*d*x^2)*e^3 + (c^2*d^2*x^8 + 8*b*c*d^2*x^6 + 4*(2*b^2 + a*c)*d^2
```

```
*x^4 + 8*a*b*d^2*x^2 + a^2*d^2)*e^2 - 2*(3*c^2*d^3*x^6 + 4*b*c*d^3*x^4 + 3*
a*c*d^3*x^2)*e)/(c^2*d^4*x^4 + a^2*x^4*e^4 + 2*(a*c*d*x^6 + a^2*d*x^2)*e^3
+ (c^2*d^2*x^8 + 4*a*c*d^2*x^4 + a^2*d^2)*e^2 + 2*(c^2*d^3*x^6 + a*c*d^3*x^
2)*e))/(c*d^3*e - b*d^2*e^2 + a*d*e^3), -1/2*arctan(2*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)*x/(c*d^2*x^2 + a*x^2*e^2 - (c*d*x^4
+ 2*b*d*x^2 + a*d)*e))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{a}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\right)dx-\int\frac{cx^4}{ade\sqrt{a+bx^2+cx^4}+ae^2x^2\sqrt{a+bx^2+cx^4}+cd^2x^2\sqrt{a+bx^2+cx^4}+cdex^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2
 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*
x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*
e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4)
 + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(x^2*e + d)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a-c\,x^4}{\left(e\,x^2+d\right)\,\left(c\,d\,x^2+a\,e\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.1020 $\int \left(x+\frac{1-x^2}{1+x}\right) dx$

Optimal. Leaf size=1

$$x$$

[Out] x

Rubi [A]
time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x$$

Antiderivative was successfully verified.

[In] Int[x + (1 - x^2)/(1 + x),x]

[Out] x

Rubi steps

$$\int \left(x+\frac{1-x^2}{1+x}\right) dx = x$$

Mathematica [A]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

[In] Integrate[x + (1 - x^2)/(1 + x),x]

[Out] x

Maple [A]
time = 0.22, size = 2, normalized size = 2.00

method	result	size
default	x	2
norman	x	2
risch	x	2

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x+(-x^2+1)/(1+x),x,method=_RETURNVERBOSE)
```

```
[Out] x
```

Maxima [A]
time = 0.27, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A]
time = 0.34, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [A]
time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x**2+1)/(1+x),x)
```

```
[Out] x
```

Giac [A]
time = 1.98, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x+(-x^2+1)/(1+x),x, algorithm="giac")
```

```
[Out] x
```

Mupad [B]
time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x - (x^2 - 1)/(x + 1),x)
[Out] x
```

3.1021 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=42

$$\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}\,(1-2x^2)}\right)}{\sqrt{3}}$$

```
[Out] arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2)
```

Rubi [C] Result contains complex when optimal does not.
time = 0.11, antiderivative size = 122, normalized size of antiderivative = 2.90, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {6874, 1121, 632, 210, 1307, 222, 1188, 385, 211}

$$\text{ArcSin}(x)-\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqr
t[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3]
)/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}}\,dx = \int \left(\frac{x}{1-x^2+x^4}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\right)dx$$

$$= \int \frac{x}{1-x^2+x^4}\,dx - \int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right) + \int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1}{\sqrt{1-x^2}\,(1-x^2+x^4)}\,dx$$

$$= \sin^{-1}(x) + \frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1-i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}} - \frac{(2i)\int \frac{1}{\sqrt{1-x^2}\left(-1+i\sqrt{3}+2x^2\right)}\,dx}{\sqrt{3}} -$$

$$= \sin^{-1}(x) + \frac{\tan^{-1}\left(\frac{-1+2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{(2i)\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}$$

$$= \sin^{-1}(x) - \frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \frac{\tan^{-1}(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 56, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) + \frac{2i\tanh^{-1}\left(\frac{(2+i)-2ix^2+2x\sqrt{1-x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[1 - x^2]] + ((2*I)*ArcTanh[((2 + I) - (2*I)*x^2 + 2*x*Sqrt[1
- x^2])/Sqrt[3]])/Sqrt[3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.22, size = 234, normalized size = 5.57

method	result

trager	$\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)+\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\,\mathrm{RootOf}\left(\_Z^2+3\right)x^2+3x}{\mathrm{RootOf}\left(\_\right.}\right.}{3}$
default	$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{i\sqrt{3}\,\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}-\frac{i\sqrt{3}\,\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctan(((-x^2+1)^(1/2)-1)/x)+1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(
1+I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/
x^2+(1-I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)+1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-
1)^2/x^2+(-1+I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^
(1/2)-1)^2/x^2+(-1-I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)+1/3*3^(1/2)*arctan(1/
3*(2*x^2-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)
```

Fricas [A]
time = 0.36, size = 73, normalized size = 1.74

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,\left(2\,x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x^2-1\right)\sqrt{-x^2+1}}{3\,\left(x^3-x\right)}\right)-2\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{x\sqrt{1-x^2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(37) = 74.

time = 2.62, size = 193, normalized size = 4.60

$$\frac{1}{2}\pi\operatorname{sgn}(x) - \frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x) + 2\arctan\left(-\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x) + 2\arctan\left(\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

Mupad [B]

time = 3.92, size = 549, normalized size = 13.07

$$\operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)-1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}} - \sqrt{1-x^2}\,i}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)-1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}} - \sqrt{1-x^2}\,i}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)+1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}} + \sqrt{1-x^2}\,i}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)+1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}} + \sqrt{1-x^2}\,i}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^
3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 -
 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2
)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^
(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)
 - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
+ 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2
)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i)
```

3.1022 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=42

$$\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}\,(1-2x^2)}\right)}{\sqrt{3}}$$

[Out] arcsin(x)-1/3*arctan(1/3*(1+4*x*(-x^2+1)^(1/2))*3^(1/2)/(-2*x^2+1))*3^(1/2)

Rubi [C] Result contains complex when optimal does not.
time = 0.26, antiderivative size = 149, normalized size of antiderivative = 3.55, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.303, Rules used = {6874, 1307, 222, 1188, 385, 211, 1265, 787, 632, 210}

$$\text{ArcSin}(x)-\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{x}{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt{-\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\,x}{\sqrt{1-x^2}}\right)}{\sqrt{3}}-\frac{x^2}{2}+\frac{1}{4}(1-x)^2+\frac{1}{4}(x+1)^2$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]

[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}}\,dx = \int \left(\frac{1}{2}(-1+x)+\frac{1+x}{2}-\frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}+\frac{x^3(1-x^2)}{1-x^2+x^4}\right)dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2-\int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4}\,dx+\int \frac{x^3(1-x^2)}{1-x^2+x^4}\,dx$$

$$= \frac{1}{4}(1-x)^2+\frac{1}{4}(1+x)^2+\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^2\right)+\int \frac{1}{\sqrt{1-x^2}}\,dx-$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)+$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{(2i)\text{Subst}\left(\int \frac{1}{-1+i\sqrt{3}-\left(-1-i\sqrt{3}\right)x^2}\,d\right.}{\sqrt{3}}$$

$$= \frac{1}{4}(1-x)^2-\frac{x^2}{2}+\frac{1}{4}(1+x)^2+\sin^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}\right)}{\sqrt{3}}-$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 56, normalized size = 1.33

$$\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)+\frac{2i\tanh^{-1}\left(\frac{(2+i)-2ix^2+2x\sqrt{1-x^2}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]
```

```
[Out] ArcTan[x/Sqrt[1 - x^2]] + ((2*I)*ArcTanh[((2 + I) - (2*I)*x^2 + 2*x*Sqrt[1
- x^2])/Sqrt[3]])/Sqrt[3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.24, size = 234, normalized size = 5.57

method	result

trager	$\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-x^2+1}+x\right)-\frac{\operatorname{RootOf}(\_Z^2+3)\ln\left(-\frac{2\operatorname{RootOf}(\_Z^2+3)x^2}{\operatorname{RootOf}}\right.}{3}$
default	$-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{i\sqrt{3}\ln\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+\frac{\left(1+i\sqrt{3}\right)\left(\sqrt{-x^2+1}-1\right)}{x}-1\right)}{6}-\frac{i\sqrt{3}\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctan(((-x^2+1)^(1/2)-1)/x)+1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/x^2+(
1+I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-1)^2/
x^2+(1-I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)+1/6*I*3^(1/2)*ln(((-x^2+1)^(1/2)-
1)^2/x^2+(-1+I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)-1/6*I*3^(1/2)*ln(((-x^2+1)^
(1/2)-1)^2/x^2+(-1-I*3^(1/2))*((-x^2+1)^(1/2)-1)/x-1)+1/3*3^(1/2)*arctan(1/
3*(2*x^2-1)*3^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

Fricas [**A**]
time = 0.36, size = 73, normalized size = 1.74

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^2-1\right)\sqrt{-x^2+1}}{3\left(x^3-x\right)}\right)-2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3
)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(37) = 74.

time = 1.43, size = 193, normalized size = 4.60

$$\frac{1}{2}\pi\operatorname{sgn}(x) - \frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x) + 2\arctan\left(-\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \frac{1}{6}\sqrt{3}\left(\pi\operatorname{sgn}(x) + 2\arctan\left(\frac{\sqrt{3}\,x\left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1\right)}{3\left(\sqrt{-x^2+1}-1\right)}\right)\right) + \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right) + \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - 1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^
2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/
6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sq
rt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/
3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sq
rt(-x^2 + 1) - 1))
```

Mupad [B]

time = 3.89, size = 549, normalized size = 13.07

$$\operatorname{asin}(x) - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)-1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}} - \sqrt{1-x^2}\,i}{\frac{\sqrt{3}}{2}-x+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i\right)} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)-1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}} - \sqrt{1-x^2}\,i}{x-\frac{\sqrt{3}}{2}+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i\right)} - \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)+1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}} + \sqrt{1-x^2}\,i}{x+\frac{\sqrt{3}}{2}-\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2}\left(-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i\right)} - \frac{\ln\left(x-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i} - \frac{\ln\left(x+\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}{\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i} + \frac{\ln\left(\frac{\frac{\left(x\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)+1\right)i}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}} + \sqrt{1-x^2}\,i}{x+\frac{\sqrt{3}}{2}+\frac{1}{2}i}\right)}{\sqrt{1-\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^2}\left(\sqrt{3}-4\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^3+1i\right)} + \frac{\ln\left(x-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i} + \frac{\ln\left(x+\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)}{-\sqrt{3}+4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^3+1i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)
```

```
[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(
1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)
^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 -
1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^
3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 -
 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2
)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^
(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)
 - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
+ 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2
)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
1/2) + 1i)
```

3.1023 $\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx$

Optimal. Leaf size=34

$$-\frac{(1-x)(1+x+x^2+x^3)^{-n}(1-x^4)^n}{1+n}$$

```
[Out] -(1-x)*(-x^4+1)^n/(1+n)/((x^3+x^2+x+1)^n)
```

Rubi [F]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]
```

Rubi steps

$$\int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx = \int (1+x+x^2+x^3)^{-n} (1-x^4)^n \, dx$$

Mathematica [A]
time = 0.09, size = 31, normalized size = 0.91

$$\frac{(-1+x)(1+x+x^2+x^3)^{-n}(1-x^4)^n}{1+n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]
[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)
```

Maple [A]
time = 0.25, size = 32, normalized size = 0.94

method	result

gosper	$\frac{(-1+x)(-x^4+1)^n (x^3+x^2+x+1)^{-n}}{1+n}$
norman	$\left(\frac{x\,\mathrm{e}^{n\ln\left(-x^4+1\right)}}{1+n}-\frac{\mathrm{e}^{n\ln\left(-x^4+1\right)}}{1+n}\right)\mathrm{e}^{-n\ln\left(x^3+x^2+x+1\right)}$
risch	$\frac{(-1+x)\left(x^3+x^2+x+1\right)^{-n}\mathrm{e}^{\frac{n\left(i\pi\,\mathrm{csgn}\left(i(-1+x)\left(x^3+x^2+x+1\right)\right)^3+i\,\mathrm{csgn}\left(i(-1+x)\left(x^3+x^2+x+1\right)\right)^2\mathrm{csgn}(i(-1+x))\pi+i\,\mathrm{csgn}\left(i(-1+x)\left(x^3+x^2+x+1\right.\right.\right.}{}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x,method=_RETURNVERBOSE)
```

```
[Out] (-1+x)/(1+n)*(-x^4+1)^n/((x^3+x^2+x+1)^n)
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.47

$$\frac{(x-1)(-x+1)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")
```

```
[Out] (x - 1)*(-x + 1)^n/(n + 1)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.91

$$\frac{(-x^4+1)^n(x-1)}{(x^3+x^2+x+1)^n(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")
```

```
[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(24) = 48.
time = 20.91, size = 73, normalized size = 2.15

$$\begin{cases} \frac{x(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} - \frac{(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} & \text{for}\ n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)
```

[Out] Piecewise((x*(1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)**n) - (1 - x**4)**n/(n*(x**3 + x**2 + x + 1)**n + (x**3 + x**2 + x + 1)**n), Ne(n, -1)), (-log(x - 1), True))

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(31) = 62$.
time = 2.58, size = 81, normalized size = 2.38

$$\frac{\frac{xe^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{(x^3+x^2+x+1)^n} - \frac{e^{\left(n\log\left(x^3+x^2+x+1\right)+n\log(-x+1)\right)}}{(x^3+x^2+x+1)^n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")

[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)

Mupad [B]
time = 3.45, size = 31, normalized size = 0.91

$$\frac{(1-x^4)^n\,(x-1)}{(n+1)\,(x^3+x^2+x+1)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^4)^n/(x + x^2 + x^3 + 1)^n,x)

[Out] ((1 - x^4)^n*(x - 1))/((n + 1)*(x + x^2 + x^3 + 1)^n)

3.1024 $\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308}}$

Optimal. Leaf size=177

$$\frac{\log\left(207380736000000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 95105071448064\right.}{}$$

```
[Out] 1/18432*ln(207380736000000000*b^8*c^4+597005697024000000*b^6*c^6*x^2+2583100
705996800000*b^5*c^7*x^3+9510507144806400000*b^4*c^8*x^4+216416873695150800
0*b^3*c^9*x^5+32462531054272512000*b^2*c^10*x^6+149587343098087735296*c^12*
x^8+5308416*(12230590464*c^10*x^6+1990656000*b^2*c^8*x^4+1105920000*b^3*c^7
*x^3+38880000*b^4*c^6*x^2+79200000*b^5*c^5*x+12203125*b^6*c^4)*(5308416*c^4
*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2))/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2107}

$$\frac{\log\left(207380736000000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 9510507144806400000b^4c^8x^4 + 2164168736951500800000b^3c^9x^5 + 32462531054272512000b^2c^{10}x^6 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}\left(12203125b^6c^4 + 79200000b^5c^5x + 38880000b^4c^6x^2 + 1105920000b^3c^7x^3 + 1990656000b^2c^8x^4 + 12230590464c^{10}x^6\right) + 149587343098087735296c^{12}x^8\right)}{18432c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x
^4],x]
```

```
[Out] Log[207380736000000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705
996800000*b^5*c^7*x^3 + 9510507144806400000*b^4*c^8*x^4 + 216416873695150080
00*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 149587343098087735296*
c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 +
5308416*c^4*x^4]*(12203125*b^6*c^4 + 79200000*b^5*c^5*x + 38880000*b^4*c^6*
x^2 + 1105920000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^
6)]/(18432*c^2)
```

Rule 2107

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px,
 x], x]*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx = \frac{\log\left(207380736000000000b^8c^4+597005\right.}{}$$

Mathematica [A]
time = 6.52, size = 182, normalized size = 1.03

$$-\frac{\log\left(-20386355871744000000000000000b^{13}c^7-58688048040247296000000000000000b^{11}c^9x^2-253929131802309427200000000000000b^{10}c^{10}x^3-934920894363048345600000000000000b^9c^{11}x^4-2127464435172803346432000000000000b^8c^{12}x^5-3191196652759205019648000000000000b^7c^{13}x^6-1470503417591441673053798400000000b^5c^{15}x^8+\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}\left(636806076825600000000000000b^{11}c^7+4132961129594880000000000000b^{10}c^8x+20289081908920320000000000000b^9c^9x^2+57711166318706688000000000000b^8c^{10}x^3+103880099373672038400000000000b^7c^{11}x^4+63823933055184100392960000000b^5c^{13}x^6\right)\right)}{18432c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416
*c^4*x^4],x]
```

```
[Out] -1/18432*Log[-20386355871744000000000000000*b^13*c^7 - 586880480402472960000
000000000*b^11*c^9*x^2 - 253929131802309427200000000000000*b^10*c^10*x^3 - 9
34920894363048345600000000000000*b^9*c^11*x^4 - 2127464435172803346432000000000
00*b^8*c^12*x^5 - 3191196652759205019648000000000000*b^7*c^13*x^6 - 147050341
7591441673053798400000000*b^5*c^15*x^8 + Sqrt[-44375*b^4 + 576000*b^3*c*x +
576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(636806076825600000000000000*b^11*c^7
+ 4132961129594880000000000000*b^10*c^8*x + 20289081908920320000000000000*b^9*
c^9*x^2 + 57711166318706688000000000000*b^8*c^10*x^3 + 103880099373672038400
00000000*b^7*c^11*x^4 + 63823933055184100392960000000*b^5*c^13*x^6)]/c^2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.68, size = 1597, normalized size = 9.02

method	result	size
default	Expression too large to display	1597
elliptic	Expression too large to display	1597

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x
,method=_RETURNVERBOSE)
```

```
[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)
*b/c)^2*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+1
0*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)
```

```
*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/
c))^(1/2)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4
+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=
4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10
*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c))^(1/2)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^
4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2
)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(c^4*(x-5/48*RootOf(_
Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,ind
ex=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(x-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)*(5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=2)*b/c*EllipticF(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/
c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+
96*_Z-71,index=2)*b/c))^(1/2),((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*
b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/48*RootOf(_Z^4+10*_Z
^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5
/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_
Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*Roo
tOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2))+(5/48*RootOf(_Z^4+10*_Z^2+9
6*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*Ellipt
icPi(((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_
Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/
c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))
^(1/2),(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*
_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c
-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c),((5/48*RootOf(_Z^4+10*_Z^2
+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)*(5/4
8*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-
71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootO
f(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,in
dex=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

Fricas [A]
time = 0.45, size = 164, normalized size = 0.93

$$\frac{\log\left(28179280429056\,c^8x^8+6115295232000\,b^2c^6x^6+4076863488000\,b^3c^5x^5+179159040000\,b^4c^4x^4+486604800000\,b^5c^3x^3+112464000000\,b^6c^2x^2+3906640625\,b^8+\left(12230590464\,c^6x^6+1990656000\,b^2c^4x^4+1105920000\,b^3c^3x^3+38880000\,b^4c^2x^2+79200000\,b^5cx+12203125\,b^6\right)\sqrt{5308416\,c^4x^4+576000\,b^2c^2x^2+576000\,b^3cx-44375\,b^4}\right)}{18432\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="fricas")
```

```
[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488
000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112
464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*
b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*
c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*
c*x - 44375*b^4))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*
b**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 530
8416*c**4*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44
375*b^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-44375\,b^4+576000\,b^3\,c\,x+576000\,b^2\,c^2\,x^2+5308416\,c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2),x)
```

```
[Out] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(
1/2), x)
```

3.1025 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx$

Optimal. Leaf size=100

$$\frac{1}{16}\log\left(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5+12288x^6+8192x^7+4096x^8+\sqrt{9+120\right.$$

```
[Out] 1/16*ln(921+2864*x+9280*x^2+13440*x^3+17024*x^4+19456*x^5+12288*x^6+8192*x^
7+4096*x^8+(512*x^6+768*x^5+960*x^4+1280*x^3+744*x^2+444*x+179)*(64*x^4+64*
x^3+64*x^2+120*x+9)^(1/2))
```

Rubi [**B**] Leaf count is larger than twice the leaf count of optimal. 243 vs. $2(100) = 200$.
time = 0.09, antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2108, 2107}

$$\frac{1}{16}\log\left(4096x^8+8192x^7+12288x^6+19456x^5+17024x^4+13440x^3+9280x^2+960\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^4+1280\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^3+744\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^2+444\sqrt{64x^4+64x^3+64x^2+120x+9}\,x+179\sqrt{64x^4+64x^3+64x^2+120x+9}+512\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^6+768\sqrt{64x^4+64x^3+64x^2+120x+9}\,x^5+2864x+921\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6
 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 4
44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x +
64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*
x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9
+ 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*
x^3 + 64*x^4]]/16
```

Rule 2107

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1/320)*(33*b^2*c + 6*a*c^2 + 40*a^2*e) - (22/5)*a*c*e*x^2 + (
22/15)*b*c*e*x^3 + (1/4)*e*(5*c^2 + 4*a*e)*x^4 + (4/3)*b*e^2*x^5 + 2*c*e^2*
x^6 + e^3*x^8}, Simp[(1/(8*Rt[e, 2]))*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px,
 x], x]*Sqrt[a + b*x + c*x^2 + e*x^4]], x]] /; FreeQ[{a, b, c, e}, x] && Eq
Q[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rule 2108

```
Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^
3 + (e_.)*(x_)^4], x_Symbol] :> Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2
*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + (d^3 - 4*c*d*e + 8*b*e^2)*(x/(8*e^
2)) - (3*d^2 - 8*c*e)*(x^2/(8*e)) + e*x^4], x], x, d/(4*e) + x], x] /; Free
Q[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*
c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*
```

```
e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]
```

Rubi steps

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}}\,dx = 4\text{Subst}\left(\int \frac{x}{\sqrt{-\frac{71}{4}+96x+40x^2+64x^4}}\,dx, x, \frac{1}{4}+x\right)$$
$$= \frac{1}{16}\log\left(921+2864x+9280x^2+13440x^3+17024x^4+19456x^5\right.$$

Mathematica [A]
time = 5.04, size = 100, normalized size = 1.00

$$-\frac{1}{16}\log\left(-921-2864x-9280x^2-13440x^3-17024x^4-19456x^5-12288x^6-8192x^7-4096x^8+\sqrt{9+120x+64x^2+64x^3+64x^4}\left(179+444x+744x^2+1280x^3+960x^4+768x^5+512x^6\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4],x]
```

```
[Out] -1/16*Log[-921 - 2864*x - 9280*x^2 - 13440*x^3 - 17024*x^4 - 19456*x^5 - 12
288*x^6 - 8192*x^7 - 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]*
(179 + 444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 1.04, size = 2992, normalized size = 29.92

method	result
trager	$\ln\left(-4096x^8-512x^6\sqrt{64x^4+64x^3+64x^2+120x+9}-8192x^7-768x^5\sqrt{64x^4+64x^3+64x^2+120x+9}\right.$
default	Expression too large to display
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=4))*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,i
ndex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*
_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_
```

Z^3+16*_Z^2+60*_Z+9,index=2))^2*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60
*_Z+9,index=3)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*Ro
otOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_
Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index
=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/
2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=2))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-
1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)*EllipticF(((1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(
1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,
index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_
Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4
*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*RootOf(4*_Z^4+8*_Z^3+16
*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4))*(
(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,inde
x=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+
9,index=2)))^(1/2)*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))^2*
((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^
3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3)-1/2*RootOf(4*_Z^4
+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=2)))^(1/2)*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2
*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(x-1/2*RootOf(4*_Z^4+8*_Z^3
+16*_Z^2+60*_Z+9,index=4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+
8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2)/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))/(1/2*Roo
tOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2
+60*_Z+9,index=1))/((x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))*(
x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=3))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9

```
,index=4)))^(1/2)*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)*Ellipt
icF(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8
*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9
,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*
_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+6
0*_Z+9,index=2)))^(1/2),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)
-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z
^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=
4))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*
_Z^3+16*_Z^2+60*_Z+9,index=3))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,in
dex=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2))+(1/2*Root
Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+
60*_Z+9,index=2))*EllipticPi(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,ind
ex=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z
^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z
+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Fricas [A]
time = 0.42, size = 97, normalized size = 0.97

$$\frac{1}{16}\log\left(-4096\,x^8-8192\,x^7-12288\,x^6-19456\,x^5-17024\,x^4-13440\,x^3-9280\,x^2-\left(512\,x^6+768\,x^5+960\,x^4+1280\,x^3+744\,x^2+444\,x+179\right)\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}-2864\,x-921\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x
^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x +
 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{4x+1}{\sqrt{64x^4+64x^3+64x^2+120x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)
[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="giac")
[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4\,x+1}{\sqrt{64\,x^4+64\,x^3+64\,x^2+120\,x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)
[Out] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```